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Activity Report 2011

Project-Team **MATHFI**

Financial mathematics

IN COLLABORATION WITH: Laboratoire d'Analyse et de Mathématiques Appliquées (LAMA), Centre d'Enseignement et de Recherche en Mathématiques et Calcul Scientifique (CERMICS)

RESEARCH CENTER
Paris - Rocquencourt

THEME
Stochastic Methods and Models

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Project-Team MATHFI

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2. Overall Objectives

2.1. Introduction

MathFi is a joint INRIA project-team with ENPC (CERMICS Laboratory) and the University Paris-Est Marne la Vallée (LAMA Laboratory), located in Rocquencourt and Marne la Vallée. <https://www-rocq.inria.fr/mathfi/>

The development of increasingly complex financial products requires the use of advanced stochastic and numerical analysis techniques. The scientific skills of the Mathfi research team are focused on probabilistic and deterministic numerical methods and their implementation, stochastic analysis, stochastic control. Main applications concern evaluation and hedging of derivative products, dynamic portfolio optimization in incomplete markets, calibration of financial models, risk management. Special attention is paid to models with jumps, stochastic volatility models, asymmetry of information. The Mathfi project team develops the software Premia dedicated to pricing and hedging options and calibration of financial models, in collaboration with a consortium of financial institutions. <https://www-rocq.inria.fr/mathfi/Premia/>.

2.2. Highlights

The Mathfi project is acknowledged as an active part of the Université Paris-Est “**Labex**” **BÉZOUT** which has been recently selected by the French ministry of research.

A new team called “MathRisk” based on the current Mathfi project team is being launched on the theme of *mathematical treatment of risk*.

2.3. Numerical methods for option pricing and model calibration

2.3.1. Simulation of stochastic differential equations

Participants: Benjamin Jourdain, Aurélien Alfonsi, Damien Lamberton, Mohamed Sbai.

Most financial models are described by SDEs. Except in very special cases, no closed-form solution is available for such equations and one has to approximate the solution via time-discretization schemes in order to compute options prices and hedges by Monte Carlo simulations. Usually this is done by using the standard explicit Euler scheme since schemes with higher order of strong convergence involve multiple stochastic integrals which are difficult to simulate. In addition, the weak order of convergence of the explicit Euler scheme can be improved by using Romberg-Richardson’s extrapolations. Nevertheless, some schemes with weak order of convergence two or more have been designed recently. The idea is either to replace the multiple Brownian integrals by discrete random variables which share their moments up to a given order or to integrate ordinary differential equations associated with the vector fields giving the coefficients of the Stochastic Differential Equation up to well-chosen random time-horizons. Another interesting new direction of investigation is the design of exact simulation schemes.

Three directions of research have been investigated in the Mathfi project. First, fine properties of the Euler scheme have been studied [74], [69], [77]. Secondly, concerning SDEs for which the Euler scheme is not feasible, A. Alfonsi [63] have proposed and analysed new schemes respectively for Cox-Ingersoll-Ross processes and for equations with locally but not globally Lipschitz continuous coefficients. Last, the team has contributed to the new directions of research described above. For CIR processes, A. Alfonsi has designed a scheme with weak order two even for large values of the volatility parameter. Adapting exact simulation ideas, B. Jourdain and M. Sbai [76] have proposed an unbiased Monte Carlo estimator for the price of arithmetic average Asian options in the Black-Scholes model.

2.3.2. Monte-Carlo simulations

Participants: Benjamin Jourdain, Aurélien Alfonsi, Damien Lamberton, Mohamed Sbai, Vlad Bally, Bernard Lapeyre, Ahmed Kebaier, Céline Labart, Jérôme Lelong, Sidi-Mohamed Ould-Aly, Lokmane Abbas-Turki, Abdelkoddousse Ahida, Antonino Zanette, El Hadj Aly Dia.

Efficient computations of prices and hedges for derivative products are major issues for financial institutions (see [80]). Monte-Carlo simulations are widely used because of their implementation simplicity and because closed formulas are usually not available. Speeding up the algorithms is a constant preoccupation in the development of Monte-Carlo simulations. The team is mainly concerned with *adaptive versions* which improve the Monte-Carlo estimator by relying only on stochastic simulations.

The team has also been active on numerical methods in models with jumps and large dimensional problems.

This activity in the MathFi team is strongly related to the development of the Premia software.

2.3.3. Model calibration

The modeling of the so called *implied volatility smile* which indicates that the Black-Scholes model with constant volatility does not provide a satisfactory explanation of the prices observed in the market has led to the appearance of a large variety of extensions of this model as the local volatility models (where the stock price volatility is a deterministic function of the price level and time), stochastic volatility models, models with jump, and so on. An essential step in using any such approach is the *model calibration*, that is, the reconstruction of model parameters from the prices of traded options. This is an inverse problem to that of option pricing and as such, typically ill-posed.

The calibration problem is yet more complex in the interest rate markets since in this case the empirical data that can be used includes a wider variety of financial products from standard obligations to swaptions (options on swaps). The underlying model may belong to the class of short rate models like Hull-White [75], [67], CIR [70], Vasicek [89] etc. or to the popular class of LIBOR (London Interbank Offered Rates) market models like BGM [68].

The choice of a particular model depends on the financial products available for calibration as well as on the problems in which the result of the calibration will be used.

The calibration problem is of particular interest for Mathfi project because due to its high numerical complexity, it is one of the domains of mathematical finance where efficient computational algorithms are most needed.

2.4. Malliavin calculus and applications in finance

Participants: Vlad Bally, Arturo Kohatsu-Higa, Agnès Sulem, Antonino Zanette.

The original Stochastic Calculus of Variations, now called the Malliavin calculus, was developed by Paul Malliavin in 1976 [81]. It was originally designed to study the smoothness of the densities of solutions of stochastic differential equations. One of its striking features is that it provides a probabilistic proof of the celebrated Hörmander theorem, which gives a condition for a partial differential operator to be hypoelliptic. This illustrates the power of this calculus. In the following years a lot of probabilists worked on this topic and the theory was developed further either as analysis on the Wiener space or in a white noise setting. Many applications in the field of stochastic calculus followed. Several monographs and lecture notes (for example D. Nualart [83], D. Bell [66] D. Ocone [85], B. Øksendal [91]) give expositions of the subject. See also V. Bally [64] for an introduction to Malliavin calculus.

