Activity Report 2014

Team GALAAD2

Géométrie , Algèbre, Algorithmes

RESEARCH CENTER
Sophia Antipolis - Méditerranée

THEME
Algorithmics, Computer Algebra and Cryptology

Table of contents

Team GALAAD2

Keywords: Algorithmic Geometry, Computer Algebra, Real Numbers

Creation of the Team: 2014 January 01.

1. Members

Research Scientists
Bernard Mourrain [Team leader, Inria, Senior Researcher, HdR]
Laurent Busé [Inria, Researcher, HdR]
Evelyne Hubert [Inria, Researcher, HdR]

Faculty Member
André Galligo [Univ. Nice, Professor, HdR]

Engineers
Nicolas Douillet [Inria]
Anaïs Ducoffe [Inria]
Hung Nguyen [Inria, until Nov 2014]

PhD Students
Marta Abril-Bucero [Univ. Nice]
Elisa Berrini [CIFRE]
Mathieu Collowald [Univ. Nice]

Post-Doctoral Fellows
Laura Saini [Inria, until Aug 2014]
Meng Wu [Inria, until Aug 2014]

Visiting Scientist
Anna Karasoulou [Visiting PhD student, until May 2014]

Administrative Assistant
Sophie Honnorat [Inria]

Other
Danny Schmitt [Inria, Master student, from Apr 2014 until Jun 2014]

2. Overall Objectives

2.1. Overall Objectives

There is a shared vision that our day life environment will increasingly interact with a digital world, populated by captors, sensors, or devices used to simplify or improve some of our activities. Digital cameras, positioning systems, mobile phones, internet web interfaces are such typical examples which are nowadays completely standard tools. Interconnected with each other, these devices are producing, exchanging or processing digital data in order to interact with the physical world. Computing is becoming ubiquitous and this evolution raises new challenges to represent, analyze and transform this digital information.

From this perspective, geometry is playing an important role. There is a strong interaction between physical and digital worlds through geometric modeling and analysis. Understanding a physical phenomenon can be done by analyzing numerical simulations on a digital representation of the geometry. Conversely developing digital geometry (as in Computer Aided Geometric Design – CAGD for short) is nowadays used to produce devices to overcome some physical difficulties (car, planes, ...). Obviously, geometry is not addressing directly problems related to storage or transmission of information, but it deals with structured and efficient representations of this information and methods to compute with these models.

Within this context, our research program aims at developing new and efficient methods for modeling geometry with algebraic representations. We don't see shapes just as set of points with simple neighbor information. In our investigations, we use richer algebraic models which provide structured and compact representation of the geometry, while being able to encode their important characteristic features.

The first challenge to be addressed is how to move from the digital world to an algebraic world. Our objective is to develop efficient methods which can transform digital data produced by cameras, laser scanners, observations or simulations into algebraic models involving few parameters. This is a way to structure the digital information and to further exploit its properties. This methodological investigations are connected with practical problems such as compression of data for exchange of geometric information, accurate description and simulation with manufactured objects, shape optimization in computer aided design, ...

A second challenge concerns operations and transformations on these algebraic representations. They require the development of dedicated techniques which fully exploit the algebraic characteristics of these representations. The theoretical foundations of our investigations are in algebraic geometry. This domain deals with the solutions of algebraic equations and its effective aspect concerns algorithms to compute and analyze them. It is an old, important and very active part of mathematics. Its combination with algorithmic developments for algebraic computation leads to new methods to treat effectively geometric problems. These investigations result in new contributions in commutative algebra, new algorithms in computer algebra, complexity analyses and/or software development for practical experimentation.

The third challenge is how to analyze and understand digital geometric data. In this approach, constructing algebraic representation and developing methods to compute with these models are the preliminary steps of our analysis process. The goal is to develop methods to extract some type of information we are searching from this data, such as topological descriptions, subdivisions in smooth components and adjacency relations, decomposition in irreducible components. The interplay between algebraic models and numerical computation is central in this activity. A main issue concerns the approximation of models and the certification of the computation.

3. Research Program

3.1. Introduction

Our scientific activity is structured according to three broad topics:

1. **Algebraic representations for geometric modeling**.
2. **Algebraic algorithms for geometric computing**,
3. **Symbolic-numeric methods for analysis**,

3.2. Algebraic representations for geometric modeling

Compact, efficient and structured descriptions of shapes are required in many scientific computations in engineering, such as "Isogeometric" Finite Elements methods, point cloud fitting problems or implicit surfaces defined by convolution. Our objective is to investigate new algebraic representations (or improve the existing ones) together with their analysis and implementations.

We are investigating representations, based on semi-algebraic models. Such non-linear models are able to capture efficiently complex shapes, using few data. However, they required specific methods to solve the underlying non-linear problems, which we are investigating.

Effective algebraic geometry is a natural framework for handling shape representations. This framework not only provides tools for modeling but it also allows to exploit rich geometric properties.

The above-mentioned tools of effective algebraic geometry make it possible to analyse in detail and separately algebraic varieties. We are interested in problems where collections of piecewise algebraic objects are involved. The properties of such geometrical structures are still not well controlled, and the traditional algorithmic geometry methods do not always extend to this context, which requires new investigations.

The use of piecewise algebraic representations also raises problems of approximation and reconstruction, on which we are working on. In this direction, we are studying B-spline function spaces with specified regularity associated to domain partitions.

