

*Team Mathfi*

*Financial Mathematics*

*Rocquencourt*

THEME 4B

*Activity*  
*R* *eport*

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## 2. Overall Objectives

MATHFI is a joint project-team with ENPC (CERMICS) and the University of Marne la Vallée, located in Rocquencourt and Marne la Vallée.

The development of increasingly complex financial products requires the use of advanced stochastic and numerical analysis techniques that pose challenging problems to mathematicians. In this field, the scientific skills of the MATHFI research team are focused on • the modeling of the price of assets by stochastic processes (diffusions with jumps, stable processes, fractional Brownian processes, stochastic volatility models), • probabilistic and deterministic numerical methods and their implementation (Monte-Carlo methods, Malliavin calculus, numerical analysis of linear and nonlinear parabolic partial differential equations), • Stochastic control, with applications to calibration of financial assets, pricing and hedging of derivative products, dynamic portfolio optimization in incomplete markets (transaction costs, discrete time hedging, portfolio constraints, partial observation).

Moreover, the MATHFI team is developing a software called PREMIA dedicated to pricing and hedging options and calibration of financial models, in collaboration with a financial consortium. Web Site : <http://cermics.enpc.fr/~premia>.

### 2.1.1. International cooperations, Tutorials and industrial relationship

- International collaborations:
  - French-Russian program on financial mathematics of the Liapounov Institute in Moscow. Web Site : <http://www-direction.inria.fr/international/liapunov.html>.
  - Collaborations with the Department of Mathematics of the University of Oslo, the University of Bath, the Universities of Rome II and III.
  - Teaching in the doctoral programs of Paris and the surrounding region (Paris I, Paris VI, Paris IX, UMLV, University of Evry, Ecole Polytechnique, ENPC).
- Industrial developments :
  - The PREMIA consortium: A consortium of financial institutions is created centered on the PREMIA option computation software. The consortium is currently composed of : Caisse des Dépôts et Consignations, Crédit Industriel et Commercial, Crédit Agricole/Indosuez, Crédit-Lyonnais, Natexis-Banques Populaires, EDF, GDF and Summit. Web Site : <http://cermics.enpc.fr/~premia>.
  - CIFRE agreements with EDF (hedging options on electricity), CIC (calibration by Monte-Carlo methods), CAI (interest rate models with stochastic volatility).

## 3. Scientific Foundations

### 3.1. Numerical methods for option pricing and hedging and model calibration

**Key words:** Monte-Carlo, Euler schemes, approximation of SDE, tree methods, quantization, Malliavin calculus, finite difference, calibration.

**Participants:** V. Bally, L. Caramellino, E. Clément, B. Jourdain, D. Lamberton, B. Lapeyre, J. Printems, A. Sulem, E. Temam, A. Zanette.

Efficient computations of prices and hedges for derivative products is a major issue for financial institutions. Although this research activity exists for more than fifteen years at both academy and bank levels, it remains a lot of challenging questions especially for exotic products pricing on interest rates and portfolio optimization with constraints.

This activity in the Mathfi team is strongly related to the development of the Premia software. It also motivates theoretical researches both on Monte-Carlo methods and numerical analysis of partial differential equations : Kolmogorov equations Hamilton-Jacobi-Bellman equations, variational and quasi-variational inequalities.

The calibration of financial models using the available information of option prices is a research subject which associates F. Bonnans of the Sydoco research team.

#### 3.1.1. Probabilistic numerical methods.

The main issues concern numerical pricing and hedging of European and American derivatives and sensibility analysis. Financial modeling is generally based on diffusion processes of large dimension (greater than 10), often degenerate. Therefore, efficient numerical methods are very difficult to implement. Monte-Carlo simulations are widely used because of their implementation simplicity. Nevertheless, efficiency issues rely on tricky mathematical problems such as accurate approximation of functionals of Brownian motion (e.g. for exotic options), use of low discrepancy sequences for nonsmooth functions .... We develop algorithms based on quantization trees and Malliavin calculus.

#### 3.1.2. Approximation of stochastic differential equations.

In the diffusion models, the implementation of Monte-Carlo methods generally requires the approximation of a stochastic differential equation, the most common being the Euler scheme. The error can then be controlled either by the  $L_P$ -norm or the probability transitions. E. Clément [63][62] uses moderate deviation type techniques. These techniques are applied to approximate hedging of options.

#### 3.1.3. Deterministic numerical methods.

We are concerned with the numerical analysis of degenerate parabolic partial differential equations, variational and quasivariational inequalities, Hamilton-Jacobi-Bellman equations especially in the case when the discrete maximum principle is not valid.

#### 3.1.4. Model calibration.

While option pricing theory deals with valuation of derivative instruments given a stochastic process for the underlying asset, model calibration is about identifying the (unknown) stochastic process of the underlying asset given information about prices of options. It is generally an ill-posed inverse problem which leads to optimisation under constraints.

### 3.2. Malliavin calculus

**Key words:** Malliavin calculus, stochastic variations calculus, sensibility calculus, greek computations.

**Participants:** V. Bally, M.C. Bavouzet, L. Caramellino, B. Jourdain, D. Lamberton, B. Lapeyre, P. Malliavin, M. Messaoud, A. Sulem, E. Temam, A. Zanette.

The original Stochastic Calculus of Variations, now called the Malliavin calculus, was developed by Paul Malliavin in 1976 [76]. It was originally designed to study the smoothness of the densities of solutions of stochastic differential equations. One of its striking features is that it provides a probabilistic proof of the

celebrated Hörmander theorem, which gives a condition for a partial differential operator to be hypoelliptic. This illustrates the power of this calculus. In the following years a lot of probabilists worked on this topic and the theory was developed further either as analysis on the Wiener space or in a white noise setting. Many applications in the field of stochastic calculus followed. Several monographs and lecture notes (for example D. Nualart [77], D. Bell [60] D. Ocone [79], B. Øksendal [82]) give expositions of the subject.

From the beginning of the nineties, applications of the Malliavin calculus in finance have appeared : In 1991 Karatzas and Ocone showed how the Malliavin calculus, as further developed by Ocone and others, could be used in the computation of hedging portfolios in complete markets [78].

Since then, the Malliavin calculus has raised increasing interest and subsequently many other applications to finance have been found, such as minimal variance hedging and Monte Carlo methods for option pricing. More recently, the Malliavin calculus has also become a useful tool for studying insider trading models and some extended market models driven by Lévy processes or fractional Brownian motion.

Let us try to give an idea why Malliavin calculus may be a useful instrument for probabilistic numerical methods. We recall that the theory is based on an integration by parts formula of the form  $E(f'(X)) = E(f(X)Q)$ . Here  $X$  is a random variable which is supposed to be "smooth" in a certain sense and non-degenerated. A basic example is to take  $X = \sigma\Delta$  where  $\Delta$  is a standard normally distributed random variable and  $\sigma$  is a strictly positive number. Note that an integration by parts formula may be obtained just by using the usual integration by parts in the presence of the Gaussian density. But we may go further and take  $X$  to be an aggregate of Gaussian random variables (think for example of the Euler scheme for a diffusion process) or the limit of such simple functionals.

An important feature is that one has a relatively explicit expression for the weight  $Q$  which appears in the integration by parts formula, and this expression is given in terms of some Malliavin-derivative operators. Let us now look at one of the main consequences of the integration by parts formula. If one considers the Dirac function  $\delta_x(y)$ , then  $\delta_x(y) = H'(y - x)$  where  $H$  is the Heaviside function and the above integration by parts formula reads  $E(\delta_x(X)) = E(H(X - x)Q)$ , where  $E(\delta_x(X))$  can be interpreted as the density of the random variable  $X$ . We thus obtain an integral representation of the density of the law of  $X$ . This is the starting point of the approach to the density of the law of a diffusion process: the above integral representation allows us to prove that under appropriate hypothesis the density of  $X$  is smooth and also to derive upper and lower bounds for it. Concerning simulation by Monte Carlo methods, suppose that you want to compute  $E(\delta_x(y)) \sim \frac{1}{M} \sum_{i=1}^M \delta_x(X^i)$  where  $X^1, \dots, X^M$  is a sample of  $X$ . As  $X$  has a law which is absolutely continuous with respect to the Lebesgue measure, this will fail because no  $X^i$  hits exactly  $x$ . But if you are able to simulate the weight  $Q$  as well (and this is the case in many applications because of the explicit form mentioned above) then you may try to compute  $E(\delta_x(X)) = E(H(X - x)Q) \sim \frac{1}{M} \sum_{i=1}^M E(H(X^i - x)Q^i)$ . This basic remark formula leads to efficient methods to compute by a Monte Carlo method some irregular quantities as derivatives of option prices with respect to some parameters (the Greeks) or conditional expectations, which appear in the pricing of American options by the dynamic programming). See the papers by Fournié et al [68] and [67] and the papers by Bally et al, Benhamou, Bermin et al., Bernis et al., Cvitanic et al., Talay and Zheng and Temam in [30].

More recently the Malliavin calculus has been used in models of insider trading. The "enlargement of filtration" technique plays an important role in the modelling of such problems and the Malliavin calculus can be used to obtain general results about when and how such filtration enlargement is possible. See the paper by P.Imkeller in [30]). Moreover, in the case when the additional information of the insider is generated by adding the information about the value of one extra random variable, the Malliavin calculus can be used to find explicitly the optimal portfolio of an insider for a utility optimization problem with logarithmic utility. See the paper by J.A. León, R.Navarro and D. Nualart in [30]).