From the beginning of the nineties, applications of the Malliavin calculus in finance have appeared : In 1991 Karatzas and Ocone showed how the Malliavin calculus, as further developed by Ocone and others, could be used in the computation of hedging portfolios in complete markets [84].

Since then, the Malliavin calculus has raised increasing interest and subsequently many other applications to finance have been found [82], such as minimal variance hedging and Monte Carlo methods for option pricing. More recently, the Malliavin calculus has also become a useful tool for studying insider trading models and some extended market models driven by Lévy processes or fractional Brownian motion.

Let us try to give an idea why Malliavin calculus may be a useful instrument for probabilistic numerical methods.

We recall that the theory is based on an integration by parts formula of the form $E(f'(X)) = E(f(X)Q)$. Here X is a random variable which is supposed to be “smooth” in a certain sense and non-degenerated. A basic example is to take $X = \sigma\Delta$ where Δ is a standard normally distributed random variable and σ is a strictly positive number. Note that an integration by parts formula may be obtained just by using the usual integration by parts in the presence of the Gaussian density. But we may go further and take X to be an aggregate of Gaussian random variables (think for example of the Euler scheme for a diffusion process) or the limit of such simple functionals.

An important feature is that one has a relatively explicit expression for the weight Q which appears in the integration by parts formula, and this expression is given in terms of some Malliavin-derivative operators.

Let us now look at one of the main consequences of the integration by parts formula. If one considers the Dirac function $\delta_x(y)$, then $\delta_x(y) = H'(y - x)$ where H is the Heaviside function and the above integration by parts formula reads $E(\delta_x(X)) = E(H(X - x)Q)$, where $E(\delta_x(X))$ can be interpreted as the density of the random variable X . We thus obtain an integral representation of the density of the law of X . This is the starting point of the approach to the density of the law of a diffusion process: the above integral representation allows us to prove that under appropriate hypothesis the density of X is smooth and also to derive upper and lower bounds for it. Concerning simulation by Monte Carlo methods, suppose that you want to compute $E(\delta_x(y)) \sim \frac{1}{M} \sum_{i=1}^M \delta_x(X^i)$ where X^1, \dots, X^M is a sample of X . As X has a law which is absolutely continuous with respect to the Lebesgue measure, this will fail because no X^i hits exactly x . But if you are able to simulate the weight Q as well (and this is the case in many applications because of the explicit form mentioned above) then you may try to compute $E(\delta_x(X)) = E(H(X - x)Q) \sim \frac{1}{M} \sum_{i=1}^M E(H(X^i - x)Q^i)$. This basic remark formula leads to efficient methods to compute by a Monte Carlo method some irregular quantities as derivatives of option prices with respect to some parameters (the *Greeks*) or conditional expectations, which appear in the pricing of American options by the dynamic programming). See the papers by Fournié et al [73] and [72] and the papers by Bally et al., Benhamou, Bermin et al., Bernis et al., Cvitanic et al., Talay and Zheng and Temam in [79].

L. Caramellino, A. Zanette and V. Bally have been concerned with the computation of conditional expectations using Integration by Parts formulas and applications to the numerical computation of the price and the Greeks (sensitivities) of American or Bermudean options. The aim of this research was to extend a paper of Reigner and Lions who treated the problem in dimension one to higher dimension - which represent the real challenge in this field. Significant results have been obtained up to dimension 5 [65] and the corresponding algorithms have been implemented in the Premia software.

Moreover, there is an increasing interest in considering jump components in the financial models, especially motivated by calibration reasons. Algorithms based on the integration by parts formulas have been developed in order to compute Greeks for options with discontinuous payoff (e.g. digital options). Several papers and two theses (M. Messaoud and M. Bavouzet defended in 2006) have been published on this topic and the corresponding algorithms have been implemented in Premia. Malliavin Calculus for jump type diffusions - and more general for random variables with locally smooth law - represents a large field of research, also for applications to credit risk problems.

More recently the Malliavin calculus has been used in models of insider trading. The “enlargement of filtration” technique plays an important role in the modeling of such problems and the Malliavin calculus can be used to obtain general results about when and how such filtration enlargement is possible. See the paper by P. Imkeller in [79]). Moreover, in the case when the additional information of the insider is generated by adding the information about the value of one extra random variable, the Malliavin calculus can be used to find explicitly the optimal portfolio of an insider for a utility optimization problem with logarithmic utility. See the paper by J.A. León, R. Navarro and D. Nualart in [79]).

A. Kohatsu Higa and A. Sulem have studied a controlled stochastic system whose state is described by a stochastic differential equation with anticipating coefficients. These SDEs can be interpreted in the sense of *forward integrals*, which are the natural generalization of the semimartingale integrals, as introduced by Russo and Valois [87]. This methodology has been applied for utility maximization with insiders.

2.5. Optimal stopping

Participants: Aurélien Alfonsi, Benjamin Jourdain, Damien Lamberton, Maxence Jeunesse, Ayeche Bouselmi.

The theory of American option pricing has been an incite for a number of research articles about optimal stopping. Our recent contributions in this field concern optimal stopping for one dimensional diffusions and American options in exponential Lévy models.

In the context of general one-dimensional diffusions, we have studied optimal stopping problems with bounded measurable payoff functions. We have obtained results on the continuity of the value function and its characterization as the unique solution of a variational inequality in the sense of distributions, both in finite and infinite horizon problems (collaboration between D. Lamberton and Michail Zervos, London School of Economics).

We have explained how to calibrate a continuous and time-homogeneous local volatility function from the prices of perpetual American Call and Put options (A. Alfonsi and B. Jourdain).

The use of jump diffusions in financial models goes back to Merton (1976). More recently, there has been a growing interest for more sophisticated models, involving Lévy processes with no diffusion part and infinite activity (see, in particular, papers by Carr, Geman, Madan and Yor). A number of results on the exercise boundary and on the so called smooth fit property have been established.

2.6. Stochastic control and backward stochastic differential equations (BSDEs)

Participants: Vlad Bally, Jean-Philippe Chancelier, Marie-Claire Quenez, Agnès Sulem.

