INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

Project-Team poems

Wave propagation: mathematical analysis and simulation

Paris - Rocquencourt

Theme : Computational models and simulation

Activity Report

2010

Table of contents

1. Team

Research Scientists

Patrick Joly [DR, Team Leader, HdR]
Eliane Bécache [CR, HdR]
Jean-David Benamou [DR]
Gary Cohen [CR, HdR]
Anne-Sophie Bonnet-Ben Dhia [DR, Team Leader, HdR]
Marc Lenoir [DR, HdR]
Christophe Hazard [CR, HdR]
Jean-François Mercier [CR]
Stéphanie Chaillat [CR, recruited since december 2010]

Faculty Members

Christine Poirier [Assistant professor, University of Versailles-Saint Quentin]
Patrick Ciarlet [Professor at ENSTA, HdR]
Eric Lunéville [Assistant professor at ENSTA]
Laurent Bourgeois [Assistant professor at ENSTA]
Sonia Fliss [Assistant professor at ENSTA]
Guillaume Legendre [Assistant professor, University of Paris Dauphine]

External Collaborators

Francis Collino [Independant Researcher]
Marc Duruflé [EPI Bacchus, INRIA Bordeaux]

Technical Staff

Colin Chambeyron [IE, CNRS]
Nicolas Kielbasiewicz [IE, CNRS]
Khefil Igue [IJD, INRIA, left POems in November 2010]

PhD Students

Morgane Bergot [Bourse CORDI INRIA, Paris IX, INRIA]
Juliette Chabassier [Bourse X, Ecole Polytechnique, INRIA]
Nicolas Chaulet [Bourse X, Ecole Polytechnique, ENSTA]
Lucas Chesnel [Bourse X, Ecole Polytechnique, ENSTA]
Julien Coatléven [Bourse X, Ecole Polytechnique, INRIA]
Jérémi Dardé [Bourse X, Ecole Polytechnique, ENSTA]
Bérangère Delourme [Bourse CEA, Paris VI, INRIA]
Benjamin Goursaud [Bourse X,Ecole Polytechnique, ENSTA]
Sébastien Impériale [Bource CEA, Paris IX, INRIA]
Lauris Joubert [Bourse MESR, Université Versailles St-Quentin, ENSTA]
Nicolas Salles [Bourse MESR, Université Orsay, INRIA]
Adrien Semin [Bourse MESR, Université Orsay, INRIA]
Alexandre Sinding [Bourse DGA, Paris IX, INRIA]
Maxence Cassier [Bourse X,Ecole Polytechnique, ENSTA]
Aliénor Burel [Bourse MESR, Université Orsay, INRIA]

Post-Doctoral Fellows

Frédérique Le Louër [Post Doc ENSTA]
Abdelkader Makhlouf [Post Doc, ANR AEROSON]
Kersten Schmidt [Post Doc, Bourse FSNS (Swiss National Science Foundation)]

Administrative Assistants

Nathalie Bonte [Secretary Inria, TR]
Annie Marchal [Secretary at ENSTA]

Others

Maxence Cassier [POems Fellowship, March - July 2010]
Aliénor Burel [POems Fellowship, March - July 2010]
Lada Vybulkova [POems Fellowship, June - August 2010]
Andrey Kuzmin [POems Fellowship, June - August 2010]
David Droste [POems Fellowship, October - December 2010]
Jacques Barret [POems Fellowship,March - July 2010]
Antoine Tonnoir [POems Fellowship,June - October 2010]

2. Overall Objectives

2.1. Introduction

The propagation of waves is one of the most common physical phenomena one can meet in nature. From the human scale (sounds, vibrations, water waves, telecommunications, radar) and to the scale of the universe (electromagnetic waves, gravity waves), to the scale of the atom (spontaneous or stimulated emission, interferences between particles), the emission and the reception of waves are our privileged way to understand the world that surrounds us.

The study and the simulation of wave propagation phenomena constitute a very broad and active field of research in the various domains of physics and engineering science.

The variety and the complexity of the underlying problems, their scientific and industrial interest, the existence of a common mathematical structure to these problems from different areas justify together a research project in Scientific Computing entirely devoted to this theme.

The project POEMS is an UMR (Unité Mixte de Recherche) between CNRS, ENSTA and INRIA (UMR 2706). The general activity of the project is oriented toward the conception, the analysis, the numerical approximation, and the control of mathematical models for the description of wave propagation in mechanics, physics, and engineering sciences.

Beyond the general objective of contributing to the progress of the scientific knowledge, four goals can be ascribed to the project:

- the development of an expertise relative to various types of waves (acoustic, elastic, electromagnetic, gravity waves, ...) and in particular for their numerical simulation,
- the treatment of complex problems whose simulation is close enough to real life situations and industrial applications,
- the development of original mathematical and numerical techniques,
- the development of computational codes, in particular in collaboration with external partners (scientists from other disciplines, industry, state companies...)

2.2. Highlights

This year has corresponded to the achievement of several research projects, which has been concretized by the defense of six PhD theses during the last term of 2010. In the domain of aeroacoustics, significant contributions have been made to better taking into account wall effects. We organized a workshop in July that gathered several specialists in this subject. In the area of electromagnetic meta-materials, a new surprising phenomenon of “black hole” has been highlighted which motivates further research about the mathematical modeling of such materials at the microscopic scale. The link between the unit of mechanics of ENSTA have been consolidated through the works on the numerical simulation of a concert piano and new exchanges about the problematic of the computation of nonlinear modes. An original contribution has been achieved on the new thematic of wave propagation in a fractal network. Finally, we have proposed a new approach to the transmission of wave across thin rapidly oscillating interfaces, through the concept of effective transmission conditions.

3. Scientific Foundations

3.1. Mathematical analysis and simulation of wave propagation

Our activity relies on the existence of mathematical models established by physicists to model the propagation of waves in various situations. The basic ingredient is a partial differential equation (or a system of partial differential equations) of the hyperbolic type that are often (but not always) linear for most of the applications we are interested in. The prototype equation is the wave equation:

$$\frac{\partial^2 u}{\partial t^2} - c^2 \Delta u = 0,$$

which can be directly applied to acoustic waves but which also constitutes a simplified scalar model for other types of waves (This is why the development of new numerical methods often begins by their application to the wave equation). Of course, taking into account more realistic physics will enrich and complexify the basic models (presence of sources, boundary conditions, coupling of models, integro-differential or non linear terms,...)

It is classical to distinguish between two types of problems associated with these models: the time domain problems and the frequency domain (or time harmonic) problems. In the first case, the time is one of the variables of which the unknown solution depends and one has to face an evolution problem. In the second case (which rigorously makes sense only for linear problems), the dependence with respect to time is imposed a priori (via the source term for instance): the solution is supposed to be harmonic in time, proportional to $e^{i\omega t}$, where $\omega > 0$ denotes the pulsation (also commonly, but improperly, called the frequency). Therefore, the time dependence occurs only through this pulsation which is given a priori and plays the rôle of a parameter: the unknown is only a function of space variables. For instance, the wave equation leads to the Helmholtz wave equation (also called the reduced wave equation) :

$$-c^2 \Delta u - \omega^2 u = 0.$$

These two types of problems, although deduced from the same physical modelization, have very different mathematical properties and require the development of adapted numerical methods.

However, there is generally one common feature between the two problems: the existence of a dimension characteristic of the physical phenomenon: the wavelength. Intuitively, this dimension is the length along which the searched solution varies substantially. In the case of the propagation of a wave in an heterogeneous medium, it is necessary to speak of several wavelengths (the wavelength can vary from one medium to another). This quantity has a fundamental influence on the behavior of the solution and its knowledge will have a great influence on the choice of a numerical method.

Nowadays, the numerical techniques for solving the basic academic and industrial problems are well mastered. A lot of companies have at their disposal computational codes whose limits (in particular in terms of accuracy or robustness) are well known. However, the resolution of complex wave propagation problems close to real applications still poses (essentially open) problems which constitute a real challenge for applied mathematicians. A large part of research in mathematics applied to wave propagation problems is oriented towards the following goals:

- the conception of new numerical methods, more and more accurate and high performing.
- the treatment of more and more complex problems (non local models, non linear models, coupled systems, ...)
- the study of specific phenomena or features such as guided waves, resonances,...
- the development of approximate models in various situations,
- imaging techniques and inverse problems related to wave propagation.

4. Application Domains

4.1. Introduction

We are concerned with all application domains where linear wave problems arise: acoustics and elastodynamics (including fluid-structure interactions), electromagnetism and optics, and gravity water waves. We give in the sequel some details on each domain, pointing out our main motivations and collaborations.

4.2. Acoustics

As the acoustic propagation in a fluid at rest can be described by a scalar equation, it is generally considered by applied mathematicians as a simple preliminary step for more complicated (vectorial) models. However, several difficult questions concerning coupling problems have occupied our attention recently.

Aeroacoustics, or more precisely, acoustic propagation in a moving compressible fluid, is for our team a new and very challenging topic, which gives rise to a lot of open questions, from the modeling until the numerical approximation of existing models. Our works in this area are partially supported by EADS (and Airbus). The final objective is to reduce the noise radiated by Airbus planes.

Vibroacoustics, which concerns the interaction between sound propagation and vibrations of thin structures, also raises up a lot of relevant research subjects. Our collaboration with EADS on this subject, with application to the comfort of the cockpits of airplanes, allowed us to develop a new research direction about time domain integral equations.

A particularly attractive application concerns the simulation of musical instruments, whose objectives are both a better understanding of the behavior of existing instruments and an aid for the manufacturing of new instruments. The modeling and simulation of the timpani and of the guitar have been carried out in collaboration with A. Chaigne of ENSTA. We intend to initiate a new collaboration on the piano.

4.3. Electromagnetism

This is a particularly important domain, first because of the very important technological applications but also because the treatment of Maxwell's equations poses new and challenging mathematical questions.

Applied mathematics for electromagnetism during the last ten years have mainly concerned stealth technology, electromagnetic compatibility, design of optoelectronic micro-components or smart materials.

Stealth technology relies in particular on the conception and simulation of new absorbing materials (anisotropic, chiral, non-linear...). The simulation of antennas raises delicate questions related to the complexity of the geometry (in particular the presence of edges and corners). Finally micro and nano optics have seen recently fantastic technological developments, and there is a real need for tools for the numerical simulation in these areas.

Our team has taken a large part in this research in the past few years. In the beginning, our activity was essentially concerned with radar furtivity (supported by the French Army and Aeronautic Companies). Now, it is evolving in new directions thanks to new external (academic and industrial) contacts:

- We have been developing since 2001 a collaboration with ONERA on EM modeling by higher order methods (theses of S. Pernet and M. Duruflé).
- As partners of ONERA, we have been selected by the CEG (a research organism of the French Army) to contribute to the development of a general computational code in electromagnetism. The emphasis is on the hybridization of methods and the possibility of incorporating specific models for slits, screens, wires,...
- Optics is becoming again a major application topic. In the past our contribution to this subject was quite important but remained at a rather academic level. Our recent contacts with the Institut d'Electronique Fondamentale (Orsay) (we have initiated with them a research program about the simulation of micro and nano opto-components) are motivating new research in this field.

- Multiscale modelling is becoming a more and more important issue in this domain. In particular, in collaboration with the LETI(CEA) in Grenoble, we are interested in simulated devices whise some of the geometric characterictics are much smaller than the wavelength.

4.4. Elastodynamics

Wave propagation in solids is with no doubt, among the three fundamental domains that are acoustics, electromagnetism and elastodynamics, the one that poses the most significant difficulties from mathematical and numerical points of view. Our activity on this topic, which unfortunately has been forced to slow down in the middle of the 90's due to the disengagement of French oil companies in matter of research, has seen a most welcomed rebound through new academic and industrial contacts.

The two major application areas of elastodynamics are geophysics and non destructive testing. A more recent interest has also been brought to fluid-stucture interaction problems.

- In geophysics, one is interested in the propagation of elastic waves under ground. Such waves appear as natural phenomena in seisms but they are also used as a tool for the investigation of the subterrain, mainly by the petroleum industry for oil prospecting (seismic methods). This constitutes an important field of application for numerical methods. Our more recent works in this area have been motivated by various research contracts with IFP (French Institute of Petroleum), IFREMER (French Research Institute for the Sea) or SHELL.
- Another important application of elastic waves is non-destructive testing: the principle is typically to use ultra-sounds to detect the presence of a defect (a crack for instance) inside a metallic piece. This topic is the object of an important cooperation with EDF (French Company of Electricity) and CEA Saclay in view on the application to the control of nuclear reactors.

  At a more academic level, we have been interested in other problems in the domain of elastic waves in plates (in view of the application to non-destructive testing) through our participation to the GDR Ultrasons. In this framework, we have developped our resarches on multi-modal methods, exact transparent conditions or shape reconstruction of plates of variable cross section.
- Finally, we have recently been led to the study of fluid-solid interaction problems (coupling of acoustic and elastic waves through interfaces) as they appear in underwater seismics (IFREMER) and stemming from ultra-sound propagation in bones (in contact with the Laboratoire d'Imagerie Paramétrique of Paris VI University).

4.5. Gravity waves

These waves are related to the propagation of the ocean swell. The relevant models are derived from fluid mechanics equations for incompressible and irrotational flows. The applications concern in large part the maritime industry, if particular the questions of the stability of ships, sea keeping problems, wave resistance,... The application we have recently worked on concerns the stabilization of ships and off-shore platforms (contract with DGA).

