IN PARTNERSHIP WITH:
CNRS
Université de Lorraine

Activity Report 2017

Project-Team ALICE

Geometry and Lighting

IN COLLABORATION WITH: Laboratoire lorrain de recherche en informatique et ses applications (LORIA)

RESEARCH CENTER
Nancy - Grand Est

THEME
Interaction and visualization

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Project-Team ALICE

Creation of the Project-Team: 2006 January 09

Keywords:

Computer Science and Digital Science:

A5.5.1. - Geometrical modeling
A5.5.2. - Rendering
A6.2.1. - Numerical analysis of PDE and ODE
A6.2.8. - Computational geometry and meshes
A8.1. - Discrete mathematics, combinatorics
A8.3. - Geometry, Topology

Other Research Topics and Application Domains:

B3.3.1. - Earth and subsoil
B5.1. - Factory of the future
B5.7. - 3D printing
B9.2.2. - Cinema, Television
B9.2.3. - Video games
B9.4.1. - Computer science
B9.4.2. - Mathematics
B9.4.3. - Physics

1. Personnel

Research Scientists

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Laurent Alonso [Inria, Researcher]
Samuel Hornus [Inria, Researcher]
Sylvain Lefebvre [Inria, Senior Researcher, HDR]
Jonàs Martínez Bayona [Inria, Researcher]
Nicolas Ray [Inria, Researcher]
Erica Schwindt [Inria, Starting Research Position]
Haichuan Song [Inria, Starting Research Position, from Sep 2017]

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Dmitry Sokolov [Univ de Lorraine, Associate Professor, HDR]
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Jean Hergel [Inria, until Jan 2017]
Maxence Reberol [Inria]
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External Collaborator

Guillaume Caumon [Univ de Lorraine, from Feb 2017]

2. Overall Objectives

2.1. Overall Objectives

ALICE is a project-team in Computer Graphics. The fundamental aspects of this domain concern the interaction of *light* with the *geometry* of the objects. The lighting problem consists in designing accurate and efficient *numerical simulation* methods for the light transport equation. The geometrical problem consists in developing new solutions to *transform and optimize geometric representations*. Our original approach to both issues is to restate the problems in terms of *numerical optimization*. We try to develop solutions that are *provably correct*, *numerically stable* and *scalable*.

To reach these goals, our approach consists in transforming the physical or geometric problem into a numerical optimization problem, studying the properties of the objective function and designing efficient minimization algorithms. Besides Computer Graphics, our goal is to develop cooperations with researchers and people from the industry, who test applications of our general solutions to various domains, comprising CAD, industrial design, oil exploration, plasma physics... Our solutions are distributed in both open-source software (Graphite, OpenNL, CGAL) and industrial software (Gocad, DVIZ).

Since 2010, we started to develop techniques to model not only virtual objects, but also real ones. Our "modeling and rendering" research axis evolved, and we generalized our results on by-example texture synthesis to the production of real objects, using 3D printers. As compared to virtual objects, this setting defines higher requirements for the geometry processing techniques that we develop, that need to be adapted to both numerical simulation and computer-aided fabrication. We study how to include *computational physics* into the loop, and simulation methods for various phenomena (*e.g.*, fluid dynamics).

3. Research Program

3.1. Introduction

Computer Graphics is a quickly evolving domain of research. These last few years, both acquisition techniques (*e.g.*, range laser scanners) and computer graphics hardware (the so-called GPU's, for Graphics Processing Units) have made considerable advances. However, despite these advances, fundamental problems still remain open. For instance, a scanned mesh composed of hundred millions of triangles cannot be used directly in real-time visualization or complex numerical simulation. To design efficient solutions for these difficult problems, ALICE studies two fundamental issues in Computer Graphics:

- the representation of the objects, *i.e.*, their geometry and physical properties;
- the interaction between these objects and light.

Historically, these two issues have been studied by independent research communities. However, we think that they share a common theoretical basis. For instance, multi-resolution and wavelets were mathematical tools used by both communities [31]. We develop a new approach, which consists in studying the geometry and lighting from the *numerical analysis* point of view. In our approach, geometry processing and light simulation are systematically restated as a (possibly non-linear and/or constrained) functional optimization problem. This type of formulation leads to algorithms that are more efficient. Our long-term research goal is to find a formulation that permits a unified treatment of geometry and illumination over this geometry.

3.2. Geometry Processing for Engineering

Keywords: Mesh processing, parameterization, splines

Geometry processing emerged in the mid-1990's as a promising strategy to solve the geometric modeling problems encountered when manipulating meshes composed of hundred millions of elements. Since a mesh may be considered to be a *sampling* of a surface - in other words a *signal* - the *digital signal processing* formalism was a natural theoretic background for this subdomain (see *e.g.*, [32]). Researchers of this domain then studied different aspects of this formalism applied to geometric modeling.

