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# 1. Team

*CALVI is a project associating Institut Elie Cartan (IECN, UMR 7502, CNRS, INRIA and Université Henri Poincaré, Nancy), Institut de Recherche Mathématique Avancée (IRMA, UMR 7501, CNRS and Université Louis Pasteur, Strasbourg) and Laboratoire des Sciences de l'Image, de l'Informatique et de la Télédétection (LSIIT, UMR 7005, CNRS and Université Louis Pasteur, Strasbourg) with close collaboration to Laboratoire de Physique des Milieux Ionisés et Applications (LPMIA, UMR 7040, CNRS and Université Henri Poincaré, Nancy).*

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## 2. Overall Objectives

### 2.1. Overall Objectives

CALVI was created in July 2003.

It is a project associating Institut Elie Cartan (IECN, UMR 7502, CNRS, INRIA and Université Henri Poincaré, Nancy), Institut de Recherche Mathématique Avancée (IRMA, UMR 7501, CNRS and Université Louis Pasteur, Strasbourg) and Laboratoire des Sciences de l'Image, de l'Informatique et de la Télédétection (LSIIT, UMR 7005, CNRS and Université Louis Pasteur, Strasbourg) with close collaboration to Laboratoire de Physique des Milieux Ionisés et Applications (LPMIA, UMR 7040, CNRS and Université Henri Poincaré, Nancy).

Our main working topic is modeling, numerical simulation and visualization of phenomena coming from plasma physics and beam physics. Our applications are characterized in particular by their large size, the existence of multiple time and space scales, and their complexity.

Different approaches are used to tackle these problems. On the one hand, we try and implement modern computing techniques like **parallel computing** and **grid computing** looking for appropriate methods and algorithms adapted to large scale problems. On the other hand we are looking for **reduced models** to decrease the size of the problems in some specific situations. Another major aspect of our research is to develop numerical methods enabling us to optimize the needed computing cost thanks to **adaptive mesh refinement** or **model choice**. Work in scientific visualization complement these topics including **visualization of multidimensional data** involving large data sets and **coupling visualization** and **numerical computing**.

## 3. Scientific Foundations

### 3.1. Kinetic models for plasma and beam physics

**Keywords:** *Vlasov equation, asymptotic analysis, beam physics, existence, kinetic models, mathematical analysis, modeling, plasma physics, reduced models, uniqueness.*

#### 3.1.1. Abstract

Plasmas and particle beams can be described by a hierarchy of models including  $N$ -body interaction, kinetic models and fluid models. Kinetic models in particular are posed in phase-space and involve specific difficulties. We perform a mathematical analysis of such models and try to find and justify approximate models using asymptotic analysis.

#### 3.1.2. Models for plasma and beam physics

The **plasma state** can be considered as the **fourth state of matter**, obtained for example by bringing a gas to a very high temperature ( $10^4$  K or more). The thermal energy of the molecules and atoms constituting the gas is then sufficient to start ionization when particles collide. A globally neutral gas of neutral and charged particles, called **plasma**, is then obtained. Intense charged particle beams, called nonneutral plasmas by some authors, obey similar physical laws.

The hierarchy of models describing the evolution of charged particles within a plasma or a particle beam includes  $N$ -body models where each particle interacts directly with all the others, kinetic models based on a statistical description of the particles and fluid models valid when the particles are at a thermodynamical equilibrium.

In a so-called *kinetic model*, each particle species  $s$  in a plasma or a particle beam is described by a distribution function  $f_s(\mathbf{x}, \mathbf{v}, t)$  corresponding to the statistical average of the particle distribution in phase-space corresponding to many realisations of the physical system under investigation. The product  $f_s dx dv$  is

the average number of particles of the considered species, the position and velocity of which are located in a bin of volume  $dx dv$  centered around  $(\mathbf{x}, \mathbf{v})$ . The distribution function contains a lot more information than what can be obtained from a fluid description, as it also includes information about the velocity distribution of the particles.

A kinetic description is necessary in collective plasmas where the distribution function is very different from the Maxwell-Boltzmann (or Maxwellian) distribution which corresponds to the thermodynamical equilibrium, otherwise a fluid description is generally sufficient. In the limit when collective effects are dominant with respect to binary collisions, the corresponding kinetic equation is the *Vlasov equation*

$$\frac{\partial f_s}{\partial t} + \mathbf{v} \cdot \frac{\partial f_s}{\partial \mathbf{x}} + \frac{q}{m} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \frac{\partial f_s}{\partial \mathbf{v}} = 0,$$

which expresses that the distribution function  $f$  is conserved along the particle trajectories which are determined by their motion in their mean electromagnetic field. The Vlasov equation which involves a self-consistent electromagnetic field needs to be coupled to the Maxwell equations in order to compute this field

$$\begin{aligned} -\frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} + \nabla \times \mathbf{B} &= \mu_0 \mathbf{J}, \\ \frac{\partial \mathbf{B}}{\partial t} + \nabla \times \mathbf{E} &= 0, \\ \nabla \cdot \mathbf{E} &= \frac{\rho}{\varepsilon_0}, \\ \nabla \cdot \mathbf{B} &= 0, \end{aligned}$$

which describes the evolution of the electromagnetic field generated by the charge density

$$\rho(\mathbf{x}, t) = \sum_s q_s \int f_s(\mathbf{x}, \mathbf{v}, t) d\mathbf{v},$$

and current density

$$\mathbf{J}(\mathbf{x}, t) = \sum_s q_s \int f_s(\mathbf{x}, \mathbf{v}, t) \mathbf{v} d\mathbf{v},$$

associated to the charged particles.

When binary particle-particle interactions are dominant with respect to the mean-field effects then the distribution function  $f$  obeys the Boltzmann equation

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{x}} = Q(f, f),$$

where  $Q$  is the nonlinear Boltzmann collision operator. In some intermediate cases, a collision operator needs to be added to the Vlasov equation.

The numerical resolution of the three-dimensional Vlasov-Maxwell system represents a considerable challenge due to the huge size of the problem. Indeed, the Vlasov-Maxwell system is nonlinear and posed in phase space. It thus depends on seven variables: three configuration space variables, three velocity space variables and time, for each species of particles. This feature makes it essential to use every possible option to find a reduced model wherever possible, in particular when there are geometrical symmetries or small terms which can be neglected.

### 3.1.3. Mathematical and asymptotic analysis of kinetic models

The mathematical analysis of the Vlasov equation is essential for a thorough understanding of the model as well for physical as for numerical purposes. It has attracted many researchers since the end of the 1970s. Among the most important results which have been obtained, we can cite the existence of strong and weak solutions of the Vlasov-Poisson system by Horst and Hunze [51], see also Bardos and Degond [32]. The existence of weak solution for the Vlasov-Maxwell system has been proved by Di Perna and Lions [40]. The state of the theory is presented in a recent book by Glassey [48].

Many questions concerning for example uniqueness or existence of strong solutions for the three-dimensional Vlasov-Maxwell system are still open. Moreover, there is a realm of approached models that need to be investigated. In particular, the Vlasov-Darwin model for which we could recently prove the existence of global solutions for small initial data [33].