B. Øksendal (Oslo University) and Agnès Sulem have written a book on Stochastic control of Jump diffusions [10]). The types of control problems covered include classical stochastic control, optimal stopping, impulse control and singular control. Both the dynamic programming method and the maximum principle method are discussed, as well as the relation between them. Corresponding verification theorems involving the Hamilton-Jacobi Bellman equation and/or (quasi-)variational inequalities are formulated. There are also chapters on the viscosity solution formulation and numerical methods. In the second edition (2007), a chapter on optimal control of stochastic partial differential equations driven by Lévy processes and a section on optimal stopping with delayed information have been added. Applications to portfolio optimization problems and insurance problems have been studied.

In the context of risk measures, M.C. Quenez (Prof Paris VII) has shown how some dynamic measures of risk can be induced by Backward Stochastic Differential Equations and A. Sulem and B. Øksendal in [90] have studied risk-indifference pricing in incomplete markets with jumps using stochastic control theory and PDE methods.

3. Application Domains

3.1. Application domains

- Option pricing and hedging
- Calibration of financial models
- Portfolio optimization
- Risk management
- Market microstructure
- Insurance-reinsurance optimization policy
- Insider modeling, asymmetry of information

4. Software

4.1. PREMIA

Participants: Antonino Zanette, Mathfi Research team, Agnès Sulem [correspondant].

Premia is a software designed for option pricing, hedging and financial model calibration. It is provided with its C/C++ source code and an extensive scientific documentation. <https://www-rocq.inria.fr/mathfi/Premia>

This Premia project keeps track of the most recent advances in the field of computational finance in a well-documented way. It focuses on the implementation of numerical analysis techniques for both probabilistic and deterministic numerical methods. An important feature of the platform Premia is the detailed documentation which provides extended references in option pricing.

Premia is thus a powerful tool to assist Research & Development professional teams in their day-to-day duty. It is also a useful support for academics who wish to perform tests on new algorithms or pricing methods without starting from scratch.

Besides being a single entry point for accessible overviews and basic implementations of various numerical methods, the aim of the Premia project is:

1. to be a powerful testing platform for comparing different numerical methods between each other;
2. to build a link between professional financial teams and academic researchers;
3. to provide a useful teaching support for Master and PhD students in mathematical finance.

- AMS: 91B28;65Cxx;65Fxx;65Lxx;65Pxx
- License: Licence Propriétaire (genuine license for the Consortium Premia)
- Type of human computer interaction: Console, interface in Nsp
- OS/Middleware: Linux, Mac OS X, Windows
- APP: The development of Premia started in 1999 and 13 are released up to now and registered at the APP agency.
- Programming language: C/C++ librairie Gtk
- Documentation: the PNL library is interfaced via doxygen
- Size of the software: 250 Mbyte, 40 Mbyte of C/C++ routines; Number of lines of code: 972000 (for the source part only)
- Publications: [1] [71] [78] [86] [88], [62]

4.1.1. Content of Premia

Premia contains various numerical algorithms (Finite-differences, trees and Monte-Carlo) for pricing vanilla and exotic options on equities, interest rate, credit and energy derivatives.

1. Equity derivatives:

The following models are considered:

Black-Scholes model (up to dimension 10), stochastic volatility models (Hull-White, Heston, Fouque-Papanicolaou-Sircar), models with jumps (Merton, Kou, Tempered stable processes, Variance gamma, Normal inverse Gaussian), Bates model.

For high dimensional American options, Premia provides the most recent Monte-Carlo algorithms: Longstaff-Schwartz, Barraquand-Martineau, Tsitsklis-Van Roy, Broadie-Glassermann, quantization methods and Malliavin calculus based methods.

Dynamic Hedging for Black-Scholes and jump models is available.

Calibration algorithms for some models with jumps, local volatility and stochastic volatility are implemented.

2. Interest rate derivatives

The following models are considered:

HJM and Libor Market Models (LMM): affine models, Hull-White, CIR++, Black-Karasinsky, Squared-Gaussian, Li-Ritchken-Sankarasubramanian, Bhar-Chiarella, Jump diffusion LMM, Markov functional LMM, LMM with stochastic volatility.

Premia provides a calibration toolbox for Libor Market model using a database of swaptions and caps implied volatilities.

3. Credit derivatives: CDS, CDO

Reduced form models and copula models are considered.

Premia provides a toolbox for pricing CDOs using the most recent algorithms (Hull-White, Laurent-Gregory, El Karoui-Jiao, Yang-Zhang, Schönbucher)

4. Hybrid products

PDE solver for pricing derivatives on hybrid products like options on inflation and interest or change rates is implemented.

5. Energy derivatives: swing options

Mean reverting and jump models are considered.

Premia provides a toolbox for pricing swing options using finite differences, Monte-Carlo Malliavin-based approach and quantization algorithms.

4.1.2. Development of the PNL Library

Here are the major contribution of J. Lelong:

1. Development of the PNL.
 - A PnlArray object has been added to create arrays of PnlObjects.
 - The implementation of the PnlList type has been changed to improve linear iteration on a list.
 - Design of a new unit test framework and backport of all the previous tests.
 - Integration of the Runge Kutta Fehlner 45 method for solving n dimensional ODEs.
 - New organisation of the manual.
 - Random number generators: new Sobol generators (32 and 64 bits), dynamic Mersenne Twister updated to version 0.6.1. All random number generators are now thread-safe.
 - Update of internal Lapack (and the corresponding shipped version of Blas) to version 3.2.1
 - Update of F2C.
 - LU factorization for tridiagonal matrices.
 - Cholesky block factorization for positive semi-definite matrices.

4.1.3. Premia design

Anton Kolotaev (ADT engineer), supervised by J. Lelong, has realized the Bindings for Premia with Python and F#; a library allowing the development of bindings to other languages; a Web interface Web for Premia. Moreover he has improved the documentation facility system of Premia and has updated the Excel interface Excel for the new versions MS Excel and MS Windows.

