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ACTIVITY REPORT

Project-Team

TONUS

TOkamaks and NUmerical Simulations

IN COLLABORATION WITH: Institut de recherche mathématique avancée (IRMA)

DOMAIN

Digital Health, Biology and Earth

THEME

Earth, Environmental and Energy Sciences

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Project-Team TONUS

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Computer sciences and digital sciences

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2 Overall objectives

TONUS has started in January 2014. It is a team of the Inria Antenna in Strasbourg. It is located in the mathematics institute (IRMA) of the University of Strasbourg.

The International Thermonuclear Experimental Reactor (ITER) is a large-scale scientific experiment that aims to demonstrate that it is possible to produce energy from fusion, by confining a very hot hydrogen plasma inside a toroidal chamber, called tokamak. In addition to physics and technology research, the design of tokamaks also requires mathematical modeling and numerical simulations on supercomputers.

The objective of the TONUS project is to deal with such mathematical and computing issues. We are mainly interested in kinetic, gyrokinetic and fluid simulations of tokamak plasmas. In the TONUS project-team we are working on the development of new numerical methods devoted to such simulations. We investigate several classical plasma models, study new reduced models and new numerical schemes adapted to these models. We implement our methods in two software projects: Selalib and SCHNAPS adapted to recent computer architectures.

We have strong relations with the CEA-IRFM team and participate in the development of their gyrokinetic simulation software GYSELA. We were involved in two Inria Project Labs, respectively devoted to tokamak mathematical modeling and high performance computing. The numerical tools developed from plasma physics can also be applied in other contexts. For instance, we collaborate with a small company in Strasbourg specialized in numerical software for applied electromagnetism. We also study kinetic acoustic models with the CEREMA and multiphase flows with EDF.

Finally, our topics of interest are at the interface between mathematics, computer science, High Performance Computing, physics and practical applications.

3 Research program

3.1 Kinetic models for plasmas

The fundamental model for plasma physics is the coupled Vlasov-Maxwell kinetic model: the Vlasov equation describes the distribution function of particles (ions and electrons), while the Maxwell equations describe the electromagnetic field. In some applications, it may be necessary to take relativistic particles into account, which leads to consider the relativistic Vlasov equation, even if in general, tokamak plasmas are supposed to be non-relativistic. The distribution function of particles depends on seven variables (three for space, three for the velocity and one for time), which yields a huge amount of computation. To these equations we must add several types of source terms and boundary conditions for representing the walls of the tokamak, the applied electromagnetic field that confines the plasma, fuel injection, collision effects, etc.

Tokamak plasmas possess particular features, which require developing specialized theoretical and numerical tools.

Because the magnetic field is strong, the particle trajectories have a very fast rotation around the magnetic field lines. A full resolution would require a prohibitive amount of computation. It is necessary to develop reduced models for large magnetic fields in order to obtain tractable calculations. The resulting model is called a gyrokinetic model. It allows us to reduce the dimensions of the problem. Such models are implemented in GYSELA and Selalib.

On the plasma boundary, the collisions can no longer be neglected. Fluid models, such as Magneto-HydroDynamics (MHD), become relevant again. For the good operation of the tokamak, it is necessary to control MHD instabilities that arise at the plasma boundary. Computing these instabilities requires special implicit numerical discretizations with excellent long time behavior.

In addition to theoretical modeling tools, it is necessary to develop numerical schemes adapted to kinetic, gyrokinetic and fluid models. Three kinds of methods are studied in TONUS: Particle-In-Cell (PIC) methods, semi-Lagrangian and fully Eulerian approaches.

3.2 Gyrokinetic models: theory and approximation

In most phenomena where oscillations are present, we can establish a three-model hierarchy: *(i)* the model parameterized by the oscillation period, *(ii)* the limit model and *(iii)* the two-scale model, possibly with its corrector. In a context where one wishes to simulate such a phenomenon where the oscillation period is small and the oscillation amplitude is not small, it is important to have numerical methods based on an approximation of the two-scale model. If the oscillation period varies significantly over the domain of simulation, it is important to have numerical methods that approximate properly and effectively the model parameterized by the oscillation period and the two-scale model. Implementing two-scale numerical methods (for instance by Frénod et al. [37]) is based on a numerical approximation of the Two-Scale model. These are called of order 0. A Two-Scale Numerical Method is called of order 1 if it incorporates information from the corrector and from the equation of which this corrector is a solution. If the oscillation period varies between very small values and values of order 1, it is necessary to have new types of numerical schemes (Two-Scale Asymptotic Preserving Schemes of order 1 or TSAPS) that preserve the asymptotics between the model parameterized by the oscillation period and the Two-Scale model with its corrector. A first work in this direction has been initiated by Crouseilles et al. [36].

3.3 Semi-Lagrangian schemes

The TONUS team, and more generally the scientific computing team at IRMA Strasbourg, has a long and recognized experience in numerical methods for Vlasov-type equations. We are specialized in both particle and phase space solvers for the Vlasov equation: Particle-in-Cell (PIC) methods and semi-Lagrangian methods. We also have a long-standing collaboration with CEA Cadarache for the development of the GYSELA software for gyrokinetic tokamak plasmas.

The Vlasov and the gyrokinetic models are partial differential equations that express the transport of the distribution function in the phase space. In the original Vlasov case, the phase space is the six-dimension position-velocity space. For the gyrokinetic model, the phase space is five-dimensional because we consider only the parallel velocity in the direction of the magnetic field and the gyrokinetic angular velocity instead of three velocity components.

A few years ago, Eric Sonnendrücker and his collaborators introduced a new family of methods for solving transport equations in the phase space. This family of methods are the semi-Lagrangian methods. The principle of these methods is to solve the equation on a grid of the phase space. The grid points are transported with the flow of the transport equation for a time step and interpolated back periodically onto the initial grid. The method is then a mix of particle Lagrangian methods and Eulerian methods. The characteristics can be solved forward or backward in time leading to the Forward Semi-Lagrangian (FSL) or Backward Semi-Lagrangian (BSL) schemes. Conservative schemes based on this idea can be developed and are called Conservative Semi-Lagrangian (CSL).

GYSELA is a 5D full gyrokinetic code based on a classical backward semi-Lagrangian scheme (BSL) [42] for the simulation of core turbulence that has been developed at CEA Cadarache in collaboration with our team [38].

More recently, we have started to apply the semi-Lagrangian methods to more general kinetic equations. Indeed, most of the conservation laws of physics can be represented by a kinetic model with a small set of velocities. Compressible fluids or MHD equations have such representations. Semi-Lagrangian methods then become a very appealing and efficient approach for solving these equations.

3.4 PIC methods

Historically PIC methods have been very popular for solving the Vlasov equations. They make it possible to solve the equations in phase space at a relatively low cost. The main disadvantage of this approach is that, due to its random aspect, it produces an important numerical noise that has to be controlled in some way, for instance by regularizations of the particles, or by divergence correction techniques in the Maxwell solver. We have a long-standing experience in PIC methods and we started implementing them in Selalib. An important aspect is to adapt the method to new multicore computers. See the work by Crestetto and Helluy [35].

3.5 Fluid and reduced kinetic models for plasmas

As already said, kinetic plasmas computer simulations are very intensive, because of the gyrokinetic turbulence. In some situations, it is possible to make assumptions on the shape of the distribution function that simplify the model. We obtain in this way a family of fluid models or reduced models.

Assuming that the distribution function has a Maxwellian shape, for instance, we obtain the Magneto-HydroDynamic (MHD) model. It is physically valid only in some parts of the tokamak (at the edges for instance). The fluid model is generally obtained from the hypothesis that the collisions between particles are strong.

However, the reduction is not necessarily a consequence of collisional effects. Indeed, even without collisions, the plasma may still relax to an equilibrium state over sufficiently long time scales (Landau damping effect).

In the fluid or reduced-kinetic regions, the approximation of the distribution function could require fewer data while still achieving a good representation, even in the collisionless regime.

Therefore, a fluid or a reduced model is a model where the explicit dependency on the velocity variable is removed. In a more mathematical way, we consider that in some regions of the plasma, it is possible to exhibit a (preferably small) set of parameters α that allows us to describe the main properties of the plasma with a generalized "Maxwellian" M. Then

$$f(x,v,t) = M(\alpha(x,t),v).$$

In this case it is sufficient to solve for $\alpha(x,t)$. Generally, the vector α is the solution of a first order hyperbolic system.

Another way to reduce the model is to try to find an abstract kinetic representation with a set of kinetic velocities as small as possible. The kinetic approach has then only a mathematical meaning. It allows solving very efficiently many equations of physics.

3.6 Numerical schemes

An efficient method for solving the reduced models is the Discontinuous Galerkin (DG) approach [40]. It is possible to make it of arbitrary order. It requires limiters when it is applied to nonlinear PDEs occurring for instance in fluid mechanics. But the reduced models that we intend to write are essentially linear. The nonlinearity is concentrated in a few coupling source terms.