Many geometric properties are, by nature, independent from the reference one chooses for performing analytic computations. This leads naturally to invariant theory. We are interested in exploiting these invariant properties, to develop compact and adapted representations of shapes.

3.3. Algebraic algorithms for geometric computing

This topic is directly related to polynomial system solving and effective algebraic geometry. It is our core expertise and many of our works are contributing to this area.

Our goal is to develop algebraic algorithms to efficiently perform geometric operations such as computing the intersection or self-intersection locus of algebraic surface patches, offsets, envelopes of surfaces, ...

The underlying representations behind the geometric models we consider are often of algebraic type. Computing with such models raises algebraic questions, which frequently appear as bottlenecks of the geometric problems.

In order to compute the solutions of a system of polynomial equations in several variables, we analyse and take advantage of the structure of the quotient ring defined by these polynomials. This raises questions of representing and computing normal forms in such quotient structures. The numerical and algebraic computations in this context lead us to study new approaches of normal form computations, generalizing the well-known Gröbner bases.

Geometric objects are often described in a parametric form. For performing efficiently on these objects, it can also be interesting to manipulate implicit representations. We consider particular projections techniques based on new resultant constructions or syzygies, which allow to transform parametric representations into implicit ones. These problems can be reformulated in terms of linear algebra. We investigate methods which exploit this matrix representation based on resultant constructions.

They involve structured matrices such as Hankel, Toeplitz, Bezoutian matrices or their generalization in several variables. We investigate algorithms that exploit their properties and their implications in solving polynomial equations.

We are also interested in the "effective" use of duality, that is, the properties of linear forms on the polynomials or quotient rings by ideals. We undertake a detailed study of these tools from an algorithmic perspective, which yields the answer to basic questions in algebraic geometry and brings a substantial improvement on the complexity of resolution of these problems.

We are also interested in subdivision methods, which are able to efficiently localise the real roots of polynomial equations. The specificities of these methods are local behavior, fast convergence properties and robustness. Key problems are related to the analysis of multiple points.

An important issue while developing these methods is to analyse their practical and algorithmic behavior. Our aim is to obtain good complexity bounds and practical efficiency by exploiting the structure of the problem.

3.4. Symbolic numeric analysis

While treating practical problems, noisy data appear and incertitude has to be taken into account. The objective is to devise adapted techniques for analyzing the geometric properties of the algebraic models in this context.

Analysing a geometric model requires tools for structuring it, which first leads to study its singularities and its topology. In many contexts, the input representation is given with some error so that the analysis should take into account not only one model but a neighborhood of models.

The analysis of singularities of geometric models provides a better understanding of their structures. As a result, it may help us better apprehend and approach modeling problems. We are particularly interested in applying singularity theory to cases of implicit curves and surfaces, silhouettes, shadows curves, moved curves, medial axis, self-intersections, appearing in algorithmic problems in CAGD and shape analysis.

The representation of such shapes is often given with some approximation error. It is not surprising to see that symbolic and numeric computations are closely intertwined in this context. Our aim is to exploit the complementarity of these domains, in order to develop controlled methods.

The numerical problems are often approached locally. However, in many situations it is important to give global answers, making it possible to certify computation. The symbolic-numeric approach combining the algebraic and analytical aspects, intends to address these local-global problems. Especially, we focus on certification of geometric predicates that are essential for the analysis of geometrical structures.

The sequence of geometric constructions, if treated in an exact way, often leads to a rapid complexification of the problems. It is then significant to be able to approximate the geometric objects while controlling the quality of approximation. We investigate subdivision techniques based on the algebraic formulation of our problems which allow us to control the approximation, while locating interesting features such as singularities.

According to an engineer in CAGD, the problems of singularities obey the following rule: less than 20% of the treated cases are singular, but more than 80% of time is necessary to develop a code allowing to treat them correctly. Degenerated cases are thus critical from both theoretical and practical perspectives. To resolve these difficulties, in addition to the qualitative studies and classifications, we also study methods of *perturbations* of symbolic systems, or adaptive methods based on exact arithmetics.

The problem of decomposition and factorisation is also important. We are interested in a new type of algorithms that combine the numerical and symbolic aspects, and are simultaneously more effective and reliable. A typical problem in this direction is the problem of approximate factorization, which requires to analyze perturbations of the data, which enables us to break up the problem.

4. Application Domains

4.1. Shape modeling

Geometric modeling is increasingly familiar for us (synthesized images, structures, vision by computer, Internet, ...). Nowadays, many manufactured objects are entirely designed and built by means of geometric software which describe with accuracy the shape of these objects. The involved mathematical models used to represent these shapes have often an algebraic nature. Their treatment can be very complicated, for example requiring the computations of intersections or isosurfaces (CSG, digital simulations, ...), the detection of singularities, the analysis of the topology, etc. Optimizing these shapes with respect to some physical constraints is another example where the choice of the models and the design process are important to lead to interesting problems in algebraic geometric modeling and computing. We propose the development of methods for shape modeling that take into account the algebraic specificities of these problems. We tackle questions whose answer strongly depends on the context of the application being considered, in direct relationship with the industrial contacts that we are developing in Computer Aided Geometric Design.

