Activity Report 2015

Project-Team APICS

Analysis and Problems of Inverse type in Control and Signal processing

RESEARCH CENTER
Sophia Antipolis - Méditerranée

THEME
Optimization and control of dynamic systems

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Project-Team APICS

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2. Overall Objectives

2.1. Research Themes

The team develops constructive, function-theoretic approaches to inverse problems arising in modeling and design, in particular for electro-magnetic systems as well as in the analysis of certain classes of signals.

Data typically consist of measurements or desired behaviors. The general thread is to approximate them by families of solutions to the equations governing the underlying system. This leads us to consider various interpolation and approximation problems in classes of rational and meromorphic functions, harmonic gradients, or solutions to more general elliptic partial differential equations (PDE), in connection with inverse potential problems. A recurring difficulty is to control the singularities of the approximants.

The mathematical tools pertain to complex and harmonic analysis, approximation theory, potential theory, system theory, differential topology, optimization and computer algebra. Targeted applications include:

- identification and synthesis of analog microwave devices (filters, amplifiers),
- non-destructive control from field measurements in medical engineering (source recovery in magneto/electro-encephalography), and paleomagnetism (determining the magnetization of rock samples).

In each case, the endeavor is to develop algorithms resulting in dedicated software.

3. Research Program

3.1. Introduction

Within the extensive field of inverse problems, much of the research by Apics deals with reconstructing solutions of classical elliptic PDEs from their boundary behavior. Perhaps the simplest example lies with harmonic identification of a stable linear dynamical system: the transfer-function f can be evaluated at a point $i\omega$ of the imaginary axis from the response to a periodic input at frequency ω. Since f is holomorphic in the right half-plane, it satisfies there the Cauchy-Riemann equation $\overline{\partial} f = 0$, and recovering f amounts to solve a Dirichlet problem which can be done in principle using, *e.g.* the Cauchy formula.

Practice is not nearly as simple, for f is only measured pointwise in the pass-band of the system which makes the problem ill-posed [67]. Moreover, the transfer function is usually sought in specific form, displaying the necessary physical parameters for control and design. For instance if f is rational of degree n, then $\overline{\partial} f = \sum_1^n a_j \delta_{z_j}$ where the z_j are its poles and δ_{z_j} is a Dirac unit mass at z_j. Thus, to find the domain of holomorphy (*i.e.* to locate the z_j) amounts to solve a (degenerate) free-boundary inverse problem, this time on the left half-plane. To address such questions, the team has developed a two-step approach as follows.

Step 1: To determine a complete model, that is, one which is defined at every frequency, in a sufficiently versatile function class (*e.g.* Hardy spaces). This ill-posed issue requires regularization, for instance constraints on the behavior at non-measured frequencies.

Step 2: To compute a reduced order model. This typically consists of rational approximation of the complete model obtained in step 1, or phase-shift thereof to account for delays. We emphasize that deriving a complete model in step 1 is crucial to achieve stability of the reduced model in step 2.

Step 1 relates to extremal problems and analytic operator theory, see Section 3.3.1. Step 2 involves optimization, and some Schur analysis to parametrize transfer matrices of given Mc-Millan degree when dealing with systems having several inputs and outputs, see Section 3.3.2.2. It also makes contact with the topology of rational functions, in particular to count critical points and to derive bounds, see Section 3.3.2. Step 2 raises further issues in approximation theory regarding the rate of convergence and the extent to which singularities of the approximant (*i.e.* its poles) tend to singularities of the approximated function; this is where logarithmic potential theory becomes instrumental, see Section 3.3.3.

Applying a realization procedure to the result of step 2 yields an identification procedure from incomplete frequency data which was first demonstrated in [73] to tune resonant microwave filters. Harmonic identification of nonlinear systems around a stable equilibrium can also be envisaged by combining the previous steps with exact linearization techniques from [30], see [14].

A similar path can be taken to approach design problems in the frequency domain, replacing the measured behavior by some desired behavior. However, describing achievable responses in terms of the design parameters is often cumbersome, and most constructive techniques rely on specific criteria adapted to the physics of the problem. This is especially true of filters, the design of which traditionally appeals to polynomial extremal problems [69], [53]. Apics contributed to this area the use of Zolotarev-like problems for multi-band synthesis, although we presently favor interpolation techniques in which parameters arise in a more transparent manner, see Section 3.2.2.

The previous example of harmonic identification quickly suggests a generalization of itself. Indeed, on identifying $\mathbb{C}$ with $\mathbb{R}^2$, holomorphic functions become conjugate-gradients of harmonic functions, so that harmonic identification is, after all, a special case of a classical issue: to recover a harmonic function on a domain from partial knowledge of the Dirichlet-Neumann data; when the portion of boundary where data are not available is itself unknown, we meet a free boundary problem. This framework for 2-D non-destructive control was first advocated in [58] and subsequently received considerable attention. It makes clear how to state similar problems in higher dimensions and for more general operators than the Laplacian, provided solutions are essentially determined by the trace of their gradient on part of the boundary which is the case for elliptic equations [1] [20], [76]. Such questions are particular instances of the so-called inverse potential problem, where a measure μ has to be recovered from the knowledge of the gradient of its potential (*i.e.*, the field) on part of a hypersurface (a curve in 2-D) encompassing the support of μ. For Laplace's operator, potentials are logarithmic in 2-D and Newtonian in higher dimensions. For elliptic operators with non constant coefficients, the potential depends on the form of fundamental solutions and is less manageable because it is no longer of convolution type. Nevertheless it is a useful concept bringing perspective on how problems could be raised and solved, using tools from harmonic analysis.

Inverse potential problems are severely indeterminate because infinitely many measures within an open set produce the same field outside this set; this phenomenon is called *balayage* [66]. In the two steps approach previously described, we implicitly removed this indeterminacy by requiring in step 1 that the measure be supported on the boundary (because we seek a function holomorphic throughout the right half-space), and by requiring in step 2 that the measure be discrete in the left half-plane (in fact: a sum of point masses $\sum_1^n a_j\delta_{z_j}$). The discreteness assumption also prevails in 3-D inverse source problems, see Section 4.2. Conditions that ensure uniqueness of the solution to the inverse potential problem are part of the so-called regularizing assumptions which are needed in each case to derive efficient algorithms.

To recap, the gist of our approach is to approximate boundary data by (boundary traces of) fields arising from potentials of measures with specific support. This differs from standard approaches to inverse problems, where descent algorithms are applied to integration schemes of the direct problem; in such methods, it is the equation which gets approximated (in fact: discretized).

Along these lines, Apics advocates the use of steps 1 and 2 above, along with some singularity analysis, to approach issues of nondestructive control in 2-D and 3-D [4], [2], [37]. The team is currently engaged in the generalization to inverse source problems for the Laplace equation in 3-D, to be described further in Section 3.2.1. There, holomorphic functions are replaced by harmonic gradients; applications are to EEG/MEG and inverse magnetization problems in Geosciences, see Section 4.2.

[1] There is a subtle difference here between dimension 2 and higher. Indeed, a function holomorphic on a plane domain is defined by its non-tangential limit on a boundary subset of positive linear measure, but there are non-constant harmonic functions in the 3-D ball, C^1 up to the boundary sphere, yet having vanishing gradient on a subset of positive measure of the sphere. Such a "bad" subset, however, cannot have interior points on the sphere.

The approximation-theoretic tools developed by Apics to handle issues mentioned so far are outlined in Section 3.3. In Section 3.2 to come, we describe in more detail which problems are considered and which applications are targeted.

3.2. Range of inverse problems

3.2.1. Elliptic partial differential equations (PDE)

Participants: Laurent Baratchart, Sylvain Chevillard, Juliette Leblond, Konstantinos Mavreas, Christos Papageorgakis, Dmitry Ponomarev.

By standard properties of conjugate differentials, reconstructing Dirichlet-Neumann boundary conditions for a function harmonic in a plane domain, when these conditions are already known on a subset E of the boundary, is equivalent to recover a holomorphic function in the domain from its boundary values on E. This is the problem raised on the half-plane in step 1 of Section 3.1. It makes good sense in holomorphic Hardy spaces where functions are entirely determined by their values on boundary subsets of positive linear measure, which is the framework for Problem (P) that we set up in Section 3.3.1. Such issues naturally arise in nondestructive testing of 2-D (or 3-D cylindrical) materials from partial electrical measurements on the boundary. For instance, the ratio between the tangential and the normal currents (the so-called Robin coefficient) tells one about corrosion of the material. Thus, solving Problem (P) where ψ is chosen to be the response of some uncorroded piece with identical shape yields non destructive testing of a potentially corroded piece of material, part of which is inaccessible to measurements. This was an initial application of holomorphic extremal problems to non-destructive control [51], [54].

Another application by the team deals with non-constant conductivity over a doubly connected domain, the set E being now the outer boundary. Measuring Dirichlet-Neumann data on E, one wants to recover level lines of the solution to a conductivity equation, which is a so-called free boundary inverse problem. For this, given a closed curve inside the domain, we first quantify how constant the solution on this curve. To this effect, we state and solve an analog of Problem (P), where the constraint bears on the real part of the function on the curve (it should be close to a constant there), in a Hardy space of a conjugate Beltrami equation, of which the considered conductivity equation is the compatibility condition (just like the Laplace equation is the compatibility condition of the Cauchy-Riemann system). Subsequently, a descent algorithm on the curve leads one to improve the initial guess. For example, when the domain is regarded as separating the edge of a tokamak's vessel from the plasma (rotational symmetry makes this a 2-D situation), this method can be used to estimate the shape of a plasma subject to magnetic confinement. This was actually carried out in collaboration with CEA (French nuclear agency) and the University of Nice (JAD Lab.), to data from *Tore Supra* [57]. The procedure is fast because no numerical integration of the underlying PDE is needed, as an explicit basis of solutions to the conjugate Beltrami equation in terms of Bessel functions was found in this case. Generalizing this approach in a more systematic manner to free boundary problems of Bernoulli type, using descent algorithms based on shape-gradient for such approximation-theoretic criteria, is an interesting prospect, still to be pursued.

The piece of work we just mentioned requires defining and studying Hardy spaces of the conjugate-Beltrami equation, which is an interesting topic by itself. For Sobolev-smooth coefficients of exponent greater than 2, they were investigated in [3], [31]. The case of the critical exponent 2 is treated in [28], which apparently provides the first example of well-posedness for the Dirichlet problem in the non-strictly elliptic case: the conductivity may be unbounded or zero on sets of zero capacity and, accordingly, solutions need not be locally bounded.

The 3-D version of step 1 in Section 3.1 is another subject investigated by Apics: to recover a harmonic function (up to an additive constant) in a ball or a half-space from partial knowledge of its gradient on the boundary. This prototypical inverse problem (*i.e.* inverse to the Cauchy problem for the Laplace equation) often recurs in electromagnetism. At present, Apics is involved with solving instances of this inverse problem arising in two fields, namely medical imaging *e.g.* for electroencephalography (EEG) or magneto-encephalography (MEG), and paleomagnetism (recovery of rocks magnetization) [2], [33], see Section 6.1. In this connection, we collaborate with two groups of partners: Athena Inria project-team, CHU La Timone, and BESA company on the one hand, Geosciences Lab. at MIT and Cerege CNRS Lab. on the other hand. The question is considerably more difficult than its 2-D counterpart, due mainly to the lack of multiplicative structure for harmonic gradients. Still, considerable progress has been made over the last years using methods of harmonic analysis and operator theory.

