



INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

Project-Team Sydoco

*SYstèmes Dynamiques, Optimisation et
Commande Optimale*

Rocquencourt

THEME NUM

Activity
R *eport*

2005

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1. Team

Head of project-team

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Administrative assistant

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Staff member

Claudia Sagastizábal [CR, détachée à l'IMPA - Rio de Janeiro]

Research scientist

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Romain Apparigliato [EDF, Logilab]

Grégory Emiel [EDF, IMPA]

Audrey Hermant [DGA, since october 05]

Julien Laurent-Varin [bourse CNES, untill september 05]

Stefania Maroso [bourse MESR]

Nadia Megdich [bourse tunisienne]

Elisabeth Ottenwaelter [IUT Paris]

Visiting scientist

Felipe Alvarez [Université du Chili, 2 weeks]

Pablo Lotito [Pladema, Argentine, 1 week]

Hector Ramirez-Cabrera [Université du Chili, 4 weeks]

Mikhail Solodov [IMPA - Rio de Janeiro, 4 weeks]

Student intern

Audrey Hermant [DGA (April-September)]

Philillipe Laugier [ENSTA (May-July)]

2. Overall Objectives

2.1. Overall Objectives

To develop new algorithms in deterministic and stochastic optimal control, and deal with associated applications, especially for aerospace trajectories and management for the power industries (hydroelectric resources, storage of gas and petroleum).

In the field of deterministic optimal control, our objective is to develop algorithms combining iterative fast resolution of optimality conditions (of the discretized problem) and refinement of discretization, through the use of interior point algorithms. At the same time we wish to study multiarcs problems (separations, rendez-vous, formation flights) which necessitates the use of decomposition ideas.

In the field of stochastic optimal control, our first objective is to develop fast algorithms for problems of dimension two and three, based on fast computation of consistent approximations as well as splitting methods. The second objective is to link these methods to the stochastic programming approach, in order to deal with problems of dimensions greater than three.

3. Scientific Foundations

3.1. Scientific Foundations

For deterministic optimal control problems there are basically three approaches. The so-called direct method consists in an optimization of the trajectory, after having discretized time, by a nonlinear programming solver

that possibly takes into account the dynamic structure; see Betts [26]. The indirect approach eliminates control variables using Pontryagin's maximum principle, and solves the resulting two-points boundary value problem by a multiple shooting method. Finally the dynamic programming approach solves the associated Hamilton-Jacobi-Bellman (HJB) equation, which is a partial differential equation of dimension equal to the number n of state variables. This allows to find the global minimum, whereas the two other approaches are local; however, it suffers from the curse of dimensionality (complexity is exponential with respect to n).

There are various additional issues: decomposition of large scale problems, simplification of models (leading to singular perturbation problems), computation of feedback solutions.

For stochastic optimal control problems there are essentially two approaches. The one based on the (stochastic) HJB equation has the same advantages and disadvantages as its deterministic counterpart. The stochastic programming approach is based on a finite approximation of uncertain events called a scenario tree (for problems with no decision this boils down to the Monte Carlo method). Their complexity is polynomial with respect to the number of state variables but exponential with respect to the number of time steps. In addition, various heuristics are proposed for dealing with the case (uncovered by the two other approaches) when both the number of state variables and time steps is large.

4. Application Domains

4.1. Application Domains

Aerospace trajectories (rockets, planes), automotive industry (car design), chemical engineering (optimization of transient phases, batch processes).

Storage and management, especially of natural and power resources, portfolio optimization.

5. Software

5.1. Software

We have presently two research softwares. The first is an implementation of interior point algorithms for trajectory optimization, and the second is an implementation of fast algorithms for bidimensional HJB equations of stochastic control.

6. New Results

6.1. Trajectory optimization

6.1.1. Analysis of junction conditions for state constrained optimal control problems

Participants: F. Bonnans, A. Hermant.