4.2. Shape processing

Many problems encountered in the application of computer sciences start from measurement data, from which one wants to recover a curve, a surface, or more generally a shape. This is typically the case in image processing, computer vision or signal processing. This also appears in computer biology where the geometry of distances plays a significant role, for example, in the reconstruction from NMR (Nuclear Magnetic Resonance) experiments, or the analysis of realizable or accessible configurations. In another domain, scanners which tend to be more and more easily used yield large set of data points from which one has to recover a compact geometric model. We are working in collaboration with groups in agronomy on the problem of reconstruction of branching models (which represent trees or plants). We are investigating the application of algebraic techniques to these reconstruction problems. Geometry is also highly involved in the numerical simulation of physical problems such as heat conduction, ship hull design, blades and turbines analysis, mechanical stress analysis. We apply our algebraic-geometric techniques in the isogeometric approach which uses the same (B-spline) formalism to represent both the geometry and the solutions of partial differential equations on this geometry.

5. New Software and Platforms

5.1. Mathemagix, a free computer algebra environment

Participant: Bernard Mourrain.

http://www.mathemagix.org/

algebra, univariate polynomial, multivariate polynomial, matrices, series, fast algorithm, interpreter, compiler, hybrid software.

MATHEMAGIX is a free computer algebra system which consists of a general purpose interpreter, which can be used for non-mathematical tasks as well, and efficient modules on algebraic objects. It includes the development of standard libraries for basic arithmetic on dense and sparse objects (numbers, univariate and multivariate polynomials, power series, matrices, etc., based on FFT and other fast algorithms). These developments, based on C++, offer generic programming without losing effectiveness, via the parameterization of the code (*template*) and the control of their instantiations.

The language of the interpreter is imperative, strongly typed and high level. A compiler of this language is available. A special effort has been put on embedding of existing libraries written in other languages like C or C++. An interesting feature is that this extension mechanism supports template types, which automatically induce generic types inside Mathemagix. Connections with GMP, MPFR for extended arithmetic, LAPACK for numerical linear algebra are currently available in this framework.

The project aims at building a bridge between symbolic computation and numerical analysis. It is structured by collaborative software developments of different groups in the domain of algebraic and symbolic-numeric computation.

In this framework, we are working more specifically on the following components:

- REALROOT: a set of solvers using subdivision methods to isolate the roots of polynomial equations in one or several variables; continued fraction expansion of roots of univariate polynomials; Bernstein basis representation of univariate and multivariate polynomials and related algorithms; exact computation with real algebraic numbers, sign evaluation, comparison, certified numerical approximation.
- SHAPE: tools to manipulate curves and surfaces of different types including parameterized, implicit with different type of coefficients; algorithms to compute their topology, intersection points or curves, self-intersection locus, singularities, ...

These packages are integrated from the former library SYNAPS (SYmbolic Numeric APplicationS) dedicated to symbolic and numerical computations. There are also used in the algebraic-geometric modeler AXEL.

Collaborators: Grégoire Lecerf, Joris van der Hoeven and Philippe Trébuchet.

5.2. Axel, a geometric modeler for algebraic objects

Participants: Nicolas Douillet, Anaïs Ducoffe [contact], Valentin Michelet, Bernard Mourrain, Hung Nguyen, Meriadeg Perrinel.

http://axel.inria.fr.

computational algebraic geometry, curve, implicit equation, intersection, parameterization, resolution, surface, singularity, topology

We are developing a software called AXEL (Algebraic Software-Components for gEometric modeLing) dedicated to algebraic methods for curves and surfaces. Many algorithms in geometric modeling require a combination of geometric and algebraic tools. Aiming at the development of reliable and efficient implementations, AXEL provides a framework for such combination of tools, involving symbolic and numeric computations.

The software contains data structures and functionalities related to algebraic models used in geometric modeling, such as polynomial parameterizations, B-splines, implicit curves and surfaces. It provides algorithms for the treatment of such geometric objects, such as tools for computing intersection points of curves or surfaces, for detecting and computing self-intersection points of parameterized surfaces, for implicitization, for computing the topology of implicit curves, for meshing implicit (singular) surfaces, etc.

The developments related to isogeometric analysis have been integrated as dedicated plugins. Optimization techniques and solvers for partial differential equations developed by R. Duvigneau (OPALE) have been connected.

The new version of the algebraic-geometric modelers based on the DTK platform is still developed in order to provide a better modularity and a better interface to existing computation facilities and geometric rendering interface. This software is intended to be multi-platform, and jobs are running nightly on the Continous Integration platform https://ci.inria.fr/ of Inria, performing builds and tests on Virtual Machines of different OS such as Fedora, Ubuntu, Windows.

AXEL is written in C++ and thanks to a wrapping system using SWIG, its data structures and algorithms can be integrated into C# programs, as well as Python and Java programs. This wrapper was used to integrate AXEL into the CAD software TopSolid, developed by Missler Company and written in C#. But it also enables AXEL to embed a Python interpreter.

Other functionalities were also added or improved: the scientific visualization was improved and it is now possible to create dynamic geometric model in AXEL.

The software is distributed as a source package, as well as binary packages for Linux, MacOSX and Windows. It is hosted at http://dtk.inria.fr/axel with some of its plugins developed on Inria's gforge server (http://gforge.inria.fr) The first version of the software has been downloaded more than 15000 times, since it is available. A new version, AXEL 2.3.1, was released at the end of this year.

Collaboration with Gang Xu (Hangzhou Dianzi University, China), Julien Wintz (Dream), Elisa Berrini (MyCFD, Sophia), Angelos Mantzaflaris (GISMO library, Linz, Austria) and Laura Saini (Post-Doc GALAAD/Missler, TopSolid).

6. New Results

6.1. Algebraic representations for geometric modeling

6.1.1. A comparison of different notions of ranks of symmetric tensors

Participants: Alessandra Bernardi, Jérome Brachat, Bernard Mourrain.