The team is further concerned with 3-D generalizations and applications to non-destructive control of step 2 in Section 3.1. A typical problem is here to localize inhomogeneities or defaults such as cracks, sources or occlusions in a planar or 3-dimensional object, knowing thermal, electrical, or magnetic measurements on the boundary. These defaults can be expressed as a lack of harmonicity of the solution to the associated Dirichlet-Neumann problem, thereby posing an inverse potential problem in order to recover them. In 2-D, finding an optimal discretization of the potential in Sobolev norm amounts to solve a best rational approximation problem, and the question arises as to how the location of the singularities of the approximant (*i.e.* its poles) reflects the location of the singularities of the potential (*i.e.* the defaults we seek). This is a fairly deep issue in approximation theory, to which Apics contributed convergence results for certain classes of fields expressed as Cauchy integrals over extremal contours for the logarithmic potential [34], [48] [5]. Initial schemes to locate cracks or sources *via* rational approximation on planar domains were obtained this way [51], [37], [41]. It is remarkable that finite inverse source problems in 3-D balls, or more general algebraic surfaces, can be approached using these 2-D techniques upon slicing the domain into planar sections [38], [7]. This bottom line generates a steady research activity within Apics, and again applications are sought to medical imaging and geosciences, see Sections 4.2, 4.3 and 6.1.

Conjectures can be raised on the behavior of optimal potential discretization in 3-D, but answering them is an ambitious program still in its infancy.

3.2.2. Systems, transfer and scattering

Participants: Laurent Baratchart, Matthias Caenepeel, Sylvain Chevillard, Martine Olivi, Fabien Seyfert.

Through contacts with CNES (French space agency), members of the team became involved in identification and tuning of microwave electromagnetic filters used in space telecommunications, see Section 4.4. The initial problem was to recover, from band-limited frequency measurements, physical parameters of the device under examination. The latter consists of interconnected dual-mode resonant cavities with negligible loss, hence its scattering matrix is modeled by a 2×2 unitary-valued matrix function on the frequency line, say the imaginary axis to fix ideas. In the bandwidth around the resonant frequency, a modal approximation of the Helmholtz equation in the cavities shows that this matrix is approximately rational, of Mc-Millan degree twice the number of cavities.

This is where system theory comes into play, through the so-called *realization* process mapping a rational transfer function in the frequency domain to a state-space representation of the underlying system of linear differential equations in the time domain. Specifically, realizing the scattering matrix allows one to construct a virtual electrical network, equivalent to the filter, the parameters of which mediate in between the frequency response and the geometric characteristics of the cavities (*i.e.* the tuning parameters).

Hardy spaces provide a framework to transform this ill-posed issue into a series of regularized analytic and meromorphic approximation problems. More precisely, the procedure sketched in Section 3.1 goes as follows:

1. infer from the pointwise boundary data in the bandwidth a stable transfer function (*i.e.* one which is holomorphic in the right half-plane), that may be infinite dimensional (numerically: of high degree). This is done by solving a problem analogous to (P) in Section 3.3.1, while taking into account prior knowledge on the decay of the response outside the bandwidth, see [9] for details.
2. A stable rational approximation of appropriate degree to the model obtained in the previous step is performed. For this, a descent method on the compact manifold of inner matrices of given size and degree is used, based on an original parametrization of stable transfer functions developed within the team [24], [9].

3. Realizations of this rational approximant are computed. To be useful, they must satisfy certain constraints imposed by the geometry of the device. These constraints typically come from the coupling topology of the equivalent electrical network used to model the filter. This network is composed of resonators, coupled according to some specific graph. This realization step can be recast, under appropriate compatibility conditions [52], as solving a zero-dimensional multivariate polynomial system. To tackle this problem in practice, we use Gröbner basis techniques and continuation methods which team up in the Dedale-HF software (see Section 3.4.1).

Let us mention that extensions of classical coupling matrix theory to frequency-dependent (reactive) couplings have been carried-out in recent years [1] for wide-band design applications.

Apics also investigates issues pertaining to design rather than identification. Given the topology of the filter, a basic problem in this connection is to find the optimal response subject to specifications that bear on rejection, transmission and group delay of the scattering parameters. Generalizing the classical approach based on Chebyshev polynomials for single band filters, we recast the problem of multi-band response synthesis as a generalization of the classical Zolotarev min-max problem for rational functions [23] [8]. Thanks to quasi-convexity, the latter can be solved efficiently using iterative methods relying on linear programming. These were implemented in the software easy-FF (see easy-FF). Currently, the team is engaged in the synthesis of more complex microwave devices like multiplexers and routers, which connect several filters through wave guides. Schur analysis plays an important role here, because scattering matrices of passive systems are of Schur type (*i.e.* contractive in the stability region). The theory originates with the work of I. Schur [72], who devised a recursive test to check for contractivity of a holomorphic function in the disk. The so-called Schur parameters of a function may be viewed as Taylor coefficients for the hyperbolic metric of the disk, and the fact that Schur functions are contractions for that metric lies at the root of Schur's test. Generalizations thereof turn out to be efficient to parametrize solutions to contractive interpolation problems [25]. Dwelling on this, Apics contributed differential parametrizations (atlases of charts) of lossless matrix functions [24], [68], [62] which are fundamental to our rational approximation software RARL2 (see Section 3.4.4). Schur analysis is also instrumental to approach de-embedding issues, and provides one with considerable insight into the so-called matching problem. The latter consists in maximizing the power a multiport can pass to a given load, and for reasons of efficiency it is all-pervasive in microwave and electric network design, *e.g.* of antennas, multiplexers, wifi cards and more. It can be viewed as a rational approximation problem in the hyperbolic metric, and the team presently gets to grips with this hot topic using multipoint contractive interpolation in the framework of the (defense funded) ANR COCORAM, see Sections 6.2 and 8.2.1.

In recent years, our attention was driven by CNES and UPV (Bilbao) to questions about stability of high-frequency amplifiers, see Section 7.2. Contrary to previously discussed devices, these are *active* components. The response of an amplifier can be linearized around a set of primary current and voltages, and then admittances of the corresponding electrical network can be computed at various frequencies, using the so-called harmonic balance method. The initial goal is to check for stability of the linearized model, so as to ascertain existence of a well-defined working state. The network is composed of lumped electrical elements namely inductors, capacitors, negative *and* positive reactors, transmission lines, and controlled current sources. Our research so far focuses on describing the algebraic structure of admittance functions, so as to set up a function-theoretic framework where the two-steps approach outlined in Section 3.1 can be put to work. The main discovery so far is that the unstable part of each partial transfer function is rational and can be computed by analytic projection, see Section 7.2.

3.3. Approximation

Participants: Laurent Baratchart, Sylvain Chevillard, Juliette Leblond, Martine Olivi, Dmitry Ponomarev, Fabien Seyfert.

3.3.1. Best analytic approximation

In dimension 2, the prototypical problem to be solved in step 1 of Section 3.1 may be described as: given a domain $D \subset \mathbb{R}^2$, to recover a holomorphic function from its values on a subset K of the boundary of D. For the discussion it is convenient to normalize D, which can be done by conformal mapping. So, in the simply connected case, we fix D to be the unit disk with boundary unit circle T. We denote by H^p the Hardy space of exponent p, which is the closure of polynomials in $L^p(T)$-norm if $1 \le p < \infty$ and the space of bounded holomorphic functions in D if $p = \infty$. Functions in H^p have well-defined boundary values in $L^p(T)$, which makes it possible to speak of (traces of) analytic functions on the boundary.

To find an analytic function g in D matching some measured values f approximately on a sub-arc K of T, we formulate a constrained best approximation problem as follows.

> (P) Let $1 \le p \le \infty$, K a sub-arc of T, $f \in L^p(K)$, $\psi \in L^p(T \setminus K)$ and $M > 0$; find a function $g \in H^p$ such that $\|g - \psi\|_{L^p(T \setminus K)} \le M$ and $g - f$ is of minimal norm in $L^p(K)$ under this constraint.

Here ψ is a reference behavior capturing *a priori* assumptions on the behavior of the model off K, while M is some admissible deviation thereof. The value of p reflects the type of stability which is sought and how much one wants to smooth out the data. The choice of L^p classes is suited to handle point-wise measurements.

To fix terminology, we refer to (P) as a *bounded extremal problem*. As shown in [36], [39], [45], the solution to this convex infinite-dimensional optimization problem can be obtained when $p \neq 1$ upon iterating with respect to a Lagrange parameter the solution to spectral equations for appropriate Hankel and Toeplitz operators. These spectral equations involve the solution to the special case $K = T$ of (P), which is a standard extremal problem [60]:

> (P_0) Let $1 \le p \le \infty$ and $\varphi \in L^p(T)$; find a function $g \in H^p$ such that $g - \varphi$ is of minimal norm in $L^p(T)$.

The case $p = 1$ is more or less open.

Various modifications of (P) can be tailored to meet specific needs. For instance when dealing with lossless transfer functions (see Section 4.4), one may want to express the constraint on $T \setminus K$ in a point-wise manner: $|g - \psi| \le M$ a.e. on $T \setminus K$, see [40]. In this form, the problem comes close to (but still is different from) H^∞ frequency optimization used in control [63], [71]. One can also impose bounds on the real or imaginary part of $g - \psi$ on $T \setminus K$, which is useful when considering Dirichlet-Neumann problems, see [65].

The analog of Problem (P) on an annulus, K being now the outer boundary, can be seen as a means to regularize a classical inverse problem occurring in nondestructive control, namely to recover a harmonic function on the inner boundary from Dirichlet-Neumann data on the outer boundary (see Sections 3.2.1, 4.2, 6.1.1). It may serve as a tool to approach Bernoulli type problems, where we are given data on the outer boundary and we *seek the inner boundary*, knowing it is a level curve of the solution. In this case, the Lagrange parameter indicates how to deform the inner contour in order to improve data fitting. Similar topics are discussed in Section 3.2.1 for more general equations than the Laplacian, namely isotropic conductivity equations of the form $\mathrm{div}(\sigma \nabla u) = 0$ where σ is no longer constant. Then, the Hardy spaces in Problem (P) are those of a so-called conjugate Beltrami equation: $\overline{\partial} f = \nu \overline{\partial f}$ [64], which are studied for $1 < p < \infty$ in [3], [12], [28], [31] and [55]. Expansions of solutions needed to constructively handle such issues in the specific case of linear fractional conductivities (occurring for instance in plasma shaping) have been expounded in [57].

Though originally considered in dimension 2, Problem (P) carries over naturally to higher dimensions where analytic functions get replaced by gradients of harmonic functions. Namely, given some open set $\Omega \subset \mathbb{R}^n$ and some $\mathbb{R}^n$-valued vector field V on an open subset O of the boundary of Ω, we seek a harmonic function in Ω whose gradient is close to V on O.

When Ω is a ball or a half-space, a substitute for holomorphic Hardy spaces is provided by the Stein-Weiss Hardy spaces of harmonic gradients [74]. Conformal maps are no longer available when $n > 2$, so that Ω can no longer be normalized. More general geometries than spheres and half-spaces have not been much studied so far.

On the ball, the analog of Problem (P) is

(P_1) Let $1 \leq p \leq \infty$ and $B \subset \mathbb{R}^n$ the unit ball. Fix O an open subset of the unit sphere $S \subset \mathbb{R}^n$. Let further $V \in L^p(O)$ and $W \in L^p(S \setminus O)$ be $\mathbb{R}^n$-valued vector fields. Given $M > 0$, find a harmonic gradient $G \in H^p(B)$ such that $\|G - W\|_{L^p(S \setminus O)} \leq M$ and $G - V$ is of minimal norm in $L^p(O)$ under this constraint.

When $p = 2$, Problem (P_1) was solved in [2] as well as its analog on a shell, when the tangent component of V is a gradient (when O is Lipschitz the general case follows easily from this). The solution extends the work in [36] to the 3-D case, using a generalization of Toeplitz operators. The case of the shell was motivated by applications to the processing of EEG data. An important ingredient is a refinement of the Hodge decomposition, that we call the *Hardy-Hodge* decomposition, allowing us to express a $\mathbb{R}^n$-valued vector field in $L^p(S)$, $1 < p < \infty$, as the sum of a vector field in $H^p(B)$, a vector field in $H^p(\mathbb{R}^n \setminus \overline{B})$, and a tangential divergence free vector field on S; the space of such divergence-free fields is denoted by $D(S)$. If $p = 1$ or $p = \infty$, L^p must be replaced by the real Hardy space or the space of functions with bounded mean oscillation. More generally this decomposition, which is valid on any sufficiently smooth surface (see Section 6.1), seems to play a fundamental role in inverse potential problems. In fact, it was first introduced formally on the plane to describe silent magnetizations supported in $\mathbb{R}^2$ (*i.e.* those generating no field in the upper half space) [33].