In the framework of the internship, and the beginning of doctoral thesis, of A. Hermant, we have studied the junction conditions for optimal control problems with only one control and one state constraint, of arbitrary order q . For these problems there exists an "alternative formulation" to the classical Pontryaguin principle, whose Hamiltonian involves the q -times derivative of the state constraint. We have clarified the question of equivalence between the classical and alternative formulations, by stating a complete set of "additional conditions". We have showed that, under weak conditions, the Jacobian of the shooting algorithm is invertible.

An INRIA report will be published at the beginning of 2006.

6.1.2. Multiarc optimization

Participants: F. Bonnans, J. Laurent-Varin.

in collaboration with N. Bérend (Onera) and Ch. Talbot (Cnes).

We report here the work done in the framework of the thesis of J. Laurent-Varin (defended in November 2005), whose subject is the numerical methods for multiarc problems, i.e., when several connected trajectories are to be optimized simultaneously (separations of stages of launchers, orbital rendez-vous). The work done previously was an implementation of an interior-point algorithm, combined with a refinement procedure, for the single arc case. We reported in 2004 a theoretical study of multiarc problems. The main effort in 2005 was the implementation of the method. We use an elimination procedure of variables associated with arcs, and obtain a reduced system whose unknowns are the variables associated with nodes.

The method has been applied to a model deriving from the FSSC16 concept of two-stages launcher. After separation, the re-usable booster comes back to Earth using an optimized path. Our algorithm was able to optimize this model.

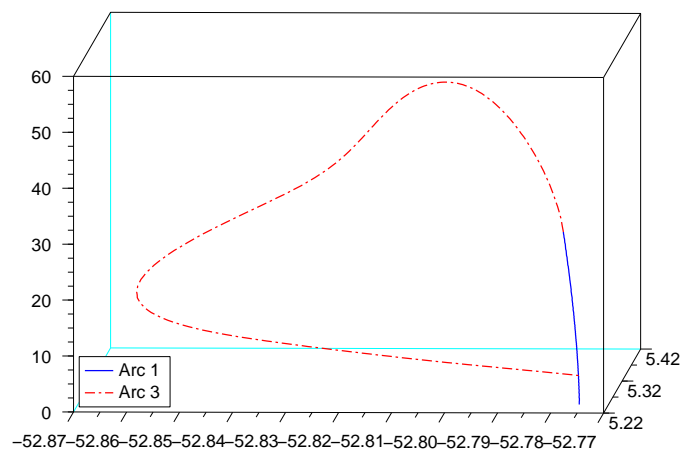


Figure 1. 3D view of optimized trajectory, FSSC16 concept

6.2. Antidiffusive schemes for first order HJB equations

Participants: N. Megdich, H. Zidani.

We study numerical schemes for HJB equations HJB coming from optimal control problems with state constraints (RDV problems, target problems of target, minimal time). When some controllability assumptions are not satisfied, the solution of the HJB equation is discontinuous and the classical schemes provide poor quality approximation, mainly around the discontinuities.

In collaboration with O. Bokanowski (U. Paris VII), we have continued the study of the Ultrabee scheme and its application for the resolution of HJB equations HJB. We have obtained a convergence result in dimension 1. The Ultrabee not being monotonous, our proof of the convergence does not use the classical arguments by Barles-Souganidis. We presented this result at ENUMATH' 05 and a corresponding preprint is in preparation.

In [19], we have studied the implementation of Ultrabee on adaptive grids. The use of such grids (coded in the form of a "quadtrees"), enables us to have the best approximation, in particular around discontinuities, while minimizing the number of meshes in the grid of calculation. Let us mention that the adaptive methods were already used for the resolution of equations HJB in the continuous case [28], [27]. The criteria of refinement and coarsening presented in these works do not work when the solution is discontinuous. In our work, we propose criteria adapted to the equations which we treat.

For the target problems, the value function takes only values 0 or 1 (0 in the points who can reach the target, and 1 elsewhere). To benefit from this particular structure, we also have developed a sparse code in 2d. The idea consists on coding only the meshes where the front lies (the border between 0 and 1). This method is inexpensive and very fast. Numerical experiments in 2d were performed and were presented at the Enumath'05 conference by O Bokanowski. The application to a probleme of atmospheric re-entry in 3d is under study.