On the other hand, the asymptotic study of the Vlasov equation in different physical situations is important in order to find or justify reduced models. One situation of major importance in Tokamaks, used for magnetic fusion as well as in atmospheric plasmas, is the case of a large external magnetic field used for confining the particles. The magnetic field tends to incurve the particle trajectories who eventually, when the magnetic field is large, are confined along the magnetic field lines. Moreover, when an electric field is present, the particles drift in a direction perpendicular to the magnetic and to the electric field. The new time scale linked to the cyclotron frequency, which is the frequency of rotation around the magnetic field lines, comes in addition to the other time scales present in the system like the plasma frequencies of the different particle species. Thus, many different time scales as well as length scales linked in particular to the different Debye length are present in the system. Depending on the effects that need to be studied, asymptotic techniques allow to find reduced models. In this spirit, in the case of large magnetic fields, recent results have been obtained by Golse and Saint-Raymond [49], [55] as well as by Brenier [37]. Our group has also contributed to this problem using homogenization techniques to justify the guiding center model and the finite Larmor radius model which are used by physicist in this setting [46], [44], [45].

Another important asymptotic problem yielding reduced models for the Vlasov-Maxwell system is the fluid limit of collisionless plasmas. In some specific physical situations, the infinite system of velocity moments of the Vlasov equations can be closed after a few of those, thus yielding fluid models.

## 3.2. Development of simulation tools

**Keywords:** *Numerical methods, Vlasov equation, adaptivity, convergence, numerical analysis, semi-Lagrangian method, unstructured grids.*

### 3.2.1. Abstract

The development of efficient numerical methods is essential for the simulation of plasmas and beams. Indeed, kinetic models are posed in phase space and thus the number of dimensions is doubled. Our main effort lies in developing methods using a phase-space grid as opposed to particle methods. In order to make such methods efficient, it is essential to consider means for optimizing the number of mesh points. This is done through different adaptive strategies. In order to understand the methods, it is also important to perform their mathematical analysis.

### 3.2.2. Introduction

The numerical integration of the Vlasov equation is one of the key challenges of computational plasma physics. Since the early days of this discipline, an intensive work on this subject has produced many different numerical schemes. One of those, namely the Particle-In-Cell (PIC) technique, has been by far the most widely used. Indeed it belongs to the class of Monte Carlo particle methods which are independent of dimension and thus become very efficient when dimension increases which is the case of the Vlasov equation posed in phase space. However these methods converge slowly when the number of particles increases, hence if the complexity of grid based methods can be decreased, they can be the better choice in some situations. This is the reason why one of the main challenges we address is the development and analysis of adaptive grid methods.

### 3.2.3. Convergence analysis of numerical schemes

Exploring grid based methods for the Vlasov equation, it becomes obvious that they have different stability and accuracy properties. In order to fully understand what are the important features of a given scheme and how to derive schemes with the desired properties, it is essential to perform a thorough mathematical analysis of scheme, investigating in particular its stability and convergence towards the exact solution.



### 3.2.4. The semi-Lagrangian method

The semi-Lagrangian method consists in computing a numerical approximation of the solution of the Vlasov equation on a phase space grid by using the property of the equation that the distribution function  $f$  is conserved along characteristics. More precisely, for any times  $s$  and  $t$ , we have

$$f(\mathbf{x}, \mathbf{v}, t) = f(\mathbf{X}(s; \mathbf{x}, \mathbf{v}, t), \mathbf{V}(s; \mathbf{x}, \mathbf{v}, t), s),$$

where  $(\mathbf{X}(s; \mathbf{x}, \mathbf{v}, t), \mathbf{V}(s; \mathbf{x}, \mathbf{v}, t))$  are the characteristics of the Vlasov equation which are solution of the system of ordinary differential equations

$$\begin{aligned} \frac{d\mathbf{X}}{ds} &= \mathbf{V}, \\ \frac{d\mathbf{V}}{ds} &= \mathbf{E}(\mathbf{X}(s), s) + \mathbf{V}(s) \times \mathbf{B}(\mathbf{X}(s), s), \end{aligned} \quad (1)$$

with initial conditions  $\mathbf{X}(t) = \mathbf{x}$ ,  $\mathbf{V}(t) = \mathbf{v}$ .

From this property,  $f^n$  being known one can induce a numerical method for computing the distribution function  $f^{n+1}$  at the grid points  $(\mathbf{x}_i, \mathbf{v}_j)$  consisting in the following two steps:

1. For all  $i, j$ , compute the origin of the characteristic ending at  $\mathbf{x}_i, \mathbf{v}_j$ , i.e. an approximation of  $\mathbf{X}(t_n; \mathbf{x}_i, \mathbf{v}_j, t_{n+1}), \mathbf{V}(t_n; \mathbf{x}_i, \mathbf{v}_j, t_{n+1})$ .
2. As

$$f^{n+1}(\mathbf{x}_i, \mathbf{v}_j) = f^n(\mathbf{X}(t_n; \mathbf{x}_i, \mathbf{v}_j, t_{n+1}), \mathbf{V}(t_n; \mathbf{x}_i, \mathbf{v}_j, t_{n+1})),$$

$f^{n+1}$  can be computed by interpolating  $f^n$  which is known at the grid points at the points  $\mathbf{X}(t_n; \mathbf{x}_i, \mathbf{v}_j, t_{n+1}), \mathbf{V}(t_n; \mathbf{x}_i, \mathbf{v}_j, t_{n+1})$ .

This method can be simplified by performing a time-splitting separating the advection phases in physical space and velocity space, as in this case the characteristics can be solved explicitly.

### 3.2.5. Adaptive semi-Lagrangian methods

Uniform meshes are most of the time not efficient to solve a problem in plasma physics or beam physics as the distribution of particles is evolving a lot as well in space as in time during the simulation. In order to get optimal complexity, it is essential to use meshes that are fitted to the actual distribution of particles. If the global distribution is not uniform in space but remains locally mostly the same in time, one possible approach could be to use an unstructured mesh of phase space which allows to put the grid points as desired. Another idea, if the distribution evolves a lot in time is to use a different grid at each time step which is easily feasible with a semi-Lagrangian method. And finally, the most complex and powerful method is to use a fully adaptive mesh which evolves locally according to variations of the distribution function in time. The evolution can be based on a posteriori estimates or on multi-resolution techniques.

### 3.2.6. Particle-In-Cell codes

The Particle-In-Cell method [36] consists in solving the Vlasov equation using a particle method, i.e. advancing numerically the particle trajectories which are the characteristics of the Vlasov equation, using the equations of motion which are the ordinary differential equations defining the characteristics. The self-fields are computed using a standard method on a structured or unstructured grid of physical space. The coupling between the field solve and the particle advance is done on the one hand by depositing the particle data on the grid to get the charge and current densities for Maxwell's equations and, on the other hand, by interpolating the fields at the particle positions. This coupling is one of the difficult issues and needs to be handled carefully.