Tasks achieved by J. Lelong:

1. New design of the enumeration type to allow the number of parameters of Premia objects to depend on the selected value within the enumeration. This change had a strong impact on the VAR system and many core functions had to be rewritten. This modification was definitely essential to improve the Nsp interface, which was broken for long as far as credit derivatives are concerned and is now working properly again.
2. Creation of the first Premia bundle for Mac OS X and automation of the building process.

3. Generic “Get”, “FGet” and “Print” functions had been introduced to simplify object creation but there were still many exceptions which were not using these generic functions. All these exceptions have been handled and now rely on the generic functions.
4. Improvement of the scripts to build the free version of Premia.
5. Integration of Cosine methods implemented by Bowen Zhang.

Tasks achieved by C. Labart:

- Improvement of the credit part of Premia : correction of memory leaks, modification of old codes to use the new copula structure. This enables to remove a large part of the code which has become hard to maintain.
- Complete rewriting of the BSDE algorithm for pricing basket options in high dimension to use a more efficient approximation technique. This has been possible thanks to the polynomial approximation tool provided by the PNL.

4.1.4. Algorithms implemented in Premia in 2011

Premia 13 was delivered to the consortium members in March 2011. It contains the following new algorithms:

4.1.4.1. Interest rate derivatives

- Pricing and hedging callable Libor exotics in forward Libor models. V. Piterbarg, *The Journal of Computational Finance* Volume 8 Number 2, Winter 2004/05
- A stochastic volatility forward Libor model with a term structure of volatility smiles. A. Piterbarg, V. (2004), *SSRN Working Paper*.
- Iterating cancelable snowballs and related exotics in a many-factor Libor model, J. Schoenmakers *RISK*, Sept. 2006, *Asia RISK* Oct. 2006
- A new approach to Libor modeling. M. Keller-Ressel, A. Papapantoleon and J. Teichmann. *Preprint 2009*
- Jump-adapted discretization schemes for Levy-driven SDEs A. Kohatsu Higa P. Tankov, *Stochastic Processes and their Applications*, to appear.

4.1.4.2. Credit risk derivatives

- R. Cont and A. Minca Recovering Portfolio Default Intensities Implied by CDO Quotes. *Mathematical Finance*, to appear
- Interacting particle systems for the computation of rare credit portfolio losses, R.Carmona, J.P. Fouque and D. Vestal, *Finance & Stochastics* Volume 13, Number 4, September, 2009.

4.1.4.3. Electricity derivatives

- Variance optimal hedging for processes with independent increments and applications. Applications to electricity market. F.Russo, S. Goutte and N. Oudjane. *Preprint 2010*

4.1.4.4. Equity derivatives

- A Novel Option Pricing Method based on Fourier-Cosine Series Expansions. F. Fang and C.W. Oosterlee. *SIAM J. Sci. Comput.* 31: 826-848 (2008)
- Pricing early-exercise and discrete barrier options by Fourier-cosine series expansions. C.W. Oosterlee and F. Fang *Numerische Mathematik* 114: 27-62 (2009)
- On The Heston Model with Stochastic Interest Rates. L.A. Grzelak and C.W. Oosterlee. *Preprint 2010*
- Pricing options under stochastic volatility: a power series approach. F.Antonelli and S.Scarlatti *Finance & Stochastics*, Volume XIII (2009), issue 1

- Gamma Expansion of the Heston Stochastic Volatility Model. P. Glasserman and K. Kim, *Finance & Stochastics*, to appear.
- Kudryavtsev O.: Wiener-Hopf techniques for Heston model.
- Multi-level Monte Carlo path simulation. M.B. Giles *Operations Research*, 56(3):607-617, 2008.
- Fourier space time-stepping for option pricing with Levy models. R. Jackson, S. Jaimungal and V. Surkov *The Journal of Computational Finance*, Volume 12 / Number 2, Winter 2008.
- Saddlepoint methods for option pricing Peter Carr and Dilip Madan *The Journal of Computational Finance*, to appear
- Saddlepoint Approximations for Affine Jump-Diffusion Models P. Glasserman and K. Kim, *Journal of Economic Dynamics and Control*, vol 33, 37-52, 2009.
- Monte Carlo for pricing Asian options in jump models. E. Dia and D. Lamberton, *Preprint*
- An Improved Convolution Algorithm for Discretely Sampled Asian Options. Cerny, A. and I. Kyriakou, *Quantitative Finance*, to appear
- Estimating Greeks in Simulating Levy-Driven Models. Paul Glasserman and Zongjian Liu. *The Journal of Computational Finance*, Volume 14/Number 2, Winter 2010.
- Computing exponential moments of the discrete maximum of a Levy process and lookback options. L. Feng and V. Linetsky. *The Journal of Computational Finance*, Volume 13, Number 4 / September, 2009
- Doubly Reflected BSDEs with Call Protection and their Approximation. J.F. Chassagneux S.Crepey. *Preprint 2010*
- Pricing Convertible Bonds with Call Protection S.Crepey A.Rahal. *The Journal of Computational Finance*, to appear.
- Empirical martingale simulation of asset prices. J.-C. Duan, and J.-G Simonato. *Management Science*(1998) 44-9, 1218-1233
- Pricing and hedging American-style options: a simple simulation-based approach. Y. Wang and R.Caffisch. *The Journal of Computational Finance*, 13-4, 2010.

4.1.5. New algorithms for the release 14 of Premia to be delivered in March 2012 to the Consortium:

4.1.5.1. Interest rate derivatives

- An n-Dimensional Markov-functional Interest Rate Model. Linus Kaisajuntti and Joanne Kennedy. *Preprint 2008*
- Picard approximation of SDEs and application to LIBOR models. A. Papapantoleon, D. Skovmand (2010). *Preprint*, TU Berlin.

4.1.5.2. Credit risk derivatives

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- Backward Stochastic Differential Equations (BSDE).
- American options in high dimension solving BSDE with penalization.

5. New Results

5.1. Numerical probability

5.1.1. Simulation of stochastic processes

Participants: Aurélien Alfonsi, Benjamin Jourdain, Arturo Kohatsu-Higa [(University of Osaka)], Abdelkoudousse Ahdida, Stefanp De Marco.