In addition, this method, when written in a special set of variables, called the entropy variables, has nice properties concerning the entropy dissipation of the model. It opens the door to constructing numerical schemes with good conservation properties and no entropy dissipation, as already used for other systems of PDEs [43, 34, 41, 39].

3.7 Matrix-free implicit schemes

In tokamaks, the reduced model generally involves a lot of different time scales. Most of them, associated to the fastest waves, are not relevant to the tokamak simulation. In order to filter them out, it is necessary to adopt implicit solvers in time. When the reduced model is based on a kinetic interpretation, it is possible to construct implicit schemes that do not impose solving costly linear systems. In addition the resulting solver is stable even at a very high CFL (Courant–Friedrichs–Lewy) number.

3.8 Electromagnetic solvers

Accurate resolution of the electromagnetic fields is essential for proper plasma simulations. Thus it is important to use efficient solvers for the Maxwell systems and its asymptotics: Poisson equation and magnetostatics.

The proper coupling of the electromagnetic solver with the Vlasov solver is also crucial for ensuring conservation properties and stability of the simulation.

Finally, plasma physics implies very different time scales. It is thus very important to develop implicit Maxwell solvers and Asymptotic Preserving (AP) schemes in order to obtain good behavior on long time scales.

3.9 Coupling

The coupling of the Maxwell equations to the Vlasov solver requires some precautions. The most important one is to control the charge conservation errors, which are related to the divergence conditions on the electric and magnetic fields. We will generally use divergence correction tools for hyperbolic systems presented for instance in [32] (and the references therein).

3.10 Implicit solvers

As already pointed out, in a tokamak, the plasma presents several different space and time scales. It is not possible in practice to solve the initial Vlasov-Maxwell model. It is first necessary to establish asymptotic models by letting some parameters (such as the Larmor frequency or the speed of light) tend to infinity. This is the case for the electromagnetic solver. To address this issue, time-implicit solvers have to be implemented, in order to provide good approximations of stationary states, or of the solution to the magnetic induction equation or to the Poisson equation.

4 Application domains

4.1 Controlled fusion and ITER

The search for alternative energy sources is a major issue for the future. Among others, controlled thermonuclear fusion in a hot hydrogen plasma is a promising possibility. The principle is to confine the plasma in a toroidal chamber, called a tokamak, and to attain the necessary temperatures to sustain nuclear fusion reactions.The International Thermonuclear Experimental Reactor (ITER) is a tokamak being constructed in Cadarache, France. This was the result of a joint decision by an international consortium including the European Union, Canada, USA, Japan, Russia, South Korea, India and China. ITER is a huge project. As of today, the budget is estimated at 22 billion euros. The first plasma shot is planned for November 2025. Many technical and conceptual difficulties have to be overcome before the actual exploitation of fusion energy. Consequently, much research has been carried out around magnetically confined fusion. Among these studies, it is important to carry out computer simulations of the burning plasma. Thus, mathematicians and computer scientists are also needed in the design of ITER. The reliability and the precision of numerical simulations allow a better understanding of the physical phenomena and thus would lead to better designs. TONUS's main involvement is in such research. The required temperatures to attain fusion are very high, of the order of a hundred million degrees. Thus it is imperative to prevent the plasma from touching the tokamak inner walls. This confinement is obtained thanks to intense magnetic fields. The magnetic field is created by poloidal coils, which generate the toroidal component of the field. The toroidal plasma current also induces a poloidal component of the magnetic field that twists the magnetic field lines. The twisting is very important for the stability of the plasma. The idea goes back to research by Tamm and Sakharov, two Russian physicists, in the 50's. Other devices are essential for the proper operation of the tokamak: a divertor for collecting the escaping particles, microwave heating for reaching higher temperatures, a fuel injector for sustaining the fusion reactions, toroidal coils for controlling instabilities, etc.

4.2 Other applications

The software and numerical methods that we develop can also be applied to other fields of physics or engineering.

- For instance, we have a collaboration with the company AxesSim in Strasbourg for the development of efficient Discontinuous Galerkin (DG) solvers on hybrid computers (CPU/GPU). The applications are electro-magnetic simulations for the conception of antennas, electronic devices or aircraft electromagnetic compatibility.

- The acoustic conception of large rooms requires huge numerical simulations. It is not always possible to solve the full wave equation and many reduced acoustic models have been developed. A popular model consists in considering "acoustic" particles moving at the speed of sound. The resulting Partial Differential Equation (PDE) is very similar to the Vlasov equation. The same modeling is used in radiation theory. We have started to work on the reduction of the acoustic particles model and realized that our reduction approach perfectly applies to this situation. A PhD with CEREMA (Centre d'études et d"expertise sur les risques, l"environnement, la mobilité et l'aménagement), started in October 2015 and ended in January 2020 (PhD of Pierre Gerhard). The objective was to investigate the model reduction and to implement the resulting acoustic model in our DG solver.

- In September 2017, we started a collaboration with EDF Chatou (PhD of Lucie Quibel, defended in September 2020) on the modeling of multiphase fluids with complex equations of state. The goal is to simulate the high temperature liquid-vapor flow occurring in a nuclear plant. Among others, we will apply our recent kinetic method for designing efficient implicit schemes for this kind of flows.

5 Highlights of the year

Members of the TONUS team have organized the FVCA10 conference (Tenth International Conference on Finite Volumes for Complex Applications), held in Strasbourg, from October 30 to November 3, 2023. There were around 110 participants, mostly from across Europe, and also from Asia and North America. This conference led to the publication of two books of proceedings: [13, 14].

5.1 Awards

- October 2023: prix Blaise Pascal 2023 de l'Académie des Sciences (Yannick Privat)

6 New software, platforms, open data

6.1 New software

6.1.1 KOUGLOFV

Name: Kinetic schemes On Unstructured Grids for Large Optimized Finite Volume simulations

Keywords: Discontinuous Galerkin, Parallel numerical solvers

Functional Description: This code, written with the RUST language, solves Maxwell's equations on a generic tetrahedral mesh, for applications in electromagnetism. The underlying numerical method is stable without a restrictive CFL condition on the time step, and the quasi-explicit resolution ensures a low computational cost. Thanks to these properties, the code is able to handle multi-scale aspects in the mesh. In addition, the code is enhanced with shared-memory parallelization.

Contact: Victor Michel-Dansac

7 New results

7.1 Numerical methods for fluids and plasma dynamics

7.1.1 Practical and theoretical works around the kinetic and Lattice Boltzmann methods

Participants: Hubert Baty *(Observatoire Astronomique de Strasbourg)*, Florence Drui *(CEA, Saclay)*, Emmanuel Franck, Philippe Helluy, Romane Hélie, Clément Flint, Christian Klingenberg *(Univ. Würzburg, Germany)*, Lukas Thanhäuser *(Univ. Würzburg, Germany)*, Kevin Guillon *(Univ. Bordeaux)*.

The work described in [3] is devoted to the simulation of magnetohydrodynamic (MHD) flows with a Lattice Boltzmann Method (LBM) that handles shock waves. The scheme is implemented on GPU using PyOpenCL and makes it possible to use very fine grids. The method is validated on complex resistive instabilities.

In [26] we develop a simple and efficient methodology for compressing data on the fly during LBM simulations. This approach makes it possible to handle simulations that would not enter the GPU memory. It is based on wavelet compression and presents a reasonable overhead.

In [29] we propose a rigorous entropy stability analysis of the LBM scheme. We also expose a new algorithm for performing an equivalent equation analysis of the kinetic scheme with over-relaxation.

In [30] we propose a simple general approach for applying boundary conditions in the LBM. The method is based on a correction that ensures entropy dissipation, while maintening scheme accuracy.

7.1.2 Extension of the hydrostatic reconstruction to moving steady solutions

Participants: Christophe Berthon *(Nantes University)*, Victor Michel-Dansac.

In [17], we focused on the numerical approximation of the weak solutions of the shallow water model over a non-flat topography. In particular, we paid close attention to steady solutions with nonzero velocity. The goal of this work was to derive a scheme that exactly preserves these stationary solutions, as well as the commonly preserved lake at rest steady solution, by proposing an extension of the well-known hydrostatic reconstruction [33]. By appropriately defining the reconstructed states at the interfaces, any numerical flux function, combined with a relevant source term discretization, produces a well-balanced scheme that preserves both moving and non-moving steady solutions. This eliminates the need to construct specific numerical fluxes. Additionally, we proved that the resulting scheme is consistent with the homogeneous system on flat topographies, and that it reduces to the hydrostatic reconstruction when the velocity vanishes. To increase the accuracy of the simulations, we proposed a linear well-balanced high-order procedure.

7.1.3 Composition schemes for the guiding-center model

Participants: Michel Mehrenberger *(Aix-Marseille Université)*, Laurent Navoret , Anh-Tuan Vu *(Aix-Marseille Université)*.