### 3.3. Stochastic Control

**Key words:** *Stochastic Control, singular and impulse control, risk-sensitive control, free boudary, Hamilton-Jacobi-Bellman, variational and quasi-variational inequalities.*



**Participants:** J.-Ph. Chancelier, D. Lefèvre, M. Mnif, M. Messaoud, B. Øksendal (Université d’Oslo), A. Sulem.

Stochastic control consists in the study of dynamical systems subject to random perturbations and which can be controlled in order to optimize some objective function.

We consider systems following controlled diffusion dynamics possibly with jumps. The objective is to optimize a criterion over all admissible strategies on a finite or infinite planning horizon. The criterion can also be of ergodic or risk-sensitive type. Dynamic programming approach leads to Hamilton-Jacobi-Bellman (HJB) equations for the value function. This equation is integrodifferential in the case of underlying jump processes. The limit conditions depend on the behaviour of the underlying process on the boundary of the domain (stopped or reflected).

Optimal stopping problems such as American pricing lead to variational inequalities of obstacle type. In the case of singular control, the dynamic programming equation is a variational inequality, that is a system of partial differential inequalities. Singular control is used for example to model proportional transaction costs in portfolio optimisation. The control process may also be of impulse type: in this case the state of the system jumps at some intervention times. The impulse control consists in the sequence of instants and sizes of the impulses. The associated dynamic programming equation is then a quasivariational inequality (QVI). This model is used for example in the case of portfolio optimisation with fixed transaction costs. Variational and quasivariational inequalities are free boundary problems. The theory of viscosity solutions offers a rigorous frame for the study of dynamic programming equations. An alternative approach to dynamic programming is the study of optimality conditions which lead to backward stochastic differential equations for the adjoint state: In [26], we provided maximum principle for the optimal control of jump diffusion processes: we prove a verification theorem by employing Arrow’s generalization of the Mangasarian sufficient condition to a general jump diffusion setting, and show the adjoint processes’ connections to dynamic programming. The result is applied to financial optimization problems.

We also study the case of partial observation optimal control problems [10].

### 3.4. Backward Stochastic Differential equations

**Key words:** BSDE.

**Participants:** M.C. Quenez, M. Kobylanski (University of Marne la Vallée).

Backward Stochastic Differential equations (BSDE) are related to the stochastic maximum principle for stochastic control problems. They also provide the prices of contingent claims in complete and incomplete markets.

The solution of a BSDE is a pair of adapted processes  $(Y, Z)$  which satisfy

$$-dY_t = f(t, Y_t, Z_t)dt - Z'_t dW_t; \quad Y_T = \xi, \quad (1)$$

where  $f$  is the driver and  $\xi$  is the terminal condition [80].

M.C. Quenez, N.El Karoui and S.Peng have established various properties of BSDEs, in particular the links with stochastic control (cf. [92], [33]). There are numerous applications in finance. For example in the case of a complete market, the price of a contingent claim  $B$  satisfies a BSDE with a linear driver and a terminal condition equal to  $B$ . This is a dynamic way of pricing which provides the price of  $B$  at all time and not only at 0. In incomplete markets, the price process as defined by Föllmer and Schweizer (1990) in [66] corresponds to the solution of a linear BSDE. The selling price process can be approximated by penalised prices which satisfy nonlinear BSDEs. Moreover nonlinear BSDEs appear in the case of big investors whose strategies affect market prices. Another application in finance concerns recursive utilities as introduced by Duffie and Epstein (1992) [65]. Such a utility function associated with a consumption rate  $(c_t, 0 \leq t \leq T)$  corresponds to the solution of a BSDE with terminal condition  $\xi$  which can be a function of the terminal wealth, and a driver  $f(t, c_t, y)$  depending on the consumption  $c_t$ . The standard utility problem corresponds to a linear driver  $f$  of

the type  $f(t, c, y) = u(c) - \beta_t y$ , where  $u$  is a deterministic, non decreasing, concave function and  $\beta$  is the discount factor.

In the case of reflected BSDEs, introduced in [86], the solution  $Y$  is forced to remain above some obstacle process. It satisfies

$$-dY_t = f(t, Y_t, Z_t)dt + dK_t - Z'_t dW_t; \quad Y_T = \xi. \quad (2)$$

where  $K$  is a nondecreasing process.

For example the price of an American option satisfies a reflected BSDE where the obstacle is the payoff. The optimal stopping time is the first time when the prices reaches the payoff [87][90].

## 4. Application Domains

### 4.1. Modelisation of financial assets

We try to find realistic models for the prices of financial assets by using stochastic volatilities, stable processes and fractional Brownian motions.

**Participants:** V. Genon-Catalot, T. Jeantheau, A. Sulem.

**Key words:** *stochastic volatility, stable law, fractional Brownian motion.*

#### 4.1.1. Statistics of stochastic volatility models.

It is well known that the Black-Scholes model, which assumes a constant volatility, doesn't completely fit with empirical observations. Several authors have thus proposed a stochastic modelisation for the volatility, either in discrete time (ARCH models) or in continuous time (see Hull and White [74]). The price formula for derivative products depend then of the parameters which appear in the associated stochastic equations. The estimation of these parameters requires specific methods. It has been done in several asymptotic approaches, e.g. high frequency [71][72][70].

#### 4.1.2. Application of stable laws in finance.

Statistical studies show that market prices do not follow diffusion prices but rather discontinuous dynamics. Stable laws seem appropriate to model cracks, differences between ask and bid prices, interventions of big investors. Moreover pricing options in the framework of geometric  $\alpha$ -stable processes lead to a significant improvement in terms of volatility smile. Statistic analysis of exchange rates lead to a value of  $\alpha$  around 1.65. A. Tisseyre has developped analytical methods in order to compute the density, the repartition function and the partial Laplace transform for  $\alpha$ -stable laws. These results are applied for option pricing in "stable" markets. (see [81]).

#### 4.1.3. Fractional Brownian Motion (FBM).

The Fractional Brownian Motion  $B_H(t)$  with Hurst parameter  $H$  has originally been introduced by Kolmogorov for the study of turbulence. Since then many other applications have been found.

If  $H = \frac{1}{2}$  then  $B_H(t)$  coincides with the standard Brownian motion, which has independent increments. If  $H > \frac{1}{2}$  then  $B_H(t)$  has a *long memory* or *strong aftereffect*. On the other hand, if  $0 < H < \frac{1}{2}$ , then  $\rho_H(n) < 0$  and  $B_H(t)$  is *anti-persistent*: positive values of an increment is usually followed by negative ones and conversely. The strong aftereffect is often observed in the logarithmic returns  $\log \frac{Y_n}{Y_{n-1}}$  for financial quantities  $Y_n$  while the anti-persistence appears in turbulence and in the behavior of volatilities in finance.

For all  $H \in (0, 1)$  the process  $B_H(t)$  is *self-similar*, in the sense that  $B_H(\alpha t)$  has the same law as  $\alpha^H B_H(t)$ , for all  $\alpha > 0$ .

Nevertheless, if  $H \neq \frac{1}{2}$ ,  $B_H(t)$  is not a semi-martingale nor a Markov process [73][61], [28][24], and integration with respect to a FBM requires a specific stochastic integration theory.

Consider the classical Merton problem of finding the optimal consumption rate and the optimal portfolio in a Black-Scholes market, but now driven by fractional Brownian motion  $B_H(t)$  with Hurst parameter

$H \in (\frac{1}{2}, 1)$ . The interpretation of the integrals with respect to  $B_H(t)$  is in the sense of Itô (Skorohod-Wick), not pathwise (which are known to lead to arbitrage). This problem can be solved explicitly by proving that the martingale method for classical Brownian motion can be adapted to work for fractional Brownian motion as well [73]. When  $H \rightarrow \frac{1}{2}+$  the results converge to the corresponding (known) results for standard Brownian motion [28]. Moreover, a stochastic maximum principle holds for the stochastic control of FBM's [61].

## 4.2. Pricing contingent claims in incomplete markets

**Key words:** *incomplete markets.*

**Participants:** M.C. Quenez, D. Lamberton, S. Njoh.

In incomplete markets, the available information may not be restricted to the underlying assets prices. Perfect hedge may not be possible for some contingent claims and pricing can not be done by arbitrage techniques. Moreover, there exists several probabilities, equivalent to the initial one  $P$ , under which the discounted prices are martingales. By duality, these probabilities are associated to various prices [88][89]. The upper bound of these prices is characterised as the smallest supermartingale which is equal to  $B$  at time  $T$ . This  $Q$ -supermartingale can be written as the difference between a  $Q$ -martingale corresponding to the discounted value of a superhedging portfolio and an optional nondecreasing process. This decomposition implies that this upper bound corresponds to the selling price defined as the lowest price for which there exists a superhedging strategy.

Approximating hedging problems appear when it is not possible to hedge with the underlying asset. The PhD dissertation of S. Njoh, supervised by D. Lamberton, concerns options on electricity. Since electricity can not be stocked, we can only try to hedge approximately by using other assets.

## 4.3. Portfolio optimisation with transaction costs

**Key words:** *Portfolio optimisation, transaction costs.*

**Participants:** T. Bielecki (Northeastern Illinois University, Chicago), J.-Ph. Chancelier, B. Øksendal (University of Oslo), S. Pliska (University of Illinois at Chicago), A. Sulem, M. Taksar (Stony Brook University New York).

We consider a model of  $n$  risky assets (called *Stocks*) whose prices are governed by logarithmic Brownian motions, which can eventually depend on economic factors and one riskfree asset (called *Bank*). Consider an investor who has an initial wealth invested in Stocks and Bank and who has ability to transfer funds between the assets. When these transfers involve transaction costs, this problem can be formulated as a singular or impulse stochastic control problem.

In one type of models, the objective is to maximize the cumulative expected utility of consumption over a planning horizon [1]. Another type of problem is to consider a model without consumption and to maximize a utility function of the growth of wealth over a finite time horizon [57]. Finally, a third class of problem consists in maximizing a long-run average growth of wealth [58].