Although many advances have been made in the geometry processing area, important problems still remain open. Even if shape acquisition and filtering is much easier than 30 years ago, a scanned mesh composed of hundred million triangles cannot be used directly in real-time visualization or complex numerical simulation. For this reason, automatic methods to convert those large meshes into higher level representations are necessary. However, these automatic methods do not exist yet. For instance, the pioneer Henri Gouraud often mentions in his talks that the *data acquisition* problem is still open [22]. Malcolm Sabin, another pioneer of the "Computer Aided Geometric Design" and "Subdivision" approaches, mentioned during several conferences of the domain that constructing the optimum control-mesh of a subdivision surface so as to approximate a given surface is still an open problem [30]. More generally, converting a mesh model into a higher level representation, consisting of a set of equations, is a difficult problem for which no satisfying solutions have been proposed. This is one of the long-term goals of international initiatives, such as the AIMShape European network of excellence.

Motivated by gridding application for finite elements modeling for oil and gas exploration, within the context of the Gocad project, we started studying geometry processing in the late 90's and contributed to this area at the early stages of its development. We developed the LSCM method (Least Squares Conformal Maps) in cooperation with Alias Wavefront [27]. This method has become the de-facto standard in automatic unwrapping, and was adopted by several 3D modeling packages (including Maya and Blender). We explored various applications of the method, including normal mapping, mesh completion and light simulation [24].

However, classical mesh parameterization requires to partition the considered object into a set of topological disks. For this reason, we designed a new method (Periodic Global Parameterization) that generates a continuous set of coordinates over the object [28]. We also showed the applicability of this method, by proposing the first algorithm that converts a scanned mesh into a Spline surface automatically [26].

We are still not fully satisfied with these results, since the method remains quite complicated. We think that a deeper understanding of the underlying theory is likely to lead to both efficient and simple methods. For this reason, in 2012 we studied several ways of discretizing partial differential equations on meshes, including Finite Element Modeling and Discrete Exterior Calculus. In 2013, we also explored Spectral Geometry Processing and Sampling Theory (more on this below).

3.3. Computer Graphics

Keywords: texture synthesis, shape synthesis, texture mapping, visibility

Content creation is one of the major challenges in Computer Graphics. Modeling shapes and surface appearances which are visually appealing and at the same time enforce precise design constraints is a task only accessible to highly skilled and trained designers.

In this context the team focuses on methods for by-example content creation. Given an input example and a set of constraints, we design algorithms that can automatically generate a new shape (geometry+texture). We formulate the problem of content synthesis as the joint optimization of several objectives: Preserving the local appearance of the example, enforcing global objectives (size, symmetries, mechanical properties), reaching user defined constraints (locally specified geometry, contacts). This results in a wide range of optimization problems, from statistical approaches (Markov Random fields), to combinatorial and linear optimization techniques.

As a complement to the design of techniques for automatic content creation, we also work on the representation of the content, so as to allow for its efficient manipulation. In this context we develop data structures and algorithms targeted at massively parallel architectures, such as GPUs. These are critical to reach the interactive rates expected from a content creation technique. We also propose novel ways to store and access content defined along surfaces [29] or inside volumes [21] [25].

The team also continues research in core topics of computer graphics at the heart of realistic rendering and realistic light simulation techniques; for example, mapping textures on surfaces, or devising visibility relationships between 3D objects populating space.

4. Application Domains

4.1. Geometric Tools for Simulating Physics with a Computer

Numerical simulation is the main targeted application domain for the geometry processing tools that we develop. Our mesh generation tools are tested and evaluated within the context of our cooperation with the Gocad consortium, with applications in oil exploration and geomechanics, through co-advised Ph.D. theses (Arnaud Botella, Julien Renaudeau). We think that the hex-dominant meshes that we generate have geometrical properties that make them suitable for some finite element analyses. We work on evaluating and measuring their impact with simple problems (heat equation, linear elasticity) and then practical applications (unfolding geological layer), with the Ph.D. thesis of Maxence Reberol.

In numerical simulation, developing discrete formulations that satisfy the conservation laws (conservation of mass, conservation of energy, conservation of momentum) is important to ensure that the numerical simulation faithfully reflects the behavior of the physics. There are interesting relations with optimal transport theory, as explained by Benamou and Brenier who developed a numerical algorithm for optimal transport that uses a fluid dynamics formulation [20]. Conversely, some dynamics can be approximated by a series of optimal transport problems, as in the Jordan-Kinderlehrer-Otto scheme [23] and in recent works by Mérigot. We started developing efficient geometric algorithms and optimisation methods that may serve as the basis for implementing these numerical methods in 3D. We started discussions / cooperation projects with Quentin Mérigot (MOKAPLAN project).