5. Software

5.1. Introduction

We are led to develop two types of software. The first category is prototype softwares : various softwares are developed in the framework of specific research contracts (and sometimes sold to the contractor) or during PhD theses. They may be also contributions to already existing softwares developed by other institutions such as CEA, ONERA or EDF. The second category is advanced software which are intended to be developed, enriched and maintained over longer periods. Such sofwares are devoted to help us for our research and/or promote our research. We have chosen to present here only our advanced softwares.

5.2. MELINA

This software has been developed under the leadership of D. Martin for several years in order to offer to the researchers a very efficient tool (in Fortran 77 and object oriented) for easily implementing finite element based original numerical methods for solving partial differential equations. It has specific and original potential in the domain of time harmonic wave problems (integral representations, spectral DtN conditions,...). Nowadays, it is fully functional in various application areas (acoustics and aeroacoustics, elastodynamics, electromagnetism, water waves). It is an open source software with on line documentation available at

http://perso.univ-rennes1.fr/daniel.martin/melina/

The software is regularly used in about 10 research laboratories (in France and abroad) and number of research papers have published results obtained with MELINA (see the Web site). Moreover, every 2 years, a meeting is organized which combines a workshop which teaches new users with presentations by existing users.

During the last four years, apart from various local improvements of the code, new functionalities have been developed:

- Higher order finite elements (up to 10^{th} order),
- Higher order quadrature formulae,
- DtN boundary conditions in 3D.

A new C++ version of the software is under development. We will take advantage of this evolution for extending the class of finite elements (mixed elements, tensor valued elements, ...).

This year, a beta version of MELINA++ has been achieved. It has been presented to the users at the Melina's days (12 -15 May 2009, Dinard, France).

5.3. MONTJOIE

This is a software for the efficient and accurate wave propagation numerical modeling in both time dependent or time harmonic regimes in various domains of application : acoustics, aeroacoustics, elastodynamics and electromagnetism . It is based essentially on the use of quadrilateral/hexaedric conforming meshes and continuous or discontinuous Galerkin approximations, The use of tensor product basis functions coupled to judicious numerical quadrature techniques leads to important gains in both computing time and memory storage. Various techniques for treating unbounded domains have been incorporated : DtN maps, local absorbing conditions, integral representations and PML's.

We have written an interface for the use of other libraries : SELDON, a C++ linear algebra library (interfaced with BLAS and LAPACK) used for iterative linear solvers, MUMPS and UMFPACK for direct linear solvers, ARPACK for eigenvalue computations. The mesh generation is not part of the code. It can be done with Modulef, Gmsh, Ghs3D or Cubit.

This code has been developed by Marc Duruflé during his PhD thesis. Some other contributors have brought more specific enrichments to the code. The on line documentation is available at

http://montjoie.gforge.inria.fr/

In the framework of M. Bergot thesis, the simulation on hybrid meshes is now possible in Montjoie. Furthermore, the parallilization has been extended to time-harmonic equations. Some nonlinear equations are tackled since recently.

6. New Results

6.1. Numerical methods for time domain wave propagation

6.1.1. Optimal High-Order Edge Finite Elements for Hybrid Meshes Using Pyramidal Elements

Participants: Morgane Bergot, Gary Cohen, Marc Duruflé.

At the end of her thesis directed by G. Cohen and M. Duruflé, after the H^1- and the discontinuous approximations, M. Bergot studied edge finite elements for hybrid meshes for $H(curl)$-approximation. Such approximation is very attractive when solving time-harmonic Maxwell's equations, since it avoids spurious modes without stabilizing terms. The first family defined by Nédélec on hexahedral, prismatic and tetrahedral edge elements have been well studied, but the hexahedral and prismatic elements does not provide an optimal convergence in $H(curl)$-norm when the elements are non-affine.

New conforming hexahedral and prismatic edge elements have been proposed, along with pyramidal elements so that the optimal convergence is ensured. We have also proposed a pyramidal edge element which is compatible with tetrahedral/hexahedral elements of the first family. Numerical results have been conducted on these elements so that we have checked the optimal convergence (when the first family exhibits a loss of an order). An exhaustive comparison with existing edge pyramidal elements in the literature (works of Nigam & Phillips, Gradinaru & Hiptmair, Zgainski, Graglia) has been performed. Numerical results have shown the absence of spurious modes when these new finite element spaces are used for Maxwell's equations. For the discretization of these spaces, we have constructed nodal basis functions as well as hierarchical functions, since both types of discretizations can be advantageous.

6.1.2. Unconditionally stable high order θ-schemes for time discretization of wave equations.

Participants: Juliette Chabassier, Sébastien Impériale.

In the context of the high order finite element discretization in space of a wave equation, leading to the following semi discrete equation : $\partial_{tt} u_h + A_h u_h = 0$ (where A_h is a semi definite positive symmetric matrix), the most classical time discretization is the explicit second order leap frog scheme (1). Modified equation technique allows to achieve higher order time discretizations, by adding terms of the form $c_k \Delta t^{2(k-1)} A_h^k u_h^n$ to the standard leap frog scheme, as illustrated by the fourth order scheme (2).

$$(1) \quad \frac{u_h^{n+1} - 2u_h^n + u_h^{n-1}}{\Delta t^2} + A_h u_h^n = 0 \qquad (2) \quad \frac{u_h^{n+1} - 2u_h^n + u_h^{n-1}}{\Delta t^2} + A_h u_h^n - \frac{\Delta t^2}{12} A_h^2 u_h^n = 0$$

Leap frog scheme — Order 4 modified equation

It is easy to show, using energy techniques, that these schemes are stable under a condition on the time step that depends on the mesh. In general, this is a very efficient approach, but when strong mesh refinement is needed (for instance in presence of singular geometries), this can lead to a very severe time step restriction. To avoid this locking effect, it is possible to use the standard second order θ-scheme (3). This scheme is implicit for $\theta > 0$ and unconditionally stable for $\theta \geq 1/4$, and the cost of the required matrix inversion is compensated by the fact that the time step can be arbitrarily big. However, the use of such big time steps induces a loss of precision. We propose a family of schemes of the form (4) based on the modified equation technique, which extend the θ-scheme to higher order time discretizations.

$$(3) \quad \frac{u_h^{n+1} - 2u_h^n + u_h^{n-1}}{\Delta t^2} + A_h \{u_h\}_{\theta_0}^n = 0 \qquad (4) \quad \frac{u_h^{n+1} - 2u_h^n + u_h^{n-1}}{\Delta t^2} + \sum_{m=1}^{p-1} \beta_m^{p-1} \Delta t^{2m} A_h^{m+1} \{u_h^n\}_{\theta_m} = 0$$

Order 2 θ-scheme — Order $2p$ θ-scheme

where $\{u_h\}_{\theta_i} = \theta_i u_h^{n+1} + (1 - 2\theta_i) u_h^n + \theta_i u_h^{n-1}$ and the β_m^p are well chosen coefficients. For particular values of these coefficients, we show unconditional stability. Dispersion analysis has enabled us to optimize the choice of the θ_i.

6.1.3. Coupling Retarded Potentials and Discontinuous Galerkin Methods for time dependent wave propagation problems

Participant: Patrick Joly.

This topic is developed in collaboration with J. Rodriguez (Santiago de Compostela), T. Abboud (IMACS) and I. Terrasse (EADS) in the framework of the contract ADNUMO with AIRBUS. Let us recall that our objective was to use time-domain integral equations (developed in particular at IMACS and EADS) as a tool for contructing transparent boundary conditions for wave problems in unbounded media. Our previous contribution of this topic concern the construction of an energy preserving method for the coupling between discontinuous Galerkin methods for the numerical approximation of the equations inside the computational domain with a space-time Galerkin approximation of the integral equations that represent the transparent boundary conditions. This work has been submitted for publication.

One drawback of the above method relies in the fact that it is based on central fluxes, which appears to generate high frequency spurious oscillations and not to provide the optimal accuracy (with respect to the local polyomial degree). In the framework of ADNUMO2, the continuation of ADNUMO, we intend to extend our work to the use of non centered fluxes, which presents the interest of introducing numerical dissipation that filter the spurious oscillations and restore the optimal accuracy in time. The difficulty lies in the time discretizationthat must be modified appropriately in order to preserve both the stability property of the initial method (via energy dissipation instead of energy conservation) and the explicit nature of the computations in the interior 3D domain.

6.1.4. Evolution problems in locally perturbed infinite periodic media

Participants: Julien Coatléven, Sonia Fliss.

This work is part of the PhD of J. Coatléven. The principles and theoretical basis of a numerical method have been developed rigorously for the treatment of linear evolution problems in locally perturbed periodic media, several geometries. In 1D or in wave-guide geometries, the idea is to take advantage of the periodicity to reduce the effective computations to small neighborhood of the perturbation, by constructing transparent boundary conditions. The method is based on a semi-discretization in time of the problem in the whole infinite media, the transparent boundary conditions being constructed through the resolution of semi-discretized unperturbed half-guide problems. The method has been extended and tested for the multiple scattering situation (several perturbations). For geometries corresponding to a line defect in an unbounded (typically a plane or a layer) periodic media, an original decomposition method based on the previous resolution for wave-guide geometries has been developed. The theoretical basis of this new method is well understood, and the method has been successfully tested numerically. An important C++ code, named Periodique, has been developed for the treatment of this new situations, and it also includes all previous ones.
We develop another approach more adapted to parabolic equations. Before studying time-domain problems in locally perturbed periodic media, we have developped a method for time-harmonic or stationary problems. A natural idea to construct the time-domain DtN operator is to apply the Laplace transform to the equation and use the previous study. For hyperbolic equations, this method is not adapted because of the cutoff frequencies for which the limiting absorption principle is not possible. However, for parabolic equations, this method seems to work. The main difficulties are the choice of an adapted discretization for the laplace variable and the determination the asymptotic behaviour of the DtN operator in the laplace domain when the laplace variable tends to $p_0 \pm \imath\infty$. This work enters in the framework of the ANR Project MicrWave, in collaboration with Institut Elie Cartan de Nancy, UMR CNRS 7502 and Laboratoire Paul Painlevé, UMR CNRS 8524.

6.1.5. Multiple scales method and convergence study of energy preserving schemes for non linear hyperbolic systems of wave equations.

Participants: Aliénor Burel, Juliette Chabassier, Patrick Joly.

Following the PhD thesis of Juliette Chabassier, we studied two aspects aside her work on the vibration of the piano string :

We created energy preserving schemes of order 1 by generalizing θ-schemes to 1D non linear hamiltonian systems of dimension n (n is the dimension of the unknowns), which can be associated to a permutation $\sigma \in \mathfrak{S}_n$ and gives the scheme $\mathbb{S}_\sigma$. More general schemes can be obtained by linear combinations of the scheme $\mathbb{S}_\sigma$, over a subset of permutations of $\mathfrak{S}_n$. If this set of permutations has certain symmetry properties, we obtain second order accuracy. This is the case where we take the average value over all permutations, which gives the most expensive scheme, called $\mathbb{S}_{\text{all}}$, or if we restrict ourselves to the set $\{\text{Id}, \sigma^*\}$, where σ^* is the permutation of $\mathfrak{S}_n$ defined as $\sigma^*(j) = \sigma^*(n+1-j),\ \forall j \in \{1, 2, ..., n\}$. This method give a new scheme called $\mathbb{S}_{\text{return}}$.

By a systematic numerical study, we compared the initial scheme $\mathbb{S}_1$ with $\mathbb{S}_{\text{all}}$ and $\mathbb{S}_{\text{return}}$ and compared their performances and convergence rates. Figure 1 presents the convergence curves, in which we see that the "return" scheme behaves like the totally permuted schemes, although it gives a much lower calculation cost.
Then, we applied two methods of asymptotic expansions for small initial data to the non linear hamiltonian systems of the piano string. First a naive method gave non physical solutions for which the amplitude of the vibration of the string was linearly time increasing, due to a "secular term". To overcome this drawback, we applied the so-called multi-scale method, whose typical use is for cases where the solution depends simultaneously of several time scales. For a solution of amplitude ε for instance, we had to deal with a solution that depends on $t,\ \varepsilon t,\ \varepsilon^2 t$, etc. This expansion introduces new parameters which can be fixed in order to kill secular terms and have a uniform time expansion for the solution of our non linear problem. The quality of this second approximation has been validated through various numerical experimentations.

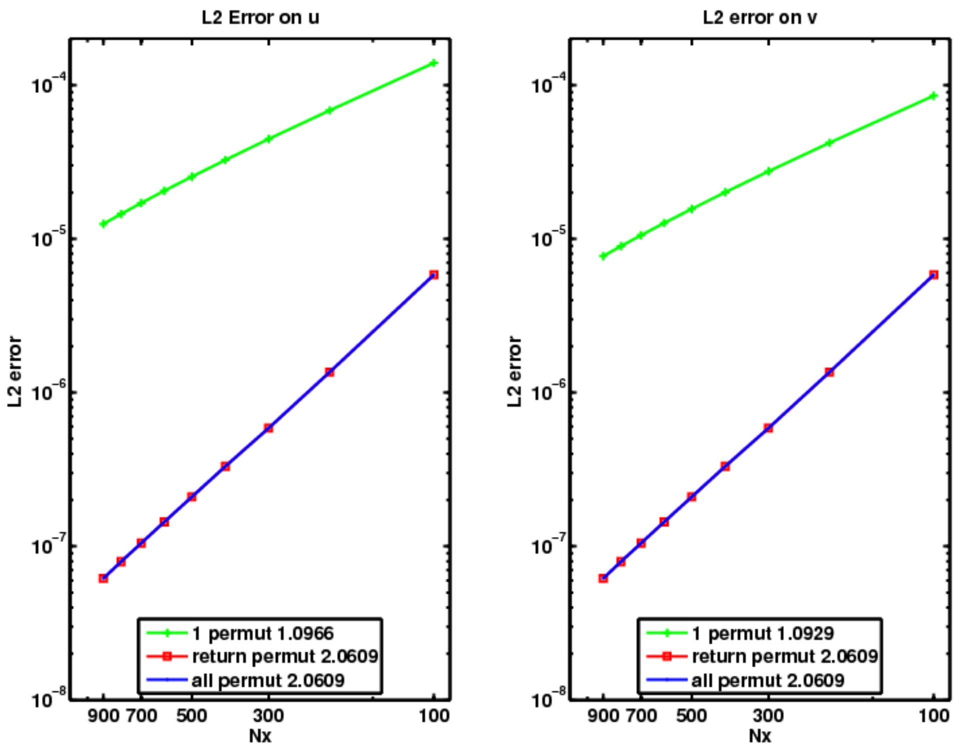

Figure 1. Convergence rate of three preserving schemes.

