Project-Team SYDOCO

SYstèmes Dynamiques, Optimisation et Commande Optimale

Rocquencourt

THEME 4A
Table of contents

1. Team 1
2. Overall Objectives 1
3. Scientific Foundations 1
4. Application Domains 2
5. Software 2
6. New Results 2
   6.1. Trajectory optimization 2
   6.2. Numerical methods for the HJB equations 2
   6.3. Variational problems in mechanics 3
   6.4. Positive semidefinite optimization 3
   6.5. Optimization with chance constraints 3
   6.6. \(W^1_\text{L}u\)-space decomposition theory Bundle methods for constrained problems 3
   6.7. Industrial applications Energy problems 4
7. Contracts and Grants with Industry 4
   7.1. Trajectory optimization 4
8. Other Grants and Activities 4
   8.1. International collaborations 4
   8.2. Scientific responsabilities outside Inria 4
   8.3. Visites et invitations de chercheurs 4
9. Dissemination 4
   9.1. Animation de la communauté scientifique 4
   9.2. Teaching 5
   9.3. Participation à des colloques, congrès 5
   9.4. Organisation de conférences et séminaires 5
10. Bibliography 5
1. Team

**Head of project-team**
- Frédéric Bonnans [DR Inria]

**Administrative assistant**
- Martine Verneuille [TR Inria]

**Staff member**
- Claudia Sagastizábal [CR, détachée à l’IMPA - Rio de Janeiro]

**Research scientist**
- Mounir Haddou [Maître de conférence, Université d’Orléans]
- Sady Maurin [IR CNRS]
- Housnaa Zidani [Enseignante chercheur, ENSTA]

**Ph. D. student**
- Sandrine Avril [bourse ONERA]
- Julien Laurent-Varin [bourse CNES]
- Stefania Maroso [bourse MESR - depuis octobre]
- Elisabeth Ottenwaelter [IUT Paris]
- Hector Ramirez-Cabrera [Université du Chili]

**Visiting scientist**
- Jose Aliste [Université du Chili, 3 mois]
- Felipe Alvarez [Université du Chili, 2 semaines]
- Radia Bessi-Fourati [ENIT - Tunis, 2 semaines]
- Abdeslem Boukhtouta [Université le Val - Québec, 3 mois]
- Henda El Fekih [ENIT - Tunis, 2 mois]
- Alexander Shapiro [Georgie tech., 1 mois]
- Mikhail Solodov [IMPA - Rio de Janeiro, 3 semaines]

2. Overall Objectives

Develop new algorithms in deterministic and stochastic optimal control, and deal with associated applications, especially for aerospace trajectories and management for the power industries (hydroelectric resources, storage of gas and petroleum).

In the field of deterministic optimal control, our objective is to develop algorithms combining iterative fast resolution of optimality conditions (of the discretized problem) and refinement of discretization, through the use of interior point algorithms. At the same time we wish to study multiarcs problems (separations, rendez-vous, formation flights) which necessitates the use of decomposition ideas.

In the field of stochastic optimal control, our first objective is to develop fast algorithms for problems of dimension two and three, based on fast computation of consistent approximations as well as splitting methods. The second objective is to link these methods to the stochastic programming approach, in order to deal with problems of dimensions greater than three.

3. Scientific Foundations

For deterministic optimal control problems there are basically three approaches. The so-called direct method consists in an optimization of the trajectory, after having discretized time, by a nonlinear programming solver that possibly takes into account the dynamic structure; see Betts [15]. The indirect approach eliminates control variables using Pontryagin’s maximum principle, and solves the resulting two-points boundary value problem by a multiple shooting method. Finally the dynamic programming approach solves the associated Hamilton-Jacobi-Bellman (HJB) equation, which is a partial differential equation of dimension equal to the number
of state variables. This allows to find the global minimum, whereas the two other approaches are local; however, it suffers from the curse of dimensionality (complexity is exponential with respect to \(n\)). There are various additional issues: decomposition of large scale problems, simplification of models (leading to singular perturbation problems), computation of feedback solutions.

For stochastic optimal control problems there are essentially two approaches. The one based on the (stochastic) HJB equation has the same advantages and disadvantages as its deterministic counterpart. The stochastic programming approach is based on a finite approximation of uncertain events called a scenario tree (for problems with no decision this boils down to the Monte Carlo method). Their complexity is polynomial with respect to the number of state variables but exponential with respect to the number of time steps. In addition, various heuristics are proposed for dealing with the case (uncovered by the two other approaches) when both the number of state variables and time steps is large.