In [2], we introduce various notions of rank for a symmetric tensor, namely: rank, border rank, catalecticant rank, generalized rank, scheme length, border scheme length, extension rank and smoothable rank. We analyze the stratification induced by these ranks. The mutual relations between these stratifications, allow us to describe the hierarchy among all the ranks. We show that strict inequalities are possible between rank, border rank, extension rank and catalecticant rank. Moreover we show that scheme length, generalized rank and extension rank coincide.

6.1.2. Dimensions and bases of hierarchical tensor-product splines

Participant: Bernard Mourrain.

In [1], we prove that the dimension of trivariate tensor-product spline space of tri-degree (m,m,m) with maximal order of smoothness over a three-dimensional domain coincides with the number of tensor-product B-spline basis functions acting effectively on the domain considered. A domain is required to belong to a certain class. This enables us to show that, for a certain assumption about the configuration of a hierarchical mesh, hierarchical B-splines span the spline space. This paper presents an extension to three-dimensional hierarchical meshes of results proposed recently by Giannelli and Jüttler for two-dimensional hierarchical meshes.

Joint work with Dmitry Berdinsky, Taiwan Kim, Oh Min-Jae, Sutipong Kiatpanichgij (Department of Naval Architecture and Ocean Engineering, Seoul, South Korea), Cesare Bracco (Dipartimento di Matematica "Giuseppe Peano", Torino, Italy), Durkbin Cho (Department of Mathematics, Dongguk, South Korea).

6.1.3. Bounds on the dimension of trivariate spline spaces: A homological approach

Participant: Bernard Mourrain.

In [8], we consider the vector space of globally differentiable piecewise polynomial functions defined on a three-dimensional polyhedral domain partitioned into tetrahedra. We prove new lower and upper bounds on the dimension of this space by applying homological techniques. We give an insight of different ways of approaching this problem by exploring its connections with the Hilbert series of ideals generated by powers of linear forms, fat points, the so-called Fröberg–Iarrobino conjecture, and the weak Lefschetz property.

Joint work with Nelly Villamizar (RICAM - Johann Radon Institute for Computational and Applied Mathematics, Linz, Austria)

6.1.4. High-quality construction of analysis-suitable trivariate NURBS solids by reparameterization methods

Participants: André Galligo, Bernard Mourrain.

High-quality volumetric parameterization of computational domain plays an important role in three-dimensional isogeometric analysis. Reparameterization techniques can improve the distribution of isoparametric curves/surfaces without changing the geometry. In [10], using the reparameterization method, we investigate the high-quality construction of analysis-suitable NURBS volumetric parameterization. Firstly, we introduce the concept of volumetric reparameterization, and propose an optimal Möbius transformation to improve the quality of the isoparametric structure based on a new uniformity metric. Secondly, from given boundary NURBS surfaces, we present a two-stage scheme to construct the analysis-suitable volumetric parameterization: in the first step, uniformity-improved reparameterization is performed on the boundary surfaces to achieve high-quality isoparametric structure without changing the shape; in the second step, from a new variational harmonic metric and the reparameterized boundary surfaces, we construct the optimal inner control points and weights to achieve an analysis-suitable NURBS solid. Several examples with complicated geometry are presented to illustrate the effectiveness of proposed methods.

Joint work with Gang Xu (College of computer - Hangzhou Dianzi University, China), Timon Rabczuk (Bauhaus-Universität Weimar, Germany).

6.1.5. Spline Spaces over Quadrangle Meshes with Complex Topologies

Participants: André Galligo, Bernard Mourrain, Meng Wu.

Motivated by Magneto Hydrodynamic (MHD) simulation with isoparametric elements method, we pursue our work on new types of spline functions defined over a quadrangular mesh, that can follow isobaric curves with node singularities. The practicability of these splines is analyzed for different geometries related to MHD simulation.

This work is done in collaboration with Boniface Nkonga (Inria, EPI CASTOR and University of Nice).

6.1.6. Parametric modeling for ship hull deformation

Participant: Elisa Berrini.

The objective of the work is to develop a parametric modeler tool, allowing consistent ship hull deformations with respect to classic naval architecture design constraints. This work will be applied in automatic shape optimization process. Two scientific problematics are addressed : 1) The parametrization of the hull: the numerical representation of the shape from a defined set of parameters; 2) The deformations of curves and surfaces: getting a new shape by modifying chosen parameters from the parameterization set. The consistency with naval architecture constraints is essential.

To produce realistic models, we want to use methods similar to naval architects' ones. The approach under development is based on the extraction and deformation of skeletons curves.

6.2. Algebraic algorithms for geometric computing

6.2.1. Resultant of an equivariant polynomial system with respect to the symmetric group

Participants: Laurent Busé, Anna Karasoulou.

Given a system of n homogeneous polynomials in n variables which is equivariant with respect to the canonical actions of the symmetric group of n symbols on the variables and on the polynomials, it is proved that its resultant can be decomposed into a product of several smaller resultants that are given in terms of some divided differences. As an application, we obtain a decomposition formula for the discriminant of a multivariate homogeneous symmetric polynomial.

This work is submitted for publication [14].

6.2.2. Delaunay Mesh Generation of NURBS Surfaces

Participant: Laurent Busé.

