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Project-Team MOÏSE

Modelling, Observations, Identification for Environmental Sciences

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1. Team

The MOISE project is a joint project between CNRS, UJF (Université Joseph Fourier, Grenoble 1), INPG (Institut National Polytechnique de Grenoble) and INRIA Rhône-Alpes. This project is located in the LMC-IMAG laboratory.

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2. Overall Objectives

2.1. Overall Objectives

MOÏSE is a research project in applied mathematics and scientific computing, focusing on the development of **mathematical and numerical methods for direct and inverse modelling in environmental applications** (mainly geophysical fluids). The scientific backdrop of this project is the **design of complex forecasting systems**, our overall applicative aim being to contribute to the improvement of such systems, especially those related to natural hazards: climate change, regional forecasting systems for the ocean and atmosphere, decision tools for floods, snow avalanches, mud or lava flows...

A number of specific features are shared by these different applications: interaction of different scales, multi-component aspects, necessity of combining heterogeneous sources of information (models, measurements, images), uniqueness of each event. The development of efficient methods therefore requires to take these features into account, a goal which covers several aspects, namely:

- mathematical and numerical modelling

- data assimilation (determistic and stochastic approaches)

- quantification of forecast uncertainties.

Pluridisciplinarity is a key aspect of the project. The part of our work more related to applications is therefore being conducted in close collaboration with specialists from the different fields involved (geophysicists, etc).

3. Scientific Foundations

3.1. Introduction

Geophysical flows generally have a number of particularities that make it more difficult to model them and that justify the development of specifically adapted mathematical and numerical methods:

- Geophysical flows are non-linear. There is often a strong interaction between the different scales of the models, and small-scale effects (smaller than mesh size) have to be modelled in the equations.
- Every geophysical episode is unique: a field experiment cannot be reproduced. Therefore the validation of a model has to be carried out in several different situations, and the role of the data in this process is crucial.
- Geophysical fluids are non closed systems, i.e. there is always interactions between the different components of the environment (atmosphere, ocean, continental water,etc.). Boundary terms are thus of prime importance.
- Geophysical flows are often modelled with the goal of providing forecasts. This has several consequences, like the usefulness of providing corresponding error bars or the importance of designing efficient numerical algorithms to perform computations in a limited time.

Given these particularities, the overall objectives of the MOÏSE project described earlier will be addressed mainly by using the mathematical tools presented in the following.

3.2. Numerical Modelling

Models allow a global view of the dynamics, consistent in time and space on a wide spectrum of scales. They are based on fluid mechanics equations and are very complex since they deal with the irregular shape of domains, and include a number of specific parameterizations (for example, to account for small-scale turbulence, boundary layers, or rheological effects). Another fundamental aspect of geophysical flows is the importance of non-linearities, i.e. the strong interactions between spatial and temporal scales, and the associated cascade of energy, which of course makes their modelling more complicated.

Since the behaviour of a geophysical fluid generally depends on its interactions with others (e.g. interactions between ocean, continental water, atmosphere and ice for climate modelling), building a forecasting system often requires **coupling different models**. Several kinds of problems can be encountered, since the models to be coupled may differ in numerous respects: time and space resolution, physics, dimensions. Depending on the problem, different types of methods can be used, which are mainly based on open and absorbing boundary conditions, multigrid theory, domain decomposition methods, and optimal control methods.

3.3. Data Assimilation and Inverse Methods

Despite their permanent improvement, models are always characterised by an imperfect physics and some poorly known parameters (e.g. initial and boundary conditions). This is why it is important to also have **observations** of natural systems. Such observations are now increasingly numerous due, in particular, to satellite techniques. However, their accuracy is not always satisfactory, and the processing of such a quantity of data can be difficult. Moreover, observations provide only a partial view of reality, localised in time and space, and sometimes only very indirectly.

Since models and observations taken separetely do not allow for a deterministic reconstruction of real geophysical flows, it is necessary to use these heterogeneous but complementary sources of information simultaneously, by using **data assimilation methods**. These tools for **inverse modelling** are based on the mathematical theories of optimal control and stochastic filtering. Their aim is to identify system parameters which are poorly known in order to correct, in an optimal manner, the model trajectory, bringing it closer to the available observations.

Variational methods are based on the minimization of a function measuring the discrepancy between a model solution and observations, using optimal control techniques for this purpose. The model inputs are then used as control variables. The Euler Lagrange condition for optimality is satisfied by the solution of the "Optimality System" (OS) that contains the adjoint model obtained by derivation and transposition of the direct model. It is important to point out that this OS contains all the available information: model, data and statistics. The OS can therefore be considered as a generalised model. The adjoint model is a very powerful tool which can also be used for other applications, such as sensitivity studies, identification, etc.

Stochastic filtering is the basic tool in the sequential approach to the problem of data assimilation into numerical models, especially in meteorology and oceanography. This approach, of a stochastic nature, is justified by the fact that the dynamical system is chaotic and thus behaves similarly to a stochastic system. Moreover, the (unknown) initial state of the system can be conveniently modelled by a random vector, and the error of the dynamical model can be taken into account by introducing a random noise term. The goal of filtering is to obtain a good approximation of the conditional expectation of the system state (and of its error covariance matrix) given the observed data. These data appear as the realizations of a random process related to the system state and contaminated by an observation noise.

The development of data assimilation methods in the context of geophysical fluids, however, is difficult for several reasons:

- the models are often strongly non-linear, whereas the theories result in optimal solutions only in the context of linear systems;
- the model error statistics are generally poorly known;

- the size of the model state variable is often quite large, which requires dealing with huge covariance matrices and working with very large control spaces;
- data assimilation methods generally increase the computational costs of the models by one or two orders of magnitude.

Such methods are now used operationally (after 15 years of research) in the main meteorological centers, but tremendous development is still needed to improve the quality of the identification, to reduce their cost, and to make them available for other types of applications.

3.4. Sensitivity Analysis - Quantification of Uncertainties

Due to the strong non-linearity of geophysical systems and to their chaotic behaviour, the dependence of their solutions on external parameters is very complex. Understanding the relationship between model parameters and model solutions is a prerequisite to design better models as well as better parameter identification. Moreover, given the present strong development of forecast systems in geophysics, the ability to provide an estimate of the uncertainty of the forecast is of course a major issue. However, the systems under consideration are very complex, and providing such an estimation is very challenging. Several mathematical approaches are possible to address these issues, using either variational or stochastic tools.

Variational approach In the variational framework, the sensitivity is the gradient of a response function with respect to the parameters or the inputs of the model. The adjoint techniques can therefore be used for such a purpose. If sensitivity is sought with respect to the observations, the adjoint of the optimality system must be used, since observations are not present in the direct model. The derivation of the optimality system for sensitivity analyses leads to the study of second-order properties: spectrum and eigenvectors of the Hessian are important information on system behaviour.

Global stochastic approach Unlike the variational approach to sensitivity, the response here is considered as the output of black-box functions, i.e. complex functions depending on many poorly-understood variables. Given such a function $f(x)$ for d-dimensional x, it is important to determine which variables, if any, dominate f and how they affect the function. Stochastic sensitivity analysis requires that the uncertainty about model inputs (parameters, initial values, boundary conditions, exogenous variables,etc.) be expressed in the form of a (joint) probability distribution of these inputs. Here, appropriate nonparametric methods for estimating joint density in high dimensional settings are required, such as nonlinear independent component analysis or other dimension-reduction methods. This is a preliminary step in sensitivity analysis.

After assessment of the uncertainty distribution of the inputs, functional ANOVA models may be used to decompose a square integrable f into a sum of lower dimensional component functions that depend on subsets of input variables. The variances of the component functions will then be used to identify important variables and interactions.

4. Application Domains

4.1. Introduction

Keywords: *Data assimilation*, *Glaciology*, *High Performance Computing*, *Hydrology*, *Inverse Problems*, *Meteorology*, *Mud Flows*, *Numerical Modelling*, *Oceanography*.

The evolution of natural systems, in the short, mid, or long term, has extremely important consequences for both the global earth system and humanity. Forecasting this evolution is thus a major challenge from the scientific, economic, and human viewpoints.

Humanity has to face the problem of **global warming**, brought on by the emission of greenhouse gases from human activities. This warming will probably cause huge changes at global and regional scales, in terms of climate, vegetation and biodiversity, with major consequences for local populations. Research has therefore been conducted over the past 15 to 20 years in an effort to model the earth's climate and forecast its evolution in the 21st century in response to anthropic action.

With regard to short-term forecasts, the best and oldest example is of course **weather forecasting**. Meteorological services have been providing daily short-term forecasts for several decades which are of crucial importance for numerous human activities.

There have also been a number of newer issues arising in recent years, one of the most important being the problem of **water resource management**. The availability of pure water is a problem of prime importance which is becoming a major concern even in countries which had no such difficulties in the past. Numerical tools play an important role in the design of hydrology management systems.

