



INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

*Project-Team commands*

*Control, Optimization, Models, Methods  
and Applications for Nonlinear Dynamical  
Systems*

*Saclay - Île-de-France*

Theme : Modeling, Optimization, and Control of Dynamic Systems

*Activity*  
*R* *eport*

2010



## Table of contents

<b>1. Team</b>	<b>1</b>
<b>2. Overall Objectives</b>	<b>1</b>
2.1. Scientific directions	1
2.2. Industrial impact	2
2.3. Highlights	2
<b>3. Scientific Foundations</b>	<b>2</b>
3.1. Historical aspects	2
3.2. Trajectory optimization	3
3.3. Hamilton-Jacobi-Bellman approach	3
<b>4. Application Domains</b>	<b>4</b>
<b>5. Software</b>	<b>4</b>
5.1. Trajectory Optimization	4
5.1.1. Shoot2.0	4
5.1.2. TOPAZE code for trajectory optimization	4
5.2. Resolution of HJB equations	5
5.2.1. SOHJB code for second order HJB equations	5
5.2.2. Sparse HJB-Ultrabee	5
5.2.3. HJB-REF	5
5.3. BiNoPe-HJ	5
5.4. BOCOP Optimal control toolbox	5
<b>6. New Results</b>	<b>5</b>
6.1. Trajectory optimization - PMP approach	5
6.1.1. Analysis of state constrained optimal control problems	5
6.1.2. Coupling the HJB and shooting method approaches	6
6.2. Trajectory optimization - HJB approach	6
6.2.1. Deterministic state constrained optimal control problems without controllability assumptions	6
6.2.2. Reachability and minimal times for state constrained nonlinear problems without any controllability assumption	6
6.2.3. Characterization of the value function of final state constrained control problems with BV trajectories	6
6.2.4. An Efficient Data Structure and Accurate Scheme to Solve Front Propagation Problems	7
6.3. Trajectory optimization	7
6.3.1. First and second order necessary conditions for stochastic optimal control problems	7
6.3.2. Error estimates for the logarithmic barrier method in linear quadratic stochastic optimal control problems	7
6.3.3. Approximation schemes for monotone systems of nonlinear second order partial differential equations: convergence result and error estimate	7
6.4. Trading of NLG using stochastic programming techniques	7
6.4.1. Energy contracts management by stochastic programming techniques	8
6.4.2. Sensitivity analysis of LNG energy contracts	8
<b>7. Contracts and Grants with Industry</b>	<b>8</b>
<b>8. Other Grants and Activities</b>	<b>8</b>
8.1. Regional Initiatives	8
8.2. National Initiatives	8
8.2.1. Organization of conferences	8
8.2.2. Talks in conferences in France	9
8.3. European Initiatives	9
8.4. International Initiatives	9

8.4.1.	Organization of international courses	9
8.4.2.	Talks in conferences outside Europe	10
8.4.3.	Short-term visits: invited professors	10
<b>9.</b>	<b>Dissemination</b> .....	<b>10</b>
9.1.	Animation of the scientific community	10
9.2.	Teaching	10
<b>10.</b>	<b>Bibliography</b> .....	<b>11</b>

# 1. Team

## Research Scientists

J. Frédéric Bonnans [Team leader, Senior Researcher, INRIA, HdR]  
Oana Serea [Associate Professor, U. de Perpignan, délégation, until September 1st 2010]  
Pierre Martinon [Junior Researcher, INRIA]

## Faculty Members

Hasnaa Zidani [Team co-leader, Associate Professor, ENSTA, HdR]  
Ariela Briani [Associate Professor, until September 1st 2010]  
Anna Désilles [Researcher, ENSTA ParisTech, since September 15, 2010]

## External Collaborator

Emmanuel Trélat [Professor, U. Orléans, until June 1st 2010, HdR]

## Technical Staff

Vincent Grélard [October 2010 - October 2011]  
Jun Yi Zhao [December 2009 - December 2011]

## PhD Students

Zhihao Cen [CIFRE fellowship supported by TOTAL, since October 1st 2008]  
Francisco Silva [CORDI fellowship, since October 1st 2007]  
Soledad Aronna [U. Rosario, Argentina, since October 1st 2008]  
Giovanni Granato [CIFRE fellowship supported by Renault, since December 1st 2009]  
Xavier Dupuis [ENS Lyon, since October 1st 2010]  
Laurent Pfeiffer [Polytechnique, since September 1st 2010]  
Zhiping Rao [Polytechnique, since September 1st 2010]

## Administrative Assistant

Wallis Filippi

## Others

Liu Wei [Internship, 3 months (June - August) 2010]  
Imene Ben Latifa [Internship, 3 months (April - June) 2010]  
Joao Saude [Internship, 6 months (March - August) 2010]  
Carina Geldhauser [Internship, 5 months (April - August) 2010]

# 2. Overall Objectives

## 2.1. Scientific directions

Commands is a team devoted to dynamic optimization, both for deterministic and stochastic systems. This includes the following approaches: trajectory optimization, deterministic and stochastic optimal control, stochastic programming, dynamic programming and Hamilton-Jacobi-Bellman equation.