6.1.6. Modeling of a grand piano

Participants: Juliette Chabassier, Patrick Joly.

This work is developed in collaboration with Antoine Chaigne (UME, ENSTA).

In the context of the modeling of a grand piano, various acoustic pieces have to be taken into account. The hammer is given an initial velocity. It strikes the strings, which vibrate not only according to a transversal motion but also to a longitudinal one. These two waves arrive at the bridge, where a slight angle induces the transmission of both the longitudinal and transversal motions to the soundboard. Thanks to this angle, we can explain the so-called "precursor at the bridge", which contains the "fantom partials" considered responsible for the particular percussive timbre of the piano. Finally, the soundboard radiates in the air, causing sound to be generated.

In order to achieve the numerical stability of the whole coupled problem, we adopt an energy technique : the stability is guaranteed as long as the numerical scheme preserves time step after time step a positive global discrete energy. Each subsystem must be energy preserving when considered alone, and in the coupled problem, the energy must circulate between the subsystems without any loss or gain.

The first years of the PhD were concentrated on the string study, with recent improvements in the comprehension of the model (see Bubu). Several energy preserving schemes have been designed and implemented. Coupling with a nonlinear hammer brings to theoretical difficulties.

A Reissner Mindlin model has been chosen for the soundboard (thick plate model). It allows us to use H^1 high order finite elements for the space discretisation. Time resolution is done with an analytic method in the modal basis, preserving a natural energy.

Sound propagation is done with standard high order finite elements, but we must consider the primitive P of the physical pressure p in order to preserve a total energy.

Lagrange multipliers methods and Schur complement have been implemented, in order to handle separately the resolutions on the strings, the soundboard and the air. This is not a priori an easy task, since the energy circulation is reciprocal.

6.1.7. Numerical solution of the fully axisymmetric Maxwell equations

Participant: Patrick Ciarlet.

A collaboration with Simon Labrunie (Nancy Univ.)
This is the conclusion of a series of researches on the theoretical and numerical solutions to the Maxwell equations in $2\frac{1}{2}$D axisymmetric settings, which began in the early 2000s, with F. Assous, N. Filonov and J. Segré as co-workers.
In previous works, the domain and the data were invariant by rotation, and as a consequence so were the electromagnetic fields. Our aim was to relax this assumption on the data. Then, following some works on the solution to the scalar Laplace equation in a similar setting with the help of the Fourier - Singular Complement Method (FSCM), we adapted this method to the case of the time-dependent, vector Maxwell equations. The FSCM is based on a Fourier-angular expansions of fields, and then truncation of the series, plus discretization with the help of the Singular Complement Method in space. Indeed, in an axisymmetric geometry, intense EM fields can be found near conical corners (on the axis), and/or near reentrant edges. Among others, the numerical analysis has been carried out for explicit and implicit discrete time schemes. A paper on this topic has been accepted for publication in Differential Equations and Applications.

6.1.8. Mathematical and numerical modeling of piezoelectric sensors.

Participants: Gary Cohen, Sébastien Impériale, Patrick Joly.

Piezoelectric sensors are widely used for ultrasonic non destructive testing as they can convert an elastodynamic wave into a difference of potential and vice versa, they are used both as emitter and receiver. The behavior of such devices is governed by the equations of piezoelectricity. Piezoelectric bars have been studied decades ago, these models are either too simple (1D Approximation) or in frequential domain, and assume a very simple geometry. We have designed a mathematical model in time domain for general piezoelectric sensors using the full mathematical model.

The equations of piezoelectricity result from a coupling with linear Maxwell equations and linear elastodynamic equations. The unknowns of the problem are both the displacement field and the electromagnetic field. To simplify the model, we use the well known electrostatic approximation which reduces the electromagnetic part of the unknowns to a scalar potential φ. We justify mathematically this approach by a limit process considering the inverse of the speed of light as a small parameter. The full system to be solved in time domain couples the classical elastodynamic equations on the solid domain (Ω_S) coupled with a Laplace equation posed a priori in the whole domain.

$$\begin{aligned} &\rho\partial_{tt}u - \mathbf{div}\ C\varepsilon(u) - \mathbf{div}\ \mathbf{e}\nabla\varphi = 0 \quad \text{in}\ \ \Omega_S, \\ &\nabla\cdot\epsilon\nabla\varphi - \nabla\cdot\mathbf{e}^T\varepsilon(u) = 0 \quad \text{in}\ \ \mathbb{R}^3. \end{aligned}$$

ρ is the density, C is the classical elastic tensor and ϵ is the permittivity. $\mathbf{e}$ is the non standard piezoelectric tensor that couples both equations. Considering the large contrast of permittivity between various materials, we restrict the domain of computation of φ to a subdomain of Ω_S. This is also justify by a limit process : small permittivities are considered as small parameters.

We wish to model both emission and reception processes. This will induce particular boundary conditions. In the emission process, a potential is applied to both side of the piezoelectric materials, this corresponds to a simple Dirichlet condition that closes the Laplace equation. During the reception process, both sides of the piezoelectric materials are connected to a resistor, the current being directly proportional to the voltage we obtain on one of the boundary a relation of the form :

$$\varphi = \partial_t \int_\Gamma \nabla\varphi\cdot\mathbf{n}.$$

We have designed a numerical method for handling the problem combining high order Galerkin finite elements and explicit time discretization. Numerical results will be presented.

6.1.9. Numerical Methods for Vlasov-Maxwell Equations

Participants: Gary Cohen, Alexandre Sinding.

There exists a large number of methods for approximating the motion of charged particles. They rely on a suitable discretization of Maxwell's and Vlasov's equations.

A. Sinding's thesis, directed by G. Cohen, is devoted to the coupling of higher order hexahedral finite elements for Maxwell's equations with a Particle in Cell (PIC) method for Vlasov's equations. It is part of a joint work between ONERA Toulouse, CEG and INRIA Rocquencourt.

Since continuous approximations seem more fitted to Vlasov's equations, an original implementation of continuous spectral finite elements for solving Maxwell's equations has been constructed. It is based on a mixed formulation of the system. In order to avoid the occurrence of spurious modes, a dissipation term corresponding to the tangential jump of the discontinuous fields is added to the formulation. This penalization term is a good way to get rid of the parasitic modes, by sending them into the complex plane with a negative imaginary part (the parasitic waves becoming evanescent).

This solver (high order, continuous fields, numerical dissipation) has given good results in 2D and 3D . An important advantage of this penalization is that it eliminates spurious modes without introducing a divergence term in the formulation so that there will be no problem with non-convex geometries (see Costabel and Dauge). In particular we don't need to use correction techniques such as the Weighted Regularization (Costabel and Dauge) or the Singular Complement Method (Ciarlet et al.) which require the knowledge of the geometry of the computational domain.

So far this technique has been analyzed and compared in 2D and 3D with other classical spectral element formulation's of the problem, and coupled with a Particle In Cell code for 2D and 2D-axisymmetric cases. The approximation of motion equations is chosen to ensure energy conservation for the complete coupled system under a stability condition, and polynomial functions such as described by Jacobs and Hesthaven are used to interpolate between charged particles and electromagnetic fields.

6.1.10. Trigonometric and wavelet basis for the approximation of the wave equation by a discontinuous galerkin method.

Participants: Sébastien Impériale, Patrick Joly, Antoine Tonnoir.

This work is a continuation on our research concerning Discontinuous Galerkin methods for the approximation of time domain wave propagation phenomena. During the internship af A. Tonnoir, new approaches for the discretization of the 1D wave equation by a Discontinuous Galerkin technique has been studied (trigonometric basis or adaptive wavelet basis). Our studies have not led to positive conclusions. We are currently studying the numerical dispersion of classical polynomial DG schemes. In particular, we aim at understanding more deeply the impact of the used of centered or off-centered fluxes on the accuracy of such schemes.

6.2. Time-harmonic diffraction problems

6.2.1. Harmonic wave propagation in locally perturbed infinite periodic media

Participants: Julien Coatléven, Sonia Fliss, Patrick Joly.

Since the past few years, we have proposed a new method for construction DtN operators for the time harmoni wave equation with absorption in the case of a single perturbation. The method is well established, rigorously justified and successfully tested in the case of absorbing media. An article describing the numerical aspects of the method is submitted at SIAM Numerical Analysis.
The treatment of non absorbing media raises complicated and new questions and requires the definition of an appropriate numerical procedure that should correspond to the continuous limiting absorption principle. The difficulties concerns the resolution of non standard integral equations whose kernels become singular when the absorption goes to 0. The ideas are a semi analytical treatment of the singularities and an enriched Galerkin formulation. The work is in progress.
This method has been already used to solve multi-scattering problems, when several bounded scatterers are present and cannot be put together in a single sufficiently small interior domain. Indeed, it has been shown that the global DtN map (defined on the boundaries of the N interior domains) can be factorized as a product of a block diagonal DtN operator and a propagation operator, each of these being computable with the numerical method that we designed for a single scaterrer. This theroretical framework extends to the situation of a perturbed interface problem or the case of multiple scattering problem with bounded and unbounded (an halfspace for example) scatterers. With this particular situation, we get close to a quasi realistic non-destructive testing simulations : we can compute the wave diffracted by obstacles embedded in a periodic structure. See figure 2

Finally, another part of J. Coatléven's PhD consists in extending the method existing for the Helmholtz equation to the more complicated situation of Maxwell's equations. The theoretical basis of this extension is now well understood for the absorbing case, the numerical testing being in progress.

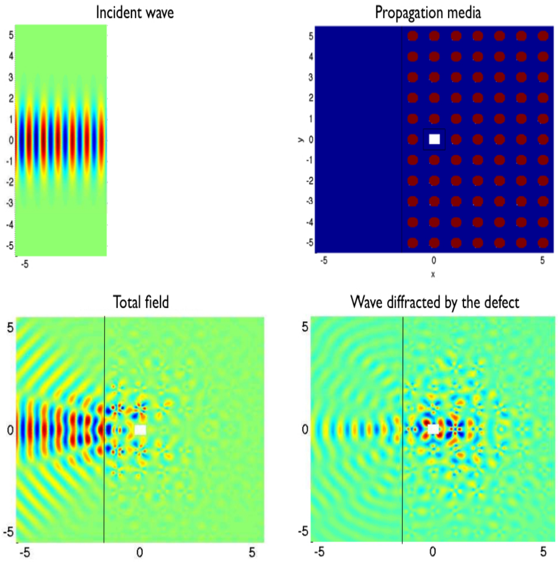

Figure 2. Wave diffracted by a defect embedded in a periodic halfspace

6.2.2. Multiscale FEM for photonic crystal bands

Participant: Kersten Schmidt.

This is a joint work with Christoph Schwab and Holger Brandsmeier (ETH Zürich, Switzerland). A Multiscale generalized hp-Finite Element Method (MSFEM) for time harmonic wave propagation in bands of locally periodic media of large, but finite extent, *e.g.*, photonic crystal (PhC) bands, has been proposed. The method distinguishes itself by its size robustness, *i.e.*, to achieve a prescribed error its computational effort does not depend on the number of periods. The proposed method shows this property for general incident fields, including plane waves incident at a certain angle to the infinite crystal surface, and at frequencies in and outside of the bandgap of the PhC. The proposed MSFEM is based on a precomputed problem adapted multiscale basis. This basis incorporates a set of complex Bloch modes, the eigenfunctions of the infinite PhC, which are modulated by macroscopic piecewise polynomials on a macroscopic FE mesh. The multiscale basis is shown to be efficient for finite PhC bands of any size, provided that boundary effects are resolved with a simple macroscopic boundary layer mesh. The MSFEM, constructed by combing the multiscale basis inside the crystal with some exterior discretisation, is a special case of the Generalised Finite Element Method (g-FEM). For the rapid evaluation of the matrix entries we introduce a size robust algorithm for integrals of quasi-periodic micro functions and polynomial macro functions. Size robustness of the present MSFEM in both, the number of basis functions and the computation time, is verified in extensive numerical experiments.

A technical report on this topic has been published as SAM report at ETH Zurich, and a paper is accepted for publication in the Journal for Computational Physics.

6.2.3. Time harmonic aeroacoustics

Participants: Anne-Sophie Bonnet-Ben Dhia, Jean-François Mercier, Abdelkader Makhlouf.

We are still working on the numerical simulation of the acoustic scattering and radiation in presence of a mean flow. This is the object of the ANR project AEROSON, in collaboration with Florence Millot and Sébastien Pernet at CERFACS, Nolwenn Balin at EADS and Vincent Pagneux at the Laboratoire d'Acoustique de l'Université du Maine. The new main recent improvements concern: the consideration of ducts with treated boundaries and the development of an alternative model to Galbrun's equation.