Just like solving problem (P) appeals to the solution of problem (P_0), our ability to solve problem (P_1) will depend on the possibility to tackle the special case where $O = S$:

(P_2) Let $1 \leq p \leq \infty$ and $V \in L^p(S)$ be a $\mathbb{R}^n$-valued vector field. Find a harmonic gradient $G \in H^p(B)$ such that $\|G - V\|_{L^p(S)}$ is minimum.

Problem (P_2) is simple when $p = 2$ by virtue of the Hardy Hodge decomposition together with orthogonality of $H^2(B)$ and $H^2(\mathbb{R}^n \setminus \overline{B})$, which is the reason why we were able to solve (P_1) in this case. Other values of p cannot be treated as easily and are currently investigated by Apics, especially the case $p = \infty$ which is of particular interest and presents itself as a 3-D analog to the Nehari problem [70].

Companion to problem (P_2) is problem (P_3) below.

(P_3) Let $1 \leq p \leq \infty$ and $V \in L^p(S)$ be a $\mathbb{R}^n$-valued vector field. Find $G \in H^p(B)$ and $D \in D(S)$ such that $\|G + D - V\|_{L^p(S)}$ is minimum.

Note that (P_2) and (P_3) are identical in 2-D, since no non-constant tangential divergence-free vector field exists on T. It is no longer so in higher dimension, where both (P_2) and (P_3) arise in connection with source recovery in electro/magneto encephalography and paleomagnetism, see Sections 3.2.1 and 4.2.

3.3.2. Best meromorphic and rational approximation

The techniques set forth in this section are used to solve step 2 in Section 3.2 and instrumental to approach inverse boundary value problems for the Poisson equation $\Delta u = \mu$, where μ is some (unknown) measure.

3.3.2.1. Scalar meromorphic and rational approximation

We put R_N for the set of rational functions with at most N poles in D. By definition, meromorphic functions in $L^p(T)$ are (traces of) functions in $H^p + R_N$.

A natural generalization of problem (P_0) is:

(P_N) Let $1 \leq p \leq \infty$, $N \geq 0$ an integer, and $f \in L^p(T)$; find a function $g_N \in H^p + R_N$ such that $g_N - f$ is of minimal norm in $L^p(T)$.

Only for $p = \infty$ and f continuous is it known how to solve (P_N) in semi-closed form. The unique solution is given by AAK theory (named after Adamjan, Arov and Krein), which connects the spectral decomposition of Hankel operators with best approximation [70].

The case where $p = 2$ is of special importance for it reduces to rational approximation. Indeed, if we write the Hardy decomposition $f = f^+ + f^-$ where $f^+ \in H^2$ and $f^- \in H^2(\mathbb{C} \setminus \overline{D})$, then $g_N = f^+ + r_N$ where r_N is a best approximant to f^- from R_N in $L^2(T)$. Moreover, r_N has no pole outside D, hence it is a *stable* rational approximant to f^-. However, in contrast to the case where $p = \infty$, this best approximant may *not* be unique.

The former Miaou project (predecessor of Apics) designed a dedicated steepest-descent algorithm for the case $p = 2$ whose convergence to a *local minimum* is guaranteed; until now it seems to be the only procedure meeting this property. This gradient algorithm proceeds recursively with respect to N on a compactification of the parameter space [29]. Although it has proved to be effective in all applications carried out so far (see Sections 4.2, 4.4), it is still unknown whether the absolute minimum can always be obtained by choosing initial conditions corresponding to *critical points* of lower degree (as is done by the RARL2 software, Section 3.4.4).

In order to establish global convergence results, Apics has undertaken a deeper study of the number and nature of critical points (local minima, saddle points...), in which tools from differential topology and operator theory team up with classical interpolation theory [42], [44]. Based on this work, uniqueness or asymptotic uniqueness of the approximant was proved for certain classes of functions like transfer functions of relaxation systems (*i.e.* Markov functions) [46] and more generally Cauchy integrals over hyperbolic geodesic arcs [49]. These are the only results of this kind. Research by Apics on this topic remained dormant for a while by reasons of opportunity, but revisiting the work [26] in higher dimension is still a worthy endeavor. Meanwhile, an analog to AAK theory was carried out for $2 \le p < \infty$ in [45]. Although not as effective computationally, it was recently used to derive lower bounds [11]. When $1 \le p < 2$, problem (P_N) is still quite open.

A common feature to the above-mentioned problems is that critical point equations yield non-Hermitian orthogonality relations for the denominator of the approximant. This stresses connections with interpolation, which is a standard way to build approximants, and in many respects best or near-best rational approximation may be regarded as a clever manner to pick interpolation points. This was exploited in [50], [47], and is used in an essential manner to assess the behavior of poles of best approximants to functions with branched singularities, which is of particular interest for inverse source problems (*cf.* Sections 3.4.2 and 6.1).

In higher dimensions, the analog of Problem (P_N) is best approximation of a vector field by gradients of discrete potentials generated by N point masses. This basic issue is by no means fully understood, and it is an exciting research prospect. It is connected with certain generalizations of Toeplitz or Hankel operators, and with constructive approaches to so-called weak factorizations for real Hardy functions [56].

Besides, certain constrained rational approximation problems, of special interest in identification and design of passive systems, arise when putting additional requirements on the approximant, for instance that it should be smaller than 1 in modulus (*i.e.* a Schur function). In particular, Schur interpolation lately received renewed attention from the team, in connection with matching problems. There, interpolation data are subject to a well-known compatibility condition (positive definiteness of the so-called Pick matrix), and the main difficulty is to put interpolation points on the boundary of D while controlling both the degree and the extremal points (peak points for the modulus) of the interpolant. Results obtained by Apics in this direction generalize a variant of contractive interpolation with degree constraint studied in [61], see Section 6.2. We mention that contractive interpolation with nodes approaching the boundary has been a subsidiary research topic by the team in the past, which plays an interesting role in the spectral representation of certain non-stationary stochastic processes [35], [32]. The subject is intimately connected to orthogonal polynomials on the unit circle, and this line of investigation has been pursued towards an asymptotic study of orthogonal polynomials on planar domains, which is an active area in approximation theory with application to quantum particle systems and Hele-Shaw flows. Section 6.4.

3.3.2.2. Matrix-valued rational approximation

Matrix-valued approximation is necessary to handle systems with several inputs and outputs but it generates additional difficulties as compared to scalar-valued approximation, both theoretically and algorithmically. In the matrix case, the McMillan degree (*i.e.* the degree of a minimal realization in the System-Theoretic sense) generalizes the usual notion of degree for rational functions.

The basic problem that we consider now goes as follows: *let* $\mathcal{F} \in (H^2)^{m \times l}$ *and* n *an integer; find a rational matrix of size* $m \times l$ *without poles in the unit disk and of McMillan degree at most* n *which is nearest possible to* $\mathcal{F}$ *in* $(H^2)^{m \times l}$. Here the L^2 norm of a matrix is the square root of the sum of the squares of the norms of its entries.

The scalar approximation algorithm derived in [29] and mentioned in Section 3.3.2.1 generalizes to the matrix-valued situation [59]. The first difficulty here is to parametrize inner matrices (*i.e.* matrix-valued functions analytic in the unit disk and unitary on the unit circle) of given McMillan degree degree n. Indeed, inner matrices play the role of denominators in fractional representations of transfer matrices (using the so-called Douglas-Shapiro-Shields factorization). The set of inner matrices of given degree is a smooth manifold that allows one to use differential tools as in the scalar case. In practice, one has to produce an atlas of charts (local parametrizations) and to handle changes of charts in the course of the algorithm. Such parametrization can be obtained using interpolation theory and Schur-type algorithms, the parameters of which are vectors or matrices ([24], [62], [68]). Some of these parametrizations are also interesting to compute realizations and achieve filter synthesis ([62], [68]). The rational approximation software "RARL2" developed by the team is described in Section 3.4.4.

Difficulties relative to multiple local minima of course arise in the matrix-valued case as well, and deriving criteria that guarantee uniqueness is even more difficult than in the scalar case. The case of rational functions of degree n or small perturbations thereof (the consistency problem) was solved in [43]. Matrix-valued Markov functions are the only known example beyond this one [27].

Let us stress that RARL2 seems the only algorithm handling rational approximation in the matrix case that demonstrably converges to a local minimum while meeting stability constraints on the approximant. It is still a working pin of many developments by Apics on frequency optimization and design.

3.3.3. Behavior of poles of meromorphic approximants

Participant: Laurent Baratchart.

We refer here to the behavior of poles of best meromorphic approximants, in the L^p-sense on a closed curve, to functions f defined as Cauchy integrals of complex measures whose support lies inside the curve. Normalizing the contour to be the unit circle T, we are back to Problem (P_N) in Section 3.3.2.1; invariance of the latter under conformal mapping was established in [4]. Research so far has focused on functions whose singular set inside the contour is polar, meaning that the function can be continued analytically (possibly in a multiple-valued manner) except over a set of logarithmic capacity zero.

Generally speaking in approximation theory, assessing the behavior of poles of rational approximants is essential to obtain error rates as the degree goes large, and to tackle constructive issues like uniqueness. However, as explained in Section 3.2.1, Apics considers this issue foremost as a means to extract information on singularities of the solution to a Dirichlet-Neumann problem. The general theme is thus: *how do the singularities of the approximant reflect those of the approximated function?* This approach to inverse problem for the 2-D Laplacian turns out to be attractive when singularities are zero- or one-dimensional (see Section 4.2). It can be used as a computationally cheap initial condition for more precise but much heavier numerical optimizations which often do not even converge unless properly initialized. As regards crack detection or source recovery, this approach boils down to analyzing the behavior of best meromorphic approximants of a function with branch points, which is the prototype of a polar singular set. For piecewise analytic cracks, or in the case of sources, we were able to prove ([4], [5], [34]), that the poles of the approximants accumulate, when the degree goes large, to some extremal cut of minimum weighted logarithmic capacity connecting the singular points of the crack, or the sources [37]. Moreover, the asymptotic density of the poles turns out to be the Green equilibrium distribution on this cut in D, therefore it charges the singular points if one is able to approximate in sufficiently high degree (this is where the method could fail, because high-order approximation requires rather precise data).

The case of two-dimensional singularities is still an outstanding open problem.

It is remarkable that inverse source problems inside a sphere or an ellipsoid in 3-D can be approached with such 2-D techniques, as applied to planar sections, see Section 6.1. The technique is implemented in the software FindSources3D, see Section 3.4.2.

3.4. Software tools of the team

In addition to the above-mentioned research activities, Apics develops and maintains a number of long-term software tools that either implement and illustrate effectiveness of the algorithms theoretically developed by the team or serve as tools to help further research by team members. We present briefly the most important of them.

3.4.1. Dedale-HF

Participant: Fabien Seyfert [corresponding participant].

http://www-sop.inria.fr/apics/Dedale/

Dedale-HF is a software dedicated to solve exhaustively the coupling matrix synthesis problem in reasonable time for the filtering community. Given a coupling topology, the coupling matrix synthesis problem (C.M. problem for short) consists in finding all possible electromagnetic coupling values between resonators that yield a realization of given filter characteristics. Solving the latter problem is crucial during the design step of a filter in order to derive its physical dimensions as well as during the tuning process where coupling values need to be extracted from frequency measurements.

Dedale-HF consists in two parts: a database of coupling topologies as well as a dedicated predictor-corrector code. Roughly speaking each reference file of the database contains, for a given coupling topology, the complete solution to the C.M. problem associated to particular filtering characteristics. The latter is then used as a starting point for a predictor-corrector integration method that computes the solution to the C.M. corresponding to the user-specified filter characteristics. The reference files are computed off-line using Gröbner basis techniques or numerical techniques based on the exploration of a monodromy group. The use of such continuation techniques, combined with an efficient implementation of the integrator, drastically reduces the computational time.