5.1.1.1. Pathwise convergence of the Euler scheme

A. Alfonsi, B. Jourdain and A. Kohatsu-Higa are studying the convergence for the Wasserstein distance of the Euler scheme towards the limit diffusion. This would be a way to analyze the weak pathwise convergence of this discretization scheme. They have obtained some promising results in the one-dimensional case.

5.1.2. Multi-dimensional models and correlation issues

A. Alfonsi and his PhD Student A. Ahdida have submitted a paper on the simulation of Wishart processes. They have introduced a new family of stochastic differential equations that are defined in the space of correlation matrices and provided high order discretization schemes for such processes [51]. They are currently trying to use this type of matrix valued processes to model the dependence between assets. In particular, they would like to calibrate Index options data. This work is still in progress. The thesis of A. Ahdida has been defended on December 1st [12].

The PhD student Lokmane Abbas Turki has worked on numerical methods for American option pricing based on Malliavin calculus and parallel implementation. He has submitted a paper (co-authored with B. Lapeyre) [50]. He is now working on the dependence of option prices with respect to correlations between stocks in multi-dimensional models.

A. Alfonsi and S. De Marco (postdoc CERMICS) have studied how some option prices (such as spread options) are modified when the correlation between stocks is increased.

5.1.3. Stochastic volatility models

Exotic options and stochastic volatility models is the subject of Sidi Mohamed OULD ALY's thesis, defended on June 16th, 2011. Sidi-Mohamed has results on the effective computation of option prices in a stochastic volatility model, in the context of variance swap modelling. He has worked out a new model, in the spirit of Bergomi's approach. This model has remarkable features in terms of tractability and calibration. S.M. Ould Aly has developed numerical methods and an original variance reduction method for models with a log normal volatility. He also has results on the monotony of option prices with respect to the correlation between the stock price and the volatility in the Heston model. Sidi Mohamed has submitted three papers.

5.2. American options

Participants: Benjamin Jourdain, Maxence Jeunesse, Damien Lamberton, Ayech Bouselmi.

5.2.1. American put option with discrete dividends.

B. Jourdain and M. Jeunesse are interested in the regularity of the optimal exercise boundary for the American Put option when the underlying asset pays discrete dividends at known times during the lifetime of the option. The dividend amounts are deterministic functions of the asset prices just before the dividend dates. B. Jourdain and M. Jeunesse have proved continuity of the exercise boundary and smooth contact for the value function under general assumptions on the dividend functions.

5.2.2. American options in exponential Lévy models

D. Lamberton and his PhD student A. Bouselmi are working on American options within multi-dimensional exponential Lévy models. They also have preliminary results on the asymptotic behaviour of the exercise boundary of the American put near maturity in the one dimensional case when the limit is strictly smaller than the strike price.

5.3. Financial issues modelling

Participants: Aurélien Alfonsi, Céline Labart, Jérôme Lelong, J. Acevedo.

5.3.1. Credit risk modelling

A. Alfonsi, C. Labart and J. Lelong are studying loss models called “stochastic local intensity models” that have been proposed in the literature. First, they are interested in proving mathematically that these models are well posed (it exists and has a unique solution). Second, they aim to provide numerical tools to sample such dynamics.

5.3.2. Limit order markets

A. Alfonsi has an active collaboration with A. Schied (Mannheim University) on limit order book models. A. Schied has visited the CERMICS two weeks in February 2011 and one week on December. They are currently studying some type of non-Markovian resilience for the limit-order book for which they are able to get the optimal execution strategy in a closed form. Moreover A. Alfonsi and his PhD student J. Acevedo study impact models for which the limit order book shape evolves along the time.

5.3.3. Control of systemic risk

Participants: Agnès Sulem, J.Ph. Chancelier, Andreea Minca.

We are interested in contagion modeling and systemic risk in financial networks. We aim to contribute in particular to the domain of *control* of such systems in order to reduce the systemic risk. We model the propagation of distress in financial systems as an epidemic on a random graph in which the nodes represent financial institutions and edges the exposure between them. Cascade dynamics may be reduced to the evolution of a multi-dimensional markov chain that corresponds to a sequential discovery of exposures and determines the size of contagion. We study the optimal intervention strategy by a lender of last resort who wants to minimize the size of contagion under budget constraints.

5.4. Stochastic control of jump processes and stochastic differential games

Participants: Agnès Sulem [in collaboration with B. Øksendal (Oslo University) and T. Zhang (Manchester University)], John Joseph Absalom Hosking.

5.4.1. Stochastic control under model uncertainty

In [58], we study optimal stochastic control problems with jumps under model uncertainty. We rewrite such problems as (zero-sum) stochastic differential games of forward-backward stochastic differential equations. We prove general stochastic maximum principles for such games, both in the zero-sum case (finding conditions for saddle points) and for the non-zero sum games (finding conditions for Nash equilibria). We then apply these results to study optimal portfolio and consumption problems under model uncertainty. We combine the optimality conditions given by the stochastic maximum principles with Malliavin calculus to obtain a set of equations which determine the optimal strategies.

In [45], we consider some robust optimal portfolio problems for markets modeled by (possibly non-Markovian) Itô–Lévy processes. Mathematically the situation can be described as a stochastic differential game, where one of the players (the agent) is trying to find the portfolio which maximizes the utility of her terminal wealth, while the other player (“the market”) is controlling some of the unknown parameters of the market (e.g. the underlying probability measure, representing a model uncertainty problem) and is trying to minimize this maximal utility of the agent. This leads to a worst case scenario control problem for the agent.

In the Markovian case such problems can be studied using the Hamilton-Jacobi-Bellman-Isaacs (HJBI) equation, but these methods do not work in the non-Markovian case. We approach the problem by transforming it to a stochastic differential game for backward stochastic differential equations (BSDE game). Using comparison theorems for BSDEs with jumps we arrive at criteria for the solution of such games, in the form of a kind of non-Markovian analogue of the HJBI equation. The results are illustrated by examples.

5.4.2. Singular stochastic control

In [59], A. Sulem and B. Øksendal study partial information, possibly non-Markovian, singular stochastic control of Itô–Lévy processes and obtain general maximum principles. The results are used to find connections between singular stochastic control, reflected BSDEs and optimal stopping in the partial information case. As an application we give an explicit solution to a class of optimal stopping problems with finite horizon and partial information. Singular control of SPDEs.