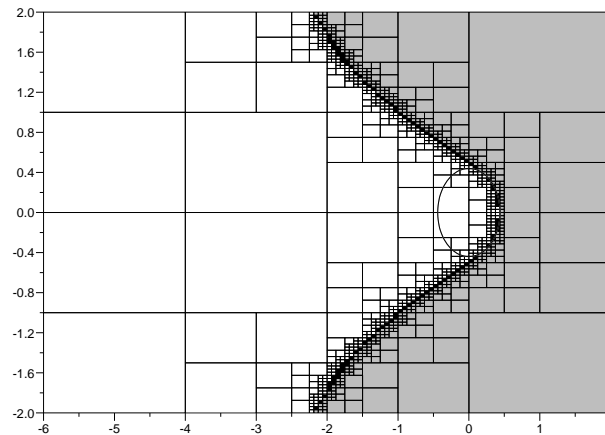


Figure 2. Bassin de capture du problème de Zermelo déterminé par le schéma UltraBee sur un maillage adaptatif de niveau de raffinement maximal 6 et 7.

6.3. Numerical methods for HJB equation

6.3.1. Markov chain approximation algorithms

Participants: H. Zidani, R. Munos.

In [CRAS-2005], we present an approximation scheme of the type of Markov chain approximation for the resolution of second order HJB equations. Our scheme is a generalization of the approximations suggested by [RM-CF]. It is based on an approximation of the characteristics and an interpolation on a grid which is not necessarily regular. We prove that when the diffusion step and the interpolation weights satisfy a 'balance conditions', the scheme is consistent (which guarantees its convergence).

6.3.2. Splitting type algorithms

Participants: F. Bonnans, H. Zidani, E. Ottenwaelter.

We have continued the analysis and implementation of splitting type algorithms for second order HJB equations. We use the Strang decomposition for obtaining second-order approximations for problems with smooth solutions. Extensive numerical results for θ schemes, combined or not with the Strang decomposition, confirm the predictins of the theory.

6.3.3. Splitting type algorithms

Participants: F. Bonnans, H. Zidani, S. Maroso.

We have established error estimates for stochastic impulse control problems. We use the techniques of Barles and Jakobsen, and the analysis of cascade problems. The error estimate combines an error estimate

for an arbitrary solution of the cascade problem, with the estimate due to Ishii of the distance between that solution and the one of the impulse problem. The results were presented in INRIA report RR 5441.

6.4. Nonlinear optimization

6.4.1. \mathcal{VU} -algorithms and theory

Participants: C. Sagastizábal, R. Mifflin (Washington State University - EUA).

For the \mathcal{VU} -superlinear bundle algorithm for convex minimization described in [12], see Report 2004, we consider a line search along the lines of [29]. We are currently developing a battery of test problems to validate the results.

This work is a follow-up of our more theoretical research on variational analysis, see [15] and [14].

6.4.2. Bundle methods for constrained optimization problems

Participants: C. Sagastizábal, M. Solodov, E. Karas, A. Ribeiro (UFPR).

In [24] we propose an algorithm for solving nonsmooth convex constrained optimization problems, which combines the ideas of the proximal bundle methods with the filter strategy for evaluating candidate points. The resulting algorithm inherits some attractive features from both approaches. On the one hand, it allows an effective control of the size of quadratic programming subproblems via the compression and aggregation techniques of the proximal bundle methods. On the other hand, the filter criterion for accepting a candidate point as the new iterate is expected to be easier to satisfy than the usual descent condition in bundle methods. Preliminary computational results, including comparisons with the (nonfilter) infeasible bundle method in [16], see Report 2004, are also reported.

6.4.3. Nonconvex proximal methods

Participants: C. Sagastizábal, W. Hare (Advanced Optimization Laboratory, Mc Master University, Canada).