Numerous other problems can also be mentioned, like **seasonal weather forecasting** (to enable powerful phenomena like an El Niño event or a drought period to be anticipated a few months in advance), **operational oceanography** (short-term forecasts of the evolution of the ocean system to provide services for the fishing industry, ship routing, defence, or the fight against marine pollution), **air pollution** prediction systems, the prediction of **floods**, or the simulation of **mud flows** and **snow avalanches** for impact studies and regional planning.

As mentioned previously, mathematical and numerical tools are omnipresent and play a fundamental role in these areas of research. In this context, the vocation of MOISE is not to carry out numerical prediction, but to address mathematical issues raised by the development of prediction systems for these application fields, in close collaboration with geophysicists.

4.2. Oceanography and the Ocean-Atmosphere System

Keywords: *Atmosphere*, *Coupling Methods*, *Data Assimilation*, *Multiresolution*, *Ocean*.

Participants: Eric Blayo, Didier Bresch, Laurent Debreu, Christine Kazantsev, Eugène Kazantsev, François-Xavier Le Dimet, Florian Lemarié, Carine Lucas, Cyril Mazauric, Maëlle Nodet, Céline Robert, Antoine Rousseau, Ehouarn Simon, Arthur Vidard.

Understanding and forecasting the ocean circulation is currently the subject of an intensive research effort by the international scientific community. This effort was primarily motivated by the crucial role of the ocean in determining the earth's climate, particularly from the perspective of global change. In addition, important recent research programs are aimed at developing operational oceanography, i.e. near real-time forecasting of ocean circulation, with applications for ship routing, fisheries, weather forecasting, etc. Another related field is coastal oceanography, dealing for example with pollution, littoral planning, or the ecosystems management. Local and regional agencies are currently very interested in numerical modelling systems for coastal areas.

Both ocean-alone models and coupled ocean-atmosphere models are being developed to address these issues. In this context, the MOISE project conducts efforts mainly on the following topics:

- *Multiresolution approaches and coupling methods*: Many applications in coastal and operational oceanography require high resolution local models. These models can either be forced at their boundaries by some known data, or be dynamically coupled with a large-scale coarser resolution model. Such model interactions require specific mathematical studies on open boundary conditions, mesh refinement methods, and coupling algorithms. The latter have also to be studied in the context of ocean-atmosphere coupled systems.
- *Advanced numerical schemes*: Most ocean models use simple finite difference schemes on structured grids. We are seeking for higher order schemes allowing both accuracy and good conservation properties, and dealing with irregular boundaries and bottom topography.
- *Parameterization and modelling of boundary layers*: A striking feature of ocean dynamics is the existence of several types of boundary layers, either lateral (near the coastlines), or vertical (near the ocean surface and bottom). Despite their relatively small size, these layers have an important role in the global dynamics, and must be accurately represented in the model. New modelling and numerical approaches to this problem are studied.

- *Data assimilation methods for ocean modelling systems*: The main difficulties encountered when assimilating data in ocean or atmosphere models are the huge dimension of the model state vector (typically 10^6-10^7), the strongly nonlinear character of the dynamics, and our poor knowledge of model error statistics. In this context, we are developing reduced order sequential and variational data assimilation methods addressing the aforementioned difficulties. We are also working on the assimilation of lagrangian data, and on the design of data assimilation methods for multiresolution models and for coupled systems.

Most of these studies are led in strong interaction with geophysicists, in particular from the Laboratoire des Ecoulements Géophysiques et Industriels (LEGI, Grenoble).

4.3. Hydrology and River Hydraulics

Keywords: *Richards equations*, *Shallow-water equations*, *coupling/superposition of models*, *data assimilation*, *floods*, *hydrology*, *sensitivity analysis*.

Participants: William Castaings, Igor Gejadze, Xijun Lai, Marc Honnorat, François-Xavier Le Dimet, Joël Marin, Jérôme Monnier, Maelle Nodet.

Water resources and floods are critical issues. They are the result of complex interactions within the water cycle between meteorology, hydrology and hydraulics. Mathematical and numerical modelling is becoming accepted as a standard engineering practice for prevention and prediction.
Concerning river hydraulics, forward models based on 1-D and 2-D shallow water equations and the corresponding industrial softwares (e.g. Telemac2D, Carima1D) are satisfying for many situations. Nevertheless for real applications, initial and boundary conditions (basically, water level and discharge) are very partially measured hence difficult to prescribe. Moreover, empirical parameters (e.g. land roughness) are calibrated manually with difficulties. Concerning soil infiltration and rainfall-runoff phenomena, on one hand forward models have still to be improved (e.g. 3D Richards equations), and on the other hand, empirical parameters are numerous and very difficult to prescribe.
Realistic and reliable numerical prediction requires an integrated approach with all components (different models coupled together and corresponding measured data), with affordable computational cost. Sensitivity analysis and data assimilation methods, that have shown their potential in other geosciences like meteorology and oceanography, are now in the forefront in hydrology. This prediction chain is far from being operational in hydrology.
The problems addressed in MOISE are related to the coupling of models, more efficient forward solvers, sensitivity analysis and data assimilation for catchment scale hydrology and river hydraulics.
The current research topics conducted in MOISE are the following:

- *Image data assimilation*. Images potentially contain a huge amount of information which could be used in conjunction with numerical models. A major difficulty is to develop convenient "observation operators”, linking images and model variables.
- *Coupling between 1D and 2D models, with data assimilation*.
- *Soil water transfer modelling*.
- *Sensitivity analysis for rainfall-runoff models*.

4.4. Mud Flows and Snow Avalanches in Mountains

Keywords: *Finite Element Method*, *Non-Newtonian Fluids*, *Saint-Venant Equations*.

Participant: Pierre Saramito.

The prevention of hazards in mountains, from snow avalanches to mud flows, requires advanced numerical simulations. Such simulations are based on sophisticated rheological and mechanical laws for describing complex materials, such as yield stress fluids and granular materials. Moreover, numerical simulations use sophisticated numerical methods: conservative finite volumes or finite elements, adaptive unstructured mesh generation, in a dynamical context. While dense snow avalanche prevention is quite advanced, the prediction of powder-snow avalanche phenomena is more delicate.

4.5. Glaciology

Keywords: *Inverse Methods*, *asymptotic analysis*, *coupling*, *data assimilation*, *glaciology*, *non-linear Stokes model*, *optimal control*.

Participants: Eric Blayo, François-Xavier Le Dimet, Bénédicte Lemieux-Dudon, Jérôme Monnier.

The study of past climate is a means of understanding climatic mechanisms. Drillings in polar ice sheets provide a huge amount of information on paleoclimates: correlation between greenhouse gases and climate, fast climatic variability during the last ice age, etc. However, in order to improve the quantitative use of the data from this archive, numerous questions remain to be answered because of phenomena occurring during and after the deposition of snow. An important research aim is therefore to optimally model ice sheets in the vicinity of drilling sites in order to improve their interpretation: age scale for the ice and for the gas bubbles, mechanical thinning, initial surface temperature and accumulation when snow is deposited, spatial origin of ice from the drilling.

Another objective is the evaluation of the state of the polar ice caps in the past, and their interactions with the other components of the earth climate, in order to forecast their evolution in the forthcoming centuries. The joint use of models and data, through data assimilation techniques, to improve system description is relatively new for the glaciological community. Therefore inverse methods have to be developed or adapted for this particular purpose.

5. Software

5.1. River Hydraulics

Participants: Igor Gejadze, Xijun Lai, Marc Honnorat, Joel Marin, Jérôme Monnier.

DASSFLOW is a river hydraulics simulation software designed for variational data assimilation. The forward model is based on the 2D shallow-water equations in conservative form with a friction term. Time-discretization is the explicit Euler scheme, space-discretization is a finite volume scheme based on a well-balanced HLLC approximate Riemann solver (including the topography term). The mesh is unstructured, mix of triangles-quadrangles. Many boundary conditions, including characteristics ones, are available. The code is written in Fortran 90, and the adjoint code is automatically generated using the automatic differentiation tool Tapenade. Benchmarks related to the forward model and to some identification problems are available. It is interfaced with a few free and commercial pre and post-processors (SIG tools, mesh generators, visualization tools), which allows using Dassflow for real data. A twin experiment mode is included.
Two versions are currently available. The first one will be published on the web in january 2007 (via a Forge tool). The second one is more experimental and includes the additional following features:

1. assimilation of Lagrangian data (local particles trajectories extracted from video images for example),
2. a weak coupling algorithm between a 1D global model with storage area and local 2D models, with assimilation of in-situ measurements,
3. assimilation of the location of fronts, based on a level set method,
4. additional transport equation (dedicated to pollutant for example).

Joël Marin joined our team in September 2006 to develop and maintain this software among others.

5.2. Adaptive Grid Refinement

Participants: Laurent Debreu, Florian Lemarié, Cyril Mazauric.