In [31], we presented composition methods for solving non-linear transport equations, like the guiding-center model. By composing direct and adjoint transport steps on different well-chosen sub time steps, we were able to construct high-order in time numerical methods. The adjoint steps being implicit, they are replaced by explicit approximated adjoint steps using a given number of fixed point iterations. This does not modify the order of accuracy. Several numerical tests, using high-order semi-Lagrangian spatial discretization, assess the performance of the method. Despite the large number of substeps per iteration, the method can be interesting since it reduces the number of copies of the unknown.

7.1.4 Thermodynamically compatible discretization of a compressible two-fluid model with two entropy inequalities

Participants: Saray Busto *(Universidade de Vigo, Spain)*, Michael Dumbser *(Università degli Studi di Trento, Italy)*, Andrea Thomann .

In [8], we have introduced a new thermodynamically compatible finite volume scheme for the symmetric hyperbolic and thermodynamically compressible (SHTC) two-fluid model of Romenski et al. that is endowed with two entropy inequalities. The new method is able to discretize the two entropy inequalities directly and obtains total energy conservation as a mere consequence of the thermodynamically compatible discretization of all other equations. The thermodynamically compatible numerical flux is based on the seminal ideas of Abgrall, which was subsequently generalized and applied to SHTC systems. Compared to previous thermodynamically compatible finite volume schemes the new approach forwarded in this paper does not require the computation of any path integral but is totally general and can be applied to arbitrary overdetermined hyperbolic and thermodynamically compatible systems that admit an extra conservation law. The proposed HTC FV schemes satisfy two discrete entropy inequalities and are provably nonlinearly stable in the energy norm. We furthermore consider arbitrary high order accurate discontinuous Galerkin (DG) schemes applied to the vanishing viscosity limit of the overdetermined system. In this case, no particular care is taken to satisfy the additional conservation law exactly at the discrete level, apart from the mere resolution of all flow features. The numerical results clearly show that the schemes proposed in this project achieve their designed order of accuracy and that in the stiff relaxation limit the numerical solution tends to the asymptotically reduced Baer–Nunziato (BN) limit system with lift forces. The influence of the lift forces has been studied separately, showing that their presence in the BN limit is necessary to achieve a good agreement with the underlying SHTC system. This work was also the object of the short communication [9].

7.1.5 Hard congestion limit of the dissipative Aw-Rascle system

Participants: Nilasis Chaudhuri *(Imperial College London)*, Laurent Navoret , Charlotte Perrin *(CNRS & Aix-Marseille Université)*, Ewelina Zatorska *(Imperial College London)*.

In [22], we analyse the famous Aw-Rascle system in which the difference between the actual and the desired velocities (the offset function) is a gradient of a singular function of the density. This leads to a dissipation in the momentum equation which vanishes when the density is zero. The resulting system of PDEs can be used to model traffic or suspension flows in one dimension with the maximal packing constraint taken into account. After proving the global existence of smooth solutions, we study the so-called "hard congestion limit", and show the convergence of a subsequence of solutions towards a weak solution of an hybrid freecongested system. In the context of suspension flows, this limit can be seen as the transition from a suspension regime, driven by lubrication forces, towards a granular regime, driven by the contact between the grains.

7.1.6 Implicit-explicit solver for a two-fluid single-temperature model

Participants: Maria Lukáčová-Medviďová *(JGU Mainz, Germany)*, Ilya Peshkov *(Università degli Studi di Trento, Italy)*, Andrea Thomann .

In [7], we have derived an analyzed a new implicit-explicit finite volume (RS-IMEX FV) scheme for a single-temperature SHTC model. We note that the two-fluid model allows two velocities and pressures. Further, it includes two dissipative mechanisms: phase pressure and velocity relaxations. In the proposed scheme these processes are treated differently. The relative velocity relaxation term is linear and is resolved as a part of the implicit sub-system, whereas the pressure relaxation is strongly nonlinear and therefore is treated separately by the Newton method. Our RS-IMEX FV method is constructed in such a way that acoustic-type waves are linearized around a suitably chosen reference state (RS) and approximated implicitly in time and by means of central finite differences in space. The remaining advective-type waves are approximated explicitly in time and by means of the Rusanov FV method. The RS-IMEX FV scheme is suitable for all Mach number flows, but in particular it is asymptotic preserving in the low Mach number flow regimes. Many multi-phase flows, such as granular or sediment transport flows, can be modeled within the single-temperature approximation. In turn, many of these flows are weakly compressible and therefore impose severe time step restrictions if solved with a time-explicit numerical scheme. Therefore, the proposed RS-IMEX FV scheme is suitable to model various environmental flows. The proposed method was tested on a number of test cases for low and moderately high Mach number flows demonstrating the capability of the scheme to properly capture both regimes. The theoretical second order accuracy of the scheme was confirmed on a stationary vortex test case. We compared the second order scheme against its first order variant which showed that the second order scheme yields more accurate approximations of discontinuities. Finally, the theoretically proven asymptotic preserving property was verified numerically.

7.1.7 Two-Dimensional Linear Implicit Relaxed Scheme for Hyperbolic Conservation Laws

Participants: Angelo Iollo *(Université de Bordeaux & Inria MEMPHIS, France)*, Gabriella Puppo *(Sapienza – Università di Roma, Italy)*, Andrea Thomann .

In [10], we present a two-dimensional extension to the linear implicit all-speed finite volume scheme for hyperbolic conservation laws based on Jin-Xin relaxation recently forwarded in [44]. It is based on stiffly accurate SDIRK (Singly-Diagonal Implicit Runge-Kutta) methods in time and a convex combination of Rusanov and centered fluxes in space making it asymptotically consistent in the low Mach number regime and allows an accurate capturing of material waves under large time steps. The scheme is numerically tested on the Euler equations and a non-linear model for elasticity in the compressible and low Mach number regime.

7.2 Around machine learning and numerical schemes

7.2.1 Enrichment of Discontinuous Galerkin bases with Physics-Informed Neural Networks

Participants: Emmanuel Franck, Victor Michel Dansac, Laurent Navoret.

In [28], we proposed to enrich Discontinuous Galerkin (DG) bases, so that the resulting scheme provides a much better approximation of steady solutions to hyperbolic systems of balance laws. The basis enrichment leverages a prior – an approximation of the steady solution – which we computed using a Physics-Informed Neural Network (PINN). We proved convergence results and error estimates, showing that the basis with prior does not change the order of convergence, and that the error constant is improved. To construct the prior, parametric PINNs were used. Numerical experiments on 1D and 2D hyperbolic systems show that the DG scheme with prior is much more accurate on steady solutions than the DG scheme without prior, while retaining the same approximation quality on unsteady solutions.

7.2.2 An optimal control framework for adaptive neural ordinary differential equations

Participants: Joubine Aghili , Olga Mula *(TU Eindhoven)*.

In [16], we proposed an iterative adaptive algorithm where we progressively refined the time discretization (i.e. increasing the number of layers). Provided that certain tolerances are met across the iterations, we prove that the strategy converges to the underlying continuous problem. One salient advantage of such a shallow-to-deep approach is that it helps to benefit in practice from the higher approximation properties of deep networks by mitigating over-parametrization issues.

7.2.3 Deep optimal control to design hybrid numerical schemes

Participants: Emmanuel Franck, Léo Bois, Laurent Navoret, Vincent Vigon.

In [19], we propose a method for constructing a neural network viscosity in order to reduce the non-physical oscillations generated by high-order Discontiuous Galerkin (DG) methods. To this end, the problem is reformulated as an optimal control problem for which the control is the viscosity function and the cost function involves comparison with a reference solution after several compositions of the scheme. The learning process is strongly based on a time propagation method which uses the ML framework to compute the gradient of the full scheme with respect to the parameters of the network at many time steps.

7.2.4 Modelling solar coronal magnetic fields with physics-informed neural networks

Participants: Hubert Baty *(Observatoire Astronomique de Strasbourg)*, Vincent Vigon .

In [4], we present a novel numerical approach aiming at computing equilibria and dynamics structures of magnetized plasmas in coronal environments. A technique based on the use of neural networks that integrates the partial differential equations of the model, and called physics-informed neural networks (PINNs), is introduced. The functionality of PINNs is explored via calculation of different magnetohydrodynamic (MHD) equilibrium configurations, and also the derivation of exact two-dimensional steady-state magnetic reconnection solutions. The advantages and drawbacks of PINNs compared to traditional numerical codes are discussed in order to propose future improvements. Interestingly, PINNs are a meshfree method in which the obtained solution and associated different order derivatives are quasi-instantaneously generated at any point of the spatial domain. We believe that our results can help to pave the way for future developments of time dependent MHD codes based on PINNs.

7.3 Reduced modeling and ML

7.3.1 Hamiltonian reduced model for wave equations

Participants: Emmanuel Franck, Guillaume Steimer, Laurent Navoret, Vincent Vigon, Raphaël Côte *(Unistra)*.

In the paper [24] we propose a non-linear reduction method for models coming from the spatial discretization of partial differential equations: it is based on convolutional auto-encoders and Hamiltonian neural networks. Their training is coupled in order to simultaneously learn the encoder-decoder operators and the reduced dynamics. Several test cases on non-linear wave dynamics show that the method has better reduction properties than standard linear Hamiltonian reduction methods.

