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Project-Team Miaou

Mathématiques et Informatique de l'Automatique et de l'Optimisation pour l'Utilisateur

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THEME 4A

Activity Report

2003

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1. Team

Head of project team
Laurent Baratchart [DR INRIA]

Vice-head of project team
Juliette Leblond [CR INRIA]

Administrative assistant
France Limouzis [AI INRIA, partial time in the project]

Staff member
José Grimm [CR INRIA]
Martine Olivi [CR INRIA]
Jean-Baptiste Pomet [CR INRIA]
Fabien Seyfert [CR INRIA]
Franck Wielonsky [IR, on leave to the Université des Sciences et Technologies de Lille]

Ph. D. Students
David Avanessoff [Fellow, INRIA]
Fehmi Ben Hassen [Co-advised, ENIT Tunis]
Alex Bombrun [Since November, 1st]
Imen Fellah [Co-advised, ENIT Tunis (in France in February and March)]
Reinhold Küstner [Fellow, TMR, until May]
Moncef Mahjoub [Co-advised, ENIT Tunis (in France in February, March, October, November)]

Scientific advisors
Andrea Gombani [LADSEB-CNR, Padova, Italy]
Jonathan Partington [University of Leeds, GB]
Edward Saff [Vanderbilt University, Nashville, USA]

Visiting scientists
Felipe Monroy [Universidad Autonoma Mexicana (Mexico), until December]
Vladimir Peller [Michigan State University, one month]
Vladimir Chetverikov [From 6 to 19, January 2003]
Bronislav Jakubczyk [From January, 26 to February, 13 2003]
Antoine Chaillet [Four months, starting April, 15]

Post-doctoral fellows
Per Enqvist [arrived April 2003, funded by a regional grant]
Mario Sigalotti [arrived November 2003, funded by a Marie Curie grant]

2. Overall Objectives

The project was terminated June the 30th, 2003. A proposal for a new project named APICS has been submitted to the steering comittee of Inria Sophia Antipolis.
The Team develops effective methods for modelling, identification and control of dynamical systems.

2.1.1. Research Themes

- Meromorphic and rational approximation in the complex domain, application to identification of transfer functions and matrices as well as singularity detection for 2-D Laplace operators. Development of software for frequency domain identification and synthesis of transfer matrices.
- Control and structure of non-linear systems: continuous stabilization, non-linear transformations (linearization, classification).

2.1.2. International and industrial partners

- Industrial collaborations with Alcatel-Space, Alcatel-R&I, CNES, IRCOM, Thomson-MX.
- Exchanges with CWI (the Netherlands), CNR (Italy), the Universities of Illinois (Urbana-Champaign), of South Florida (Tampa), of California (San Diego), of Alabama (Mobile), of Minnesota (Minneapolis), of Vanderbilt (Nashville), of Padova (Italy), of Beer Sheva (Israel), of Leeds (GB), of Maastricht and of Amsterdam (The Netherlands), of TU-Wien (Austria), of TFH-Berlin (Germany), of Kingston (Canada), of Szegëd (Hungary), of Colorado School of Mines, of CINVESTAV (Mexico), ENIT (Tunis), VUB (Belgium).
- The project is involved in a NATO Collaborative Linkage Grant (with Vanderbilt University and ENIT-LAMSIN), in the ACI "Obs-Crev" (with the Teams Caiman and Odyssée from Inria-Sophia Antipolis, among others), in the ERCIM "Working Group Control and Systems Theory", in the TMR-ERNSI and TMR-NCN European research networks.

3. Scientific Foundations

3.1. Identification and deconvolution

Let us first introduce the subject of Identification in some generality.

Abstracting in the form of mathematical equations the behavior of a phenomenon is a step called *modeling*. It typically serves two purposes: the first is to describe the phenomenon with minimal complexity for some specific purpose, the second is to *predict* its outcome. This is general practice in most applied sciences, be it for design, control or prediction, although it is generally thought of as yet another optimization problem.

As a general rule, the user imposes the model to fit a parameterized form that reflects one's own prejudice,knowledge of the underlying physical system, and the algorithmic effort consented. Looking for such a trade-off usually raises the question of approximating the experimental data by the prediction of the model when the latter is subject to external excitations assumed to be the cause of the phenomenon under study. The ability to solve this approximation problem, which is often non-trivial and ill-posed, often conditions the practical usefulness of a given method.

It is when the predictive potential of a model is to be assessed that one is led to *postulate* the existence of a *true* functional correspondence between data and observations, thereby entering the field of *identification* itself. The predictive power of a model can be expressed in various manners all of which are attempts to measure the difference between the true model and the observations. The necessity of taking into account the difference between the observed behavior and the computed behavior induces naturally the notion of *noise* as a corrupting factor of the identification process. This noise incorporates into the model, and can be handled in a deterministic mode, where the quality of an identification algorithm is its robustness to small errors. This notion is that of well-posedness in numerical analysis or stability of motion in mechanics. The noise however is often considered to be random, and then the true model is estimated by averaging the data. This notion allows approximate but reasonnably simple descriptions of complex systems whose mechanisms are not well known but plausibly antagonistic. Note that, in any case, some *assumptions* on the noise are required in order to justify the approach (it has to be small in the deterministic case, and must satisfy some independence and ergodicity properties in the stochastic case). These assumptions can hardly be checked in practice, so that the satisfaction of the end-user is the final criterion.

Hypothesizing an exact model also results in the possibility of choosing the data in a manner suited for identifying a specific phenomenon. This often interacts in a complex manner with the *local* character of the model with respect to the data (for instance a linear model is only valid in a neighborhood of a point).

We now turn to the activity of the team proper in identification. Although the subject, on the academic level, has been the realm of the stochastic paradigm for more than twenty years, it is in a deterministic approach to identification of linear dynamical systems (i.e. 1-D convolution processes) based on approximation in the complex domain, that the Team made perhaps its most original contributions. Naturally, the deep links stressed by the spectral theorem between time and frequency domains induce well-known parallels between function theory and probability, and the work of the Miaou-project can be partly recast from the stochastic viewpoint. However, the issue was rather tackled by translating the problem of identification into an inverse problem, namely the reconstruction, from boundary data, of an analytic function in a domain of the plane. For convolution equations in dimension one—that is, ordinary differential equations possibly in infinite dimensional spaces —such a translation is provided by the Fourier transform. For certain elliptic partial differential equations in dimension two, Identification is also connected to analytic continuation, but this time it is the form of the fundamental solution that introduces holomorphy, especially in the case of the Laplacian whose solutions are logarithmic potentials.

The data are considered without postulating an exact model, but we simply look for a convenient approximation to the data in a range of frequency representing the working conditions of the underlying system. A prototypical example that illustrates our approach is the harmonic identification of dynamical systems which is widely used in the engineering practice, where the data are the responses of the system to periodic excitations in its band-width. We look for a stable linear model that describes correctly the behavior in this band-width, although the model can be inaccurate at high frequencies (that can seldom be measured). In most cases, we also want this model to be rational, of suitable degree, either because this degree is imposed by the physical significance of the parameters, or because it must remain of reasonably low order to allow the efficient use of the model for control, estimation or simulation. Other structural constraints, arising from the physics of the phenomenon to be modeled, often superimpose on the model. Note that, in this approach, no statistics are used for the errors, which can originate from corrupted measurements or from the limitated validity of the linear hypothesis.

We distinguish between an identification step (called non-parametric in a certain terminology) that is provided with an infinite dimensional model, and an approximation step in which the order is reduced subject to certain specific constraints on the considered system. The first step typically consists, mathematically speaking, in reconstructing a function, analytic in the right half-plane, knowing its pointwise values on a portion of the imaginary axis, in other terms, to make the principle of analytic continuation effective on the boundary of the analyticity domain. This is a classical question which is ill-posed (inverse Cauchy problem for the Laplace equation) that we embed into a family of well-posed extremal problems. The second step is typically a rational or meromorphic approximation procedure (but approximating families other than rational functions may be considered) in a space of functions analytic in a simply connected open subset, say the right half-plane in the case of harmonic identification. To make the best possible use of the allowable number of parameters, or to priviledge some specific physical parameters of the system, it is generally important, in the second step, to compute optimal or nearly optimal approximants. Rational approximation in the complex plane is a classical and difficult problem, for which only few effective methods exist. In relation to system theory, two main difficulties arise: the necessity of controlling the poles of the approximants (to ensure the stability of the model), and the need to handle matrix-valued functions in the case where the system has several inputs and outputs.

Rational approximation in the L^p sense to a transfer function on the imaginary axis (i.e the boundary of the right half-plane) acquires a particular significance in this context for $p = 2$ and $p = \infty$. If $p = 2$, it corresponds to parametric identification of minimum variance when the system is fed with white noise input (the case of colored noise corresponds to weighted approximation), and it also corresponds to the minimization of the $L^2 \to L^\infty$ error in operator norm in the time domain. If $p = \infty$, the approximation consists in minimizing the power transfer $L^2 \to L^2$ of the error (both in the time and frequency domains for the Fourier transform is an isometry). These problems contribute a generalization (both rational and matrix-valued) of Szegö theory on orthogonal polynomials, that seems the most natural frame work for setting out many optimization problems related to linear system identification.

We shall explain in more detail the above two steps in the sub-paragraphs to come. For convenience, we shall approach them on the circle rather than the line, which is the framework for discrete-time rather than continuous-time systems. The two frameworks are mathematically equivalent *via* a Möbius transform.

3.1.1. Analytic approximation of incomplete boundary data

Participants: Laurent Baratchart, José Grimm, Birgit Jacob [University of Leeds (GB)], Juliette Leblond, Jean-Paul Marmorat [CMA, École des Mines], Jonathan Partington, Fabien Seyfert.

Key words: *meromorphic approximation*, *frequency-domain identification*, *extremal problems*.

The title refers to the construction of a convolution model of infinite dimension from frequency data in some bandwidth Ω and some reference gauge outside Ω. The class of models consists of stable transfer functions *i.e.* analytic in the domain of stability, be it the half-plane, the disk, etc), and possibly also transfer functions with finitely many poles in the domain of stability *i.e,* convolution operators corresponding to linear differential or difference equations with finitely many unstable modes. This issue arises in particular for the design and identification of linear dynamical systems, and in certain inverse problems for the Laplacian in dimension two.

Since the question under study may occur on the boundary of planar domains of various shapes when it comes to inverse problems, it is common practice to normalize this boundary once and for all, and to apply in each particular case a conformal transformation to bring back to the normalized situation. The normalized contour chosen here is the unit circle. We denote by D the unit disk, by H^p the Hardy space of exponent p, R_N is the set of all rational functions having at most N poles in D, and $C(X)$ is the set of continuous functions on X. We are looking for a function in $H^p + R_N$, taking on an arc K of the unit circle values that are close to some experimental data, and satisfying on $T \setminus K$ some gauge constraints, so that a prototypical Problem is:

(P) Let $p \geq 1$, $N \geq 0$, K be an arc of the unit circle T, $f \in L^p(K)$, $\psi \in L^p(T \setminus K)$ and $M > 0$; find a function $g \in H^p + R_N$ such that $\|g - \psi\|_{L^p(T \setminus K)} \leq M$ and such that $g - f$ is of minimal norm in $L^p(K)$ under this constraint.

In order to impose pointwise constraints in the frequency domain (for instance if the considered models are transfer functions of loss-less systems, see section 4.3.2), one may wish to express the gauge constraint on $T \setminus K$ in a more subtle manner, depending on the frequency (see section 6.7):

(P') Let $p \geq 1$, $N \geq 0$, K be an arc of the unit circle T, $f \in L^p(K)$, $\psi \in L^p(T \setminus K)$ and $M \in L^p(T \setminus K)$; find a function $g \in H^p + R_N$ such that $|g - \psi| \leq M$ a.e. on $T \setminus K$ and such that $g - f$ is of minimal norm in $L^p(K)$ under this constraint.

Problem (P) is an extension to the meromorphic case, and to incomplete data, of classical analytic extremal problems (obtained by setting $K = T$ and $N = 0$), that generically go under the name *bounded extremal problems*. These have been introduced and intensively studied by the Team, distinguishing the case $p = \infty$ [38] from the cases $1 \leq p < \infty$, among which the case $p = 2$ presents an unexpected link with the Carleman reconstruction formulas [35].

Deeply linked with Problem *(P)*, and meaningful for assessing the validity of the linear approximation in the considered pass-band, is the following completion Problem:

(P'') Let $p \geq 1$, $N \geq 0$, K an arc of the unit circle T, $f \in L^p(K)$, $\psi \in L^p(T \setminus K)$ and $M > 0$; find a function $h \in L^p(T \setminus K)$ such that $\|h - \psi\|_{L^p(T \setminus K)} \leq M$, and such that the distance to $H^p + R_N$ of the concatenated function $f \vee h$ is minimal in $L^p(T)$ under this constraint.

A version of this problem where the constraint depends on the frequency is:

(P''') Let $p \geq 1$, $N \geq 0$, K an arc the unit circle T, $f \in L^p(K)$, $\psi \in L^p(T \setminus K)$ and $M \in L^p(T \setminus K)$; find a function $h \in L^p(T \setminus K)$ such that $|h - \psi| \leq M$ a.e. on $T \setminus K$, and such that the distance to $H^p + R_N$ of the concatenated function $f \vee h$ is minimal in $L^p(T)$ under this constraint.

Let us mention that Problem *(P'')* reduces to Problem *(P)* that in turn reduces, although implicitly, to an extremal Problem without constraint, (i.e. a Problem of type *(P)* where $K = T$) that is denoted conventionally by *(P_0)*. In the case where $p = \infty$, Problems *(P')* and *(P''')* can viewed as special cases of *(P)* and *(P'')* respectively, but if $p < \infty$ the situation is different. On can also chose different exponents p on K and $T \setminus K$ (the Problem is then said to be of mixed type), and this comes up naturally when identifying lossless systems where the constraint $|h| \leq 1$ must hold at each point while the data, whose signal-to-noise ratio is small on the ends of the bandwidth, are better approximated in the L^2 sense. Mixed Problems have begun to be studied within the Team *cf.* module 6.7. One has to stress the perhaps counter-intuitive fact that these have in general no solution if unless the gauge constraintis accounted for, that is, if one sets formally $M = +\infty$. For instance, considering Problem *(P'')*, a function given by its trace on a subset K of positive measure on the unit circle can always be extended in such a manner as to be arbitrarily close, on K, to a function analytic in the disk; however, it goes to infinity in norm on $T \setminus K$ when the approximation error goes to zero, unless we are in the ideal case where the initial data are *exactly* the trace on K of an analytical function. The phenomenon illustrates the ill-posedness of the analytic continuation on the boundary of the analyticity domain.

The solution to *(P_0)* is classical if $p = \infty$: it is given by the Adamjan-Arov-Krein (in short: AAK) theory. If $p = 2$ and $N = 0$, then *(P_0)* reduces to an orthogonal projection. AAK theory plays a great rôle in showing the existence and uniqueness of the solution to *(P'')* when $p = \infty$, under the assumption that the concatenated function $f \vee \psi$ belongs to $H^\infty + C(T)$, and to compute this solution by solving iteratively a spectral problem relative to a family of Hankel operators whose symbols depend implicitly from the data. The robust convergence of this algorithm in separable Hölder-Zygmund classes has been established [37]. In the Hilbertian case $p = 2$, again for $N = 0$, the solution of *(P)* is obtained by solving a spectral equation, this time for a Toeplitz operator, depending linearly on a parameter λ that plays the rôle of a Lagrange multiplier and makes the dependence of the solution implicit in M. The ill-posed character of the analytic continuation described above is to the effect that, if the data are not exactly analytic, the approximation error on K tends to 0 if, and only if, the constraint M on $T \setminus K$ goes to infinity [35]. This phenomenon can be quantified in Sobolev or meromorphic classes of functions f, and asymptotic estimates of the behavior of M and of the error respectively can be obtained, based on a constructive diagonalization scheme for Toeplitz operators due to Rosenblum and Rovnyak, that makes the spectral theorem effective [12]. These results indicate that the error decreases much faster, as M increases, if the data have a holomorphic extension to a neighborhood of the unit disk, this being conceptually interesting for discriminating between nearly analytic data and those that are not close to a linear stable model; from the point of view of effective computing arises the problem of representing the functions through expansions that are specifically adapted to the underlying geometry, for instance, rational bases whose poles cluster at the endpoints of K. Research in this direction is in its infancy.