AGRIF (Adaptive Grid Refinement In Fortran, [54], [53]) is a Fortran 90 package for the integration of full adaptive mesh refinement (AMR) features within a multidimensional finite difference model written in Fortran. Its main objective is to simplify the integration of AMR potentialities within an existing model with minimal changes. Capabilities of this package include the management of an arbitrary number of grids, horizontal and/or vertical refinements, dynamic regridding, parallelization of the grids interactions on distributed memory computers. AGRIF requires the model to be discretized on a structured grid, like it is typically done in ocean or atmosphere modelling. As an example, AGRIF is currently used in the following ocean models: MARS (a coastal model developed at IFREMER-France), ROMS (a regional model developed jointly at Rutgers and UCLA universities, [21]), OPA-NEMO ocean modelling system (a general circulation model used by the French and European scientific community).
AGRIF is licensed under a GNU (GPL) license and can be downloaded at its web site (http://www-lmc.imag.fr/MOISE/AGRIF/index.html). More than two hundred downloads of the software have been done during the last year.

5.3. Rheolef: a C++ Finite Element Environment

Keywords: *C++*, *adaptive meshes*, *finite elements*, *numerical simulation*, *partial derivative equations*.

Participant: Pierre Saramito.

Home page: http://www-lmc.imag.fr/lmc-edp/Pierre.Saramito/rheolef
Current stable version: 5.18
The license is GPL.
Rheolef is a computer environment that serves as a convenient laboratory for computations, involving finite element-like methods. It provides a set of unix commands and `C++` algorithms and classes.
Classes covers first the classic graph data structure for sparse matrix formats and finite element meshes. A higher level of abstraction is provided by classes related to approximate finite element spaces, discrete fields and bilinear forms.

Current applications cover:

- Poisson problems in 1D, 2D and 3D with P1 or P2 elements,
- Stokes problems in 2D and 3D, with P2-P1 or P1 bubble-P1 elements,
- linear elasticity in 2D and 3D, with P1 and P2 elements, including the incompressible and nearly incompressible elasticity,
- characteristic method for convection-difusion, time-dependent problems and Navier-Stokes equations,
- self-adaptive mesh based for 2D problems,
- axisymmetric problems,
- multi-regions and non-constant coefficients.

Input and output support various file formats for meshes generators and numerical data visualization systems (mayavi and vtk, plotmtv, gnuplot).
Both reference manual and user's guide are available. Distributions are available both in source form as a tar.gz pack and as binaries (debian and rpm packs).

6. New Results

6.1. Ocean Modelling

6.1.1. Mathematical Modelling of the Ocean Dynamics

Participants: Didier Bresch, Christine Kazantsev, Carine Lucas, Maëlle Nodet, Antoine Rousseau.

6.1.1.1. Small-scale induced effects in the oceans

Ocean bottom topography and coastlines vary over a wide range of scales, the smallest ones being unresolved in numerical computations. There is a need for the development of simplified models that implicitly account for the impact of the small-scale topography on the large-scale ocean circulation. We have recently developed such nonlinear models in idealized cases for the quasi-geostrophic system and the lake equations. This leads for instance to some nonlinear PDEs which govern the western boundary layer, extending in some sense the linear one proposed by H.W. Munk (Munk layer). It also provides some models with memory effects. Recently a mathematical derivation has been performed using two-scale convergence technics and defect measures control. We have also worked on the effect of fast oscillating topography in quasi-geostrophic equations and proposed some numerical implementation tools to simulate such flows. We are now looking at more general cases (other PDEs, more general topography and coastlines) and are trying to tranfer these developments to actual applications (collaboration with geophysicists at LEGI).

6.1.1.2. Regularity around some PDEs in geophysics

It is well known that the Investigation of regularity results for PDEs derived from geophysics is not straight forward in the case of non smooth domains. In a recent work, we have proposed with G. Métivier an extended Youdovitch's method for the inviscid lake equation in bounded domains with degenerated topography. To obtain such a result, we have established the first (to our knowledge) L^p regularity result with explicit constant control on weighted elliptic system.

6.1.1.3. Mathematical justification of asymptotics

The mathematical justifications of formal asymptotics encountered in geophysics are not straight forward. They depend on the domain, the data, and the PDEs that are considered. For instance, we have established that one has to add a corrector to the Sverdrup relation in the case of western intensification of currents for domains with islands. We have also considered rotating fluids in a cylinder, showing that the Rossby waves are damped if the lateral section is not a disk (this result is linked to the Schiffer conjecture). This work concerns ill-prepared data. We have also obtained the first justification of the planetary geostrophic equations from the Primitive Equations. This problem concerns singular perturbations with an operator which is not skew-symmetric as it usually is in mathematical papers.

6.1.1.4. Influence of the viscous terms (Reynolds closure)

In the framework of compressible flows, we have introduced viscous models which provide additional behavior of the density close to vacuum. We actually proved that, if some compatibility conditions between the viscosities are satisfied, some additional regularity information on a quantity involving the density is available. We obtained a non-trivial equality deduced from the special structure of the momentum equations. This result allows for instance to justify the link between the viscous shallow water equations and the viscous quasi-geostrophic equations or the viscous lake equations. Using this new mathematical entropy that has been discovered by D. Bresch and B. Desjardins (called now BD entropy in the litterature), we obtained this year the first result of global existence of weak solutions to the full compressible Navier-Stokes equations with heat conductivity. This extends the result obtained by J. Leray on incompressible flows and P.–L. Lions on barotropic compressible flows. This gives an answer to this open problem. Note that this technic leads also to the existence of global weak solutions for barotropic compressible Navier-Stokes equations with non-constant viscosity coefficients. It completes, in some sense, the result by P.–L. Lions since the two approaches covers different cases.

Recently, C. Lucas has performed some studies on the effect of Cauchy tensors and boundary conditions on the derivation of shallow water equations. She proved for instance that some terms have been omitted in some previous studies made by other authors.

6.1.1.5. Numerical simulations and finite volume schemes

In the framework of a regional project devoted to viscous shallow water equations and their application to environment, we have developed sedimentation and sub-marine avalanches models and established well balanced finite volume schemes (in collaboration with E. Fernandez Nieto, and more generally Sevilla and Malaga teams). We also developed appropriate finite volume schemes for pollutant propagation.

6.1.1.6. Regularity results for the linear Primitive Equations of the ocean

We have considered the linear Primitive Equations of the ocean in the three-dimensional space, with periodic (on the horizontal) and Dirichlet (on the vertical) boundary conditions. Thanks to Fourier transforms we were able to compute explicitly the pressure term. We then stated existence, unicity and regularity results for the linear time-depending Primitive Equations, with low-regularity right-handside (see [18]).

6.1.1.7. Open boundary conditions for the inviscid primitive equations

A. Rousseau pursued his work started in [55] on the design of open boundary conditions for the Primitive Equations of the atmosphere and the ocean in the absence of viscosity. When the viscosity is present, the mathematical theory has been pretty much studied since the 90's by many authors. In the absence of viscosity, little progress has been made since pionneering papers of the 70's.
In earlier works, A. Rousseau collaborated with R. Temam (Indiana University) and J. Tribbia (NCAR) to investigate these equations in 2 dimensions. In [24], numerical computations of the 2D model have been performed (see figure 1), which confirm the well-posedness of the so-called open boundary conditions.
The 3D model is now under investigation, both from the theoretical and numerical viewpoints.

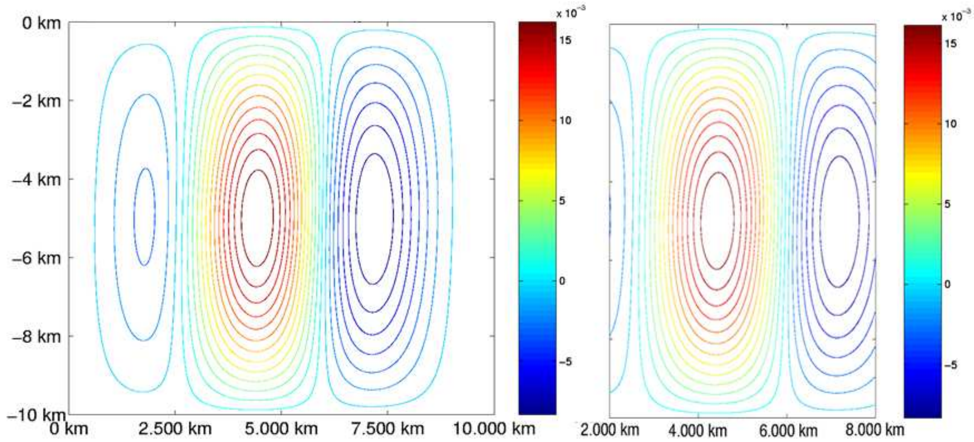

Figure 1. The solution of the PEs computed with transparent boundary conditions (right) match very well the reference solution (left) in the subdomain 2000-8000 km.

6.1.2. High-Order Schemes for a Numerical Model of the Black Sea

Participant: Eugène Kazantsev.