7.3.2 Hyperbolic reduced model for Vlasov-Poisson equation with Fokker-Planck collision

Participants: Emmanuel Franck, Ibtissem Lannabi *(Univ Pau)*, Laurent Navoret, Youssouf Nasseri, Giuseppe Parasiliti *(INRIA Ange)*, Guillaume Steimer.

The paper [27] proposes a reduced model to simulate the one-dimensional Vlasov-Poisson equation. The model provides the space-time dynamics of a few macroscopic quantities constructed following the Reduced Order Method (ROM) in the velocity variable: the compression is thus applied to the semi-discretization of the Vlasov equation. To gain efficiency, a Discrete Empirical Interpolation Method (DEIM) is applied to the compressed non-linear Fokker-Planck operator. Furthermore, we propose a correction to the reduced collision operator which ensures that the reduced moments satisfy an Euler-type system. Numerical simulations of the reduced model show that the model can capture the plasma dynamics in different collisional regimes and initial conditions at a low cost.

7.4 Other applications

While the main focus of the numerical tools we develop is plasma physics, they can also be used for other applications. We list below four such applications.

7.4.1 Micromagnetic simulations of the size dependence of the Curie temperature in ferromagnetic nanowires and nanolayers

Participants: Clémentine Courtès , Matthieu Boileau , Raphaël Côte *(Université de Strasbourg)*, Paul-Antoine Hervieux *(Université de Strasbourg)*, Giovanni Manfredi *(Université de Strasbourg)*.

In [25], we solved the Landau-Lifshitz-Gilbert equation in the finite-temperature regime, where thermal fluctuations are modeled by a random magnetic field whose variance is proportional to the temperature. We obtained Curie temperatures T_C that are in line with the experimental values for cobalt, iron and nickel and for finite-sized objects such as nanowires (1D) and nanolayers (2D), we study the variances of the Curie temperature with respect to the smallest size of the system. Moreover, optimization and parallelization of the python code solving the Landau-Lifshitz-Gilbert equation in the finite-temperature regime led to a 100-time acceleration of the code. We were therefore able to study the effect of the size of the material on the magnetization, in particular the value of the Curie temperature.

7.4.2 Minimal time of magnetization switching in small ferromagnetic ellipsoidal samples

Participants: Raphaël Côte *(Université de Strasbourg)*, Clémentine Courtès , Guillaume Ferrière *(Inria Lille)*, Yannick Privat .

Considering a ferromagnetic material of ellipsoidal shape, the associated magnetic moment then has two asymptotically stable opposite equilibria, of the form $\pm\overline{m}$. In order to use these materials for memory storage purposes, it is necessary to know how to control the magnetic moment. In [23], we use as a control variable a spatially uniform external magnetic field and consider the question of flipping the magnetic moment, i.e., changing it from the $+\overline{m}$ configuration to the $-\overline{m}$ one, in minimal time. Of course, it is necessary to impose restrictions on the external magnetic field used. We therefore include a constraint on the L^∞ norm of the controls, assumed to be less than a threshold value U. We show that, generically with respect to the dimensions of the ellipsoid, there is a minimal value of U for this problem to have a solution. We then characterize it precisely. Finally, we investigate some particular configurations associated to geometries enjoying symmetry properties and show that in this case the magnetic moment can be controlled in minimal time without imposing a threshold condition on U. This type of phenomenon (existence of a minimum time only if the control is powerful enough and non-controllability otherwise) seems new and leads to interesting extensions for more complex systems.

7.4.3 Reduced modeling and optimal control of epidemiological individual-based models with contact heterogeneity

Participants: Clémentine Courtès, Emmanuel Franck, Killian Lutz, Laurent Navoret, Yannick Privat.

Modeling epidemics using classical population-based models suffers from shortcomings that so-called individual-based models are able to overcome. They are able to take into account heterogeneity features, such as super-spreaders, and describe the dynamics involved in small clusters. In return, such models often involve large graphs which are expensive to simulate and difficult to optimize, both in theory and in practice. By combining the reinforcement learning philosophy with reduced models, we propose in [5] a numerical approach to determine optimal health policies for a stochastic individual-based model taking into account heterogeneity in the population. More precisely, we introduce a deterministic reduced population-based model involving a neural network, designed to faithfully mimic the local dynamics of the more complex individual-based model. Then the optimal control is determined by sequentially training the network until an optimal strategy for the population-based model succeeds in also containing the epidemic when simulated on the individual-based model. After describing the practical implementation of the method, several numerical tests are proposed to demonstrate its ability to determine controls for models with contact heterogeneity.

7.4.4 Numerical analysis of a shape optimization algorithm

Participants: Antonin Chambolle *(CEREMADE & Inria MOKAPLAN, Paris, France)*, Idriss Mazari-Fouquer *(CEREMADE, Paris, France)*, Yannick Privat .

In [20, 21], we prove a new result in shape optimization: the convergence of the fixed-point (also called thresholding) algorithm in three optimal control problems under large volume constraints. This algorithm was introduced by Céa, Gioan and Michel, and is of constant use in the simulation of $L^\infty - L^1$ optimal control problems. In this paper we consider the optimisation of the Dirichlet energy, of Dirichlet eigenvalues and of certain non-energetic problems. Our proofs rely on new diagonalisation procedure for shape Hessian matrices in optimal control problems, which leads to local stability estimates.

7.4.5 Optimal scenario for road evacuation in an urban environment

Participants: Emmanuel Franck , Laurent Navoret , Mickael Bestard, Yannick Privat
.

How to free a road from vehicle traffic as efficiently as possible and in a given time, in order to allow, for example, emergency vehicles to pass? In [18], we are interested in this question which we reformulate as an optimal control problem. We consider a macroscopic road traffic model on networks, semi-discretized in space and decide to give ourselves the possibility to control the flow at junctions. Our target is to smooth the traffic along a given path within a fixed time. A sparsity constraint is imposed on the controls, in order to ensure that the optimal strategies are feasible in practice. We perform an analysis of the resulting optimal control problem, proving the existence of an optimal control and deriving optimality conditions, which we rewrite as a single functional equation. We then use this formulation to derive a new mixed algorithm interpreting it as a mix between two methods: a descent method combined with a fixed point method allowing global perturbations. We verify with numerical experiments the efficiency of this method on examples of graphs, first simple, then more complex. We highlight the efficiency of our approach by comparing it to standard methods. We propose an open source code implementing this approach in the Julia language.

7.4.6 Spontaneous rotations in epithelia as an interplay between cell polarity and RhoA activity at boundaries

Participants: Simon Lo Vecchio *(IGBMC, Strasbourg)*, Olivier Pertz *(Cellular Dynamics lab, Berne)*, Marcela Szopos *(MAP5, Paris)*, Laurent Navoret, Daniel Riveline *(IGBMC, Strasbourg)*.

Directed flows of cells in vivo are essential in morphogenesis. They shape living matter in phenomena involving cell mechanics and regulations of the acto-myosin cytoskeleton. However the onset of coherent motion during collective cell migration is still poorly understood. In [6] we show that coherence is set by spontaneous alignments of cell polarity by designing cellular rings of controlled dimensions. A tug-of-war between opposite polarities dictates the onset of coherence, as assessed by tracking live cellular shapes and motions in various experimental conditions. In addition, we identify an internally driven constraint by cellular acto-myosin cables at boundaries as essential to ensure coherence. Moreover, active force is generated as evaluated by the high RhoA activity. Its contribution is required to trigger coherence as shown by our numerical simulations based on a novel Vicsek-type model including free active boundaries. Altogether, spontaneous coherent motion results from basic interplay between cell orientations and active cables at boundaries.

8 Bilateral contracts and grants with industry

8.1 Bilateral contracts with industry

Participants: Yannick Privat .

Collaboration with start-up Renaissance Fusion on the Stellacage project, co-funded by INRIA (Project Leader: Mario Sigalotti).

8.2 Bilateral grants with industry

Participants: Philippe Helluy .

We collaborate with EDF through the CIFRE thesis of Gauthier Lazare in the optimization of nuclear plant simulation software. The optimization is realized through algorithmic improvements but also with software acceleration based on machine learning.