We emphasize that *(P)* has many analogs, equally interesting, that occur in different contexts connected to conjugate functions. For instance one may consider the following extremal Problem, germane to Problem *(P)*, in the Hilbertian context $p = 2$ and $N = 0$, where the constraint on the approximant is expressed in terms of its imaginary part:

Let $f \in L^2(K)$, $\psi \in L^2(T \setminus K)$ and $M > 0$; find a function $g \in H^2$ such that $\|Img - \psi\|_{L^2(T \setminus K)} \leq M$ and such that $g - f$ has minimal norm in $L^2(K)$.

Existence and uniqueness of the solution have been established in [57] as well as the foundations of a constructive procedure to solve for this Problem. Note that, to the Toeplitz operator that characterizes the solution of *(P)* when $p = 2$ and $N = 0$, is superimposed here a Hankel operator, see section 6.8 for still another extension. This type of constraints is particularly suited to inverse problems for the Laplacian, cf sections 4.2 and 6.9, where one does not know the real part of the solution on the boundary (for instance because of local measurements of temperature or electrical potential).

In the non-Hilbertian case, where $p \neq 2, \infty$, but still $N = 0$, the solution of *(P)* can be deduced from that of *(P_0)* in an manner analogous to the case $p = 2$, though the situation is a bit more tricky concerning duality, because one remains in a convex setup (in infinite dimension, of course), for which local optimization methods can be applied.

If $p < \infty$ and $N > 0$, there is up to now no algorithmic solution to Problem *(P_0)* which is proved convergent. However, the progress that were made allow us to conceive a coherent picture of the main issues and to develop rather efficient numerical schemes whose global convergence has been proved for prototypical classes of functions in Approximation theory. The essential features of the approach are summarized below.

First of all, in the case $p = 2$ and $N > 0$ which is of particular importance, Problem *(P_0)* can be reduced to that of rational approximation which is described in more details in section 3.1.2. Here, the link with classical interpolation theory, orthogonal polynomials, and logarithmic potentials is strong and fruitful. Second, a general AAK theory in L^p has been proposed which is relatively complete for $p \geq 2$ [43]. Although it does not have, for $p \neq \infty$, the computational power of the classical theory, it has better continuity properties and stresses a continuous link between rational approximation in H^2 (see section 3.1.2) and meromorphic approximation in the uniform norm, allowing one to use, in either context, the techniques available from the other. Hence, similar to the case $p = \infty$, the best meromorphic approximation with at most n poles in the disk of a function $f \in L^p(T)$ is obtained from the singular vectors of the Hankel operator of symbol f between the spaces H^s and H^2 with $1/s + 1/p = 1/2$, the error being here again equal to the $(n+1)$st singular number of the operator. This generalization has a strong topological nature and relies on the theory of critical points of Ljusternik-Schnirelman as well as on the particular geometry of the Blaschke products of given degree. Among the common features of this family of problems, the deepest one is perhaps the following: the critical point equations express non-Hermitian orthogonality of the denominator (i.e. the polynomial whose zeroes are the poles of the approximant) against polynomials of lower degree, for a complex measure that depends however on this denominator (because the problem is non-linear). This allows one to extend the index theorem to the case $2 \leq p \leq \infty$ [30] and to tackle the uniqueness problem, to study asymptotic errors, and also, combined with classical techniques of potential theory, to characterize the asymptotic behavior of the poles of the approximants for functions with connected singularities that are of particular interest for inverse problems (cf. section 3.1.3). In the light of these results, and although many questions remain open, one can expect algorithmic progress concerning (P_0) for $N > 0$ and $p \geq 2$ in the forthcoming years. As a consequence, the transition from (P_0) to (P) should follow the same lines as in the analytic case [63].

The case where $1 \leq p < 2$ remains largely open, especially from the constructive point of view, because if the approximation error can still be interpreted in terms of singular values, the Hankel operator takes an abstract form not permitting for a functional identification of its singular vectors. These values of p are not simply an academic exercise: the L^1 criterion induces the operator norm $L^\infty \to L^\infty$ in the frequency domain, which is interesting for damping perturbations. It is possible that some appropriate duality links the case $p < 2$ to the case $2 < p$, but this has not yet been established.

3.1.2. Scalar rational approximation

Participants: Laurent Baratchart, Reinhold Küstner, Juliette Leblond, Martine Olivi, Edward Saff, Herbert Stahl, Franck Wielonsky.

Key words: *rational approximation*, *critical point*, *orthogonal polynomials*.

Rational approximation is the second step mentioned in section 3.1 and we first approach it in the scalar case, for complex-valued functions (as opposed to matrix-valued ones). The Problem can be stated as:

Let $1 \leq p \leq \infty$, $f \in H^p$ and n an integer; find a rational function without poles in the unit disk, and of degree at most n that is nearest possible to f in H^p.

The most important values of p, as indicated in the introduction, are $p = \infty$ and $p = 2$. In the latter case, the orthogonality between Hardy spaces of the disk and of the complement of the disk (the last one being restricted to functions that vanish at infinity to exclude the constants) makes rational approximation equivalent to meromorphic approximation, i.e. we are back to Problem *(P)* of section 3.1.1 with $p = 2$ and $K = T$. Although no demonstrably convergent algorithm is known for a single value of p, the Miaou project has designed a steepest-descent algorithm for the case $p = 2$ whose convergence to a *local minimum* is guaranteed in theory, and it is the first satisfying this property. Roughly speaking, it is a gradient algorithm, proceeding recursively with respect to the order n of the approximant, that uses the particular geometry of the problem in order to restrict the search to a compact region of the parameter space [1]. This algorithm can determine several local *minima* if there are, thus allowing one to compare between them. If there is no local *maximum*, a property which is satisfied when the degree is large enough, it happens that every local *minimum* can be obtained from an initial condition of lower order. It is not proved, however, that the absolute *minimum* can always be obtained using the strategy of the hyperion or RARL2 software (*cf.* sections 5.1 and 5.3) that consists in choosing the collection of initial points corresponding to critical points of lower degree; note that we do not know of a counter-example either, still assuming that there is no *maximum*, so there is room for a conjecture at this point.

It is only fair to say that the design of a numerically efficient algorithm whose convergence to the best approximant would be proved is the most important problem from a practical perspective. However, the algorithms developed by the team seem rather effective and although their global convergence has not been established. *A contrario*, it is possible to consider an elimination algorithm when the function to approximate is rational, in order to find all critical points, since the problem is algebraic in this case. This method is surely convergent, since it is exhaustive, but one has to compute the roots of an algebraic system with n variables of degree N, where N is the degree of the function to approximate and there can be as many as N^n solutions among which it is necessary to distinguish those that are coefficients of polynomials having all their roots in the unit disk; the latter indeed are the only ones that generate critical points. Despite the increase of computing capacity, such a procedure is still unrealistic granted that realistic values of n and N would be like a tenth and a couple of hundreds (cf. section 4.3.2).

To prove or disprove the convergence of the above-described algorithms, and to check them against practical situations, the team has undergone a long-haul study of the number and nature of critical points, depending on the class of functions to be approximated, in which tools from differential topology and operator theory team up with classical approximation theory. The study of transfer functions of relaxation systems (*i.e.* Markov functions) was initiated in [6] and more or less completed in [44], as well as the case of e^z (the prototype of an entire function with convex Taylor coefficients) and the case of meromorphic functions (à la Montessus de Ballore) [5]. After these studies, a general principle has emerged that links the nature of the critical points in rational approximation to the regularity of the decrease of the interpolation errors with the degree, and a methodology to analyze the uniqueness issue in the case where the function to be approximated is a Cauchy integral on an open arc (roughly speaking these functions cover the case of singularities of dimension one that are sufficiently regular, *cf.* section 3.1.3) has been developed. This methodology relies on the localization of the singularities *via* the analysis of families of non-Hermitian orthogonal polynomials, to obtain strong estimates of the error that allow one to evaluate its relative decay. Note in this context an analogue of the Gonchar conjecture, that uniqueness ought to hold at least for infinitely many values of the degree. Another uniqueness criterion has been obtained [43] for rational functions, inspired from the spectral techniques of AAK theory. This result is interesting in that it is not asymptotic and does not require pointwise estimates of the error; however, it assumes a rapid decrease of the errors and the current formulation calls for further investigation.

The introduction of a weight in the optimization criterion is an interesting issue induced by the necessity to balance the information one has at the various frequencies. For instance in the stochastic theory, minimum variance identification leads to weight the error by the inverse of the spectral density of the noise. It is worth noting that most approaches to frequency identification in the engineering practice consists of posing a least-square minimization problem, and to weigh the terms so as to obtain a suitable result using a generic optimization toolbox. In this way we are led to consider minimizing a criterion of the form:

$$\left\| f - \frac{p_m}{q_n} \right\|_{L^2(d\mu)} \tag{1}$$

where, by definition,

$$\|g\|^2_{L^2(d\mu)} = \frac{1}{2\pi} \int_{-\pi}^{\pi} |g(e^{i\theta})|^2 d\mu(\theta),$$

and μ is a positive finite measure on T, p_m is a polynomial of degree less or equal to m and q_n a monic polynomial of degree less or equal to n. Such a problem is nicely put when μ is absolutely continuous with respect to the Lebesgue measure and has invertible derivative in L^∞. For instance when μ is the squared modulus of an invertible analytic function, introducing μ-orthogonal polynomials instead of the Fourier basis makes the situation similar to the non-weighted case, at least if $m \geq n-1$ [8]. The corresponding algorithm was implemented in the hyperion software (see section 5.1). The analysis of the critical points equations in the weighted case gives various counter-examples to unimodality in maximum likelihood identification [59].

Another kind of rational approximation, that arises in several design problems where only constraints on the modulus are seeked, consists of approximating the module of a function by the module of a rational function, that is, solving for

$$\min \left\| |f| - \left| \frac{p_n}{q_n} \right| \right\|_{L^p(T)}.$$

This problem is strongly related to the previous ones; in fact, it can be reduced to a convergent series of standard rational approximation problems. Note also that if $p = \infty$, and if moduli are squared, *i.e.* if the feasibility of

$$\left\| |f|^2 - \left| \frac{p_n}{q_n} \right|^2 \right\|_{L^\infty(T)} < \varepsilon,$$

is required, one can use the Féjèr-Riesz characterization of positive trigonometric polynomials on the unit as squared moduli of algebraic polynomials to approach this issue as a convex problem in infinite dimension. This constitutes another fundamental direction for dealing with rational approximation in modulus that arises naturally in filter design problems.

3.1.3. Asymptotic behavior of poles of meromorphic approximants and inverse problems for the Laplacian

Participants: Laurent Baratchart, Edward Saff, Herbert Stahl, Reinhold Küstner, Vilmos Totik [univ. Szeged and Scien. Acad., Hungary].

Key words: *singularity detection*, *free boundary inverse problems*, *meromorphic approximation*, *rational approximation*, *orthogonal polynomials*, *discretization of potentials*.

We want here to study the behavior of poles of optimal meromorphic approximants in L^p norm on a closed contour, to functions defined by Cauchy integrals of measures whose support lies inside the contour. If one normalizes the contour to be the unit circle, which is no restriction in principle thanks to conformal mapping but raises of course difficult questions from the constructive point of view for domains whose shape is not standard (for instance polygonal or elliptic), we find ourselves again in the framework of sections 3.1.1 and 3.1.2. The research so far has focused on functions that are analytic on and outside the contour, and have singularities on an open arc inside the contour.

Generally speaking, the behavior of poles is particularly important in meromorphic approximation for the analysis of the error decrease with the degree and for most constructive aspects like uniqueness, so that everything here could take place in section 3.1.1. However, it is the original motivation of the team to consider this issue in connection with the approximation of the solution to a Dirichlet-Neumann problem, so as to extract information on the singularities of that solution. This way to tackle a free boundary problem, classical in every respect but still widely open, illustrates the approach of the team to certain inverse problems, and gives rise to an active direction of research at the crossroads of function theory, potential theory and orthogonal polynomials.

As a general rule, critical point equations for these problems express that the polynomial whose roots are the poles of the approximant is a non-Hermitian orthogonal polynomial with respect to some complex measure on the singular set of the function to be approximated. New results were obtained over the last three years concerning the location of such zeroes, and the approach to inverse problem for the Laplacian that we outline in this section appears to be attractive when the singularities are one-dimensional, for instance in the case of a cracked domain (see section 4.2). In case the crack is sufficiently smooth, the approach in question is in fact equivalent to meromorphic approximation of a function with two branch points, and one has been able to prove [40][34] that the poles of the approximants accumulate in a neighborhood of the geodesic hyperbolic arc that links the enpoints of the crack [3]. Moreover the asymptotic density of the poles is nothing but the equilibrium distribution on the geodesic arc of the Green potential and it charges the end points, that are *de facto* well localized if one is able to compute sufficiently many zeros (this is where the method is not fully constructive). It is interesting to note that these results apply also, and even more easily, to the detection of monopolar and dipolar sources, a case where poles as well as logarithmic singularities exist. The case of more general cracks (for instance formed by a finite union of analytic arcs) requires the analysis of the situation where there the number of branch points is finite but arbitrary. It is conjectured that the poles tend to the contour $\mathcal{C}$ that links the end points of these analytic arcs while minimizing the capacity of the condenser $(T, \mathcal{C})$, where T is the exterior boundary of the domain (see section 6.6). The conjecture is confirmed numerically and has been actually proved in the case where the locus of minimal capacity is *connected*; this covers a large number of interesting cases, including the case of general polynomial cracks, or of cracks consisting of sufficiently smooth arcs. This breakthrough, we hope, will constitute a substantial progress towards a proof of the general case. It would of course be very interesting to know what happens when the crack is "absolutely non analytic", a limiting case that can be interpreted as that of an infinite number of branch points, and on which very little is known. Concerning the problem of a general singularity, in the light of what precedes, one can formulate the following conjecture: if f is analytic outside and on the exterior boundary of a domain D and if K is the minimal compact set included in D that minimizes the capacity of the condensor (T, K) under the constraint that f is analytic and single-valued outside K (it exists, it is unique, and we assume it is of positive capacity in order to avoid degenerated cases), then every limit point (in the weak star sense) of the sequence ν_n of probability measures having equal mass at each pole of an optimal meromorphic approximant (with at most n poles) of f in $L^p(T)$ *has its support in* K *and sweeps out to the boundary of* K *as the equilibrium measure on* K *of the condensor* (T, K). Yet this conjecture is far from being solved.

Results of this type open new perspectives in non-destructive control (see section 4.2), in that they link issues of current interest in approximation theory (the behavior of zeroes of non-Hermitian orthogonal polynomials) to some classical inverse problems for which it a dual approach is proposed: to approximate boundary conditions and not the equation. Note that the problem of finding a crack suggests non-classical variants of rational and meromorphic approximation where the residues of the approximants must satisfy some constraints in order to take into account the boundary conditions, normal or tangential, along the singularity. In fact, the afore-mentioned results dealing with (unconstrained) meromorphic approximation lead to identify a deformation of the crack (the arc of minimal capacity that links its end points) rather than the crack itself, which is valuable to initialize a heavier direct method but which is not conclusive by itself. In order to limit the deformation which is due to the fact that we did not keep track of the limiting-conditions (especially the fact that the jump across the crack is real), one may consider approximating the complexified solution F of a Neumann problem in a cracked domain D by a meromorphic function of the type $\sum_{j=1}^{n} a_j/(z - z_j) + g(z)$, where g is analytic in D, under the constraint that $\sum_{k \neq j} a_k/(z_j - z_k) + g(z_j)$ is real for each j; in effect, if the poles z_j are distributed along an arc, the above sum is a discrete estimation of the Hilbert transform of the measure defining the function, and enforcing that it is zero should help satisfying the Neumann condition along the arc. Such modifications of the initial problem are only beginning to be considered within the team.

We conclude by mentioning that the problem of approximating, by a rational or meromorphic function, in the L^p sense on the boundary of a domain, the Cauchy transform of a real measure, localized inside the domain, can be viewed as an optimal discretization problem for a logarithmic potential according to a criterion involving a Sobolev norm. This formulation can be generalized to higher dimensions, even if the computational power of complex analysis is no longer there, and this makes for a long-term research project with a much wider range of applicability. The case of sources in dimension three in a spherical geometry, can for instance, be attacked with the above 2D techniques when applied to planar sections (see section 6.9).

3.1.4. Matrix-valued rational approximation

Participants: Laurent Baratchart, Andrea Gombani, Martine Olivi, José Grimm.

Key words: *rational approximation*, *inner matrix*, *reproducing kernel space realization theory*.

Matrix-value approximation is necessary for handling systems with several inputs and outputs, and generates substantial additional difficulties with respect to scalar approximation, theoretically as well as algorithmically.