4.2. Fabrication

Our work around fabrication and additive manufacturing finds applications in different fields. Our algorithms for fast geometric computations on solids (boolean operations, morphological operations) are useful to model a variety of shapes, from mechanical engineering parts to prosthetics for medical applications.

Our by-example techniques allow for simpler modeling and processing of very intricate geometries and therefore also find applications in art and design, for unusual shapes that would be very difficult to obtain otherwise. Extensions of these techniques also find applications for reproducing naturally occurring micro-structures from a scanned sample.

5. Highlights of the Year

5.1. Highlights of the Year

Sylvain Lefebvre initiated the team creation process for MFX (Matter from Graphics), a new team that will focus on synthesizing and designing complex shapes for additive manufacturing.

Jonàs Martínez have been awarded an ANR JCJC 2017 project entitled MuFFin (Microstructures Procedurales et Stochastiques pour la Fabrication Fonctionnelle). MuFFin aims at contributing a unified pipeline for the efficient and scalable synthesis, visualization, and modeling of additively manufactured microstructures with tailored macroscopic physical behavior. In an interdisciplinary effort, MuFFin will blend together computer and material science perspectives to deliver an integrated approach that is both computationally and physically sound.

6. New Software and Platforms

6.1. Graphite

Graphite: The Numerical Geometry Workbench
KEYWORDS: 3D modeling - Numerical Geometry - Texturing - Lighting - CAD - Visualization
SCIENTIFIC DESCRIPTION: Graphite is an experimental 3D modeler, built in top of the Geogram programming library. It has data structures and efficient OpenGL visualization for pointsets, surfacic meshes (triangles and polygons), volumetric meshes (tetrahedra and hybrid meshes). It has state-of-the-art mesh repair, remeshing, reconstruction algorithms. It also has an interface to the Tetgen tetrahedral mesh generator (by Hang Si). This year, Graphite3 was released. It is a major rewrite, based on Geogram, with increased software quality standards (zero warnings on all platforms, systematic documentation of all classes / all functions / all parameters, dramatically improved performances). It embeds Geogram (and optionally Vorpaline) with an easy-to-use Graphic User Interface.
FUNCTIONAL DESCRIPTION: Graphite is a dedicated software platform in numerical geometry that enables, among other things, 3D modelling and texture baking.

- Participants: Bruno Lévy, David Lopez, Dobrina Boltcheva, Jeanne Pellerin, Nicolas Ray and Samuel Hornus
- Contact: Bruno Lévy
- URL: http://alice.loria.fr/software/graphite

6.2. GEOGRAM

GEOGRAM : A functions library for geometric programming
KEYWORD: 3D modeling

FUNCTIONAL DESCRIPTION: GEOGRAM is a programming library with a set of basic geometric algorithms, such as search data structures (AABB tree, Kd tree), geometric predicates, triangulations (Delaunay triangulation, Regular triangulation), intersection between a simplicial mesh and a Voronoi diagram (restricted Voronoi diagram). GEOGRAM also includes a code generator for predicates (PCK: Predicate Construction Kit) and an efficient implementation of expansion arithmetics in arbitrary precision. GEOGRAM is shipped with WARPDRIVE, the first program that computes semi-discrete optimal transport in 3D.

- Participant: Bruno Lévy
- Contact: Bruno Lévy
- URL: http://alice.loria.fr

6.3. OpenNL

Open Numerical Library
KEYWORDS: 3D modeling - Numerical algorithm
SCIENTIFIC DESCRIPTION: Open Numerical Library is a library for solving sparse linear systems, especially designed for the Computer Graphics community. The goal for OpenNL is to be as small as possible, while offering the subset of functionalities required by this application field. The Makefiles of OpenNL can generate a single .c + .h file, very easy to integrate in other projects. The distribution includes an implementation of the Least Squares Conformal Maps parameterization method.
FUNCTIONAL DESCRIPTION: Open Numerical Library is a library for solving sparse linear systems, especially designed for the Computer Graphics community. The goal for OpenNL is to be as small as possible, while offering the subset of functionalities required by this application field.

RELEASE FUNCTIONAL DESCRIPTION: * OpenMP parallel solver * more compact data structures, X2 acceleration * SuperLU weak coupling (dynamically loads SuperLU .so if available) (latest version available as part of geogram http://alice.loria.fr/software/geogram/doc/html/index.html)

- Participants: Bruno Lévy, Nicolas Ray and Rhaleb Zayer
- Contact: Bruno Lévy
- URL: http://alice.loria.fr/index.php/software/4-library/23-opennl.html

6.4. IceSL

KEYWORD: Additive manufacturing
FUNCTIONAL DESCRIPTION: IceSL allows to model complex shapes through CSG boolean operations. Objects can be directly prepared and sent to a 3d printer for fabrication, without the need to compute an intermediate 3D mesh.