Our aim is to derive new and powerful algorithms for solving numerically these problems, with applications in several industrial fields. While the numerical aspects are the core of our approach it happens that the study of convergence of these algorithms and the verification of their well-posedness and accuracy raises interesting and difficult theoretical questions, such as, for trajectory optimization: qualification conditions and second-order optimality condition, well-posedness of the shooting algorithm, estimates for discretization errors; for the Hamilton-Jacobi-Bellman approach: accuracy estimates, strong uniqueness principles when state constraints are present, for stochastic programming problems: sensitivity analysis.

## 2.2. Industrial impact

For many years the team members have been deeply involved in various industrial applications. The Commands team itself has dealt since its foundation in 2007 with two main types of applications:

- Space vehicle trajectories, in collaboration with CNES, the French space agency,
- Production, management, storage and trading of energy resources (in collaboration with EDF, GDF and TOTAL).
- Energy management for hybrid vehicles (in collaboration with Renault).

We give more details in the Application domain section.

## 2.3. Highlights

This year is the last one of the "Opale" project (launchers trajectory optimization) with CNES that has been the occasion of several fruitful collaborations over the last years.

In Fall 2010 we have seen the defence of the PhD thesis of Francisco Silva, and the one of the Habilitation thesis of Hasnaa Zidani (let us mention that Oana Serea who was in "delegation" until September passed her habilitation thesis in December). We have also seen the beginning of the PhD theses of Xavier Dupuis, Laurent Pfeiffer and Zhiping Rao.

Several projects started this year: ALMA (analysis of leukaemia models, Digiteo DIM), BOCOP (optimal control Toolbox, INRIA), BiNoPe-HJ (parallel calculus toolbox for high dimensional HJB equations, with the SME HPC-Project), and others will start next year: SADCO (Sensitivity Analysis for Deterministic Controller Design, a Marie Curie European network), and "Energy optimization and control" (within the CIRIC project of research center in Chile).

# 3. Scientific Foundations

## 3.1. Historical aspects

The roots of deterministic optimal control are the "classical" theory of the calculus of variations, illustrated by the work of Newton, Bernoulli, Euler, and Lagrange (whose famous multipliers were introduced in [66]), with improvements due to the "Chicago school", Bliss [41] during the first part of the 20th century, and by the notion of relaxed problem and generalized solution (Young [74]).

*Trajectory optimization* really started with the spectacular achievement done by Pontryagin's group [72] during the fifties, by stating, for general optimal control problems, nonlocal optimality conditions generalizing those of Weierstrass. This motivated the application to many industrial problems (see the classical books by Bryson and Ho [47], Leitmann [68], Lee and Markus [67], Ioffe and Tihomirov [63]). Since then, various theoretical achievements have been obtained by extending the results to nonsmooth problems, see Aubin [37], Clarke [48], Ekeland [55].

*Dynamic programming* was introduced and systematically studied by R. Bellman during the fifties. The HJB equation, whose solution is the value function of the (parameterized) optimal control problem, is a variant of the classical Hamilton-Jacobi equation of mechanics for the case of dynamics parameterized by a control variable. It may be viewed as a differential form of the dynamic programming principle. This nonlinear first-order PDE appears to be well-posed in the framework of *viscosity solutions* introduced by Crandall and Lions [50], [51], [49]. These tools also allow to perform the numerical analysis of discretization schemes. The theoretical contributions in this direction did not cease growing, see the books by Barles [39] and Bardi and Capuzzo-Dolcetta [38].

## 3.2. Trajectory optimization

The so-called *direct methods* consist in an optimization of the trajectory, after having discretized time, by a nonlinear programming solver that possibly takes into account the dynamic structure. So the two main problems are the choice of the discretization and the nonlinear programming algorithm. A third problem is the possibility of refinement of the discretization once after solving on a coarser grid.

