INRIA

INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

Project-Team NUMOPT

Numerical Optimization

Rhône-Alpes

THEME 4A

Activity Report

2003

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1. Team

This project has ceased its activities in June 2003.

Head of project-team

Claude Lemaréchal [senior research scientist]

administrative assistant

Françoise de Coninck [jointly with IS2, Helix]

Staff members Inria

François Oustry [junior research scientist (on secondment since VIII 1 2001)]

Pierre-Brice Wieber [junior research scientist]

Ph. D. student

Jérôme Malick [ENS fellowship]

2. Overall Objectives

Numopt is concerned with various aspects of algorithms for numerical optimization:

- *Research*, which can be motivated by present application problems, or by important activity in the international scientific community;
- *Development* of optimization algorithms, motivated by general classes of problems;
- *Applications*, i.e. counsultancy to partners in industry or in other branches of science, for specific problems they are faced with.

Generally speaking, we deal mostly with: nonsmooth optimization, positive semidefinite programming, links between combinatorial and continuous optimization.

3. Scientific Foundations

Key words: *optimization*, *numerical algorithm*, *convexity*, *Lagrangian relaxation*, *SDP relaxation*.

This project deals with the numerical optimization of a function f of n variables over a domain $D \subset \mathbb{R}^n$, say

$$\min f(x)\,, \quad \text{with } x = \{x_1, ..., x_n\} \in D\,. \tag{1}$$

We specialize in various situations, listed by decreasing order of theoretical level:

1. Case where first derivatives of f are discontinuous. Algorithms exist (*bundle, analytic center*) and are applied to problems of various origins. Our research deals with acceleration of these algorithms, and this implies a suitable generalization of the concept of *second order* at a point where first derivatives do not exist.
2. *Combinatorial problems*, where D is a finite set, typically a subset of $\{0, 1\}^n$. We have no special competence in this large field; but it turns out that *convex analysis* plays there a useful role, although still misappreciated by the scientific community (*Lagrangian, SDP relaxations* and others). Here we are at the corner point of three domains: continuity, combinatorics, convexity.
3. Eigenvalue problems, or *positive semidefinite* optimization (SDP). Here $\mathbb{R}^n$ is actually $\mathbb{R}^{m(m+1)/2}$, the space of symmetric matrices. Typically, f is then the maximal eigenvalue function (or, which is similar, f is linear but D is the set of positive semidefinite matrices). In this subject, we work in two directions: applications (robust control, combinatorial optimization – item 1. above –,...) with SDP *relaxation*, and methodology, solving SDP problems via nonsmooth optimization (item 1.), which thus complements *interior-point* methods.

4. More "classical" problems, where D is either the whole of $\mathbb{R}^n$, or defined by *constraints* $c_i(x) \leq 0$, f and the c_i's being smooth; n is possibly large (10^5 and more). Here we act most of the time as consultants, on various levels: modelling, choosing an approach, suggesting suitable software (*Modulopt*, Estime or Mocoa team, or also external library).

4. Application Domains

Optimization exists in virtually all economic sectors. Simulation tools can be used to optimize the system they simulate. Another domain is parameter *identification* (Idopt or Estime teams), where the deviation between measurements and theoretical predictions must be minimized. Accordingly, giving an exhaustive list of applications is impossible. Some past domains where Inria has been involved in the past, possibly through the former Promath team: production management, geophysics, finance, molecular modelling, robotics, networks, astrophysics, crystallography,...

5. Software

Essentially two possibilities exist to distribute our software: library programs (say Modulopt codes), communicated either freely or not, depending on what they are used for, and on the other hand specific software, developed for a given application.

The following optimization codes have been developed in the framework of the former Promath project.

5.1.1. Code M1QN3

Optimization without constraints for problems with many variables ($n \geq 10^3$, has been used for $n = 10^6$). Technically, uses a limited-memory BFGS algorithm with Wolfe's line-search (see [2] or [1] for the terminology).
Participants: Jean-Charles Gilbert [Estime team – partner], Claude Lemaréchal [partner].