- Participants: Frédéric Claux, Jean Hergel, Jérémie Dumas, Jonas Martinez-Bayona, Samuel Hornus and Sylvain Lefebvre
- Contact: Sylvain Lefebvre
- URL: http://shapeforge.loria.fr/icesl/

6.5. LibSL

Simple Library For Graphics
KEYWORDS: 3D - Graphics
FUNCTIONAL DESCRIPTION: LibSL is a toolbox for rapid prototyping of computer graphics algorithms, under both OpenGL, DirectX 9 - 10, Windows and Linux.

- Participant: Sylvain Lefebvre
- Contact: Sylvain Lefebvre

6.6. 3DPrintScaffoldings

KEYWORDS: 3D - 3D modeling - Additive manufacturing
FUNCTIONAL DESCRIPTION: Support generation for additive manufacturing. Optimizes scaffolding made of vertical pillars and horizontal bars that are optimized to use minimal material, be easily removed and support the part at all stages of the fabrication process.

- Participants: Jean Hergel, Jérémie Dumas and Sylvain Lefebvre
- Partner: Université de Lorraine
- Contact: Sylvain Lefebvre
- URL: http://shapeforge.loria.fr/icesl/

6.7. VORPALINE

VORPALINE mesh generator
KEYWORDS: 3D modeling - Unstructured heterogeneous meshes
FUNCTIONAL DESCRIPTION: VORPALINE is a surfacic and volumetric mesh generator, for simplicial meshes (triangles and tetrahedra), for quad-dominant and hex-dominant meshes.

- Participant: Bruno Lévy
- Contact: Bruno Lévy
- URL: http://alice.loria.fr/index.php/erc-vorpaline.html

7. New Results

7.1. Geometric foundations

7.1.1. Detecting the Intersection of Two Convex Shapes by Searching on the 2-sphere

Participant: Samuel Hornus.

We take a look at the problem of deciding whether two convex shapes intersect or not. We do so through the well known lens of Minkowski sums and with a bias towards applications in computer graphics and robotics. We describe a new technique that works explicitly on the unit sphere, interpreted as the sphere of directions. In extensive benchmarks against various well-known techniques, ours is found to be slightly more efficient, much more robust and comparatively easy to implement. In particular, our technique is compared favourably to the ubiquitous algorithm of Gilbert, Johnson and Keerthi (GJK), and its decision variant by Gilbert and Foo. We provide an in-depth geometrical understanding of the differences between GJK and our technique and conclude that our technique is probably a good drop-in replacement when one is not interested in the actual distance between two non-intersecting shapes.

The work was published in the journal Computer-Aided Design (special issue for the Proceedings of Solid and Physical Modeling: SPM'17) [9]. The paper has received a best paper award (2nd place) at the SPM conference.

7.1.2. Decomposition of a Hexahedron into a Set of Tetrahedra

Participant: Laurent Alonso.

This year was marked by some works on the combinatorial decomposition of a generic hexahedron in a set of nonintersecting tetrahedra up to symmetries: it is well known that there are only six decompositions of a cube into tetrahedra ; we show that there are at most 1360 potential different decompositions of a hexahedron and at least 1357 are geometrically valid. Additional work is in progress in order to show that the last 3 remaining decompositions do not correspond to valid geometrical solutions.

7.1.3. Hash-based CSG Evaluation on GPU

Participants: Cédric Zanni, Sylvain Lefebvre.

We have developed a new evaluation scheme for Constructive Solid Geometry (CSG) modeling that is well adapted to modern GPU. The approach falls into the category of screen space techniques and can handle a large range of geometric representation. The proposed method relies on the idea of hashing in order to reduce the memory footprint for the processing of a given ray in the scene (e.g. for discovering which part of the space is within or outside the object) while allowing the evaluation of the CSG in amortized constant time. This memory reduction in turn allows to subdivide the space in order to apply progressively the rendering algorithm, ensuring that required data fit in the graphic memory. This improvement over previous approach allows to handle objects of higher complexity during both modeling and slicing for additive manufacturing.

7.2. Geomety processing

7.2.1. Hexahedral Meshes: Generation, Simulation, Evaluation

Participants: Maxence Reberol, Nicolas Ray, Dmitry Sokolov, Bruno Lévy.

We continued working on the generation of the so-called hexahedral (or hexahedral-dominant) meshes. It is believed that these meshes are more efficient (both in terms of required space and computational time) for certain physics and numerical simulation. However, they are much more difficult to generate, and no fully automatic method currently exists. It is a huge problem in the industry, that uses days/weeks/months of manual work to generate them, because they are preferred in certain domains (fluids, mechanics, wave propagation). In this research, we aim at answering the following questions:

- How can we generate a hexahedral (or hex-dominant) mesh in a fully automatic manner?
- How can we evaluate the quality of this mesh, suitability for numerical simulation?