In the matrix case, the McMillan degree (*i.e.* the degree of a minimal realization in the System-Theoretic sense) generalizes the degree. Hence the problem reads: *Let $1 \leq p \leq \infty$, $\mathcal{F} \in (H^p)^{m\times l}$ and n an integer; find a rational matrix of size $m \times l$ without poles in the unit disk and of McMillan degree at most n nearest possible to $\mathcal{F}$ in $(H^p)^{m\times l}$.* To fix ideas, we may define the L^p norm of a matrix as the p-th root of the sum of the p powers of the norms of its entries.

The main interest of Miaou so far lies in the case $p = 2$. Then, the scalar approximation algorithm designed in the scalar case generalizes to the matrix-valued situation [7]. The first difficulty consists here in the parametrization of transfer matrices of given McMillan degree n, and the inner matrices (*i.e.* matrix-valued functions that are analytic in the unit disk and unitary on the circle) of degree n enter the picture in an essential manner: they play the role of the denominator in a fractional representation of transfer matrices using the so-called Douglas-Shapiro-Shields factorization. The set of inner matrices of given degree has a manifold structure that allows to use differential tools as in the scalar case. In practice, one has to produce a good atlas of charts (parameterizations valid in a neighborhood of a point), and one must handle changes of chart in the course of the algorithm. The tangential Schur algorithm [27] provides us with such a parameterization and allowed the team to develop two rational approximation codes. The first one is integrated in the hyperion software (see section 5.1) that oparates on transfer matrices, while the other is developed under the matlab interpreter, goes by the name of RARL2, and works with realizations. Both have been tested under contract against 2×2 matrix-valued data built from measurements done by the CNES (branch of Toulouse), IRCOM, and Alcatel Space, and are the object of sections 7.1 and 7.2. They give high quality results [2] in all cases encountered so far. These codes are of daily use by Alcatel space and IRCOM, coupled with simulation software like EMXD, to design physical coupling parameters for the synthesis of hyperfrequency filters made of resonant cavities.

In this application, obtaining physical couplings requires the computation of realizations, also called internal representation in system theory. Among the parameterizations obtained via the Schur algorithm, some have a particular interest from this viewpoint [62]. They lead to a simple and robust computation of balanced realizations and form the basis of the RARL2 algorithm.

Problems relative to multiple local minima are naturally also present as in the scalar case, but deriving criteria that guarantee uniqueness is much more difficult than in the scalar case. The case of rational functions of the proper degree already uses rather heavy machinery [4], and that of matrix-valued Markov functions, that are the first example beyond rational function has made progress only very recently (*cf.* section 6.5).

In practice, a method similar to the one used in the scalar case, has been developed to generate local minima at a given order from those at lower order. In short, one sets out a matrix of degree n by perturbation of a matrix of degree $n - 1$ where the drop in degree is due to a pole-zero cancellation. There is an important difference between polynomial representations of transfer matrices and their realizations: the former lead to an embedding in a ambient space of rational matrices that allows a differentiable extension of the criterion on a neighborhood of the initial definition manifold, but not the latter (the boundary is strongly singular). Generating initial conditions in a recursive way is more delicate in terms of realizations, and some basic questions on the boundary behaviour of the gradient vector field are still open.

Let us stress that the algorithms mentioned above are first to handle rational approximation in the matrix case in a way that converges to local minima, while meeting stability constraints on the approximant.

3.1.5. Linear parametric identification

Participants: Laurent Baratchart, Manfred Deistler [TU Wien, Au], Reinhold Küstner, Martine Olivi.

Key words: *rational approximation*, *parametric identification*, *topology of rational matrices*, *critical points*.

The asymptotic study of likelihood estimators is a natural companion to the research on rational approximation described above. The context is ultra-classical. Given a discrete process $y(t)$ with values in $\mathbf{R}^p$, and another process with values in $\mathbf{R}^m$, we check for an explanation of y in terms of u as a finite order linear model:

$$\hat{y}(t) = Hu(t) + Le(t),$$

where e is a white noise with p components, uncorrelated to u, assumed to represent the uncertainty in $y(t)$, and where the transfer matrix $[L \quad H]$ that links $(e \quad u)^t$ to $\hat{y}$ is rational and stable of McMillan degree n, the matrix L being also of stable inverse (among all noises with same covariance, and given innovation, we chose those whose spectral factor has minimum phase). The number n is, by definition, the order of the model. If we only suppose that $[H \quad L]$ belongs to the Hardy space H^2 and that L is outer (this means stably invertible in some sense), such a representation is in fact general for *regular* (i.e. purely non-deterministic) stationary processes. Identification in this context appears then as a rational approximation problem for which the classical theory makes a trade-off between two antagonist factors, namely the bias error on the one hand that decreases when n increases and the variance error on the other hand that increases with n since the dispersion is amplified with the number of parameters. This is the stochastic version of the complexity versus precision alternative which is all-pervasive in modeling.

If one introduces now as a new variable the rational matrix R defined by

$$R = \begin{pmatrix} L & H \\ 0 & I_m \end{pmatrix}^{-1}$$

and if T stands for the first block-row, normalizing the variance of the noise to be identity, the maximum likelihood estimator is asymptotically equivalent, when the sample size increases, to the minimization of

$$\|T\|_\Lambda^2 = \mathbf{Tr} \left\{ \frac{1}{2\pi} \int_0^{2\pi} T(e^{i\theta})\, d\Lambda(\theta)\, T^*(e^{i\theta}) \right\}, \tag{2}$$

where Λ is the spectral measure of the process $(y \quad u)^t$ (which positive and matrix-valued) and where $\mathbf{Tr}$ indicates the trace. If we further restrict the class of models by assuming that we deal with white noise, that is if $L = I_m$, one obtains a weighted rational approximation problem corresponding to the minimization of the variance on the output error. If moreover u itself is (observed) white noise, the situation becomes that of 3.1.4.

Formulation (2) shows that stochastic identification aims at a twofold generalization, both rational and matrix-valued, of the Szegö theory of orthogonal polynomials on the circle, and this sets up a link with classical function theory.

The consistency problem arises from the fact that the measure Λ is not available, so that one has to estimate (2) from time averages of the observed samples, assuming that the process is ergodic. The question is then to decide whether the argument of the minimum of the estimated functional tends to that of (2) when the sample size increases, and what is the speed of convergence. The most significant result here is perhaps the one asserting that if there exists a functional model linking u to y (*i.e.* u is indeed the cause of the phenomenon), and without assuming compactness of the class of models [56], then consistency holds under weak ergodicity conditions and persistent excitation assumptions. An analogous of the law of large numbers indicates, in this context, that convergence is in the order of $1/\sqrt{N}$, where N is the sample size.

In the preceding result, consistency holds in the sense of pointwise convergence of the estimates on the manifold of transfer functions of given size and order. One contribution of the Miaou team has been to show that the result holds even if we do not postulate a causal dependency between inputs and outputs, the measure Λ being simply defined as the weak limit of the covariances. A second contribution is that this convergence holds uniformly with all its derivatives on each compact subset of the manifold of models, thereby drawing a path between the algorithmic behavior of the rational approximation problem (number and nature of critical points, decrease of error, behavior of the poles) and that of the minimization of empirical means. This allows one to translate in terms of asymptotic behavior of the estimators virtually all properties that are uniform with respect to the order of the approximants, and without having to assume that the “true” systems belongs to the class of models. Let us mention for instance that uniqueness of a critical point in H^2 rational approximation, in the case where the system to approximate is nearly rational of degree n, implies [4] uniqueness of a local minimizer for the output error when the input is a white noise, asymptotically almost surely on every compact, when the density of y with respect to u is nearly rational of degree n. In the case of relaxation systems, with one input-output, that is, if the transfer function is a Markov function, we obtain, in the light of the results exposed in module 3.1.2, the same conclusion when the order of approximation is large enough. This is the first known case of unimodularity where the "true" system does not belong to the class of models. An application to the localization of the poles of rational estimates of the output error of a long memory system was derived from this [33]. Here, we are faced again with the question, already mentioned in the introduction, of how to expand functions in bases that are adapted to the singularities of the spectral density of long memory processes. We believe this research direction is worth exploring.

3.2. Structure and control of non-linear systems

In order to control a system, one generally relies on a model, obtained from *a priori* knowledge like physical laws or experimental observations. In many applications, one is satisfied with a linear approximation around a design point or a trajectory. It is however very important to study non-linear systems (or models) and their control for the following reasons. First, some systems have, near interesting working points, a linear approximation that is non-controllable so that linearization is ineffective, even locally. Secondly, even if the linearized model is controllable, one may wish to extend the working domain beyond the validity domain of the linear approximation. Work described in module 3.2.1 proceeds from this problematics. Finally, some control problems, such as path planning, are not of local nature and cannot be answered by a linearly approached model. The structural study described in module 3.2.2 has for purpose to exhibit invariants that can be used, either for reducing the study to simpler systems or for being used as a foundation of a non-linear identification theory, that would give informations on model classes to be used in the case where there is no *a priori* reliable information, and that black-box linear identification is not satisfactory. The success of the linear model, in control or in identification, has its cause in the fine understanding one has of it; in the same fashion, a better mastery of invariants of non-linear models for some transformations is a prerequisite to a true theory of non-linear identification and control. In what follows, all non-linear systems are supposed to have a state space of finite dimension.

3.2.1. Continuous stabilization

Key words: *control*, *stabilization of non-linear systems*, *non-linear control*, *non holonom mechanical system*.

Participants: Ludovic Faubourg [univ. of Bourgogne and CNES], Andreï Ivanov, Jean-Baptiste Pomet.

Stabilization by continuous state feedback — or output feedback, that is, the partial information case — consists of designing a control that is a smooth (at least continuous) function of the state and such that a design point (or a trajectory) is asymptotically stable, for the closed system. One can consider this as a weakened version of the optimal control problem: to compute a control that optimizes exactly a given criterion (for instance to go somewhere in minimal time) leads in general to a very irregular dependence on the state; stabilization is a *qualitative* objective (to go somewhere asymptotically) less constraining than minimization of a criterion, and leaves of course more latitude and allows to impose for instance a lot of regularity. Stabilization problems are often solved, at least near a regular design point, by well-mastered control theory methods; the methods studied here deal with the behavior near points where linear methods are inefficient (non-controllable linear approximation) or tend to master the behavior on a larger zone in the state space. A very important question is the robustness of the stability: in fact, control laws depend heavily on the structure of the model and asymptotic stability conservation for nearby structures or parameter values is not granted. We shall explain hereafter two research directions followed by the Team.

3.2.1.1. Periodic stabilisation of non-linear systems.

It is known that a certain number of non-linear systems, although controllable, cannot be stabilized by a control that is a continuous function of the state alone [48]. One can of course, for these systems, relax the continuity requirement using for instance non-continuous control feedbacks obtained from minimal-time optimization, but a more recent idea consists of looking after continuous feedbacks (better, smooth ones) loosening the constraints that the control depends only on space and allowing a *time* dependency, for instance periodic.

Researches in the Team, with collaboration from the Icare Team, have played an important role in establishing these results [9].

3.2.1.2. Control Lyapunov functions.

Lyapunov functions are a well-known tool for the study of the stability of non-control dynamic systems. For a control system a *Control Lyapunov Function* is a Lyapunov function for the system closed by a given command. This can be expressed by a differential inequality that is called the "Artstein equation [29]", that looks like the Hamilton-Jacobi-Bellmann equation but is largely under-determined. One can deduce, from the knowledge of a control Lyapunov function, stabilizing continuous state-space feedbacks easily.

We are interested in the Team in obtaining control Lyapunov functions. This can be the first step in synthesizing a stabilizing control, but even when a stabilizing control is already known, obtaining a control Lyapunov function can be very useful for studying robustness of the stabilization, or for modifying the initial control law to a more robust one; also if one has to deal with a problem where it is important to optimize a criterion, and that the optimal solution is hard to compute, one can look for a control Lyapunov function that is "near" the solution of the optimization problem, and that leads to a stabilizing control easier to work on, and of a cost (in the sense of the criterion) not far from the optimum.

Recent work in the Team has consisted, starting from objects that are "nearly" control Lyapunov functions, and that are explicitly constructible, or at least easily described, in distorting them, constructively, into control Lyapunov functions, or, on the contrary, depending on the case, to show that such a construction is impossible. In [50], these objects are either functions of type first integrals [51] that cannot be made decreasing, or functions that have the desired properties, but are not smooth [52].

Note that these constructions are exploited in the study requested by Alcatel Space (see module 7.3), where choice is left between the use of optimal control techniques or stabilization.

3.2.2. Transformations and equivalences of non-linear systems and models

Participants: David Avanessoff, Laurent Baratchart, Monique Chyba [UC Santa Cruz (USA)], Jean-Baptiste Pomet.

Key words: *non-linear control*, *non-linear feedback*, *classification*, *non-linear identification*.

A *static feedback* transformation of a dynamical control system is a (non-singular) reparametrization of control, depending on the state, and possibly, a change of coordinates in the state space. A *dynamic feedback* transformation of a dynamic control system consists in a dynamic extension (adding new states, and assigning then a new dynamics) followed by a state feedback on the augmented system.

- From the point of view of the control, the interest of these transformations is that a command that allows to satisfy some objectives on the transformed system can be used to control the original system including the possibly extended dynamics in the controller. Of course the favorable case is when the transformed system has a structure that can more easily be exploited than the original one, for instance a linear controllable system.
- From the point of view of identification and modeling, in the non-linear case, the interest is as mentioned above, either to derive qualitative invariants that can support the choice of a non-linear model given some observations, or to contribute to a classification of non-linear systems that is missing sorely today for elaborating real methods in non-linear identification.

These two problems studied in the Team are now developped.

3.2.2.1. Dynamic linearization.

The problem of dynamic linearization, still unsolved, is that of finding explicit conditions on a system for the existence of a dynamical feedback that would make it linear.

These last years [53], the following property of control systems has been emphasized: for some systems, included linear systems, there exists a given number of functions of the state and the derivatives of the control, that are related by no differential equation and that “parameterize all trajectories”. This property and its importance in control, has been brought in light in [53], where it is called *differential flatness*, the above mentioned functions being called *flat* or *linearizing functions*, and it was shown, roughly speaking, that a system is differentially flat if, and only if, it can be converted to a linear system by dynamic feedback. On one hand, this property of the set of trajectories has in itself an interest at least as important for control than the equivalence to a linear system, and on the other hand it gives a handle for tackling the problem of dynamic linearization, namely to find linearizing functions.

An important question remains still open: how can one algorithmically decide that a given system has or not such functions, i.e. is dynamically linearizable or not? This problem is both difficult and important for non-linear control. For systems with four states and two controls, whose dynamic is affine in the control (these are the lowest dimensions for which the problem is really non-trivial), necessary and sufficient conditions [10] for the existence of linearizing functions depending on the state and the control (but not the derivatives of the control) are surely explicit, but point to the complexity of the question.

From the algebraic-differential point of view, the module of differentials of a controllable system is free and of finite dimension over the ring of differential polynomials in d/dt with coefficients in the space of functions of the system, and for which a basis can be explicitly constructed [28]. The question is to find out if it has a basis made of closed forms, that is, locally exact. Expressed in this way, it is an extension of the classical integrability theorem of Frobenius to the case where coefficients are differential operators. Together with stability by exterior differentiation (the classical condition), further conditions are required here to ascertain the degree of the solutions is finite, the mean-term goal is to obtain a formal and implementable algorithm, able to decide whether or not a given system is flat on a regular point. One can also consider sub-problems with their own interests, such as deciding flatness with a given pre-compensator, or characterizing “formal” flatness that would correspond to a weak interpretation of the differential equation, and also localizing these questions to a neighborhood of an equilibrium point.

3.2.2.2. Topological Equivalence

In what precedes, we have not taken into account the degree of *smoothness* of the transformations under consideration.

In the case of dynamical systems without control, it is well known (Hartman-Grobman theorem) that, away from degenerate (non hyperbolic) points, if one requires the transformations to be merely continuous, every system is *locally* equivalent to a linear system in a neighborhood of an equilibrium. It is tempting thus, in the frame of a classification of *control* systems, to look for such equivalence modulo non-differentiable transformations and to hope bring about some robust “qualitative” invariants and perhaps stable normal forms. An equivalent of the Hartman-Grobman theorem for control systems would say for instance, that outside a “rare” class of models (for instance, those whose linear approximation is non-controllable), and locally near fixed values of the state and the control, no qualitative phenomenon can distinguish a non-linear system from a linear one, all non-linear phenomena being hence either of global nature or singularities. Such a statement is wrong: if a system is locally equivalent to a controllable linear system via a bi-continuous transformation—a local homeomorphism in the state-control space—it is *also* equivalent to this same controllable linear system via a transformation that is as smooth as the system itself, at least in the neighborhood of a regular point (in the sense where the rank of the control system is locally constant), see [21] for details; *a contrario*, under weak regularity conditions, linearization can be done by non-causal transformations (see the same report) whose structure remains unclear, but take a concrete sense when the entries are generated by a finite dimensional dynamics.