In the *full discretization approach*, general Runge-Kutta schemes with different values of control for each inner step are used. This allows to obtain and control high orders of precision, see Hager [59], Bonnans [44]. In an interior-point algorithm context, controls can be eliminated and the resulting system of equation is easily solved due to its band structure. Discretization errors due to constraints are discussed in Dontchev et al. [54]. See also Malanowski et al. [69].

In the *indirect* approach, the control is eliminated thanks to Pontryagin's maximum principle. One has then to solve the two-points boundary value problem (with differential variables state and costate) by a single or multiple shooting method. The questions are here the choice of a discretization scheme for the integration of the boundary value problem, of a (possibly globalized) Newton type algorithm for solving the resulting finite dimensional problem in  $\mathbb{R}^n$  ( $n$  is the number of state variables), and a methodology for finding an initial point.

For state constrained problems the formulation of the shooting function may be quite elaborated [42], [43]. As initiated in [58], we focus more specifically on the handling of discontinuities, with ongoing work on the geometric integration aspects (Hamiltonian conservation).

## 3.3. Hamilton-Jacobi-Bellman approach

This approach consists in calculating the value function associated with the optimal control problem, and then synthesizing the feedback control and the optimal trajectory using Pontryagin's principle. The method has the great particular advantage of reaching directly the global optimum, which can be very interesting, when the problem is not convex.

*Characterization of the value function* From the dynamic programming principle, we derive a characterization of the value function as being a solution (in viscosity sense) of an Hamilton-Jacobi-Bellman equation, which is a nonlinear PDE of dimension equal to the number  $n$  of state variables. Since the pioneer works of Crandall and Lions [50], [51], [49], many theoretical contributions were carried out, allowing an understanding of the properties of the value function as well as of the set of admissible trajectories. However, there remains an important effort to provide for the development of effective and adapted numerical tools, mainly because of numerical complexity (complexity is exponential with respect to  $n$ ).

*Numerical approximation for continuous value function* Several numerical schemes have been already studied to treat the case when the solution of the HJB equation (the value function) is continuous. Let us quote for example the Semi-Lagrangian methods [57], [56] studied by the team of M. Falcone (La Sapienza, Rome), the high order schemes WENO, ENO, Discrete galerkin introduced by S. Osher, C.-W. Shu, E. Harten [60], [61], [62], [70], and also the schemes on nonregular grids by R. Abgrall [36], [35]. All these schemes rely on finite differences or/and interpolation techniques which lead to numerical diffusions. Hence, the numerical solution is unsatisfying for long time approximations even in the continuous case.

One of the (nonmonotone) schemes for solving the HJB equation is based on the Ultrabee algorithm proposed, in the case of advection equation with constant velocity, by Roe [73] and recently revisited by Després-Lagoutière [53], [52]. The numerical results on several academic problems show the relevance of the antidiffusive schemes. However, the theoretical study of the convergence is a difficult question and is only partially done.

*Optimal stochastic control problems* occur when the dynamical system is uncertain. A decision typically has to be taken at each time, while realizations of future events are unknown (but some information is given on their distribution of probabilities). In particular, problems of economic nature deal with large uncertainties (on prices, production and demand). Specific examples are the portfolio selection problems in a market with risky

and non-risky assets, super-replication with uncertain volatility, management of power resources (dams, gas). Air traffic control is another example of such problems.

Sometimes this value function is smooth (e.g. in the case of Merton's portfolio problem, Oksendal [75]) and the associated HJB equation can be solved explicitly. Still, the value function is not smooth enough to satisfy the HJB equation in the classical sense. As for the deterministic case, the notion of viscosity solution provides a convenient framework for dealing with the lack of smoothness, see Pham [71], that happens also to be well adapted to the study of discretization errors for numerical discretization schemes [64], [40].

*Numerical approximation* for optimal stochastic control problems. The numerical discretization of second order HJB equations was the subject of several contributions. The book of Kushner-Dupuis [65] gives a complete synthesis on the chain Markov schemes (i.e. Finite Differences, semi-Lagrangian, Finite Elements, ...). Here a main difficulty of these equations comes from the fact that the second order operator (i.e. the diffusion term) is not uniformly elliptic and can be degenerated. Moreover, the diffusion term (covariance matrix) may change direction at any space point and at any time (this matrix is associated the dynamics volatility).