In the context of Maxence Reberol's Ph.D. thesis, we developed new algorithms to answer both questions. To answer the first question, in [17], building on our previous results based on global parameterization, we proposed a method to mesh the globality of the domain, by isolating the singular zones of the parameterization and meshing them with a separate algorithm. To answer the second question, in [16], we proposed a new method to estimate the distance between two Finite Element simulations obtained from two different computational meshes / function bases. We started using our algorithm to compare the rate of convergence of the method as a function of element size h with different PDEs (Poisson, linear elasticity) using different function bases (tetrahedral: P1, P2, P3, hexahedral: Q1, Q2, Q3).

7.2.2. Surface Reconstruction

Participants: Dobrina Boltcheva, Bruno Lévy.

We developed a new algorithm [7] for surface reconstruction. Our algorithm is equivalent to Delaunay-based reconstruction, it computes the Delaunay triangulation restricted to an object computed from the input pointset. The object is a set of disks centered on the input points and perpendicular to estimated normals. The algorithm is fast and memory efficient, because the only used global data structure is a Kd-tree. Applications are demonstrated with a parallel implementation on a multicore processor, and a version for hand-held devices.

7.2.3. Geometric Algorithms for 3D modeling in Geo-sciences

Participants: Bruno Lévy.

We developed RingMesh [13], an application layer around our Geogram library, specialized for 3D modeling in Geo-sciences. The RingMesh library uses the mesh data structure and basic algorithms in Geogram to offer 3D modeling primitives well-suited to geosciences. It has datatypes to efficiently represent complicated 3D models of the underground, with the topological relationships between the interfaces (horizons and faults), as well as interfaces to 3D mesh generation softwares.

7.3. Additive Manufacturing

7.3.1. Iterative Carving for Self-supporting 3D Printed Cavities

Participants: Samuel Hornus and Sylvain Lefebvre.

This work explores the printing of shapes with as little material as possible, mostly with a view toward minimizing fabrication time for large pieces. In particular, it aims at modeling a structure of thin sheets inside a volume in such a way that the sheets and the boundary of the volume can be 3D-printed as is, without internal support.

The work is an adaptation of the technique developed earlier fo other applications in 3D printing and achieved state-of-the-art results. It is available as an Inria technical report [14].

7.3.2. Optimal Discrete Slicing

Participant: Sylvain Lefebvre.

This work is a collaboration with Marc Alexa and Kristian Hildebrand from TU Berlin. We developed a novel algorithm to compute the optimal decomposition of a 3D shape into layers of varying thickness, in a discrete setting. This answers a long standing problem in additive manufacturing. Our approach computes all optimal solutions for any number of slices by formulating the optimization as a dynamic programming problem. We developed efficient algorithms for both computing the geometric errors within each slice (based on volume difference) as well as for the optimizer. Our technique is the first to provide a provably optimal result and outperforms all existing heuristics. The work has been published in ACM TOG [5], presented at SIGGRAPH 2017 and is fully implemented within IceSL, available for public download.

7.3.3. Fabricable Tile Decors

Participants: Sylvain Lefebvre and Jonàs Martínez.

We propose a modeling technique to produce large objects whose surface is composed of user-provided decorative tiles. Such objects are very inefficient to 3D print as they occupy a large volume while in fact using little material. On low end printers they require large amounts of support structures which are difficult to remove. We propose a decomposition of the input shape into sets of planar patches that can print flat and can be later assembled into stable structures. This work is a collaboration with Hong Kong University in the context of the PrePrint3D associated team. It was published in ACM TOG [8] and presented at SIGGRAPH Asia 2017.

7.3.4. Visualizing and Fabricating Complex Internal Structures

Participant: Sylvain Lefebvre.

This work considers efficient display and manufacturing of extremely detailed internal structures described by implicit (procedural) indicator functions [15]. We describe a technique for their progressive rendering when the structures fill an envelope provided as a 3D mesh. We also describe how to efficiently extract slices for additive manufacturing, in a process that is both computationally and memory efficient. This work was presented at the Visual Analytics conference (Moscow, 2017) and is under submission to the Scientific Visualization journal.

7.3.5. Orthotropic k-nearest Foams for Additive Manufacturing

Participants: Jonàs Martínez, Haichuan Song, Jérémie Dumas, Sylvain Lefebvre.

We proposed a novel metamaterial with controllable, freely orientable, orthotropic elastic behavior – orthotropy means that elasticity is controlled independently along three orthogonal axes, which leads to materials that better adapt to uneven, directional load scenarios, and offer a more versatile material design primitive. The fine-scale structures are generated procedurally by a stochastic process, and resemble a foam. This work has been published in ACM TOG [12], and presented at SIGGRAPH 2017.