Dedale-HF has been licensed to, and is currently used by TAS-España.

3.4.2. FindSources3D

Participants: Juliette Leblond [corresponding participant], Jean-Paul Marmorat, Nicolas Schnitzler.

This work is conducted in collaboration with Maureen Clerc and Théo Papadopoulo from the Athena EPI.

FindSources3D is a Matlab software program dedicated to the resolution of inverse source problems in electroencephalography (EEG) (see http://www-sop.inria.fr/apics/FindSources3D/en/index.html). From pointwise measurements of the electrical potential taken by electrodes on the scalp, FindSources3D estimates pointwise dipolar current sources within the brain in a spherical model. Following the scheme described in Section 4.2, see also Sections 5.2 and 6.1.1, after a first data transmission "cortical mapping" step, it makes use of best rational approximation on 2-D planar cross-sections and of the software RARL2 (see Section 3.4.4) in order to locate singularities [7]. From those planar singularities, the 3-D sources are estimated in a last step.

The program is currently being tested by BESA company (Munich). Our purpose is to distribute FindSources3D to teams in partner-hospitals (like la Timone, Marseille). It has a CeCILL license.

3.4.3. Presto-HF

Participants: Jean-Paul Marmorat, Martine Olivi, Fabien Seyfert [corresponding participant].

Presto-HF is a toolbox dedicated to low-pass parameter identification for microwave filters https://project.inria.fr/presto-hf/. In order to allow the industrial transfer of our methods, a Matlab-based toolbox has been developed, dedicated to the problem of identification of low-pass microwave filter parameters. It allows one to run the following algorithmic steps, either individually or in a single stroke:

- Determination of delay components caused by the access devices (automatic reference plane adjustment),
- Automatic determination of an analytic completion, bounded in modulus for each channel,
- Rational approximation of fixed McMillan degree,
- Determination of a constrained realization.

For the matrix-valued rational approximation step, Presto-HF relies on RARL2. Constrained realizations are computed using the Dedale-HF software. As a toolbox, Presto-HF has a modular structure, which allows one for example to include some building blocks in an already existing software.

The delay compensation algorithm is based on the following assumption: far off the pass-band, one can reasonably expect a good approximation of the rational components of S11 and S22 by the first few terms of their Taylor expansion at infinity, a small degree polynomial in 1/s. Using this idea, a sequence of quadratic convex optimization problems are solved, in order to obtain appropriate compensations. In order to check the previous assumption, one has to measure the filter on a larger band, typically three times the pass band.

This toolbox has been licensed to, and is currently used by Thales Alenia Space in Toulouse and Madrid, Thales airborne systems and Flextronics (two licenses). XLIM (University of Limoges) is a heavy user of Presto-HF among the academic filtering community and some free license agreements have been granted to the microwave department of the University of Erlangen (Germany) and the Royal Military College (Kingston, Canada).

3.4.4. *RARL2*

Participants: Jean-Paul Marmorat, Martine Olivi [corresponding participant].

RARL2 (Réalisation interne et Approximation Rationnelle L2) is a software for rational approximation (see Section 3.3.2.2) http://www-sop.inria.fr/apics/RARL2/rarl2.html.

RARL2 computes a stable rational L2-approximation of specified order to a given L2-stable (L2 on the unit circle, analytic in the complement of the unit disk) matrix-valued function. This can be the transfer function of a multivariable discrete-time stable system. RARL2 takes as input either:

- its internal realization,
- its first N Fourier coefficients,
- discretized (uniformly distributed) values on the circle. In this case, a least-square criterion is used instead of the L^2 norm.

It thus performs model reduction in the first or the second case, and leans on frequency data identification in the third. For band-limited frequency data, it could be necessary to infer the behavior of the system outside the bandwidth before performing rational approximation (see Section 3.2.2).
An appropriate Möbius transformation allows to use the software for continuous-time systems as well.

The method is a steepest-descent algorithm. A parametrization of MIMO systems is used, which ensures that the stability constraint on the approximant is met. The implementation, in Matlab, is based on state-space representations.

RARL2 performs the rational approximation step in the software tools PRESTO-HF (see Section 3.4.3) and FindSources3D (see Section 3.4.2). It is distributed under a particular license, allowing unlimited usage for academic research purposes. It was released to the universities of Delft and Maastricht (the Netherlands), Cork (Ireland), Brussels (Belgium), Macao (China) and BITS-Pilani Hyderabad Campus (India).

3.4.5. *Sollya*

Participant: Sylvain Chevillard [corresponding participant].

This software is developed in collaboration with Christoph Lauter (LIP6) and Mioara Joldeş (LAAS).

Preliminary remark: The coming of Sylvain Chevillard in the team in 2010 resulted in Apics hosting a research activity in certified computing, centered on the software *Sollya*. On the one hand, Sollya is an Inria software which still requires some tuning to a growing community of users. On the other hand, approximation-theoretic methods at work in Sollya are potentially useful for certified solutions to constrained analytic problems described in Section 3.3.1. However, developing Sollya is not a long-term objective of Apics.

Sollya is an interactive tool where the developers of mathematical floating-point libraries (libm) can experiment before actually developing code. The environment is safe with respect to floating-point errors, *i.e.* the user precisely knows when rounding errors or approximation errors happen, and rigorous bounds are always provided for these errors.

Among other features, it offers a fast Remez algorithm for computing polynomial approximations of real functions and also an algorithm for finding good polynomial approximants with floating-point coefficients to any real function. As well, it provides algorithms for the certification of numerical codes, such as Taylor Models, interval arithmetic or certified supremum norms.

It is available as a free software under the CeCILL-C license at http://sollya.gforge.inria.fr/.

4. Application Domains

4.1. Introduction

Application domains are naturally linked to the problems described in Sections 3.2.1 and 3.2.2. By and large, they split into a systems-and-circuits part and an inverse-source-and-boundary-problems part, united under a common umbrella of function-theoretic techniques as described in Section 3.3.

4.2. Inverse source problems in EEG

Participants: Laurent Baratchart, Juliette Leblond, Jean-Paul Marmorat, Christos Papageorgakis, Nicolas Schnitzler.

This work is conducted in collaboration with Maureen Clerc and Théo Papadopoulo from the Athena EPI.

Solving overdetermined Cauchy problems for the Laplace equation on a spherical layer (in 3-D) in order to extrapolate incomplete data (see Section 3.2.1) is a necessary ingredient of the team's approach to inverse source problems, in particular for applications to EEG, see [7]. Indeed, the latter involves propagating the initial conditions through several layers of different conductivities, from the boundary shell down to the center of the domain where the singularities (*i.e.* the sources) lie. Once propagated to the innermost sphere, it turns out that traces of the boundary data on 2-D cross sections coincide with analytic functions with branched singularities in the slicing plane [38][5]. The singularities are related to the actual location of the sources, namely their moduli reach in turn a maximum when the plane contains one of the sources. Hence we are back to the 2-D framework of Section 3.3.3, and recovering these singularities can be performed *via* best rational approximation. The goal is to produce a fast and sufficiently accurate initial guess on the number and location of the sources in order to run heavier descent algorithms on the direct problem, which are more precise but computationally costly and often fail to converge if not properly initialized.

Numerical experiments obtained with our software FindSources3D give very good results on simulated data and we are now engaged in the process of handling real experimental data (see Sections 3.4.2 and 6.1), in collaboration with the Athena team at Inria Sophia Antipolis, neuroscience teams in partner-hospitals (la Timone, Marseille), and the BESA company (Munich).

4.3. Inverse magnetization problems

Participants: Laurent Baratchart, Sylvain Chevillard, Juliette Leblond, Konstantinos Mavreas, Dmitry Ponomarev.

Generally speaking, inverse potential problems, similar to the one appearing in Section 4.2, occur naturally in connection with systems governed by Maxwell's equation in the quasi-static approximation regime. In particular, they arise in magnetic reconstruction issues. A specific application is to geophysics, which led us to form the Inria Associate Team "IMPINGE" (Inverse Magnetization Problems IN GEosciences) together with MIT and Vanderbilt University. A recent collaboration with Cerege (CNRS, Aix-en-Provence), in the framework of the ANR-project MagLune, completes this picture, see Section 8.2.2.

To set up the context, recall that the Earth's geomagnetic field is generated by convection of the liquid metallic core (geodynamo) and that rocks become magnetized by the ambient field as they are formed or after subsequent alteration. Their remanent magnetization provides records of past variations of the geodynamo, which is used to study important processes in Earth sciences like motion of tectonic plates and geomagnetic reversals. Rocks from Mars, the Moon, and asteroids also contain remanent magnetization which indicates the past presence of core dynamos. Magnetization in meteorites may even record fields produced by the young sun and the protoplanetary disk which may have played a key role in solar system formation.

For a long time, paleomagnetic techniques were only capable of analyzing bulk samples and compute their net magnetic moment. The development of SQUID microscopes has recently extended the spatial resolution to sub-millimeter scales, raising new physical and algorithmic challenges. This associate team aims at tackling them, experimenting with the SQUID microscope set up in the Paleomagnetism Laboratory of the department of Earth, Atmospheric and Planetary Sciences at MIT. Typically, pieces of rock are sanded down to a thin slab, and the magnetization has to be recovered from the field measured on a parallel plane at small distance above the slab.

Mathematically speaking, both inverse source problems for EEG from Section 4.2 and inverse magnetization problems described presently amount to recover the (3-D valued) quantity m (primary current density in case of the brain or magnetization in case of a thin slab of rock) from measurements of the vector potential:

$$\int_{\Omega} \frac{\operatorname{div} m(x')\, dx'}{|x-x'|}\,, \tag{1}$$

outside the volume Ω of the object. The difference is that the distribution m is located in a volume in the case of EEG, and on a plane in the case of rock magnetization. This results in quite different identifiability properties, see [33] and Section 6.1.2, but the two situations share a substantial Mathematical common core. .

4.4. Identification and design of microwave devices

Participants: Laurent Baratchart, Sylvain Chevillard, Jean-Paul Marmorat, Martine Olivi, Fabien Seyfert.

This is joint work with Stéphane Bila (XLIM, Limoges).

One of the best training grounds for function-theoretic applications by the team is the identification and design of physical systems whose performance is assessed frequency-wise. This is the case of electromagnetic resonant systems which are of common use in telecommunications.

In space telecommunications (satellite transmissions), constraints specific to on-board technology lead to the use of filters with resonant cavities in the microwave range. These filters serve multiplexing purposes (before or after amplification), and consist of a sequence of cylindrical hollow bodies, magnetically coupled by irises (orthogonal double slits). The electromagnetic wave that traverses the cavities satisfies the Maxwell equations, forcing the tangent electrical field along the body of the cavity to be zero. A deeper study of the Helmholtz equation states that an essentially discrete set of wave vectors is selected. In the considered range of frequency, the electrical field in each cavity can be decomposed along two orthogonal modes, perpendicular to the axis of the cavity (other modes are far off in the frequency domain, and their influence can be neglected).

Each cavity (see Figure 1) has three screws, horizontal, vertical and midway (horizontal and vertical are two arbitrary directions, the third direction makes an angle of 45 or 135 degrees, the easy case is when all cavities show the same orientation, and when the directions of the irises are the same, as well as the input and output slits). Since screws are conductors, they behave as capacitors; besides, the electrical field on the surface has to be zero, which modifies the boundary conditions of one of the two modes (for the other mode, the electrical field is zero hence it is not influenced by the screw), the third screw acts as a coupling between the two modes. The effect of an iris is opposite to that of a screw: no condition is imposed on a hole, which results in a coupling between two horizontal (or two vertical) modes of adjacent cavities (in fact the iris is the union of two rectangles, the important parameter being their width). The design of a filter consists in finding the size of each cavity, and the width of each iris. Subsequently, the filter can be constructed and tuned by adjusting the

Figure 1. Picture of a 6-cavities dual mode filter. Each cavity (except the last one) has 3 screws to couple the modes within the cavity, so that 16 quantities must be optimized. Quantities such as the diameter and length of the cavities, or the width of the 11 slits are fixed during the design phase.

screws. Finally, the screws are glued once a satisfactory response has been obtained. In what follows, we shall consider a typical example, a filter designed by the CNES in Toulouse, with four cavities near 11 GHz.