### 3.2.7. Maxwell's equations in singular geometry

#### 3.2.7.1. The Singular Complement Method

The solutions to Maxwell's equations are *a priori* defined in a function space such that the curl and the divergence are square integrable and that satisfy the electric and magnetic boundary conditions. Those solutions are in fact smoother (all the derivatives are square integrable) when the boundary of the domain is smooth or convex. This is no longer true when the domain exhibits non-convex *geometrical singularities* (corners, vertices or edges).

Physically, the electromagnetic field tends to infinity in the neighbourhood of the reentrant singularities, which is a challenge to the usual finite element methods. Nodal elements cannot converge towards the physical solution. Edge elements demand considerable mesh refinement in order to represent those infinities, which is not only time- and memory-consuming, but potentially catastrophic when solving instationary equations: the CFL condition then imposes a very small time step. Moreover, the fields computed by edge elements are discontinuous, which can create considerable numerical noise when the Maxwell solver is embedded in a plasma (e.g. PIC) code.

In order to overcome this dilemma, a method consists in splitting the solution as the sum of a *regular* part, computed by nodal elements, and a *singular* part which we relate to singular solutions of the Laplace operator, thus allowing to calculate a local analytic representation. This makes it possible to compute the solution precisely without having to refine the mesh.

This *Singular Complement Method* (SCM) had been developed [30] and implemented [31] in plane geometry.

An especially interesting case is axisymmetric geometry. This is still a 2D geometry, but more realistic than the plane case; despite its practical interest, it had been subject to much fewer theoretical studies [34]. The non-density result for regular fields was proven [38], the singularities of the electromagnetic field were related to that of modified Laplacians [27], and expressions of the singular fields were calculated [28]. Thus the SCM was extended to this geometry. It was then implemented by F. Assous (now at Bar-Ilan University, Israel) and S. Labrunie in a PIC-finite element Vlasov-Maxwell code [29].

#### 3.2.7.2. Other results, extensions.

As a byproduct, space-time regularity results were obtained for the solution to time-dependent Maxwell's equation in presence of geometrical singularities in the plane and axisymmetric cases [47], [28].

## 3.3. Large size problems

**Keywords:** *GRID, Parallelism, code transformation, domain decomposition.*

### 3.3.1. Abstract

The applications we consider lead to very large size computational problems for which we need to apply modern computing techniques enabling to use efficiently many computers including traditional high performance parallel computers and computational grids.

### 3.3.2. Introduction

The full Vlasov-Maxwell system yields a very large computational problem mostly because the Vlasov equation is posed in six-dimensional phase-space. In order to tackle the most realistic possible physical problems, it is important to use all the modern computing power and techniques, in particular parallelism and grid computing.

### 3.3.3. Parallelization of numerical methods

An important issue for the practical use of the methods we develop is their parallelization. We address the problem of tuning these methods to homogeneous or heterogeneous architectures with the aim of meeting increasing computing resources requirements.

Most of the considered numerical methods apply a series of operations identically to all elements of a geometric data structure: the mesh of phase space. Therefore these methods intrinsically can be viewed as a

data-parallel algorithm. A major advantage of this data-parallel approach derives from its scalability. Because operations may be applied identically to many data items in parallel, the amount of parallelism is dictated by the problem size.

Parallelism, for such data-parallel PDE solvers, is achieved by partitioning the mesh and mapping the submeshes onto the processors of a parallel architecture. A good partition balances the workload while minimizing the communications overhead. Many interesting heuristics have been proposed to compute near-optimal partitions of a (regular or irregular) mesh. For instance, the heuristics based on space-filling curves [50] give very good results for a very low cost.

Adaptive methods include a mesh refinement step and can highly reduce memory usage and computation volume. As a result, they induce a load imbalance and require to dynamically distribute the adaptive mesh. A problem is then to combine distribution and resolution components of the adaptive methods with the aim of minimizing communications. Data locality expression is of major importance for solving such problems. We use our experience of data-parallelism and the underlying concepts for expressing data locality [56], optimizing the considered methods and specifying new data-parallel algorithms.

As a general rule, the complexity of adaptive methods requires to define software abstractions allowing to separate/integrate the various components of the considered numerical methods (see [54] as an example of such modular software infrastructure).

Another key point is the joint use of heterogeneous architectures and adaptive meshes. It requires to develop new algorithms which include new load balancing techniques. In that case, it may be interesting to combine several parallel programming paradigms, i.e. data-parallelism with other lower-level ones.

Moreover, exploiting heterogeneous architectures requires the use of a runtime support associated with a programming interface that enables some low-level hardware characteristics to be unified. Such runtime support is the basis for heterogeneous algorithmics. Candidates for such a runtime support may be specific implementations of MPI such as MPICH-G2 (a grid-enabled MPI implementation on top of the GLOBUS tool kit for grid computing [43]).

Our general approach for designing efficient parallel algorithms is to define code transformations at any level. These transformations can be used to incrementally tune codes to a target architecture and they warrant code reusability.

### 3.4. Scientific visualization of plasmas and beams

Visualization of multi-dimensional data and more generally of scientific data has been the object of numerous research projects in computer graphics. The approaches include visualization of three-dimensional scalar fields looking at iso-curves and iso-surfaces. Methods for volume visualization, and methods based on points and flux visualization techniques and vectorial fields (using textures) have also been considered. This project is devoted to specific techniques for fluids and plasmas and needs to introduce novel techniques for the visualization of the phase-space which has more than three dimensions.

Even though visualization of the results of plasma simulations is an essential tool for the physical intuition, today's visualization techniques are not always well adapted tools, in comparison with the complexity of the physical phenomena to understand. Indeed the volume visualization of these phenomena deals with multidimensional data sets and sizes nearer to terabytes than megabytes. Our scientific objective is to appreciably improve the reliability of the numerical simulations thanks to the implementation of suitable visualization techniques. More precisely, to study these problems, our objective is to develop new physical, mathematical and data-processing methods in scientific visualization: visualization of larger volume data-sets, taking into account the temporal evolution. A global access of data through 3D visualization is one of the key issues in numerical simulations of thermonuclear fusion phenomena. A better representation of the numerical results will lead to a better understanding of the physical problems. In addition, immersive visualization helps to extract the complex structures that appear in the plasma. This work is related to a real integration between numerical simulation and scientific visualization. Thanks to new methods of visualization, it will be possible to detect the zones of numerical interest, and to increase the precision of calculations in these

zones. The integration of this dynamical side in the pipeline “simulation then visualization” will not only allow scientific progress in these two fields, but also will support the installation of a unique process “simulation-visualization”.

## 4. Application Domains

### 4.1. Thermonuclear fusion

**Keywords:** *ITER, Inertial fusion, laser-matter interaction, magnetic fusion, particle accelerators.*

Controlled fusion is one of the major prospects for a long term source of energy. Two main research directions are studied: magnetic fusion where the plasma is confined in tokamaks using large external magnetic field and inertial fusion where the plasma is confined thanks to intense laser or particle beams. The simulation tools we develop apply for both approaches.

Controlled fusion is one of the major challenges of the 21st century that can answer the need for a long term source of energy that does not accumulate wastes and is safe. The nuclear fusion reaction is based on the fusion of atoms like Deuterium and Tritium. These can be obtained from the water of the oceans that is widely available and the reaction does not produce long-term radioactive wastes, unlike today’s nuclear power plants which are based on nuclear fission.