For solving stochastic control problems, we studied the so-called Generalized Finite Differences (GFD), that allow to choose at any node, the stencil approximating the diffusion matrix up to a certain threshold [46]. Determining the stencil and the associated coefficients boils down to a quadratic program to be solved at each point of the grid, and for each control. This is definitely expensive, with the exception of special structures where the coefficients can be computed at low cost. For two dimensional systems, we designed a (very) fast algorithm for computing the coefficients of the GFD scheme, based on the Stern-Brocot tree [45].

## 4. Application Domains

### 4.1. Application Domains

We have mainly contributed to the following fields

1. Trajectory optimization for space launcher problems (with CNES).
2. Trading of Liquefied Natural Gas (with TOTAL).
3. Energy management for hybrid vehicles (with Renault).

## 5. Software

### 5.1. Trajectory Optimization

#### 5.1.1. Shoot2.0

**Participants:** J. Gergaud, P. Martinon.

The SHOOT2.0 package implements an indirect method for optimal control problems, and is a complete rewrite of the previous SHOOT software. New features include a parallel (OpenMP) grid shooting for an easier initialization, as well as the generic handling of mixed state-control constraints. The software also retains the automatic switching detection and embedded continuation of the previous version. Additional features under development include the numerical minimization of the Hamiltonian and the handling of pure state constraints. The package has been used for the practical solving of trajectory optimization problems for space launchers, and is available at <http://www.cmap.polytechnique.fr/~martinon/codes.html>.

#### 5.1.2. TOPAZE code for trajectory optimization

**Participants:** J.F. Bonnans, J. Laurent-Varin [CNES].



Developed in the framework of the PhD Thesis of J. Laurent-Varin, supported by CNES and ONERA. Implementation of an interior-point algorithm for multiarc trajectory optimization, with built-in refinement. Applied to several academic, launcher and reentry problems.

## 5.2. Resolution of HJB equations

### 5.2.1. *SOHJB code for second order HJB equations*

**Participants:** J.F. Bonnans, E. Ottenwaelter, H. Zidani.

Developped since 2004 in C++ for solving the stochastic HJB equations in dimension 2. The code is based on the Generalized Finite Differences, and includes a decomposition of the covariance matrices in elementary diffusions pointing towards grid points. The implementation is very fast and was mainly tested on academic examples.

### 5.2.2. *Sparse HJB-Ultrabee*

**Participants:** O. Bokanowski, E. Cristiani, H. Zidani.

Developped in C++ for solving HJB equations in dimension 4. This code is based on the Ultra-Bee scheme and an efficient storage technique with sparse matrices.

### 5.2.3. *HJB-REF*

**Participants:** O. Bokanowski, N. Forcadel, J. Zhao, H. Zidani.

Developped in C++ for solving HJB equations. This code does not depend on the dimension of the problem.

## 5.3. BiNoPe-HJ

**Participants:** O. Bokanowski, P. Fiorini [HPC-Project], N. Forcadel, T. Porchet [HPC-Project], J. Zhao, H. Zidani [Chair].

This is a project of toolbox in parallel calculus for solving high dimensional HJB equations. The project gathers mathematicians (F. Bonnans, O. Bokanowski, N. Forcadel, H. Zidani, J. Zhao), and researchers in computer science (P. Fiorini, T. Porcher) from he SME HPC-Project. This project has also the support of Inria (DTI), and takes now the form of an I-Lab.

## 5.4. BOCOP Optimal control toolbox

**Participants:** P. Martinon [Chair], J.F. Bonnans, V. Grélard.

This is an “ADT” (action of software development) project of toolbox in optimal control involving P. Martinon (chairman of the board, F. Bonnans, and V. Grélard. Its kernel will be based on open source software and in particular the IPOPT and ADOL-C facilities from COIN-OR, as well as the Maxima computer algebra system. The project began in October 2010. A white paper has been issued in December 2010.

# 6. New Results

## 6.1. Trajectory optimization - PMP approach

### 6.1.1. *Analysis of state constrained optimal control problems*

**Participants:** F. Bonnans, C. de la Vega [U. Buenos Aires].

We consider the optimal control problem of a class of integral equations with initial and final state constraints, as well as running state constraints. We prove Pontryagin’s principle, and study the continuity of the optimal control and of the measure associated with first order state constraints. We also establish the Lipschitz continuity of these two functions of time for problems with only first order state constraints. The results were published as an INRIA report 7257 (2010) and in a journal [20].