4. Application Domains

Aerospace trajectories (rockets, planes), automotive industry (car design), chemical engineering (optimization of transient phases, batch processes). Storage and management, especially of natural and power resources, portfolio optimization.

5. Software

We have presently two research softwares. The first is an implementation of interior point algorithms for trajectory optimization, and the second is an implementation of fast algorithms for bidimensional HJB equations of stochastic control.

6. New Results

6.1. Trajectory optimization

Participants: Sandrine Avril, Frédéric Bonnans, Mounir Haddou, Julien Laurent-Varin.

In collaboration with Nicolas Bérend (Onera-Châtillon) and Christophe Talbot (Direction des Lanceurs, Cnes) in the framework of the thesis of S. Avril, we have studied a two level formulation of the problem of optimal transfer for low-thrust satellites, where the upper level corresponds to the slowly varying orbital parameters. In the framework of the thesis of J. Laurent-Varin, a first version of the interior point algorithm code, based on an Runge-Kutta discretization of arbitrary order, ran for a simplified ascent problem (of a space rocket). This was published in the 5th International Conference on Launcher Technology. We have obtained theoretical results on asymptotically optimal refinement. They will be published in the 16th IFAC Symposium on Automatic Control in Aerospace, St Petersburg, 2004.

6.2. Numerical methods for the HJB equations

Participants: Frédéric Bonnans, Stefania Maroso, Elisabeth Ottenwaelter, Housnaa Zidani.

In the framework of the thesis of E. Ottenwaelter, we have continued the study of fast solvers for two dimensional HJB equations of stochastic control. The relations with a classical concept of arithmetic, namely the Stern-Brocot, have been clarified; they allow a computation of the coefficients of the scheme (at a given point of the state grid and for a given control) in \(O(p)\) iterations, where \(p\) is the size of the stencil. The method has been implemented; the numerical experiments confirmed the theory. The corresponding Inria report will appear in January 2004.

During the internship of S. Maroso (April-July 2003) we have studied the possibility of fast solvers for three dimensional HJB equations of stochastic control. This amounts to the computation of the projection onto a cone in \(R^6\), and hence, is more difficult. We concentrated on schemes using only the closest neighbouring
points. If consistency holds, we could improve an explicit decomposition scheme due to Arturo Pratt (U. Chile). If a projection is needed we have computed a reduced basis on which coefficients must be (but an iterative procedure is needed in order to compute these coefficients). In the framework of the thesis of S. Maroso, started in September 2003, we are currently studying the literature on error estimates for the HJB equations of stochastic control.

6.3. **Variational problems in mechanics**

**Participants:** Frédéric Bonnans, Radhia Bessi Fourati [Enit-Tunisie], Hichem Smaoui [Enit-Tunisie].

We have ended the study on the minimization of potential energy for static equilibrium problems and performed various numerical tests of a relaxation type decomposition scheme. A more or less complete second order sensitivity analysis was performed. The Inria report has been published this year as well as the corresponding paper in [3].

6.4. **Positive semidefinite optimization**

**Participants:** Frédéric Bonnans, Hector Ramirez-Cabrera.

We have obtained some necessary or sufficient conditions for strong regularity (in the sense of Robinson, Math. Oper. Res. 1980) of positive semidefinite optimization problems. We also started a similar study for second order cone problems.

6.5. **Optimization with chance constraints**

**Participants:** Frédéric Bonnans, Agnès Sulem [Mathfi Project], René Aïd [EDF R & D].

In the framework of the internship of N. Boulanger (Mastere ENS Ulm and DEA Paris IX Dauphine) and motivated by the problem of management of water resources for hydraulic energy, we have discussed the optimization of Markov chains with constraints of VaR (value at risk) type and the associated duality gap.

6.6. **\(VU\)-space decomposition theory Bundle methods for constrained problems**

**Participant:** Claudia Sagastizábal.

Consider the problem of minimizing a nonsmooth function that is lower semicontinuous and subdifferentially regular. In [6] we extended the class of functions with “primal dual gradient structure” (pdg) to Lipschitz continuous functions. We showed that, under certain assumptions of transversality, such functions have a \(U\)-Hessian, which is related to the second-order epiderivative; see also [8]. We also showed that pdg functions are “partially smooth”, as defined by A. Lewis.