Near the resonance frequency, a good approximation to the Helmholtz equations is given by a second order differential equation. Thus, one obtains an electrical model of the filter as a sequence of electrically-coupled resonant circuits, each circuit being modeled by two resonators, one per mode, the resonance frequency of which represents the frequency of a mode, and whose resistance accounts for electric losses (surface currents) in the cavities.

This way, the filter can be seen as a quadripole, with two ports, when plugged onto a resistor at one end and fed with some potential at the other end. One is now interested in the power which is transmitted and reflected. This leads one to define a scattering matrix S, which may be considered as the transfer function of a stable causal linear dynamical system, with two inputs and two outputs. Its diagonal terms $S_{1,1}$, $S_{2,2}$ correspond to reflections at each port, while $S_{1,2}$, $S_{2,1}$ correspond to transmission. These functions can be measured at certain frequencies (on the imaginary axis). The filter is rational of order 4 times the number of cavities (that is 16 in the example on Figure 2), and the key step consists in expressing the components of the equivalent electrical circuit as functions of the S_{ij} (since there are no formulas expressing the lengths of the screws in terms of parameters of this electrical model). This representation is also useful to analyze the numerical simulations of the Maxwell equations, and to check the quality of design, in particular the absence of higher resonant modes.

In fact, resonance is not studied via the electrical model, but via a low-pass equivalent circuit obtained upon linearizing near the central frequency, which is no longer conjugate symmetric (*i.e.* the underlying system may no longer have real coefficients) but whose degree is divided by 2 (8 in the example).

In short, the strategy for identification is as follows:

- measuring the scattering matrix of the filter near the optimal frequency over twice the pass band (which is 80MHz in the example).
- Solving bounded extremal problems for the transmission and the reflection (the modulus of he

response being respectively close to 0 and 1 outside the interval measurement, cf. Section 3.3.1). This provides us with a scattering matrix of order roughly 1/4 of the number of data points.

- Approximating this scattering matrix by a rational transfer-function of fixed degree (8 in this example) via the Endymion or RARL2 software (cf. Section 3.3.2.2).
- A realization of the transfer function is thus obtained, and some additional symmetry constraints are imposed.
- Finally one builds a realization of the approximant and looks for a change of variables that eliminates non-physical couplings. This is obtained by using algebraic-solvers and continuation algorithms on the group of orthogonal complex matrices (symmetry forces this type of transformation).

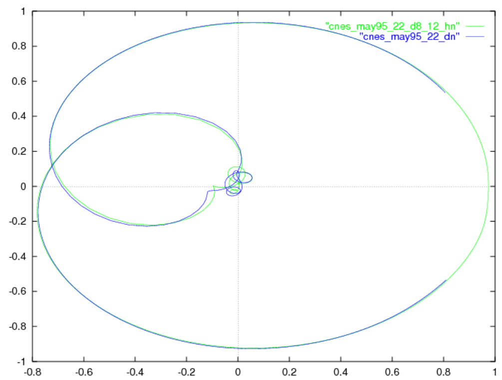

Figure 2. Nyquist Diagram. Rational approximation (degree 8) and data - S_{22}.

The final approximation is of high quality. This can be interpreted as a confirmation of the linearity assumption on the system: the relative L^2 error is less than 10^{-3}. This is illustrated by a reflection diagram (Figure 2). Non-physical couplings are less than 10^{-2}.

The above considerations are valid for a large class of filters. These developments have also been used for the design of non-symmetric filters, which are useful for the synthesis of repeating devices.

The team also investigates problems relative to the design of optimal responses for microwave devices. The resolution of a quasi-convex Zolotarev problems was proposed, in order to derive guaranteed optimal multi-band filter responses subject to modulus constraints [8]. This generalizes the classical single band design techniques based on Chebyshev polynomials and elliptic functions. The approach relies on the fact that the modulus of the scattering parameter $|S_{1,2}|$ admits a simple expression in terms of the filtering function $D = |S_{1,1}|/|S_{1,2}|$, namely

$$|S_{1,2}|^2 = \frac{1}{1 + D^2}.$$

The filtering function appears to be the ratio of two polynomials p_1/p_2, the numerator of the reflection and transmission scattering factors, that can be chosen freely. The denominator q is obtained as the unique stable unitary polynomial solving the classical Feldtkeller spectral equation:

$$qq^* = p_1 p_1^* + p_2 p_2^*.$$

The relative simplicity of the derivation of a filter's response, under modulus constraints, owes much to the possibility of forgetting about Feldtkeller's equation and express all design constraints in terms of the filtering function. This no longer the case when considering the synthesis N-port devices for $N > 3$, like multiplexers, routers power dividers or when considering the synthesis of filters under matching conditions. The efficient derivation of multiplexers responses is among the team's recent investigation, where techniques based on constrained Nevanlinna-Pick interpolation problems are being considered (see Section 6.2).

Through contacts with CNES (Toulouse) and UPV (Bilbao), Apics got further involved in the design of amplifiers which, unlike filters, are active devices. A prominent issue here is stability. A twenty years back, it was not possible to simulate unstable responses, and only after building a device could one detect instability. The advent of so-called *harmonic balance* techniques, which compute steady state responses of linear elements in the frequency domain and look for a periodic state in the time domain of a network connecting these linear elements *via* static non-linearities made it possible to compute the harmonic response of a (possibly nonlinear and unstable) device [75]. This has had tremendous impact on design, and there is a growing demand for software analyzers.

There are two types of stability involved. The first is stability of a fixed point around which the linearized transfer function accounts for small signal amplification. The second is stability of a limit cycle which is reached when the input signal is no longer small and truly nonlinear amplification is attained (*e.g.* because of saturation). Work by the team so far has been concerned with the first type of stability, and emphasis is put on defining and extracting the "unstable part" of the response, see Section 7.2.

5. New Software and Platforms

5.1. Dedale-HF

Recent developments allow to use Dedale-HF in combination with Presto-HF and in replacement of the former software RGC. A circuit optimizer has also been added to handle specific coupling topologies, the admissible set of which is not known in terms of a simple polynomial description.

5.2. FindSources3D

A new (Matlab) version of the software that automatically performs the estimation of the quantity of sources is being built (see Section 3.4.2). It uses an alignment criterion in addition to other clustering tests for the selection. Also, the team benefit from an "Action de Développement Technologique" (ADT Inria) BOLIS, 2014-2016, and of the young engineer N. Schnitzler at half-part of the time. The aim is to get from FindSources3D a modular, ergonomic, accessible and interactive platform, providing a convenient graphical interface and a tool that can be easily distributed and used, for medical imaging (EEG, MEG, EIT) or other applications (like inverse source problems in planetary sciences, see Section 6.1.3). Modularity is now granted, though still in progress (using the tools dtk, Qt, still with compiled Matlab libraries; translation in C++ will be continued). The related version of the software now offers a detailed and nice visualization of the data and tuning parameters, of the processing steps and of the computed results (using VTK).

6. New Results

6.1. Inverse problems for Poisson-Laplace equations

Participants: Laurent Baratchart, Sylvain Chevillard, Juliette Leblond, Konstantinos Mavreas, Christos Papageorgakis, Dmitry Ponomarev.

This section is concerned with inverse problems for 3-D Poisson-Laplace equations, among which source recovery issues. Though the geometrical settings differ in Sections 6.1.1 and 6.1.2, the characterization of silent sources (those giving rise to a vanishing field) is one common problem to both which has been resolved in the magnetization setup [33].

6.1.1. Inverse problems in medical imaging

This work is conducted in collaboration with Jean-Paul Marmorat and Nicolas Schnitzler, together with Maureen Clerc and Théo Papadopoulo from the Athena EPI.

In 3-D, functional or clinical active regions in the cortex are often modeled by pointwise sources that have to be localized from measurements taken by electrodes on the scalp of an electrical potential satisfying a Laplace equation (EEG, electroencephalography). In the works [38][5] on the behavior of poles in best rational approximants of fixed degree to functions with branch points, it was shown how to proceed via best rational approximation on a sequence of 2-D disks cut along the inner sphere, for the case where there are finitely many sources (see Section 4.2).

In this connection, a dedicated software FindSources3D (see Section 3.4.2) is being developed, in collaboration with the team Athena and the CMA. We continued this year algorithmic developments, prompted by a fruitful collaboration with the firm BESA, namely automatic detection of the number of sources (which was left to the user until recently). It appears that, in the rational approximation step, *multiple* poles possess a nice behavior with respect to branched singularities. This is due to the very physical assumptions on the model (for EEG data, one should consider *triple* poles). Though numerically observed in [7], there is no mathematical justification so far why multiple poles generate such strong accumulation of the poles of the approximants. This intriguing property, however, is definitely helping source recovery. It is used in order to automatically estimate the "most plausible" number of sources (numerically: up to 3, at the moment). Further, a modular and ergonomic platform version of the software is under development.

In connection with these and other brain exploration modalities like electrical impedance tomography (EIT), we are now studying conductivity estimation problems. This is the topic of the PhD research work of C. Papageorgakis (co-advised with the Athena project-team and BESA GmbH). In layered models, it concerns the estimation of the conductivity of the skull (intermediate layer). Indeed, the skull was assumed until now to have a given isotropic constant conductivity, whose value can differ from one individual to another. A preliminary issue in this direction is: can we uniquely recover and estimate a single-valued skull conductivity from one EEG recording? This has been established in the spherical setting when the sources are known, see [17]. Situations where sources are only partially known and the geometry is more realistic than a sphere are currently under study. When the sources are unknown, we should look for more data (additional clinical and/or functional EEG, EIT, ...) that could be incorporated in order to recover both the sources locations and the skull conductivity. Furthermore, while the skull essentially consists of hard bone part that may be assumed to have constant electrical conductivity, it also contains spongy bone compartments. These two distinct parts of the skull possess quite different conductivities. The influence of that second value on the overall model is now being studied [19].

6.1.2. Inverse magnetization issues in the thin-plate framework

This work is carried out in the framework of the "équipe associée Inria" IMPINGE, comprising Eduardo Andrade Lima and Benjamin Weiss from the Earth Sciences department at MIT (Boston, USA) and Douglas Hardin, Michael Northington and Edward Saff from the Mathematics department at Vanderbilt University (Nashville, USA).

Localizing magnetic sources from measurements of the magnetic field away from the support of the magnetization has been the fundamental issue under investigation by IMPINGE. The goal was to determine magnetic properties of rock samples (*e.g.* meteorites or stalactites) from fine field measurements close to the sample that can nowadays be obtained using SQUIDs (superconducting quantum interference devices). Currently, rock samples are cut into thin slabs and the magnetization distribution is considered to lie in a plane, which makes for a somewhat less indeterminate framework than EEG as regards inverse problems because "less" magnetizations can produce the same field (for the slab has no inner volume). Note however that EEG data consist of values of the normal current and of the associated potential, while in the present setting only values of the normal magnetic field are measured.

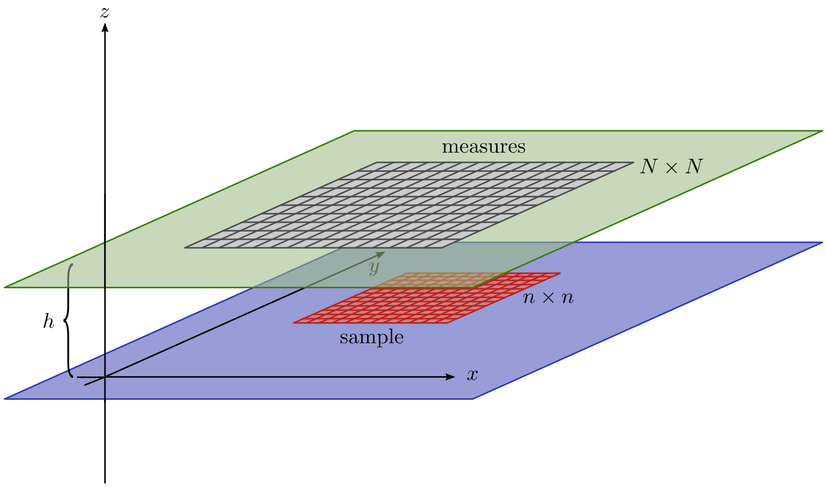

Figure 3. Schematic view of the experimental setup

Figure 3 presents a schematic view of the experimental setup: the sample lie on a horizontal plane at height 0 and its support is included in a rectangle. The vertical component B_3 of the field produced by the sample is measured on points of a horizontal $N \times N$ rectangular grid at height h.