### 6.1.2. Coupling the HJB and shooting method approaches

**Participants:** O. Bokanowski, P. Martinon, H. Zidani.

In optimal control, there is a well-known link between the Hamilton-Jacobi-Bellman (HJB) equation and Pontryagin's Minimum Principle (PMP). Namely, the costate (or adjoint state) in PMP corresponds to the gradient of the value function in HJB. We investigate from the numerical point of view the possibility of coupling these two approaches to solve control problems. First a rough approximation of the value function is computed by the HJB method, and then used to obtain an initial guess for the PMP method. The advantage of our approach over other initialization techniques (such as continuation or direct methods) is to provide an initial guess close to the global minimum. Numerical tests have been conducted over simple problems involving multiple minima, discontinuous control, singular arcs and state constraints ([24]).

Thanks to our recent achievements on the HJB (efficient numerical methods) and PMP approaches (numerical minimization of the Hamiltonian, grid shooting), we were able to apply this coupling method to a realistic space launcher problem (Ariane 5), in the framework of a research contract with the Cnes (french space agency).

## 6.2. Trajectory optimization - HJB approach

### 6.2.1. Deterministic state constrained optimal control problems without controllability assumptions

**Participants:** O. Bokanowski, N. Forcadel, H. Zidani.

In the paper [18], we consider nonlinear optimal control problems with constraints on the state of the system. We are interested in the characterization of the value function without any controllability assumption. In the unconstrained case, it is possible to derive a characterization of the value function by means of a Hamilton-Jacobi-Bellman (HJB) equation. This equation expresses the behavior of the value function along the trajectories arriving or starting from any position  $x$ . In the constrained case, when no controllability assumption is made, the HJB equation may have several solutions. Our first result aims to give the precise information that should be added to the HJB equation in order to obtain a characterization of the value function. This result is very general and holds even when the dynamics is not continuous and the state constraints set is not smooth. On the other hand we study also some stability results for relaxed or penalized control problems.

### 6.2.2. Reachability and minimal times for state constrained nonlinear problems without any controllability assumption

**Participants:** O. Bokanowski, N. Forcadel, H. Zidani.

In [16], we consider a target problem for a nonlinear system under state constraints. We give a new continuous level-set approach for characterizing the optimal times and the backwardreachability sets. This approach leads to a characterization via a Hamilton-Jacobi equation, without assuming any controllability assumption. We also treat the case of time-dependent state constraints, as well as a target problem for a two-player game with state constraints. Our method gives a good framework for numerical approximations, and some numerical illustrations are included in the paper.

### 6.2.3. Characterization of the value function of final state constrained control problems with BV trajectories

**Participants:** A. Briani, H. Zidani.

This work aims to investigate a control problem governed by differential equations with Radon measure as data and with final state constraints. By using a known reparametrization method by Dal Maso and Rampazzo, we obtain that the value function can be characterized by means of an auxiliary control problem of absolutely continuous trajectories, involving time-measurable Hamiltonian. We study the characterization of the value function of this auxiliary problem and discuss its numerical approximations. A preprint corresponding to this paper is available [32].

### 6.2.4. An Efficient Data Structure and Accurate Scheme to Solve Front Propagation Problems

**Participants:** O. Bokanowski, E. Cristiani, H. Zidani.

In [14], we are interested in some front propagation problems coming from control problems in  $d$ -dimensional spaces, with  $d \geq 2$ . As opposed to the usual level set method, we localize the front as a discontinuity of a characteristic function. The evolution of the front is computed by solving an Hamilton-Jacobi-Bellman equation with discontinuous data, discretized by means of the antidissipative Ultra Bee scheme. We develop an efficient dynamic storage technique suitable for handling front evolutions in large dimension. Then we propose a fast algorithm, showing its relevance on several challenging tests in dimension  $d = 2, 3, 4$ . We also compare our method with the techniques usually used in level set methods. Our approach leads to a computational cost as well as a memory allocation scaling as  $O(N_n b)$  in most situations, where  $N_n b$  is the number of grid nodes around the front. Moreover, we show on several examples the accuracy of our approach when compared with level set methods.

## 6.3. Trajectory optimization

### 6.3.1. First and second order necessary conditions for stochastic optimal control problems

**Participants:** F. Bonnans, F. Silva.

In this work we consider a stochastic optimal control problem with convex control constraints. Using the variational approach, we are able to obtain first and second order expansions for the state and cost function, around a local minimum. This fact allows us to prove general first order necessary condition and, under a geometrical assumption over the constraint set, second order necessary conditions are also established. The result is published as report [30].