9 Partnerships and cooperations

9.1 International research visitors

9.1.1 Visits of international scientists

Other international visits to the team

Gero Schnücke

Status: post-Doc

Institution of origin: Friedrich-Schiller-Universität Jena

Country: Germany

Dates: 09/2023 (1 month)

Context of the visit: working with Andrea Thomann

Mobility program/type of mobility: PROCOPE grant

Michael Dumbser

Status: reasearcher

Institution of origin: Università degli Studi di Trento

Country: Italy

Dates: 10/2023 (1 week)

Context of the visit: working with Andrea Thomann

Mobility program/type of mobility: using Andrea Thomann's PEPS grant

Sandra May

Status: reasearcher

Institution of origin: Uppsala universitet

Country: Sweden

Dates: 10/2023 (3 days)

Context of the visit: working with Andrea Thomann

Mobility program/type of mobility: using Andrea Thomann's PEPS grant

Davide Ferrari

Status: PhD student

Institution of origin: Università degli Studi di Trento

Country: Italy

Dates: 09/2023 – 12/2023 (3 months)

Context of the visit: working with Andrea Thomann

Mobility program/type of mobility: research visit to validate a European doctorate

9.1.2 Visits to international teams

Research stays abroad

Andrea Thomann

Visited institution: Johannes-Gutenberg University

Country: Germany

Dates: 03/2023 (1 week)

Context of the visit: Research visit Maria Lukáčová-Medviďová

Mobility program/type of mobility: research stay

Andrea Thomann

Visited institution: Johannes-Gutenberg University

Country: Germany

Dates: 09/2023 (1 week)

Context of the visit: Research visit with Maria Lukáčová-Medviďová

Mobility program/type of mobility: research stay

9.2 European initiatives

9.2.1 Other european programs/initiatives

- **Machine Learning for Kinetic equations**
  - *Acronym*: ANR-DFG MILK
  - *Duration*: 01/2022 – 12/2024
  - *Coordinator*: E. Franck, Inria Antenna in Strasbourg & E. Sonnendrücker, Technische Universität München (TUM)
  - *Partners*: Inria Antenna in Strasbourg & TUM
  - *Participants*: Clémentine Courtès, Emmanuel Franck, Laurent Navoret
  - *Abstract*: Kinetic models are accurate descriptions of interacting particle systems in physics. However, their numerical resolution is often too demanding, as they are defined in the large-dimensional position/velocity phase space and involve multi-scale dynamics. For this reason, reduced models have been developed that represent optimal trade-offs between numerical cost and modeling completeness. In general, this reduction is carried out in two ways. The first is based on asymptotic models that filter out fast dynamics and are obtained when a small parameter tends towards zero (collision/oscillation limit). The second, called reduced order modeling, consists in finding a smaller representation of the problem able to describe the dynamics (POD). The main objective of this project is to design new reduced order models that are more efficient than classical ones, based on machine learning techniques applied to kinetic data. Ensuring the stability of the obtained models will be a key point of these studies.

9.3 National initiatives

9.3.1 National projects

- **MOdeling and SImulation of COmplex Ferromagnetic systems**:
  - *Acronym*: ANR MOSICOF
  - *Duration*: 10/2021 – 10/2025
  - *Coordinator*: S. Labbé, Sorbonne Université
  - *Partners*: Sorbonne université, Université de Pau et des Pays de l'Adour, Université de Strasbourg
  - *Participants*: Clémentine Courtès, Yannick Privat
  - *Abstract*: During the last decade, promising applications of ferromagnetic materials have emerged in the domains of nanoelectronics (spintronic) and data storage: complex ferromagnetic systems are increasingly used for digital data recording and logic devices. They reduce the energy storage cost while improving the performance of the devices. The goal of this proposal is to bring together mathematicians and physicists around the understanding of the properties of ferromagnetism. One of the main objectives is to highlight and treat new multi-physics models, allowing for optimization and control of the magnetizations, and to

simulate the phenomena in a more efficient and less expensive way. We wish to develop approaches leading to mathematically justified and physically relevant solutions for the analysis and optimization of these materials, and which could ultimately lead to implementation on devices.

- **Modelling the effect of Apoptosis on Epithelium FLUidity**
  - *Acronym*: ANR MAPEFLU
  - *Duration*: 03/2023 – 02/2027
  - *Coordinator*: Laurent Navoret
  - *Partners*: Institut Pasteur, Université Paris Université de Strasbourg (IRMA, IGBMC)
  - *Participants*: Laurent Navoret, Roxana Sublet
  - *Abstract*: Epithelia have a viscoelastic behaviour: they respond as solids over short times and as fluids over large times. This fluidity plays an essential role in morphogenesis and tissue deformation. At the cellular scale, fluidity is achieved by the remodelling of junctions between cells due to their interactions but also by cell division and death. However, the contribution of apoptosis to fluidity has been little studied and remains unclear since cell death is also associated with local elastic constraints. Our project first aims at developing a novel particle model, describing cell cycles and the polarities interactions (Vicsek-like model), to assess the impact of cell death rate on tissue fluidity. The construction of this model will be strongly guided by comparisons with in vitro (MDCK cells) and in vivo (Drosophila pupa) experiments. From this particle model, a hydrodynamic model will be rigorously derived and simulations based on this new macroscopic description will be utilized to improve the understanding of tissue dynamics. The present study will thus provide a generic model, consistent with the experimental data and allowing one of the first systematic assessments of the role of apoptosis in tissues.
- **PEPR IA: PC IA-EDP**:
  - *Acronym*: PEPR IA/ PC IA-EDP
  - *Duration*: 01/09/2023 – 31/08/2027
  - *Coordinator*: Antonin Chambolle
  - *Partners*: Univ. Paris-Dauphine (PSL), Univ. Paris-Cité, Sorbonne Univ., Univ. Paris-Saclay, Univ. Toulouse, Univ. Lyon (CNRS), Univ. Bordeaux, Univ. Côte d'Azur, CREST (ENSAE/Institut Polytechnique de Paris) and Univ. Strasbourg
  - *Participants*: Emmanuel Franck, Clémentine Courtès, Vincent Vigon, Victor Michel-Dansac, Joubine Aghili, Philippe Helluy, Laurent Navoret
  - *Abstract*: The PEPR IA is a large national project on IA. The PC IA-PDE is a project funded by the ANR, which gathers ten major French institutions involved in developing the mathematical analysis of AI, the study of optimization in machine learning, as well as in developing machine learning for numerical analysis and scientific computing. We will study the link between modern IA methods and optimal control, optimal transport, PDE and numerical analysis. The team is involved in the optimal control aspect.
- **PEPR Numpex: PC Exa-MA**:
  - *Acronym*: PEPR Numpex/ PC Exama
  - *Duration*: 01/09/2023 – 31/08/2027
  - *Coordinator*: Christophe Prud'Homme et Hélène Barucq
  - *Partners*: Unistra, INRIA, Sorbonne University, Polytechnique, CEA
  - *Participants*: Emmanuel Franck, Clémentine Courtès, Vincent Vigon, Victor Michel-Dansac, Joubine Aghili, Philippe Helluy, Laurent Navoret

– *Abstract*: The Exa-MA project focuses on the Exascale aspects of digital methods, guaranteeing their adaptability to existing and future hardware. It is also a cross-disciplinary project, proposing methods and tools in which modelling, data and AI, through algorithms, are central. The team is mainly involved in the WP2 on reduced modeling and ML technics but also in the WP1 on numerical methods.

10 Dissemination

10.1 Promoting scientific activities

10.1.1 Scientific events: organisation

Member of the organizing committees

- Organizing committee of Finite Volumes for Complex Applications, FVCA10, Strasbourg, 30/10/2023 – 03/11/2023 (Joubine Aghili, Clémentine Courtès, Emmanuel Franck, Philippe Helluy, Victor Michel-Dansac, Laurent Navoret, Andrea Thomann)
- Organizing committee of CEMRACS2023, CIRM, Marseille, 17/07/2023 – 23/08/2023 (Victor Michel-Dansac)
- Organizing committee of the ANR MOSICOF workshop, Strasbourg, 11/01/2023 – 12/01/2023 (Clémentine Courtès, Yannick Privat)
- Organizer of the Fifth IRMA-EDF Workshop on multiphase flows, Strasbourg, 30/5/23 – 31/5/23 (Philippe Helluy).

Reviewer

- 31st European Signal Processing Conference, EUSIPCO 2023 (Antoine Deleforge)
- Finite Volumes for Complex Applications 10, FVCA10 (Joubine Aghili, Clémentine Courtès, Emmanuel Franck, Philippe Helluy, Victor Michel-Dansac, Laurent Navoret, Andrea Thomann)
- 2023 IEEE International Conference on Acoustics, Speech and Signal Processing, ICASSP2023 (Antoine Deleforge)
- The Fortieth International Conference on Machine Learning, ICML2023 (Antoine Deleforge)
- Neural Information Processing Systems, NeurIPS 2023 (Antoine Deleforge)
- IEEE Workshop on Applications of Signal Processing to Audio and Acoustics, WASPAA 2023 (Antoine Deleforge)

10.1.2 Journal

Member of the editorial boards

- EURASIP Journal on Audio, Speech and Music Processing (Antoine Deleforge)
- Mathematical Control and Related Fields (Yannick Privat)
- Computational and Applied Mathematics (Yannick Privat)
- Numerical Algebra, Control & Optimization (Yannick Privat)
- Advances in Continuous and Discrete Models (Yannick Privat)
- Journal of Optimization, Theory and Applications (Yannick Privat)
- AIMS Applied Mathematics books (Yannick Privat)