In [13] we show that when strong transversality is satisfied, there exists a \(C^2\) trajectory leading to \(\pi\) and an associated subdifferential that is \(C^1\). As a result, there exists a space decomposition mapping that is \(C^1\) and a second order expansion of \(f\) on the trajectory. For \(\pi\) a minimizer, we give conditions on \(f\) to ensure that for any point near \(\pi\) its corresponding proximal point is on the trajectory. This purely theoretical result is fundamental for future minimization algorithms and their implementations, since it is known that a sequence of null steps from a bundle mechanism can approximate proximal points with any desired accuracy.

Suppose the function is not pdg structured, but has additional properties of prox-regularity and prox-boundedness. In [9] we make use of \(VU\)-space decomposition theory to connect three minimization-oriented objects. These objects are \(U\)-Lagrangians obtained from minimizing a function over \(V\)-space, proximal points depending on minimization over \(\mathbb{R}^n = U \oplus V\), and epi-derivatives determined by lower limits associated with epigraphs. We relate second-order epi-derivatives of a function to the Hessian of its associated \(U\)-Lagrangian. We also show that the function’s proximal points are on a trajectory determined by certain \(V\)-space minimizers.

In [14] we show that the use of a parametrized penalty function in nonsmooth convex optimization can be avoided without using the relatively complex filter methods. In particular, we propose an approach which appears to be more direct and easier to implement, in the sense that it is closer to the well-developed
unconstrained bundle methods. Preliminary computational results are also reported. Joint work with P. Rey (DII, U. de Chile) and M. Solodov (IMPA).

6.7. Industrial applications Energy problems

Participant: Claudia Sagastizábal.

In [7] we consider the inclusion of commitment of thermal generation units in the optimal management of the Brazilian power system. By means of Lagrangian relaxation we decompose the problem and obtain a nondifferentiable dual function that is separable. We solve the dual problem with a bundle method. Our purpose is twofold: first, bundle methods are the methods of choice in nonsmooth optimization when it comes to solve large-scale problems with high precision. Second, they give good starting points for recovering primal solutions. We use an inexact augmented Lagrangian technique to find a near-optimal primal feasible solution. We assess our approach with numerical results.

7. Contracts and Grants with Industry

7.1. Trajectory optimization

We have agreements of cooperation with Onera and CNRS concerning the studies on transfer or orbits for low-thrust satellites, and optimal trajectories for future launchers.

8. Other Grants and Activities

8.1. International collaborations

- Brazil: Collaboration with IMPA. With C. Sagastizábal we have continued the work on bilevel optimization.
- Canada: Collaboration with U. Laval: with A. Boukhtouta, work on the optimization of management of water.
- Chile: Collaboration with DIM: with F. Alvarez and his student J. Aliste we have started to work on the optimal reentry by a HJB approach.

8.2. Scientific responsibilities outside Inria

F. Bonnans is Vice President for Publications of SMAI, the French Applied Mathematics Society. This means supervising three scientific journals, one proceedings series, and a series of books. It was decided this year to launch another series of books for Master studies.

8.3. Visites et invitations de chercheurs

C. Sagastizábal (IMPA - Brazil), A. Boukhtouta (U. Laval - Canada), F. Alvarez and J. Aliste (DIM - Chile), H. Smaoui and R. Bessi Fourati (ENIT - Tunisie).

9. Dissemination

9.1. Animation de la communauté scientifique

9.2. Teaching

- F. Bonnans/ Professeur chargé de cours, Ecole polytechnique and Course on Continuous Optimization, DEA "OJME" Optimisation, Jeux et Modélisation en Economie, Université Paris VI et Paris X.

9.3. Participation à des colloques, congrès


- 5ème congrès de la Société Française de Recherche Opérationnelle et d’Aide à la Décision. 26-28 février - Avignon - France. F. Bonnans "Optimisation de la conception de moteur hybride".


9.4. Organisation de conférences et séminaires


10. Bibliography

Books and Monographs


Articles in referred journals and book chapters


**Miscellaneous**


[12] **N. Boulanger.** *Optimisation sous contrainte de risque.* 2003, Rapport de stage de DEA.


**Bibliography in notes**