In [57], A. Sulem, B. Øksendal and T. Zhang study general singular control problems for random fields given by a stochastic partial differential equation (SPDE). They show that under some conditions the optimal singular control can be identified with the solution of a coupled system of SPDE and a *kind of reflected backward* SPDE (R BSPDE). They also establish existence and uniqueness of solutions of R BSPDEs.

5.4.3. Optimal control with delay

In [44], we study optimal control problems for (time-)delayed stochastic differential equations with jumps. We establish sufficient and necessary stochastic maximum principles for an optimal control of such systems. The associated adjoint processes are shown to satisfy a (time-) advanced backward stochastic differential equation (ABSDE). Several results on existence and uniqueness of such ABSDEs are shown. The results are illustrated by an application to optimal consumption from a cash flow with delay.

In [48], we will prove a sufficient necessary stochastic maximum principles for the optimal control of SPDEs with delay and study associated time-advanced backward stochastic partial differential equations.

5.4.4. Stochastic differential games

In [55], J. Hosking has constructed a stochastic maximum principle (SMP) which provides necessary conditions for the existence of Nash equilibria in a certain form of N-agent stochastic differential game (SDG) of a mean-field type. The information structure considered for the SDG is of a possible asymmetric and partial type. To prove our SMP we use a spike-variation approach with adjoint representation techniques, analogous to that of S. Peng in the optimal stochastic control context. In our proof we apply adjoint representation procedures at three points. The first-order adjoint processes are defined as solutions to certain mean-field backward stochastic differential equations (BSDEs), and second-order adjoint processes of a first type are defined as solutions to certain BSDEs. Second order adjoint processes of a second type are defined as solutions of backward stochastic equations of a type that we introduce in this paper, and which we term conditional mean-field BSDEs. From the resulting representations, we show that the terms relating to these second-order adjoint processes of the second type are of an order such that they do not appear in our final SMP equations.

5.5. Risk measures, BSDEs with jumps and nonlinear expectations

Participants: Agnès Sulem, Marie-Claire Quenez, Z. Chen [Shandong University].

In the Brownian case, links between dynamic risk measures and Backward Stochastic Differential Equations (BSDEs) have been established. A. Sulem and M.-C. Quenez are exploring these links in the case of stochastic processes with jumps. They have extended some comparison theorems for BSDEs with jumps, and provided a representation theorem of convex dynamic risk measures induced by BSDEs with jumps. They study optimal stopping problems for (non necessarily) convex dynamic risk measures induced by BSDEs with jumps and establish their connections with *Reflected* BSDEs with jumps. They also study the case of model ambiguity and its relation with mixed control/optimal stopping problems.

There are two classes of nonlinear expectations, one is the Choquet expectation given by Choquet (1955), the other is the Peng's g -expectation given by Peng (1997) via backward differential equations (BSDE). Recently, Peng raised the following question: can a g -expectation be represented by a Choquet expectation? In [26], A. Sulem and Z. Chen provide a necessary and sufficient condition on g -expectations under which Peng's g -expectation can be represented by a Choquet expectation for some random variables (Markov processes). It is well known that Choquet expectation and g -expectation (also BSDE) have been used extensively in the pricing of options in finance and insurance. Our result also addresses the following open question: given a

BSDE (g -expectation), is there a Choquet expectation operator such that both BSDE pricing and Choquet pricing coincide for all European options? Furthermore, the famous Feynman-Kac formula shows that the solutions of a class of (linear) partial differential equations (PDE) can be represented by (linear) mathematical expectations. As an application of our result, we obtain a necessary and sufficient condition under which the solutions of a class of nonlinear PDE can be represented by nonlinear Choquet expectations [26].

5.6. Malliavin calculus and applications

5.6.1. Lower bounds for the density of functionals on the Wiener space

In collaboration with: B. Fernandez and A. Meda from the University of Mexico, V. Bally gave a lower bound for general Itô processes to remain in a tube around a given curve. This is done under some ellipticity assumption in [21]. Now, with L. Caramellino (University Tor Vergata, Rome) he investigates the case of a diffusion processes which satisfies the Hörmander condition.

5.6.2. Malliavin Calculus for Poisson Point Processes and applications

V. Bally and E. Clément (Université Paris-Est Marne la Vallée) study the density of the law of the solution of a stochastic equation with jumps, which has discontinuous coefficients [18], [19]. Moreover, with N. Fournier (university of Creteil), V. Bally obtained results on the smoothness of the law of a bidimensional Boltzman equation [22].

5.6.3. Riesz transform and regularity of the law of a random variable

The idea of using the Riesz transform in order to study the regularity of the law of a random variable appears in former works of P. Malliavin and A. Thalmaier. In collaboration with L. Caramellino (University Tor Vergata, Rome) we gave regularity results using this tool [17].

6. Contracts and Grants with Industry

6.1. Contracts with Industry

Consortium Premia presently composed of CALYON, Société Générale, Natixis, and Pricing Partners.

6.2. Grants with Industry

- Chair “Risques financiers”, Fondation du Risque: 2007-2012
Partners: Ecole des Ponts ParisTech, Ecole Polytechnique, Société Générale.
- Fondation Natixis grant
- Fondation Axa grant

7. Partnerships and Cooperations

7.1. Regional Initiatives

- **Pôle Finance Innovation.**

Project “Credinext” on credit risk derivatives (2009-2012).

Partners: Thomson Reuters, Lunalogic, Pricing Partners, Ecole Polytechnique, INRIA, ENPC, Université Paris-Est Marne la Vallée.

7.2. National Initiatives

ANR ANR-08-BLAN, Program: Big'MC (Issues in large scale Monte Carlo). (2009-2012).

Partners ENST, ENPC, University Paris-Dauphine.

7.3. European Initiatives

7.3.1. Collaborations in European Programs, except FP7

Eurostars Program "Transparency in Financial Markets" (OSEO grant).