Dynamic programming lead to variational and quasivariational inequalities which are studied theoretically by using the theory of viscosity solutions and numerically by finite difference approximations and policy iteration type algorithms.

The case of fixed costs is studied in [11] and [84]. In [43] we develop methods of risk sensitive impulsive control theory in order to solve an optimal asset allocation problem with transaction costs and a stochastic interest rate.

In the case of jump diffusion markets, dynamic programming lead to integrodifferential equations. In the absence of transaction costs, the problem can be solved explicitly [69]: the optimal portfolio is to keep the fraction invested in the risky assets constantly equal to some optimal value. In the case of proportional transaction costs, there exists a *no transaction region*  $D$  with the shape of a cone with vertex at the origin, such that it is optimal to make no transactions as long as the position is in  $D$  and to sell stocks at the rate of

local time (of the reflected process) at the upper/left boundary of  $D$  and purchase stocks at the rate of local time at the lower/right boundary [91]. These results generalize the results obtained in the no jump case.

## 5. Software

### 5.1. Development of the software PREMIA for financial option computations

**Key words:** *pricer, options, pricing, hedging, calibration.*

**Participants:** V. Bally, L. Caramellino, B. Jourdain, B. Lapeyre, A. Sulem, E. Temam, A. Zanette.

We develop a software called PREMIA designed for pricing and hedging options on assets and interest rates and for calibration of financial models and we have created a bank consortium on this project. Premia is concentrated on derivatives with rigorous numerical treatment and didactic inclination [75]. It contains the most recent algorithms published in the mathematical finance literature with their detailed description. The target is to reach on the first hand the market makers, and the PHD students in finance or mathematical finance.

Premia is developed in collaboration with a consortium of financial institutions or departments: It is presently composed of: Caisse des Dépôts et Consignations, Crédit Lyonnais, Crédit Agricole-Indosuez, Crédit Industriel et Commercial, Natexis-Banques Populaires, EDF, GDF and the Summit Society. See <http://cermics.enpc.fr/~premia>.

History of PREMIA:

The development of Premia started in 1999. There exists now 5 releases.

Premia1, 2 and 4 contain finite difference algorithms, tree methods and Monte Carlo methods for pricing and hedging European and American options on stocks in the Black-Scholes model in one and two dimension.

Premia3 is dedicated to Monte Carlo methods for American options in large dimension. Moreover, there is an interface with the software Scilab [64].

Premia5 and 6 contain more sophisticated algorithms such as quantization methods for American options [21][22] and methods based on Malliavin calculus both for European and American options [68][67]. Moreover models have been extended to some underlying processes with jumps, local volatility and stochastic volatility. Some calibration algorithms have been implemented.

Premia1, 2, 3, 4 and 5 have been delivered to the bank consortium in May 1999, December 1999, February 2001, February 2002, February 2003 respectively. Premia6 will be delivered in February 2004. The release Premia7, under development in 2004, will be dedicated to derivative products on interest rates.

Premia2 and soon Premia3 can be downloaded from the web site.

## 6. New Results

### 6.1. Monte Carlo methods and stochastic algorithms

**Participants:** B. Arouna, B. Lapeyre, N. Moreni.

#### 6.1.1. Variance reduction methods in Monte Carlo simulations

Under the supervision of Bernard Lapeyre, Bouhari Arouna is a PhD student working on Stochastic Approximations and Monte Carlo Methods. His purpose is to provide tractable methods of variance reduction in Monte Carlo estimation of expectations (integrals) and to prove associated theoretical results.

With Bernard Lapeyre, Nicola Moreni is studying variance reduction techniques for option pricing based on Monte Carlo simulation. In particular, in a joint project with the University of Pavia (Italy), he applies path integral techniques to the pricing of path-dependent European options. He has also deals with a variance reduction technique for the Longstaff-Schwartz algorithm for American option pricing [45]. This technique is based on importance sampling and the Girsanov theorem and can be applied to the field of interest rate models.

## 6.2. Approximation of stochastic differential equations

**Participants:** V. Bally, E. Clément, D. Lamberton, B. Lapeyre, J. Guyon, V. Lemaire.

During Arturo Kohatsu-Higa's visit to the university of Marne la Vallée in June, a joint work with E. Clément, V. Bally and D. Lamberton started. In particular, a connection between the error in the Euler scheme and backward stochastic differential equations was established. This work is still in progress. J. Guyon has started a PhD thesis on the Euler scheme for the approximation of diffusion with jumps. From Talay and Tubaro, it is known that when  $(X_t, t \geq 0)$  is a diffusion and  $f$  is smooth, then

$$\mathbb{E}[f(X_T)] - \mathbb{E}[f(X_T^n)] = \frac{C(f)}{n} + O\left(\frac{1}{n^2}\right).$$

Julien tries to extend this result to general  $f$ 's by controlling the linear mapping  $f \mapsto C(f)$  and the remainder. This is done by bounding the density of  $X_T$  from above by using Malliavin calculus. He plans to use this approach to deal with discontinuous  $X$ 's.

Moreover, Vincent Lemaire has obtained new results on explicit Euler schemes for some diffusions with locally Lipschitz coefficients.

## 6.3. Malliavin calculus for jump diffusions

**Key words:** *Malliavin calculus, jump diffusions.*

**Participants:** V. Bally, M.P. Bavouzet, M. Messaoud.

One of the main applications of the classical Malliavin calculus is to prove that under some non degeneracy and regularity assumptions on the coefficients, the law of a diffusion process is absolutely continuous with respect to the Lebesgue measure. Properties of the density are also obtained. The main tool in the calculus is an integration by parts formula which is strongly related to the Gaussian law (because the diffusion process is a functional of the Brownian motion).

Vlad Bally and M.P. Bavouzet are working now on the Malliavin calculus for Poisson point processes, in order to give sufficient conditions under which a diffusion with jumps has an absolutely continuous law and to study the properties of the density. A large literature is already known on this topic but the method we try to settle is different from the ones already used. The key point is the extension of the integration by parts formula to general random variables with smooth density (instead of Gaussian random variables in the Brownian case). The random variables on which the calculus is based may be the amplitude of the jumps or the jump times. Moreover we have to define all the differential operators which are needed to this extended formula (Malliavin derivatives, Skorohod integral...).

On the other hand, M.P. Bavouzet and M. Messaoud use the Malliavin calculus for Poisson processes in order to compute sensitivities (like the Delta for example) for European options with underlying jump diffusions. Our methodology follows the line opened by Fournié, Lasry, Lebuchou, Lions and Touzi for continuous diffusions.

## 6.4. Algorithms for optimal control and portfolio optimization

**Key words:** *Optimal control.*

**Participants:** H. Pham (Université Paris 7), G. Pagès, J. Printems.

We implement numerical methods in order to solve portfolio optimization problems in the framework of stochastic control. We are concern with the numerical computation of optimal hedging strategies in incomplete markets. We consider two illustrating examples: (i) the case when the investor can trade only one asset among  $d$  risky assets, (ii) and the case when the risky asset  $\{S_t\}$  ( $d = 1$ ) together with the volatility process  $\{\sigma_t\}$  follows a stochastic volatility model. In both cases, the objective is to hedge according to a quadratic criterion with the available risky assets. Since the controls do not appear in the risky asset model, we can easily simulate

them and compute their optimal quantization before proceeding with a dynamical programming procedure. See [53].

## 6.5. Functional quantization and Asian options

**Key words:** *quantization, Asian options.*

**Participants:** G. Pagès, J. Printems.

We are working on quadratic functional quantization and its application to the pricing of Asian options. The aim of optimal quantization is to study the best  $L^2$  approximation of Hilbert valued random variables taking at most  $N$  values. In a former framework (see e.g. [21]), the Hilbert space was  $\mathbf{R}^d$ . Recently, Gilles Pagès and Graf Luschgy (Journal of Functional Analysis **196**, 2002) investigated the case of an infinite dimensional Hilbert space (e.g.  $L^2([0, T])$ ). This approach allows us to study the numerical quantization of the Brownian motion from a functional point of view by considering the Brownian motion as a random variable taking values in  $L^2([0, T])$ . Similar approach can be considered for other Gaussian and non Gaussian processes. Unfortunately, the resulting quantization error has a bad rate with respect to  $N$ , namely  $1/\log(N)$ . Nevertheless, numerical computations tell us that things behave better than expected.

In a financial framework, functional quantization is helpful when dealing with options with “non Markovian” payoffs, that is payoffs depending on the whole trajectory of the asset price process, such as time average (Asian) options or maximum (lookback) options. In these cases, we can approximate the value of the option by the usual numerical integration in a functional space considering the asset price process as some  $H$ -valued random variable rather than as a  $\mathbf{R}^d$ -valued process. Associated numerical study can be found in [54].

## 6.6. Quantization methods for filtering of nonlinear systems

**Key words:** *nonlinear filtering, Zakai equation.*

**Participants:** B. Saussereau (University of Franche-Comté, Besançon), J. Printems.

We study the filtering of nonlinear systems from a numerical point of view: we want to compute the conditional expectations of signals when the observation process and/or the dynamic of the signal is not linear by means of a spectral approximation of the Zakai equation. Sergey Lototsky, Remigijus Mikulevicius and Boris Rozovskii (SIAM J. Control Optimization **35**, 1997) have proposed a spectral approach of nonlinear filtering by means of the Chaos expansion of the Wiener process. Numerical experiments based on this approach together with a quantization method provide promising results (see [55]).