We introduce a method for isotropic triangle meshing of NURBS surfaces. Based on Delaunay filtering and refinement, our approach departs from previous work by meshing in embedding space instead of parametric space. The meshing engine relies upon a novel line/surface intersection test, based on the matrix-based implicit representation of NURBS surfaces and numerical methods in linear algebra such as singular value and eigenvalue decompositions. A careful treatment of degenerate cases makes our approach robust to intersection points with multiple pre-images. In addition to ensure both approximation accuracy and mesh quality, our approach is seamless as it does not depend on the initial decomposition into NURBS patches, and is oblivious to the parameterization of the patches. Removing such dependencies provides us with a means to reliably mesh across patches with greater control over mesh sizing and shape of the elements.

This work was done in collaboration with Jingjing Shen and Neil Dodgson from Cambridge University and Pierre Alliez from TITANE.

6.2.3. Toric Border Basis

Participant: Bernard Mourrain.

In [11], we extend the theory and the algorithms of Border bases to systems of Laurent polynomial equations, defining "toric" roots. Instead of introducing new variables and new relations to saturate by the variable inverses, we propose a more efficient approach which works directly with the variables and their inverse. We show that the commutation relations and the inversion relations characterize toric border bases. We explicitly describe the first syzygy module associated to a toric border basis in terms of these relations. Finally, a new border basis algorithm for Laurent polynomials is described and a proof of its termination is given for zero-dimensional toric ideals.

Joint work with Philippe Trébuchet (LIP6 - UPMC).

6.2.4. Border Basis relaxation for polynomial optimization

Participants: Marta Abril-Bucero, Bernard Mourrain.

A relaxation method based on border basis reduction which improves the efficiency of Lasserre's approach is proposed to compute the optimum of a polynomial function on a basic closed semi-algebraic set. A new stopping criterion is given to detect when the relaxation sequence reaches the minimum, using a sparse flat extension criterion. We also provide a new algorithm to reconstruct a finite sum of weighted Dirac measures from a truncated sequence of moments, which can be applied to other sparse reconstruction problems. As an application, we obtain a new algorithm to compute zero-dimensional minimizer ideals and the minimizer points or zero-dimensional G-radical ideals. Experimentations show the impact of this new method on significant benchmarks. See [12].

6.2.5. *Flat extensions in $*$-algebra*

Participant: Bernard Mourrain.

The objective of this work is to develop a flat extension characterization on moment matrices in the non-commutative case. We give a flat extension theorem for positive linear functionals on $_*$-algebras. The theorem is applied to truncated moment problems on cylinder sets, on matrices of polynomials and on enveloping algebras of Lie algebras. See [17].

Joint work with Konrad Schmüdgen, University of Leipzig, Germany.

6.3. Symbolic-Numeric Analysis

6.3.1. *Cubatures, and related problems, with symmetry*

Participants: Mathieu Collowald, Evelyne Hubert.

We address the computation of cubature formulae as a moment problem. Symmetry by finite groups arise naturally for cubatures. We developed the algebraic results to use the symmetry in order to reduce the number of parameters and the size of the matrices involved in the flat extension.

6.3.2. *Quantitative Equidistribution for the Solutions of Systems of Sparse Polynomial Equations*

Participant: André Galligo.

For a system of Laurent polynomials $f_1, ..., f_n \in \mathbb{C}[x_1^{\pm 1}, ..., x_n^{\pm 1}]$ whose coefficients are not too big with respect to its directional resultants, we show in [6] that the solutions in the algebraic torus $(\mathbb{C}^*)^n$ of the system of equations $f_1 = \cdots = f_n = 0$, are approximately equidistributed near the unit polycircle. This generalizes to the multivariate case a classical result due to Erdös and Turán on the distribution of the arguments of the roots of a univariate polynomial. We apply this result to bound the number of real roots of a system of Laurent polynomials, and to study the asymptotic distribution of the roots of systems of Laurent polynomials over $\mathbb{Z}$ and of random systems of Laurent polynomials over $\mathbb{C}$.

Joint work with Carlos D'Andrea (DM-UBA - Departamento de Matemática, Spain), Martin Sombra (ICREA & Universitat de Barcelona, Spain).

7. Bilateral Contracts and Grants with Industry

7.1. Bilateral Grants with Industry

7.1.1. *Algebraic-geometric methods for design and manufacturing*

This collaboration between Inria and Missler in the context of Carnot program, aims at developing algebraic-geometric computational techniques for the control of machining tools. It focuses on the problem of pocket manufacturing and the computation of medial axis and of offsets of planar regions with piecewise algebraic boundaries. An integration of plugins related to AXEL platform into the CAGD modeler TOPSOLID developed by Missler is planned. Laura Saini is involved in this collaboration.

8. Partnerships and Cooperations

8.1. National Initiatives

8.1.1. GEOLMI

GEOLMI - Geometry and Algebra of Linear Matrix Inequalities with Systems Control Applications - is an ANR project working on topics related to the Geometry of determinantal varieties, positive polynomials, computational algebraic geometry, semidefinite programming and systems control applications.

The partners are LAAS-CNRS, Univ. de Toulouse (coordinator), LJK-CNRS, Univ. Joseph Fourier de Grenoble; Inria Sophia Antipolis Méditerranée; LIP6-CNRS Univ. Pierre et Marie Curie; Univ. de Pau et des Pays de l'Adour; IRMAR-CNRS, Univ. de Rennes.

More information available at http://homepages.laas.fr/henrion/geolmi.