Treated boundaries

We have extended the time harmonic equation of Galbrun to take into account acoustically treated boundaries. Such boundaries are generally described by the Myers boundary condition. Since this condition is naturally expressed in terms of Galbrun's unknown, the displacement $\mathbf{u}$, Galbrun's equation easily extends to treated boundaries. When introducing a finite element discretization of the extended equation of Galbrun, to select the outgoing solution Perfectly Matched Layers have been introduced. Two difficulties have to be faced:

- Let us recall that the original equation of Galbrun leads to a non coercive problem. For rigid boundaries, we have obtained well-posedness by considering an augmented variational formulation. But this approach does not work anymore for treated boundaries.
- The joint presence of a complexe impedance and of PML leads to more restrictive conditions on the complex parameters in the PML than in the rigid case.

Some premilinary numerical tests indicate that, under the restrictive conditions of the second point, the first point is not very sensitive. For the first point, the determination of a theoretical solution was the object of Abdelkader Makhlouf's postdoc. A first attempt was to introduce a weighted augmentation of Galbrun's equation. However this leads to work in a fonctional space for which compact embedding in L^2 fails. The solution we have developed recently consists in regularizing the Myers boundary condition by adding higher order derivatives of the displacement to get a well-posed problem. Numerical validations are under progress at Cerfacs. We have organized in june 2010 a workshop called "Myers condition" where these results have been presented.

Alternative to Galbrun's model

For 3D configurations Galbrun's equation requires to introduce many unknowns. Therefore to facilitate the treatment of 3D problems we have considered the alternative model of Goldstein's equations. It is well known that when the flow $\mathbf{v}_0$ and the source are potential, the acoustic perturbations are also potential and satisfy a simple scalar model. The velocity potential φ is solution of a modified Helmholtz's equation, with variable coefficients linked to the flow. For a general flow, this model is slightly modified and is called Goldstein's equations. A new vectorial unknown ξ has to be introduced, satisfying a transport equation coupled to the velocity potential. φ satisfies the same modified Helmholtz's equation than in the potential flow case, in which ξ is added as a source term. Note that since the transport equation is very similar to the equation satisfied by the vorticity $\psi = \mathrm{curl}\mathbf{u}$, used to regularize Galbrun's equation, the analysis of Goldstein's equations is based on tools we have already developed for Galbrun's model. The advantage of Goldstein's formulation compared to Galbrun's model is that the vectorial unknown vanishes in the areas where the flow is potential: indeed this unknow is simply linked to Galbrun's unknown by the relation $\xi = (\mathrm{curl}\mathbf{v}_0) \wedge \mathbf{u}$. Since realistic flows are mainly potential, the non-potential areas being located near the boundaries or behind obstacles, this alternative model is a good tool to measure the influence of non curl-free areas on the acoustic propagation. On the other side the advantage of Galbrun's model is to use the displacement which is a natural unknown in boundary conditions. Goldstein's model is naturally linked to the velocity, which is not a good unknown for instance when coupling an elastic medium with a fluid in flow.
We have developed a method combining finite element discretization of Goldstein's model with the introduction of PML to bound the calculation domain. Testing comparisons with Galbrun's equation for a mixing layer and a jet flow have been initiated.

6.2.4. *Modeling of meta-materials in electromagnetism*

Participants: Anne-Sophie Bonnet-Ben Dhia, Patrick Ciarlet, Lucas Chesnel.

A collaboration with Eric Chung (Chinese Univ. of Hong Kong).
Meta-materials can be seen as particular media whose dielectric and/or magnetic constant are negative, at least for a certain range of frequency. This type of behaviour can be obtained, for instance, with particular periodic structures. Of special interest is the transmission of an electromagnetic wave between two media with opposite sign dielectric and/or magnetic constants. As a matter of fact, applied mathematicians have to address challenging issues, both from the theoretical and the discretization points of view.
The first topic we considered a few years ago was: when is the (simplified) scalar model well-posed in the classical H^1 framework? It turned out this issue could be solved with the help of the so-called T-coercivity framework. While numerically, we proved that the (simplified) scalar model could be solved efficiently by the most "naive" discretization, still using T-coercivity.
Recently, we have been able to provide optimality conditions for the T-coercivity to hold in general 2D and 3D geometries, which involve explicit estimates in simplified geometries together with localization arguments. We also showed that the problem can be solved with the help of a Discontinuous Galerkin discretization, which allows one to approximate both the field and its gradient (with E. Chung). Last, we are currently investigating the case of a 2D corner which can be ill-posed (in the classical H^1 framework). Using the Mellin transform, we show that a radiation condition at the corner has to be imposed to restore well-posedness. Indeed there exists a wave which takes an infinite time to reach the corner: this "black hole" phenomenon is observed in other situations (elastic wedges for example).
As a second topic, we considered the study of the transmission problem in a 3D electromagnetic setting from a theoretical point of view: to achieve well-posedness of this problem, we had to proceed in several steps, proving in particular that the space of electric fields is compactly embedded in L^2. For that, we assumed some regularity results on the interface. We are investigating how to remove this assumption, to be able (for instance) to solve the problem around an interface with corners. It turns out the T-coercivity framework can be applied once more. In the process, we recover more compact embedding results. Last, using a different approach, based on regular-singular splittings of fields, we proved some optimal compact imbeddings results in 2D geometries.

6.2.5. Numerical computation of diffraction problems

Participants: Marc Lenoir, Nicolas Salles.

The dramatic increase of the efficiency of the variational integral equation methods for the solution of scattering problems must not hide the difficulties remaining for an accurate numerical computation of some influence coefficients, especially when the panels are close and almost parallel.
The formulas have been extended to double layer potentials and, for self influence coefficients, to affine basis functions. Their efficiency for the solution of Maxwell equations has been proved in the framework of a collaboration with CERFACS. The redaction of a paper devoted to the case of parallel panels is currently in progress.

6.2.6. Second Order Numerical MicroLocal Analysis for high frequency problem

Participants: Jean-David Benamou, Francis Collino.

Given local frequency domain Wave data, the proposed improvement of the NMLA algorithm gives a pointwise estimate of the the number of rays, their slowness vectors and corresponding wavefront curvature. With time domain wave data and assuming the source wavelet is given, the method also estimates the traveltime. We produced theoretical and numerical results on synthetic data that demonstrates both the effectiveness and the robustness of the new method. Comparisons with competing algorithms tends to show the superiority of the new method.

6.3. Absorbing boundary conditions and absorbing layers

6.3.1. Propagation in non uniform open waveguides

Participants: Anne-Sophie Bonnet-Ben Dhia, Benjamin Goursaud, Christophe Hazard.

We have completed our theoretical study about the junction of two uniform waveguides. We have considered a two-dimensional problem for which time-harmonic wave propagation is described by the scalar Helmholtz equation. The main difficulty in the modeling of the scattering problem lies in the choice of conditions which characterize the outgoing behavior of a scattered wave. We have used *modal radiation conditions* which extend the classical conditions used for closed waveguides. They are based on the generalized Fourier transforms which diagonalize the transverse contributions of the Helmholtz operator on both sides of the junction. We have proved the existence and uniqueness of the solution, which seems to be the first result in this context. Our approach combines and extends two techniques. On one hand, we use the ideas developed for the problem of the scattering by a defect located in a uniform open waveguide, which was the object of a collaboration with Lahcene Chorfi (University of Annaba, Algeria) and Ghania Dakhia (University of Biskra, Algeria), presented in the previous activity reports. On the other hand, we also use the technique developed a few year ago in the lab by Anne-Sophie Bonnet-Ben Dhia and Axel Tillequin in the case of an abrupt junction (along a line perpendicular to the direction of propagation). The originality of our approach lies in the proof for uniqueness which combines a natural property related to energy fluxes with an argument of analyticity with respect to the generalized Fourier variable. The difficulties we have encountered last year about this latter argument have been finally overcome. This work is the object of an article which has been submitted to SIAM Journal on Applied Mathematics.

6.3.2. Leaky modes and PML techniques for non-uniform waveguides

Participants: Anne-Sophie Bonnet-Ben Dhia, Benjamin Goursaud, Christophe Hazard.

This topic was initiated in the framework on the ANR SimNanoPHot (with the Institut d'Electronique Fondamentale, Orsay), about the simulation of tapers in integrated optics, or more generally varying cross section open waveguides. Our motivation was to study the possible use of the so-called *leaky modes* in the numerical simulation of such devices. Using an infinite PML surrounding the core of the waveguide, which amounts to a complex stretching of spatial coordinates, the leaky modes appear as the eigenvalues of the transverse component of the stretched Helmholtz operator defined in a section. Using a PML of finite width yields a numerical approximation of the leaky modes. But, as noticed in the previous report, the results can be very polluted, if the PML is far from the core. Here, we prove an exponential behavior of the norm of the resolvent of the Helmholtz operator with respect to the distance between the core and the PML, which explains the instability described in the previous report. Moreover, we study the possibility of using a so called *non orthogonal PML*, so as to minimize the distance between the core and the PML in non canonical cases. This work is a part of the PhD thesis of Benjamin Goursaud, defended December 8th 2010.

6.3.3. Exact bounded PML's with singularly growing absorption in the time domain

Participant: Eliane Bécache.

In collaboration with Andres Prieto, we have restarted our work on exact finite length PMLs, i. e. PML models using a singular damping function. This work is an extension to the time domain of the work done in the frequency domain by Bermudez et al. A paper is in preparation.

6.3.4. On the stability in PML corner domains

Participant: Eliane Bécache.

In collaboration with Andres Prieto, we are finalizing our work on the stability of the discretization of PMLs in the corners. A paper is in preparation.

6.3.5. High order Absorbing Boundary Conditions for elastodynamics

Participant: Eliane Bécache.

Our collaboration on high order absorbing boundary condition with Dan Givoli is going on, in particular for elastodynamics. We first considered the case of isotropic media for which there are already stability questions to understand, which are related to the discretization. We also started to consider the anisotropic case, which involves difficulties already for the design of ABC on the continuous level.

6.3.6. Wave propagation on infinite trees

Participants: Patrick Joly, Adrien Semin.

This topic corresponds to the subject of the second part of the PhD thesis of Adrien Semin, which has been defended on November 24th.

We consider the propagation of waves in a graph. The model is obtain as the limit model for acoustic wave propagation in the junction of thin 2D or 3D tubes when the wavelength is large with respect to the transverse dimensions of the tubes. This model, which has been rigorously justified via asymptotic analysis (see the previous activity report) consists in the 1D wave equation along each branch of the tree together with so called Kirchhoff transmission conditions at the nodes of the graph.

We have been more particularly interested in the case of infinite trees (but of finite size) seen as formal limit of finite trees with a very large number of generations . The first difficulty consists in giving a precise sense to the solution of the wave equation on such infinite trees and in particular to give a precise sense to the notion of "boundary condition at infinity" in the tree. This can de done by a variational approach related to the introduction of an appropriate functional framework (weighted Sobolev spaces on infinite trees), which allows us to define properly the solutions of the problem with Dirichlet or Neumann boundary conditions at infinity.

The second difficulty concerns the numerical approximation of the problem : is it possible to restrict the effective calculations to finite trees by an appropriate truncation problem. We have considered the case where we assume that the tree has a particular structure : after a certain finite number of generations, the truncated subtrees are self-similar p-adic contractive trees (in other words the initial tree is of fractal nature). In this case, one can replace each of this subtree by a transparent DtN like condition involving a non local in time DtN operator Λ whose Fourier symbol $\Lambda(\omega)$ (where ω denotes the - possibly complex- frequency) can be characterized as one particular meromorphic solution of a (non standard) quadratic functional equation. The main properties of this equation, which deeply depends on two characteristic numbers linked to the geometry of the fractal subtree, have been investigated and a numerical algorithm has been designed for the determination of $\Lambda(\omega)$. Furthermore, using a low frequency Taylor approximation of $\Lambda(\omega)$, we have been able to propose approximate (first and second order) local DtN operators that provide, as it has been put in evidence via numerical experiments, a good accuracy provided that the wavelength is large enough with respect of the length of the first branch of the truncated subtree.

Our future developments on the subjects will concern some theoretical questions that remain unsolved (including the question of error estimates), a more detailed analysis on the structure of $\Lambda(\omega)$ (singularities) and the improvement of the approximation of $\Lambda(\omega)$ to derive more accurate local transparent boundary conditions (using rational approximations for instance).

6.3.7. A reduced basis approach for perfectly matched layers in the presence of backward guided waves

Participants: Anne-Sophie Bonnet-Ben Dhia, Guillaume Legendre.

It is well known that Perfectly Matched Layers cannot be used in a waveguide in presence of backward propagating waves. What we have shown is that an accurate post-treatment of computations using usual PMLs allows to recover the right solution. The method is very cheap and easy to implement since it only requires several inversions of a linear system, with the same matrix and different right hand sides. Very good results have been obtained in 2D configurations : the error is reduced to the level of the finite element error and seems to be independent of the parameters of the PML. Finally, the method can even been used without PMLs : it can be seen as a way to build an approximation of the DtN map. The implementation of the method in 3D is in progress.

6.4. Waveguides, resonances, and scattering theory

6.4.1. A new approach for the numerical computation of non linear modes of vibrating systems

Participants: Anne-Sophie Bonnet-Ben Dhia, Jean-François Mercier.