The proximal point mapping is the basis of many optimization techniques for convex functions. By means of variational analysis, the concept of proximal mapping was recently extended to nonconvex functions that are prox-regular and prox-bounded. In such a setting, the proximal point mapping is locally Lipschitz continuous and its set of fixed points coincide with the critical points of the original function. This suggests that the many uses of proximal points, and their corresponding proximal envelopes (Moreau Envelopes), will have a natural extension from Convex Optimization to Nonconvex Optimization. For example, the inexact proximal point methods for convex optimization might be redesigned to work for nonconvex functions. In order to begin the practical implementation of proximal points in a nonconvex setting, a first crucial step would be to design efficient methods of approximating nonconvex proximal points. This would provide a solid foundation on which future design and analysis for nonconvex proximal point methods could flourish.

In [23] we present a methodology based on the computation of proximal points of piecewise affine models of the nonconvex function. These models can be built with only the knowledge obtained from a black box providing, for each point, the function value and one subgradient. Convergence of the method is proven for the class of nonconvex functions that are prox-bounded and lower- \mathcal{C}^2 and encouraging preliminary numerical testing is reported.

6.5. Industrial applications

Participants: C. Sagastizábal, E. Finardi (UFSC), E. da Silva (Tractbel Brasil), R. Marcató (Light Brasil).

In [11] we consider the inclusion of hydro-thermal unit-commitment in the optimal management of the Brazilian power system, as well as a long term generation and interconnection expansion planning problem.

Modern generation expansion planning problems take into account not only satisfaction of future demand but also government imposed limits on pollution produced by coal based power plants. In [18], [25] we model these features with a two-stage stochastic programming problem with recourse that can be solved by decomposition methods. We present results for different demand scenarios, corresponding to low, medium,

and high economic future growth, as well as different environmental cases, with stringent and non-stringent emission limits for the thermal plants composing the electric mix.

7. Contracts and Grants with Industry

7.1. Trajectory optimization

We have agreements of cooperation with Onera and CNRS concerning the studies on transfer or orbits for low-thrust satellites, and optimal trajectories for future launchers.

8. Other Grants and Activities

8.1. International collaborations

- With Felipe Alvarez, from CMM and Universidad de Chile, Santiago de Chile, F. Bonnans and J. Laurent-Varin have worked on the analysis of logarithmic penalty for optimal control problems. F. Bonnans and H. Ramirez have published the INRIA Research report 5293.
- With Claudia Sagastizábal, IMPA, Rio de Janeiro : we are currently analysing some approaches for stochasting programming, with application to the production of electricity.

8.2. Visiting Scientists

C. Sagastizábal and Mikhail Solodov (IMPA - Brazil), F. Alvarez and Hector Ramirez-Cabrera (DIM - Chile), Pablo Lotito (Argentina).

9. Dissemination

9.1. Teaching

- F. Bonnans : Professeur chargé de cours, Ecole Polytechnique, and Course on Continuous Optimization, Mastere de Math. et Applications, Filière "OJME", Optimisation, Jeux et Modélisation en Economie, Université Paris VI.
- S. Maroso
 - Chargé des TD et des TP du cours "Optimisation Différentiable: Théorie et Algorithmes" du prof. J.Ch. Gilbert pour la 2^{ème} année de l'ENSTA. Nombre d'heures totale (TD+TP) en 2005: 10 (6 TD + 4 TP).
- H. Zidani - Enseignant chercheur à l'ENSTA (70h)
 1. 'Optimisation quadratique': cours de tronc commun en 1^{ère} année ENSTA.
 2. 'Propagation de fronts': cours de spécialité en 3^e année ENSTA, Master 'titre'
 3. 'Contrôle optimal et équations Hamilton-Jacobi-Bellman': cours de spécialité en 3^e année ENSTA.
 4. Encadrement du PPL (projet personnel en laboratoire) de Phillippe Laugier, Avril-Juillet 2005.
- N. Megdich (15h)
 1. TD et TP d'optimisation, 1^{ère} année à l'ENSTA