8.1.2. ANEMOS

ANEMOS - Advanced Numeric for ELMs (Edge Localized Mode) : Modeling and Optimized Schemes - is an ANR project devoted to the numerical modelling study of such ELM control methods as Resonant Magnetic Perturbations (RMPs) and pellet ELM pacing both foreseen in ITER. The goals of the project are to improve understanding of the related physics and propose possible new strategies to improve effectiveness of ELM control techniques. The study of spline spaces for isogemetric finite element methods is proposed in this context.

The partners are IRFM, CEA, Cadarache; JAD, University of Nice - Sophia Antipolis; Inria, Bacchus; Maison de la Simulation CEA-CNRS-Inria-University of Orsay- University of Versailles St Quentin.

8.2. European Initiatives

8.2.1. FP7 & H2020 Projects

8.2.1.1. TERRIFIC

Title: Towards Enhanced Integration of Design and Production in the Factory of the Future through Isogeometric Technologies

Type: COOPERATION (ICT)

Defi: PPP FoF: Digital factories: Manufactoring design and product lifecycle manage

Instrument: Specific Targeted Research Project (STREP)

Duration: September 2011 - August 2014

Coordinator: SINTEF, Oslo (Norway)

Others partners:

Alenia Aeronautica (Italy); Inria Méditerranée (France); Jozef Kepler universitet, Linz (Austria); JOTNE, Oslo (Norway); MAGNA, Steyr (Austria); Missler Software (France); Siemens AG (Germany); Technische Universität Kaiserslautern (Germany); University of Pavia (Italy).

See also: http://terrific-project.eu

Abstract: The project aims at significant improvement of the interoperability of computational tools for the design, analysis and optimization of functional products. An isogeometric approach is applied for selected manufacturing application areas (cars, trains, aircrafts) and for computer-aided machining. Computer Aided Design (CAD) and numerical simulation algorithms are vital technologies in modern product development, yet they are today far from being seamlessly integrated. Their interoperability is severely disturbed by inconsistencies in the mathematical approaches used. Efficient feedback from analysis to CAD and iterative refinement of the analysis model is a feature of isogeometric analysis, and would be an essential improvement for computer-based design optimization and virtual product development. Our vision is to provide and disseminate tangible evidence of the performance of the isogeometric approach in comparison to traditional ones in four important application areas as well as addressing interoperability and other issues that necessarily arise in a large-scale industrial introduction of isogeometry.

8.3. International Initiatives

8.3.1. Participation In other International Programs

We have a bilateral collaboration between Galaad and the University of Athens-DIT team ERGA, headed by Ioannis Emiris for the period August 2013-August 2014. It is supported by both Inria and the University of Athens.

Title: Algebraic algorithms in optimization

Abstract: In the past decade, algebraic approaches to optimization problems defined in terms of multivariate polynomials have been intensively explored and studied in several directions. One example is the work on semidefinite optimization and, more recently, convex algebraic geometry. This project aims to focus on algebraic approaches for optimization applications in the wide sense. We concentrate on specific tools, namely root counting techniques, the resultant, the discriminant and non-negative polynomials, on which the two teams have extensive collaboration and expertise. We examine applications in convex algebraic geometry as well as to a newer topic for the two teams, namely game theory. A common thread to these approaches is to exploit any (sparse) structure.

We participate to a bilateral collaboration between France and Spain which is supported as a PICS from CNRS. The Spanish partner is the University of Barcelona (J. Burgos, C. D'Andrea, Martin Sombra) and the French partners are The university of Caen (F. Amoroso, M. Weimann), the University of Paris 6 (M. Chardin, P. Philippon) and GALAAD (L. Busé).

Title: Diophantine Geometry and Computer Algebra

Abstract: This project aims at exploring interactions between diophantine geometry and computer algebra by stimulating collaborations between experts in both domains. The research program focus on five particular topics : toric varieties and height, equidistribution, Diophantine geometry and complexity, Factorization of multivariate polynomials by means of toric geometry and study of singularities of toric parameterizations.

8.4. International Research Visitors

8.4.1. Visits of International Scientists

Chandrajit Bajaj, professor at University of Austin, Texas, USA, September 14-28.

Nicolàs Botbol, researcher CONICET, University of Buenos Aires, Argentina, March 10-23.

Philippe Trébuchet, LIP6, University of Paris 6, France, May 4-11.

Nelly Villamizar, researcher at RICAM, University of Linz, Austria, February 19-26.

8.4.2. Visits to International Teams

8.4.2.1. Research stays abroad

Evelyne Hubert was invited to participate to the program on *Inverse Moment Problems: the Crossroads of Analysis, Algebra, Discrete Geometry and Combinatorics* at the Institute for Mathematical Science at the National University of Singapore (December 1013 - January 2014).

9. Dissemination

9.1. Promoting Scientific Activities

9.1.1. Scientific events organisation

9.1.1.1. Member of the organizing committee

Evelyne Hubert was invited to organize the workshop *Symbolic Analysis* at the conference Foundation of Computational Mathematics in Montevideo (December 2014).

9.1.2. Scientific events selection

9.1.2.1. Member of the conference program committee

Evelyne Hubert was a member of the program committee of the ACM conference ISSAC 2014 in Kobe (July 2014).

Bernard Mourrain was a member of the program committee of the ACM conference SNC 2014 in Shanghai (July 2014).

9.1.3. Journal

9.1.3.1. Member of the editorial board

Evelyne Hubert is an associate editor of the Journal of Symbolic Computation (since 2007).

Bernard Mourrain is an associate editor of the Journal of Symbolic Computation.