The above considerations call for the following question, important for modeling control systems: are there local "qualitative" differences between the behavior of a non-linear system and its linear approximation in the case the latter is controllable?

4. Application Domains

4.1. Introduction

The activity of the team focuses on two bottom lines, namely optimization in the frequency domain on the one hand, and the control of systems governed by differential equations on the other hand. Therefore one can distinguish between two main families of applications: one dealing with design and inverse problems for diffusive and resonant systems, and one dealing with control of cetain mechanical or optical systems. For applications of the first type, approximation techniques as described in module 3.1.1 allow one to deconvolve linear equations, analyticity being the result of either the use of Fourier transforms or the harmonic character of the equation itself. Concerning the second type of applications, they mostly concern the control of systems that are "poorly" controllable, for instance low thrust satellites or optical regenerators. We describe all these applications below in more detail.

4.2. Geometric inverse problems for the Laplacian

Participants: Laurent Baratchart, Amel Ben Abda [ENIT, Tunis], Fehmi Ben Hassen, Slim Chaabane, Imen Fellah, Mohamed Jaoua [ENIT, Tunis], Moez Kallel, Reinhold Küstner, Juliette Leblond, Moncef Mahjoub, Edward Saff, Franck Wielonsky.

Key words: *inverse problem*, *Laplace equation*, *non destructive control*, *tomography*.

Localizing cracks, pointwise sources or occlusions in a two-dimensional material, using thermal, electrical, or magnetic measurements on its boundary is a classical inverse problem. It arises when studying fatigue of structures, behavior of conductors, or else magneto-encephalography as well as the detection of buried objects (mines, etc). However, no really efficient algorithm has emerged so far if no initial information on the location or on the geometry is known, because numerical integration of the inverse problem is very unstable. The presence of cracks in a plane conductor, for instance, or of sources in a cortex (modulo a conversion of 3D data to 2D, see later) can be expressed as an analyticity defect of the solution of the associated Dirichlet-Neumann problem, and may in principle be approached using techniques of best rational or meromorphic approximation on the boundary of the object (see sections 3.1.1 to 3.1.3 and 6.9). The realistic case where data are available only on a part of the boundary is a typical example of application of the analytic and meromorphic extension techniques developed earlier.

The 2D approach proposed here consists in constructing, from measured data on a subset K of the boundary Γ of a plane domain D, the trace on Γ of a function F which is analytic in D except for a possible singularity across some subset $\gamma \subset D$ (typically: a crack). One can then use the approximation techniques described above in order to:

- extend F to all Γ if the data are incomplete (it may happen that $K \neq \Gamma$) if the boundary is not fully accessible to measurements), in order to identify for instance an unknown Robin coefficient, see [13], [49] where stability properties of the procedure are established;
- detect the presence of a defect γ in a computationally efficient manner; [45];
- obtain information on the location of γ [34][31], [15].

Thus, inverse problems of geometric type that consist in finding an unknown boundary from incomplete data can be approached this way [3], possibly in combination with other techniques [45]. Preliminary numerical experiments have yielded excellent results and it is now important to process real experimental data, that the team is currently busy collecting. In particular, contacts with the Odyssée Team of Inria Sophia Antipolis (within the ACI "Obs-Cerv") should provide 3D magneto-encephalographic data, and we already studied how to extract 2D informations from them, see section 6.9. The team is also in contact with companies in order to get 2D (or cylindrical 3D) data issued from engineering practice.

Among the research perspectives opened by these applications, there lies a non-classical approximation problem where residues would be constrained so as to incorporate in the structure of the approximant some features inherited from the fact that we have to estimate a logarithmic potential with a boundary condition, see module 3.1.3. Experiments have been carried out with real residues for a straight crack, which indeed indicate a critical configuration on the crack. However, parametrizing through poles and residues produces global singularities that are undesirable, hence we need to adopt another parametrization based on the coefficients of the polynomials; this requires further study.

In the long term, we envisage generalizing this type of methods to the case of problems with variable conductivity coefficients, as well as to the Helmholtz equation. Using convergence properties of approximation algorithms in order to establish stability results for some of these inverse problems is also an appealing direction for future research.

4.3. Identification and design of resonant systems

Key words: *telecommunications*, *multiplexing*, *filtering device*, *hyperfrequency*, *surface waves*.

One of the best training ground for the research of the team in function theory is the identification and design of physical systems for which the linearity assumption is well-satisfied in the working range of frequency, and whose specifications are made in frequency domain. Resonant systems, acoustic or electromagnetic, are prototypical examples of common use in telecommunications. We shall be more specific on two examples below.

4.3.1. Design of surface acoustic wave filters

Participants: Laurent Baratchart, Andrea Gombani, José Grimm, Martine Olivi.

Surface acoustic waves filters are largely used in modern telecommunications especially for cellular phones. This is mainly due to their small size and low cost. Unidirectional filters, formed of SPUDT transducers that contain inner reflectors (cf. Figure 1), are increasingly used in this technological area. The design of such filters is more complex than traditional ones.

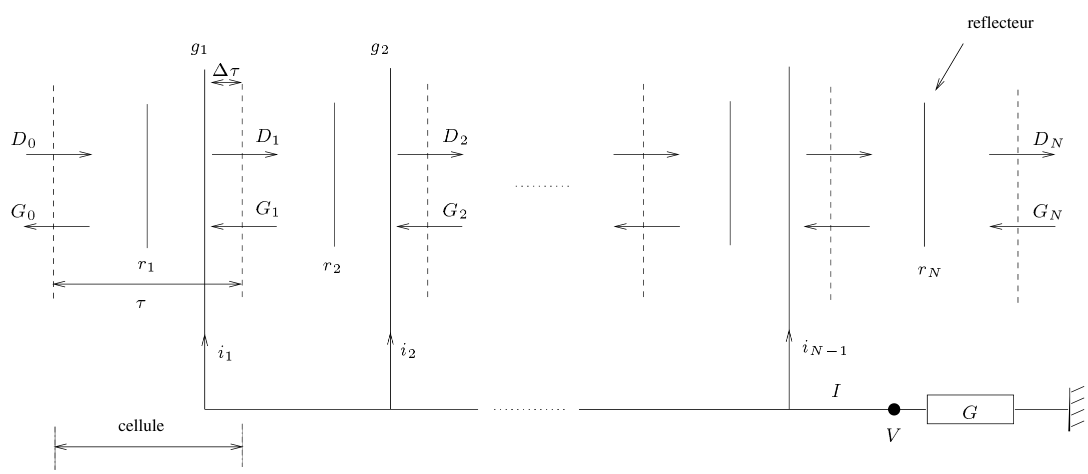

Figure 1. Transducer model.

We are interested here in a filter formed of two SPUDT transducers (Figure 2). Each transducer is composed of cells of the same length τ each of which contains a reflector and all but the last one contain a source

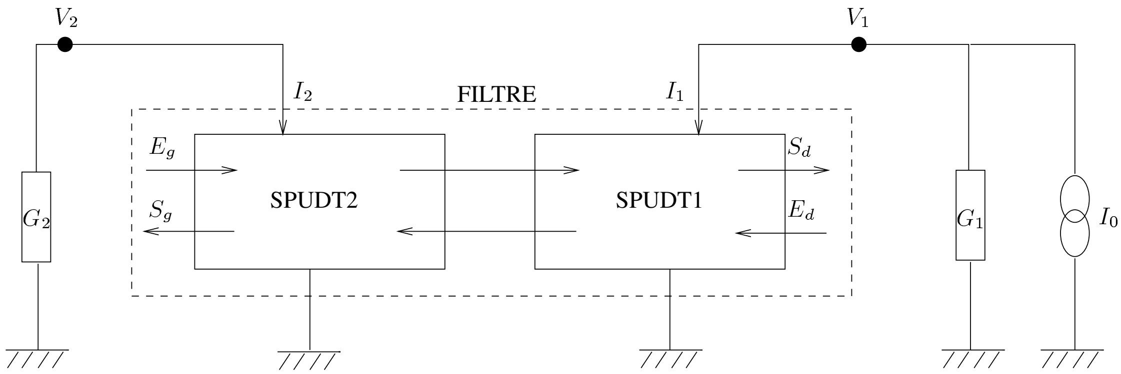

Figure 2. Configuration of the filter

(Figure 1). These sources are all connected to an electrical circuit, and cause electro-acoustic interactions inside the piezo-electric medium. In the transducer SPUDT2 represented on Figure 2, the reflectors are positioned with respect to the sources in such a way that near the central frequency, almost no wave can emanate from the transducer to the left ($S_g \approx 0$), this being called unidirectionality. In the right transducer SPUDT1, reflectors are positioned in a symmetric fashion so as to obtain unidirectionality to the left.

Specifications are given in the frequency domain on the amplitude and phase of the electrical transfer function. This function expresses the power transfer and can be written as

$$E(r,g) = 2\,\frac{V_2}{I_0} = \frac{2\sqrt{G_1 G_2}\quad Y_{12}}{Y_{12}Y_{21} - (Y_{11}+G_1)(Y_{22}+G_2)},$$

where Y is the admittance of the coupling:

$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}.$$

The design problem consists in finding the reflection coefficients r and the source efficiency in both transducers so as to meet the specifications.

The transducers are described by analytic transfer functions called mixed matrices, that link input waves and currents to output waves and potentials. Physical properties of reciprocity and energy conservation endow these matrices with a rich mathematical structure that allows one to use approximation techniques in the complex domain (see module 7.1) according to the following steps:

- describe the set $\mathcal{E}$ of electrical transfer functions obtainable from the model,
- set out the design problem as a *rational approximation problem* in a normed space of analytic functions:

$$\min_{E\in\mathcal{E}} \|D - E\|,$$

  where D is the desired electrical transfer,
- use a rational approximation software (see modules 5.1 and 5.3) to identify the design parameters.

The first item, is the subject of ongoing research. It connects the geometry of the zeroes of a rational matrix to the existence of an inner symmetric extension without increase of the degree (reciprocal Darlingtom synthesis). Let us mention that the interest of the team for this application started through a collaboration with Thomson Microsonics in 1999.

4.3.2. Hyperfrequency filter identification

Participants: Laurent Baratchart, Stéphane Bila, José Grimm, Jean-Paul Marmorat [CMA-EMP], Fabien Seyfert.

In the domain of space telecommunications (satellite transmissions), constraints specific to onboard technology lead filters with resonant cavities to be used in the hyperfrequency range. These filters are used for multiplexing (before or after amplification), and consist of a sequence of cylindrical hollow bodies, magnetically coupled by irises (orthogonal double slits). The electromagnetic wave that traverses the cavities satisfies the Maxwell equations, forcing the tangent electrical field along the body of the cavity to be zero. A deeper study (of the Helmholtz equation) states that essentially only a discrete set of wave vectors is selected. In the considered range of frequency, the electrical field in each cavity can be seen as being decomposed along two orthogonal modes, perpendicular to the axis of the cavity (other modes are far away, and their influence can be neglected).

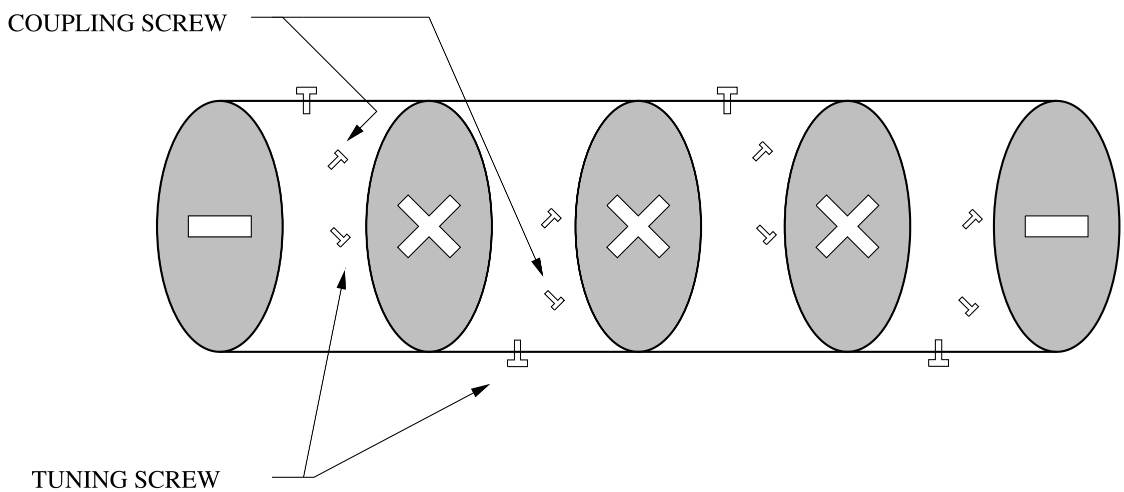

Figure 3. Schematic 4-cavities dual mode filter. Each cavity has 3 screws to couple the modes within the cavity, so that there are 12 quantities that should be optimized. Quantities like the diameter and length of the cavities, or the width of the 8 slits are fixed in the design phase.

Each cavity (see Figure 3) has three screws, horizontal, vertical and midway (horizontal and vertical are two arbitrary directions, the third direction makes an angle of 45 or 135 degrees, the easy case is when all the cavities have the same orientation, and when the directions of the irises are the same, as well as the input and output slits). Since the screws are conductors, they act more or less as capacitors; on the other hand, the electrical field on the surface has to be zero, which modifies the boundary conditions of one of the two modes (for the other mode, the electrical field is zero, and hence is not influenced by the screw), the third screw acts as a coupling between the two modes. The effect of the iris is to the contrary of a screw: no condition is imposed where there is a hole, which results in a coupling between two horizontal (or two vertical) modes of adjacent cavities (in fact the iris is the union of the rectangles, the important parameter being their width). The design of a filter consists of finding the size of each cavity, and the width of each iris. After that, the filter can be constructed, and tuned by adjusting the screws. Finally, the screws are glued. In what follows, we shall consider a typical example, a filter designed by the CNES in Toulouse, with four cavities near 11 Ghz.

Near the resonance frequency, a good approximation of the Maxwell equations is given by the solution of a second order differential equation. One obtains thus an electrical model for our filter as a sequence of electrically-coupled resonant circuits, and each circuit will be modeled by two resonators, one per mode, whose resonance frequency represents the frequency of a mode, and whose resistance represent the electric losses (current on the surface).

In this way, the filter can be seen as a quadripole, with two ports, when plug on a resistor at one end and fed with some potential at the other. We are then interested in the power transmitted and reflected. This leads to defining a scattering matrix S, that can be considered as the transfer function of a stable causal linear dynamical system, with two inputs and two outputs. Its diagonal terms $S_{1,1}$, $S_{2,2}$ correspond to reflections at each port, while $S_{1,2}$, $S_{2,1}$ correspond to transmission. These functions can be measured at certain frequencies (on the imaginary axis). The filter is rational of order 4 times the number of cavities (that is 16 in the example), and the key step consists in expressing the components of the equivalent electrical circuit as a function of the S_{ij} (since there are no formulas for expressing the length of the screws in terms of parameters of this electrical model). On the other hand, this is also useful for the design of the filter, for analyzing numerical simulations of the Maxwell equations, and for checking the design, particularly the absence of higher resonant modes.

In reality, the resonance is not studied via the electrical model, but via a low pass equivalent obtained upon linearizing near the central frequency, which is no longer conjugate symmetric (*i.e.* he underlying system may not have real coefficients) but whose degree is divided by 2 (8 in the example).

In short, the identification strategy is as follows:

- measuring the scattering matrix of the filter near the optimal frequency over twice the pass band (which is 80Mhz in the example).
- solving bounded extremal problems, in H^2 norm for the transmission and in Sobolev norm for the reflection (the module of he response being respectively close to 0 and 1 outside the interval measurement) cf. module 3.1.1. This gives a scattering matrix of order roughly 1/4 of the number of data points.
- Then one rationally approximate with fixed degree (8 in the occurrence) via the hyperion software cf. module 3.1.4 and 5.1.
- A realization of the transfer function is thus obtained, and some symmetry constraints are added here.
- Finally one builds a realization of the approximant and he looks for a change of variables that kills non-physical couplings. This is obtained by using algebraic-solvers and continuation algorithms on the group of orthogonal complex matrices (the symmetry forces this kind of change of basis).