### 6.3.2. Error estimates for the logarithmic barrier method in linear quadratic stochastic optimal control problems

**Participants:** F. Bonnans, F. Silva.

We consider a linear quadratic stochastic optimal control problem with non-negativity control constraints. The latter are penalized with the classical logarithmic barrier. Using a duality argument and the stochastic minimum principle, we provide an error estimate for the solution of the penalized problem which is the natural extension of the well known estimate in the deterministic framework. The result is published as report [30].

### 6.3.3. Approximation schemes for monotone systems of nonlinear second order partial differential equations: convergence result and error estimate

**Participants:** A. Briani, F. Camilli [Univ. La Sapienza], H. Zidani.

Within a collaboration with F. Camilli, we consider approximation schemes for monotone systems of fully nonlinear second order partial differential equations. We first prove a general convergence result for monotone, consistent and regular schemes. This result is a generalization to the well known framework of Barles-Souganidis, in the case of scalar nonlinear equation. Our second main result provides the convergence rate of approximation schemes for weakly coupled systems of Hamilton-Jacobi-Bellman equations. Examples including finite difference schemes and Semi-Lagrangian schemes are discussed.

## 6.4. Trading of NLG using stochastic programming techniques

**Participants:** F. Bonnans, Z. Cen, T. Christel [Total Gaz].

### 6.4.1. Energy contracts management by stochastic programming techniques

We consider the problem of optimal management of energy contracts, with bounds on the local (time step) amounts and global (whole period) amounts to be traded, integer constraint on the decision variables and uncertainty on prices only. After building a finite state Markov chain by using vectorial quantization tree method, we rely on the stochastic dual dynamic programming (SDDP) method to solve the continuous relaxation of this stochastic optimization problem. An heuristic for computing sub optimal solutions to the integer optimization problem, based on the Bellman values of the continuous relaxation, is provided. Combining the previous techniques, we are able to deal with high-dimension state variables problems. Numerical tests applied to realistic energy markets problems have been performed. The results have been published in [33].

### 6.4.2. Sensitivity analysis of LNG energy contracts

We consider a model of medium-term commodity contracts management. Randomness takes place only in the prices on which the commodities are exchanged whilst state variable is multi-dimensional. In [33], we proposed an algorithm to deal with such problem, based on quantization of random process and a dual dynamic programming type approach. We obtained accurate estimates of the optimal value and a suboptimal strategy from this algorithm. In this paper, we analyse the sensitivity with respect to parameters driving the price model. We discuss the estimate of marginal price based on the Danskin's theorem. Finally, some numerical results applied to realistic energy market problems have been performed. Comparisons between results obtained in [33] and other classical methods are provided and evidence the accuracy of the estimate of marginal prices. The objective is to check how the optimal value changes when random process parameters, exactly the forward price in our model changes. This point is crucial for risk management and hedging strategy. Here, we give a partial response by applying Danskin theorem, which demands highly accuracy on optimal decision policy. We compare our algorithm with another traditional algorithm on the simple example of a swing option.

## 7. Contracts and Grants with Industry

### 7.1. Contracts with Industry

1. INRIA - RENAULT, *Energy management for hybrid vehicles*, PhD. fellowship of G. Granato, Dec 2009 - Nov 2012. Involved researchers: F. Bonnans, **H. Zidani**.
2. INRIA - TOTAL, *Trading of Liquefied Natural Gas*, PhD fellowship (CIFRE) of Y. Cen, Dec 2008 - Dec 2011. Involved researchers: **F. Bonnans**.
3. INRIA - CNES, R&T *Trajectory optimization for climbing problems*, 2009-2010. Involved researchers: O. Bokanowski, P. Martinon, **H. Zidani**.
4. INRIA - HPC PROJECT, *Bibliothèque numérique de calcul parallèle: Equations HJB*, Dec 2009 - Dec 2011. Involved researchers: O. Bokanowski, F. Bonnans, N. Forcadel, **H. Zidani**.
5. ENSTA-DGA, *Optimisation de trajectoires pour des systèmes dynamiques de grande taille*, Jan 2010 - Dec 2010. Involved researchers: O. Bokanowski, **H. Zidani**.