Reviewer - reviewing activities

- Applied Mathematics and Computation (Clémentine Courtès, Emmanuel Franck)
- Calcolo (Joubine Aghili, Emmanuel Franck)
- Communications on Applied Mathematics and Computation (Andrea Thomann)
- Computer Physics Communications (Victor Michel-Dansac)
- ESAIM: Mathematical Modelling and Numerical Analysis (Victor Michel-Dansac)
- ESAIM: Proceedings (Laurent Navoret)
- IEEE Signal Processing Letters (Antoine Deleforge)
- IEEE/ACM Transactions in Audio, Speech and Language Processing (Antoine Deleforge)
- IEEE Transactions on Signal Processing (Antoine Deleforge)
- International Journal for Numerical Methods in Engineering (Andrea Thomann)
- Journal of Advances in Aircraft and Spacecraft Science (Andrea Thomann)
- Journal of Computational Physics (Emmanuel Franck, Philippe Helluy, Laurent Navoret, Victor Michel-Dansac, Andrea Thomann)
- Journal of Engineering Mathematics (Joubine Aghili)
- Journal of the Acoustical Society of America (Antoine Deleforge)
- Journal of Scientific Computing (Victor Michel-Dansac, Andrea Thomann)
- MathSciNet (Victor Michel-Dansac)
- Nonlinearity (Clémentine Courtès)
- SIAM Journal on Mathematical Analysis (Laurent Navoret)
- SIAM Journal on Scientific Computing (Philippe Helluy, Victor Michel-Dansac, Andrea Thomann)
- zbMath (Victor Michel-Dansac)

10.1.3 Talks

- 12/2023: Colloquium “Physique et Mathématique”, Strasbourg, France (Laurent Navoret)
- 12/2023: AIAE’23: 3rd International Workshop on Artificial Intelligence and Augmented Engineering, Orsay, France (Victor Michel-Dansac)
- 12/2023: Group seminar of Prof. Christiane Helzel, Heinrich-Heine Universität Düsseldorf, Germany (Andrea Thomann)
- 11/2023: Journées Mathrice, Paris, France (Matthieu Boileau)
- 11/2023: Finite Volumes for Complex Applications, FVCA10, Strasbourg, France (Andrea Thomann)
- 10/2023: International Conference on Applied Mathematics and Mechanics (ICAMM 2023), Indore, Madhya Pradesh, India (online) (Victor Michel-Dansac)
- 09/2023: 3rd Inria-DFKI European Summer School on AI (IDESSAI 2023), Sophia-Antipolis, France (Victor Michel-Dansac)
- 08/2023: Kolloquium der Mechanik, TU Darmstadt, Germany (Andrea Thomann)

- 06/2023: Workshop "Around numerical analysis of lattice Boltzmann methods, Palaiseau, France (Philippe Helluy)
- 06/2023: "22ème Journée Calcul Scientifique et Modélisation Mathématique d'Amiens", Amiens, France (Philippe Helluy)
- 06/2023: NumHyp 2023, Bordeaux, France (Victor Michel-Dansac)
- 06/2023: Séminaire X/Stra, Strasbourg, France (Vincent Vigon)
- 04/2023: Lebesgue Doctoral Meeting, Nantes, France (Clémentine Courtès)
- 03/2023: Seminar, univ. Dijon, France (Yannick Privat)
- 03/2023: Univ. Lisbon, Portugal (Yannick Privat)
- 01/2023: PDE Masterclass, Nancy, France (Clémentine Courtès)
- through 2023: many talks in the Machine Learning workgroup (Matthieu Boileau, Clémentine Courtès, Emmanuel Franck, Laurent Navoret, Victor Michel-Dansac, Yannick Privat, Vincent Vigon)

10.1.4 Leadership within the scientific community

- Member of the IEEE Technical Committee on Audio and Acoustic Signal Processing (Antoine Deleforge)

10.1.5 Research administration

- Board member of "GDR Calcul" (Matthieu Boileau)
- Commission régionale d'interclassement des ingénieurs et techniciens CNRS (Matthieu Boileau)
- Member of the CdT (commission locale de Développement Technologique) Inria Research Center at Université de Lorraine (Matthieu Boileau)
- Member of the Conseil Scientifique de l'Ecole Doctorale MS2I, Université de Strasbourg (Matthieu Boileau)
- Member of the Conseil Scientifique de l'ITI IRMIA++ (Matthieu Boileau)
- Committee for an associate professor (section 26), University of Strasbourg (Clémentine Courtès)
- 09/2023 – now: Organization of the internal Seminar at Strasbourg University (G. Ancona, Clémentine Courtès, Y. le Floch, X. Zeng)
- 09/2021 – now: Organization of the PDE Seminar at Strasbourg University (Joubine Aghili, Clémentine Courtès and Victor Michel-Dansac)
- Elected member of the "commission des mathématiciens", University of Strasbourg (Clémentine Courtès, Laurent Navoret)
- 04/2021 – now: nominated member of the Inria Research Center at Université de Lorraine "comité de centre" (Clémentine Courtès)
- Referent for research data, Inria Research Center at Université de Lorraine (Antoine Deleforge)
- 01/2021 – now: Organization of the Machine Learning working group at Strasbourg University (Emmanuel Franck)
- Member of the board of "GDR Calcul" (Emmanuel Franck)
- 11/2023 – now: Member of CNU Section 26 (Emmanuel Franck)

- Co-responsable du GT IA du PEPR NumPex (Emmanuel Franck)
- Comex Inria Nancy-Grand-Est (Philippe Helluy)
- Elected member of the “Commission de la Recherche” of the University of Strasbourg (Philippe Helluy)
- 09/2018 – 09/2023: Head of the CNRS Lab IRMA “Institut de Recherche Mathématique Avancée”, UMR 7501 (Philippe Helluy)
- Elected member of the experts committee, UFR Math-Info, Université de Strasbourg (Laurent Navoret)
- Responsable formation, Institut Thématique Interdisciplinaire IRMIA++, Université de Strasbourg (Laurent Navoret)
- Member of the jury for the AMIES company thesis prize (Yannick Privat)
- 11/2019 – now: Member of CNU Section 26 (Yannick Privat)
- 04/2019 – 09/2023: Member of the IRMA expert committee (Yannick Privat)
- Jan. 2023-Aug. 2023: Head of the “Modeling and Control” team at IRMA (Yannick Privat)
- 2023-2027: Local leader (University of Strasbourg) of the PEPR Numerical analysis, optimal control and optimal transport for AI (Yannick Privat)
- Committee for an full professor (section 26), univ. Clermont Auvergne (Yannick Privat)

10.2 Teaching - Supervision - Juries

10.2.1 Teaching

- Licence 1, “Maths pour les sciences 2”, 64h lectures + problem sessions, Strasbourg University, France (Joubine Aghili)
- Licence 1, “Mathematics for biology”, 12h lectures + 23h exercise sessions, Strasbourg University, France (Michaël Gutnic)
- Licence 1, “Mathematics for health studies”, 36h, Strasbourg University, France (Michaël Gutnic)
- Licence 2, “Applied numerical analysis”, 18h, Strasbourg University, France (Clémentine Courtès)
- Licence 2, “Object-Oriented Programming in Python”, 16h lectures (Emmanuel Franck, Victor Michel-Dansac) + 16h computer labs (Victor Michel-Dansac), Strasbourg University, France
- Licence 2, “Scientific computing”, 20h lectures + 17h computer labs, Strasbourg University, France (Michaël Gutnic)
- Licence 2 Sciences et Société, “Introduction à l’apprentissage automatique ”, 12.5h, Strasbourg University, France (Laurent Navoret)
- Licence 2, “Numerical resolution techniques”, 16h computer labs, Strasbourg University, France (Andrea Thomann)
- “Probability and Statistics”, 30h, CNAM, France (Michaël Gutnic)
- Licence 3, “Calcul scientifique et analyse numérique”, 56h lectures + computer labs, Strasbourg University, France (Joubine Aghili)
- Licence 3, “Numerical analysis techniques 1”, 32h lectures + problem sessions + computer labs, Strasbourg University, France (Clémentine Courtès)