7.4. International Initiatives

7.4.1. Conferences, Seminars, Invitations

7.4.1.1. Conferences

- A. Alfonsi:
 1. "Market Impact and Manipulation Strategies", Modeling and managing financial risks, Paris, 10-13 January 2011
 2. "Optimal execution in Limit Order Books", Vancouver, ICIAM 2011, High Frequency Modeling & Algorithmic Trading minisymposium, 20 July 2011.
 3. "Efficient simulation schemes for some multidimensional stochastic volatility models", Vancouver, ICIAM 2011, Session MS368: Computational Methods in Finance, 21 July 2011.
 4. "Efficient simulation schemes for some multidimensional stochastic volatility models", Leicester, ENUMATH 2011, 9 September 2011.
 5. "Optimal execution and price manipulations in limit order book models", Annual Financial Market Liquidity Conference Budapest, Hungary, 10 November 2011.
- V. Bally:
 1. Rough path and numerical integration methods. University of Marbourg, Germany, September 21-23, 2011. "Lower bounds for tube under a local first order Hörmander condition".
 2. Stochastic analysis, Levy processes and BSDE's, University of Innsbruck, Austria, October 3-7, 2011. "Regularization properties for the 2D homogeneous Boltzman equation without cutoff".
 3. Seventh Seminar on Stochastic Analysis, Random Fields and Applications May 23-27, 2011, Centro Stefano Franscini, Ascona, Switzerland. "Regularization properties for the 2D homogeneous Boltzman equation without cutoff."
 4. International Conference on Malliavin Calculus and Stochastic Analysis in Honor of Professor David Nualart. March 19-21, 2011, Kansas University, USA. "Regularization properties for the 2D homogeneous Boltzman equation without cutoff".
- B. Jourdain:
 1. Conference Modeling and Managing Financial Risks, Paris, 10-13 January, High order discretization schemes for stochastic volatility models
 2. Seventh Seminar on Stochastic Analysis, Random Fields and Applications, Ascona, 23-27 May, High order discretization schemes for stochastic volatility models
- J. Hosking:

1. Groupe de travail Méthodes Stochastiques et Finance, Université Paris-Est Marne-la-Vallée. “ Conditions for the non-degeneracy of solutions to pure jump Lévy process driven stochastic differential equations, 21 January 2011.
- A. Sulem:
 1. Conference on Stochastic Analysis and Applications, Växjö, Sweden, June 2011
 2. Applied Probability Society Conference, Invited session on “Stochastic Control in Finance”, Stockholm, July 2011;
 3. Stochastic Analysis Conference in Ascona, Suisse, Mai 2011

7.4.1.2. Seminars

- A. Alfonsi:
 - "Exact and High order discretization schemes for Wishart processes and their affine extensions", (April at Evry, May at Mannheim).
- B. Jourdain
 - Applied mathematics seminar of the collège de France, 18 march, Robust variance reduction techniques for Gaussian random vectors
 - Probability theory, Statistics and control seminar at ENSTA, 6 april, Adaptive variance reduction for Gaussian random vectors
 - Mathematical Finance, Numerical probability and Statistics of random processes working group Paris 6, 20 october, Exercise boundary of the American put option in the Black-Scholes model with discrete dividends
- C. Labart:
 - Seminar at Institut Fourier, May 2011.
- J. Lelong:
 - Seminar of the University of Montpellier II (Institut de mathématiques et de modélisation de Montpellier)
- A. Sulem:
 - ENSTA Seminar, April 2011.

7.4.1.3. Invitations

A. Alfonsi: University Mannheim, by Alexander Schied (23rd to the 25th of May)

7.4.2. Visits of International Scientists

7.4.2.1. Invited Professors

Lucia Caramellino (University Tor Vergata, Rome); A. Kohatsu Higa (University of Osaka); A. Schied (Mannheim University).

7.4.2.2. Internship

- Ignacio Orrego[Ecole Polytechnique, 3rd year]. Supervisor: A. Sulem and S. Ould Aly; Subject: multi-dimensional Markov-functional interest rate models.
- Phuong Nguyen[Ecole Polytechnique, 3rd year]. Supervisor: A. Sulem and S. Ould Aly; Subject: European Option Pricing in a stochastic volatility model
- Nicolas Baby[ENSTA, 2nd year]; Supervisor: C. Labart on “Numerical methods for solving BSDEs” (2 months).

8. Dissemination

8.1. Animation of the scientific community

- A. Alfonsi: Co-organizer of the working group seminar of MATHFI “Méthodes stochastiques et finance”.
- V. Bally: Member of the scientific committee of the congress of SMAI 2011, du 23 au 27 mai 2011.
- B. Jourdain:
 1. Deputy head of the doctoral school ICMS, university Paris-Est
 2. report on the PhD dissertation "Some aspects of optimal quantization and applications to finance" by Sylvain Corlay
 3. report on the PhD dissertation "Non-parametric calibration of some financial models" by Rémi Tachet des Combes
- D. Lamberton:
 1. "Associate Editor" of *Mathematical Finance*, co-editor of *ESAIM P&S*.
 2. In charge of the master program “Mathématiques et Applications” (Universities of Marne-la-Vallée, Créteil and Evry, and Ecole Nationale des Ponts et Chaussées).
 3. Directeur de l’UFR de mathématiques, Université Paris-Est Marne-la-Vallée.
 4. PhD report on the thesis of Paul Gassiat: Modélisation du risque de liquidité et méthodes de quantification appliquées au contrôle stochastique séquentiel.
- A. Sulem:

Associate editor of:

 - *SIAM Journal on Financial Mathematics (SIFIN)* (since its creation in 2008)
 - *International Journal of Stochastic Analysis (IJSA)* (since 2009)
 - *Journal of Mathematical Analysis and Applications (JMAA)*(since 2011)

Examinor of the thesis of Paul Gassiat, December 2011, Université Paris Diderot.