## 8. Other Grants and Activities

### 8.1. Regional Initiatives

- F. Bonnans: Coorganizer of the *Séminaire Parisien d'Optimisation*: <http://www.ann.jussieu.fr/plc/spo.html>

### 8.2. National Initiatives

#### 8.2.1. Organization of conferences

F. Bonnans: Coorganizer of the *Conference of the FIME laboratory*, HEC, Jouy-en-Josas, June 28-29, 2010.

### 8.2.2. Talks in conferences in France

- S. Aronna:
  - (i) *Second order sufficient conditions for a linear Optimal Control Problem*. MODE 2010 (Conference on optimization and decision making of Société française de Mathématiques Appliquées et Industrielles). Limoges, France. March 24th. 2010.
  - (ii) *Second order conditions for bang-singular extremals*. Séminaires des doctorants COMMANDS (INRIA) et Optimisation et Commands (UMA-ENSTA). Paris, France. June 18th. 2010.
  - (iii) *Second order conditions for bang-singular extremals*. Séminaires des doctorants Université Paris VI. Paris, France. June 21st. 2010.
- F. Bonnans:
  - (i) *Interior-point algorithms for optimal control problems*. Convex analysis, optimization and applications. Les Houches, Jan. 5-8, 2010.
  - (ii) *Interior-point algorithms for optimal control problems*. Advanced methods and perspectives in nonlinear optimization and control. Toulouse, STAE Foundation, Feb. 3-5, 2010.
  - (iii) *Optimal control of state constrained integral equations*. JBHU 2010: Analyse Variationnelle, Optimisation et Applications. Bayonne, October 25-27, 2010.
- Z. Cen:
  - (i) *LNG contract managements: optimal decision and sensitivity analysis*. SMAI-MODE, Limoges, March 24-26, 2010.
  - (ii) *Stochastic dual dynamic programming technique in finance : pricing and sensitivity analysis – from swing to LNG portfolio*. Workshop on numerical methods in finance, Bordeaux, June 1-2, 2010. <http://alea.bordeaux.inria.fr/index.php/component/content/article/35>.
- P. Martinon:
  - (i) *Contrôle optimal et Principe de Pontryagin: tirer meilleur parti de la puissance de calcul disponible*. Journées MODE 2010, Limoges, France, 24-26 Mars, 2010.
- F. Silva:
  - (i) *Second order necessary optimality conditions for stochastic optimal control problems*. SMAI-MODE, Limoges, March 24-26, 2010.

## 8.3. European Initiatives

### 8.3.1. Talks in conferences in Europe

- F. Silva: *Asymptotic expansions for interior penalty solutions of some control constrained optimal control problems* Summer school Optimal Control of Partial Differential Equations at Cortona, Italy. July, 12-17, 2010.

## 8.4. International Initiatives

### 8.4.1. Organization of international courses

F. Bonnans: coorganizer and lecturer of the CIMPA course on dynamic optimization, Tandil (Argentina), 30/8/10-10/9/10.

H. Zidani: lecturer of the CIMPA course on dynamic optimization, Tandil (Argentina), 30/8/10-10/9/10.

### 8.4.2. Talks in conferences outside Europe

- F. Bonnans:  
*Second order conditions for bang-singular extremals*. International conference on continuous optimization (ICCOPT), Santiago, Chile. July 26-29, 2010.
- Z. Cen:  
(i) *Stochastic dual dynamic programming and its application in energy contracts portfolio optimization*. International conference on continuous optimization (ICCOPT), Santiago, Chile. July 26-29, 2010.  
(ii) *Cutting plane technique and its application in energy contracts portfolio optimization*. 12th International Conference on Stochastic Programming. August 14-20, 2010, Halifax, Nova Scotia.
- G. Granato:  
*Stochastic Optimization of Hybrid Systems with Activation Delay and Decision Lag*. International conference on continuous optimization (ICCOPT), Santiago, Chile. July 26-29, 2010.
- F. Silva:  
(i) *Second order necessary optimality conditions for stochastic optimal control problems*. International conference on continuous optimization (ICCOPT), Santiago, Chile. July 26-29, 2010.
- H. Zidani:  
*State-constrained optimal control problems lacking controllability assumptions*. 9th Brazilian Conference on Dynamics, Control and Their Applications (DINCON), Sao Paulo, Brazil (Keynote speaker). June 07-11, 2010.

### 8.4.3. Short-term visits: invited professors

- Andrei Dmitruk (Moscow State University), 2 weeks.
- Pablo Lotito (U. Tandil, Argentina), 2 weeks.
- Mohamed Mnif (ENIT, Tunis), 3 weeks.
- Lars Grüne (University of Bayreuth), 1 week.
- Antonio Siconolfi (University La Sapienza, Rome 1), 2 weeks.