Over the previous years, we mainly focused on developing techniques to recover magnetizations with rather sparse support. To this end, we set up a heuristic procedure to recover sparse magnetizations, based on iterative truncation of the support of the recovered magnetization. In this heuristics, magnetizations were represented by dipoles placed at the points of a regular rectangular $n \times n$, which seemed general enough a model class to correctly approximate the magnetizations commonly encountered in samples.

The procedure turned out to be poor when trying to recover the magnetization itself, due to the severe ill-posedness of the problem and the unexpected existence of magnetizations that produce almost no field at the height where measurements are performed, although the corresponding magnetic distributions strongly differ from truly silent distributions. Nevertheless, whenever the support could be significantly shrunk while keeping the error small (*i.e.*, explaining the data satisfactorily), estimates of the net moment so far, based on the dipolar model obtained by inversion, have been good.

This suggests that recovering the net moment and recovering the magnetization are rather different problems, the first one being less ill-posed than the second. Although the information provided by the net moment of the sample seems to be much weaker than knowing the full magnetic distribution, its importance has been emphasized by the geophysicists at MIT for at least three reasons:

- It yields important geological information on the sample in particular to estimate the magnitude of the ambient magnetic field at the time the rock was formed.
- It can be estimated independently to some extent, using a magnetometer, thereby allowing one to cross-validate the approach.
- From a computation point of view, knowledge of the net moment should lead to numerically stable reconstruction of an equivalent unidirectional magnetization. The support of the latter would provide us with valuable information to test for unidirectionality of the true magnetization, which is an important question to physicists in connection with rocks history and formation.

This year, we addressed the problem of directly recovering the net moment, without recourse to full inversion. Indeed, the latter is rather inefficient as it requires using a cluster and even then, for some samples, days of evaluation in order to obtain only a coarse estimate of the net moment. This research effort led us to investigate three different and complementary approaches.

First, we improved over Fourier based techniques previously designed by reformulating the problem with the help of the Kelvin transform. This gave us an asymptotic expansion of the net moment involving, at the first order, the integrals $\iint B_3(x,y,h)\,\mathrm{d}x\mathrm{d}y$, $\iint x\,B_3(x,y,h)\,\mathrm{d}x\mathrm{d}y$ and $\iint y\,B_3(x,y,h)\,\mathrm{d}x\mathrm{d}y$, computed on a disc with large radius. Although the method is promising, the computations are quite involved and we did not manage yet to obtain higher-order terms. This is a part of D. Ponomarev PhD work.

In parallel, and based on the results obtained with Fourier transform, we investigated a second approach, consisting in directly computing asymptotic expansions of the above integrals, on several domains (namely, the 2-D balls of radius R for the 1, 2 and ∞ norm, that are squares, disks, diamonds). In all cases, we get

$$\iint x\,B_3(x,y,h)\,\mathrm{d}x\mathrm{d}y = \alpha\langle m_1\rangle + \beta(\langle t_1\,m_3\rangle - h\langle m_1\rangle)/R + \mathcal{O}(1/R^3),$$

where $\langle m_1\rangle$ is the moment of the first component m_1 of the magnetization and $\langle t_1\,m_3\rangle$ is the first moment of m_3 with respect to the first variable. The constants α and β depend on the domain where the integral is computed. Therefore, an appropriate linear combination of the integrals computed on the different domains allows us to compute $\langle m_1\rangle$ with an accuracy of $\mathcal{O}(1/R^3)$. Similar results are obtained for $\langle m_2\rangle$ and $\langle m_3\rangle$ with the other integrals. Preliminary numerical experiments confirm the practical usability of these formulas in order to recover the moment of magnetizations. A research report is currently being written to sum up these results.

Finally, a third more ambitious approach has been investigated. As an attempt to generalize the previous expansions, our initial question was: given measurement of B_3, say on a square, find a function $\phi(x,y)$ such that $\iint \phi(x,y)B_3(x,y)\,\mathrm{d}x\mathrm{d}y$ is a the best possible estimate of the net moment components $\langle m_i\rangle$ $(i=1,2,3)$. This problem does not admit a solution because, for any $\epsilon > 0$, there exists a function ϕ_ϵ allowing to estimate the moment with an error bounded by ϵ. However, when ϵ tends to zero, the function ϕ_ϵ is expected to have strong oscillations, which hinders an accurate computation of $\iint \phi(x,y)B_3(x,y)\,\mathrm{d}x\mathrm{d}y$ since B_3 is only known on a discrete grid of points. We therefore expressed the problem as a bounded extremal problem (see Section 3.3.1): to find the best ϕ_ϵ (with the smallest possible error value ϵ) under the constraint that $\|\nabla\phi_\epsilon\|_2 \le M$. Here, M is a user-defined parameter. We proved theoretical results regarding this bounded extremal problem (existence and uniqueness of a solution, characterization of its solution as a solution of integro-differential equation) and we are currently designing a numerical procedure to compute it. An article on this topic is in preparation.

Still in the course of D. Ponomarev's PhD research, the study of a 2D spectral problem for the truncated Poisson operator in planar geometry has been pursued. It is a simplified formulation of the relation between the magnetization and the magnetic potential (of which the magnetic field is the gradient) and is expected to produce an efficient representation basis (the eigenfunctions of the magnetization-to-field operator). This is a long-standing problem. Noteworthy properties of solutions have been obtained through connections with other spectral problems and asymptotic reductions for large and small values of the main parameters (distance h from the measurement plane to the sample support and sample support size), yielding approximate solutions by means simpler integral equations and ODEs.

The year 2015 was the last of our "équipe associée" IMPINGE with the MIT and Vanderbilt University. The final report is available on the web page of the associate team [2]. This collaboration is currently supported in part by a MIT-France seed funding from the US side, and we applied for a three-years extension of the associate team.

[2] http://www-sop.inria.fr/apics/IMPINGE/

6.1.3. Inverse magnetization issues from sparse spherical data

The team APICS is a partner of the ANR project MagLune concerning Lunar magnetism, associated to the Geophysics and Planetology Department of Cerege, CNRS, Aix-en-Provence (see Section 8.2.2). Measurements of the remanent magnetic field of the Moon let geoscientists think that the Moon used to have a magnetic dynamo for some time, but the exact process that triggered and fed this dynamo is not yet understood, much less why it stopped. In particular, the Moon is too small to have a convecting dynamo like the Earth has. The overall goal of the project is to devise models to explain how this dynamo phenomenon was possible on the Moon.

To this end, the geophysicists from Cerege will go to NASA to perform some measurements on samples brought back from the Moon by Apollo missions. The samples are kept inside bags with a protective atmosphere, and geophysicists are not allowed to open the bags, nor to take out the samples from NASA facilities. Therefore, measurements must be performed with some rudimentary instrument and our colleagues from Cerege designed a specific magnetometer. This device allows them to obtain measurements of the components of the magnetic field produced by the sample, at some discrete set of points located on disks belonging to three cylinders (see Figure 4).

This collaboration started this year and some preparatory work was necessary fix conventions used by our colleagues from Cerege in order to handle their measurements. During his Master 2 internship, Konstantinos Mavreas has developed a method based on rational approximation, using the same ideas as those underlying the FindSources3D tool (see Sections 3.4.2 and 6.1.1), for the case where the field produced by the sample can be well explained by a single magnetic dipole, whose position and moment are unknown. See his report [3]. Konstantinos Mavreas is now engaged in a PhD within APICS and will extend these results to the case of several dipoles.

6.2. Matching problems and their applications

Participants: Laurent Baratchart, Martine Olivi, David Martinez Martinez, Fabien Seyfert.

This is collaborative work with Stéphane Bila (Xlim, Limoges, France), Yohann Sence (Xlim, Limoges, France), Thierry Monediere (Xlim, Limoges, France), Francois Torrès (Xlim, Limoges, France).

Filter synthesis is usually performed under the hypothesis that both ports of the filter are loaded on a constant resistive load (usually 50 Ohm). In complex systems, filters are however cascaded with other devices, and end up being loaded, at least at one port, on a non purely resistive frequency varying load. This is for example the case when synthesizing a multiplexer: each filter is here loaded at one of its ports on a common junction. Thus, the load varies with frequency by construction, and is not purely resistive either. Likewise, in an emitter-receiver, the antenna is followed by a filter. Whereas the antenna can usually be regarded as a resistive load at some frequencies, this is far from being true on the whole pass-band. A mismatch between the antenna and the filter, however, causes irremediable power losses, both in emission and transmission. Our goal is therefore to develop a method for filter synthesis that allows us to match varying loads on specific frequency bands, while enforcing some rejection properties away from the pass-band.

Figure 5 shows a filter with scattering matrix S, plugged at its right port on a frequency varying load with reflexion parameter $L_{1,1}$. If the filter is lossless, simple algebraic manipulations show that on the frequency axis the reflexion parameter satisfies:

$$|G_{1,1}| = \left| \frac{S_{2,2} - \overline{L_{1,1}}}{1 - S_{2,2} L_{1,1}} \right| = \delta(G_{1,1}, S_{2,2}).$$

[3] http://www-sop.inria.fr/members/Konstantinos.Mavreas/main.pdf

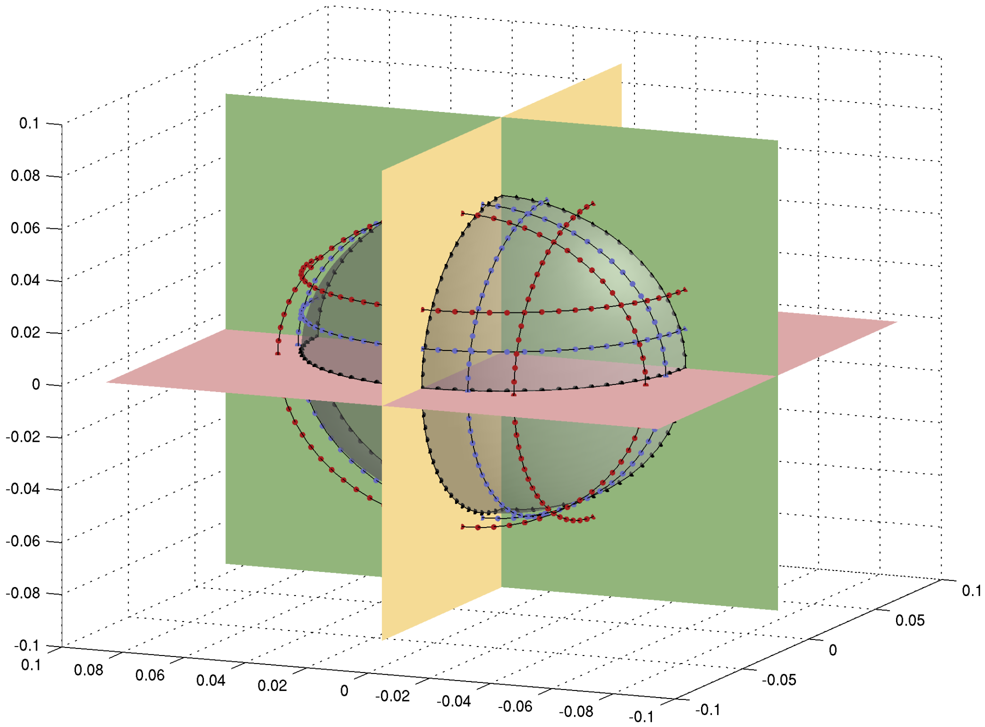

Figure 4. Typical measurements obtained with the instrument of Cerege. Discrete measurements of the field are performed on three cylinders. On each cylinder, the magnetic field $\mathbf{B}$ *is expressed as a component* B_h *co-linear with the axis of the cylinder, and a component* $\mathbf{B_s}$ *parallel to a section of the cylinder.* $\mathbf{B_s}$ *is itself decomposed as a tangential component* B_τ *and a normal component* B_n*, with respect to the circle given by the intersection of the cylinder with the corresponding section. At black points* B_n *is measured, at blue points* B_h *is measured, and at red points* B_τ *is measured.*