The final approximation is of high quality. This can be interpreted as a validation of the linearity hypothesis for the system: the relative L^2 error is less than 10^{-3}. This is illustrated by a reflection diagram (Figure 4). Non-physical coupling are less than 10^{-2}.

The above considerations are valid for a large class of filters. These developments have also been used for the design of unsymmetric filters, useful for the synthesis of repeating devices.

The team extends today its investigations, to the design of output multiplexors (OMUX) that couple several filters of the previous type on a manifold. The objective is to establish a global model for the behavior that takes into account

- within each channel the coupling between the filter and the Tee that connects it to the manifold,
- the coupling between two consecutive channels.

The model is obtained upon chaining the transfer matrices associated to the scattering matrices. It mixes rational elements and complex exponentials (because of the delays) and constitutes an extension of the previous framework. Under contract with the CNES (see 7.1), the team has started a study of the design with gauge constraints, based on function theoretical tools.

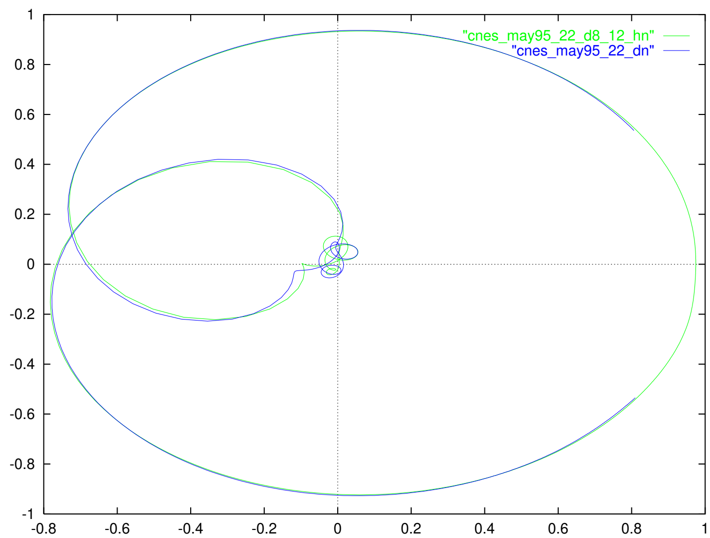

Figure 4. Nyquist Diagram. Rational approximation (degree 8) and data - S_{22}

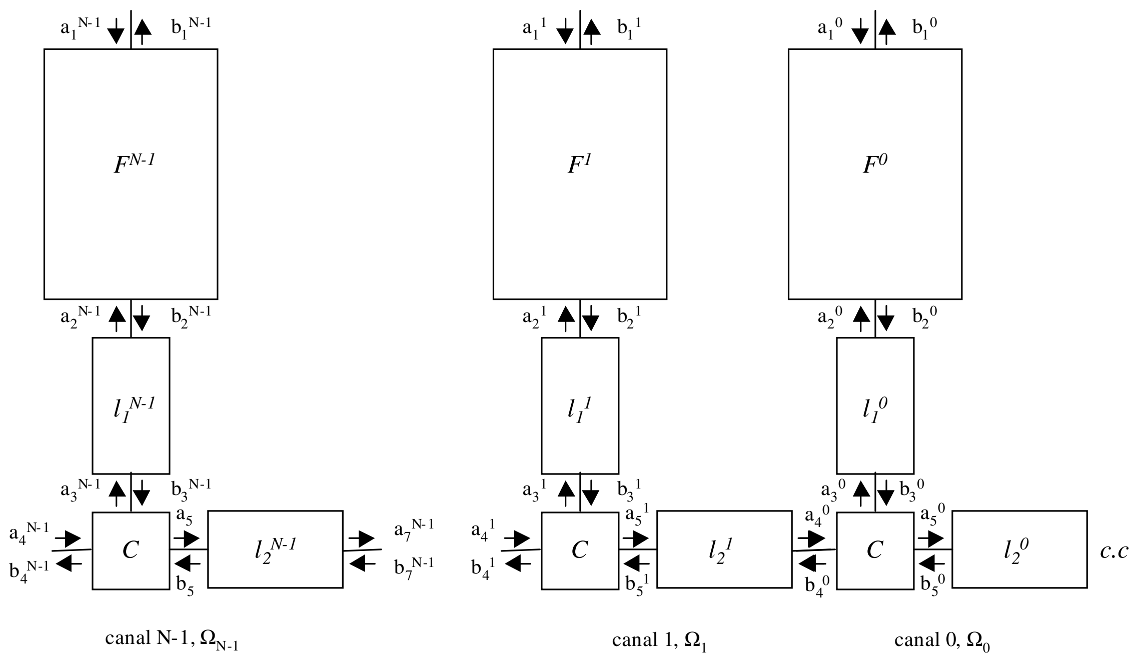

Figure 5. N filters on a manifold. Quantities a and b are incoming and outcoming waves, boxes with an l stand for an adjustable line-length, the 'C' in the square box represents the heart of the Tee (3 by 3 transfer matrix); the big rectangles with an F represent the filters, they are connected to the amplifiers, the bottom row (with the ls and Cs) is the manifold, connected to the antenna on the left, terminated by a cc (short-circuit) on the right.

4.4. Spatial mechanics

Participants: Ludovic Faubourg [univ. of Bourgogne and CNES], Jean-Baptiste Pomet.

Key words: *spatial mechanics*, *satellite*, *orbital control*, *telecommunications*.

The use of satellites in telecommunication networks motivates a lot of research in the area of signal and image processing. Problems of spatial mechanics and satellite control are also vital to these new technologies. For instance, fuel represents half of the total mass of the satellite, which is an obstacle to the missioning charge (devices for telecommunications, image processing surveillance, etc), since the total mass is limited by the capacity of the launchers.

Hence it is natural to seek more efficient propulsion means. Progress in physics permit today effective "electrical" propulsion modes (ionic engines, plasma, etc) that have a better efficiency, but a much smaller instantaneous thrust than traditional chemical rockets. This raises difficult control problems, whose study by the team is carried out in collaboration with Alcatel-Space Cannes, see module 7.3.

Note that spatial mechanics is a domain that poses a great deal of delicate control problems, due to the extreme conditions and long lease of life of satellites.

4.5. Non-linear Optics

Participants: Alex Bombrun, Jean-Baptiste Pomet, Fabien Seyfert.

Key words: *Optics*, *3R regeneration*, *optical fibers*, *networks*, *telecommunications*.

The increased capacity of numerical channels in information technology is a major industrial challenge. The most performing means nowadays for transporting the signals from a server to the user and backwards is via optic fibers. The use of this medium at the limit of its time of response causes new control problems in order to maintain a safe signal, both in the fibers and in the routing and regeneration devices.

The team has been associated, under contract with Alcatel R&I (see module 7.4), in the control of the "all-optic" regenerators.

4.6. Transformations and equivalence of non-linear systems

Participants: Laurent Baratchart, Jean-Baptiste Pomet, David Avanessoff.

Key words: *path planning*, *mobile cybernetics*, *identification*.

The works presented in module 3.2.2 are upstream from applications. However, beyond the fact that deciding whether a given system is linear modulo an adequate compensator is clearly conceptually useful, the use of "flat outputs" for path planning has a great interest, see for instance the European Control Conference [60]. Moreover, as indicated in section 3.2, a better understanding of the invariants on non-linear systems under feedback would lead to considerable progress in identification.

5. Software

5.1. The hyperion software

Participants: José Grimm [manager], Fabien Seyfert, Franck Wielonsky.

There was no major development concerning the hyperion software this year. It was used in research contracts with CNES and Alcatel Space, as well as for numerical tests in crack detections.

On the other hand, we started to create a library named `bibapics`, a set of `matlab`-callable routines, that offers the same functionalities as hyperion, and is compatible with its system of batch files. It uses XML as language for descriptions of tasks.

5.2. The Tralics software

Participant: José Grimm [manager].

The development of a LaTeX to XML translator, named Tralics was continued. For more details, see module 6.2. TRALICS was sent to the APP in December 2002. Its IDDN number is InterDepositDigitalNumber = IDDN.FR.001.510030.000.S.P.2002.000.31235. Binary versions are available for Linux, Solaris, Windows and Mac-OS X. Its web page is http://www-sop.inria.fr/miaou/tralics.

5.3. The RARL2 software

Participants: Jean-Paul Marmorat, Martine Olivi [manager].

RARL2 (Réalisation interne et Approximation Rationnelle L2) is a software for rational approximation (see module 3.1.4). Its web page is http://www-sop.inria.fr/miaou/RARL2/rarl2.html. This software takes as input a stable transfer function of a discrete time system represented by

- either it internal realization
- or its N first Fourier coefficients
- or discretized values on the circle

It computes a best approximant (local minimum) *stable, of given McMillan degree*, in the L^2 norm.

It is somehow related to the arl2 function of hyperion (see module 5.1) and differs in the way it represents the systems: a polynomial representation is used in hyperion, while RARL2 uses a realization, this being very interesting in some cases. It is implemented in MATLAB. This software handles *multi-variable* systems (with several inputs and several outputs), and uses a parameterization that has the following advantages

- it handles only *stable systems*, so that the result is necessarily stable,
- it allows the use of differential tools, and can identify uniquely a system,
- it is well-conditioned, and computations are cheap.

An iterative research strategy on the degree of the local minima, similar in principle to that of arl2, increases the chance of obtaining the absolute minimum (see module 6.3) by generating, in a structured manner, several initial conditions. Contrary to the polynomial case, we are in a singular geometry on the boundary of the manifold on which minimization takes place, which forbids the extension of the criterion to the ambient space. We have thus to take into account a singularity on the boundary of the approximation domain, and it is not possible to compute a descent direction as being the gradient of a function defined on a larger domain, although the initial conditions obtained from minima of lower order are on this boundary. Thus, determining a descent direction is nowadays, to a large extent, a heuristic step. This step works well in the cases handled up to now, but research are under way in order to make this step ruly algorithmic.

5.4. The RGC software

Participants: Fabien Seyfert, Jean-Paul Marmorat.

The RGC software (Réalisation interne à géométrie contrainte) has no web page.

The identification of filters modeled by an electrical circuit that was developed inside the team (see module 4.3.2) leads to compute the electrical parameters of the filter. This means finding a particular realization (A, B, C, D) of the model given by the rational approximation step. This 4-tuple must satisfy constraints that come from the geometry of the equivalent electrical network and translate into some of the coefficients in (A, B, C, D) being zero. Among the different geometries of coupling, there is one called “the arrow form” [47] which is of particular interest since it is unique for a given transfer function and also easily computed. The computation of this realization is the first step of RGC. However if the desired realization is not in arrow form, one can show that it can be deduced by an orthogonal change of basis (in general complex). In this case, RGC starts a local optimization procedure that reduces the distance between the arrow form and the target, using successive orthogonal transformations. This optimization problem on the group of orthogonal matrices is non-convex and has a lot of local and global minima. In fact, there is not always uniqueness of the realization of the filter in the given geometry. Moreover, it is often interesting to know all the solutions of the problem, because the designer cannot be sure, in many cases, which one is being handled, and sometimes the assumptions on the reciprocal influence of the resonant modes are not well satisfied. Today, apart from the particular case where the arrow form is the desired form (this happens frequently up to degree 6) the RGC software gives no guarantee to obtain a single realization that satisfies the prescribed constraints. Work is in progress, see section 6.12.

5.5. PRESTO-HF

Participant: Fabien Seyfert.

PRESTO-HF: a toolbox dedicated to lowpass parameter identification for hyperfrequency filters http://www-sop.inria.fr/miaou/Fabien.Seyfert/Presto_web_page/presto_pres.html

In order to allow the industrial transfer of our methods, a Matlab-based toolbox has been developed, dedicated to the problem of identification of low-pass hyperfrequency filter parameters. It allows to run the following algorithmic steps, one after the other, or all together in a single sweep:

- determination of delay components, that are caused by the access devices (automatic reference plane adjustment);
- automatic determination of an analytic completion, bounded in module for each channel, (see module 6.11);
- rational approximation, of fixed McMillan degree;
- determination of a constrained realization.

For the matrix-valued rational approximation stage Presto-HF relies either on hyperion (Unix or Linux only) or RARL2 (platform independent), both rational approximation engines are developed within the team. Constrained realizations are computed by the RGC software. As a toolbox, Presto-HF has a modular structure, which allows one for example to include some building blocks in an already existing software.

The delay compensation algorithm is based on the following strong assumption: far off the passband, one can reasonably expect a good approximation of the rational components of S_{11} and S_{22} by the first few terms of their Taylor expansion at infinity, a small degree polynomial in $1/s$. Using this idea, a sequence of quadratic convex optimization problems are solved, in order to obtain appropriate compensations. In order to check the previous assumption, one has to measure the filter on a larger band, typically three times the pass band.

This toolbox is currently used by Alcatel Space in Toulouse.

6. New Results

6.1. Tools for producing the Activity Report

Participants: José Grimm, Bruno Marmol [DISC], Marie-Pierre Durollet [DISC].

Key words: *Perl*, *XML*, *module*, *configure*, *make*.

The great novelty in the RAWEB2002 (Scientific Annex to the Annual Activity Report of Inria), was the use of XML as intermediate language, and the possibility of bypassing LaTeX (one Inria Team acted as a guinea pig, or beta-tester, depending on your point of view). A working group, formed of M.P. Durollet, J. Grimm, L. Pierron, and I. Vatton (not forgetting A. Benveniste, J.-P. Verjus and J.-C. Le Moal) is in charge of the definition of the tools; in 2003, B. Marmol joined the group, he is in charge of the dissemination of the package.

The first step of this new writing scheme has been to put on the Web, for the year 2001, together with the HTML version (obtained by Latex2HTML) and the PostScript version (obtained by LaTeX), a Pdf version, obtained independently via the XML route. The second step was, in 2002, to produce the HTML, PostScript and Pdf versions using an XML intermediate representation. Finally, for the year 2003, authors of the RAWEB are assumed to produce the XML by themselves (using the same tools as in 2002, of course).

This XML version of 2001 was obtained via a Perl script [55]. This script used some various tool, like Ω, ltx2x, bibtex, Perl, etc. It is nowadays replaced by the Tralics software, described in module 6.2.

One important issue was the choice of the DTD (*document type definition*). One one hand, it should follow the pseudo-DTD as defined for the RAWEB since five years (the Activity Report is a set of modules, with contributors, key-words, etc), and on the other hand, it must be as close as possible to standards DTDs. We have decided to use a variant of the TEI (*text encoding initiative*, see http://www.tei-c.org/) for the text, MathML for the mathematics, and an ad-hoc DTD for the bibliography. This DTD was not modified in 2003: first of all, we do not have enough experience (some people look closely at the DTD because they have to use it and this will give us new ideas for the next year), second, we do not want to change everything every year (in 2003, the RAWEB is in English, so that keyword values have been translated, but we left the original keyword names), and finally, we still do not know a good DTD for the bibliography.

The translation from XML to HTML is done via an XSLT style sheet and the Gnome tools (xsltproc being an efficient processor). The main difficulty comes from the mathematics: we have decided to translate all formulas into images, (in the case `$x+\alpha$`, only the α is converted) as follows: a dedicated Perl script extracts from the XML file all formulas, and converts them to a set of pages in a dvi file (we use here the same algorithm for converting the XML to PostScript). Each page is converted to an image via pstoimg, which is a Perl code, part of latex2html. For the next year, we anticipate to find a solution that avoids the need to install of the whole latex2html bundle.

The translation of the XML text to a Pdf or PostScript document is a two-phase process: first a style sheet is used, that converts the XML into an XSL-FO document, by adding some formatting instructions (in this phase, we explain for instance that the text font should be Times). This file is formatted by TEX or pdfTEX, thanks to the xmltex package that teaches to TEX the subtleties of XML and utf-8 encoding, and two packages for the XSL-FO and MathML commands. Some commands have been rewritten, improving the rendering of formulas like $\lim_{x\to 0}\sin^2(x)$ and $\binom{n}{m}$.

6.2. Tralics: a Latex to XML Translator

Participant: José Grimm.

Key words: *Scanner*, *parsing*, *validation*.

The TRALICS software is a C++ written LATEX to XML translator, based upon a Perl script that was used for the raweb, and described in [55]. It was presented at the EuroTEX conference in Brest, [16]. One use of the software is shown on figure 6. Since the XML source of the raweb is strongly constrained, Tralics can also be used as a raweb validator: it refuses commands like `\section`, and emits warnings for bibliographic entries that are not of the current year; it can also generate a draft version of the PostScript output that does not require the XML tools to be installed. On the other hand, Tralics knows of over one thousand commands (included those forbidden by the raweb), and is linked to the preview-latex package of David Kastrup.