## 6.7. Multiple stopping time problems

**Participants:** M.C. Quenez, M. Kobylanski, E. Rouy.

We are working on multiple stopping time problems. For example, in the bidimensional case, the optimization problem can be written as

$$\sup_{\tau^1, \tau^2 \in T_0} E[\psi(\tau^1, \tau^2)] \quad (3)$$

where  $T_0$  is the set of stopping times smaller than  $T$  and the reward function  $\psi : (t, s, \omega) \mapsto \psi(t, s, \omega)$  is a positive random function defined on  $\mathbf{R}_+ \times \mathbf{R}_+ \times \Omega$  such that for each  $(t, s)$ ,  $\psi(t, s)$  is  $\mathcal{F}_{t \vee s}$ -measurable. For example  $\psi(t, s)$  can be given by

$$\psi(t, s) = \Psi(S_t^1, S_s^2),$$

where  $\Psi$  is a deterministic function and  $S^1, S^2$  are two price processes. We have obtained a characterization of the value function of this problem as the value function of a classical optimal stopping time problem, where the



reward function is the maximum of two functions which are also the value functions of two optimal stopping time problems. We give a characterization of the optimal stopping times and prove their existence under some regularity assumptions. In the Markovian case, the value function is a solution of a PDE system. We study two examples: First, we consider the case of an investor who can invest in two risky assets. At time 0, he holds one asset  $S^1$  and one asset  $S^2$ . Then, at time  $\theta_1$  ( $\in T_0$ ), he can sell the first asset and invest the remain amount in the second asset, or either sell the second asset and invest the remain amount in the first one. At time  $\theta_2$ , he transfers his wealth into a bond until time  $T$ . The problem is to determine the optimal strategy which maximizes the expected utility of terminal wealth. Second, we study the problem of pricing a call option on this financial instrument and show that in the Markovian case, the price is associated to a system of PDEs.

We also consider some options like digital options on two or more assets. In this case, the reward function is not regular and we can not apply the previous results. This problem is under study.

## 6.8. Calibration of financial asset models

**Key words:** *calibration, inverse problems, Dupire equation.*

**Participants:** V. Bally, M. Messaoud, B. Jourdain, L. Nguyen, J. Printems, E. Temam.

### 6.8.1. Calibration of local volatility model via entropy minimization

Participants: Messaoud, Printems.

We have investigated an algorithm proposed by Avellaneda et al in [59] for calibrating volatility surfaces via relative-entropy minimization. The model used is a trinomial tree. The idea consists in starting from a prior diffusion then modifying it until the model fits the market price. The updating procedure is based on a Kullback-Leiber information criterion, which minimizes the relative entropy between two different diffusions. The numerical computations show surprisely that convex regularization such as entropy minimization together with a suitable choice of the algorithm parameters seem sufficient whereas we would have expected  $H^1$  regularization as usually admitted. This work will be published and implemented in the software Premia6.

### 6.8.2. Calibration of asset models.

Participants: L.Nguyen, B.Jourdain, A.Alfonsi.

Laurent Nguyen has studied the calibration of a jump diffusion asset model. More precisely, he considers an a-priori model given by a stochastic differential equation with jumps. He suggests to look for the equivalent martingale measure which minimizes the relative entropy with respect to this a-priori model among those giving the good price to the calibration options. Of course, to ensure absolute continuity, the local volatility function is preserved and only the jump intensity is calibrated. Using optimal control techniques, he proves existence of the solution of the calibration problem [15].

Aurélien Alfonsi has just started his PhD thesis. He aims to improve the stability calibration techniques with respect to time.

## 6.9. Howard algorithms for quasi-variational inequalities

**Key words:** *impulse stochastic control, quasi-variational inequalities, Howard algorithm.*

**Participants:** J.Ph. Chancelier, M. Messaoud, A. Sulem.

J.-Ph. Chancelier, B. Øksendal and A. Sulem studied in [83] and [85] the problem of a consumer-investor wishing to select an optimal portfolio with respect to utility maximisation criteria in the presence of fixed and proportional transaction costs. The authors characterize the value function as the unique viscosity solution of a quasi-variational-inequality and propose a numerical method based on an iterative sequence of variational inequalities. M. Messaoud has studied a two stage Howard algorithm and proved that this iterative algorithm converges to the viscosity solution of the QVI.

## 6.10. Regularity and representation of viscosity solutions of partial differential equations via backward stochastic differential equations

**Key words:** *stochastic integrals, Brownian motion, stochastic differential equations, distributional derivative, forward backward stochastic differential equations.*

**Participants:** M. N'ZI (University of Cocody, Abidjan), A. Sulem, Y. Ouknine (University of Marrakech).

We study the regularity of the viscosity solution of a quasilinear parabolic partial differential equation with Lipschitz coefficients by using its connection with a forward backward stochastic differential equation (in short FBSDE) and we give a probabilistic representation of the generalized gradient (derivative in the distribution sense) of the viscosity solution. This representation is a kind of nonlinear Feynman-Kac formula. The main idea is to show that the FBSDE admits a unique linearized version interpreted as its distributional derivative with respect to the initial condition. If the diffusion coefficient of the forward equation is uniformly elliptic, we approximate the FBSDE by smooth ones and use Krylov's estimate to prove the convergence of the derivatives. In the degenerate case, we use techniques of Bouleau-Hirsch on absolute continuity of probability measures.

## 6.11. Partial observation control of jump processes in an anticipating environment

**Key words:** *Partial observation control, Lévy processes.*

**Participants:** B. Øksendal (Oslo University), A. Sulem.

In [38], we study a controlled stochastic system whose state  $X(t)$  at time  $t$  is described by a stochastic differential equation driven by Lévy processes with filtration  $\{\mathcal{F}_t\}_{t \in [0, T]}$ . The system is *anticipating*, in the sense that the coefficients are assumed to be adapted to a filtration  $\{\mathcal{G}_t\}_{t \geq 0}$ , where  $\mathcal{F}_t \subseteq \mathcal{G}_t$  for all  $t \in [0, T]$ . The corresponding anticipating stochastic differential equation is interpreted in the sense of *forward integrals*, which are the natural generalization of the semimartingale integrals. The admissible controls are assumed to be adapted to a filtration  $\{\mathcal{E}_t\}_{t \in [0, T]}$ , such that  $\mathcal{E}_t \subseteq \mathcal{F}_t$  for all  $t \in [0, T]$ . The general problem is to maximize a given performance functional of this system over all admissible controls. This is a *partial observation stochastic control problem in an anticipating environment*. Examples of applications include stochastic volatility models in finance, insider influenced financial markets and stochastic control of systems with delayed noise effects. Some specific cases from finance, involving optimal portfolio with logarithmic utility, are solved explicitly.

## 6.12. Optimal investment models with stochastic volatilities, jump diffusion processes and portfolio constraints

**Participant:** M. Mnif.

**Key words:** *Optimal investment, portfolio constraints.*

We are interested in the problem of optimal investment with stochastic volatilities and portfolio constraints on amount in the case of a multidimensional model. We model the risky assets by jump diffusion processes and we consider an exponential utility function. The dynamic programming approach leads to a characterization of the value function as a viscosity solution of the highly nonlinear Bellman equation. Thanks to a transformation, we reduce the nonlinearity of the Bellman equation to a semilinear equation. A verification theorem is stated and relates the solution of this semilinear equation to our stochastic control problem. We prove also the existence of a smooth solution to this semilinear equation. We present an example which shows the importance of this reduction for numerical studying for the value function and the optimal portfolio.

## 6.13. Information and stochastic control

**Participant:** D. Lefèvre.



**Key words:** *information, control.*

Given  $T \in (0, \infty)$  and  $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{0 \leq t \leq T}, \mathcal{P})$  a given probability space satisfying the usual conditions on which a one dimensional Brownian Motion  $(B_t)_{0 \leq t \leq T}$  is defined, consider the following stochastic optimal control problem,

Maximize over all  $u \in \mathcal{A}$

$$J(x) = E^x [g(X(T))], \quad (4)$$

where the evolution of the system state  $(X_t)_{0 \leq t \leq T}$  is governed by the controlled stochastic differential equation

$$dX_t = b(X_t, u_t)dt + \sigma(X_t, u_t)dB_t, \quad t \geq 0. \quad (5)$$

In the above,  $b, \sigma : \mathbf{R} \times U \rightarrow \mathbf{R}$  and  $g : \mathbf{R} \rightarrow \mathbf{R}$  are some given maps with  $U$  being a given compact metric space. The process  $u$  is called the control representing the policy of the decision-makers and  $\mathcal{A}$  denotes the family of admissible control processes.

One of the main difference between deterministic and stochastic optimal control theory is the adaptation constraint required on the policies  $u$ . In the stochastic case, relevant information, as specified by the filtration  $(\mathcal{F}_t)_{0 \leq t \leq T}$ , is acquired with the passage of time. In other words, at any time instant the controllers know only what has happened up to that moment but cannot foretell what is going to happen afterwards. This nonanticipative restriction in mathematical terms is that  $u$  must be  $(\mathcal{F}_t)_{0 \leq t \leq T}$ -adapted.

Following an idea from R.J. Wets in the context of stochastic programming, we allow a wider class of possibly anticipative controls and formulate the adaptedness requirement on  $u$  as an equality constraint which we can write :

$$(I - \Pi)u = 0,$$

where  $\Pi$  denotes the projection onto the subspace of adapted processes and  $I$  the identity matrix.

Hence, our stochastic optimal control problem (4)-(5) may be seen as a non-adapted constrained control problem of the following type

Maximize

$$E^x [g(X_T)], \quad (6)$$

where

$$dX_t = b(X_t, u_t)dt + \sigma(X_t, u_t)dB_t, \quad t \geq 0, \quad (7)$$

over all measurable control processes  $u(t, \omega)$  under the constraint

$$(I - \Pi)u = 0. \quad (8)$$

Our general goal is twofold : we want to define an incremental value of information for the stochastic optimal control problem (4)-(5) and compute explicitly the ‘‘Expected Value of Perfect Information’’, a quantity introduced by Dempster in the 80’s.