7.3.6. Color Fused Filament Fabrication

Participants: Haichuan Song, Sylvain Lefebvre.

Traditional filament printers cannot truly reproduce colored objects. The best current techniques rely on a form of dithering exploiting occlusion, that was only demonstrated for shades of two base colors and that behaves differently depending on surface slope. We explored a novel approach for 3D printing colored objects, capable of creating controlled gradients of varying sharpness. Our technique exploits off-the-shelves nozzles that are designed to mix multiple filaments in a small melting chamber, obtaining intermediate colors once the mix is stabilized. The key idea is to divide each input layer into a set of sublayers, each having a different constant color. By locally changing the thickness of the sublayers, we change the color that is perceived at a given location. By optimizing the choice of colors of each sublayer, we further improve quality and allow the use of different numbers of input filaments. We demonstrate our results by building a functional color printer using low cost, off-the-shelves components. Using our tool a user can paint a 3D model and directly produce its physical counterpart, using any material and color available for fused filament fabrication. This work has been submitted and is available at [18].

7.3.7. Anti-aliasing for Fused Filament Deposition

Participants: Haichuan Song, Sylvain Lefebvre.

Layered manufacturing inherently suffers from staircase defects along surfaces that are gently sloped with respect to the build direction. Reducing the slice thickness improves the situation but also largely increases the print time. We proposed a simple yet effective technique to improve the print accuracy for layered manufacturing by filament deposition. It better reproduces the geometry of sloped surfaces without increasing the print time. The key idea is to perform a local anti-aliasing, working at a sub-layer accuracy to produce slightly curved deposition paths and reduce approximation errors. We further split and order paths to minimize defects due to the extruder nozzle shape, avoiding any change to the existing hardware. We apply and analyze our approach on 3D printed examples, showing that our technique greatly improves surface accuracy and silhouette quality while keeping the print time nearly identical. This work has been published in the Computer Aided Design (CAD) journal [19].

8. Bilateral Contracts and Grants with Industry

8.1. Bilateral Contracts with Industry

We developed a collaboration with a local company regarding additive manufacturing technologies. This contract allowed us to host two interns (Mélanie Siret and Jimmy Etienne), both supervised by Sylvain Lefebvre. The topic is confidential.

9. Partnerships and Cooperations

9.1. Regional Initiatives

9.1.1. CPER (2014-2020) 50 k

Sylvain Lefebvre coordinates a work package for the CPER 2014-2020. It involves several members of ALICE as well as laboratories within the Nancy area (Institut Jean Lamour, LRGP, ERPI). Our goal is to consider the interaction between software and material in the additive manufacturing process, with a focus on filament-based printers.

9.1.2. PIC (2015-2017) 150 k

The PIC project (Polymères Innovants Composites) is a collaboration between Inria, Institut Jean Lamour and Ateliers Cini, funded by Région Lorraine. The goal is to develop a new additive manufaturing process using filaments of composite materials with applications in mechanical engineering and the medical domain. Our goal in the project is to provide novel ways to deposit the filament that is better suited to the considered materials and improves the quality of the final parts.

9.2. National Initiatives

9.2.1. EXPLORAGRAM

Inria exploratory project EXPLORAGRAM (in cooperation with MOKAPLAN): We explored new algorithms for computational optimal transport. The project allowed us to hire a post-doc for 18 months (Erica Schwindt). She worked on the semi-discrete algorithm, and its application to the simulation of fluid-structure interactions. The project allowed to strengthen the cooperation with MOKAPLAN. It also allowed us to start exploring new cooperations, with Institut d'Astrophysique de Paris, on early universe reconstruction.

9.2.2. ANR MAGA (2016-2020)

We participate to the ANR MAGA (ANR-16-CE40-0014) on the Monge Ampere equation and computational geometry. In this ANR project, we cooperate with Quentin Merigot and other researchers of the MOKAPLAN Inria team on new computational methods for optimal transport.

9.2.3. ANR ROOT (2016-2020)

We participate to the Young Researcher ANR ROOT (ANR-16-CE23-0009) on Optimal Transport for computer graphics, with Nicolas Bonneel (CNRS Lyon) as Principal Investigator. In the context of this project, we develop a new symmetric algorithm for semi-discrete optimal transport that optimizes for both the location of the samples and their Lagrange multipliers. An ENS training period will start in Jan. 2018 (Agathe Herrou), hosted in Nancy.