5.1.2. Code M2QN1

Optimization with simple bound-constraints for (small) problems: D is a parallelotope in $\mathbb{R}^n$. Uses BFGS with Wolfe's linesearch and active-set strategy.
Participant: Claude Lemaréchal.

5.1.3. Code N1CV2

Minimization without constraints of a convex nonsmooth function by a proximal bundle method ([4], [2], [1]).
Participants: Claude Lemaréchal [partner], Claudia Sagastizábal.

5.1.4. Modulopt

In addition to codes such as above, the Modulopt library contains application problems, synthetic or from the real world. It is a field for experimentation, functioning both ways: to assess a new algorithm on a set of test-problems, or to select among several codes one best suited to a given problem.
Participants: Jean-Charles Gilbert [Estime team – partner], Claude Lemaréchal [partner].

6. New Results

6.1. Eigenvalue optimization

Participant: Jerôme Malick.

An important class of optimization problems is semi-definite programming, in which the control variables involve a symmetric matrix, subject to being positive semi-definite; the standard problem is SDP linear programming. Motivated by applications in finance, we have started to work on the quadratic version of SDP [14]:

$$\min \|X - M_0\|^2 \,, \quad X \succcurlyeq 0 \,, \quad \langle M_j, X \rangle = b_j \,,\ j = 1, ..., m \,,$$

where the M_j's are given symmetric matrices (see Numopt Activity Report 2001). We have resumed this activity in two directions:

On the theoretical side, we continue the work of Scott Miller, relating the concept of $\mathcal{U}$-Lagrangian with differential geometry (§6.1 in Numopt's Activity Report 2002). In fact, the above quadratic SDP problem can be solved via a Newton-like method [15], the generalized Hessian being computed using works of Lewis-Sendov [13]. We have implemented this method and the present work would validate it theoretically.

With P. N'Diaye and S. Volle, from Raise Partner, we work on applications of quadratic SDP on three levels: expertise and transfer of knowledge, validation of Raise Partner's software, robust solution of Markovitz problem and multifactor analysis. We also work on new relaxations of combinatorial problems, based on quadratic SDP relaxations instead of the traditional one, as in [7].

6.2. Nonsmooth optimization for column generation

Participant: Claude Lemaréchal.

Column generation, a rather common technique in combinatorial optimization, is traditionally implemented via the algorithm of Dantzig-Wolfe, which is a particular nonsmooth-optimization algorithm (among the worst; see [6] for more explanations).

With the Universities of Bordeaux (F. Vanderbeck, O. Briant) and Geneva (J.-Ph. Vial) we have started a collaborative work, to graft alternative algorithms (bundle, accpm) in BapCod [16], a software for Branch & Price. Our bundle code is being thus grafted and preliminary tests have been conducted. They indicate similar behaviours for small problems (cutting stock, lot-sizing) and a total collapse of Dantzig-Wolfe for bigger ones (Held-Karp relaxation of TSP).

This work is funded by the Inria new investigation grant ODW, extending over 2003-2004.

6.3. Applications: Electrical production

Participant: Claude Lemaréchal.

The work mentioned last year (Numopt Activity Report 2002, §6.3, see also §6.12) has been concretized by [10], submitted to Mathematical Programming. We plan to insert in the bundle code a tolerance to inaccurate oracles; we will use for this a theory developed by K.C. Kiwiel [12]. The resulting code will be useful to accept difficult constraints in the model. It will also be useful in the framework of §6.2 above, for Lagrangian relaxations with NP-hard subproblems.

9. Dissemination

9.1. Dissemination of software

Dissemination of codes from the Modulopt library (collaboration with Estime team; upon request only[1]). Besides, SG2QN (a variant of M2QN1 accepting numerical computation of derivatives) is used at Univ. Lille (C. Brezinski, identification in an input-output system).