One of the obvious ways to reduce the number of discretisation points of a numerical model and to reproduce the impact of small scales on the large scale processes consists in using improved numerical techniques, namely high order schemes. This kind of schemes is widely used atmospheric modelling. In particular, compact schemes of fourth, fifth and higher order have shown their great ability to reproduce complex physical phenomena in models at relatively low resolution.

However, the major difficulty in using high-order schemes in ocean modelling is due to the presence of boundary conditions. Indeed, periodical boundaries in atmospheric models greatly simplify the implementation of high-order compact schemes.

It was shown previously that the use of high order schemes in a shallow water model for large oceanic scales results in a deformed solution due to a lack of resolution in the boundary layers. However, when the shallow water model is integrated in a smaller rectangular domain, of typical size of some internal sea or lake, the solution possesses no boundary layer, and the advantages of high order schemes become visible. In that case, the solution of the model obtained with fourth order mass-conservative scheme is comparable to the solution with classical second order scheme with three times higher spatial resolution. This fact indicates that high order schemes for some realistic internal sea model should be implemented.

The fourth order scheme was applied to the model of the Black sea developed in the Marine Hydrophysic Institute of the Ukrainian Academy of Science. However, the particular approximation of the boundary conditions that insures the mass conservation in the case of a square domain is no longer valid in a complex domain like the Black sea. The total water mass decreases in the experiment. New approximation of the boundary conditions is proposed in order to ensure the mass conservation, but this approximation is first order only. The whole scheme is then of second order in this case and the solution is very similar to the reference solution obtained with the classical second order scheme.

A paper analyzing this problem is in preparation.

6.1.3. Coupling Methods for Oceanic and Atmospheric Models

Participants: Eric Blayo, Laurent Debreu, Florian Lemarié, Joel Marin, Elise Nourtier, Céline Robert.

The implementation of high-resolution local models can be performed in several ways. An usual way consists in designing a local model, and in using some external data to force it at its open boundaries. These data can be either climatological or issued from previous simulations of a large-scale coarser resolution model. The main difficulty in that case is to specify relevant open boundary conditions (OBCs).

Following our preceding work, we have implemented characteristic (in the hyperbolic sense) OBCs in two different 3-D primitive equation ocean models, MARS (collaboration with F. Vandermeirsch, IFREMER Brest) and OPA-NEMO, in realistic configurations. Our results confirmed the fact that characteristics-based OBCs are relevant for the barotropic (i.e. depth-averaged) dynamics, and lead to clear improvements with regard to usual OBCs. For the baroclinic (i.e. fully 3-D) part of the dynamics, characteristic OBCs are based on a local decomposition into vertical modes. Such OBCs had never been implemented before in such realistic models. Our first results lead to improved, or at least similar, results with regard to usual OBCs. However numerous aspects require additional investigation, in order to optimize the different parameters involved in the implementation. This work is presently being continued.
Another way of designing such local models consists in coupling a high resolution local model with some coarser resolution outer model. Such a coupling between two models with possibly different resolutions, numerics, and even physics, can be performed within the framework of global-in-time Schwarz domain decomposition methods. However, the efficiency of these algorithms is strongly dependent on interface conditions. In collaboration with applied mathematicians from LAGA - Paris 13 (L. Halpern, C. Japhet, V. Martin, E. Audusse), we carry on mathematical studies on the development of absorbing conditions for the usual systems of equations encountered in ocean and atmosphere modelling. The absorbing conditions approach can be seen as a generalization of the characteristics method. Exact absorbing conditions, avoiding any reflection of errors on the boundaries, have been determined for different kind of tracer equations (with either laplacian or bi-laplacian diffusion operators), and for the 2-D linearized shallow-water system, taking into account both advection and Coriolis terms. These exact conditions are non local both in space and

in time, and need therefore to be approximated. Two approaches have been investigated for this purpose: (1) optimization of the convergence rate of the Schwarz algorithm (by controlling parameters involved in the expression of the boundary condition), (2) approximation based on mathematical coherence constraints and/or physical properties. Efficient boundary conditions have been computed by the first approach for tracer equations, and by the second approach for the 2-D linearized shallow-water system.

Many applications in coastal and operational oceanography require high resolution local models, for which ocean-atmosphere interactions must be properly taken into account. The algorithms currently applied to couple ocean and atmosphere usually provide approached solutions with some regularity troubles. In this context, domain decomposition methods provide flexible and efficient tools for coupling models with non-conforming time and space discretizations. In order to apply this kind of methods to ocean-atmosphere coupling we must consider not only primitive equations but also parameterizations of small-scale physical processes near air-sea interface. The basic idea consists in controling the non conformity at the interface by using Schwarz algorithms with physically relevant interface conditions determined by our knowledge of surface boundary layer processes. On top of that for transmission conditions involving time derivatives, discontinuous Galerkin method could be used to get higher order schemes in time.

A realistic application making use of these new concepts will be implemented. It will consist in a 2D vertical y-z configuration with MAR (Modèle Atmosphérique Régional) and ROMS (Regional Ocean Modelling System) numerical models, for the simulation of Hadley cell circulation and sub-equatorial currents.

These works are partially supported by the national MERCATOR program and by an ANR project.

6.2. Data Assimilation Methods for Ocean Models

Participants: Didier Auroux, Eric Blayo, Laurent Debreu, Eugène Kazantsev, François-Xavier Le Dimet, Maëlle Nodet, Ehouarn Simon, Arthur Vidard.

6.2.1. Assimilation of Lagrangian Data

Participant: Maëlle Nodet.

This work is motivated by the Argo program, which aims at deploying a network of 3000 profiling floats over the world ocean. Argo is part of the international GODAE experiment (Global Ocean Data Assimilation Experiment). These profilers drift at a typical depth of 1500m, and perform a vertical profile of temperature and salinity measurements every ten days. Their position is known every ten days, which gives a set of lagrangian data. We have developed a variational method in order to assimilate such data. Twin experiments were performed within the OPAVAR model, in an idealized configuration. We have tested the impact on the assimilation of the parameters of the floats network: impact of the frequency of the data, drifting depth, number of floats, noise on the observations, initial distribution of the floats, see [19] and [40]. We also implemented a method for the simultaneous assimilation of floats positions and another type of data, namely vertical profiles of temperature. The complementarity of these two data types has been assessed. For example, figure 2 shows the horizontal velocity and temperature fields (u, v, T) at level 2 (around 400 meters deep): on the left are the reference state outputs, which we want to reconstruct, in the middle are the control state outputs, which we obtain whithout data assimilation, and on the right are the analysed state outputs, which we obtain after assimilation of profiles and positions data. We can see that the data assimilation system is able to reconstruct accurately the reference state.

This work was granted by the Mercator program.

6.2.2. Variational data assimilation for locally nested models.

Participants: Eric Blayo, Laurent Debreu, Ehouarn Simon.

The objectives are to study the mathematical formulation of variational data assimilation for locally nested models and to conduct numerical experiments for validation.

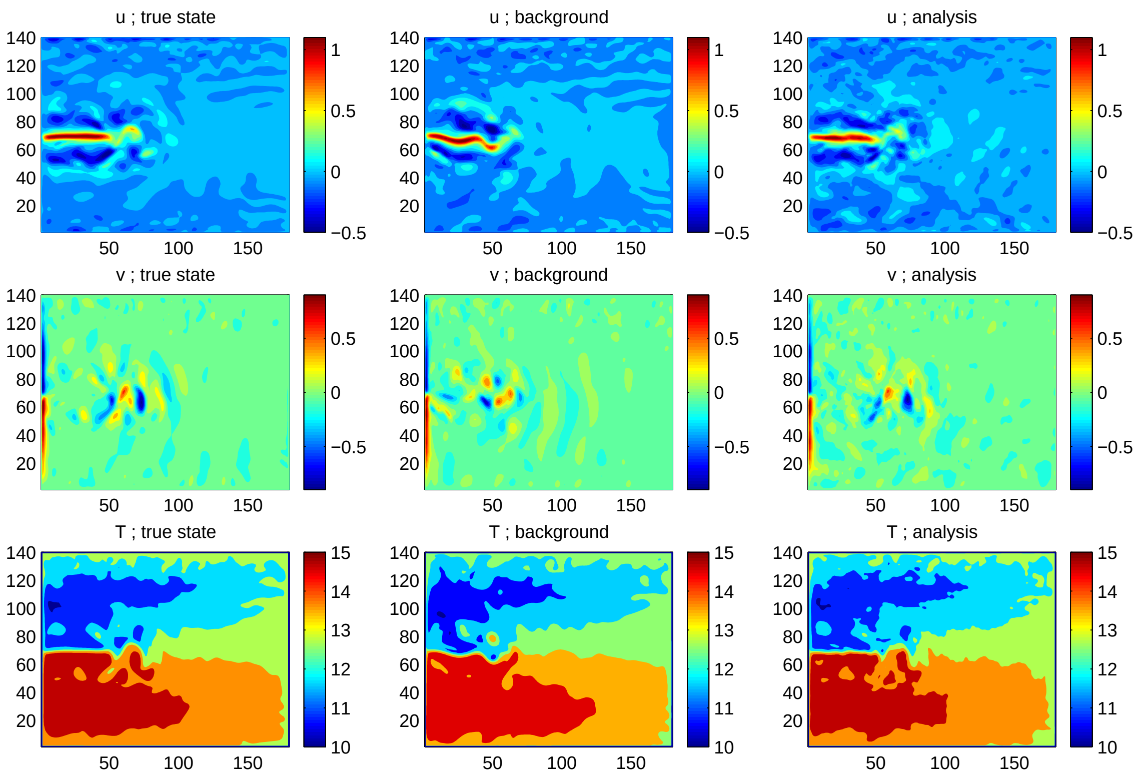

Figure 2. The solution of the PEs computed with transparent boundary conditions (right) match very well the reference solution (left) in the subdomain 2000-8000 km.