8.2. Teaching

- A. Alfonsi:
 1. “Modéliser, Programmer et Simuler”, second year course at the Ecole des Ponts.
 2. “Calibration, Volatilité Locale et Stochastique”, third-year course at ENSTA (Master with Paris I).
 3. “Traitement des données de marché : aspects statistiques et calibration”, lecture for the Master at UPEMLV.
 4. “Mesures de risque”, Master course of UPEMLV and Paris VI.
- V. Bally:
 1. Master 2 of the University Marne la Vallée:
 - Malliavin Calculus and numerical applications in finance
 - Probabilistic methods for risk analysis.
 - Taux d’intérêt
- B. Jourdain :

1. Course "Probability theory and statistics", first year ENPC
 2. Course "Introduction to probability theory", 1st year, Ecole Polytechnique
 3. Course "Stochastic numerical methods", 3rd year, Ecole Polytechnique
 4. projects in finance and numerical methods, 3rd year, Ecole Polytechnique
- B. Jourdain, B. Lapeyre: course "Monte-Carlo methods in finance", 3rd year ENPC and Master Recherche Mathématiques et Application, University of Marne-la-Vallée
 - J.-F. Delmas, B. Jourdain: course "Jump processes with applications to energy markets", 3rd year ENPC and Master Recherche Mathématiques et Application, university of Marne-la-Vallée
 - D. Lamberton:
 1. Second year of Licence de mathématiques (probability), Université Paris-Est Marne-la-Vallée.
 2. Master course "Calcul stochastique et applications en finance", Université Paris-Est Marne-la-Vallée.
 - A. Sulem:
 1. **Master Course, Université Paris IX-Dauphine**, Département MIDO (Mathématiques et Informatique de la Décision et des Organisations), Master MASEF, (21 h) *Finite difference methods in Finance*
 2. **Master of Mathematics, Université du Luxembourg**, 15h, December 2011. *Numerical Methods in Finance*.

8.3. PhD

8.3.1. PhD defense

- **Sidi Mohamed Ould Aly**, *Exotic options and stochastic volatility models*. Thesis defense was on June 16th, 2011, Université Paris-Est Marne-la-Vallée. Adviser: D. Lamberton.
- **Ahdida Abdelkoddousse**, *Wishart processes and correlation modeling in large dimension*. Defense on the 1st of December 2011. Adviser: A. Alfonsi.
- **Andrea Minca**, *Modélisation mathématique de la contagion de défaut; Mathematical modeling of financial contagion* Adviser: A. Sulem (30%) and Rama Cont, (Bourse Fondation Natixis): Thesis defense was at Université Pierre et Marie Curie (Paris 6) September 5, 2011. Actual Position: Assistant Professor, School of Operations Research and Information Engineering, Cornell University.

8.3.2. PhD in progress

- **José Infante Acevedo** (from Oct. 2009). *Liquidity risk and limit order books modelling*, ENPC. Adviser: A. Alfonsi
- **Lokmane Abbas Turki** (3rd year, started in March 2009, *Modelling of correlation in high dimensions and numerical methods*. This thesis is funded by Credinext. Université Paris-Est Marne-la-Vallée Advisers: D. Lamberton and B. Lapeyre.
- **Ayech Bouselmi** (3rd year, started in October 2009). *Lévy processes and multi-dimensional models in finance*. Allocataire de recherche, Université Paris-Est Marne-la-Vallée. Adviser: D. Lamberton
- **Jing Chen**, *Non linear expectations and Backward SDEs*, (Shandong University grant, INRIA. Adviser: A. Sulem (started September 2011).
- **Maxence Jeunesse**: *Study of some numerical methods in finance* Adviser: B. Jourdain

- **Victor Rabiet**, *Malliavin calculus for jump diffusions*. (3rd year, started in October 2009), ENS Cachant, Adviser: V. Bally

9. Bibliography

Major publications by the team in recent years

- [1] *Numerical Methods implemented in the Premia Software*, 2009, Bankers, Markets, Investors, Introduction by A. Sulem and A. Zanette.
- [2] A. ALFONSI, A. FRUTH, A. SCHIED. *Optimal execution strategies in limit order books with general shape functions*, in "Quantitative Finance", 2009, vol. 10, n^o 2, p. 143-157, DOI:10.1080/14697680802595700.
- [3] A. ALFONSI, B. JOURDAIN. *Exact volatility calibration based on a Dupire-type Call-Put duality for perpetual American options*, in "Nonlinear Differential Equations and Applications", 2009, vol. 16, n^o 4, p. 523-554.
- [4] V. BALLY, M.-. BAVOUZET, M. MESSAOUD. *Computations of Greeks using Malliavin Calculus in jump type market models*, in "Annals of Applied Probability", 2007, vol. 17, p. 33-66.
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- [7] A. KOHATSU-HIGA, A. SULEM. *Utility maximization in an insider influenced market*, in "Mathematical Finance", 2006, vol. 16, n^o 1, p. 153-179.
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Publications of the year

Doctoral Dissertations and Habilitation Theses

- [12] A. AHDIDA. *Wishart processes and correlation modeling in large dimension*, ENPC, Université Paris-Est, December 1st 2011.
- [13] A. MINCA. *Modélisation mathématique de la contagion de défaut; Mathematical modeling of financial contagion*, Université Pierre et Marie Curie (Paris 6), September 5 2011.

- [14] S. M. OULD ALY. *Modélisation de la courbe de variance et modèles à volatilité stochastique ; Forward variance modelling and stochastic volatility models*, Université Paris Est, June 16 2011.

Articles in International Peer-Reviewed Journal

- [15] V. BALLY, M. CABALLERO, B. FERNANDEZ, N. EL-KAROUI. *Reflected BSDE's, PDE's and Variational Inequalities*, in "Bernouilli", 2011, accepted for publication.
- [16] V. BALLY, L. CARAMELLINO. *Positivity and lower bounds for the density of Wiener functionals*, in "Stochastics", 2011, arXiv:1004.5269.
- [17] V. BALLY, L. CARAMELLINO. *Riesz transform and integration by parts formulas for random variables*, in "Stochastic Processes and their Applications", 2011, vol. 121, p. 1332-1355.
- [18] V. BALLY, E. CLÉMENT. *Integration by parts formulas and applications to equations with jumps*, in "PTRF", 2011, n^o 151, p. 613-657.
- [19] V. BALLY, E. CLÉMENT. *Integration by parts formulas with respect to jump times and stochastic differential equations*, in "Stochastic Analysis", 2011.
- [20] V. BALLY, S. DE MARCO. *Some estimates in extended Stochastic Volatility models of Heston type*, in "Risk and Decision Analysis", 2011, vol. 2, n^o 4, p. 195-206.
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