9.1.3.2. Reviewer

Laurent Busé wrote reviews for the ISSAC 2014 conference and for the following journals: Graphical Models, Linear Algebra and its Applications, Foundations of Computational Mathematics, Transactions on Visualization and Computer Graphics, Computer Aided Geometric Design, Journal of Algebra, Expert Systems with Applications, Proceedings of the London Mathematical Society, Journal of Symbolic Computation and Vietnam Journal of Mathematics. He also wrote reviews for the NSA Mathematical Sciences Grant Program.

Evelyne Hubert wrote reviews for the Journal of Symbolic computation, the Foundation of Mathematical Computing journal, the Journal of Pure and Applied Algebra, Mathematics of Computations, Plos One, Theoretical Computer Science, and for the conference proceedings of ISSAC and SNC. She was also part of the selection committee for Fulbright grants.

Bernard Mourrain wrote reviews for the followings journals: Journal of Symbolic computation, Foundation of Mathematical Computing journal, Journal of Pure and Applied Algebra, Linear Algebra and its Applications, Applicable Algebra in Engineering, Communication and Computing, Computer Aided Geometric Design, Theoretical Computer Science, Computing Surveys, SIAM Journal on Matrix Analysis and Applications, and for the conference proceedings of ISSAC and SNC, MTNS.

9.2. Teaching - Supervision - Juries

9.2.1. Teaching

Licence: Mathieu Collowald, Maths (Exercices), 25h ETD, L1 SV, University of Nice-Sophia Antipolis.

Licence: Mathieu Collowald, Discrete maths (Exercices), 18h ETD, L1 I, University of Nice-Sophia Antipolis.

Licence: Mathieu Collowald, Algebra1 (Exercices), 40h ETD, L1 I, University of Nice-Sophia Antipolis.

Licence: Mathieu Collowald, Algebra2 (Exercices), 40h ETD, L1 I, University of Nice-Sophia Antipolis.

Licence: Marta Abril Bucero, Statistiques (Exercices), 20h ETD, L2 I, University of Nice-Sophia Antipolis.

Licence: Marta Abril Bucero, Analyse (Exercices), 40h ETD, L2 PC, University of Nice-Sophia Antipolis.

Master : Laurent Busé, Curves and Surfaces, 66h ETD, M1, EPU of the university of Nice-Sophia Antipolis.

Master : Laurent Busé, Elimation matrices for geometric modeling, 40h ETD, M2, math department of the university of Nice-Sophia Antipolis.

Master : Laurent Busé, Elimination matrices and applications, 18h ETD, M2, math department of the university of Buenos Aires, Argentina.

Master: Bernard Mourrain, Algorithmique des Courbes et Surfaces, M2 ACSYON, Limoges. 6h.

Doctoral courses: Bernard Mourrain, Effective Algebraic Geometry and Applications. Central European University, Budapest. 12h.

Doctoral courses: Bernard Mourrain, Sparse modelling, tensor decomposition, algorithms and applications. Summer School on An Interdisciplinary Approach to Tensor Decomposition, Trento Italy. 6h.

9.2.2. Supervision

PhD : Marta Abril Bucero, Moment matrices, real algebraic geometry and polynomial optimization, supervised by Bernard Mourrain, university of Nice-Sophia Antipolis, defended December 12th.

PhD in progress: Elisa Berrini, Parametric modeling for ship hull deformation and optimization. CIFRE with MyCFD. Started in January 2014. Supervised by Bernard Mourrain.

PhD in progress : Mathieu Collowald, Integral representation of shapes for feature conservation or extraction, started in 2011, supervised by Evelyne Hubert.

Laurent Busé supervised :

- The master thesis of Danny Schmitt, “Représentations matricielles des courbes et hypersurfaces rationnelles”, university of Nice-Sophia Antipolis,
- Anna Karasoulou, 3 months during its PhD thesis at the university of Athens under the supervision of I. Emiris, “Discriminant of a symmetric homogeneous polynomial”.
- Jingjing Shen, co-supervised with P. Alliez, 3 months during its PhD thesis at Cambridge under N. Dodgson, “Isotropic triangle meshing of nurbs models”.

9.2.3. Juries

Laurent Busé was a reviewer and a member of the PhD committee of Yacine Bouzidi, “Résolution de systèmes bivariés et toplogie de courbes planes” under Sylvain Lazard and Marc Pouget, defended March 18th 2014 at the university of Lorraine (Inria Nancy), France.

Laurent Busé was a member of the PhD committee of Claire Tête, “Profondeur, dimension et résolutions en algèbre commutative: quelques aspects effectifs” under Lionel Ducos, defended October 21st 2014 at the university of Poitiers, France.

Laurent Busé was a reviewer and a member of the PhD committee of Kamran Lamei, “Fonction de Hilbert non standard et nombres de Betti gradués des puissances d’idéaux” under Marc Chardin, defended December 18th 2014 at the university Paris 6, France.

Bernard Mourrain was a reviewer and member of the PhD committee of Jeremy Veysset, “Anisotropic mesh adaptation and stabilized finite elements methods for solving conjugate heat transfers and turbulent flows” supervised by E. Hachem and Th. Coupez, September 29th, Mines ParisTech.

Bernard Mourrain was a reviewer and member of the PhD committee of Heidi Dahl, “Improved blends between primitive surface” under the supervision of Rimvydas Krasauskas, October 23th, University of Oslo.