The main philosophy of Tralics is to have the same parser as TEX, but the same semantics as LATEX. This means that commands like `\chardef`, `\catcode`, `\ifx`, `\expandafter`, `\csname`, etc., that are not described in the LATEX book and not implemented in translators like latex2html, tth, hévéa, etc., are recognised by Tralics. This year we added constructions like `\endlinechar`, `\read`, `\uppercase`, `\endinput`, which are less used, and a bit tricky. Note that a construction like `\ifdim\wd0>0pt\fi` is recognised by the parser, but there is no way to change the size of the box number zero, so that the test is always false.

Some commands (like `\dump` or `\patterns` are not implemented, because they neither affect parsing nor produce an output. All commands that produce a dvi output in LATEX have been implemented as commands that produce XML code, for instance basic commands like (`\chapter`, `\it`, etc), environnments (figure, table, notes), mathematics, and of course all commands needed by the Raweb (for instance, "topics" management). There are some unresolved problems: for instance, Tralics understands only basic array specifications (r, l, c, and bar, not p or @), non-math material in a math formula is rejected (unless it is formed of characters only),

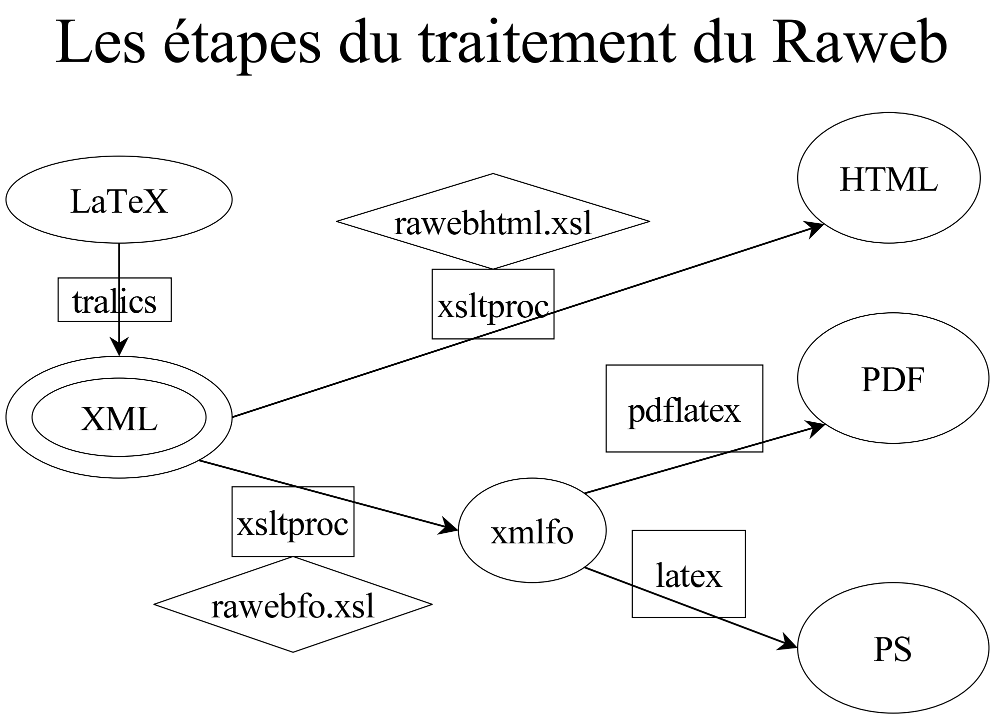

Figure 6. A slide that explains how the raweb operates. Rectangular boxes contain tools, diamond-shape boxes are style sheets, and ellipses contain language names; the name XML is in a double ellipse, it is the central object. The Perl script that handles the math formulas is not shown here; it uses tools borrowed from latex2html.

a figure environment should contain only graphics together with a single caption, commands defined by the picture environment are translated (but refused by the style sheet). Finally, because it is too complicated to parse the result of Bibtex, we decided to use our own bibtex-to-latex translator (this is not the best solution).

For more information, see the Tralics web page.

6.3. Parametrizations of matrix-valued lossless functions

Participants: Andrea Gombani, Bernard Hanzon [Univ. Libre (VU) of Amsterdam], Jean-Paul Marmorat, Martine Olivi, Ralf Peeters [Univ. of Maastricht].

These parametrization issues have been studied for several years in the project. Atlases of charts have been derived from a matrix Schur algorithm associated with Nevanlinna-Pick interpolation data. In a chart, a lossless function can be represented by a balanced realization computed as a product of unitary matrices. Moreover, an adapted chart for a given lossless function can be built from a realization in Schur form. Such a parametrization presents a lot of advantages : it ensures identifiability, takes into account the stability constraint and preserves the order and presents a nice numerical behavior. This parametrization has been used in the software RARL2 which deals with rational approximation in L^2 norm.

The natural framework for these studies is that of complex functions while, in most applications, systems are real-valued and their transfer functions have real coefficients. We may of course restrict our parametrization by imposing real interpolation data, but in this case our strategy to find an adapted chart from the Schur does not work anymore. In order to preserve all the nice properties of the previous parametrization, it appears that we must consider a more general interpolation problem, that is the contour integral interpolation problem of Nudelman. Doing this, we can follow the previous approach, and build an atlas of charts for real lossless function (of fixed degree and size), which allow for a recursive construction of balanced realizations and such that the Schur real form provides an adapted chart. This new results have been presented at the CDC03 [18]. We hope that this general framework will allow us to parametrize other subclasses of function, in particular symmetric lossless functions which naturaly arise in connection with the physical principle of reciprocity.

6.4. The mathematics of Surface Acoustic Wave filters

Participants: Laurent Baratchart, Per Enqvist, Andrea Gombani, Martine Olivi.

Surface Acoustic Waves (in short: SAW) filters consist in a series of transducers which transmit electrical power by means of surface acoustic waves propagating on a piezoelectric medium. They are usually described by a mixed scattering matrix which relates acoustic waves, currents and voltages. By reciprocity and energy conservation, these transfers must be either lossless, contractive or positive real, and symmetric. In the design of SAW filters, the desired electrical power transmission is specified. An important issue is to characterize the functions that can actually be realized for a given type of filter. In any case, these functions are Schur and can be completed into a conservative matrix with an increase of at most 2 of the McMillan degree, this matrix describing the global behavior of the filter. Such a completion problem is known as Darlington synthesis and has always a solution for any higher McMillan degree in the rational case if the symmetry condition is of no concern. However in our case, additional constraints arise from the geometry of the filter as the symmetry and certain interpolation condition. In [22] and [32], we give a complete mathematical description of such devices, and we provide some realization for the relevant tranfer-functions while giving he solution to the Darlington synthesis in the symmetric case with preserved McMillan degree.

6.5. Rational and Meromorphic Approximation

Participants: Laurent Baratchart, Reinhold Küstner, Vasilyi Prokhorov [Univ. Alabama, Mobile], Edward Saff, Herbert Stahl, Pascale Vitse.

Meromorphic approximation of Markov functions in the L^1 sense made progress in recent years concerning the error rate and the pole distribution, and it has been the object of further study in connection with minimal Blaschke products in $L^2(\mu)$ where μ is a positive measure with support on a segment in $(-1, 1)$ whose Cauchy transform generates the associated Markov function. In particular, sharp asymptotics for these Blaschke products were derived in [41]. Moreover, this work naturally induces some links between meromorphic approximation of Markov functions and the n-widths of the unit ball of H^p in $L^q(\mu)$, because the extremal functions are essentially the singular vectors of the Hankel operator associated to the approximation problem with exponent s such that $1/s + 2/p = 1$. When $p \geq q$, we were able to generalize the asymptotics obtained by Fisher and Stessin for the n-width and we derived the corresponding orthogonality conditions. These are interesting because they are similar to those that arise in rational approximation in L^2-norm, translated to the segment rather than the circle. This work has given rise to a publication [42]

The matrix version of a Markov function is the Cauchy transform of a positive matrix valued measure. For those, it has been proved this year that a best L^2 rational approximant is again Markov. This is an important step towards studying rates of convergence and uniqueness issues, that was achieved using the critical point equations based on the Douglas-Shapiro-Shields factorization of the approximant [4] and the Potapov decomposition of the inner factor of the approximant, plus some transversality theory to show the best approximant generically has right and left inner factors that are transpose from each other (the approximant is assumed to be conjugate-symmetric).

Back to scalar-valued functions, a natural generalization of Markov functions is the class of Cauchy integrals with respect to some complex measure supported on symmetric contours for the Green potential in the unit disk, *i.e.* the Green potential has equal normal derivative on either side of the contour. Thus the generalization is twofold: symmetric contours generalize the segment and complex measures generalize positive ones. Such Cauchy integrals were studied by H. Stahl who showed the convergence in capacity of Padé approximants for them [64], and subsequently by Gonchar and Rachmanov who established the exact geometric rate in uniform approximation to these [54] after Parfenovs proof of the Gonchar conjecture [61]. We have generalized their result by showing, under weak conditions, that the upper limit of the n-th root of the meromorphic approximation error with n poles in L^p is less or equal to $e^{1/2C}$ where C is the Green capacity of the support of the measure in the unit disk. These results are based on the assessment of the asymptotic behaviour of the poles of the approximants using Hankel operator techniques, and they are linked with those concerning asymptotics of poles of functions with branch points described in module 6.6. An article is currently under writing on the subject.

6.6. Asymptotic behavior of poles

Participants: Laurent Baratchart, Reinhold Küstner, Edward Saff, Herbert Stahl, Vilmos Totik [univ. Szeged and Acad. of Sciences, Hongary].

It has been shown in [43] that the denominators of best rational of meromorphic approximants in the L^p norm of a closed curve, say the unit circle T for definiteness, satisfy for $p \geq 2$ a non-Hermitian orthogonality relation with respect to some complex measure for functions described by a Cauchy integral on a curve γ (locus of singularities) contained in the unit disk D. When γ is a real segment, it is also established that under very weak conditions, the poles of the best rational approximants in $L^2(T)$ converge to the minimal segment of the real axis that contains the support of the measure as long as it is regular enough. These conditions require that the argument of the measure to be of bounded variation, but any support in $(-1, 1)$ is allowed, as long as the density of the measure is not too small. These conditions were generalized to the case of a varying weight, provided that its modulus of continuity is bounded by a function continuous at zero. In any case, using conformal transformations, this applies to the case where γ is a geodesic arc, and is sufficient for analyzing, in the sense of weak-star convergence of counting measures, the asymptotic behaviour of the poles of best approximants to algebraic functions with two branch points: the asymptotic distribution of the poles, if γ does not intersect T, is then the equilibrium distribution of the condenser $(T, \mathcal{C})$ where $\mathcal{C}$ is the geodesic arc that joins the branch points.

According to what precedes, the poles of the best rational or meromorphic approximants of the “complex solution” of the Laplacian on a cracked domain converge if the crack is “analytic enough”, to the geodesic arc that joins its endpoints, with a density that charges these endpoints (since it is a property of the equilibrium measure). This gives substantial information on the location of the crack. The case of definitely more general cracks, for instance piecewise polynomial, under suitable regularity conditions for the data to analyze, can be reduced to the case of a function with a finite number, but maybe more than two, branch points.

After having conjectured that, for a finite but arbitrary number of branch points, the asymptotic pole distribution is the equilibrium distribution on the continuum $\mathcal{M}$ that contains these points and that minimizes the capacity of the capacitor $(T, \mathcal{M})$, see module 3.1.3, we have proved in 2002 that this conjecture holds in the case of a connected continuum. The proof does not use orthogonality but rather the minimizing character of the optimal approximants as well as some symmetry properties of the arc of minimal capacity with respect to ring conformal applications combined to the boundedness of the Cauchy transform on the Carleson curves. This year, the case of a non-connected continuum was solved in the case $p = \infty$, and considerable progress was also made for $2 \leq p < \infty$, though the final result is not yet fully established.

6.7. Extremal problems with pointwise constraints

Participants: Laurent Baratchart, Juliette Leblond, Fabien Seyfert.

The study of the Problem *(P')* defined in section 3.1.1 has been carried out in the case where $p = 2$, $\psi = 0$, and the function M is in L^∞ of $T \setminus K$ and bounded from below almost everywhere by a strictly positive constant. Together with the existence and uniqueness of the solution, we have proved that the constraint is saturated pointwise, that is $|g| = M$ a.e. on $T \setminus K$, this being perhaps counter-intuitive. We obtained fixed point equations that characterize the solution, involving the resolvant of a Toeplitz operator, but with a multiplier that is here a function [39]. The study of the convergence of an iterative scheme is under examination, the goal being its implementation in the hyperion software. Note that if we approach the multiplier by a step function, we get a string of spectral equations similar to these used for solving Problem *(P)*.

An algorithm that consists in discretizing the modulus constraint and using Lagrange duality-based optimization techniques as in section 6.11 has already been implemented and performs satisfactorily.

6.8. Extremal problems with real constraints

Participants: Juliette Leblond, Jean-Paul Marmorat, Jonathan R. Partington.

Another generalization of problem *(P)* in the analytic framework where $N = 0$, that also extends the extremal problems in H^2 with constraint on the imaginary parts stated in section 3.1.1, is the following:

Let $f \in L^2(K)$, $\psi \in L^2(T \setminus K)$ and $M > 0$; find a function $g \in H^2$ such that $\|Im g - \psi\|_{L^2(T \setminus K)} \leq M$ and such that $g - f$ has minimal norm in $L^2(K)$.

Let $p \geq 1$, K be an arc of the unit circle T, $f \in L^p(K)$, $\psi \in L^p(T \setminus K)$, and $\alpha, \beta, M > 0$; fin a function $g \in H^p$ such that $\alpha \|Re(g - \psi)\|_{L^p(T \setminus K)} + \beta \|Im(g - \psi)\|_{L^p(T \setminus K)} \leq M$ and such that $Re(g - f)$ is of minimal norm in $L^p(K)$ under this constraint.

This is a natural formulation for issues concerning Dirichlet-Neumann problem for the Laplace operator, see sections 4.2 and 6.9, where data and physical prior information concern real (or imaginary) parts of analytic functions.

For $p = 2$, existence and uniqueness of the solution have been established in [58] as well as a constructive solution procedure which, in addition to the Toeplitz operator that characterizes the solution of *(P)* in the case $p = 2$ and $N = 0$, involves a Hankel operator.

Situations with other values of p will be considered, as well as a suitable general weighted formulation of constrained extremal problems on T.

6.9. Inverse Problems for 2D and 3D Laplacian

Participants: Laurent Baratchart, Amel Ben Abda [ENIT, Tunis], Fehmi Ben Hassen, Slim Chaabane [ENIT, Tunis], Imen Fellah, Mohamed Jaoua, Juliette Leblond, Moncef Mahjoub, Jean-Paul Marmorat, Jonathan R. Partington.

Key words: *inverse problems*, *Laplacian*, *non destructive control*, *tomography*.

The fact that 2D harmonic functions are real parts of analytic functions allows one to tackle issues in singularity detection and geometric reconstruction from boundary data of solutions to Laplace equations using the meromorphic and rational approximation tools developed by the team. Some electrical conductivity defaults can be modeled by pointwise sources inside the considered domain. In dimension 2, the question made significant progress last year. In this situation, the singularities of the function (of the complex variable) which is to be reconstructed from boundary measures are poles (case of dipolar sources) or logarithmic singularities (case of monopolar sources). Hence, the behavior of the poles of the rational or meromorphic approximants, described in modules 3.1.1 to 3.1.3, allow one to efficiently locate their position. This, together with corresponding software implementation, is part of the subject of the Ph.D. thesis of F. Ben Hassen and a paper is in preparation [31], where the related situation of small inhomogeneities connected to mine detection is also approached.

In 3D, epileptic regions in the cortex are often represented by pointwise sources that have to be localized from potential measures on the scalp of a potential difference, that is the solution of a Laplace equation (EEG, electoencephalography). Note that the head is here modeled as a sequence of spherical layers. This inverse EEG problem is the object of a collaboration between the Miaou and Odyssée Teams through the ACI "Obs-Cerv". A nice breakthrough has been done this year which makes it possible now to process via best rational approximation on a sequence of 2D disks along the sphere [15] and [36]. The point here is that, up to an additive function harmonic in the 3D ball, the trace of potential on each boundary circle coincides with that of a function with branchpoints as singularities in the associated disk. The behaviour along the family of disks of the poles of their best rational approximants on each circle is strongly linked to the location of the sources, using properties discussed in sections 3.1.3 and 6.6. (in the particular case of a unique source, we end up with a rational function); this is still under study. Constructive and numerical aspects of the expected procedures (harmonic 3D projection, Riesz transformation, spherical harmonics) have neen studied and encouraging results are already available on numerically computed data. We need now to handle the Cauchy problem raised by several layers of different (although constant) conductivities in order to treat experimental data from EEG (see also below).