The state equations of the optimality system have been written for the general case of two embedded grids, for which several kinds of control (initial conditions, boundary conditions) have been proposed. Both one way and two way interactions have been studied. However, the problem of the selection of the assimilated observations for such multi-grids system remains open.Theoretical investigations on this crucial problem are part of the end of Ehouarn Simon's PhD thesis.

Numerical experiments in 2D shallow water models, in the case of observations available only on the fine local grid, show promising results. More complex experiments, in particular in presence of non linear grid interactions, will be performed.

Finally, we will also compare this approach with a direct application of local multigrid methods for the solution of the optimality system.
This work is granted by the french DGA.

6.2.3. Variational Data Assimilation for the Identification of an Optimal Topography

Participant: Eugène Kazantsev.

Following a previous study on the sensitivity of the solution of an ocean model to the representation of the bottom topography, we focused our attention on the data assimilation procedure, which allows us to find an optimal topography pattern, corresponding to some optimality criterion.

The procedure is similar to data assimilation for the identification of an optimal initial condition of a model. In both cases we look for the model solution that minimizes the distance to the set of observations. However, the control parameter in this case is the model bottom topography rather than the initial condition.

This assimilation procedure has been tested for the simple barotropic ocean model that was previously used in the sensitivity study. Special attention was payed to the choice of the assimilation interval and of the amount of assimilated external information. Artificial observations generated by the same model were used in experiments. This allowed us to estimate the precision of the reconstruction of the topography pattern.

In order to estimate the influence of measurement errors that are present in real data, we performed a set of experiments of assimilation of polluted fields. The model with modified parameters is used to produce artificial observations in this case. The perturbation consists in a white noise added to the field at each grid point of the field. We first modify the model initial conditions, simulating the interpolation or assimilation errors. Second, we perturb the forcing of the model in order to simulate the different physics for real observations. And third, we add a white noise to the artificial observations at all grid-points and at all time steps in order to simulate the measurements errors.

The obtained results show the linear dependency of the error in the identified topography on the amplitude of the pollution. Hence, we can get the optimal topography with a good accuracy by assimilating observations.

The influence of the space grid of the model was also considered. When the artificial observational data are generated by a high resolution model, they contain small scale information that cannot be assimilated by a low resolution model. The problem of interpolation and smoothing of the high resolution data was discussed and the assimilation error was analyzed.

6.2.4. Development of a Variational Data Assimilation System for OPA9/NEMO

Participant: Arthur Vidard.

A new version of the french ocean model OPA (Ocean PArallèle), ocean component of the NEMO (Nucleus for European Modelling of the Ocean) framework was released in 2005. For the previous version of the OPA model (8.2) a variational data assimilation system, OPAVAR, was developed mainly by A. Weaver at CERFACS. However the OPA 9 model has been completely rewritten in Fortran 90 and the code structure is significantly different from the previous versions, making it quite difficult to update OPAVAR .

In early 2006 a collaborative project was initiated between MOISE project (A. Vidard), CERFACS (A. Weaver) and ECMWF (K. Mogensen) to develop NEMOVAR, a variational system for NEMO/OPA9.

Since a large community is interested in variational data assimilation with OPA9, we built a working group (coordinated by A. Vidard) in order to bring together various OPAVAR user-groups with diverse scientific interests (ranging from singular vector and sensitivity studies to specific issues in variational assimilation), and to get technical and scientific support from Inria Sophia (Automatic adjoint derivation, TROPICS project) and ECMWF (Parallelization). This project aims at avoiding duplication of effort, and in the long term at developing a common NEMOVAR platform.
This working group gets financial support by LEFE-Assimilation and Mercator National Programs.

6.2.5. Variational Data Assimilation with Control of Model Error

Participant: Arthur Vidard.

One of the main limitation of the current operational variational data assimilation techniques is that they assume the model to be perfect mainly because of computing cost issues. Numerous researches have been carried out to reduce the cost of controlling model errors by controlling the correction term only in certain privileged directions or by controlling only the systematic and time correlated part of the error.

Both the above methods consider the model errors as a forcing term in the model equations. Trémolet (2006) describes another approach where the full state vector (4D field : 3D spatial + time) is controlled. Because of computing cost one cannot obviously control the model state at each time step. Therefore, the assimilation window is split into sub-windows, and only the initial conditions of each sub-window are controlled, the junctions between each sub-window being penalized. One interesting property is that, in this case, the computation of the gradients, for the different sub-windows, are independent and therefore can be done in parallel.

Y. Trémolet (ECMWF) spent one month visiting the MOISE project and we successfully implemented the control of the full state vector within the OPAVAR framework. There are still however some unresolved numerical issues, mainly the need of the inverse of some operators that prevents us to use it in a realistic framework yet. In the meantime we developed another version of this scheme using a simpler shallow-water model and the PALM coupler to illustrate its nice parallelization properties described above.

6.2.6. Data Assimilation for Coupled Models

Participant: Arthur Vidard.

One can see the method described in the previous section as a coupling of several assimilation windows with perfect model hypothesis. This can then be extended to the coupling of different models.

This is not a simple task: it involves the gathering of very heterogeneous information multi-scales, multi-fluids, multi-phases and numerous datatype)

This research is still at a really preliminary stage. To begin with, we developed a coupled toy model using Lorenz equations which behaves somewhat like a tropical ocean-atmosphere system, reproducing some El Niño-like behavior. We plan now to test some of our ideas on this simple system.

6.2.7. A Nudging-Based Data Assimilation Method: the Back and Forth Nudging

Participant: Didier Auroux.

The back and forth nudging algorithm, recently introduced in collaboration with J. Blum (University of Nice) in [52], consists in solving first the standard forward nudging equation and then the direct system backwards in time with a feedback term which is opposite to the one introduced in the forward equation. The "initial" condition of this backward resolution is the final state obtained by the standard nudging method. After resolution of this backward equation, one obtains an estimate of the initial state of the system. We repeat these forward and backward resolutions until convergence of the algorithm.

This work has been motivated by the fact that it is usually tiresome to derive the adjoint model. Our algorithm does not need to linearize the system, nor to have a powerful optimization algorithm. The backward system is not the adjoint equation but the direct system, with an extra feedback term that stabilizes the resolution of this ill-posed backward problem [52].

We have recently compared this algorithm to the 4D-VAR method on toy models such as the Lorenz' and 1D viscous Burgers' equations, but also on a layered quasi-geostrophic ocean model. The numerical convergence of the BFN algorithm is always achieved in a few iterations, and in the case of twin experiments, it provides a very good estimation of the initial condition. This work is partly presented in [4].

We already proved the theoretical convergence for a linear compact operator, provided that some hypothesis on the spatial distribution of the observations and the observation operator were satisfied. We are currently working on the theoretical point of view for nonlinear models.

6.3. Hydrology and River Hydraulics

Participants: William Castaings, Igor Gejadze, Xijun Lai, Marc Honnorat, François-Xavier Le Dimet, Jérôme Monnier.

6.3.1. Lagrangian Data Assimilation for River Hydraulic Models

Participants: Marc Honnorat, François-Xavier Le Dimet, Jérôme Monnier.

M. Honnorat began his PhD in October 2003, addressing the problem of the assimilation of image type data for flood models.
Dynamic images (a series of still images, or a video) of a river flow contain informations that can be used in a data assimilation process for the identification of model parameters, such as topography and land roughness.
Numerical experiments have shown that observed trajectories of passive individual tracers on the free surface of a river flow (lagrangian data) can bring valuable information on the flow velocity for the reconstruction of inflow discharge, very local topography and manning coefficients (see Figure 3). Such computations based on video images of flows in a canal are under consideration (collaboration with N. Rivière, LMFA Lyon, and E. Huot, CLIME team-project). The next step of this approach would be to consider the assimilation of a water front, or coherent structures evolving on the surface flow. Computations based on times-series front images are under consideration (collaboration with E. Mémin and E. Arnaud from IRISA, project-team VISTA).