A collaboration with Cyril Touzé and Kerem Ege (Unité de Mécanique, ENSTA).
The simulation of vibrations of large amplitude of thin plates or shells requires the expensive solution of a non-linear finite element model. The main objective of the proposed study is to develop a reliable numerical method which reduces drastically the number of degrees of freedom. The main idea is the use of the so-called *non-linear modes* to project the dynamics on invariant subspaces, in order to generate accurate reduced-order models. Cyril Touzé from the Unité de Mécanique of ENSTA has derived an asymptotic method of calculation of the non-linear modes for both conservative and damped systems. But the asymptotically computed solution remains accurate only for moderate amplitudes. This motivates the present study which consists in developing a numerical method for the computation of the non-linear modes, without any asymptotic assumption. This is the object of a collaboration with Cyril Touzé, and the first results have been obtained during the post-doc of Kerem Ege in the Unité de Mécanique of ENSTA. The partial differential equations defining the invariant manifold of the non-linear mode are seen as a vectorial transport problem : the variables are the amplitude and the phase (a, φ) where the phase φ plays the role of the time. A finite difference scheme is used and an iterative algorithm is written, to take into account the 2π periodicity in φ which is seen as a constraint. An adjoint state approach has been introduced to evaluate the gradient of the coast function. The method has been validated in a simple example with two degrees of freedom.

6.4.2. Finite Element Simulations of Multiple Scattering in Acoustic Waveguides

Participants: Eric Lunéville, Jean-François Mercier, Andrey Kuzmin.

We are still working on the multiple-scattering in a waveguide. The aim is to build a fast numerical method to determine the acoustic field scattered by many rigid obstacles. In the past we had developed a method to reduce the calculation domain just to narrow neighbourhoods of the scatterers. This was achieved by coupling Finite Element in the narrow area to the integral representation of the scattered field. We used the Green function of the waveguide, which naturally expresses as a slowly converging series and thus which requires a long time to be evaluated. We have extended the method to the use of the Green function of the free space (in 2D), faster to evaluate. This was the aim of the three months internship of a russian student, Andrey Kuzmin's, from July to September 2010. The extension is not straightforward since many additional integral terms in the variational formulation have to be evaluated: on one hand on the boundary of the waveguide and on the other hand on two artificial boundaries introduced to select the outgoing solution. This work required a theoretical understanding of the problem, a numerical implementation task (fortran, C++) and a numerical interpretation of the results in terms of efficiency (computation time and accuracy).

6.4.3. Efficient Computation of Photonic Crystal Waveguide Modes with Dispersive Material

Participant: Kersten Schmidt.

This is a joint work with Roman Kappeler (ETH Zürich, Switzerland). The optimization of PhC waveguides is a key issue for successfully designing PhC devices. Since this design task is computationally expensive, efficient methods are demanded. The available codes for computing photonic bands are also applied to PhC waveguides. They are reliable but not very efficient, which is even more pronounced for dispersive material. We have presented a method based on higher order finite elements with curved cells, which allows to solve for the band structure taking directly into account the dispersiveness of the materials. This have been accomplished by reformulating the wave equations as a linear eigenproblem in the complex wave-vectors k. For this method, we have demonstrated the high efficiency for the computation of guided PhC waveguide modes by a convergence analysis.

An article on this topic has been published in Optics Express.

6.4.4. Study of lineic defect in periodic media

Participants: Sonia Fliss, Lada Vybulkova.

We study line defects (i.e. the perturbation is infinite in one dimension) in periodic media. In optics, such defects are created to construct an (open) waveguide to concentrate light. The existence and the computation of the eigenmodes is a crucial issue. This is related to a seladjoint eigenvalue problem associated to a PDE in an unbounded domain (namely in the directions orthogonal to the line defect), which makes both the analysis and the computation hard.
During the internship of Lada Vybulkova, we adapt the DtN approach developed for scattering problems and offer a rigorously justified alternative to existing methods such as the fictitious sources superposition method or the super-cell method. Compared to the latter method, with the DtN method, we can reduce the numerical computation to a small neighborhood of the defect independently from the confinement of the computed guided modes. Moreover, as the method is exact, we improve the accuracy for non well-confined guided modes. Obviously, there is a price to be paid : the reduction of the problem leads to a nonlinear eigenvalue problem, of a fixed point nature, but we already have much experience with this type of problem.

6.5. Asymptotic methods and approximate models

6.5.1. Asymptotic analysis and approximate models for thin and periodic interfaces

Participants: Bérangère Delourme, Patrick Joly.

This topic corresponds to the subject of the PhD thesis of B. Delourme whose defense will be on december 2010. It is developed in collaboration with the CEA-Grenoble (LETI) and H.Haddar (INRIA-Saclay DEFI). It is dedicated to the study of asymptotic models associated with the scattering of electromagnetic waves from a complex periodic structure. More precisely, this structure is made of a dielectric ring that contains two layers of wires winding around it. We are interested in situations where the thickness of the ring and the distance between two consecutive wires are very small compared to the wavelength of the incident wave and the diameter of the ring. One easily understands that in those cases, direct numerical computations of the solution would become prohibitive as the small scale (denoted by δ) goes to 0, since the used mesh would need to accurately follow the geometry of the heterogeneities.

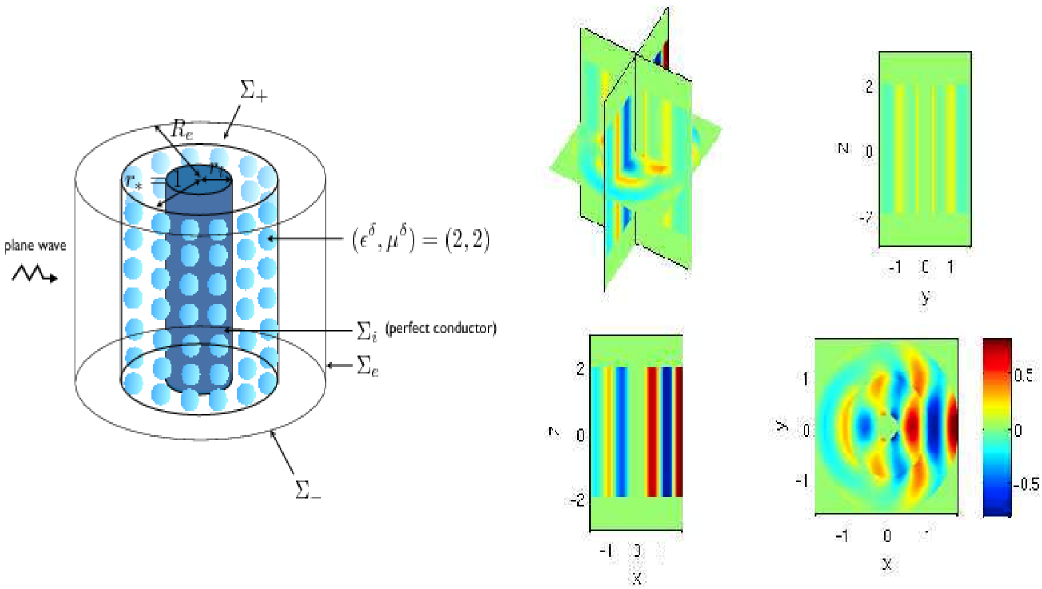

Figure 3. Approximate solution for $\delta = 0.2$ (diffracted field)

The main objective of this work is to derive and to justify approximate models where the periodic ring is replaced by effective transmission conditions. The numerical discretization of approximate problems is expected to be much less expensive that the exact one, since the used med has no longer to be constrained by the small scale.
In a first investigation, we have studied a simplified 2D case. We have constructed and justified a complete and explicit expansion of the solution with respect to the small parameter δ and we have derived stable approximate models. These models have been theoretically and numerically validated. This year, we have studied the complete 3D Maxwell case: we are interesting in the resolution of Maxwell equation in a domain made of a thin periodic ring put into an homogeneous medium. The ring is periodic in two directions (θ and z) ; the periods in θ and z and the thickness of the ring are of the same order δ which is very small compared to the wavelength. As in the 2D problem, the way to obtain approximate transmission models is divided in two main steps : we first write an asymptotic expansion of the solution with respect to the small scale δ. Then we deduce from this expansion an approximate model. From both hand computations and functional analysis points of view, the 3D Maxwell case in much more involved than the 2D one. Indeed, to construct the asymptotic expansion, we have to solve non-standard electrostatic problems posed in an infinite strip. Then, to prove the stability of our approximate model (which is essential to obtain error estimates), we encounter new difficulties: it is not obvious to prove compactness properties and consequently to show that our approximate problem is well posed using the Fredholm alternative. To overcome this difficulty, we have established the existence of a particular Helmholtz decomposition which is adapted to our transmission problem. Finally we have validated our model (in the cartesian case but also the cylindrical case) by numerical simulations. The numerical experiments are done by the 'montjoie' code with the help of Marc Duruflé.

6.5.2. *Approximate models in aeroacoustics*

Participants: Anne-Sophie Bonnet-Ben Dhia, Patrick Joly, Lauris Joubert.

We have first finalized our work on a simplified model for the propagation of acoustic waves in a duct in the presence of a laminated flow:

- An article about the stability analysis of the model in function of the Mach profile has been accepted for publication in M3AS.
- We had developed a general method for obtaining a quasi-analytic representation of the solution that results into a priori estimates. This quasi-analytic representation of the solution has been exploited numerically and the comparison with results obtained by discretizing the full model (Galbrun's equations) has been done. An article has been accepted for publication in CiCP.

The second aspect we have first developed, is the construction of effective boundary conditions for taking into account boundary layers in aeroacoustics. In the case of a rigid wall, we have proposed an effective condition. Using a Kreiss analysis we have shown that the approximate problem is well-posed provided conditions on the Mach profile. This approximate condition has the practical disadvantage to be nonlocal with respect to the normal coordinate inside the boundary layer. One can obtain a local condition after approximating the exact Mach profile by a piecewise linear profile. The implementation is in progress.
We have next considered the case of an impedant wall. Neglecting the boundary layer leads to the Myers condition which is shown to be possibly ill-posed. It is for example always the case for subsonic flows. Via asymptotic expansions, we have proposed a first order approximate condition. As it is a perturbation of order 1 of the Myers condition, we can not expect our approximate problem to be stable. However, it appears that it is well-posed as soon as Z (the impedance) is great enough, provided the same conditions as previously on the Mach profile.

All of this will be soon published in the PhD thesis of Lauris Joubert which has been defended the 26th of november.

6.5.3. *Impedance boundary conditions for the aero-acoustic wave equations in the presence of viscosity*

Participants: Bérangère Delourme, Jacques Barret, Patrick Joly, Kersten Schmidt.

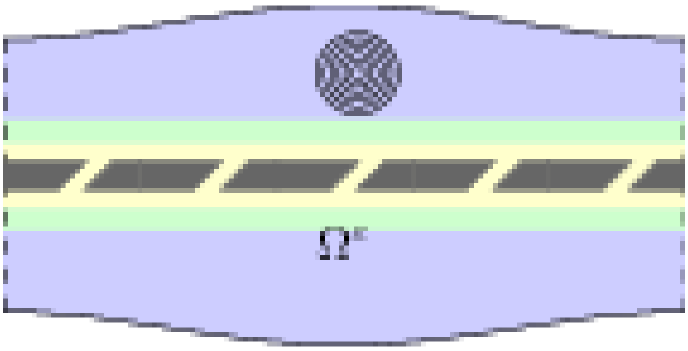

Figure 4. Decomposition of the solution in its near field (yellow) and far field (blue), which are both defined in an overlapping zone (green).

This is a joint work with Sébastien Tordeux (INSA Toulouse, France). In compressible fluids the propagating sound can be described by linearised and perturbed Navier-Stokes equations. This project is dedicated to the case of viscous fluid without and with mean flow. By multiscale expansion and matched asymptotic expansion we are deriving impedance boundary conditions taking into account the viscosity of the fluid. For the case of a plane wall impedance boundary conditions have been derived without a mean flow and in the presence of a laminar dominant mean flow. For a wall with periodic perforations impedance transmission conditions of first order have been derived using surface homogenisation and matched asymptotic expansion. The viscosity and the shape of the perforations enter the impedance conditions as a constant.

6.5.4. Robust families of transmission conditions of high order for thin conducting sheets

Participant: Kersten Schmidt.

This is a joint work with Alexey Chernov (University Bonn, Germany). Three families of transmission conditions of different order are proposed for thin conducting sheets in the eddy current model. Resolving the thin sheet by a finite element mesh is often not possible. With these transmission conditions only the middle curve, but not the thin sheet itself, has to be resolved by a finite element mesh. The families transmission conditions are derived by asymptotic expansion for small sheet thicknesses ε, where each family result from a different asymptotic framework. In the first asymptotic framework the conductivity remains constant, scales with $1/\varepsilon$ in the second and with $1/\varepsilon^2$ in the third. The different asymptotics lead to different limit conditions, namely the vanishing sheet, a non-trivial borderline case, and the impermeable sheet, as well as different transmission conditions of higher orders. We investigated the stability, the convergence of the transmission conditions as well as their robustness. Robust transmission conditions provide accurate approximation for a wide range of sheet thicknesses and conductivities. We introduce an ordering with respect to the robustness, and observe that the condition derived for the $1/\varepsilon$ asymptotics is the most robust limit condition, contrary to higher orders, where the transmission conditions derived for the $1/\varepsilon^2$ asymptotics turn out to be most robust.

6.6. Imaging and inverse problems

6.6.1. Quasi-reversibility

Participants: Laurent Bourgeois, Jérémi Dardé.