9.2. Conference and workshop committees, invited conferences

- Journées d'Analyse Numérique et Optimisation (JANO'08). Decembre 14-16 2005, Rabat (Maroc). Invitation au minisymposium 'Mathématiques pour la finance'. H. Zidani.
- Regional meeting on "Sciences et Technologies pour l'Information et la Communication" (STIC-AmSud), Santiago de Chile, december 2005. C. Sagastizábal.
- LatinAmerican Congress on Electric Generation and Transmission, CLAGTEE, Mar del Plata (Argentina), november 2005. C. Sagastizábal.
- Computer Methods and Applications. Krakow (Cracovie), 14-16 nov. 2005. Minisymposium Optimal Control : Talk by F. Bonnans.
- Optimisation Frameworks for Industrial Applications EDF Clamart, 19-21 octobre 2005. Talk by S. Maroso.
- International Workshop on Optimization Frameworks for Industrial Applications "Confront & Share Methods: Efficiencies and Limits", EDF, Clamart (France), october 2005. C. Sagastizábal.
- VII International Seminar on Optimization, Lima, Peru, october 2005. C. Sagastizábal.
- 56th IAF (International Astronautical Congress), Fukuoka, 17-21 Oct 2005. Talk by J. Laurent-Varin.
- CONFERENCE ON OPTIMIZATION UNDER UNCERTAINTIES, Heidelberg (Germany) September 28-30, 2005 (COUCH 2005) University of Heidelberg. Talk by S. Maroso.
- I South Pacific Conference on Mathematics, Nouvelle Calédonie (France), august 2005. C. Sagastizábal.
- 22nd IFIP TC 7 Conference on System Modeling and Optimization Turin, Italy, July 18-22, 2005. Talks by J. Laurent-Varin, S. Maroso et N. Megdiche.
- Enumath'05. July 18-22, 2005 - Santiago de compostella (Spain). Invited talk by H. Zidani.
- VI Brazilian Workshop on Continuous Optimization, Goiania (Brazil), july 2005. C. Sagastizábal.
- Shanghai International Workshop on Optimization (IWOS 2005), May 28-30, 2005, Tongji University, Shanghai. F. Bonnans, invited speaker.
- 2nd Congrès National de Mathématiques Appliquées, 23 au 27 mai 2005, VVF Evian. Talk by J. Laurent-Varin.
- TAM-TAM'05 : Second colloque sur les Tendances dans les Applications Mathématiques en Tunisie, Algérie, Maroc. Tunis, 26-28 Avril 2005. N. Megdich : "An adaptative antidissipative method for optimal control problems"
- Journée thématique : Optimisation et systèmes dynamiques. Limoges, 18 mars 2005. Talk by F. Bonnans.
- French Latino American Conference on Applied Mathematics and Optimization, FLACAM05, Santiago de Chile, January 14-18,2005. Talks by C. Sagastizábal. Mini course by F. Bonnans.

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Major publications by the team in recent years

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- [2] J. BONNANS, M. HADDOU. *Asymptotic analysis of congested communication networks*, in "Mathematics of Operations Research", Rapport de Recherche Inria 3133, 1997, n° 25-3, 2000, p. 409-426, <http://www.inria.fr/rrrt/rr-3133.html>.
- [3] J. BONNANS, A. SHAPIRO. *Perturbation analysis of optimization problems*, Springer-Verlag, 2000.
- [4] J. BONNANS, H. ZIDANI. *Consistency of Generalized Finite Difference Schemes for the Stochastic HJB Equation*, in "SIAM J. Numerical Analysis", vol. 41, n° 3, 2003, p. 1008-1021.

Books and Monographs

- [5] G. DI PILLO, M. ROMA (editors). *Non convex optimization and its applications series*, ISBN: 0-387-30063-5, to appear in 2006, Springer Verlag.
- [6] J. BONNANS, P. ROUCHON. *Commande et optimisation de systèmes dynamiques*, Editions de l'Ecole Polytechnique, 2005, www.polymedia.polytechnique.fr.

Articles in refereed journals and book chapters

- [7] J. BONNANS, J. LAURENT-VARIN. *Computation of order conditions for symplectic partitioned Runge-Kutta schemes with application to optimal control*, in "Numerische Mathematik", to appear.
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- [19] O. BOKANOWSKI, N. MEGDICH, H. ZIDANI. *An adaptative antidissipative method for optimal control problems*, Rapport de Recherche, n° 5770, INRIA, 2005, <http://www.inria.fr/rrrt/rr-5770.html>.
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