- Licence 3, “Scientific computing”, 65h lectures + exercise sessions, Strasbourg University, France (Clémentine Courtès)
- Licence 3, “Nonlinear Optimization”, 61h lectures + exercise sessions, Strasbourg University, France (Clémentine Courtès)
- Licence 3, “Advanced C++”, 20h lectures (Emmanuel Franck) + 34h computer labs (Emmanuel Franck), Strasbourg University, France
- Licence 3, “Numerical analysis techniques 2”, 34h, Strasbourg University, France (Philippe Helluy)
- Licence 3, “Introduction to C++”, 34h, Strasbourg University, France (Philippe Helluy)
- Master 1, “Projet”, 7h supervision, Strasbourg University, France (Joubine Aghili)
- Master 1, “Méthodes Numériques”, 8h computer labs, Strasbourg University, France (Joubine Aghili)
- Master 1, “Calcul scientifique 2”, 28h lectures, Strasbourg University, France (Joubine Aghili)
- Master 1, “Parallel Computing”, 20h, Strasbourg University, France (Matthieu Boileau)
- Master 1, “Artificial Intelligence, Machine Learning, Deep Learning”, 24h, Télécom Physique Strasbourg, France (Antoine Deleforge)
- Master 1, “Machine Learning”, 4h lectures + 4h computer labs, Strasbourg University, France (Emmanuel Franck)
- Master 1 Pure Mathematics, “Scientific computing”, 13h lectures (Philippe Helluy) + 14h computer labs (Philippe Helluy & Victor Michel-Dansac), Strasbourg University, France
- Master 1 Applied Mathematics, “Scientific computing”, 35h, Strasbourg University, France (Laurent Navoret)
- Master 1, “Optimization”, 6h computer labs, Strasbourg University, France (Clémentine Courtès)
- Master 1, “Optimization”, 62.5h, Strasbourg University, France (Laurent Navoret)
- Master 1 Applied Mathematics, “Random models”, 35h, Strasbourg University, France (Vincent Vigon)
- Master 1 Applied Mathematics, “Data mining”, 35h, Strasbourg University, France (Vincent Vigon)
- Master 1 Applied Mathematics, “Signal processing”, 35h, Strasbourg University, France (Vincent Vigon)
- Master 1 DUAS, “Data science”, 50h, Strasbourg University, France (Vincent Vigon)
- Master 1 & 2, “Initiation to Machine Learning”, 24h, Télécom Physique Strasbourg, France (Antoine Deleforge)
- Master 2, “Projet”, 7h supervision, Strasbourg University, France (Joubine Aghili)
- Master 2 Agrégation, “Calcul scientifique”, 10h lectures, Strasbourg University, France (Joubine Aghili)
- Master 2 Cell Physics, “Basics in mathematics”, 24h, Strasbourg University, France (Laurent Navoret)
- Master 2 Cell Physics, “Math for living matter”, 6h, Strasbourg University, France (Laurent Navoret)
- Master 2 Agrégation, “Scientific computing”, 28h, Strasbourg University, France (Philippe Helluy)
- Master 2 Agrégation, “Scientific computing”, 38h, Strasbourg University, France (Laurent Navoret)
- Master 2 Agrégation, “Probability”, 50h, Strasbourg University, France (Vincent Vigon)
- Master 2 Pure Mathematics, “Nonlinear evolution PDEs”, 30h, Strasbourg University, France (Clémentine Courtès)

- Master 2 Pure Mathematics, “Hydrodynamic limits of system of interacting particles”, 32h, Strasbourg University, France (Laurent Navoret)
- Master 2 Applied Mathematics, “Numerical methods for hyperbolic PDEs”, 35h, Strasbourg University, France (Laurent Navoret)
- Master 2, “Data Science”, 20h, Strasbourg University, France (Matthieu Boileau)
- Master 2, “Numerical methods for hyperbolic PDEs”, 35h, Strasbourg University, France (Philippe Helluy)
- Master 2, “Structure-preserving methods for Hamiltonian ODEs”, 14h, Strasbourg University, France (Victor Michel-Dansac)
- Master 2, “Variational Analysis and Planification”, 30h, Strasbourg University, France (Yannick Privat)
- Master 2, “Optimal control”, 70h, Strasbourg University, France (Yannick Privat)
- Master 2 Applied Mathematics, “Signal processing and deep learning”, 35h, Strasbourg University, France (Vincent Vigon)
- Master 2 Applied Mathematics, “Graphs and applications”, 35h, Strasbourg University, France (Vincent Vigon)
- Diplôme d’université, “Interdisciplinary seminars”, 8h, Strasbourg University, France (Laurent Navoret)
- Training Course, “Basics for scientific python”, 28h, Urfist (Unité Régionale de Formation à l’Information Scientifique et Technique), France (Matthieu Boileau)

10.2.2 Supervision

- 09/2022 – now: Dean of the Mathematics and Computer Science faculty (Directeur de l’UFR Mathématique et Informatique) in Strasbourg university (Michaël Gutnic)
- PhD completed in April 2023 [15]: **R. Hélie**, “Relaxation methods for kinetic models in plasma physics.”. Matthieu Boileau, Emmanuel Franck, Philippe Helluy and Laurent Navoret.
- PhD completed in December 2023: **M. Bestard**, “Optimal control for numerical simulation in physics plasma.”. Emmanuel Franck, Laurent Navoret and Yannick Privat.
- PhD completed in December 2023: **L. Bois**, “Machine learning for numerical methods. Application in plasma physics.”. Philippe Helluy, Emmanuel Franck, Laurent Navoret and Vincent Vigon.
- PhD in progress: **C. Flint**, “Efficient data compression for high-performance PDE solvers.”. Beginning: October 2020. Philippe Helluy and S. Genaud (Inria CAMUS).
- PhD in progress: **T. Sprunck**, “Can one hear the shape of a room?”. Beginning: September 2021. Antoine Deleforge and Yannick Privat.
- PhD in progress: **N. Victorion**, “Numerical methods assisted by deep learning for wave problems”. Beginning: September 2021. H. Barucq (Inria MAKUTU) and Emmanuel Franck.
- PhD in progress: **G. Steimer**, “Model reduction using deep learning for plasma physics”. Beginning: October 2021. R. Côte, Emmanuel Franck and Laurent Navoret
- PhD in progress: **M. Pallanque**, “Radiative transfer models for astrophysical code”. Beginning: September 2022. P. Ocvirk (Strasbourg Observatory) and Emmanuel Franck
- PhD in progress: **G. Lazare**, “Simulations performantes en thermo-hydraulique des cœurs avec THYC”. Beginning: November 2022. Philippe Helluy and EDF Chatou.

- PhD in progress: **A. Bélières–Frendo**, “Shape optimization through learning”. Beginning: December 2023. C. Prud’homme (Strasbourg University), C. Dapogny (CNRS & Grenoble University) and Victor Michel-Dansac
- PhD in progress: **K. Lutz**, “Optimal control of Lindblad equations”. Beginning: September 2023. Emmanuel Franck & Yannick Privat
- PhD in progress: **C. Schnoebelen**, “Geometric methods for wave equations”. Beginning: October 2023. Emmanuel Franck & Laurent Navoret & E. Ophstein (IRMA)
- PhD in progress: **R. Sublet**, “Mathematical models for collective cell dynamics”. Beginning: October 2023. Laurent Navoret & M. Szopos (Univ. Paris Cité)
- PhD in progress: **R. San Roman**, “Self supervised disentangled representation learning of audio data for compression and generation”. Beginning: June 2022. Y. M. Adi (Meta AI), Antoine Deleforge, R. Serizel (MULTISPEECH) and G. Synnaeve (Meta AI)
- PhD in progress: **S. Dilungana**, “New Algorithms for Automated Room Acoustic Diagnosis”. Beginning: October 2020. Antoine Deleforge, C. Foy (CEREMA) and S. Faisan (Université de Strasbourg)

10.2.3 Juries

- Examiner in the PhD jury of Mouna Kassan, Université de Pau et des Pays de l’Adour (Clémentine Courtès)
- Examiner in the PhD jury of Xiaoyu Bie, Université de Grenoble (Antoine Deleforge)
- Reviewer for the PhD thesis of Jai Kumar “Machine learning for numerical processing and analysis of kinetic and fluid simulations of fusion plasmas”, Université Aix-Marseille CEA-Cadarache (Emmanuel Franck)
- Reviewer for the PhD thesis of A. Al Kheir “Étude de quelques problèmes d’optimisation de forme spectrale”, univ. Pau et des pays de l’Adour, June 2023 (Yannick Privat)
- Examiner, reviewer or president of the following PhD juries: T. Bellotti, Palaiseau, June 2023; S. Maazioui, July 2023, Paris XIII; G. Jomée, Marseille, November 2023; K. Guillon, Bordeaux,November 2023; Axel Maupoux, Toulouse, December 2023; Lucas Tallois, December 2023, Bordeaux; M. Bestard, December 2023, Strasbourg (Philippe Helluy)
- Examiner of the following HDR Jury: E. Franck, Strasbourg, January 2023; A. Charguéraud, Strasbourg, February 2023 (Philippe Helluy)

10.3 Popularization

10.3.1 Education

- stage d’observation de 3e : L. Avon, 12/2023 (Matthieu Boileau)

10.3.2 Interventions

- Conference in a 4th grade class as part of the "Semaine des mathématiques », Sébastien Brant college, Eschau (Matthieu Boileau)
- Conference in front of parents of students as part of Ose La Recherche., Le Vaisseau, Strasbourg (Matthieu Boileau)
- 06/2023 : scientific workshop for the "MathC2+" week, for high school students (Clémentine Courtès)
- 05-06/2023 : research talks aimed at high school students (Clémentine Courtès)
- 02/2023 : talk aimed at PhD students on scientific careers (Clémentine Courtès)

- since 08/2022 : organization of the mathematics and computing week "Les Cigognes" for high school girls (Clémentine Courtès, M. Duflot-Kremer, A. de Roton, P. Py, S. Tapie)
- "Comment fonctionnent les modèles de language (ChatGPT, etc.)". Talk in front of colleagues from other fields (medecine, literature). July 2023, April 2023 (Philippe Helluy)

11 Scientific production

11.1 Major publications

[1] L. Bois, E. Franck, L. Navoret and V. Vigon. 'A neural network closure for the Euler-Poisson system based on kinetic simulations'. In: *Kinetic and Related Models* (2021). URL: https://hal.archives-ouvertes.fr/hal-02965954.