## 9. Dissemination

### 9.1. Animation of the scientific community

- F. Bonnans:  
(i) Co-editor of the Journal “Series on Mathematics and its Applications” of the Annals of The Academy of Romanian Scientists (AOSR), since 2009.  
(ii) Corresponding Editor, “ESAIM:COCV” (Control, Optimisation and Calculus of Variations),  
(iii) Associate Editor of “Applied Mathematics and Optimization”, and “Optimization, Methods and Software”.
- F. Bonnans is chair of the board of the SMAI-MODE group.
- E. Trélat is Associate Editor of “ESAIM:COCV” (Control, Optimisation and Calculus of Variations) and of “International Journal of Mathematics and Statistics”.

### 9.2. Teaching

- F. Bonnans: Professeur Chargé de Cours, Ecole Polytechnique. M2 (second year of Master studies): two courses on *Stochastic programming* and *Numerical methodes for partial differential equations in finance*, 54 h (both courses in common Master studies with the University of Paris VI). M1: comanager of the Applied Mathematics program, supervisor in internships in applied mathematics.
- P. Martinon: Teaching Assistant, (i) Ensta ParisTech: *Quadratic Optimization* (15h) and *Introduction à Matlab* (20h). (ii) ENSAE: *Numerical Analysis* (16h).
- F. Silva: Ensta ParisTech: Teaching Assistant in the course Quadratic Optimization, 16 hours.
- Z. Rao: Teaching Assistant at Ensta ParisTech. (i) Introduction to automatic (15h) (ii) Markov chains (15h).
- H. Zidani: Professeur Chargée de Cours à l'Ensta ParisTech. (i) 1st year *Continuous optimisation* (22h), (ii) 3rd year and Master MMMEF Paris 1, course on Numerical methods for finance (25h), (iii) 3rd year and Master "Modélisation et Simulation de Versailles St-Quentin et de l'INSTN", course on *Numerical methods for front propagation* (22h), (iv) M2 MIME Univ. Orsay: *Optimal control* (22h).

## 10. Bibliography

### Major publications by the team in recent years

- [1] O. BOKANOWSKI, B. BRUDER, S. MAROSO, H. ZIDANI. *Numerical approximation for a superreplication problem under gamma constraints*, in "SIAM. Num. Analysis.", 2009, vol. 47(3), p. 2289–2320.
- [2] O. BOKANOWSKI, N. MEGDICH, H. ZIDANI. *Convergence of a non-monotone scheme for Hamilton-Jacobi-Bellman equations with discontinuous data*, in "Numerische Mathematik / Numerical Mathematics", 2010, vol. 115, n<sup>o</sup> 1, p. 1–44, <http://hal.inria.fr/inria-00193157>.
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- [7] N. BÉREND, J. F. BONNANS, J. LAURENT-VARIN, M. HADDOU, C. TALBOT. *An Interior-Point Approach to Trajectory Optimization*, in "J. Guidance, Control and Dynamics", 2007, vol. 30, n<sup>o</sup> 5, p. 1228-1238.
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- [9] P. MARTINON, J. F. BONNANS, J. LAURENT-VARIN, E. TRÉLAT. *Numerical study of optimal trajectories with singular arcs for an Ariane 5 launcher*, in "J. Guidance, Control, and Dynamics", 2009, vol. 32, n<sup>o</sup> 1, p. 51-55.

## Publications of the year

### Doctoral Dissertations and Habilitation Theses

- [10] F. SILVA. *Interior penalty approximation for optimal control problems. Optimality conditions in stochastic optimal control theory*, Ecole Polytechnique, 2010, Defense on November 29.
- [11] H. ZIDANI. *Commande optimale déterministe et stochastique*, University Pierre et Marie Curie, 2010, Habilitation Thesis, Habilitation à Diriger des Recherches.

### Articles in International Peer-Reviewed Journal

- [12] S. ADAM, J. F. BONNANS, R. PARAISY, S. VEYRAT. *Application of convex lexicographical optimization to the balance of GRTgaz gas grid*, in "J. Global Optimization", 2010, Online First (July 2010), DOI: 10.1007/s10898-010-9567-z.
- [13] J. ANDRÉ, J. F. BONNANS. *Optimal features of gas transmission networks*, in "Optimization and Engineering", 2010, Published as Online First, DOI 10.1007/s11081-009-9094-2..
- [14] O. BOKANOWSKI, E. CRISTIANI, H. ZIDANI. *An efficient data structure to solve front propagation problems*, in "Journal of Scientific Computing", 2010, vol. 42, n<sup>o</sup> 2, p. 251–273, <http://hal.inria.fr/inria-00273977>.
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