This PhD is supervised by J. Monnier and F.-X. Le Dimet; it is funded by CNES and CNRS.

6.3.2. Coupling 1D-2D Models and Data Assimilation

Participants: Igor Gejadze, Jérôme Monnier.

I. Gejadze, researcher at university of Strathclyde UK, spent 15 months in our project-team (from april 2005). He defined with J. Monnier a new Joint-Assimilation-Coupling algorithm. This consists in superposing weakly a 2D local shallow water model to a 1D net-global shallow-water model with storage areas. This algorithm has been implemented into the DassFlow software (see section 5.1); it should be useful to improve operational 1D-net global model with storage areas when overflowing occur (superposition of local 2D models over flooded areas), see Figure 4.

6.3.3. Assimilation of Flood Satellite Images

Participant: Xijun Lai.

X. Lai, researcher at Nanjing Institute of Geography and Limnology, Chinese Academy of sciences, spent 10 months in our project-team (from february 2006). He defined and implemented, with J. Monnier, the contribution of satellite images of a flood event into a variational data process. To this end, they defined a modified cost function and adapted a temporal strategy to the classical variational data assimilation algorithms. This has been implemented into the Dassflow software (see 5.1) and applied to real data (flooding of Moselle river), see Figure 5. This is a joint work with R. Hostache and C. Puech, Cemagref Montpellier / Maison de la télédétection.

6.3.4. Soil Water Transfer

Participant: Jérôme Monnier.

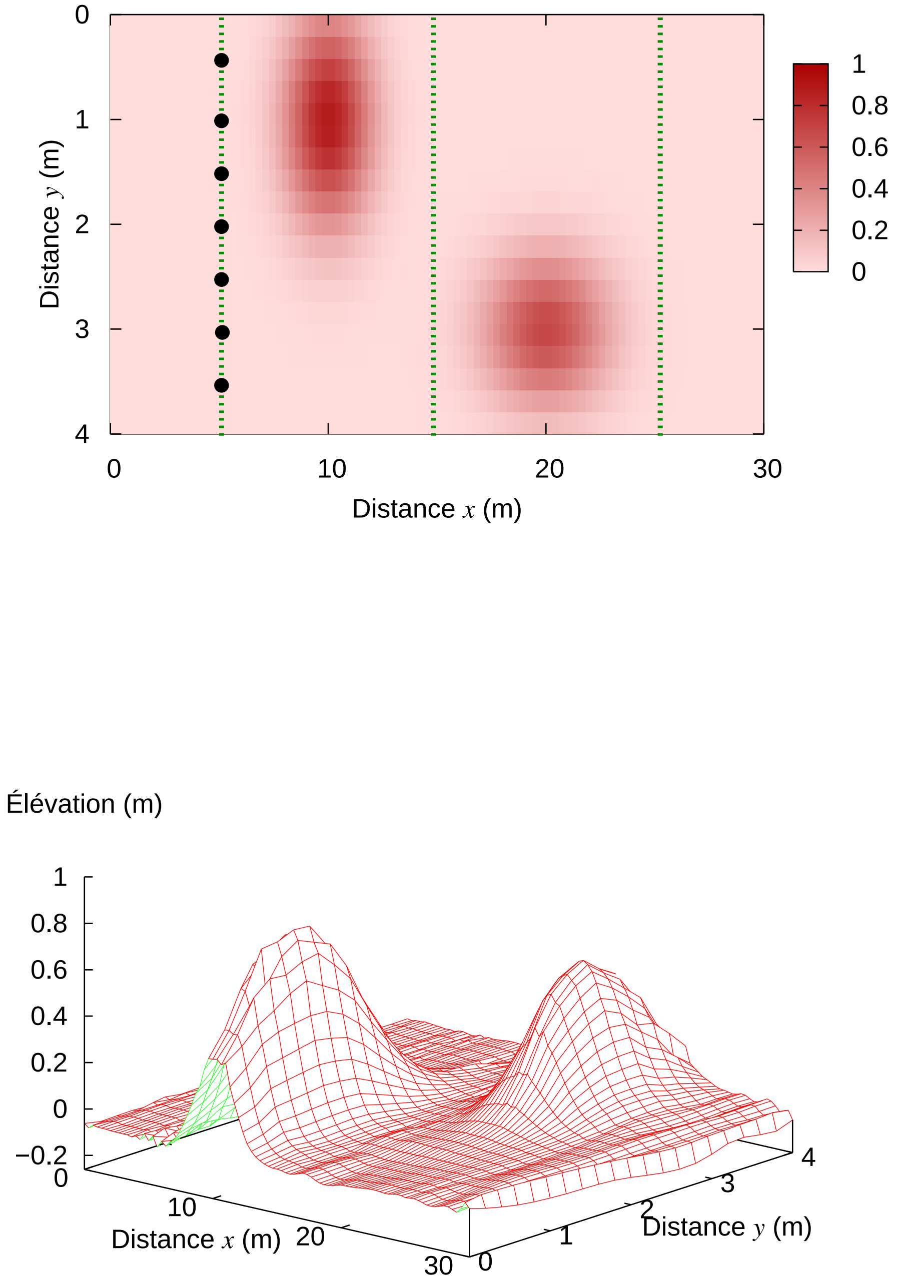

Figure 3. Top: Passive markers flowing over a local bathymetry. Bottom: Bathymetry identified after data assimilation.

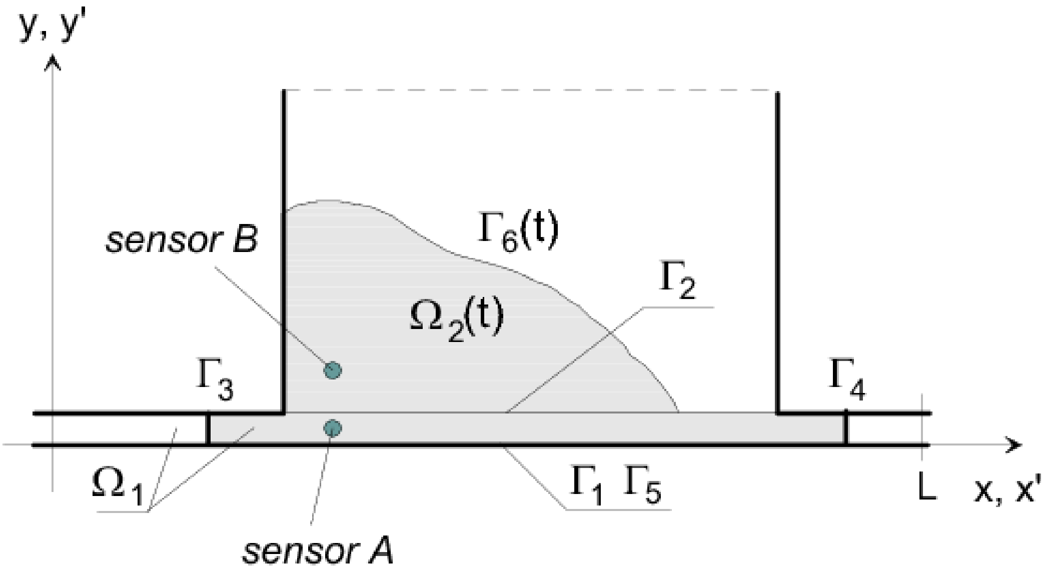

Figure 4. Simple layout. Superposition of a local 2D model over a flooded area and joined data assimilation.

We started a new collaboration with I. Braud (Cemagref Lyon) and S. Anquetin (LTHE Grenoble) focused on 3D fully distributed soil water modelling. During her 6 months Ensimag engineer internship (co-supervisor: J. Monnier), E. Neveu studied and implemented a finite solver for the Richards equations including saturated and unsaturated cases. This research topic will be continued through a Master degree internship co-supervised by M. Nodet (spring 2007), and a one-year postdoc (Cemagref Lyon and LTHE grant).

6.3.5. Adjoint sensitivity analysis and parameter estimation for hydrological modelling

Participants: William Castaings, François-Xavier Le Dimet.

During his PhD, W. Castaings investigated the potential of variational methods for some critical issues underlying the modelling of catchment scale hydrology. Adjoint sensitivity analysis and parameter estimation were carried out for both infiltration-excess and saturation-excess hydrological models. The MARINE infiltration-excess model from the Toulouse Fluids Mechanics Institute (IMFT) and a version of TOPMODEL from the Laboratory of Environmental Hydrology (LTHE) were used. The practical implementation of the approach is largely facilitated by the advent of very efficient automatic differentiation tools such as TAPENADE developed by INRIA/TROPICS. Even if the mathematical/algorithmic representations of hydrological processes contain thresholds (i.e. conditional statements in the control flow) affecting the domain of validity of the computed derivatives (only piecewise differentiability), they were found very informative and very efficient in driving bound constrained quasi-newton optimization algorithms (N2QN1 from INRIA/ESTIME).