Two major research approaches are followed towards the objective of fusion based nuclear plants: magnetic fusion and inertial fusion. In order to achieve a sustained fusion reaction, it is necessary to confine sufficiently the plasma for a long enough time. If the confinement density is higher, the confinement time can be shorter but the product needs to be greater than some threshold value.

The idea behind magnetic fusion is to use large toroidal devices called tokamaks in which the plasma can be confined thanks to large applied magnetic field. The international project ITER<sup>1</sup> is based on this idea and aims to build a new tokamak which could demonstrate the feasibility of the concept.

The inertial fusion concept consists in using intense laser beams or particle beams to confine a small target containing the Deuterium and Tritium atoms. The Laser Mégajoule which is being built at CEA in Bordeaux will be used for experiments using this approach.

Nonlinear wave-wave interactions are primary mechanisms by which nonlinear fields evolve in time. Understanding the detailed interactions between nonlinear waves is an area of fundamental physics research in classical field theory, hydrodynamics and statistical physics. A large amplitude coherent wave will tend to couple to the natural modes of the medium it is in and transfer energy to the internal degrees of freedom of that system. This is particularly so in the case of high power lasers which are monochromatic, coherent sources of high intensity radiation. Just as in the other states of matter, a high laser beam in a plasma can give rise to stimulated Raman and Brillouin scattering (respectively SRS and SBS). These are three wave parametric instabilities where two small amplitude daughter waves grow exponentially at the expense of the pump wave, once phase matching conditions between the waves are satisfied and threshold power levels are exceeded. The illumination of the target must be uniform enough to allow symmetric implosion. In addition, parametric instabilities in the underdense coronal plasma must not reflect away or scatter a significant fraction of the incident light (via SRS or SBS), nor should they produce significant levels of hot electrons (via SRS), which can preheat the fuel and make its isentropic compression far less efficient. Understanding how these deleterious parametric processes function, what non uniformities and imperfections can degrade their strength, how they saturate and interdepend, all can benefit the design of new laser and target configuration which would minimize their undesirable features in inertial confinement fusion. Clearly, the physics of parametric instabilities must be well understood in order to rationally avoid their perils in the varied plasma and illumination conditions which will be employed in the National Ignition Facility or LMJ lasers. Despite the thirty-year history of the field, much remains to be investigated.

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<sup>1</sup><http://www.iter.gov.fr/index.php>

Our work in modelling and numerical simulation of plasmas and particle beams can be applied to problems like laser-matter interaction in particular for particle accelerators, the study of parametric instabilities (Raman, Brillouin), the fast ignitor concept in the laser fusion research. Another application is devoted to the development of Vlasov gyrokinetic codes in the framework of the magnetic fusion programme in collaboration with the Department of Research on Controlled Fusion at CEA Cadarache. Finally, we work in collaboration with the American Heavy Ion Fusion Virtual National Laboratory, regrouping teams from laboratories in Berkeley, Livermore and Princeton on the development of simulation tools for the evolution of particle beams in accelerators.

## 4.2. Nanophysics

Kinetic models like the Vlasov equation can also be applied for the study of large nano-particles as approximate models when ab initio approaches are too costly.

In order to model and interpret experimental results obtained with large nano-particles, ab initio methods cannot be employed as they involve prohibitive computational times. A possible alternative resorts to the use of kinetic methods originally developed both in nuclear and plasma physics, for which the valence electrons are assimilated to an inhomogeneous electron plasma. The LPMIA (Nancy) possesses a long experience on the theoretical and computational methods currently used for the solution of kinetic equation of the Vlasov and Wigner type, particularly in the field of plasma physics.

Using a Vlasov Eulerian code, we have investigated in detail the microscopic electron dynamics in the relevant phase space. Thanks to a numerical scheme recently developed by Filbet et al. [42], the fermionic character of the electron distribution can be preserved at all times. This is a crucial feature that allowed us to obtain numerical results over long times, so that the electron thermalization in confined nano-structures could be studied.

The nano-particle was excited by imparting a small velocity shift to the electron distribution. In the small perturbation regime, we recover the results of linear theory, namely oscillations at the Mie frequency and Landau damping. For larger perturbations nonlinear effects were observed to modify the shape of the electron distribution.

For longer time, electron thermalization is observed: as the oscillations are damped, the center of mass energy is entirely converted into thermal energy (kinetic energy around the Fermi surface). Note that this thermalization process takes place even in the absence of electron-electron collisions, as only the electric mean-field is present.

## 5. Software

### 5.1. Vador

**Keywords:** *2D and axisymmetric geometry, PFC method, Vlasov, beam simulation, conservative, plasma simulation, positivity preserving.*

**Participants:** Francis Filbet [correspondant], Eric Sonnendrücker.

The development of the Vador code by Francis Filbet started during his PhD thesis. It solves the Vlasov equation on a uniform grid of phase-space. The two-dimensional version (four dimensions in phase-space) uses cartesian geometry and the Positive Flux Conservative (PFC) method [42], that is perfectly conservative and enables to preserve the positivity of the distribution function. The axisymmetric version is based on the use of the invariance of the canonical momentum and uses a semi-Lagrangian method following the characteristics exactly at the vicinity of  $r = 0$ . The method is described in [41]. It has been applied as well for plasma as for beam simulations.

The code is available at the following address:

[http://www.univ-orleans.fr/SCIENCES/MAPMO/membres/filbet/index\\_vad.html](http://www.univ-orleans.fr/SCIENCES/MAPMO/membres/filbet/index_vad.html)

## 5.2. Obiwan

**Keywords:** *Vlasov, adaptive, interpolet, multiresolution, semi-Lagrangian.*

**Participants:** Nicolas Besse, Michaël Gutnic, Matthieu Haefelé, Guillaume Latu [correspondant], Eric Sonnendrücker.

Obiwan is an adaptive semi-Lagrangian code for the resolution of the Vlasov equation. It has up to now a cartesian 1Dx-1Dv version and a 2Dx-2Dv version. The 1D version is coupled either to Poisson's equation or to Maxwell's equations. The grid adaptivity is based on a multiresolution method using Lagrange interpolation as a predictor to go from one coarse level to the immediately finer one. This idea amounts to using the so-called interpolating wavelets.

## 5.3. Yoda

**Keywords:** *Vlasov, adaptive, hierarchical finite elements, multiresolution, semi-Lagrangian.*

**Participants:** Olivier Hoenen [correspondant], Michel Mehrenberger, Isabelle Metzmeier, Eric Violard.

YODA is an acronym for Yet anOther aDaptive Algorithm. The sequential version of the code was developed by Michel Mehrenberger and Martin Campos-Pinto during CEMRACS 2003. The development of a parallel version was started by Eric Violard in collaboration with Michel Mehrenberger in 2003. It is currently continued with the contributions of Olivier Hoenen and Isabelle Metzmeier. It solves the Vlasov equation on a dyadic mesh of phase-space. The underlying method is based on hierarchical finite elements. Its originality is that the values required for interpolation at the next time step are determined in advance. In terms of efficiency, the method is less adaptive than some other adaptive methods (multi-resolution methods based on interpolating wavelets as examples), but data locality is improved. The implementation is generic  $n$  dimensional ( $2n$ -dimensions in phase-space).