Before going further, the first issue one should address is how to correctly define a solution for the anticipative stochastic differential equation (7). Indeed, when nonanticipativity on the policies  $u$  is relaxed, the system equation (7) cannot be solved within the Itô theory because  $\sigma(X_t, u_t)$  will no longer be an adapted process. Therefore, to establish the well-posedness of the constrained optimal control problem (6)-(8), we will use some results from A. Kohatsu-Higa and J. Leon on anticipative stochastic differential equations of Stratonovich type.

Next we ask : since additional information leads only to better decisions by the controllers, it is clear that there will be a price to pay for it. But how much ? We address this question by studying the Lagrange multiplier  $(\lambda)_{0 \leq t \leq T}$  associated with the information constraint (8). The classical Lagrangian theory asserts that this Lagrange multiplier has an interpretation as a price system for small violations of the constraint - in our case small anticipative perturbations of the nonanticipative controls  $u$ . Hence,  $\lambda_t$  may be seen at the time instant  $t$  as the cost to be paid for acquiring information in advance. We are thus able to define an incremental value of information and we hope to rely it to the so-called “shadow price” in economic theory.

Moreover, the reformulated optimal control problem (6)-(8) allows us to solve (4)-(5) in the “clairvoyant” case, when  $\lambda_t = 0, 0 \leq t \leq T$ , i.e. when controls are based on the complete past and future of the noise  $(B_t)_{0 \leq t \leq T}$ . The difference between the nonanticipative and anticipative value functions of (4)-(5) is the so-called “Expected Value of Perfect Information” or EVPI in Dempster’s terminology. It corresponds to the reduction in cost that it is possible if the usual nonanticipativity constraint on the controls is waived. Hence, the Expected Value of Perfect Information may be interpreted as a measure of risk : the larger the EVPI is, the larger the lack of information for the controllers is in deciding to pursue some uncertain decisions.

## 6.14. Stochastic optimisation and application in reinsurance

**Participants:** M. Mnif, A. Sulem.

In [44], we study the optimal reinsurance policy and dividends distribution of an insurance company under excess of loss reinsurance. The insurer gives part of its premium stream to another company in exchange of an obligation to support the difference between the amount of the claim and some retention level. The objective of the insurer is to maximise the expected discounted dividends. We suppose that in the absence of dividend distribution, the reserve process of the insurance company follows a compound Poisson process. We first prove existence and uniqueness results for this optimisation problem by using singular stochastic control methods and the theory of viscosity solutions. We then compute the optimal strategy of reinsurance, the optimal strategy of dividends pay-out and the value function by solving the associated integro-differential Hamilton-Jacobi-Bellman Variational Inequality numerically in the special case of a Poisson process with constant intensity.

## 6.15. Statistics of stochastic volatility models

**Participants:** V. Genon-Catalot, T. Jeantheau, C. Larédo.

We study parameter estimation for stochastic volatility models, when this volatility is driven by a stochastic differential equation or depends on the infinite past of the considered process.

## 6.16. Optimisation with risk constraints

**Participants:** F. Bonnans [Sydoco project], N. Boulanger, R. Aid [EDR R&D], A. Sulem.

In the framework of the intership of N. Boulanger (Mastere ENS Ulm and DEA Paris IX Dauphine) we have discussed the problem of Markov chain optimisation under constraints of VaR (value at risk) and CVar (conditional value at risk). This study is motivated by the problem of optimal management of water resources for hydraulic energy with risk control and is done in collaboration with the department R&D of the EDF company.

## 6.17. Filtration enlargements and applications in the credit universe

**Participant:** Y. Elouerkhaoui.

In the credit universe the choice of filtration plays a central role in multi-name problems. Enlarging the economic state-variables filtration by observing the default process of all available credits in the universe has some profound implications on the dynamics of intensities. Indeed, a sudden default in one credit triggers jumps in the spreads of all the other names. In this framework, we have derived the default times' density function in the pure CJD model. We have established the conditional expectations' formula w.r.t. the enlarged filtration. This result generalises the one-dimensional formula in Jeanblanc and Rutkowski (2000). We have studied the equivalence between the copula approach and the CJD model. We have shown that conditional jump diffusions imply an implicit copula function between the default times. Conversely, following the work of Schonbucher and Schubert (2001), we have shown that a copula model implies a CJD dynamic on the enlarged filtration.

## 6.18. Calibration of copula models for the pricing of default correlation products

**Participant:** Y. Elouerkaoui.

The Gaussian copula model introduced by Li (2000) for the pricing of correlation products can be seen as the Black-Scholes of default correlation. It is easy to use and there is a one-to-one mapping between the Gaussian correlation and the basket price. This model assumes that correlation of the underlying portfolio is constant. However, by using the prices of quoted CDO tranches in the market (e.g. tranches on the TRAC-X index) and inverting the Gaussian copula formula, we find that the implied correlation depends on the subordination level of each tranche. To model this smile effect properly, we introduce the Marshall-Olkin copula. This type of model has been used in reliability theory (see Barlow and Proshan (1981)) and for credit related problems by Duffie and Singleton (1998). We have developed a robust methodology to calibrate this model systematically. We have found that the MO default distribution is multi-modal, whereas the Gaussian copula only admits a unique mode. Each mode of the distribution can then be used to reproduce the correlation smile curve.

## 7. Contracts and Grants with Industry

### 7.1. Consortium Premia

**Participants:** V. Bally, M. Barton Smith, L. Caramellino, J Da Fonseca, B. Jourdain, B. Lapeyre, A. Sulem, E. Temam, A. Zanette.

The consortium Premia is centered on the development of the pricer software Premia. It is presently composed of the following financial institutions or departments: Caisse des Dépôts et Consignations, Crédit Lyonnais, Crédit Agricole-Indosuez, Crédit Industriel et Commercial, Natexis-Banques Populaires, EDF, GDF and the Summit Society.

See <http://cermics.enpc.fr/~premia>

### 7.2. EDF: hedging issues for options on electricity markets

**Participants:** S. Njoh, D. Lamberton.

Cifre agreement between the University of Marne la Vallée and the EDF company.

### 7.3. CIC: Calibration by Monte-Carlo methods

**Participants:** L. Nguyen, B. Jourdain, B. Lapeyre.

Cifre agreement ENPC-CIC.

### 7.4. CAI: Interest rate models with stochastic volatility stochastique

**Participants:** S. Hémon, D. Lamberton, M.C. Quenez.

Cifre agreement between University of Marne la Vallée and CAI.

## 7.5. International Relations

Financial Mathematic Joint Projet, French-Russian Institute Liapunov. Scientific leaders A Sulem and A. Shiryaev.

Web Site : <http://www-direction.inria.fr/international/liapunov.html>

## 9. Dissemination

### 9.1. Seminars organisation

- B. Jourdain, M.C. Kammerer-Quenez : organization of the seminar on stochastic methods and finance, Marne-la-Vallée
- J. Da Fonseca, E. Gobet, B. Jourdain, E. Temam, B. Lapeyre : course "Monte-Carlo methods in finance", june 16-17
- J. Printems: organization of a seminar on numerical aspects of calibration in finance based at INRIA<sup>1</sup>.

### 9.2. Teaching

- B. Arouna
  - Assistant teaching at the University of Marne La Vallée. (for 2nd year university students in Mathematics and Physics)
  - Assistant teaching in C programming at the University of Marne La Vallée for DEA (doctoral program).
- V. Bally, B. Jourdain, M.C.Kammerer-Quenez
  - course on "Mathematical methods for finance", 2nd year ENPC.
- B. Jourdain :
  - course on "Probability theory", first year ENPC
  - course on "Probabilistic tools in finance", 2nd year ENPC
- J.F. Delmas, B. Jourdain : course on "Random models", 2nd year ENPC
- M. Delasnerie, B. Jourdain, H. Regnier : course on "Monte-Carlo methods in finance", DEA Probabilités et Applications, university Paris VI
- J.P. Chancelier, B. Jourdain, A. Sulem : course "Numerical methods for financial models", DEA doctoral program Analyse et Systemes Aléatoires, university of Marne-la-Vallée.
- D. Lamberton :
  - Course undergraduate students in economy and mathematics, University of Marne-la-Vallée.
  - Course on "Stochastic calculus and applications in finance", graduate program, University of Marne-la-Vallée.
- B. Lapeyre
  - Course on Modelisation and Simulation, ENPC.
  - Projects and courses in Finance, Majeure de Mathématiques Appliquées, École Polytechnique.
  - Course on Monte-Carlo methods and stochastic algorithms, doctoral program in Random analysis and systems, University of Marne la Vallée.
- D. Lefèvre

<sup>1</sup>See [http://www.univ-paris12.fr/www/labos/cmup/homepages/printems/INRIA\\_gt.html](http://www.univ-paris12.fr/www/labos/cmup/homepages/printems/INRIA_gt.html)

- i. "Simulation and Applied Stochastic Processes in Discrete Time", Master of Science (1st year), Spring 2003, University of Evry Val d'Essonne,
  - ii. "Financial Mathematics", Licence in Economics (3rd year), Spring 2003, University of Evry Val d'Essonne,
  - iii. "Stochastic Calculus", Master of Financial Engineering (2nd year), Winter 2003, University of Evry Val d'Essonne,
  - iv. "Simulation and Engineering Probability and Statistics", Master of Science (1st year), Winter 2003, University of Evry Val d'Essonne.
- M. Mnif  
Statistics for undergraduate students, University Paris7.
  - N. Moreni  
- March to June 2003: Teaching Assistant, University of Marne-la-Vallée, Mathematics, DEUG Science and Technology, 1st year of undergraduate program.
  - M.C. Quenez  
- Courses for undergraduate students in mathematics, Université Marne la Vallée  
- Course on recent mathematical developments in finance, graduate program, University of Marne-la-Vallée,  
- Introductory course on financial mathematics, ENPC.
  - A. Sulem  
- Course on numerical methods in finance, DEA MASE and EDPA doctoral program , University Paris 9 Dauphine
  - E. Temam  
- "Numerical finance", ENSTA  
- "Stochastic process applied to Continuous finance", exercices, University Paris 7.