We have continued our works on the method of quasi-reversibility to solve second order ill-posed Cauchy problems for an elliptic equation (as they appear in standard inverse problems). This constitutes part of the subject of the PhD thesis of J. Dardé. In particular, we have developped a new strategy to identify obstacles in a domain from partial Cauchy data on the boundary of such domain. This strategy uses a coupling between the method of quasi-reversibility and a level set method. Too types of level set method have been studied in the case of an obstacle which is characterized by a Dirichlet boundary condition $u = 0$. The first one relies on the solution of an eikonal equation, while the second one relies on the solution of a simple Poisson equation. The second one turns out to be particularly easy to implement and very efficient. Some theoretical justifications have been provided for convergence of each method. We have also extended this strategy to the case of non standard boundary conditions such as $|\nabla u| = 1$. Such condition arises in the field of mechanical engineering, precisely in the problem of identification of plastic zones from simultaneous measurements of displacements and forces on the boundary. We have shown that such method enables us to identify some defects like cracks via the plastic zone they create. This may be useful in the field of Non Destructive Testing of elastoplastic media.

6.6.2. Linear sampling method

Participants: Laurent Bourgeois, Frédérique Le Louër, Eric Lunéville.

We have extended some previous work concerning the Linear Sampling Method in an acoustic waveguide to the case of an elastic waveguide. This is the subject of the Post Doc of F. Le Louër. The main issue relies on the fact that we can no longer use a family of transverse modes in order to project the displacement field. To overcome such issue, one possibility consists in using a family of two vector fields formed with particular combinations of displacement and stress components, the so-called mixed variables, for which a bi-orthogonality relationship in the transverse section has been proved (Fraser relationship). We have used propagating modes based on these vector fields as incident waves and the corresponding scattered waves measured at long distance in order to retrieve unknown obstacles, in the framework of the Linear Sampling Method. To this end we have derived some reciprocity relationship and some factorization of the near field operator that are specifically adapted to the framework of mixed variables. Numerical experiments have shown the efficiency of the method in the case of hard obstacles and cracks.

6.6.3. Inverse scattering with generalized impedance boundary condition

Participants: Laurent Bourgeois, Nicolas Chaulet.

This work is a collaboration between POEMS and DEFI projects (more precisely Houssem Haddar) and constitutes the subject of the PhD thesis of N. Chaulet. In the context of acoustics in the harmonic regime, we have first considered the problem of identification of some Generalized Impedance Boundary Conditions (GIBC) at the boundary of an object (which is supposed to be known) from far field measurements associated with a single incident plane wave at a fixed frequency. The GIBCs are approximate models for thin coatings or corrugated surfaces. We have addressed the theoretical questions of uniqueness, stability, as well as the numerical reconstruction of these boundary functions via a gradient method. We have secondly considered the problem of recovering both the obstacle and the boundary impedances with lots of incident waves, proving some uniqueness results also in this case. Concerning numerical reconstruction, one of the key step is the computation of the partial derivative of the cost function with respect to the obstacle, which is a difficult question in the presence of functional impedances. Some numerical experiments have shown the efficiency of our gradient method.

6.6.4. Detection of targets using time-reversal

Participants: Maxence Cassier, Patrick Joly, Christophe Hazard.

Time-reversal was one of the research fields of POEMS a few years ago, which was abandoned in the last years. This subject comes back in the context of a collaboration with EADS (Innovation Works) and a new PhD thesis (Maxence Cassier). The question we are interested in follows the works which were made in the lab about DORT method (French acronym for Decomposition of the Time-Reversal Operator), which concerns time-harmonic waves. It was shown in particular that for small and distant scatterers, the number of nonzero eigenvalues of this operator coincide with the number of scatterers. Moreover each eigenvector associated with such an eigenvalue provides a signal which focuses selectively on one scatterer. Our first study is related to the following question: can we take advantage of these selective focusing properties for time-harmonic waves to generate a time-dependent wave that focuses on one scatterer not only in space, but also in time, in other words, a wave that 'hits hard at the right spot'? We have given a preliminary numerical answer to this question using an asymptotic two-dimensional model of wave propagation with pointwise scatterers. The justification of this model is the object of an article in preparation.

6.7. Other topics

6.7.1. Linear elasticity

Participant: Patrick Ciarlet.

A collaboration with Philippe Ciarlet (City Univ. of Hong Kong), Oana Iosifescu (Montpellier II Univ.), Stefan Sauter (Univ. Zürich) and Jun Zou (Chinese Univ. of Hong Kong).
We study linear elasticity problems using the intrinsic approach, which amounts to considering the linearized strain tensor field as the primary unknown.
After a number of theoretical studies devoted to the solution of linearized elasticity problems via the St-Venant approach, we focused instead on the Donati approach. We focused on the pure traction problem and the pure displacement problem of three-dimensional linearized elasticity and showed that, in both cases, the intrinsic approach can lead to a quadratic minimization problem constrained by Donati-like relations. With the help of the Babuska-Brezzi inf-sup condition, we showed that the minimizer of the constrained minimization problem is the first argument of the saddle-point of an ad hoc Lagrangian, so that the second argument of this saddle-point is the Lagrange multiplier (associated with the corresponding constraints). A paper on this topic has been accepted for publication in M3AS.

6.7.2. Numerical resolution of smooth solutions for Non-linear second order Geometric equations in Lagrangian coordinates

Participant: Jean-David Benamou.

This new method applies to a very general class of second order equation (including also linear like Poisson). Our target applications are mean curvature motion, Monge Ampere equation and the Monge Kantorovitch optimal transport problem, it also applies to "Eikonal" equations in Electrocardiology mixing classical first order Eikonal speed and curvature motion. The method mixes a simple parameterization of the lagrangian motion of the level sets of the solution and the level set PDE itself.

7. Contracts and Grants with Industry

7.1. Contract POEMS-CEA-LIST-1

Participant: Gary Cohen.

G. Cohen perticipates to Projet CASSIS headed by the LIST laboratory of CEA and funded by the EADS Fundation which started in June 2008. This project aims to simulate elastic waves in thin layered anisotropic media for non-destructive testing. In collaboration with E. Demaldent, who will start a post-doc at Inria in the beginning of 2009, G. Cohen must provide a code based on spectral element methods to model these waves.

7.2. Contract POEMS-CEA-LIST-2

Participant: Anne-Sophie Bonnet-Ben Dhia.

This contract is about the scaterring of elastic waves by a stiffener in an anisotropic plate.

7.3. Contract POEMS-ONERA-CE Gramat

Participants: Gary Cohen, Marc Duruflé, Morgane Bergot.

In collaboration with ONERA-DEMR, G. Cohen participates with M. Bergot to the FEMGD project funded by CEG (Centre d'Études de Gramat), which started in 2004. This project is devoted to the construction of a software using spectral discontinuous Galerkin methods for Maxwell's equations. This project came to an end in December 2008.

7.4. Contract POEMS-CE Gramat

Participants: Gary Cohen, Alexandre Sinding.

In collaboration with ONERA-DEMR, G. Cohen participates with A. Sinding to the NADEGE project funded by CEG (Centre d'Études de Gramat), which started in September 2008. This project is devoted to the construction of a software based on FEMGD for solving Vlasov-Maxwell's equations by a PIC method.

7.5. Contract POEMS-Airbus

Participant: Patrick Joly.

This contract (Project ADNUMO) is about the hybridation of time domain numerical techniques in aeroacoustics (Linearized Euler equations).

7.6. Contract POEMS-EADS

Participants: Patrick Joly, Christophe Hazard, Maxence Cassier.

This contract is about the Detection of targets using time-reversal (Maxence Cassier's internship).

7.7. National Initiatives

- GDR Ultrasons: this GDR, which regroups more than regroup 15 academic and industrial research laboratories in Acoustics and Applied Mathematics working on nondestructive testing. It has been renoveled this year with the participation of Great Britain.
- ANR (RNTL) project *MOHYCAN: MOdélisation HYbride et Couplage semi-ANalytique pour la simulation du CND*.

  Topic: *On the coupling of the finite element code ATHENA with the semi-analitic code CIVA. Non-destructif testing.* Collaborators: CEA-LIST (main contact), EDF and CEDRAT.
- ANR project *AEROSON: Simulation numérique du rayonnement sonore dans des géométries complexes en présence d'écoulements réalistes*

8. Other Grants and Activities

8.1. Visiting researchers

- Eric Chung, Professor at Chinese University of Hong Kong, China.

9. Dissemination

9.1. Animation of the scientific community

- A. S. Bonnet-Ben Dhia is the Head of the Electromagnetism Group at CERFACS (Toulouse)

- A. S. Bonnet-Ben Dhia is in charge of the relations between l'ENSTA and the Master "Dynamique des Structures et des Systèmes Couplés (Responsible : Etienne Balmes)".
- A. S. Bonnet-Ben Dhia is presidente of the "Conseil scientifique de l'Institut des sciences de l'ingénierie et des systèmes (INSIS-CNRS)".
- P. Ciarlet is an editor of DEA (Differential Equations and Applications) since July 2008
- G. Cohen is a scientific expert of ONERA.
- P. Joly is a member of the scientific committee of CEA-DAM.
- P. Joly is a member of the Hiring Committee of Ecole Polytechnique in Applied Mathematics.
- P. Joly is a member of the Post Docs Commission of INRIA Rocquencourt.
- P. Joly is a member of the Scientific Committee of the Seminar in Applied Mathematics of College de France (P. L. Lions).
- P. Joly is an editor of the journal Mathematical Modeling and Numerical Analysis.
- P. Joly is a member of the Book Series Scientific Computing of Springer Verlag.
- P. Joly is an expert for the MRIS (Mission pour l'Innovation et la Recherche Scientifique) of DGA (Direction Générale de l'Armement)
- P. Joly is a scientific expert for the "Fondation de Recherche pour l'Aéronautique et l'Espace" in the thematic "Mathématiques Appliquées au domaine de l'Aéronautique et Espace".
- M. Lenoir is a member of the Commission de Spécialistes of CNAM.
- M. Lenoir is in charge of Master of Modelling and Simulation at INSTN.
- E. Lunéville is the Head of UMA (Unité de Mathématiques Appliquées) at ENSTA.
- The Project organizes the monthly Seminar Poems (Coordinators: J. Chabassier, N. Chaulet)

9.2. Teaching

- Eliane Bécache
  - *Introduction à la théorie et l'approximation de l'équation des ondes*, ENSTA-Paris et Master 2 UVSQ
  - *Introduction à la discrétisation des équations aux dérivées partielles*, ENSTA, Paris
  - *Compléments sur la méthode des éléments finis*, ENSTA, Paris
  - *Cours sur les PML*, formation du Collège Polytechnique, Paris
- Anne-Sophie Bonnet-Ben Dhia
  - *Outils élémentaires d'analyse pour les équations aux dérivées partielles. MA102*, Cours de Tronc Commun de 1ère année à l'ENSTA
  - *Propagation d'ondes*, Cours commun au Master de Dynamique des Structures et des Systèmes Couplés et à l'Option de Mécanique (filière VO) de l'Ecole Centrale de Paris
  - *Propagation dans les guides d'ondes. C7-3*, Cours de 3ème année à l'ENSTA. En collaboration avec Eric Lunéville.
  - *Théorie spectrale des opérateurs autoadjoints et application aux guides optiques. MAE21*, Cours de 2ème année à l'ENSTA. En collaboration avec Christophe Hazard et Jean-François Mercier.
- Laurent Bourgeois
  - *Outils élémentaires pour l'analyse des EDP*, ENSTA, Paris
  - *Variable complexe*, ENSTA, Paris

- Aliénor Burel
  - *Cours de mathématiques niveau TS*, D.A.E.U. Formation continue , Université Paris-Sud XI, Orsay.
  - *Colles en classe préparatoire H.E.C.*, Lycée Bessières, Paris 18e.
- Maxence Cassier
  - *Equations différentielles et introduction à l'automatique*, ENSTA, Paris (1st year)
  - *Tutorat pour les cours de mathématiques appliquées de première année de l'ENSTA ParisTech*, ENSTA, Paris (1st year)
- Juliette Chabassier
  - *Math315 : Calcul scientifique, initiation à MATLAB*, Université Paris Sud, Orsay, license (3rd year)
  - *Math315 : Calcul scientifique, travaux pratiques sur MATLAB*, Université Paris Sud, Orsay, license (3rd year)
  - *Math266 : Algèbre III et géométrie, travaux dirigés en petite classe*, Université Paris Sud, Orsay, license (2nd year)
  - *Math154 : Compléments d'algèbre et d'analyse*, Université Paris Sud, Orsay, license (1st year)
- Nicolas Chaulet
  - *Equations différentielles et introduction à l'automatique*, ENSTA-Paristech, Paris
  - *Introduction à la discrétisation des équations aux dérivées partielles*, ENSTA-Paristech, Paris
- Lucas Chesnel
  - *Méthode des éléments finis*, ENSTA, Paris (2nd year)
  - *Fonction d'une variable complexe*, ENSTA, Paris (2nd year)
  - *Introduction au calcul scientifique pour les admis sur titres*, ENSTA, Paris (2nd year)
- Patrick Ciarlet
  - *The finite element method*, ENSTA (2nd year)
  - *Distributed computing: a theoretical viewpoint*, ENSTA (3rd year), and Master "Modeling and Simulation" (2nd year)
  - *Maxwell's equations and their discretization*, ENSTA (3rd year), and Master "Modeling and Simulation" (2nd year)
  - *Computational and Applied Mathematics*, The Chinese University of Hong Kong, Hong Kong, China (2nd year)
  - *Electromagnetics: physical, mathematical and numerical aspects*, The Chinese University of Hong Kong, Hong Kong, China (PostGraduate year)
- Julien Coatléven
  - *Méthode des éléments finis*, ENSTA, Paris (2nd year)
  - *Systèmes dynamiques : Stabilité et commande*, ENSTA, Paris (1st year)
  - *Introduction à la discrétisation des équations aux dérivées partielles*, ENSTA, Paris (1st year)
- Gary Cohen