In the 2D case again, but with incomplete data, the geometric problem of finding, in a constructive way, an unknown (insulating) part of the boundary of a domain is considered in the Ph.D. thesis of I. Fellah. Approximation and analytic extension techniques described in section 3.1.1 together with numerical conformal transformations of the disk provide here also interesting algorithms as well as stability properties for the inverse problem under consideration.

Finally, solving Cauchy problems analytic on an annulus or on a spherical layer is also a necessary ingredient of the methodology, since it is involved in the propagation of initial conditions from the boundary to the center of the domain, where singularities are seeked, when this domain is formed of several homogeneous layers of different conductivities (as in the EEG problem above). On a 2D annulus, this issue, which is the main theme of the PhD thesis of M. Mahjoub, arises when identifying a crack in a tube or a Robin coefficient on its inner skull. It can be formulated as a completion problem on the boundary of a doubly connected domain, which allows us to get both numerical algorithms and stability results in this framework [46], thereby generalizing the simply connected situation [49], [13].

6.10. Local linearization of control systems

Participants: David Avanessoff, Laurent Baratchart, Jean-Baptiste Pomet.

In [20], we account for some novel constructions that are relevant to characterize flatness of nonlinear control systems (see "dynamic equivalence" in section 4.6). These are tools for analyzing some overdetermined systems of PDEs for which neither the number of independent variables nor the order is *a priori* fixed. A notion of "very" formal integrability was introduced, and the equations arising when characterizing flatness are proved to have this property.

Also, the final version of our results on topological linearization (see "topological equivalence" in section 4.6) became available [21].

6.11. Analytic Extension with polynomial values

Participant: Fabien Seyfert.

We study here the problem of analytic extension of pointwise frequency measurements of a dissipative linear system, where the strong assumption is that the unknown part is well-modeled by a polynomial in $1/s$ ($s = iw$). Let $\{w_i, S(w_i)\}$ be the sequence of measurements, J the interval defined by $J = [\min_i(w_i), \max_i(w_i)]$ and contains 0. We define a continuous function $S(w)$ for $w \in J$ via interpolation (for example, using splines) between points of measure. Let I be a set of frequencies for which we believe that the polynomial model is valid. Typically I is consists of two batches of measurements near the boundary of J, for instance

$$I = \{w_k, |w_k| \geq w_c\}.$$

We want to solve the following problem:

$$\min_p \psi(p) = \textstyle\sum_{w_k \in I} |S(w_k) - p(\frac{1}{w_k})|^2$$

$$\left\{ \begin{array}{l} p \in C_n[x] \\ ||P_{\overline{H}^2}(S \vee p)||^2 \leq E_c \\ \forall w \in J_c \ |p(1/w)|^2 \leq 1 \end{array} \right. \tag{3}$$

where J_c represents the complementary of J, and $\vee$ the concatenation operator ($S \vee p$ denotes the function defined by S on J and $p(1/w)$ on J_c). The notation $\overline{H}^2$ stands for the orthogonal of the Hardy space of the right half-plane associated with the measure $dw/(1 + w^2)$. In other words, we are looking for the polynomial completion that takes best account for the data at frequencies in I, and satisfies two constraints: one on the norm of the unstable part of the completed data, and the other on the dissipative character of the extension. It can be shown that if I has more than $n + 1$ points, and under the condition that there exists a feasible point, then (3) has a unique solution p_0. Moreover, if there exists a feasible point that satisfies strictly the inequalities in (3), then there exist real non-negative numbers $\lambda_0 \cdots \lambda_{2n+1}$ and $x_1 \cdots x_{2n+1} \in W = [1/\min_i(w_i), 1/\max_i(w_i)]$ such that

$$\left\{ \begin{array}{l} \lambda_0(||P_{\overline{H}^2}(S \vee p)||^2 - E_c) = 0 \\ \forall k \in \{1 \cdots 2n+1\} \ \lambda_k(|p(x_k)| - 1) = 0 \end{array} \right.$$

and such that p_0 is the unique minimum of the following convex optimization problem:

$$\min_{p \in C[x]} \sum_{w_k \in I} |S(w_k) - p(\frac{1}{w_k})|^2 + \lambda_0 ||P_{\overline{H}^2}(S \vee p)||^2 + \sum_{k=1}^{2n+1} \lambda_k |p(x_k)|^2.$$

Two problems arise however, in order to make the previous construction effective:

- localization of the x_i
- tuning of the multipliers λ_i

In order to obtain an estimation of the x_i, we have chosen to discretize the module constraint at m points, for which a Chebichev distribution is shown to be well-suited. One shows then that the solution of the discretized problem converges to p_0 for large m. Moreover, fine estimates of the errors relative to the respect of the module constraint are available as functions of m.

Concerning the tuning of the Lagrange multipliers, we decided to solve the dual problem of concave maximization associated to the dicretized version of (3). The constraints in this maximization problem are linear positivity constraints of the multipliers. The computation of the gradient and the Hessian associated to this problem allowed the implementation of an efficient algorithm for solving (3) inside the PRESTO-HF software. Note also that techniques similar to those proposed here are under study, in order to merge them with the solution of the problem explained in section 6.7.

6.12. Exhaustive determination of constrained realizations corresponding to a transfer function

Participants: Laurent Baratchart, Jean Charles Faugere [project SPACES, Rocquencourt], Fabien Seyfert.

We studied in some generality the case of parameterized linear systems characterized by the following classical state space equation,

$$\begin{aligned} \dot{x}(t) &= A(p)x(t) + B(p)u(t) \\ y(t) &= C(p)x(t) \end{aligned} \tag{4}$$

where $p = \{p_1, \cdots p_r\}$ is a finite set of r parameters and $(A(p), B(p), C(p))$ are matrices whose entries are polynomials (over the field $\mathbb{C}$) of the variables $p_1 \cdots p_r$. For a parameterized system σ and $p \in \mathbb{C}^r$ we call $\pi_\sigma(p)$ the transfer function of the system $\sigma(p)$. Some important questions in filter synthesis concern the determination of the following parameterized sets

$$\begin{aligned} p \in \mathbb{C}^{r_1},\ E_{\sigma_1}(p) &= \{q \in \mathbb{C}^{r_1}, \pi_{\sigma_1}(q) = \pi_{\sigma_1}(p)\} \\ p \in \mathbb{C}^{r_2},\ E_{\sigma_1,\sigma_2}(p) &= \{q \in \mathbb{C}^{r_1}, \pi_{\sigma_1}(q) = \pi_{\sigma_2}(p)\} \end{aligned} \tag{5}$$

General results were obtained about these sets, in particular a necessary and sufficient condition ensuring their cardinals are finite. In the special case of coupled-resonators an efficient algebraic formulation has been derived which allowed us to compute $E_{\sigma(p)}$ for nearly all common filter geometries. The latter formulation breaks in particular a natural symmetry of the problem and this in turn simplifies tremendously the computation of a Gröbner basis of the associated algebraic system.

Our next goal is to build a software package for users of the filtering community which implements our ideas (the package almost already exists but in a prototypic form). Theoretically there remains a striking question concerning the generic existence of a "real solution" to the filter realization problem when starting from a loss-less transfer. Results have already been obtained in this direction for particular coupling geometries but we conjecture a much more general property to hold.

6.13. Frequency Approximation and OMUX design

Participants: Laurent Baratchart, Jean-Paul Marmorat [CMA-EMP], Fabien Seyfert.

An OMUX (Output MUltipleXor) can be modeled in the frequency domain by chaining of scattering matrices of filters as those described in section 4.3.2, connected in parallel to a common access *via* a wave guide, see figure 5. The problem of designing the OMUX so as to satisfy gauge constraints is then naturally translated into a set of constraints on the values of the scattering matrices and phase shift introduced by the guides in the considered bandwidth.

In a first step, in order to be able to test our methods and to compare them with the tuning done by Alcatel Space, we have designed an OMUX simulator on a matlab platform. The direct approach, as used by the manufacturer, is of course to couple this simulator with an optimizer, in order to reduce transmission and reflection wherever they are too large. This is what we have done, using the matlab optimizer, choosing an integral L^p criterion, with $p = 16$, something that is near the uniform norm, but still differentiable. We have also explicitly computed the gradient of the criterion, and this generated an important improvement of the computational time (say, by a factor of ten), helpful for further developments. However, the results are similar to those obtained by Alcatel Space on a non-satisfactory example. We thus believe that this problem requires a more specific approach.

Thus, we have observed, that, for each frequency, the constraints can be interpreted as a sequence of conditions that concern each channel one after the other, and express that the reflection, evaluated at this frequency, belongs to a disk whose center and radius depend on the channels and the lengths of the guides that are not adjacent to the considered one. The hyperbolic geometry comes into play naturally, *via* the chaining formulas, and it produces a relative decoupling between the different parameters (channel length and filter). In particular, this shows that the tuning of each filter and each length should be possible in a diagonal manner, if we had an efficient rational approximation algorithm with pointwise constraints (the approximant should be Schur). This is an interesting question, both for applications and in itself, that will be studied in the future.

As a result, we should be able to construct a multi-phased tuning procedure, first relaxed, channel after channel, then global, using a quasi-Newton method. Note the the discretizations in frequency of the integral criterion and the near periodicity of the exponentials (that express the delays) interact in a complex manner, and generate numerous local minima.

7. Contracts and Grants with Industry

7.1. Contract CNES-IRCOM-INRIA

Contract n^{o}1 03 E 1034

In the framework of a contract that links CNES, IRCOM and Inria, whose objective is to realize a software package for identification and design of hyperfrequency devices, the work of Inria has been

- modeling and analysis of an IMUX, see module 4.3.2,
- study of the structure and computation of the coupling parameters associated to physical parameters for a given geometry. (see module 6.12),
- turbo-engine for hyperion,
- modeling and algorithmic analysis of an OMUX see module 4.3.2.

In this contract, we promised version 0.57 of hyperion to both partners. This contract has been renewed in 2003.

7.2. Contract Alcatel Space (Toulouse)

Selling of a license of hyperion, RARL2 and RGC.

7.3. Contract Alcatel Space (Cannes)

Contract n^{o}1 01 E 0726.

This contract started in 2001, for three years. The objective is to find control laws for posting spaceships (satellites) with new generation engines, that have excellent throughput, but a very low thrust.

7.4. Contract Alcatel CIT

Contract n^{o}1 02 E 0517. This was a one year contract, that ended formally in February, 2003.

Subject. Digital signals in optic fiber networks need some “regeneration” and also converting from a wavelength to another. The most powerful way is to avoid decoding the signal and regenerate in a purely analogic way using nonlinear optic components. The device under consideration was based on a SOA-MZI (Mac Zendher Interferometer using Semi-conductor Optical Amplifiers). Its tuning is very delicate, and is very sensitive to variations of the input signal (and these variations do occur). The goal was to set up a control procedure for such a device, to compensate for variations in the input signal. A regulation for a simpler device was already available, and a multivariable control was needed here.

Outcome. We have contributed to develop a control law that performed well on the laboratory experiments. Alcatel decided to file a patent [26] concerning this control procedure. The main reason of this success was a modeling effort.

8. Other Grants and Activities

8.1. Scientific Committees

L. Baratchart is member of the editorial board of *Computational Methods in Function Theory*.

8.2. National Actions

Together with project-teams Caiman and Odyssée (INRIA-Sophia Antipolis, ENPC), the University of Nice (J.A. Dieudonné lab.), CEA, CNRS-LENA (Paris), and a few French hospitals, we are part of the national action **ACI Masse de données « OBS-CERV »**, 2003-2006 (inverse problems, EEG).

The **region PACA** (Provence Alpes Côte d'Azur) is partially supporting the post-doctaral stay of Per Enquist until May, 2004. We also obtained a (modest) grant from the region for exchanges with SISSA Trieste (Italy), 2003-2004.

8.3. Actions Funded by the EC

The Team is member of the **TMR network** *European Research Network on System Identification* (ERNSI), see http://www.cwi.nl/~schuppen/ernsi/ernsihp.html. This formally ended in February. A new proposal of a Research Training Network (RTN) has been submitted to the EC.

The team obtained a **Marie Curie EIF** (Intra European Fellowship) FP6-2002-Mobility-5-502062, for 24 months (2003-2005). This finances Mario Sigalotti's post-doc.

The Team is a member of the **Marie Curie multi-partner training site** *Control Training Site*, number HPMT-CT-2001-00278, 2001-2005. See http://www.supelec.fr/lss/CTS/.

The project is member of Working Group Control and System Theory of the **ERCIM** consortium, see http://www.ladseb.pd.cnr.it/control/ercim/control.html.

8.4. Extra-european International Actions

NATO CLG (Collaborative Linkage Grant), PST.CLG.979703, « Constructive approximation and inverse diffusion problems », with Vanderbilt Univ. (Nashville, USA) et le LAMSIN-ENIT (Tunis, Tu.), 2003-2005.

8.5. Exterior research visitors

In addition to the "Scientific advisors" and to the "Visiting scientists" listed in section 1, the following scientists visited us in 2003.

- Mohamed Jaoua (Lamsin-ENIT, Tunis).
- Herbert Stahl (TU Berlin).
- Nejat Olgac, Univ. of Connecticut (Mechanical Engineering), "On Linear Time Invariant, Time Delayed Systems (Lti-Tds)".
- Emmanuelle Crepeau, Université Paris Sud (candidate CR2 2003).
- Bronislaw Jakubczyk, Académie des Sciences de Pologne, Varsovie, "Classification des systèmes de contrôle sur le plan et de leurs bifurcations",
- Pascale Vitse, Université Laval à Québec, "Une approche tensorielle au problème de la couronne opératorielle".
- Pascale Vitse, Université de Besançon, "Interpolation Libre Par Des Polynômes De Degré Fixé".
- Grégoire Charlot, SISSA, Trieste (Italie), (candidat CR2 2003), "Contrôle optimal pour les systèmes quantiques à n niveaux d'énergie".
- Maureen Clerc, INRIA, Team Odyssée, "l'électro-encephalographie - problème direct et inverse".
- Tarek Hamel, Laboratoire LSC FRE-CNRS 2494, Univ. d'Evry Val d'Essonne, "Modélisation et stabilisation d'un drone à 4 voilures tournantes".
- Vladimir Peller, Michigan State University, Mini cours sur la Théorie analytique des opérateurs à valeurs vectorielles (AAK matriciel).
- Benedicte Dujardin, Observatoire de la Côte d'Azur, "Polynômes orthogonaux de Szego et approximation rationnelle."

9. Dissemination

9.1. Teaching

Courses

- D. Avanessoff gave lectures in general mathematics at University of Nice - Sophia Antipolis.
- L. Baratchart, DEA Géométrie et Analyse, LATP-CMI, Univ. de Provence (Marseille).
- J. Leblond teaches mathematics in the 12-15 cycle of Montessori les Pouces Verts.

Trainees

- Antoine Chaillet, « Fonction de Lyapunov contrôlée pour le transfert d'orbite avezc rendez-vous en faible poussée » (control Lyapunov functions for low thrust orbital transfer). DEA Université Paris-sud (Orsay).

Ph.D. Students

- David Avanessoff, « Linéarisation dynamique des systèmes non linéares et parametrage de l'ensemble des solutions » (dynamic linearization of non linear control systems, and parameterization of all trajectories).
- Fehmi Ben Hassen, « Localisation de sources ponctuelles par approximation rationnelle et méromorphe », co-tutelle with Lamsin-ENIT (Tunis).
- Alex Bombrun, « Commande optimale, feedback, et tranfert orbital de satellites » (optimal control, feedback, and orbital transfert for low thrust satttelite orbit transfer)
- Imen Fellah, "Data completion in Hardy classes and applications to inverse problems", co-tutelle with Lamsin-ENIT (Tunis).

Ph.D. thesis defended

- Reinhold Küstner, "Asymptotic Zero Distribution of Orthogonal Polynomials with respect to Complex Measures having Argument of Bounded Variation", May 27, 2003.

9.2. Community service

F. Wielonsky is on leave to the University of Lille.

J.-B. Pomet is in charge of organizing a seminar on control and identification.

L. Baratchart is a member of the "bureau" of the CP (Comité des Projets) of INRIA-Sophia Antipolis.

J. Grimm is a member of the CUMI (Comité des utilisateurs des moyens informatiques) of the Research Unit of Sophia Antipolis.

J. Leblond is part of the Colors Committee of INRIA-Sophia Antipolis.

J.-B. Pomet is a representative at the "comité de centre".