## 5.4. Brennus

**Keywords:** *Maxwell, Particle-In-Cell (PIC), Vlasov, axisymmetric, beam simulation, finite volume, plasma simulation, unstructured grids.*

**Participants:** Pierre Navaro [correspondant], Eric Sonnendrücker.

The Brennus code is developed in the framework of a contract with the CEA Bruyères-Le-Châtel. It is based on a first version of the code that was developed at CEA. The new version is written in a modular form in Fortran 90. It solves the two and a half dimensional Vlasov-Maxwell equations in cartesian and axisymmetric geometry and also the 3D Vlasov-Maxwell equations. It can handle both structured and unstructured grids. Maxwell's equations are solved on an unstructured grid using a finite volume type method in 2D and Nedelec finite elements in 3D. On the 2D and 3D structured meshes Yee's method is used. The Vlasov equations are solved using a particle method. The coupling is based on traditional PIC techniques.

# 6. New Results

## 6.1. Reduced modelling of plasmas

**Participants:** Simon Labrunie, Pierre Bertrand.

In collaboration with J.A. Carrillo (Universitat Autònoma de Barcelona) we have studied mathematically a reduced kinetic model for laser-plasma interaction. Global existence and uniqueness of solutions and the stability of certain equilibria were obtained [7].

Despite its one-dimensional character, this system is strongly non-linear and already embeds some features of higher-dimensional, relativistic Vlasov-Maxwell systems. Thus it had been subject to many physical [35] and computational investigations by fluid [53] and kinetic semi-Lagrangian [52] methods. The study of its qualitative properties by numerical simulation is still an objective in the long run.

## 6.2. Convergence of numerical schemes

**Participants:** Nicolas Besse, Michel Mehrenberger.

The convergence of several semi-Lagrangian numerical schemes for the one-dimensional Vlasov-Poisson equations has been proved and error estimates given for high order schemes using symmetric Lagrange interpolation, B-Spline interpolation or wavelet interpolation[5].

We also developed and proved, in collaboration with D. Kröner (Freiburg, Germany), the convergence of a locally divergence free discontinuous Galerkin finite element method for the induction equations of the MHD system [4].

## 6.3. 2D semi-Lagrangian Vlasov solver on unstructured grids

**Participants:** Nicolas Besse, Eric Sonnendrücker.

We developed, in collaboration with Jacques Segré from CEA Saclay, a semi-Lagrangian Vlasov-Poisson solver based on a 2D unstructured mesh of the physical space where a uniform mesh in velocity space is superposed at each grid point [6].

## 6.4. A parallel 1D Vlasov solver using a Wavelet based Adaptive Mesh Refinement

**Participants:** Michaël Gutnic, Matthieu Haefelé, Guillaume Latu.

A first sequential simulator was developed using a numerical scheme based on wavelets. From one time step to the other, two advections in variables  $x$  and  $v$  are performed on a given grid containing  $N$  points. Only a percentage  $p$  of all the points  $(x,v)$  are effectively advected in the considered grid, because the use of wavelet allows us to guess the remaining points, if necessary. For small values of  $p$ , one can expect to reduce the total computing cost because the advection concerns only  $p*N$  points instead of  $N$ . The accuracy of computations is controlled by a parameter that has a direct influence on  $p$ .

Nevertheless, the use of this method has an overhead since we have to manage the data structure of wavelet coefficients after each advection. In order to obtain an efficient application, we considered the complexity of the advection algorithm, and of the overhead. We performed a reduction of the overhead by several ways. We first reduced the complexity in memory consumption and improved the use of cache memory. We replaced the data structure used to keep the wavelet coefficient (a hashtable) with a sparse data structure that has better properties in term of access time (for reading and writing).

We described the parallelization at a medium-grain level of an adaptive numerical simulator that solves the 1D Vlasov-Poisson system using a splitting technique (in  $v$  direction and in  $x$  direction). Because access patterns to sparse data are complex and very large, a shared memory architecture was targeted for this application. Almost every steps of the original algorithm were parallelized thanks to OpenMP programming paradigm and we obtained that each computing thread worked on a block of local data most of the time. The scalability is quite good but strongly linked with the sparse representation of physical phenomena. In adapting the block size for each thread, we could find a better data distribution to reduce the load imbalance. But a difficulty comes from the lack of facilities for data placement with OpenMP. This work was published in [17].

## 6.5. An adaptive 1D Vlasov-Maxwell solver

**Participants:** Nicolas Besse, Guillaume Latu.

A current work consists in adapting this 1D parallel solver with an electromagnetic field solver based on Maxwell's equations instead of Poisson's equation. In this model, where the relativistic Vlasov equation is used, the splitting method does not yield constant coefficient advections in each direction and is therefore not used. Important new developments had to be made in the code to be able to handle this situation. A characteristic feature of the numerical experiments performed with this code was the use of a finer mesh in  $x$ -direction than in  $v$ -direction.

## 6.6. An adaptive parallel 2D Vlasov solver

**Participants:** Guillaume Latu, Eric Sonnendrücker.

Another development of the adaptive code based on interpolating wavelets is its extension to 4 phase space dimensions in order to be able to handle more realistic physical situations. A first version of a sequential program has been developed this year. In this new simulator, we consider a large phase space domain. The use of an adaptive scheme leads to a severe reduction of memory usage. We are considering the parallelization of this application on a shared memory architecture to shorten the simulation run-time. The goal of this high-performance simulator is to target realistic problems in beam physics or plasma physics that involve two dimensional geometries.

## 6.7. Data-access optimizations in Yoda for a 4D phase space

**Participants:** Olivier Hoenen, Michel Mehrenberger, Eric Violard.

We continued our improvements of the code Yoda which implements an adaptive method based on hierarchical finite elements. The method falls into three phases at each time step: mesh prediction, mesh evaluation, i.e., calculation of the approximate solution at every node, and mesh compression. The code was first optimized for a 2D phase space. For a higher number of dimensions of the phase space, the evaluation phase becomes far more time consuming than the other ones making the optimization of data-access in this phase crucial. Therefore we reconsidered the data structures and indexing initially used to represent the underlying dyadic mesh. Our works focussed on the efficient implementation of the dyadic mesh for a 4D phase space. We elaborated a sparse data structure associated with a suitable indexing which intensively uses memory cache and an original mechanism which also improves data locality. This mechanism is similar to an additional “software” memory cache. It keeps in memory the values necessary for interpolating backward advected points. Experiments show the advantages of these techniques. Performances were improved by more than a factor 2 in most cases [18].

## 6.8. A new finite element method based on an Hermite reconstruction (YodaH)

**Participants:** Olivier Hoenen, Michel Mehrenberger, Eric Sonnendrücker, Eric Violard.

We designed a new adaptive method for the Vlasov equation. Like the method of Yoda, it is based on finite elements and a dyadic mesh. It aims at improving the accuracy for a given cost compared to the method implemented in Yoda. The method was designed for a 2D phase space, but it can be extended to higher dimensional space. In 2D, each (square) cell is associated with 16 “degrees of freedom” (3 at each vertice and 1 on the middle of each edge). We call “degree of freedom” the value of the approximate solution or its derivative (gradient) in  $x$  or  $v$ . The interpolation operator is defined as the unique  $C^1$  cubic spline on the triangulated square. The triangularisation is obtained by drawing in both diagonals. We implemented this method in C++ and validated it on the example of a rotating gaussian distribution function.