### 9.3. PhD defences

- M. Mnif, Université Paris 7, January 6th 2003.  
"Stochastic control with constraints and application in insurance"  
Advisors: H. Pham and A Sulem
- L. Nguyen, ENPC, December 18, 2003.  
"Calibration via relative entropy minization and jump models".  
Advisor: B. Jourdain
- S. Njoh, Université Marne la Vallée, July 2003, CIFRE agreement with EDF.  
"Hedging issues for options on electricity markets"  
Advisor: D. Lamberton

## 9.4. PhD advising

- V. Bally and E. Gobet (Ecole Polytechnique, CMAP)
  - Stephane Menozzi, Université Paris VI. (2nd year)
  - "Approche numérique probabiliste des problèmes d'EDPs avec bord"
- V. Bally and D. Lamberton
  - Ahmed Kbaier (2nd year), Grant of Université de Marne-la-Vallée.
  - " Approximation of SDE"
- V. Bally and A. Sulem
  - M.P. Bavouzet (2nd year), Grant Université Paris Dauphine and INRIA.
  - "Malliavin calcul with jumps and application in Finance".
- B. Jourdain
  - Laurent Nguyen (defended in December 2003), (CIFRE agreement CIC-ENPC).
  - "Calibration via relative entropy minimization and jump models",
  - Aurélien Alfonsi: (1st year), ENPC
  - "Stable calibration of asset models"
- D. Lamberton
  - Samuel Njoh , Université Marne la Vallée, Cifre agreement with EDF, defended in July 2003.
  - "Hedging issues for options on electricity markets"
  - Etienne Chevalier (3rd year), Grant Université de Marne-la-Vallée.
  - "Exercice boundaries for American options"
- D. Lamberton and M.C. Quenez
  - Sandrine Hénon (3rd year), Cifre agreement with Crédit Agricole Indosuez, Université Marne la Vallée.
  - "Modelling interest rates with stochastic volatility"
- D. Lamberton and Gilles Pagès
  - Vincent Lemaire (3rd year) , Grant Université de Marne-la-Vallée.
  - "Approximation of the invariant measure of a diffusion by a Euler scheme with decreasing steps"
- B. Lapeyre
  - Bouhari Arouna, ENPC, 3rd year " Stochastic algorithms, variance reduction"
  - Nicola Moreni, ENPC, 1st year :
  - "Intégrales de chemin et méthodes de Monte-Carlo en finance"
  - Ralf Laviollette (2nd year), ENS Cachan.
  - "Convergence rate of approximation schemes of SDE for trajectory functionals"
- J.F. Delmas and B. Lapeyre
  - Julien Guyon, ENPC 1st year:
  - "Convergence rate in Euler schemes for stochastic differential equations with jumps".
- M.C. Quenez
  - Benoit Jottreau,
  - "Problèmes de risque de défaut et modèles de taux d'intérêt".
- A. Sulem
  - David Lefèvre (4th year), Université Paris-Dauphine
  - "Utility maximisation in partial observation"
  - Marouen Messaoud (2th year), Université Paris-Dauphine
  - "Stochastic control, Calibration and Malliavin calculus with jumps"
  - Youssef Elouerkhaoui: (UBS Londres, Citibank from November)(2nd year)
  - "Incomplete issues in credit markets"

- A. Sulem and P. Carpentier (ENSTA)  
Cyrille Strugarek, Cifre agreement ENPC–EDF, 1st year.  
"Optimisation of portfolio of energy and financial assets in the electricity market"

## 9.5. Internship advising

- B. Jourdain
  - Yael Beer-Gabel, "Weighted Monte-Carlo calibration of asset models", scientific training period ENPC (April to June).
  - Benoît Le, "Computation of basket option prices using sparse tensor products", PFE Institut Galilée, University Paris 13 (October to December).
- B. Lapeyre
  - Mougad Oumgari, DEA University Marne la Vallée.
  - Gildas Gbaguidi, DEA University Marne la Vallée.
- A. Sulem
  - Nicolas Boulanger (with F. Bonnans, projet Sydoco and R. Aid EDF): Optimisation of hydraulic energy with risk constraints, (Mastere ENS Ulm and DEA Paris IX Dauphine)

## 9.6. Participation to workshops, conferences and invitations

- B. Arouna
  - Talk on "'An adaptive Monte Carlo method and application in finance", seminar on Stochastic methods in Finance, February 2003, University of Marne la Vallée.
- V. Bally
  - February 2003: invited talk on "The quantization method for pricing and hedging American options" in the Swiss Seminar of Probabilities in Geneva.
  - April 2003: invited talk at the University Tor Vergata Rome, and work with Lucia Caramellino , Fabio Antonelli and Sergio Scarlatti.
  - May 2003 : invited by the University of Florence to give a course on the Monte Carlo Methods.
  - August 2003: invited to the Universidad Nacional Autonoma de Mexico to work with Begonia Fernandez and Anna Mela.
  - September 2003: short course (6 hours) on "Interest Rate Theory" in a summer school organized by the Rumanian Institute of Mathematics, in Bucharest.
  - September 2003: participation to the congress of Potential Tehory organised in Bucharest by the Roumanian Institute of Mathematics. talk on "Lower bounds for the density of locally elliptic Ito processes".
  - October 2003: talk on "Lower bounds for the density of locally elliptic Ito processes" at the University of Nanterre and the University of Marne la Vallee.
- Y. Elouerkhaoui
  - "A New Technique for Measuring Default Correlation in a Conditional Jump Diffusion Framework", ICBI Global Derivatives & Risk Management, Barcelona, 21-22 May 2003.
  - "Conditional Jump Diffusion Principles to Select the Appropriate Copula and Calibrate it", ICBI Annual Credit Risk Forum, Mandelieu, Cannes, 25-26 June 2003.
- B. Jourdain
  - Blaise Pascal international conference on financial modeling, July 1-3, Villiers le Mahieu.
  - "Calibration issues in finance", Dieudonne-Omega financial seminar, Nice, October 16

- D. Lambertson
  - Sur l'utilité des bandits en finance. Seminaire Bachelier, Paris, March 2003.
  - Approximation de mesures invariantes. Journées de Probabilités. Toulouse, September 2003.
- B. Lapeyre
  - invited to the "Blaise Pascal international conference on financial modeling", July 1-3, Villiers le Mahieu.
- D. Lefèvre
  - January 31, 2003, University of Marne-la-Vallée, Seminar : "On the relation between deterministic and stochastic optimal control theory".
- Nicola Moreni
  - September 2003: Poster Session of "A non-linear World: the Real World", Second International Conference on Frontier science, Pavia.
- M.C. Quenez
  - talk on "portfolio optimisation with a priori multiple probabilities", Aquila conference on mathematical and finance, May 2003 and university of Evry, March 2003.
  - talk on "Reflected Quadratic Backward SDE and application to American option and pricing with a priori multiple probabilities", Bachelier seminar, IHP, Paris.
  - Three talks on interest rate, seminar "Stochastic methods and finance", University of Marne-La-Vallée, May-June 2003 .
- J. Printems
  - International Conference Blaise Pascal, July 2003, Villiers-le-Maheu.
  - Seminar of Applied Mathematics at the University of Clermont-Ferrand, September 2003.
  - Seminar of Mathematics at the University of Rennes, October 2003.
- A. Sulem
  - Invited talk on "risk sensitive impulse ergodic control" for the conference on *Stochastic Processes and Applications to Mathematical Finance*, Mars 2003 , BKC, Ritsumeikan University, Kusatsu, Japan.
  - Invited talk for the seminar in the honour of A. Bensoussan, April 2003, Université Paris-Dauphine.
  - First opponent of the PhD thesis of V.Zakamouline on "Portfolio Optimisation with transaction costs ", NHH, Norwegian School of Economics and Business Administration, Bergen, September 2003.
- E. Temam
  - Empirical semi-groups and calibration: May, 16th, 2003: Séminaire Bachelier
  - March, 20th, 2003: Working group of the Mathfi project
  - September, 17th, 2003: International Workshop "Differential Systems and Financial Mathematics": Calibration Problem
  - October, 9th, 2003: working group "Numerical Probabilities", University Paris 6

## 9.7. Miscellaneous

- D. Lambertson
  - "Associate Editor" of the journal *Mathematical Finance*.
  - Head of the doctoral program DEA "Stochastic Analysis and Systems", Universities of Marne-la-Vallée, Créteil, Evry and ENPC).