[2] C. Courtès, D. Coulette, E. Franck and L. Navoret. 'Vectorial kinetic relaxation model with central velocity. Application to implicit relaxations schemes'. In: *Communications in Computational Physics* 27.4 (Apr. 2020). DOI: 10.4208/cicp.OA-2019-0013. URL: https://hal.archives-ouvertes.fr/hal-01942317.

11.2 Publications of the year

International journals

[3] H. Baty, F. Drui, P. Helluy, E. Franck, C. Klingenberg and L. Thanhäuser. 'A robust and efficient solver based on kinetic schemes for Magnetohydrodynamics (MHD) equations.' In: *Applied Mathematics and Computation* 440 (Mar. 2023), p. 127667. DOI: 10.1016/j.amc.2022.127667. URL: https://hal.science/hal-02965967.

[4] H. Baty and V. Vigon. 'Modelling solar coronal magnetic fields with physics-informed neural networks'. In: *Monthly Notices of the Royal Astronomical Society* 527 (2024), pp. 2575–2584. DOI: 10.1093/mnras/stad3320. URL: https://insu.hal.science/insu-04295010.

[5] C. Courtès, E. Franck, K. Lutz, L. Navoret and Y. Privat. 'Reduced modelling and optimal control of epidemiological individual-based models with contact heterogeneity'. In: *Optimal Control Applications and Methods* (2023). DOI: 10.1002/oca.2970. URL: https://hal.science/hal-03664271.

[6] S. Lo Vecchio, O. Pertz, M. Szopos, L. Navoret and D. Riveline. 'Spontaneous rotations in epithelia as an interplay between cell polarity and boundaries'. In: *Nature Physics* (8th Jan. 2024). DOI: 10.1038/s41567-023-02295-x. URL: https://hal.science/hal-03427805.

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[8] A. Thomann and M. Dumbser. 'Thermodynamically compatible discretization of a compressible two-fluid model with two entropy inequalities'. In: *Journal of Scientific Computing* 97.1 (27th Aug. 2023), p. 9. DOI: 10.1007/s10915-023-02321-3. URL: https://hal.science/hal-04089966.

Conferences without proceedings

[9] M. Dumbser, S. Busto and A. Thomann. 'On Thermodynamically Compatible Finite Volume Schemes for Overdetermined Hyperbolic Systems'. In: FVCA10. Vol. 433. Springer Proceedings in Mathematics & Statistics. Strasbourg, France, 13th Oct. 2023, pp. 103–110. DOI: 10.1007/978-3-031-40860-1_11. URL: https://hal.science/hal-04410976.

[10] A. Iollo, G. Puppo and A. Thomann. 'Two-Dimensional Linear Implicit Relaxed Scheme for Hyperbolic Conservation Laws'. In: FVCA 2023. Vol. 433. Springer Proceedings in Mathematics & Statistics. Strasbourg, France: Springer Nature Switzerland, 13th Oct. 2023, pp. 171–179. DOI: 10.1007/978-3-031-40860-1_18. URL: https://hal.science/hal-04375673.

[11] N. Victorion, H. Barucq, H. Calandra, F. Faucher and E. Franck. 'Optimized Finite Differences methods and Machine Learning to reduce numerical dispersion for the wave equation'. In: Journées Ondes Sud-Ouest (JOSO). Toulouse, France, 14th Mar. 2023. URL: https://hal.science/hal-04398557.

[12] N. Victorion, H. Barucq, H. Calandra, E. Franck and F. Faucher. 'Improving the efficiency of acoustic wave solver with optimized FD andby coupling FEM with Machine Learning'. In: Rencontre Ondes et et leurs Applications (ROA). PAU, France, 29th June 2023. URL: https://hal.science/hal-04398588.

Scientific book chapters

[13] E. Franck, J. Fuhrmann, V. Michel-Dansac and L. Navoret. 'Preface'. In: *Finite Volumes for Complex Applications X—Volume 1, Elliptic and Parabolic Problems*. Vol. 432. Springer Proceedings in Mathematics & Statistics. Springer Nature Switzerland, 30th Sept. 2023. DOI: 10.1007/978-3-031-40864-9. URL: https://hal.science/hal-04270231.

[14] E. Franck, J. Fuhrmann, V. Michel-Dansac and L. Navoret. 'Preface'. In: *Finite Volumes for Complex Applications X—Volume 2, Hyperbolic and Related Problems*. Vol. 433. Springer Proceedings in Mathematics & Statistics. Springer Nature Switzerland, 12th Oct. 2023. DOI: 10.1007/978-3-031-40860-1. URL: https://hal.science/hal-04270236.

Doctoral dissertations and habilitation theses

[15] R. Hélie. 'Relaxation scheme for the simulation of plasmas in tokamaks'. Université de Strasbourg, 27th Mar. 2023. URL: https://theses.hal.science/tel-04034510.

Reports & preprints

[16] J. Aghili and O. Mula. *An optimal control framework for adaptive neural ODEs*. 19th Apr. 2023. URL: https://hal.science/hal-02897466.

[17] C. Berthon and V. Michel-Dansac. *A fully well-balanced hydrodynamic reconstruction*. 27th Apr. 2023. URL: https://hal.science/hal-04083181.

[18] M. Bestard, E. Franck, L. Navoret and Y. Privat. *Optimal scenario for road evacuation in an urban environment*. 20th Oct. 2023. URL: https://hal.science/hal-04253010.

[19] L. Bois, E. Franck, L. Navoret and V. Vigon. *An optimal control deep learning method to design artificial viscosities for Discontinuous Galerkin schemes*. 21st Sept. 2023. URL: https://hal.science/hal-04213057.

[20] A. Chambolle, I. Mazari-Fouquer and Y. Privat. *Stability of optimal shapes and convergence of thresholding algorithms in linear and spectral optimal control problems*. 24th June 2023. URL: https://hal.science/hal-04140177.

[21] A. Chambolle, I. Mazari-Fouquer and Y. Privat. *Stability of optimal shapes and convergence of thresholding algorithms in linear and spectral optimal control problems: Supplementary material*. 25th June 2023. URL: https://hal.science/hal-04140334.

[22] N. Chaudhuri, L. Navoret, C. Perrin and E. Zatorska. *Hard congestion limit of the dissipative Aw-Rascle system*. 5th June 2023. URL: https://hal.science/hal-03786853.

[23] R. Côte, C. Courtès, G. Ferriere and Y. Privat. *Minimal time of magnetization switching in small ferromagnetic ellipsoidal samples*. 6th Jan. 2023. URL: https://hal.science/hal-03931579.

[24] R. Côte, E. Franck, L. Navoret, G. Steimer and V. Vigon. *Hamiltonian reduction using a convolutional auto-encoder coupled to an Hamiltonian neural network*. 10th Nov. 2023. URL: https://hal.science/hal-04237799.

[25] C. Courtès, M. Boileau, R. Côte, P. A. Hervieux and G. Manfredi. *Micromagnetic simulations of the size dependence of the Curie temperature in ferromagnetic nanowires and nanolayers*. 23rd Dec. 2023. URL: https://hal.science/hal-04364178.

[26] C. Flint and P. Helluy. *Reducing the memory usage of Lattice-Boltzmann schemes with a DWT-based compression.* 17th Feb. 2023. URL: https://inria.hal.science/hal-03990880.

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[28] E. Franck, V. Michel-Dansac and L. Navoret. *Approximately well-balanced Discontinuous Galerkin methods using bases enriched with Physics-Informed Neural Networks.* 17th Oct. 2023. URL: https://hal.science/hal-04246991.

[29] K. Guillon, R. Hélie and P. Helluy. *Stability analysis of the vectorial lattice-Boltzmann method.* 12th Feb. 2023. URL: https://hal.science/hal-03986533.

[30] R. Hélie and P. Helluy. *Stable second order boundary conditions for kinetic approximations.* 2nd June 2023. URL: https://hal.science/hal-04115275.

[31] M. Mehrenberger, L. Navoret and A.-T. Vu. *Composition schemes for the guiding-center model.* 18th Mar. 2023. URL: https://hal.science/hal-04035921.

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[32] C. Altmann, T. Belat, M. Gutnic, P. Helluy, H. Mathis, E. Sonnendrücker, W. Angulo and J.-M. Hérard. ‘A local time-stepping Discontinuous Galerkin algorithm for the MHD system’. In: *Modélisation et Simulation de Fluides Complexes - CEMRACS 2008.* Marseille, France, July 2009. DOI: 10.1051/proc/2009038. URL: https://hal.inria.fr/inria-00594611.

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