9.2. Teaching

– Ensimag 2nd year "Optimization" (C. Lemaréchal, V. Acary (Bipop team), lectures 9h, tutoring 9h).
– École de Physique, Chimie, Électronique de Lyon, 2nd year "Numerical optimization" (C. Lemaréchal, lectures 16h);

[1] http://www-rocq.inria.fr/estime/modulopt

– University of Paris 1, DEA MMME (C. Lemaréchal, lectures 8h).
– Artelys Lectures "Combinatorics 2", Paris, November 2003 (C. Lemaréchal, lectures 4h).

9.3. Participation to conferences, seminars, invitations

– 3rd FNRS Cycle Meeting in Mathematical Programming, Han sur Lesse (Belgium), January 2003 (C. Lemaréchal, 2 invited lectures).
– Roadef2003, Avignon, February 2003 (C. Lemaréchal, session organizer; J. Malick, 1 lecture).
– 7th Workshop on Combinatorial Optimization, Aussois, March 2003 (C. Lemaréchal, 1 lecture).
– 11th Days of Groupe Mode, Pau, March 2003 (J. Malick, 1 poster).
– Int. Symp. on Math. Prog. 2003, Copenhagen, August 2003 (J. Malick, 1 lecture).
– OR2003, Heidelberg, September 2003 (C. Lemaréchal, 1 lecture)
– Seminar at EdF.

10. Bibliography

Major publications by the team in recent years

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[4] J.-B. Hiriart-Urruty, C. Lemaréchal. *Convex Analysis and Minimization Algorithms.* Springer Verlag, Heidelberg, 1993, Two volumes.

[5] F. Oustry. *The $\mathcal{U}$-Lagrangian of the maximum eigenvalue function.* in « Siam J. on Optimization », number 2, volume 9, 1999, pages 526-549.

[6] C. Lemaréchal. *Lagrangian relaxation.* M. Jünger, D. Naddef, editors, in « Computational Combinatorial Optimization », Springer Verlag, Heidelberg, 2001, pages 115-160.

[7] C. Lemaréchal, F. Oustry. *Semi-definite relaxations and Lagrangian duality with application to combinatorial optimization.* Rapport de Recherche, number 3710, Inria, 1999, http://www.inria.fr/rrrt/rr-3710.html.

[8] C. Lemaréchal, F. Oustry, C. Sagastizábal. *The $\mathcal{U}$-Lagrangian of a convex function.* in « Transactions of the AMS », number 2, volume 352, 2000, pages 711-729.

Articles in referred journals and book chapters

[9] C. Lemaréchal. *The omnipresence of Lagrange.* in « 4OR », number 1, volume 1, 2003, pages 7,25.

Internal Reports

[10] L. DUBOST, R. GONZALEZ, C. LEMARÉCHAL. *A primal-proximal heuristic applied to the unit-commitment problem.* RR, number 4978, Inria, 2003, http://www.inria.fr/rrrt/rr-4978.html.

Miscellaneous

[11] J. MALICK. *A dual approach to semidefinite least-squares.* 2003, Accepted for publication in SIAM Journal on Matrix Analysis and Applications.

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[12] K. KIWIEL. *A proximal bundle method with approximate subgradient linearizations.* 2002, Systems Research Institute, Warsaw.

[13] A. LEWIS, H. SENDOV. *Twice differentiable spectral functions.* in « SIAM Journal on Matrix Analysis and Applications », number 2, volume 23, 2001, pages 368-386.

[14] J. MALICK. *An Efficient Dual Algorithm to Solve Conic Least-Square Problems.* RR 4212, Inria, 2001, http://www.inria.fr/rrrt/rr-4212.html.

[15] L. QI. *Superlinearly convergent approximate Newton methods for LC^1 optimization problems.* in « Mathematical Programming », number 3, volume 64, 1994, pages 277-294.

[16] F. VANDERBECK. *On Dantzig-Wolfe decomposition in integer programming and ways to perform branching in a branch-and-price algorithm.* in « Operations Research », number 1, volume 48, 2000, pages 111-128.