  - *Méthodes numériques pour les équations des ondes*, Master 2, Université de Paris-Dauphine
- Jérémi Dardé
  - *Algorithmique et programmation*, Université Paris Diderot Paris 7
- Sonia Fliss
  - *Méthode des éléments finis*, ENSTA, Paris (2nd year)
  - *Simulation Numérique*, ENSTA, Paris (2nd year)
  - *Introduction à la discrétisation des équations aux dérivées partielles*, ENSTA, Paris (1st year)
- Benjamin Goursaud
  - *Fonctions de variable complexe*, ENSTA, Paris
  - *Introduction à Matlab*, ENSTA, Paris
  - *Analyse Numérique et Optimisation*, Ecole Polytechnique, Palaiseau
- Christophe Hazard
  - *Outils élémentaires d'analyse pour les EDP*, 1ère année, ENSTA Université Paris XI
  - *Théorie spectrale des opérateurs auto-adjoints et applications aux guides optiques*, 3ème année, ENSTA, Paris
- Sébastien Impériale
  - *Introduction aux équations aux d ériv ées partielles et à leur approximation num érique*, ENSTA, Paris
  - *Programmation scientifique*, ENSTA, Paris
  - *Galerkin discontinu et équation modifiée*, Collège polytechnique.
- Patrick Joly
  - *Introduction à la discrétisation des équations aux dérivées partielles*, ENSTA, Paris
  - *Outils élémentaires d'analyse pour les EDP*, ENSTA, Paris
  - *Méthodes volumiques et couches PML pour les problèmes de propagation d'ondes en régime transitoire*, Collège Polytechnique.
- Lauris Joubert
  - *Chaîne de Markov*, ENSTA, Paris.
- Marc Lenoir
  - *Fonctions de variable complexe*, ENSTA, Paris
  - *Equations intégrales*, ENSTA, Paris
  - *Théorie spectrale*, Master Modélisation et Simulation, UVSQ
- Eric Lunéville
  - *Introduction au calcul scientifique*, Cours de 2ème année à l'ENSTA, Paris
  - *Programmation scientifique et simulation numérique*, Cours de 2ème année à l'ENSTA, Paris
  - *Propagation dans les guides d'ondes*, Cours de 3ème année à l'ENSTA, Paris
- Jean-François Mercier

- *Outils élémentaires d'analyse pour les équations aux dérivées partielles*, Travaux dirigés de 1ère année à l'ENSTA
- *Variable complexe*, Travaux dirigés de 2ème année à l'ENSTA
- *Fluides compressibles*, Travaux dirigés de 2ème année à l'ENSTA
- *Théorie spectrale des opérateurs autoadjoints et application aux guides optiques. MAE21*, Cours de 2ème année à l'ENSTA. En collaboration avec Anne-Sophie Bonnet-Ben Dhia et Christophe Hazard.

- Nicolas Salles
  - *Math256 Analyse de Fourier pour la physique* , Licence 2ème année, Université Paris XI Orsay
  - *Math311 Systèmes Linéaires (Matlab)*, Licence 3ème année, Université Paris XI Orsay

9.3. Participation in Conferences, Workshops and Seminars

- Morgane Bergot
  - *Higher-Order Pyramidal Finite Elements for Electromagnetics*, ESCO 2010, Pilsen (Tcheque Republic), July 2010
  - *Pyramidal Finite Elements for Hybrid Meshes*, CANUM 2010, Carcans-Maubuisson (France), June 2010
  - *Éléments finis pour maillages hybride*, Journée des doctorants du CEA Gramat, Gramat (France), June 2010
- Anne-Sophie Bonnet-Ben Dhia
  - *Time-harmonic electromagnetism in presence of interfaces between classical materials and metamaterials*, Computational Workshop Electromagnetism and Acoustics, Oberwolfach, Deutschland, February 2010.
  - *A way to use Perfectly Matched Layers in the presence of backward guided elastic waves (Keynote)*, ECCM 2010, Paris, May 2010.
- Laurent Bourgeois
  - *Coupling quasi-reversibility and level set methods : the inverse obstacle problem revisited*, Workshop on Inverse Problems for Waves: Methods and Applications, Ecole Polytechnique de Palaiseau, March 2010
  - *Couplage quasi-reversibilité/lignes de niveau: le problème inverse de l'obstacle revisité*, Séminaire LMAC, UTC Compiègne, April 2010
  - *About quantification of unique continuation for elliptic equations: the case of non-smooth boundary*, Trimestre contrôle des EDP (journée "Problèmes inverses"), November 2010
- Juliette Chabassier
  - *Transitoires de piano et non linéarité des cordes : mesures et simulations*, Congrès Français d'Acoustique, Lyon, April 2010
  - *Modeling and numerical simulation of a nonlinear system of piano strings coupled to a soundboard*, International Congress on Acoustics, Sydney, Australia, August 2010
  - *Numerical simulation of a concert piano*, 5th workshop on numerical methods for evolution equations, Heraklion, Crete, September 2010
  - *Modélisation d'un piano de concert*, Journées Jeunes Chercheurs en Audition, Acoustique musicale et Signal audio, Paris, November 2010

  - *Modélisation d'un piano de concert*, Séminaire des doctorants, CERMICS, École Nationale des Ponts et Chaussées, Paris, December 2010

- Nicolas Chaulet
  - *Inverse scattering for generalized impedance boundary conditions*, Inverse problems for waves : methods and applications, Ecole Polytechnique Palaiseau, March 28-29 2010

- Lucas Chesnel
  - *Electromagnetic wave propagation at classical material/metamaterial interfaces - Mathematical aspects*, Metamaterials: Applications, Analysis and Modeling, Los Angeles, January 25 - 29, 2010

- Gary Cohen
  - *Higher-order numerical methods for Maxwell's equations*, TAU, November 9, 2010

- Jérémi Dardé
  - *On identification of defects from boundary measurements: the case of elastoplastic media*, IV European Conference on Computational Mechanics, Paris, May 2010
  - *Une méthode level-set pour résoudre le problème inverse de l'obstacle*, Congrès National d'Analyse Numérique 2010, Carcans-Maubuisson, June 2010

- Bérangère Delourme
  - *Approximate models for wave propagation across a thin periodic ring*, WONAPDE, Concepcion, January 2010.
  - *Modéles approchés pour l'étude de la diffraction par une couche mince périodique*, Séminaire du projet DeFI, INRIA-Saclay, March 2010.
  - *Construction de conditions de transmission approchées pour modéliser la diffraction par une structure périodique*, séminaire du laboratoire de métrologie et Instrumentation du Laboratoire Central des Ponts et Chaussées, Paris, July 2010.
  - *Modèles asymptotiques pour les couches minces périodiques*, groupe de travail applications des mathématiques,ENS Cachan-Bretagne, November 2010.
  - *Modèles et asymptotiques des interfaces minces et périodiques*, séminaire de Mathématiques Appliquées, Institut de Mathématiques de Bordeaux, December 2010.

- Sonia Fliss
  - *Transparent boundary conditions in periodic media*, Séminaire de Sciences numériques pour la Mécanique, Ecole Centrale Paris, February 2010.
  - *Transparent boundary conditions in periodic media*, International Symposium on Maxwell Equations: Theoretical and Numerical Issues with Applications, Fudan University, Shanghaï, July 2010.
  - *Transparent boundary conditions in periodic media*, Séminaire d'Analyse Numérique, IRMAR, Rennes, October 2010.

- Benjamin Goursaud
  - *Étude mathématique et numérique de guides d'ondes ouverts non uniformes, par approche modale*, Soutenance de thèse, Paris, 2010
  - *Analyse mathématique de la jonction de deux guides d'ondes ouverts*, Poster, Quarantième Congrès National d'Analyse Numérique, Carcans-Maubuisson, June 2010

- Sébastien Impériale

  - *Mathematical and numerical modelling of piezoelectric sensors.*, WONAPDE, Concepcion, Chile, January 2010
  - *Modélisation par éléments finis mixtes spectraux de capteurs piézoélectriques.*, Congrès Français d'Acoustique, Lyon, April 2010
  - *θ-schémas d'ordre élevé inconditionnellement stables pour la discrétisation temporelle de l'équation des ondes*, CANUM, Carcan-Maubuisson, June 2010
- Patrick Joly
  - *Numerical Modelling of Multiple-Scattering Problems in Periodic Media*, Conference WONAPDE, Concepcion, Chili, January 2010
  - *Energy preserving schemes for hamiltonian systems of nonlinea wave equations. Application to piano strings*, Workshop Intégration numérique géométrique des systèmes hamiltoniens, Dinard, January 2010
  - *Schémas numériques préservant l'énergie pour les systèmes hamiltoniens d'équations d'ondes. Application aux cordes de piano* , Séminaire DEFI, Ecole Polytechnique, Palaiseau, March 2010
  - *Schémas numériques préservant l'énergie pour les systèmes hamiltoniens d'équations d'ondes. Application aux cordes de piano*, Séminaire Collège de France, Paris, April 2010
  - *Mathematical and numerical modeling of piezoelectric sensors*, Conference SIMAI 2010, MiniSymposium Computational Electromagnetism and Industrial Applications, Cagliari, Italie, June 2010
  - *About the construction of impedance boundary conditions taking into account thin boundary layers*, Workshop "Effective boundary conditions in aeroacoustics", ENSTA, Paris, June 2010
  - *Mathematical and numerical models for wave propagation in infinite trees*, 5rd Workshop on Numerical Solution of Evolution Problems, Heraklion, Crete, September 2010
  - *Modèles approchés pour la propagation d'ondes en milieux présentant des échelles d'espace petites devant la longueur d'onde*, Méthodes Mathématiques pour l'imagerie, Journée des GDR Ondes et MSPC, Institut Henri Poincaré, Paris, September 2010
  - *Approximate boundary conditions for thin boundary layers in time domain aeroacoustics*, DAMTP Seminar, Cambridge University, England, November 2010
  - *Effective boudary conditions for the propagation of waves across thin periodic interfaces*, Workshop Multiscale Methods and Imaging, Inauguration de la Chaire Schlumberger, IHES, Paris, November 2010
  - *Mathematical and numerical models for wave propagation in infinite trees*, University of Santiago de Compostela, Spain, December 2010
- Lauris Joubert
  - *On an approximate model for acoustic wave propagation in a thin duct*, Workshop "Effective Boundary Conditions in Aeroacoustics", Paris, June 2010,
  - *Modélisation asymptotique des conditions de paroi en aéroacoustique*, CANUM, Carcans-Maubuisson, May 2010.
- Frédérique Le Louër
  - *Sur l'imagerie des guides d'ondes à l'aide des modes de Lamb*, GDR Ondes et GDR MSPC, Modèles mathématiques pour l'imagerie, Institut Henri Poincaré, September 2010.

  - *Shape derivatives of boundary integral operators in electromagnetic scattering*, International Symposium on Maxwell Equations: Theoretical and Numerical Issues with Applications, Fudan University, Shanghaï, July 2010.
  - *Shape derivatives of boundary integral operators in electromagnetic scattering*, Minisymposium on Advanced methods in Computational Electromagnetics, ECCM 2010, Paris, May 2010.

- Eric Lunéville
  - *Mathematical modeling of a discontinuous Myers condition*, Workshop "Effective Boundary Conditions in Aeroacoustics", Paris, June 2010,
  - *Méthode de Linear Sampling pour les guides d'ondes*, Séminaire d'Analyse Numérique, Nancy, November 2010.

- Jean-François Mercier
  - *Numerical simulation of Multiple Scattering in Acoustic Ducts, comparison to analytical models*, ICNAAM 2010 minisymposium "Wave scattering in acoustics and elastodynamics", Island of Rhodes, Greece, September 2010
  - *Simulations numériques de la multidiffusion acoustique en conduit, comparaison avec des modèles analytiques* , 10^{e} Congrès de la Société Française d'Acoustique, Lyon, April 2010
  - *Multiple Scattering in Acoustic Ducts: numerical simulations and comparison to analytical models*, The 6^{th} conference of the GDR-US 2501 : Reseach on Ultrasound Propagation for NDT jointly with the 10^{th} Anglo-French Physical Acoustics Conference, Lake District, UK, January 2010

- Adrien Semin
  - *Propagation of acoustic waves in fractal networks*, Oberwolfach Seminar, February 2010.
  - *Propagation of acoustic waves in junction of thin slots*, Applied and Numerical Analysis Seminar, University of Crete, Heraklion, November 2010.
  - *Propagation of acoustic waves in junction of thin slots*, 6th Singular Days Conference, April 2010.
  - *Wave propagation model in self-similar trees* CANUM, Carcan-Maubuisson, June 2010.

- Kersten Schmidt
  - *Viscous Acoustic Equations in periodically perforated chamber - A Modelling by Surface Homogenisation and Matched Asymptotic Expansions*, Seminar of modelling and scientific computation at INRIA Paris-Rocquencourt, Rocquencourt, France, 7th December 2010.
  - *Viscous Acoustic Equations in periodically perforated chamber - A Modelling by Surface Homogenisation and Matched Asymptotic Expansions*, Seminar of Scientific computation and modelisation, Institute de Mathematique Bordeaux, Bordeaux, France, 30th September 2010.
  - *Impedance transmission conditions for thin conducting sheets*, ACE'2010, Zurich, Switzerland, 6th July 2010.
  - *Viscous impedance conditions for acoustic waves derived by asymptotic methods*, Workshop "Effective Boundary Conditions in Aeroacoustics", Paris, France, 30th June 2010.
  - *Viscous Acoustic Equations in periodically perforated chamber*, Singular Days 2010, Berlin, Germany, 1st May 2010.
  - *High order transmission conditions for conductive thin sheets - asymptotic expansion versus optimal basis*, Seminar of the DEFI team, INRIA Saclay and École Polytechnique, Palaiseau, France, 26th April 2010.