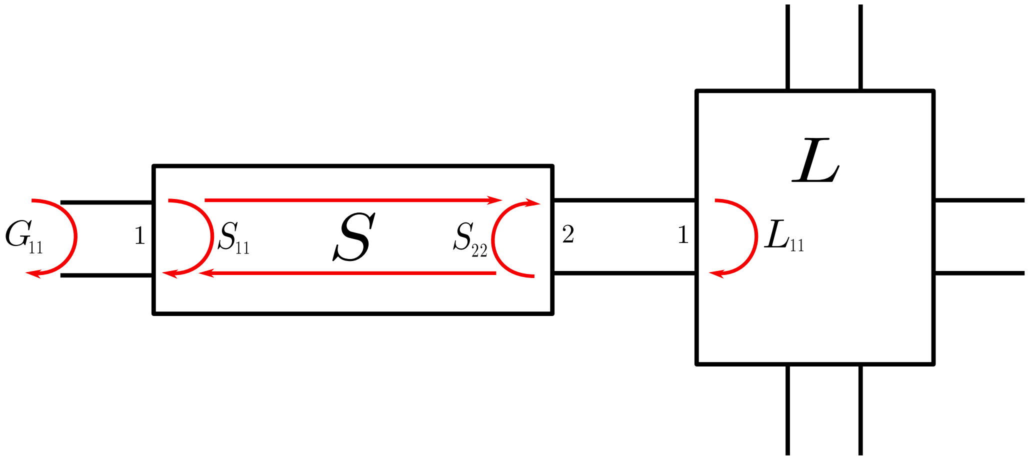

Figure 5. Filter plugged on a system with reflexion coefficient L_{11}

The matching problem of minimizing $|G_{1,1}|$ amounts therefore to minimize the pseudo-hyperbolic distance δ between the filter's reflexion parameter $S_{2,2}$ and the load's reflexion $L_{1,1}$, on a given frequency band. On the contrary enforcing a rejection level on a stop band, amounts to maintaining the value of $\delta(L_{1,1}, S_{2,2})$ above a certain threshold on this frequency band. For a broad class of filters, namely those that can be modeled by a circuit of n coupled resonators, the scattering matrix S is a rational function of McMillan degree n in the frequency variable. The matching problem thus appears to be a rational approximation problem in the hyperbolic metric.

6.2.1. Approach based on interpolation

When the degree n of the rational function $S_{2,2}$ is fixed, the hyperbolic minimization problem is non-convex and led us to seek methods to derive good initial guesses for classical descent algorithms. To this effect, if $S_{2,2} = p/q$ where p, q are polynomials, we considered the following interpolation problem $\mathcal{P}$: given n frequency points $w_1 \cdots w_n$ and a transmission polynomial r, to find a monic polynomial p of degree n such that:

$$j = 1..n, \qquad \frac{p}{q}(w_j) = \overline{L_{1,1}(w_j)}$$

where q is the unique monic Hurwitz polynomial of degree n satisfying the Feldtkeller equation

$$qq^* = pp^* + rr^*,$$

which accounts for the losslessness of the filter. The frequencies (w_k) are perfect matching points, as $\delta(S_{2,2}(w_k), L_{1,1}(w_k)) = 0$ holds, while the real zeros (x_k) of r are perfect rejection points (i.e. $\delta(S_{2,2}(x_k), L_{1,1}(x_k)) = 1$). The interpolation problem is therefore a point-wise version of our original matching-rejection problem. The monic restriction on p and q ensures the realisability of the filter in terms of coupled resonating circuits. If a perfect phase shifter is added in front of the filter, realized for example with a transmission line on a narrow frequency band, these monic restrictions can be dropped and an interpolation point w_{n+1} added, thereby yielding another interpolation problem $\widehat{\mathcal{P}}$. Our main result, states that $\mathcal{P}$ as well as $\widehat{\mathcal{P}}$ admit a unique solution. Moreover the evaluation map defined by $\psi(p) = (p/q(x_1), \cdots, p/q(x_n))$ is a homeomorphism from monic polynomials of degree n onto $\mathbb{D}^n$ ($\mathbb{D}$ the complex open disk), and ψ^{-1} is a diffeomorphism on an open, connected, dense set of $\mathbb{D}^n$. This last property has shown crucial for the design of an effective computational procedure based on continuation techniques. Current implementation of the latter tackles instances of $\mathcal{P}$ or $\widehat{\mathcal{P}}$ for $n = 10$ in less than 0.1 sec, and allows for us a recursive use of this interpolation framework in multiplexer synthesis problems. We presented these techniques at the European Microwave Week 2015 in the workshop dedicated to "Recent Advances in the Synthesis of Microwave Filters and Multiplexers". The detailed mathematical proofs can be found in [21] and will be submitted shortly. On a related topic, namely the de-embedding of filters in multiplexers, our work has been published in [13].

6.2.2. Uniform matching and global optimality considerations

The previous interpolation procedure provides us with a matching/rejecting filtering characteristics at a discrete set of frequencies. This can serve as a starting point for heavier optimization procedures where the matching and rejection specifications are expressed uniformly over the bandwidth. Although the practical results thus obtained have shown to be quite convincing, we have no proof of their global optimality. This led us to seek alternative approaches able to asses, at least in simple cases, global optimality of the derived response. Following the approach of Fano and Youla, we considered the problem of a designing a 2×2 lossless frequency response, under the condition that a specified load can be "unchained" from one of its port. This classically amounts to set interpolation conditions on the response at the transmission zeros of the Darlington extension of the load. When the load admits a rational representation of degree 1, and if the transmission zeros of the overall system are fixed, then we were able to show that the uniform matching problem over an interval reduces to a convex minimization problem with convex constraints over the set of non-negative polynomials of given degree. In this case, which is already of some practical interest for antenna matching (antenna usually exhibit a single resonance in their matching band which is reasonably approximated at order 1), it is therefore possible to perform filter synthesis with a guarantee on the global optimality of the obtained characteristics. Procedures to derive the solution are currently being investigated, and lie at the heart of our contribution to the ANR-project Cocoram.

6.3. Unambiguous de-embedding of filters

Participants: Matthias Caenepeel, Martine Olivi, Fabien Seyfert.

This work was conducted in collaboration with Yves Rolain (VUB, Brussels, Belgium)

Coupling topologies that admit multiple realizations may lead to ambiguous de-embedding tuning procedures where distinct coupled resonator circuits are identified from the same measurements. This is for example the case of the well-known coupling topologies in triplets, quadruplets and extended boxed. If no additional measurements are performed on the DUT (device under tuning), the different solutions to the coupling matrix synthesis problem are undistinguishable, as they yield similar scattering responses. We therefore studied specific tuning strategies to discriminate among them. The later uses a sequence of measurements of the DUT, obtained after varying some discriminating tuning parameters of the filter and testing for coherence of the extracted circuits. This work was presented by Matthias Caenepeel at IMS 2015 in Phoenix [15] and at the EuMC 2015 in Paris [16]. In a similar vein Matthias is currently developing techniques taking advantage of the differential information provided by EM solvers in order to compute the Jacobian matrix of the identified coupling matrix(ces) with respect to the geometrical parameters of the filter.

6.4. Orthogonal Polynomials

Participant: Laurent Baratchart.

We studied this year the asymptotic behavior of the orthonormal polynomials P_n with respect to a non-negative weight w on a simply connected planar domain Ω:

$$\int_\Omega P_n \overline{P}_k w \, dm = \delta_{n,k},$$

with $\delta_{n,k}$ the Kronecker symbol. We proved that if Ω has boundary $\partial\Omega$ of class $C^{1,\alpha}$, $\alpha > 0$, and if w converges in some appropriate sense to a boundary function $w_1 \in L^p(\partial\Omega)$ while not vanishing "too much" at the boundary, then

$$P_n(z) = \left(\frac{n+1}{\pi}\right)^{1/2} z^n S^-_{w_1}(\Phi(z))\Phi^n(z)\Phi'(z)\{1+o(1)\}$$

outside the convex hull of Ω, with Φ the conformal map from the complement of Ω onto the complement of the unit disk normalized so that $\Phi'(\infty) = \infty$, and $S^-_{w_1}$ the so-called exterior Szegő function of w_1.

This generalizes considerably known asymptotics on analytic domains with Hölder smooth non vanishing weights [10]. The proof rests on some Hardy space theory, conformal mapping and $\overline{\partial}$ techniques. An exposition of the result was given at the conference *Orthogonal and Multiple Orthogonal Polynomials*, August 9-14 2015, Oaxaca (Mexico). An article is being written to report on this result.

6.5. Asymptotics of Rational Approximants

Participant: Laurent Baratchart.

This is joint work with M. Yattselev (IUPUI).

We studied best rational approximants in the *sup* norm to an analytic function f on compact set K of the analyticity domain Ω with connected complement. We showed that if the function can be continued analytically except over a set of logarithmic capacity zero comprising at most finitely many branchpoints, then the n-th root of the approximation error converges as n goes large to $e^{-2/C}$, with C the minimal Green capacity in $\mathbb{C} \setminus K$ of a compact set E outside of which f is single valued. Moreover, if $C > 0$, the normalized counting measure of the poles converges to the Green equilibrium distribution on E. We are currently considering the case of infinitely many branchpoints so as to get a somewhat final result on weak asymptotics in rational approximation to functions with polar singular set.

The proof rests on a blend of AAK-theory and potential theory.

7. Bilateral Contracts and Grants with Industry

7.1. Contract CNES-Inria-XLIM

This contract (reference Inria: 7066, CNES: 127 197/00) involving CNES, XLIM and Inria, focuses on the development of synthesis algorithms for N-ports microwave devices. The objective is to derive analytical procedures for the design of multiplexers and routers, as opposed to "black box optimization" which is usually employed in this field (for $N \geq 3$). Emphasis at the moment bears on so-called "star-topologies".

7.2. Contract CNES-Inria-UPV/EHU

This contract (reference CNES: RS14/TG-0001-019) involving CNES, University of Bilbao (UPV/EHU) and Inria aims at setting up a methodology for testing the stability of amplifying devices. The work at Inria is concerned with the design of frequency optimization techniques to identify the unstable part of the linearized response and analyze the linear periodic components.

7.3. Contract BESA GmbH-Inria

This is a research agreement between Inria (Apics and Athena teams) and the German company BESA [4], which deals with head conductivity estimation and co-advising of the doctoral work of C. Papageorgakis, see Section 6.1.1. BESA is funding half of the corresponding research grant, the other half is supported by Region PACA (BDO), see Section 8.1.

[4] http://www.besa.de/

7.4. Flextronics

Flextronics, active in the manufacturing of communication devices all over the world, bought two sets of licenses for Presto-HF and Dedale-HF. Deployment of our tools in their production facilities for wireless communication units is being studied.

8. Partnerships and Cooperations

8.1. Regional Initiatives

Contract Provence Alpes Côte d'Azur (PACA) Region - Inria, BDO (no. 2014-05764) funding the research grant of C. Papageorgakis, see Sections 6.1.1, 7.3.

8.2. National Initiatives

8.2.1. ANR COCORAM

The ANR (Astrid) project COCORAM (Co-design et co-intégration de réseaux d'antennes actives multi-bandes pour systèmes de radionavigation par satellite) started January 2014. We are associated with three other teams from XLIM (Limoges University), geared respectively towards filters, antennas and amplifiers design. The core idea of the project is to realize dual band reception an emission chains by co-conceiving the antenna, the filters, and the amplifier. We are specifically in charge of the theoretical design of the filters, matching the impedance of a bi-polarized dual band antenna. This represent a perfect training ground to test, apply and adapt our work on matching problems (see Section 6.2).