It was shown that understanding the relationship between control and diagnostic variables is an essential step to be carried out before the assimilation of observations for state or/and parameter estimation. Adjoint sensitivity brings a significant contribution to explain and correct the lack of fit of hydrological models, identify potential deficiencies in model structure and formulation, provide guidance for model reduction and parameterization, analyze the information content of available observations and lastly describe the subspace from the original control space driving predictive uncertainty.

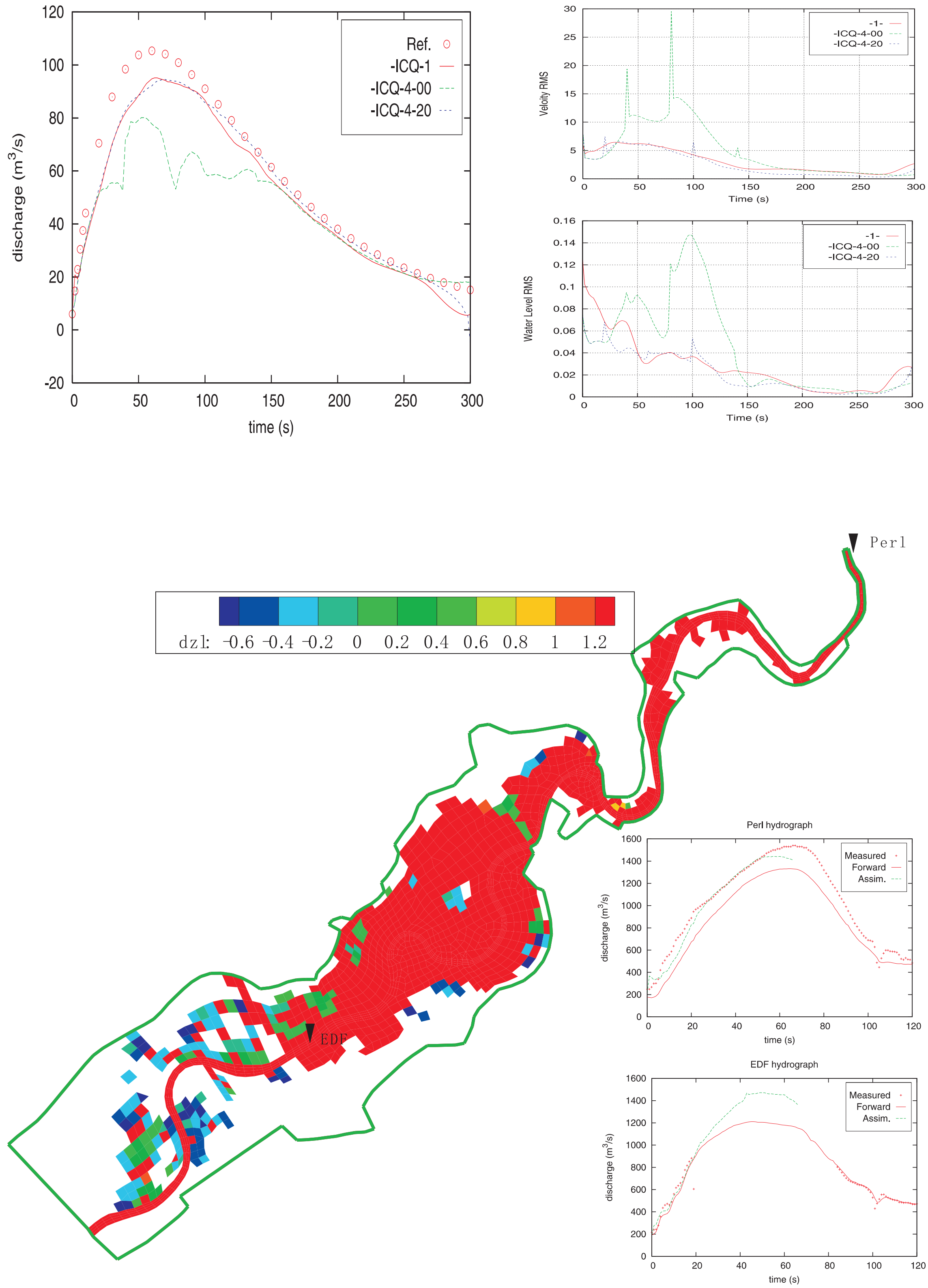

Figure 5. Application to Moselle river. Upper panel: Discharge identified using different temporal strategies. Lower panel: Difference between true and identified solutions.

While the classical parameter analysis techniques were under-used in hydrology, adjoint based-derivatives enable the use of powerful gradient-based optimization algorithms. For distributed models, the curse of dimensionality associated to sparse observations of the physical system lead to ill-posed inverse problems. The shortcomings of the heuristics commonly used for the regularization of the inverse problem (empirical and excessive reduction of the control space, Tikhonov regularization...) are highlighted and the development of a reduced-order strategy using the truncated singular value decomposition of the jacobian matrix is initiated.

Interrelated issues like state and parameter estimation, sensitivity and uncertainty analysis have received growing attention from the hydrological community but little regard was paid to the contribution of deterministic methods. The obtained results are very encouraging and open new trends for addressing number of issues related to hydrological modelling.

6.4. Snow Avalanches and Mud Flows

Participant: Pierre Saramito.

Our work is split into four axes: i) theoretical modelling of snow avalanches (aerosols) (with LEGI), ii) development of numerical solvers, iii) validation using experimental results (with CEMAGREF), and iv) visualization (virtual reality) (with GRAVIR).

Moreover, we are presently developing robust higher order schemes, in order to solve efficiently such time dependent flow problems, since simple implicit first order schemes are often insufficient for such problems.

6.5. Glaciology

Participants: Eric Blayo, Jérôme Monnier, Bénédicte Lemieux-Dudon.

In collaboration with O. Gagliardini, F. Parrenin and C. Ritz from LGGE - Grenoble, we started to study: a) the coupling between a local 3D model and a large scale asymptotic model; b) the sensitivity and identification of boundary values at open boundaries of the 3D local model.

The final aim is to obtain an efficient calibration of the numerical ice flow models used in the interpretation of the deep ice cores recently obtained in Antarctica. The interpretation (especially the dating) of these ice cores is based on thermomechanical ice flow models. However these ice flow models require improvements to make the most of the numerous data recorded in the ice cores.

The local 3D model ice flow ("local" meaning in the vicinity of the drilling location) consists in the quasi-static Stokes equations with non-linear mechanical behavior and a free surface dynamics. The large-scale global model (in the isothermal case) is based on an asymptotic analysis of the full Stokes model.

A first approach to couple both models will be addressed during a Master degree internship (spring 2007), with the support of our new associate engineer for development.

The inverse problem addressed consists in identifying reliable boundary values on the (artificial) domain boundary. Given some observations (surface elevation of the ice sheet, surface velocities at a few locations and isochrones layers), what are the boundary conditions leading to a model response corresponding to the observations? This inverse problem will be tackled during a one year postdoc (in 2007) and mainly using methods of optimal control for PDEs systems.

Another problem of interest addresses the simultaneous dating of several ice cores. Consistency constraints can then be added to the inverse procedure, in order to improve the final global dating. This aspect is the subject of the PhD thesis of B. Lemieux-Dudon, which began this year.

7. Contracts and Grants with Industry

7.1. National Contracts

Ongoing in 2006:

- A 2-year contract with MERCATOR on the thematic "coupling of ocean models": see 6.1.3.
- A 2-year contract with MERCATOR on the thematic "Assimilation of lagrangian data in OPAVAR": see 6.2.1.

8. Other Grants and Activities

8.1. Regional Actions

8.1.1. Regional Projects

J. Monnier is the leader of the regional project (Région Rhône-Alpes) "**Numerical Prevention for Floods**" 2003-2006, which conducts researches in data assimilation, coupling and parameter fitting methods, and uncertainties propagation: see 6.3. The collaborators are applied mathematicians (INRIA team-project MOISE), hydrologists (Cemagref Lyon and LMFA Lyon), image analysts (Cemagref Montpellier / Maison de la Télédétection and INRIA team-project CLIME), computer scientists (team-project TROPICS), and research engineers (Sogreah Company). During the last three years, three invited researchers have been funded by Région Rhône-Alpes (10 months each).

E. Blayo is the co-leader (with F. Desprez, ENS Lyon) of the regional project (Région Rhône-Alpes) "**Distributed High-Performance Computing**" 2005-2010, which conducts researches both in software and mathematical aspects of grid-computing. This project involves a dozen of regional research teams.

F.X. Le Dimet is responsible for numerical modelling within a regional project (Région Rhône-Alpes) "**Envirhonalp**" 2005-2010. This project aims at gathering physicists, engineers and applied mathematicians to provide improved modelling and decision tools for environmental processes.