We aim to design more flexible methods in the sense that the number of freedom degrees could be changed depending on the required accuracy. We investigate the use of “discontinuous finite elements” to achieve this goal. The idea is that the interpolation polynomial may be defined by any number of freedom degrees.

## 6.9. Parallelization of YodaH

**Participants:** Olivier Hoenen, Michel Mehrenberger, Eric Violard.

We designed a parallel version of code YodaH for a shared memory machine. The code has been written using OpenMP directives and validated on the machine available at IRMA (composed of 12 processors sparc ultra-4). In this implementation, the prediction phase is performed by using backward advections only, allowing the three phases of the algorithm to be performed independently on each piece of the mesh. Therefore only one scan of the mesh is done at each time step. A recursive procedure treats all levels of each mesh piece. This procedure enhances data locality but adds new challenges in terms of load balancing. We used a two-level *dynamic array*, i.e., an array whose elements may be themselves arrays, to keep mesh values and cells.



Computational domain is distributed amongst threads according to the  $x$ -axis. First experiments have shown good scalability of the parallel code.

## 6.10. Coupling particles with a Maxwell solver in PIC codes

**Participants:** Régine Barthelmé, Eric Sonnendrücker.

When using the classical charge and current deposition algorithms in PIC codes, the continuity equation  $\partial_t \rho + \nabla \cdot J = 0$  is not satisfied at the discrete level. Therefore when using only Ampere and Faraday's laws to compute the electromagnetic field, Gauss's law  $\nabla \cdot E = \rho$  is violated over long time computations yielding unphysical results. Specific current deposition techniques need to be introduced. The most widely used is that of Villasenor and Buneman which works for linear deposition algorithms. We extended this method to higher order deposition schemes and to non uniform meshes. This work was published in [3].

## 6.11. High order finite element method for the wave equation

**Participants:** Sébastien Jund, Stéphanie Salmon, Eric Sonnendrücker.

In the frame of the DFG/CNRS project "Noise Generation in Turbulent Flows", we need to develop very precise solvers for the acoustics wave equation on unstructured grids. This solver will then be coupled to an Euler solver to compute the noise generation.

In collaboration with our partners from the University of Stuttgart in Germany we compared the efficiency of high order solvers based on continuous finite elements to high order solvers based on the discontinuous Galerkin method.

In order for the finite element solvers to be efficient, we developed a new strategy to lump the mass matrix which can be applied at any order. It has already been implemented for  $\mathbb{P}_k$  elements with  $k \leq 6$ . For obtaining high order accuracy in time, we chose an ADER procedure which allows us to replace time derivatives by space derivatives and then to keep the right order of accuracy. We have also tried diagonally implicit Runge-Kutta schemes to keep reasonable CFL number. First comparisons between discontinuous Galerkin methods and our method with these new advances are in progress.

## 6.12. Two-scale simulation of Maxwell and acoustics equations

**Participants:** Sébastien Jund, Stéphanie Salmon, Eric Sonnendrücker.

We are developing a numerical method which is able to deal with two different space scales. The source term will be defined on the smaller scale and be localized in the simulation domain and the acoustic or electromagnetic waves will be propagating on the larger scale. The numerical method will be based on an idea of Rappaz and Wagner using two different finite element spaces for the two-scales. We shall adapt this idea to the Maxwell and acoustics equations using Raviart-Thomas-Nedelec finite elements. This work in progress is performed in collaboration with Hyam Abboud and Hamdi Zorgati from Laboratoire J.L. Lions (University of Paris 6).

## 6.13. Maxwell's equation in singular geometry

**Participant:** Simon Labrunie.

We are carrying on the improvement and extension of our *Singular Complement Method* (SCM) in some simple, but genuinely three-dimensional situations, namely prismatic and axisymmetric domains with arbitrary data. After a thorough study of the electrostatic case [22], [8], [23], we are now tackling the general case. In collaboration with B. Nkemzi (University of Buea, Cameroon), we defined a new version of the method for Maxwell's equations in a model case [26]. The convergence rate of the method is now optimal and independent of the geometry. Further work (with B. Nkemzi and P. Ciarlet's team at ENSTA) will consist in extending these results to time-dependent, three-dimensional cases. This could be done, for instance, by embedding the SCM in the general approach of [39]. Later on, approximate symmetries (i.e. domains that are nearly axisymmetric

if one neglects small peripheral parts) could be treated by using perturbative expansion techniques in addition to this method.

## 6.14. Domain decomposition for the resolution of nonlinear equations

**Participant:** Jean Roche.

This a joint work with N. Alaa, Professor at the Marrakech Cadi Ayyad University.

The principal objective of this work is to study existence, uniqueness and present a numerical analysis of weak solutions for a quasi-linear elliptic problem that arises in biological, chemical and physical systems. Various methods have been proposed for study the existence, uniqueness, qualitative properties and numerical simulation of solutions. We were particularly interested in situations involving irregular and arbitrarily growing data ([1]).

Another approach studied here is the numerical approximation of the solution to the problem. The most important difficulties are in this approach the uniqueness and the blowup of the solution.

The general algorithm for numerical solution of this equations is one application of the Newton method to the discretized version of the problem. However, in our case the matrix which appears in the Newton algorithm is singular. To overcome this difficulty we introduced a domain decomposition to compute an approximation of the iterates by the resolution of a sequence of problems of the same type as the original problem in subsets of the given computational domain. This domain decomposition method coupled with a Yosida approximation of the nonlinearity allows us to compute a numerical solution.

In previous work we consider the 1-d case[1], during the last year we consider the case 2-d where the gradient dependent non-linearity is quadratic[16].

## 6.15. Rendering large models using view-dependent texture mapping

**Participant:** Jean-Michel Dischler.

We have proposed an extension of View-Dependent Texture Mapping (VDTM) allowing rendering of very complex geometric meshes at high frame rates without usual blurring or skinning artifacts. We combine a hybrid geometric and image-based representation of a given 3D object to speed-up rendering at the cost of a little loss of visual accuracy. During a precomputation step, we store an image-based version of the original mesh by simply and quickly computing textures from viewpoints positionned around it by the user. During the rendering step, we use these textures in order to map on the fly colors and geometric details onto the surface of a low-polygon-count version of the mesh. Real-time rendering is achieved while combining up to three viewpoints at a time, using pixel shaders. No parameterization of the mesh is needed and occlusion effects are taken into account while computing on the fly the best viewpoints for a given pixel. This work was published in [19].