Bernard Mourrain was a reviewer and member of the PhD committee of Jules Svartz, “Résolution des Systèmes Polynomiaux de Dimension Zéro” under the supervision of J.C. Faugère, October 30th, University Pierre et Marie Curie, Paris.

Bernard Mourrain was supervisor and member of the PhD committee of the PhD thesis of Marta Abril-Bucero, “Moment matrices, real algebraic geometry and polynomial optimization”, December 12th, University of Nice-Sophia Antipolis.

10. Bibliography

Publications of the year

Articles in International Peer-Reviewed Journals

[1] D. BERDINSKY, T. KIM, C. BRACCO, D. CHO, B. MOURRAIN, O. MIN-JAE, S. KIATPANICHGIJ. *Dimensions and bases of hierarchical tensor-product splines*, in "Journal of Computational and Applied Mathematics", February 2014, vol. 257, pp. 86-104 [*DOI :* 10.1016/J.CAM.2013.08.019], https://hal.inria.fr/hal-00876557

[2] A. BERNARDI, J. BRACHAT, B. MOURRAIN. *A comparison of different notions of ranks of symmetric tensors*, in "Linear Algebra and its Applications", November 2014, vol. 460, pp. 205–230, https://hal.inria.fr/hal-00746967

[3] N. BOTBOL, L. BUSÉ, M. CHARDIN. *Fitting ideals and multiple points of surface parameterizations*, in "Journal of Algebra", December 2014, vol. 420, pp. 486-508 [*DOI :* 10.1016/J.JALGEBRA.2014.07.028], https://hal.inria.fr/hal-00874221

[4] L. BUSÉ. *Implicit matrix representations of rational Bézier curves and surfaces*, in "Computer-Aided Design", 2014, vol. 46, pp. 14-24, Presented at the SIAM conference of Geometric and Physical Modeling (GD/SPM), November 11-14, 2013 at Denver, Colorado, USA [*DOI :* 10.1016/J.CAD.2013.08.014], https://hal.inria.fr/hal-00847802

[5] L. BUSÉ, J.-P. JOUANOLOU. *On the discriminant scheme of homogeneous polynomials*, in "Mathematics in Computer Science", June 2014, vol. 8, n° 2, pp. 175-234, arXiv reference : arXiv:1210.4697 [*DOI :* 10.1007/S11786-014-0188-7], https://hal.inria.fr/hal-00747930

[6] A. GALLIGO, C. D'ANDREA, M. SOMBRA. *Quantitative Equidistribution for the Solutions of Systems of Sparse Polynomial Equations*, in "American Journal of Mathematics", 2014, https://hal.inria.fr/hal-00918250

[7] B. MOURRAIN. *On the dimension of spline spaces on planar T-meshes*, in "Mathematics of Computation", March 2014, vol. 83, pp. 847-871, https://hal.inria.fr/inria-00533187

[8] B. MOURRAIN, N. VILLAMIZAR. *Bounds on the dimension of trivariate spline spaces: A homological approach*, in "Mathematics in Computer Science", June 2014, vol. 8, n° 2, pp. 157-174 [*DOI :* 10.1007/S11786-014-0187-8], https://hal.inria.fr/hal-00879100

[9] E. MUSSO, E. HUBERT. *Lagrangian Curves in Affine Symplectic 4-space*, in "Acta Applicandae Mathematicae", December 2014, vol. 134, n° 1, pp. 133-160, https://hal.inria.fr/hal-00818823

[10] G. XU, B. MOURRAIN, A. GALLIGO, T. RABCZUK. *High-quality construction of analysis-suitable trivariate NURBS solids by reparameterization methods*, in "Computational Mechanics", November 2014, vol. 54, n° 5, pp. 1303-1313 [*DOI :* 10.1007/S00466-014-1060-Y], https://hal.archives-ouvertes.fr/hal-00922544

International Conferences with Proceedings

[11] B. MOURRAIN, P. TREBUCHET. *Toric Border Basis*, in "Proceedings of the 39th International Symposium on Symbolic and Algebraic Computation (ISSAC)", Kobe, Japan, ACM New York, NY, USA, July 2014, pp. 343-350 [*DOI :* 10.1145/2608628.2608652], https://hal.inria.fr/hal-00994683

Other Publications

[12] M. ABRIL BUCERO, B. MOURRAIN. *Border Basis relaxation for polynomial optimization*, June 2014, https://hal.inria.fr/hal-00981546

[13] M. ABRIL BUCERO, B. MOURRAIN. *Exact relaxation for polynomial optimization on semi-algebraic sets*, July 2014, https://hal.inria.fr/hal-00846977

[14] L. BUSÉ, A. KARASOULOU. *Resultant of an equivariant polynomial system with respect to the symmetric group*, July 2014, https://hal.inria.fr/hal-01022345

[15] I. EMIRIS, B. MOURRAIN, E. TSIGARIDAS. *Separation bounds for polynomial systems*, January 2015, https://hal.inria.fr/hal-01105276

[16] J. D. HAUENSTEIN, B. MOURRAIN, A. SZANTO. *Certifying isolated singular points and their multiplicity structure*, January 2015, https://hal.inria.fr/hal-01107541

[17] B. MOURRAIN, K. SCHMÜDGEN. *Flat extensions in $_*$-algebras*, June 2014, https://hal.inria.fr/hal-01009909

[18] M. WU, B. MOURRAIN, A. GALLIGO, B. NKONGA. *Spline Spaces over Quadrangle Meshes with Complex Topologies*, February 2014, https://hal.inria.fr/hal-00952455