- *What are... Concepts of High-order Finite Element Methods?*, Mathematics Graduate Colloquium Zurich, Zurich, Switzerland, 20th April 2010.
- *High order transmission conditions for conductive thin sheets - asymptotic expansion versus optimal basis*, Swiss Numerics Day, Zurich, Switzerland, 16th April 2010.
- *High order transmission conditions for conductive thin sheets - Asymptotic Expansions versus Thin Sheet Bases*, Seminar at Université Paris VI, Paris, France, 29th March 2010.
- *High-Order Transmission Conditions of Thin Conducting Sheets by Thin Sheet Bases*, GAMM'10, Karlsruhe, Germany, 23th March 2010.
- *Modelling in Electromagnetics with the hp-FEM and Asymptotic Expansions*, Seminar at Université de Rennes 1, Rennes, France, 11th March 2010.
- *Modelling in Electromagnetics and Nanophotonics by hp-FEM, Multiscale FEM and Asymptotic Expansions*, Seminar at UPC Barcelona, Barcelona, Spain, 19th February 2010.

- Alexandre Sinding
  - *Méthodes d'éléments finis mixtes spectraux pour la résolution des équations de Vlasov-Maxwell*, Journée des doctorants DGA, CEA Gramat, May 2010.

9.4. Miscellanous

The textbook entitled "The Finite Element Method. From Theory to Practice. II. Complements" (in French) by Eliane Bécache, Patrick Ciarlet, Christophe Hazard and Eric Lunéville, has been published (2010) in the Series Les Cours, Coll. ENSTA.

10. Bibliography

Publications of the year

Doctoral Dissertations and Habilitation Theses

[1] M. BERGOT. *High-Order Finite Elements for Hybrid Meshes - Application to the Resolution of Time-Harmonic and Time-Dependent Linear Hyperbolic Systems*, Université Paris-Dauphine, 11 2010.

[2] J. DARDÉ. *Méthodes de quasi-réversibilité et de lignes de niveau appliquées aux problèmes inverses elliptiques.*, Université Paris Diderot-Paris 7, 12 2010.

[3] B. DELOURME. *Modèles et asymptotiques des interfaces fines et périodiques en électromagnétisme*, Université Pierre et Marie Curie, 12 2010.

[4] B. GOURSAUD. *Étude mathématique et numérique de guides d'ondes ouverts non uniformes, par approche modale*, Ecole Polytechnique, 12 2010.

[5] L. JOUBERT. *Approches asymptotiques et numériques pour la propagation du son dans des écoulements fortement cisaillé*, Ecole Polytechnique, 11 2010.

[6] A. SEMIN. *Propagation d'ondes dans des jonctions de fentes minces*, Université de Paris-Sud 11, 11 2010.

Articles in International Peer-Reviewed Journal

[7] C. AMROUCHE, P. CIARLET, P. CIARLET. *Weak vector and scalar potentials. Applications to Poincaré's theorem and Korn's inequality in Sobolev spaces with negative exponents.*, in "Analysis and Applications", 2010, vol. 8, p. 1–17.

[8] V. BARONIAN, A.-S. BONNET-BENDHIA, E. LUNÉVILLE. *Transparent boundary conditions for the harmonic diffraction problem in an elastic waveguide*, in "J. Comput. Appl. Math", 2010, vol. 234(6), p. 1945–1952, http://dx.doi.org/10.1016/j.cam.2009.08.045.

[9] J.-D. BENAMOU, B. FROESE, A. OBERMAN. *Two Numerical Methods for the Elliptic Monge-Ampère Equation*, in "ESAIM: M2AN", 2010, vol. 44-4, p. 737–758.

[10] M. BERGOT, G. COHEN, M. DURUFLE. *Higher-Order Finite Elements for Hybrid Meshes Using New Nodal Pyramidal Elements*, in "Journal of Scientific Computing", Mar 2010, vol. 42, n^{o} 3, p. 345–381 [*DOI :* 10.1007/S10915-009-9334-9], http://hal.inria.fr/hal-00454261/en.

[11] A.-S. BONNET-BENDHIA, P. CIARLET, C. M. ZWÖLF. *Time harmonic wave diffraction problems in materials with sign-shifting coefficients*, in "J. Comput. Appl. Math", 2010, vol. 234(6), p. 1912–1919, http://dx.doi.org/10.1016/j.cam.2009.08.041.

[12] A.-S. BONNET-BENDHIA, J.-F. MERCIER, F. MILLOT, S. PERNET. *A low Mach model for time harmonic acoustics in arbitrary flows*, in "J. Comput. Appl. Math.", 2010, vol. 234(6), p. 1868–1875, http://dx.doi.org/10.1016/j.cam.2009.08.038.

[13] L. BOURGEOIS. *About stability and regularization of ill-posed elliptic Cauchy problems: the case of $C^{1,1}$ domains*, in "M2AN", 2010, vol. 44-4, p. 715–735, http://dx.doi.org/10.1051/m2an/2010016.

[14] L. BOURGEOIS, J. DARDÉ. *A duality-based method of quasi-reversibility to solve the Cauchy problem in the presence of noisy data*, in "Inverse Problems", 2010, vol. 26, 095016(21pp).

[15] L. BOURGEOIS, J. DARDÉ. *A quasi-reversibility approach to solve the inverse obstacle problem*, in "Inverse Problems and Imaging", 2010, vol. 4-3, p. 351–377.

[16] L. BOURGEOIS, J. DARDÉ. *About stability and regularization of ill-posed elliptic Cauchy problems: the case of Lipschitz domains*, in "Applicable Analysis", 2010, vol. 89/11, p. 1745–1768, http://dx.doi.org/10.1080/00036810903393809.

[17] L. BOURGEOIS, H. HADDAR. *Identification of generalized impedance boundary conditions in inverse scattering problems*, in "Inverse Problems and Imaging", 2010, vol. 4-1, p. 19–38.

[18] É. BÉCACHE, D. GIVOLI, T. HAGSTROM. *High Order Absorbing Boundary Conditions for Anisotropic and Convective Wave Equations*, in "JCP", 2 2010, vol. 229, issue 4.

[19] J. CHABASSIER, P. JOLY. *Energy preserving schemes for nonlinear Hamiltonian systems of wave equations: Application to the vibrating piano string*, in "Computer Methods in Applied Mechanics and Engineering", 2010, vol. 199, p. 2779-2795 [*DOI :* 10.1016/J.CMA.2010.04.013], http://hal.inria.fr/inria-00534473/en.

[20] P. CIARLET, P. CIARLET, O. IOSIFESCU, S. SAUTER, J. ZOU. *A Lagrangian approach to intrinsic linearized elasticity*, in "C. R. Acad. Sci. Paris, Ser. I", 2010, vol. 348, p. 587–592.

[21] P. CIARLET, F. LEFÈVRE, S. LOHRENGEL, S. NICAISE. *Weighted regularization for composite materials in electromagnetism*, in "Math. Mod. Num. Anal.", 2010, vol. 44, p. 75–108.

[22] P. CIARLET, C. SCHEID. *Electrowetting of a 3D drop: numerical modelling with electrostatic vector fields*, in "Math. Mod. Num. Anal.", 2010, vol. 44, p. 647–670.

[23] J. COATLÉVEN, C. ALTAFINI. *A kinetic mechanism inducing oscillations in simple chemical reactions networks*, in "Mathematical Biosciences and Engineering", 4 2010, vol. 7(2), p. 303–314.

[24] S. FLISS, E. CASSAN, D. BERNIER. *New approach to describe light refraction at the surface of a photonic crystal*, in "JOSA B", 7 2010, vol. 27, p. 1492–1503.

[25] P. JOLY, R. WEDER. *Analysis of Acoustic Wave Propagation in a Thin Moving Fluid*, in "SIAM Journal of Applied Mathematics", 2010, vol. 70, p. 2449–2472.

[26] E. LUNÉVILLE, J.-F. MERCIER. *Finite Element Simulations of Multiple Scattering in Acoustic Waveguides*, in "Waves in Random and Complex Media", 2010, vol. 20(4), p. 615–633, http://dx.doi.org/10.1080/17455031003753000.

[27] D. RABINOVICH, D. GIVOLI, É. BÉCACHE. *Comparison of High-Order Absorbing Boundary Conditions and Perfectly Matched Layers in the Frequency Domain*, in "Int. J. Numer. Meth. Biomed. Engng", 2010, vol. 26, p. 1351–1369.

[28] K. SCHMIDT, R. KAPPELER. *Efficient Computation of Photonic Crystal Waveguide Modes with Dispersive Material*, in "Opt. Express", 3 2010, vol. 18, p. 7307–7322.

[29] K. SCHMIDT, S. TORDEUX. *Asymptotic modelling of conductive thin sheets*, in "Z. Angew. Math. Phys.", 8 2010, vol. 61, p. 603–626, http://dx.doi.org/10.1007/s00033-009-0043-x.

[30] K. SCHMIDT, S. TORDEUX. *Asymptotic modelling of conductive thin sheets*, in "Zeitschrift für angewandte Mathematik und Physik", 2010, vol. 61, n^{o} 4, p. 603-626 [*DOI :* 10.1007/S00033-009-0043-X], http://hal.inria.fr/inria-00527608/en.

International Peer-Reviewed Conference/Proceedings

[31] A.-S. BONNET-BENDHIA. *Time-harmonic electromagnetism in presence of interfaces between classical materials and metamaterials*, in "Oberwolfach Reports", Report No.10/2010 of Mathematisches Forschungsinstitut Oberwolfach "Computational Electromagnetism and Acoustics", Oberwolfach., 2010, http://dx.doi.org/10.4171/OWR/2010/10.

[32] A.-S. BONNET-BENDHIA, C. HAZARD, B. GOURSAUD, A. PRIETO. *A multimodal method for non-uniform open waveguides*, in "Physics Procedia", International Congress on Ultrasonics, Universidad de Santiago de Chile, 2010, vol. 3(1), p. 497–503.

National Peer-Reviewed Conference/Proceedings

[33] J. CHABASSIER, A. CHAIGNE, P. JOLY. *Transitoires de piano et non-linéarités des cordes : mesures et simulations*, in "10ème Congrès Français d'Acoustique", France Lyon, SOCIÉTÉ FRANÇAISE D'ACOUSTIQUE - SFA (editor), 2010, http://hal.inria.fr/hal-00539771/en.

[34] B. OUEDRAOGO, E. REDON, J.-F. MERCIER. *Opérateur DtN pour les guides cylindriques à paroi traitée en présence d'un écoulement uniforme*, in "10ème Congrès Français d'Acoustique", France Lyon, SOCIÉTÉ FRANÇAISE D'ACOUSTIQUE - SFA (editor), 2010, http://hal.inria.fr/hal-00539663/en.

Scientific Books (or Scientific Book chapters)

[35] É. BÉCACHE, P. CIARLET, C. HAZARD, E. LUNÉVILLE. *La Méthode des Eléments Finis. De la Théorie à la Pratique. II. Compléments*, Coll. Les Cours, Les Presses de l'ENSTA, 288 pages, 11 2010.

[36] M. LENOIR. *Théorie spectrale et applications - Généralités et opérateurs compacts*, in "Encyclopédie des techniques de l'ingénieur", 2010, vol. AF-567.

Research Reports

[37] L. BOURGEOIS, N. CHAULET, H. HADDAR. *Identification of generalized impedance boundary conditions: some numerical issues*, INRIA, Nov 2010, n^{o} RR-7449, http://hal.inria.fr/inria-00534042/en.

[38] J. CHABASSIER, P. JOLY. *Energy Preserving Schemes for Nonlinear Hamiltonian Systems of Wave Equations. Application to the Vibrating Piano String.*, INRIA, Jan 2010, n^{o} RR-7168, http://hal.inria.fr/inria-00444470/en.

[39] B. DELOURME, H. HADDAR, P. JOLY. *Approximate Models for Wave Propagation Across Thin Periodic Interfaces*, INRIA, Feb 2010, n^{o} RR-7197, http://hal.inria.fr/inria-00456200/en.

[40] K. SCHMIDT, S. TORDEUX. *High order transmission conditions for thin conductive sheets in magneto-quasistatics*, INRIA, Apr 2010, n^{o} RR-7254, http://hal.inria.fr/inria-00473213/en.

[41] A. SEMIN, P. JOLY. *Study of propagation of acoustic waves in junction of thin slots*, INRIA Research Report, 4 2010.

[42] A. SEMIN, P. JOLY. *Study of propagation of acoustic waves in junction of thin slots*, INRIA, Apr 2010, n^{o} RR-7265, http://hal.inria.fr/inria-00476322/en.

Other Publications

[43] P. CIARLET, S. LABRUNIE. *Numerical solution of Maxwell's equations in axisymmetric domains with the Fourier Singular Complement Method*, 2010, 37 pages, http://hal.inria.fr/hal-00365391/en.

[44] M. COSTABEL, F. LE LOUËR. *Shape derivatives of boundary integral operators in electromagnetic scattering*, 2010, http://hal.inria.fr/hal-00453948/en.

[45] M. DURUFLE, S. ISRAWI. *A Numerical Study of Variable Depth KdV Equations and Generalizations of Camassa-Holm-like Equations*, 2010, http://hal.inria.fr/hal-00454495/en.