9.3. European Initiatives

9.3.1. FP7 & H2020 Projects

9.3.1.1. SHAPEFORGE

Title: ShapeForge: By-Example Synthesis for Fabrication

Program: FP7 (ERC Starting Grant)

Duration: December 2012 - November 2017

Coordinator: Inria

Inria contact: Sylvain Lefebvre

Despite the advances in fabrication technologies such as 3D printing, we still lack the software allowing for anyone to easily manipulate and create useful objects. Not many people possess the required skills and time to create elegant designs that conform to precise technical specifications. 'By–example' shape synthesis methods are promising to address this problem: New shapes are automatically synthesized by assembling parts cutout of examples. The underlying assumption is that if parts are stitched along similar areas, the result will be similar in terms of its low–level representation: Any small spatial neighborhood in the output matches a neighborhood in the input. However, these approaches offer little control over the global organization of the synthesized shapes, which is randomized. The ShapeForge challenge is to automatically produce new objects visually similar to a set of examples, while ensuring that the generated objects can enforce a specific purpose, such as supporting weight distributed in space, affording for seating space or allowing for light to go through. These properties are crucial for someone designing furniture, lamps, containers, stairs and many of the common objects surrounding us. The originality of our approach is to cast a new view on the problem of 'by–example' shape synthesis, formulating it as the joint optimization of 'by–example' objectives, semantic descriptions of the content, as well as structural and fabrication objectives. Throughout the project, we will consider the full creation pipeline, from modeling to the actual fabrication of objects on a 3D printer. We will test our results on printed parts, verifying that they can be fabricated and exhibit the requested structural properties in terms of stability and resistance.

9.3.1.2. ICEXL

Title: IceXL: Advanced modeling and slicing software for additive manufacturing

Program: FP7 (ERC Proof of Concept)

Duration: November 2016 - February 2018

Coordinator: Inria

Inria contact: Sylvain Lefebvre

The ICEXL Proof of Concept projects aims at further developing our software IceSL and its industrial potential. We have released several new major features than allowed the software to gain visibility (as shown by a strong increase in downloads towards the end of 2017, 1500+ downloads in November). We have teamed with a selected number of industrial partners to work towards industrial use, and have ongoing discussions regarding technology transfer and licensing.

9.4. International Initiatives

9.4.1. Inria Associate Teams Not Involved in an Inria International Lab

9.4.1.1. PREPRINT3D

Title: Model Preparation for 3D Printing

International Partner (Institution - Laboratory - Researcher):

HKU (Hong Kong, China) - Department of Computer Science (CS) - Wenping Wang

Start year: 2017

We seek to develop novel ways to prepare and model objects for 3D printing which better take into account limitations of the fabrication processes as well as real-world properties such as the mechanical strength of the printed object. This is especially important when targeting an audience which is not familiar with the intricacies of industrial design. We target complex, intricate shapes such as models of vegetation and highly detailed meshes, as well as models with thin walls such as architectural models.

9.4.2. Inria International Partners

9.4.2.1. Informal International Partners

Jean-Francois Remacle (University of Louvain, Belgium), we cooperate on hexahedral-dominant meshing (visits, students exchange). Our former Ph.D. student Jeanne Pellerin is doing a post-doc in his lab.

9.5. International Research Visitors

9.5.1. Visits of International Scientists

Li-Yi Wei visited us from 05/04/17 to 18/04/17 to work on the topic of element based topology optimization with Jérémie Dumas, Jonàs Martínez and Sylvain Lefebvre. This work was submitted to SIGGRAPH but not accepted, we plan to resubmit it early 2018.

9.5.1.1. Internships

Sylvain Lefebvre supervised Mélanie Siret for a 3 months internship, as well as Jimmy Etienne for a 6 months internship.

9.5.2. Visits to International Teams

B. Lévy and Nicolas Ray visited Jean-Francois Remacle (U. Louvain, Belgium). B. Lévy visited Jan Obloj (Oxford, U.K.).

10. Dissemination

10.1. Promoting Scientific Activities

10.1.1. Scientific Events Organisation

10.1.1.1. General Chair, Scientific Chair

We organized a free workshop on February 2017 (http://shapeforge.loria.fr/workshop/) in-between the PhD defenses of Jean Hergel and Jérémie Dumas. International jury members gave talks; the workshop was open to the public. We recorded all the talks and made them available online (https://www.youtube.com/playlist?list=PLfUzw-QfoyuWLPJpCCp6xwTAPMIjt7Q-k).

10.1.1.2. Member of the Organizing Committees

B. Lévy is an elected member of the steering committee of Shape Modeling Association (since 2017).

10.1.2. Scientific Events Selection

10.1.2.1. Chair of Conference Program Committees

Sylvain Lefebvre was program co-chair for SMI 2017. B. Lévy was conference co-chair of WSOM 2017.