## 6.16. Tokamak plasma visualization

**Participants:** Jean-Michel Dischler, Florence Zara.

The study of plasmas requires dedicated algorithms, allowing physicians to visualize and thus interpret related data sets. We developed an efficient volume rendering method designed for Gyrokinetic simulations. It is based on a Raymarching approach that benefits from the latest functionalities brought by graphics programmable hardware. We exploit the geometrical properties of the tokamak, a physical device used to create plasmas, in order to achieve a rendering at interactive framerates. We compared our method with the latest Gyrokinetic simulations visualization technique and demonstrated an improved efficiency. This work is now going to be improved to enhance quality and interactivity especially for dealing with large datasets.

## 6.17. Multidimensionnal visualization tools

**Participants:** Jean-Michel Dischler, Matthieu Haeefele.

The development of a new interactive visualization technique arrives to its term. This work is focusing on the visualization of a 4D function: the distribution function resulting from a 4D plasma simulation. Since it is not possible to display directly this function, the idea consists in exploring the dataset interactively by using 2D slices. Four 2D slices of the dataset are displayed at the same time, and these four slices intersect each other in one point that we called the "focus" point. Then, the exploration of the dataset is performed by moving this focus point interactively. The challenge consists in refreshing the different slices at interactive frame rates according to the position of the focus point. Hierarchical finite elements bases are used to compress the 4D dataset, and the needed 2D slices are extracted directly from the compressed data, which is therefore locally decompressed in real-time. This method has been tested and currently works on little datasets (128MB). We are now working on the improvement of the method to visualize very large datasets coming from complete parallel simulations.

## 6.18. Full wave modeling of lower hybrid current drive in tokamaks

**Participants:** Pierre Bertrand, Jean Roche.

This work is performed in collaboration with Yves Peysson (DRFC, CEA Cadarache). The goal of this work is to develop a full wave method to describe the dynamics of lower hybrid current drive problem in tokamaks. The wave dynamics may be accurately described in the cold plasma approximation, which supports two independent modes of propagation, the slow wave which correspond to a cold electrostatic plasma wave, and the fast wave, namely the whistler mode. Because of the simultaneous presence of the slow and fast propagation branches a vectorial wave equation must be solved. The wave equation is obtained from the Maxwell equations with a time harmonic approximation. We consider a toroidal formulation of the Maxwell equations. Simulations will be carried out using domain decomposition techniques.

## 6.19. Numerical experiments of stimulated Raman scattering using semi-lagrangian Vlasov-Maxwell codes

**Participants:** Alain Ghizzo, Pierre Bertrand, Thierry Réveillé.

We have investigated Vlasov-Maxwell numerical experiments for realistic plasmas in collaboration of the group of Dr B. Afeyan of Polymath research Inc. (in an international collaboration program of the Department of Energy of USA). Our studies so far indicate that a promising way to deter these undesirable processes is by instigating the externally controlled creation of large amplitude plasma fluctuations making the plasma an inhospitable host for the growth of coherent wave-wave interactions. The area where we plan to focus most of our attention is in Vlasov-Maxwell (semi-lagrangian) simulations in 1D. In several works, see for example [2], the nonlinear evolution of the electron plasma waves which have been generated by optical mixing (pump plus probe beams) is investigated to understand the kinetic effects that saturate the growth of these modes. Both the electron plasma wave and ion acoustic wave generation and SRS interaction problems are treated in great detail. Fluid and kinetic degrees of freedom to saturate SRS and to limit the growth of the optical mixing generated waves will be elucidated by Vlasov simulations.

In the thesis of Michel Albrecht-Marc defended this year, a semi-lagrangian Vlasov-Maxwell code was used for studying the wave-particle interactions and the dynamics of trapped particles met in the laser-matter interaction and in particular in the Stimulated Raman Scattering.

A new project of ANR (Agence Nationale de la Recherche) has been accepted this year for three years and concerns *the study of wave-particle interaction for Vlasov plasmas*. In this project our main working topic is modelling and numerical simulation in hot plasma physics and we focus on the physical behavior of complex systems. The project involves cross-interactions between plasma physicists of LPMIA of Nancy and of the University of Pisa in Italy (F. Califano), applied mathematiciens of the Calvi Group and computer scientists. The use of Vlasov codes to physical problems such as the relativistic Vlasov-Maxwell system in inertial fusion or the gyrokinetic Vlasov model in magnetic fusion will be carried out.

## 6.20. Ultrafast electron dynamics in thin metallic films

**Participants:** Paul-Antoine Hervieux, Giovanni Manfredi.

In the past year, we focused on the study of the ultrafast electron dynamics in thin metallic films. Self-consistent simulations of the electron dynamics and transport were performed using a semiclassical Vlasov-Poisson model. The Vlasov equation was solved with an accurate Eulerian scheme that preserves the fermionic character of the electron distribution. The stability properties of the numerical technique were tested by preparing the system in its ground state and letting it evolve self-consistently without any perturbation. By definition, the ground state is a stationary solution of the Vlasov-Poisson system and should remain stable under the time evolution. However, PIC codes show a rather quick deterioration of the Fermi-Dirac ground state, which relaxes to a Boltzmann distribution in a few electron plasmon cycles. With our Eulerian code, no departure from the Fermi-Dirac equilibrium could be detected for times as long as  $\omega_{pet} = 2000$ , corresponding to more than 300 plasmon cycles. The total energy was conserved within an error of less than 0.05%.

Several interesting physical results were obtained. We observed that, although the thermodynamical properties of the ground state are accurately described by the bulk theory, the dynamical properties are strongly influenced by the finite size of the system and the presence of surfaces. Our results also showed that: (i) heat transport is ballistic and occurs at a velocity close to the Fermi speed; (ii) after the excitation energy has been absorbed by the film, slow nonlinear oscillations appear, with a period proportional to the film thickness: these oscillations were attributed to nonequilibrium electrons bouncing back and forth on the film surfaces; (iii) except for trivial scaling factors, the above transport properties are insensitive to the excitation energy and the initial electron temperature. Finally, the coupling to the ion dynamics and the impact of electron-electron collisions was also investigated.

The existence of the nonlinearly oscillating regime described above prompted us to analyze the possibility of boosting energy absorption in the film by optically exciting the electron gas. In order to check this conjecture, a time-oscillating and spatially uniform laser (electric) field was applied to the film. In general, we observed that the field energy was partially absorbed by the electron gas in the form of thermal energy. Maximum absorption occurred when the period of the external field matched the period of the nonlinear oscillations, which, for sodium films, lies in the infrared range. Possible experimental implementations were discussed.

This work was published in a series of articles [10], [11], [12].

## 7. Contracts and Grants with Industry

### 7.1. CEA Bruyères-Le-Châtel, PIC code

**Participants:** Pierre Navaro, Eric Sonnendrücker.

The object of the contract is the development of an efficient parallel Particle-In-Cell (PIC) solver for the numerical resolution of the two configuration space - three momentum space dimensions Vlasov-Maxwell equations in cartesian and axisymmetric geometries. This code is written in a modular way using the fortran 90 language, so that it will be easy to add more physics.

### 7.2. CEA Bruyères-Le-Châtel, simulation of particle beams

**Participants:** Matthieu Haefele, Eric Sonnendrücker, Xinting Zhang.

The object of this contract is the development of efficient Vlasov-Poisson solvers based on a phase space grid for the study of intense particle beams. This contract focused on the development of new confinement fields and new particle distributions for the VADOR solver. We also continued our development of new visualization techniques.