8.2.2. ANR MagLune

The ANR project MagLune (Magnétisme de la Lune) has been approved July 2014. It involves the Cerege (Centre de Recherche et d'Enseignement de Géosciences de l'Environnement, joint laboratory between Université Aix-Marseille, CNRS and IRD), the IPGP (Institut de Physique du Globe de Paris) and ISTerre (Institut des Sciences de la Terre). Associated with Cerege are Inria (Apics team) and Irphe (Institut de Recherche sur les Phénomènes Hors Équilibre, joint laboratory between Université Aix-Marseille, CNRS and École Centrale de Marseille). The goal of this project (led by geologists) is to understand the past magnetic activity of the Moon, especially to answer the question whether it had a dynamo in the past and which mechanisms were at work to generate it. Apics participates in the project by providing mathematical tools and algorithms to recover the remanent magnetization of rock samples from the moon on the basis of measurements of the magnetic field it generates. The techniques described in Section 6.1 are instrumental for this purpose.

8.3. European Initiatives

8.3.1. Collaborations with Major European Organizations

Apics is part of the European Research Network on System Identification (ERNSI) since 1992.

System identification deals with the derivation, estimation and validation of mathematical models of dynamical phenomena from experimental data.

8.4. International Initiatives

8.4.1. Inria Associate Teams not involved in an Inria International Labs

8.4.1.1. IMPINGE

Title: Inverse Magnetization Problems IN GEosciences.

International Partner (Institution - Laboratory - Researcher):

MIT - Department of Earth, Atmospheric and Planetary Sciences (United States) - Benjamin Weiss

Start year: 2013

See also: http://www-sop.inria.fr/apics/IMPINGE/

The purpose of the associate team IMPINGE is to develop efficient algorithms to recover the magnetization distribution of rock slabs from measurements of the magnetic field above the slab using a SQUID microscope (developed at MIT). The US team also involves a group of Mathematicians at Vanderbilt Univ.

8.4.2. Inria International Partners

8.4.2.1. Declared Inria International Partners

MIT-France seed funding is a competitive collaborative research program ran by the Massachusetts Institute of Technology (Cambridge, Ma, USA). Together with E. Lima and B. Weiss from the Earth and Planetary Sciences dept. at MIT, Apics obtained two-years support from the above-mentioned program to run a project entitled: "Development of Ultra-high Sensitivity Magnetometry for Analyzing Ancient Rock Magnetism"

NSF Grant L. Baratchart, S. Chevillard and J. Leblond are external investigators in the NSF Grant 2015-2018, "Collaborative Research: Computational methods for ultra-high sensitivity magnetometry of geological samples" led by E.B. Saff (Vanderbilt Univ.) and B. Weiss. (MIT).

8.5. International Research Visitors

8.5.1. Visits of International Scientists

- Andrea Gombani (IEIIT-CNR, Padova, Italy, February 16-27).
- Michael Northington (Vanderbilt University, Nashville, Tennessee, USA, July 21-30).
- Vladimir Peller (Michigan State Univ., East Lansing, USA, September 2-30).
- Eduardo Lima (MIT, Boston, Massachusetts, USA, September 6-12).
- Isabella Sanders (MIT, Boston, Massachusetts, USA, September 6-12).

8.5.1.1. Internships

- Konstantinos Mavreas, Master 2 Computational Biology - UNSA (5 months), Dipole localization in Moon rocks from sparse magnetic data.

8.5.2. Visits to International Teams

8.5.2.1. Research stays abroad

L. Baratchart was a visiting scientist at Indiana University-Purdue University at Indianapolis (IUPUI), November 2015.

8.6. List of international and industrial partners

- Collaboration under contract with Thales Alenia Space (Toulouse, Cannes, and Paris), CNES (Toulouse), XLIM (Limoges), University of Bilbao (Universidad del País Vasco / Euskal Herriko Unibertsitatea, Spain), BESA company (Munich), Flextronics.
- Regular contacts with research groups at UST (Villeneuve d'Asq), Universities of Bordeaux-I (Talence), Orléans (MAPMO), Aix-Marseille (CMI-LATP), Nice Sophia Antipolis (Lab. JAD), Grenoble (IJF and LJK), Paris 6 (P. et M. Curie, Lab. JLL), Inria Saclay (Lab. Poems), Cerege-CNRS (Aix-en-Provence), CWI (the Netherlands), MIT (Boston, USA), Vanderbilt University (Nashville USA), Steklov Institute (Moscow), Michigan State University (East-Lansing, USA), Texas A&M University (College Station USA), Indiana University-Purdue University at Indianapolis, Politecnico di Milano (Milan, Italy), University of Trieste (Italy), RMC (Kingston, Canada), University of Leeds (UK), of Maastricht (the Netherlands), of Cork (Ireland), Vrije Universiteit Brussel (Belgium), TU-Wien (Austria), TFH-Berlin (Germany), ENIT (Tunis), KTH (Stockholm), University of Cyprus (Nicosia, Cyprus), University of Macau (Macau, China), SIAE Microelettronica (Milano).

- The project is involved in the GDR-project AFHP (CNRS), in the ANR (Astrid program) project COCORAM (with XLIM, Limoges, and DGA), in the ANR (Défis de tous les savoirs program) project MagLune (with Cerege, IPGP, ISTerre, Irphe), in a MIT-France collaborative seed funding, in the Associate Inria Team IMPINGE (with MIT, Boston), and in a NSF grant (with Vanderbilt University and MIT).

9. Dissemination

9.1. Promoting Scientific Activities

- L. Baratchart gave a talk at the International Instrumentation and Measurement Technology Conference (I2MTC IEEE), May 11-14 2015, Pisa (Italy), at the AMS-EMS-SPM meeting, June 10-13 2015, Porto (Portugal), at the 10-th ISAAC congress (International Society for Analysis, its Applications and Computation), August 3-8 2015, Macau (China); he was a colloquium speaker at IUPUI, November 2015.
- M. Caenepeel gave a talk at the IMS 2015 in Phoenix [15] and at the EuMC 2015 in Paris [16].
- S. Chevillard gave a talk at the 27th IFIP TC7 Conference on System Modelling and Optimization, Sophia Antipolis (July 2015).
- J. Leblond presented a communication at the seminar “Mathématiques pour l’Analyse des Données” (MAD), Nice, May 21.
- C. Papageorgakis presented posters at the 1st International Conference on Mathematical Neuroscience (ICMNS, Juan les Pins, Jun.) [17] and at the International Conference on Basic and Clinical Multimodal Imaging (BACI, Utrecht, the Netherlands, September) [18], [19].
- D. Ponomarev gave talks at the seminar “Modèles et Algorithmes Déterministes” of Lab. Jean Kuntzmann, Univ. J. Fourier, Grenoble (May 28) and at the 27th IFIP TC7 Conference on System Modelling and Optimization, Sophia Antipolis (July 2015).
- F. Seyfert gave a talk at the European Microwave Week 2015 in the workshop dedicated to "Recent Advances in the Synthesis of Microwave Filters and Multiplexers", Paris, France

9.1.1. Scientific events organisation

L. Baratchart and J. Leblond organized a special session on “Inverse Elliptic Problems” at the 27th IFIP TC7 Conference on System Modelling and Optimization, Sophia Antipolis (July 2015).

K. Mavreas and C. Papageorgakis are the PhD students in charge of the PhD students Seminar within the Research Center (since September).

9.1.1.1. Member of the conference program committees

L. Baratchart was on the Program Committee of the 17th IFAC Symposium on System Identification (SYSID 2015), Beijing, China, October 19-21, 2015.

F. Seyfert was a member of the technical committee of the conference "International Workshop on Microwave Filters" in Toulouse, France, March 23-25 2015, http://www.iwmf2015.com/

9.1.2. Journal

9.1.2.1. Member of the editorial boards

L. Baratchart is a member of the Editorial Boards of the journals *Constructive Methods and Function Theory* and *Complex Analysis and Operator Theory*.

9.1.2.2. Reviewer - Reviewing activities

L. Baratchart was a reviewer for several journals including, *SIAM Journal on Analysis, Inverse Problems, Journal of Approximation Theory, Annales de l’Institut Fourier*.

S. Chevillard was a reviewer for the journal *ACM Transactions on Mathematical Software*.

J. Leblond was a reviewer for the journals *Applied Mathematical Modelling, International Journal of Computer Mathematics*.

F. Seyfert was a reviewer of the journal *IEEE Microwave Transaction on Theory and Techniques*

9.1.3. Invited talks

L. Baratchart was an invited speaker at the Workshop on Blaschke Products and Function Theory, July 29-31 2015, Hong-Kong (China), and at the conference Orthogonal and Multiple Orthogonal Polynomials, August 9-14 2015, Oaxaca (Mexico). He was a plenary speaker at the “Journées du Gdr Analyse Fonctionnelle, Harmonique et Probabilités”, November 30-December 4 2015, Luminy (France).

J. Leblond was invited to give a communication at the conference “Harmonic Analysis, Function Theory, Operator Theory and Applications” (in honor of J. Esterle), Bordeaux, Jun. 1-4.

F. Seyfert gave a mini course on "Advanced Filter Synthesis" at the "International Workshop on Microwave Filters" in Toulouse, France, March 23-25 2015

9.1.4. Research administration

S. Chevillard is representative at the “comité de centre” and at the “comité des projets” (Research Center Inria-Sophia).

J. Leblond is an elected member of the “Conseil Scientifique” and of the “Commission Administrative Paritaire” of Inria. She is in charge of the mission “Conseil et soutien aux chercheurs” within the Research Center. She is also a member of the “Conseil Académique” of the Univ. Côte d’Azur (UCA).

9.2. Teaching - Supervision - Juries

9.2.1. Teaching

Colles: S. Chevillard is giving “Colles” at Centre International de Valbonne (CIV) (2 hours per week).

9.2.2. Supervision

PhD in progress: D. Ponomarev, Inverse problems for planar conductivity and Schrödinger PDEs, since Nov. 2012 (advisors: J. Leblond, L. Baratchart).

PhD in progress: M. Caenepeel, The development of models for the design of RF/microwave filters, since Feb. 2013 (advisors: Y. Rolain, M. Olivi, F. Seyfert).

PhD in progress: C. Papageorgakis, Conductivity model estimation, since Oct. 2014 (advisors: J. Leblond, M. Clerc, B. Lanfer).

PhD in progress: K. Mavreas, Inverse source problems in planetary sciences: dipole localization in Moon rocks from sparse magnetic data, since Oct. 2015 (advisors: S. Chevillard, J. Leblond).

9.2.3. Juries

L. Baratchart was a reviewer for the “Mémoire d’habilitation à diriger des Recherches” of Rachid Zarouf, Univ. Aix-Marseille, December 2015.

J. Leblond was a member of the “jury d’admission du concours CR” of Inria (Jun.).

9.3. Popularization

- M. Olivi is president of the Committee MASTIC (Commission d’Animation et de Médiation Scientifique) https://project.inria.fr/mastic/. She is responsible for Scientific Mediation.

10. Bibliography

Major publications by the team in recent years

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Publications of the year

Articles in International Peer-Reviewed Journals

[11] L. BARATCHART, S. CHEVILLARD, T. QIAN. *Minimax principle and lower bounds in H^2-rational approximation*, in "Journal of Approximation Theory", 2015, In Press. This is the corrected proof as pubished online by the journal [*DOI :* 10.1016/J.JAT.2015.03.004], https://hal.inria.fr/hal-00922815

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[15] M. CAENEPEEL, F. SEYFERT, Y. ROLAIN, M. OLIVI. *Microwave Filter Design based on Coupling Topologies with Multiple Solutions*, in "IMS (International Microwave Symposium)", Phoenix, United States, May 2015, https://hal.inria.fr/hal-01165533

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