## 10. Bibliography

### Major publications by the team in recent years

- [1] M. AKIAN, J.L. MENALDI, A. SULEM. *On an Investment-Consumption model with transaction costs*. in « SIAM J. Control and Optim. », number 1, volume 34, January, 1996, pages 329-364.
- [2] E. CLÉMENT. *Pseudo-moderate deviations in the Euler method for real diffusion processes*. in « Stochastics and Stochastics Reports », number 1-2, volume 72, 2002, pages 109-127.
- [3] B. JOURDAIN, C. MARTINI. *American prices embedded in European prices*. in « Annales de l'IHP, analyse non linéaire », number 1, volume 18, 2001, pages 1-17.
- [4] D. LAMBERTON. *Brownian Optimal Stopping and Random Walks*. in « Applied Mathematics and Optimization », volume 45, 2002, pages 283-324.
- [5] D. LAMBERTON. *Error Estimates for the Binomial Approximation of American Put Options*. in « Annals of Applied Probability », number 1, volume 8, 1998, pages 206-233.
- [6] D. LAMBERTON, B. LAPEYRE. *Une introduction au calcul stochastique appliqué à la finance*. edition Collection Mathématiques et Applications, Ellipses, 1992, traduction anglaise: *An introduction to stochastic calculus applied to finance*, Chapman and Hall, 1996.
- [7] D. LAMBERTON, L.C.G. ROGERS. *Optimal stopping and embedding*. in « Journal of Applied Probability », volume 37, 2000, pages 1143-1148.
- [8] B. LAPEYRE, E. PARDOUX, R. SENTIS. *Introduction aux méthodes de Monte-Carlo*. edition Collection Mathématiques et Applications, Springer Verlag, 1997.
- [9] B. LAPEYRE, E. TEMAM. *Competitive Monte-Carlo Methods for the Pricing of Asian Options*. in « Journal of Computational Finance », number 1, volume 5, 2001, pages 39-57.
- [10] D. LEFÈVRE. *An introduction to Utility Maximization with Partial Observation*. in « Finance », volume 23, 2002, <http://www.inria.fr/rrrt/rr-4183.html>.
- [11] B. ØKSENDAL, A. SULEM. *Optimal Consumption and Portfolio with both fixed and proportional transaction costs*. in « SIAM J. Control and Optim. », number 6, volume 40, 2002, pages 1765–1790.
- [12] E. GOBET, E. TEMAM. *Discrete time hedging errors for options with irregular payoffs*. in « Finance and Stochastics », number 3, volume 5, 2001, pages 357-367.

### Books and Monographs

- [13] B. LAPEYRE, A. SULEM, D. TALAY. *Understanding Numerical Analysis for Option Pricing*. Cambridge University Press, to appear.

## Doctoral dissertations and “Habilitation” theses

- [14] M. MNIF. *Quelques applications du contrôle stochastique en finance et en assurance*. Ph. D. Thesis, Université Paris 7, January, 2003.
- [15] L. NGUYEN. *Calibration via relative entropy minimization*. Ph. D. Thesis, ENPC, December, 2003.
- [16] S. NJOH. *Hedging issues for options on electricity markets*. Ph. D. Thesis, University Marne la Vallée, July, 2003.

## Articles in referred journals and book chapters

- [17] B. AROUNA. *Robbins-Monro algorithms and Variance reduction in finance*. in « Journal of Computational Finance », number 2, volume 7, Winter 2003/2004.
- [18] V. BALLY. *The Central Limit Theorem for a nonlinear algorithm based on quantization*. in « Proceedings of the Royal Society », <http://www.inria.fr/rrrt/rr-4629.html>, to appear.
- [19] V. BALLY, G. PAGÈS. *A quantization algorithm for solving multidimensional discrete-time Optimal Stopping problems*. in « Bernoulli », number 6, volume 9, December, 2003.
- [20] V. BALLY, G. PAGÈS. *Error analysis of the quantization algorithm for obstacle problems*. in « Stoch. Processes and their Applications », number 1, volume 106, 2003, pages 1–40.
- [21] V. BALLY, G. PAGÈS, J. PRINTEMS. *First order schemes in the numerical quantization method*. in « Mathematical Finance », number 1, volume 13, January, 2003, pages 1–16.
- [22] V. BALLY, E. PARDOUX, L. STOICA. *Backward stochastic differential equations associated to symmetric Markov processes*. in « Potential Theory », <http://www.inria.fr/rrrt/rr-4425.html>, to appear.
- [23] V. BALLY, B. SAUSSEREAU. *A compactness criterion on the Sobolev-Wiener space and application to stochastic PDE's*. in « Functional Analysis », to appear.
- [24] F. BIAGINI, B. ØKSENDAL, A. SULEM, N. WALLNER. *An Introduction to white noise theory and Malliavin Calculus for Fractional Brownian Motion*. in « Proc. Royal Society », number 2041, volume 460, 2004, special issue on stochastic analysis and applications.
- [25] J. CVITANIC, A. LAZRAC, M. QUENEZ, F. ZAPARETO. *Incomplete Information with recursive preferences*. in « International Journal of Theoretical and Applied Finance », to appear.
- [26] N. C. FRAMSTAD, B. ØKSENDAL, A. SULEM. *A maximum principle for stochastic control of jump diffusions*. in « Journal of Optimization Theory and Applications », forthcoming.
- [27] V. GENON-CATALOT, T. JEANTHEAU, C. LAREDO. *Consistency of conditional likelihood estimators for stochastic volatility models*. in « Scandinavian Journal of Statistics », number 2, 2003, pages 297–316.

- [28] Y. HU, B. ØKSENDAL, A. SULEM. *Optimal consumption and portfolio in a Black-Scholes market driven by fractional Brownian motion*. in « Infinite Dimensional Analysis, Quantum Probability and Related Topics », number 4, volume 40, 2003, pages 519–536.
- [29] T. JEANTHEAU. *A link between complete models with stochastic volatility and ARCH models*. in « Finance and stochastics », to appear.
- [30] D. LAMBERTON, B. LAPEYRE, A. SULEM. *Application of Malliavin Calculus to Finance*. in « special issue of Mathematical Finance », January, 2003.
- [31] D. LAMBERTON, G. PAGÈS, P. TARRÈS. *When can the two-armed Bandit algorithm be trusted?*. in « Annals of Applied Probability », to appear.
- [32] D. LAMBERTON, S. VILLENEUVE. *Critical price near maturity for an American option on a dividend paying stock*. in « Annals of Applied Probability », volume 13, 2003, pages 800–815.
- [33] A. LAZRAK, M.C. QUENEZ. *Generalized Stochastic Differential Utility*. in « Math. and oper. research », number 1, volume 28, 2003, pages 154–180.
- [34] M. N’ZI, Y. OUKNINE, A. SULEM. *Regularity and representation of viscosity solutions of PDEs via BSDEs*. in « Stochastic processes and their applications », accepted for publication.
- [35] E. TEMAM. *Analysis of error with Malliavin Calculus: application to hedging*. in « Mathematical Finance », number 1, volume 13, 2003, pages 201–214.
- [36] N. EL KAROUI, S. PENG, M. QUENEZ. *A dynamic maximum principle for the optimization of recursive utilities under constraints*. in « Annals of Applied Probability », to appear.

## Publications in Conferences and Workshops

- [37] M. C. QUENEZ. *Optimal portfolio in a multiple-priors model*. in « Proceedings Ascona Conference 2002 », to appear.
- [38] B. ØKSENDAL, A. SULEM. *Partial observation control in an anticipating environment*. in « Proceedings of the Kolmogorov Conference », Moscou, 2003.

## Internal Reports

- [39] V. BALLY. *An elementary introduction to Malliavin calculus*. Research Report, number 4718, Inria, Rocquencourt, February, 2003, <http://www.inria.fr/rrrt/rr-4718.html>.
- [40] V. BALLY. *Lower bounds for the density of locally elliptic diffusion processes*. Research Report, number 4887, Inria, Rocquencourt, August, 2003, <http://www.inria.fr/rrrt/rr-4887.html>.
- [41] V. BALLY, L. CARAMELLINO, A. ZANETTE. *Pricing and hedging American options by Monte Carlo method using a Malliavin calculus approach*. Research report, number RR 4804, INRIA, april, 2003, <http://www.inria.fr/rrrt/rr-4804.html>.

- [42] V. BALLY, E. TEMAM. *Empirical semigroups and calibration*. Research report, number RR 4873, INRIA, July, 2003, <http://www.inria.fr/rrrt/rr-4873.html>.
- [43] T. BIELECKI, J.PH. CHANCELIER, S. PLISKA, A. SULEM. *Risk sensitive portfolio optimisation with transaction costs*. Research report, Cermics, ENPC, Champs sur Marne, November, 2003, <http://cermics.enpc.fr/reports/CERMICS-2003/CERMICS-2003-258.pdf>, under revision for Computational Finance.
- [44] M. MNIF, A. SULEM. *Optimal risk control and dividend pay-outs under excess of loss reinsurance*. Research report, number RR 5010, INRIA, November, 2003, <http://www.inria.fr/rrrt/rr-5010.html>.
- [45] N. MORENI. *Pricing American Options: A Variance reduction Technique for the Longstaff-Schwartz Algorithm*. Research report, number 2003-256, CERMICS/ENPC, Novembre, 2003.

## Miscellaneous

- [46] B. AROUNA. *Adaptative Monte Carlo Method, A Variance Reduction technique*. submitted to Monte Carlo Methods and Applications.
- [47] V. BALLY, B. FERNANDEZ, A. MELA. *Balls evaluations for locally elliptic diffusion processes*. working paper.
- [48] Y. ELOUERKHAOUI. *Correlated Defaults: the Conditional Jump Diffusion Framework*. Working Paper.
- [49] Y. ELOUERKHAOUI. *Credit Derivatives: Basket Asymptotics*. Working Paper.
- [50] Y. ELOUERKHAOUI. *Credit Risk: Correlation with a Difference*. Working Paper, 2003, Submitted to Risk Magazine.
- [51] M. KOBYLANSKI, M. C. QUENEZ, E. ROUY. *Multiple stopping time problems*. preprint, 2003.
- [52] D. LAMBERTON. *Approximation of the invariant measure of a diffusion: the case of a weakly mean reverting drift*. To appear in Stochastics and dynamics.
- [53] G. PAGÈS, H. PHAM, J. PRINTEMS. *An Optimal Markovian Quantization Algorithm for Multidimensional Stochastic Control Problems*. submitted, 2003.
- [54] G. PAGÈS, J. PRINTEMS. *Numerical aspects of quadratic functional quantization: pricing Asian options*. Working paper, 2003.
- [55] J. PRINTEMS, B. SAUSSEREAU. *Quadratic quantization and nonlinear filtering from a spectral approach*. Working paper, 2003.
- [56] B. ØKSENDAL, A. SULEM. *Applied Stochastic Control of Jump Diffusions*. Book project, Preliminary version (202 pages), Universitext, Springer Verlag.