D. Bresch is the leader of the regional project (région Rhône Alpes) "aspects mathématiques autour des équations de Saint-Venant visqueuses et applications à l'environnement" 2004–2006, which conducts researches in parametrization effects around shallow water equations: topography effects, stress tensor effects and boundary conditions effects. It involves researchers from MOISE and from the LAMA (Chambéry).

8.1.2. Collaborations with Various Regional Research Teams

- MEOM (Modélisation des Écoulements Océaniques à Moyenne échelle) team from Laboratoire d'Écoulements Géophysiques et Industriels (Grenoble): oceanography: see 6.2.
- Laboratoire de Transferts en Hydrologie et Environnement (Grenoble): see 6.3.
- Cemagref Lyon, Department Hydrology and Hydraulics: 6.3.
- LEGI, PIM project-team (Particules Interfaces Microfluidique): 6.2.
- LGGE, Laboratoire de Glaciologie, Géophysique et Environnement: 6.5.

8.2. National Actions

8.2.1. Interactions with other INRIA Projects or Actions

Participants	**INRIA Project**	**Research topic**	**Link**
J. Monnier	CLIME	Image Processing and Data Assimilation	6.3
A. Rousseau	OMEGA	Downscaling Stochastic Methods	See OMEGA project
J. Monnier A. Vidard	TROPICS	Adjoint code automatic differentiation (TAPENADE) and operational inverse mode	6.3 6.2
J. Monnier	VISTA	Image Processing	6.3.1

8.2.2. Collaborations with other Research Teams in France

Participants	Research Team	Research topic	Link
[1]	MERCATOR-Ocean	Ocean Modelling and Data Assimilation	6.1 6.2
A. Vidard	Centre Européen de Recherche et de Formation Avancée en Calcul Scientifique (Toulouse)	Ocean Data Assimilation	6.2.4
[2]	Laboratoire d'Analyse, Géométrie et Applications (Paris 13)	Domain Decomposition and Coupling Methods	6.1.3
[3]	IFREMER Brest	Ocean Modelling	6.1.3
L. Debreu F. Lemarié	IRD Brest	Ocean Modelling Atmosphere-Ocean Coupling	6.1.3
FX Le Dimet	Laboratoire de Météorologie Dynamique (ENS Paris)	Data Assimilation for Environment	6.2
M. Honnorat, X. Lai, F.X. Le Dimet, J. Monnier	Cemagref Montpellier, Maison de la Télédétection	Data Assimilation of Flood Satellite Images	6.3.3
D. Bresch	ENS Lyon (E. Grenier), ENS Paris (D. Gérard-Varet), CEA (B. Desjardins)	Mathematical Modelling of Ocean Dynamics	6.1.1

[1] : E. Blayo, L. Debreu, M. Nodet, C. Robert, A. Vidard.

[2] : E. Blayo, L. Debreu, F. Lemarié, J. Marin, C. Robert.

[3] : E. Blayo, L. Debreu, F. Lemarié, J. Marin, M. Nodet, E. Nourtier, C. Robert.

8.2.3. Participation to National Research Groups (GdR) CNRS

D. Bresch, C. Lucas and A. Rousseau participate to the GdR EAPQ (Amplitude Equations and Qualitative Properties). This GdR ended in 2006, and was pursued by MOAD (Modelling Asymptotics and nonlinear Dynamics).
D. Bresch co-organises with E. Dumas (Institut Fourier, Grenoble) a special session related to the GdR Chant (managed by F. Castella) on complex fluids and free surface.

8.2.4. Other National Actions

- F.-X. Le Dimet is in charge of the project ASSIMAGE, devoted to the assimilation of images in numerical models in the framework of the ACI “Données Massives”. ASSIMAGE will end in december, 2006, and will be pursued by ADDISA (ANR) in early 2007. D. Auroux will be also involved in ADDISA.
- M. Nodet and D. Auroux are involved in Jacques Blum's project "Un nouvel observateur: le back and forth nudging (BFN) - Études théoriques, numériques et applications" supported by INSU-LEFE.
- A. Vidard leads a project gathering multiple partners in France and UK on the topic "Variational Data Assimilation for the NEMO/OPA9 Ocean Model". This project is granted by INSU-LEFE.
- D. Bresch holds a contract with CEA Bruyères le Chatel on multiphasic flows with B. Desjardins.
- D. Bresch is the leader of a "ACI Jeunes chercheurs" "Étude mathématique et numérique des effets de paramétrisations en océanographie".
- Numerous members of the team are also supported by IDRIS (french national supercomputing center) and get computing hours on parallel and vectorial supercomputers.

8.3. European Actions

- MOÏSE is a partner of the european MERSEA project (http://www.mersea.eu.org). This project is led by IFREMER, and aims at developing a European system for operational oceanography (participants : E. Blayo, L. Debreu, C. Robert).
- J. Monnier is involved in a joint work with E. Fernandez-Nieto (Dept. Differential Equations and Numerical Analysis, University of Sevilla). He also collaborates with I. Gejadze (Dept. of Civil Engineering, University of Strathclyde, Scotland).
- D. Bresch collaborates with F. Guillen-Gonzalez and E. Fernandez-Nieto in Sevilla. He also collaborates with J. Videman in Lisboa.
- A. Vidard collaborates with ECMWF (Reading, UK) on the development of a variational data assimilation system for the NEMO ocean model. Yannick Trémolet from ECMWF spent one month visiting our team. He worked with A. Vidard on implementing new ideas about control of model errors in variational data assimilation with the OPA ocean general circulation model.
- F.X. Le Dimet is a member of the ECMI Educational Board (European Consortium for Mathematics in Industry).

8.4. International Actions

- F.-X. Le Dimet is in charge of an action (ECCO-NET) of cooperation with Russia (Institute of Numerical Mathematics of the Russian Academy of Sciences) and Ukrain (Institute of Oceanography of the Ukrainian Academy of Sciences). The theme of this cooperation is the data assimilation for geophysical flows.
- There also exists a strong cooperation on this theme with China (Institute of Atmospheric Physics of the Chinese A.S.) and Vietnam (Institute of Mathematics and Institute of Mechanics of the Vietnamese A.S.).
- MOISE is a "Équipe-Associée" with Florida State University (under the direction of Y. Hussaini and F.-X. Le Dimet)
- MOISE is in charge of a contract of cooperation France-MIT.
- MOISE belongs to the SARIMA project for cooperation in computer Sciences and Applied Mathematics between France and Africa (one Ph.D. student starting in 2006).
- J. Monnier collaborates with X. Lai (Nanjing Institute of Geography and Limnology, Chinese Academy Sciences) on data assimilation of flood satellite images.
- M. Nodet visited Thomas Haine (Earth and Planetary Sciences department) at the Johns Hopkins University (Baltimore USA) from March to August 2006. She still collaborates with Tom Haine about "Estimating Arctic/Subarctic exchanges thanks to Data Assimilation". In the framework of the MIT-France program, she visited MIT (Cambridge USA), Earth and Planetary Sciences Department and worked with Patrick Heimbach, Carl Wunsch and the MITgcm development team for one week in November.

9. Dissemination

9.1. Scientific Community Dissemination

- D. Bresch is a member of the scientific committee of GdR CNRS MOAD (Modélisation, Asymptotique et Dynamique) who is chaired by S. Benzoni Gavage.
- D. Bresch is also a member of the scientific committee of the workshop organized in October 2006 in CIRM Marseille by F. Boyer and D. Lannes: "Actual mathematical challenges in fluid mechanics".

- E. Blayo is a member of the scientific committee of Mercator (French national program for operational oceanography).
- E. Blayo is also a member of the scientific committee of LEFE-Assimilation Program.

9.2. Teaching

9.2.1. Teaching in Grenoble University

Half of the team members are faculty, and give lectures in the Master in applied mathematics of the Joseph Fourier University and the Institut National Polytechnique de Grenoble (ENSIMAG). The non-faculty (INRIA/CNRS) members of the project also participate to teaching activities.

9.2.2. Lectures Given in International Schools and Foreign Universities

- D. Bresch is an invited lecturer in Université du Liban by Rafaat Talhouk, december 2006.
- E. Blayo gave a lecture in the International Summer School on Coastal Ocean Modelling in Toulon, september 2006.

9.3. Conferences and Workshops

- The members of the team have participated to various conferences and workshops (see the bibliography).
- F.X. Le Dimet co-organized the following summer schools or tutorials on Data Assimilation for Geosciences:
  - Institute of Mechanics, Hanoi, Vietnam, March 2006,
  - Universidade de Chile, Santiago, Chile, October 2006,
  - Université de Ahomey, Bénin, November 2006.
- J. Monnier organised a one day international workshop entitled "Numerical Modelling for Floods" at LMC-IMAG, supported by the regional project Rhône-Alpes (8th february 2006).
- We are organizing a 3 days internal seminar (18-20th december 2006).
- D. Bresch co-organises with P. Noble (Univ Lyon 1) a special session on shallow water and hydraulics in Hyp2006 (international conference) in Lyon.

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