10.1.2.2. Member of the Conference Program Committees

B. Lévy was program committee member of Eurographics 2017, SGP 2017, SPM 2017

10.1.2.3. Reviewer

Members of the team were reviewers for SoCG'17, Eurographics, SIGGRAPH, Computer Aided Design, Pacific Graphics, and SPM.

10.1.3. Journal

10.1.3.1. Member of the Editorial Boards

- B. Lévy is a member of the editorial board of ACM Transactions on Graphics.
- B. Lévy is a member of the editorial board of Graphical Models (Elsevier)
- B. Lévy is a member of the editorial board of Computer Graphics and Applications
- Sylvain Lefebvre was associated editor for ACM Transactions on Graphics.

10.1.3.2. Reviewer - Reviewing Activities

Members of the team were reviewers for Computer Aided Design (CAD),Discrete Applied Mathematics (Elsevier), Transactions on Visualization and Computer Graphics (IEEE), and Computers & Graphics (Elsevier).

10.1.4. Invited Talks

B. Lévy gave an invited keynote talk at Eurographics 2017

10.1.5. Research Administration

Samuel Hornus was a moderator of the CDT (Technological Development Commission) of CRI Nancy.

10.2. Teaching - Supervision - Juries

10.2.1. Teaching

Licence: Samuel Hornus, Module « mathématiques appliquées à l'informatique » (MAI). 64h eq. TD, niveau L3. à Télécom Nancy, France.

Licence: Cédric Zanni, Informatique 2, 40h ETD, L3, École des Mines de Nancy, France.

Licence: Cédric Zanni, Informatique 1, 20h ETD, L3, École des Mines de Nancy, France.

Master: Nicolas Ray, Initiation à la recherche, 10 heures, M1, université de Lorraine, France.

Master: Sylvain Lefebvre, Programmation pour le jeux vidéo, 30h ETD, Ecole des Mines de Nancy, France.

Master: Sylvain Lefebvre, Introduction au parallélisme et au graphisme, 9h ETD, ENSG Nancy, France.

Master: Sylvain Lefebvre, Introduction à la fabrication additive, 9h ETD, ENSEM Nancy, France.

Master: Cédric Zanni, Database essentials, 24h ETD, M1, École des Mines de Nancy, France.

Master: Cédric Zanni, Techniques de l'animation et du jeu vidéo, 27h ETD, M1, École des Mines de Nancy, France.

Master: Cédric Zanni, Software Engineering, 18h ETD, M1, École des Mines de Nancy, France.

Master: Cédric Zanni, Operating System, 17h ETD, M1, École des Mines de Nancy, France.

Master: Cédric Zanni, Introduction au C/C++, 34h ETD, M1, École des Mines de Nancy, France.

Master: Cédric Zanni, UML, 4h ETD, M1, École des Mines de Nancy, France.

Master: B. Lévy teaches Numerical Methods in École nationale supérieure de géologie de Nancy, France (12h).

Master: B. Lévy teaches Algorithmic Gems in École des Mines de Nancy, France (8h).

10.2.2. Supervision

Ph.Ds in progress supervised by B. Lévy:

- Maxence Reberol, Finite elements on hex-dominant meshes, third year. Funding: ERC ShapeForge and région Lorraine. Defense: February 15th 2018.
- Pierre Anquez, Robust meshing of geological data, consortium GOCAD, second year, co-supervised with Guillaume Caumon.
- Julien Renaudeau, Constrained geo-modelling using implicit functions, second year, Cifre Schlumberger, co-supervised with Guillaume Caumon.

Jérémie Dumas and Jean Hergel, supervised by Sylvain Lefebvre, defended their PhD theses in February 2017. Jérémie Dumas received the University of Lorraine award for the best 2017 PhD thesis in Computer Science.

10.2.3. Juries

Sylvain Lefebvre was "rapporteur" for the PhD of Benoit Arbelot (Grenoble Universities, defended April 7 2017), for the PhD thesis of Luigi Malomo (CNR Pisa) and Christian Santoni (Computational Design Lab, University of Rome).

10.3. Popularization

Haichuan Song, Sylvain Lefebvre and Salim Perchy presented IceSL at a booth at the Maker Faire Rome 2017 (http://www.makerfairerome.eu/en/). This is a major event with 100K+ visitors every year. We presented the software to students, general public and 3D printing enthusiasts. We received a Maker of Merit award for our project.

Sylvain Lefebvre participated to the radio program "La méthode scientifique" on France Culture, aired on November 8 2017 (https://www.franceculture.fr/emissions/la-methode-scientifique/la-methode-scientifique-mercredi-8-novembre-2017).

B. Lévy gave an invited talk at "Forum Art Inovation" organized by IRCAM, Paris, Pompidou center (16-03-2017): http://medias.ircam.fr/stream/ext/video/files/2017/04/04/Vertigo_16mars2017_%20BrunoLevy_GB.mov.webm.

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