### 7.3. National initiatives

#### 7.3.1. ANR Projects

Calvi members are involved in two ANR projects.

- ANR Masse de données : MASSIM project (leader J.-M. Dischler). Simulation and visualization of problems involving large data sets in collaboration with O. Coulaud (project Scalapplix).
- Non thematic ANR. Study of wave-particle interaction for Vlasov plasmas (leader A. Ghizzo) In collaboration with F. Califano from the University of Pisa in Italy.

### 7.3.2. Participation to GdR Research groups from CNRS

The members of Calvi participate actively in the following GdR:

- GdR Groupement de recherche en Interaction de particules (GRIP, CNRS 2250): This research group is devoted to the modelling and simulation of charged particles. It involves research teams from the fields of Partial Differential Equations and Probability.
- GdR équations Cinétiques et Hyperboliques : Aspects Numériques, Théoriques, et de modélisation (CHANT, CNRS 2900): This research group is devoted to the modelling and numerical simulation of hyperbolic and kinetic equations.

## 7.4. European initiatives

### 7.4.1. RTN HYKE: HYperbolic and Kinetic Equations

The HYKE network is a Research Training Network (RTN) financed by the European Union in the 5th Framework Programme "Improving the Human Potential" (IHP). It puts together the major European teams involved in the mathematics of conservation laws and kinetic theory. See the HYKE web page for more details <http://www.hyke.org>.

### 7.4.2. DFG/CNRS project "Noise Generation in Turbulent Flows"

This projects involves several French and German teams both in the applied mathematics and in the fluid dynamics community. Its aim is the development of numerical methods for the computation of noise generated in turbulent flows and to understand the mechanisms of this noise generation.

The project is subdivided into seven teams each involving a French and a German partner. Our german partner is the group of C.-D. Munz at the University of Stuttgart. More details can be found on the web page [http://www.iag.uni-stuttgart.de/DFG-CNRS/index\\_fr.htm](http://www.iag.uni-stuttgart.de/DFG-CNRS/index_fr.htm)

## 8. Dissemination

### 8.1. Leadership within scientific community

#### 8.1.1. Conferences, meetings and tutorial organization

- Eric Sonnendrücker co-organized (with Denis Talay and Thierry Goudon) the workshop "Simulation numérique du transport de particules, méthodes particulières" of GdR GRIP which took place at INRIA Sophia-Antipolis March 21-23 <http://math.univ-lille1.fr/~goudon/sophia.html>.
- Stéphanie Salmon and Eric Sonnendrücker co-organized (with Claus-Dieter Munz and Michael Dumbser) the summer research center CEMRACS 2005 which took place from July 18 to August 26 in Marseille. <http://smai.emath.fr/cemracs/cemracs05/>.
- Francis Filbet and Eric Sonnendrücker organized the CEA-EDF-INRIA school on kinetic equations. Applications to beam and plasma physics (September 19-22). <http://www.inria.fr/actualites/colloques/cea-edf-inria/2005/ecoleequacin.html>.
- Michaël Gutnic, Stéphanie Salmon and Eric Sonnendrücker co-organized (with François Castella) the workshop on numerical methods for kinetic, hyperbolic and Hamilton-Jacobi equations from GdR CHANT which took place at IRMA in Strasbourg November 23-25. <http://www-irma.u-strasbg.fr/~sonnen/WorkshopCHANT05.html>.

### 8.1.2. Administrative duties

- Jean-Michel Dischler is the vice-head of the LSIT laboratory of CNRS and University Louis Pasteur in Strasbourg.
- Jean-Michel Dischler is vice-president of the Eurographics french chapter association and member of the professional board of Eurographics.
- Jean Rodolphe Roche is the head of the Mathematics Science department of the "École Supérieure des Sciences et Technologies de l'Ingénieur de Nancy" .
- Eric Sonnendrücker is the head of the Center of studies in parallel computing and visualization of the University Louis Pasteur in Strasbourg, which makes parallel computing resources and a workbench for immersed visualization available to the researchers of the University.
- Eric Sonnendrücker is a member of the National Committee of Universities (26th section: applied mathematics).

## 8.2. Teaching

- *Jean-Michel Dischler* taught a graduate course (DEA) entitled "Rendering and visualization" in DEA of Computer Science at the University Louis Pasteur of Strasbourg.
- *Jean Rodolphe Roche* taught a DESS course entitled "Parallel Architecture and Domain Decomposition Method" in DESS IMOI of the University Henri Poincaré-Nancy I.
- *Simon Labrunie and Jean Rodolphe Roche* taught an optional graduate course (DEA) entitled "Numerical Analysis of Hyperbolic Problems" in DEA of Mathematics at the University Henri Poincaré (Nancy I).
- *Eric Sonnendrücker* taught an optional graduate course (DEA) entitled "Numerical methods for the Vlasov equation" in DEA of Mathematics at the University Louis Pasteur of Strasbourg.

## 8.3. Ph. D. Theses

### 8.3.1. Ph. D. defended in 2005

1. Régine Barthelmé, *Couplage* Université Louis Pasteur, Strasbourg, 8 July 2005. Jury: J.C. Adam (referee), P. Ciarlet, Jr (referee), V. Komornik (internal referee), J.P. Croisille, Eric Sonnendrücker (advisor).
2. Michel Albrecht-Marc (LPMIA) *Etude cinétique de l'instabilité raman en plasma inhomogène par simulation numérique Vlasov semi-lagrangienne*, UHP Nancy- 1, 30 September 2005. Jury: J.M. Rax, V. Tikhonchuk, E. Lefebvre, A. Ghizzo, P. Bertrand, B. Weber.

### 8.3.2. Ph. D. in progress

1. Alexandre Mouton, *Multiscale approximation of the Vlasov equation*. Advisors: Emmanuel Frénod and Eric Sonnendrücker.
2. Olivier Gèneveaux, *Realistic visualization of fluid - solid body interactions*. Advisor: Jean-Michel Dischler.
3. Matthieu Haefele, *High performance visualization of particle beams*. Advisors: Jean-Michel Dischler and Eric Sonnendrücker.
4. Sébastien Jund, *Mathematical and numerical analysis of weakly compressible flows*. Advisors: Stéphanie Salmon and Eric Sonnendrücker.
5. Isabelle Metzmeier, *Conservative adaptive methods for the Vlasov equation*. Advisor: Eric Sonnendrücker.
6. Olivier Hoenen, *Parallelization of adaptive methods for the Vlasov equation*. Advisor: Eric Violard.

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- [3] R. BARTHELMÉ. *Conservation de la charge dans les codes PIC*, in "C. R. Acad. Sci. Paris, Ser. I", vol. 341, n° 11, 2005, p. 689–694.
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- [17] M. GUTNIC, M. HAEFELE, G. LATU. *A Parallel Vlasov solver using a Wavelet based Adaptive Mesh Refinement*, in "2005 International Conference on Parallel Processing (ICPP'2005), 7th Workshop on High Perf. Scientific and Engineering Computing", IEEE Computer Society Press, 2005, p. 181–188.
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