Several members of the team have participated in the Direction (co-director: J. Leblond), Scientific (L. Baratchart), and Organization (J. Grimm, F. Limouzis) Committees of the CNRS-INRIA summer school "Harmonic analysis and rational approximation: their rôles in signals, control and dynamical systems theory", Porquerolles, september. http://www-sop.inria.fr/miaou/anap03/index.en.html

The whole team has also been deeply involved in establishing and writing down the proposition of a new project-team, named Apics.

9.3. Conferences and workshops

Talks, courses, sessions, software demonstrations at the CNRS-INRIA summer school "Harmonic analysis and rational approximation: their rôles in signals, control and dynamical systems theory", Porquerolles, september. http://www-sop.inria.fr/miaou/anap03/index.en.html

J. Grimm gave a talk about Tralics at Eurotex 2003 (Brest)

David Avanessoff and Mario Sigalotti gave talks at the "2nd Junior European Meeting Control Theory and Stabilization", Torino, It.

J. Leblond was invited to give a talk at the Applied Analysis Seminar, LATP, Univ. Provence (Aix-Marseille I), at the MEEG Workshop in UTC, Compiègne, "Inverse problems in medical imaging : sources localization for EEG/MEG" and the Infinite Dimensional Dynamical Systems (IDDS), Exeter, UK.

M. Olivi has given a lecture at CDC 2003, Maui, Hawaii (USA), 9-12 December.

F. Seyfert gave a talk at the "Journées nationales du calcul formel 2003" about the use of computer algebra based methods for the exhaustive computation of couplings parameters, at "Advances in constructive approximation (Nashville)" about the mixed (L^2, L^∞) bounded extremal problem and at the "IMS 2003 (Philadelphia)" about the determination of a rational stable model from measured scattering data.

L. Baratchart was an invited speaker at "Advances in Constructive Approximation" Conference , May 2003, Vanderbilt University (Tenessee), and at the colloquium of Michigan State University (East Lansing) in March 2003.

F. Wielonsky delivered a talk at the workshop "Complex Analysis and Inverse Problems", December 15-19, I.H.P. (Paris).

10. Bibliography

Major publications by the team in recent years

[1] L. BARATCHART, M. CARDELLI, M. OLIVI. *Identification and rational L^2 approximation: a gradient algorithm.* in « Automatica », volume 27, 1991, pages 413-418.

[2] L. BARATCHART, J. GRIMM, J. LEBLOND, M. OLIVI, F. SEYFERT, F. WIELONSKY. *Identification d'un filtre hyperfréquence par approximation dans le domaine complexe.* Rapport technique, number RT-219, Inria, 1998, http://www.inria.fr/rrrt/rt-0219.html.

[3] L. BARATCHART, J. LEBLOND, F. MANDRÉA, E. SAFF. *How can meromorphic approximation help to solve some 2D inverse problems for the Laplacian?.* in « Inverse Problems », volume 15, 1999, pages 79–90.

[4] L. BARATCHART, M. OLIVI. *Critical points and error rank in best H^2 matrix rational approximation of fixed McMillan degree.* in « Constructive Approximation », volume 14, 1998, pages 273-300.

[5] L. BARATCHART, E. B. SAFF, F. WIELONSKY. *A criterion for uniqueness of a critical point in H^2 rational approximation.* in « Journal d'Analyse », volume 70, 1996, pages 225-266.

[6] L. BARATCHART, F. WIELONSKY. *Rational approximation in the real Hardy space H_2 and Stieltjes integrals: a uniqueness theorem.* in « Constructive Approximation », volume 9, 1993, pages 1-21.

[7] P. FULCHERI, M. OLIVI. *Matrix rational H^2-approximation: a gradient algorithm based on Schur analysis.* in « SIAM J. on Control & Optim. », volume 36, 1998, pages 2103-2127.

[8] J. LEBLOND, M. OLIVI. *Weighted H^2 approximation of transfer functions.* in « Math. of Control, Signals & Systems (MCSS) », volume 11, 1998, pages 28-39.

[9] J.-B. POMET. *Explicit Design of Time-Varying Stabilizing Control Laws for a Class of Controllable Systems without Drift.* in « Syst. & Control Lett. », volume 18, 1992, pages 147-158.

[10] J.-B. POMET. *On Dynamic Feedback Linearization of Four-dimensional Affine Control Systems with Two Inputs.* in « Control, Optimization, and the Calculus of Variations (COCV) », volume 2, June, 1997, pages 151-230, http://www.edpsciences.com/cocv/.

Doctoral dissertations and "Habilitation" theses

[11] R. KÜSTNER. *Asymptotic Zero Distribution of Orthogonal Polynomials with respect to Complex Measures having Argument of Bounded Variation.* Ph. D. Thesis, Université de Nice, April, 2003.

Articles in referred journals and book chapters

[12] L. BARATCHART, J. GRIMM, J. LEBLOND, J. R. PARTINGTON. *Approximation and interpolation in H^2: Toeplitz operators, recovery problems and error bounds.* in « Integral Equations and Operator Theory », volume 45, 2003, pages 269–299.

[13] S. CHAABANE, M. JAOUA, J. LEBLOND. *Parameter identification for Laplace equation and approximation in Hardy classes.* in « J. of Inverse and Ill-posed Problems », number 11, volume 11, 2003, pages 33–57.

Publications in Conferences and Workshops

[14] D. AVANESSOFF, L. BARATCHART, J.-B. POMET. *In the integrability of the equations of "flat outputs" of control systems.* in « AMAM (Applied Math. & Applic. of Math.) », SMAI-EMS-SMF, Nice, February, 2003.

[15] L. BARATCHART, A. BEN ABDA, F. BEN HASSEN, J. LEBLOND. *Sources identification using meromorphic approximation.* in « AMAM (Applied Math. & Applic. of Math.) », SMAI-EMS-SMF, Nice, February, 2003.

[16] J. GRIMM. *Tralics, a LaTeX to XML Translator.* in « Proceedings of Eurotex », 2003.

[17] X. LITRICO, J.-B. POMET. *Nonlinear modelling of a long river stretch.* in « European Control Conference ECC'03, Cambridge (UK) », September, 2003.

[18] M. OLIVI, J.-P. MARMORAT, B. HANZON, R. PEETERS. *Schur parametrizations and balanced realizations of real discrete-time stable all-pass systems.* in « CDC 2003 », Maui, Hawaii, December, 2003.

[19] F. SEYFERT, J.-P. MARMORAT, L. BARATCHART, S. BILA, J. SOMBRIN. *Extraction of Coupling Parameters For Microwave Filters: Determination of a Stable Rational Model from Scattering Data.* 2003.

Internal Reports

[20] D. AVANESSOFF, L. BARATCHART, J.-B. POMET. *Sur l'intégrabilité (très) formelle d'une partie des équations de la platitude des systèmes de contrôle.* Rapport de recherche, number 5045, INRIA, December,

2003, http://www.inria.fr/rrrt/rr-5045.html.

[21] L. BARATCHART, M. CHYBA, J.-B. POMET. *On the Grobman-Hartman theorem for control systems.* Rapport de recherche, number 5040, INRIA, December, 2003, http://www.inria.fr/rrrt/rr-5040.html, submitted to J. of differential equations.

[22] L. BARATCHART, P. ENQVIST, A. GOMBANI, M. OLIVI. *Surface Acoustic Wave Filters, Unitary Extensions and Schur Analysis.* Technical report, number 44, Mittag-Lefler Institute, 2003.

[23] F. MONROY-PÉREZ, A. ANZALDO-MENESES. *Integrability of nilpotent sub-Riemannian structures.* Rapport de recherche, number 4836, INRIA, May, 2003, http://www.inria.fr/rrrt/rr-4836.html.

[24] F. MONROY-PÉREZ, A. ANZALDO-MENESES. *The nilpotent $(n, n(n+1)/2)$ sub-Riemannian problem.* Technical report, number 4990, INRIA, November, 2003, http://www.inria.fr/rrrt/rr-4990.html.

[25] C. ROMERO-MELÉNDEZ, F. MONROY-PÉREZ, J.-P. GAUTHIER. *On complexity and motion planning for co-rank one sub-Riemannian metrics.* Technical report, number 4882, INRIA, July, 2003, http://www.inria.fr/rrrt/rr-4882.html.

Miscellaneous

[26] EUROPEAN PATENT NO. 03292257.7-. European patent office, September, 2003, Title: “wavelength converter”. Applicant/proprietor: Alcatel. Inventors: B. Lavigne, O. Leclerc, J.-P. Moncelet, A. Bombrun, F. Seyfert, J.-B. Pomet.

Bibliography in notes

[27] D. ALPAY, L. BARATCHART, A. GOMBANI. *On the Differential Structure of Matrix-Valued Rational Inner Functions.* in « Operator Theory : Advances and Applications », volume 73, 1994, pages 30-66.

[28] E. ARANDA-BRICAIRE, C. H. MOOG, J.-B. POMET. *An Infinitesimal Brunovsky Form for Nonlinear Systems with Applications to Dynamic Linearization.* in « Banach Center Publications », volume 32, 1995, pages 19-33.

[29] Z. ARTSTEIN. *Stabilization with relaxed control.* in « Nonlinear Analysis TMA », volume 7, 1983, pages 1163-1173.

[30] L. BARATCHART. *Rational and meromorphic approximation in L^p of the circle: System-theoretic motivations, critical points and error rates.* in « Computational Methods in Function Theory (CMFT'97) », World Scientific Publishing Co., N. PAPAMICHAEL, S. RUSCHEWEYH, E. SAFF, editors, pages 1-34, 1998.

[31] L. BARATCHART, A. BEN ABDA, F. BEN HASSEN, J. LEBLOND. *Pointwise sources recovery and approximation.* in preparation.

[32] L. BARATCHART, P. ENQVIST, A. GOMBANI, M. OLIVI. *Surface Acoustic Wave Filters, Unitary extensions and Schur analysis.* in « Mittag-Leffler publications », submitted.

[33] L. BARATCHART, R. KÜSTNER. *Pole behaviour in identification.* in « 39th IEEE Conf. on Decision and Control (CDC) », Sydney (Australie), December, 2000.

[34] L. BARATCHART, R. KÜSTNER, F. MANDRÉA, V. TOTIK. *Pole distribution from orthogonality.* 2002, en préparation.

[35] L. BARATCHART, J. LEBLOND. *Hardy approximation to L^p functions on subsets of the circle with $1 \leq p < \infty$.* in « Constructive Approximation », volume 14, 1998, pages 41-56.

[36] L. BARATCHART, J. LEBLOND, J.-P. MARMORAT. *Sources identification in 3D balls using meromorphic approximation in 2D disks.* in preparation.

[37] L. BARATCHART, J. LEBLOND, J. R. PARTINGTON. *Problems of Adamjan-Arov-Krein type on subsets of the circle and minimal norm extensions.* in « Constructive Approximation », volume 16, 2000, pages 333-357.

[38] L. BARATCHART, J. LEBLOND, J. PARTINGTON. *Hardy approximation to L^∞ functions on subsets of the circle.* in « Constructive Approximation », volume 12, 1996, pages 423-435.

[39] L. BARATCHART, J. LEBLOND, F. SEYFERT. *A pointwise constraint for H^2 approximation on subsets of the circle.* en préparation.

[40] L. BARATCHART, F. MANDREA, E. SAFF, F. WIELONSKY. *Asymptotic behaviour of poles of rational and meromorphic approximants: Application to 2D inverse problems for the Laplacian.* en préparation.

[41] L. BARATCHART, V. A. PROKHOROV, E. B. SAFF. *Asymptotics for minimal Blaschke productsand Best L^1 meromorphic approximants of Markov functions.* in « Computational Methods and Function Theory », number 2, volume 1, 2002, pages 501-520.

[42] L. BARATCHART, V. A. PROKHOROV, E. B. SAFF. *On Blaschke products associated with n-widths.* in « Journal of Approximation Theory », 2004, to appear.

[43] L. BARATCHART, F. SEYFERT. *An L^p analog to AAK theory for $p \geq 2$.* in « J. Funct. Anal. », number 1, volume 191, 2002, pages 52–122.

[44] L. BARATCHART, H. STAHL, F. WIELONSKY. *Asymptotic uniqueness of best rational approximants of given degree to Markov functions in L^2 of the circle.* in « Constr. Approx. », number 1, volume 17, 2001, pages 103–138.

[45] A. BEN ABDA, M. KALLEL, J. LEBLOND, J.-P. MARMORAT. *Line-segment cracks recovery from incomplete boundary data.* in « Inverse problems », number 4, volume 18, 2002, pages 1057-1077.

[46] A. BEN ABDA, J. LEBLOND, M. MAHJOUB, J. PARTINGTON. *Analytic extensions on circular domains and Cauchy-type inverse problems.* in preparation.

[47] S. BILA, D. BAILLARGEAT, M. AUBOURG, S. VERDEYME, P. GUILLON, F. SEYFERT, J. GRIMM, L. BARATCHART, C. ZANCHI, J. SOMBRIN. *Direct Electromagnetic Optimization of Microwave Filters.* in

« IEEE Microwave Magazine », volume 1, 2001, pages 46-51.

[48] R. W. BROCKETT. *Asymptotic Stability and Feedback Stabilization.* in « Differential Geometric Control Theory », series Prog. Math., volume 27, Birkäuser, pages 181-191, Basel-Boston, 1983.

[49] S. CHAABANE, I. FELLAH, M. JAOUA, J. LEBLOND. *Logarithmic stability estimates for a Robin coefficient in 2D Laplace inverse problems.* en préparation.

[50] L. FAUBOURG. *Construction de fonctions de Lyapunov contrôlées et stabilisation non-linéaire.* Ph. D. Thesis, Univ. de Nice - Sophia Antipolis, December, 2001.

[51] L. FAUBOURG, J.-B. POMET. *Control Lyapunov functions for homogeneous "Jurdjevic-Quinn" systems.* in « Control, Optimization, and the Calculus of Variations (COCV) », volume 5, 2000, pages 293-311, http://www.edpsciences.com/cocv/.

[52] L. FAUBOURG, J.-B. POMET. *Nonsmooth functions and uniform limits of control Lyapunov functions.* in « 41st IEEE Conf. on Decision and Control », Las Vegas (USA), December, 2002.

[53] M. FLIESS, J. LÉVINE, P. MARTIN, P. ROUCHON. *Flatness and Defect of Nonlinear Systems: Introductory Theory and Examples.* in « Int. J. of Control », volume 61, 1995, pages 1327–1361.

[54] A. GONCHAR, E. RAKHMANOV. *Equilibrium distributions and the degree of rational approximation of analytic functions.* in « Math. USSR Sbornik », volume 176, 1989, pages 306-352.

[55] J. GRIMM. *Outils pour la manipulation du rapport d'activité.* Technical report, number RT-0265, Inria, 2002, http://www.inria.fr/rrrt/rt-0265.html.

[56] E. HANNAN, M. DEISTLER. *The Statistical Theory of Linear Systems.* Wiley, 1988.

[57] B. JACOB, J. LEBLOND, J. R. PARTINGTON. *A constrained approximation problem arising in parameter identification.* in « Linear Algebra and its Applications », volume 351-352, 2002, pages 487-500.

[58] J. LEBLOND. *Solution of inverse diffusion problems by analytic approximation with real constraints.* in preparation.

[59] J. LEBLOND, E. SAFF, F. WIELONSKY. *Weighted H_2 rational approximation and consistency properties.* in « Numerische Mathematik », number 3, volume 90, 2002, pages 521-554, DOI 10.1007/s002110100281.

[60] P. MARTIN, R. M. MURRAY, P. ROUCHON. *Flat Systems.* in « European Control Conference, Plenary Lectures and Mini-Courses », G. BASTIN, M. GEVERS, editors, pages 211-264, 1997.

[61] O. PARFENOV. *Estimates of the singular numbers of a Carleson operator.* in « Math USSR Sbornik », number 2, volume 59, 1988, pages 497-514.

[62] R. PEETERS, B. HANZON, M. OLIVI. *Balanced parametrizations of discrete-time all-pass systems and the tangential Schur algorithm.* in « Proc. of the European Control Conference (cd-rom) », Karlsruhe (Allemagne),

September, 1999.

[63] F. SEYFERT. *Problèmes extrémaux dans les espaces de Hardy, Application à l'identification de filtres hyperfréquences à cavités couplées.* Ph. D. Thesis, Ecole de Mines de Paris, 1998.

[64] H. STAHL. *The convergence of Padé approximants to functions with branch points.* in « J. of Approximation Theory », volume 91, 1997, pages 139–204.