## Bibliography in notes

- [57] M. AKIAN, A. SULEM, P. SÉQUIER. *A finite horizon multidimensional portfolio selection problem with singular transactions*. in « Proceedings CDC », pages 2193-2198, News Orleans, Décembre, 1995, Vol.3.
- [58] M. AKIAN, A. SULEM, M. TAKSAR. *Dynamic optimisation of long term growth rate for a portfolio with transaction costs - The logarithmic utility case*. in « Mathematical Finance », number 2, volume 11, Avril, 2001, pages 153–188.
- [59] M. AVELLANEDA, C. FRIEDMAN, R. HOLMES, D. SAMPIERI. *Calibrating volatility surfaces via relative-entropy minimization*. in « Appl. Math. Finance », number 4, 1997.
- [60] D. BELL. *The Malliavin Calculus*. series Pitman Monographs and Surveys in Pure and Applied Math., number 34, Longman and Wiley, 1987.
- [61] F. BIAGINI, Y. HU, B. ØKSENDAL, A. SULEM. *A stochastic maximum principle for processes driven by fractional Brownian motion*. in « Stochastic Processes and their applications », volume 100, 2002, pages 233 - 253, [http://www.math.uio.no/eprint/pure\\_math/2000/24-00.html](http://www.math.uio.no/eprint/pure_math/2000/24-00.html).
- [62] E. CLÉMENT. *Pseudo-moderate deviations in the Euler method for real diffusion processes*. in « Stochastics and Stochastics Reports », volume 72, 2002, pages 109-127.
- [63] E. CLÉMENT. *Inequalities of moderate deviation type in the Euler method for S.D.E. : the example of the geometric Brownian motion*. in « Stochastics and Stochastics Reports », volume 67, 1999, pages 287-307.
- [64] P. COHORT. *Monte–Carlo methods for Pricing American Style Options*. Technical report, 2001, part of the documentation of Premia 3.
- [65] D. DUFFIE, L.G. EPSTEIN. *Stochastic differential utility and asset pricing*. in « Econometrica », volume 60, 1992, pages 353-394.
- [66] H. FÖLLMER, M. SCHWEIZER. *Hedging of contingent claims under incomplete information. Applied Stochastic Analysis*. in « Stochastic Monographs », number 5, 1991, pages 389-414, (M.H.A. Davis and R.J. Elliott, eds), Gordon and Beach.
- [67] E. FOURNIÉ, J-M. LASRY, J. LEBUCHOUX, P-L. LIONS. *Applications of Malliavin calculus to Monte Carlo methods in Finance, II*. in « Finance & Stochastics », number 5, volume 2, 2001, pages 201-236.
- [68] E. FOURNIÉ, J-M. LASRY, J. LEBUCHOUX, P-L. LIONS, N. TOUZI. *An application of Malliavin calculus to Monte Carlo methods in Finance*. in « Finance & Stochastics », number 3, volume 4, 1999, pages 391-412.
- [69] N. FRAMSTAD, B. ØKSENDAL, A. SULEM. *Optimal Consumption and Portfolio in a Jump Diffusion Market*. in « proceedings du Workshop on Mathematical Finance », Inria, Paris, 1998, <http://www.nhh.no/for/dp/1999/index.htm>.

- [70] V. GENON-CATALOT, T. JEANTHEAU, C. LAREDO. *Stochastic volatility models as hidden Markov models and statistical applications*. in « Bernoulli », number 6, volume 6, 2000, pages 1051-1079.
- [71] V. GENON-CATALOT, T. JEANTHEAU, C. LAREDO. *Limit Theorems for Discretely Observed Stochastic Volatility Models*. in « Bernoulli », number 3, volume 4, 1998, pages 283-303.
- [72] V. GENON-CATALOT, T. JEANTHEAU, C. LAREDO. *Parameter Estimation for Discretely Observed Stochastic Volatility Models*. in « Bernoulli », number 5, volume 5, 1999, pages 858-872.
- [73] Y. HU, B. ØKSENDAL, A. SULEM. S. ET AL, editor, *Mathematical Physics and Stochastic Analysis*. World Scientific, 2000, chapter Optimal portfolio in a fractional Black & Scholes market, pages 267-279, [http://www.math.uio.no/eprint/pure\\_math/1999/13-99.html](http://www.math.uio.no/eprint/pure_math/1999/13-99.html), Essays in Honour of Ludwig Streit.
- [74] J. HULL, A. WHITE. *Efficient Procedures for Valuing European and American Path-Dependent Options*. in « Journal of Derivatives », volume 1, 1993, pages 21-31.
- [75] B. LAPEYRE, C. MARTINI. *The Premia project*. in « ERCIM news », 2000.
- [76] P. MALLIAVIN. *Stochastic calculus of variations and hypoelliptic operators*. in « Proc.Inter.Symp. on Stoch.Diff. Equations », Wiley 1978, pages 195-263, Kyoto, 1976.
- [77] D. NUALART. *The Malliavin Calculus and Related Topics*. Springer-Verlag, 1995.
- [78] D. OCONE, I. KARATZAS. *A generalized representation formula with application to optimal portfolios*. in « Stochastics and Stochastic Reports », volume 34, 1991, pages 187-220.
- [79] D. OCONE. *A guide to the stochastic calculus of variations*. in « Stochastic Analysis and Related Topics », Lecture Notes in Math.1316, H. KOERZLIOGLU, S. ÜSTÜNEL, editors, pages 1-79, 1987.
- [80] M. QUENEZ. N. KAROUI, L. MAZLIAK, editors, *Backward Stochastic Differential Equations*. edition Collection Pitman, series Research Notes in Mathematics Series, volume 364, Longman, 1997, chapter Stochastic control and BSDE's.
- [81] A. TISSEYRE. *Stabilité et Finance*. Ph. D. Thesis, Université Paris Dauphine, 1999.
- [82] B. ØKSENDAL. *An Introduction to Malliavin Calculus with Applications to Economics*. in « Lecture Notes from a course given 1996 at the Norwegian School of Economics and Business Administration (NHH) », September, 1996, NHH Preprint Series.
- [83] B. ØKSENDAL, A. SULEM. *Optimal Consumption and Portfolio with both fixed and proportional transaction costs: A Combined Stochastic Control and Impulse Control Model*. in « SIAM J. Control and Optim. », number 6, volume 40, 2002, pages 1765-1790.
- [84] J-PH. CHANCELIER, B. ØKSENDAL, A. SULEM. *Combined stochastic control and optimal stopping, and application to numerical approximation of combined stochastic and impulse control*. in « Stochastic Financial Mathematics », volume 237, Steklov Math. Inst., A. SHIRYAEV, editor, pages 149 –173, Moscou, 2002,

[http://www.math.uio.no/eprint/pure\\_math/2000/16-00.html](http://www.math.uio.no/eprint/pure_math/2000/16-00.html).

- [85] J.PH. CHANCELIER, B. ØKSENDAL, A. SULEM. *Combined stochastic control and optimal stopping, and application to numerical approximation of combined stochastic and impulse control*. in « Stochastic Financial Mathematics ».
- [86] N. EL KAROUI, C. KAPOUDJIAN, E. PARDOUX, S. P. S., M.C.QUENEZ. *Reflected solutions of Backward SDE's and related obstacle problems for PDE's"*. in « The Annals of Probability », number 2, volume 25, 1997, pages 702-737.
- [87] N. EL KAROUI, E. PARDOUX, M-C.QUENEZ . L. ROGERS, D. TALAY, editors, *Numerical methods in Finance*. Cambridge University Press, 1997, chapter Reflected BSDE's and American options, pages 215-231.
- [88] N. EL KAROUI, M.C. QUENEZ. *Programmation dynamique et évaluation des actifs contingents en marché incomplet*. in « C.R.Acad.Sci.Paris », volume 331, 1991, pages 851-854.
- [89] N. EL KAROUI, M.C. QUENEZ. *Dynamic programming and pricing of a contingent claim in an incomplete market*. in « SIAM Journal on Control and optimization », number 1, volume 33, 1995, pages 29-66.
- [90] N. EL KAROUI, M.C. QUENEZ. *Non-linear Pricing Theory and Backward Stochastic Differential Equations*. in « Financial Mathematics », series Lectures Notes in Mathematics, volume 1656, Springer, W.J. Runggaldier, editor, 1997, Bressanone,1996.
- [91] N.C. FRAMSTAD, B. ØKSENDAL, A. SULEM. *Optimal Consumption and Portfolio in a Jump Diffusion Market with Proportional Transaction Costs*. in « Journal of Mathematical Economics », volume 35, 2001, pages 233-257, [http://www.math.uio.no/eprint/pure\\_math/2000/26-00.html](http://www.math.uio.no/eprint/pure_math/2000/26-00.html).
- [92] N.EL KAROUI, S. PENG, M.C. QUENEZ. *Backward Stochastic Differential Equations in Finance*. in « Mathematical Finance », number 1, volume 7, January, 1997, pages 1-71.