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**Université Nice - Sophia  
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Activity Report 2019

**Project-Team MCTAO**

Mathematics for Control, Transport and  
Applications

IN COLLABORATION WITH: Institut Mathématique de Bourgogne, Laboratoire Jean-Alexandre Dieudonné (JAD)

RESEARCH CENTER  
**Sophia Antipolis - Méditerranée**

THEME  
**Optimization and control of dynamic  
systems**



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## Project-Team MCTAO

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#### Computer Science and Digital Science:

A5.10.3. - Planning  
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A6.1.1. - Continuous Modeling (PDE, ODE)  
A6.1.5. - Multiphysics modeling  
A6.2.1. - Numerical analysis of PDE and ODE  
A6.2.6. - Optimization  
A6.4. - Automatic control  
A6.4.1. - Deterministic control  
A6.4.3. - Observability and Controlability  
A6.4.4. - Stability and Stabilization  
A6.4.6. - Optimal control  
A6.5. - Mathematical modeling for physical sciences  
A8.2.3. - Calculus of variations  
A8.12. - Optimal transport

#### Other Research Topics and Application Domains:

B2.6. - Biological and medical imaging  
B2.7.2. - Health monitoring systems  
B5.2.3. - Aviation  
B5.2.4. - Aerospace  
B5.6. - Robotic systems

## 1. Team, Visitors, External Collaborators

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## 2. Overall Objectives

### 2.1. Control, Transport and Dynamics

Our goal is to develop methods in geometric control theory for finite-dimensional nonlinear systems, as well as in optimal transport, and to transfer our expertise through real applications of these techniques.

Our primary domain of industrial applications in the past years is space engineering, namely designing trajectories in space mechanics using optimal control and stabilization techniques: transfer of a satellite between two Keplerian orbits, rendez-vous problem, transfer of a satellite from the Earth to the Moon or more complicated space missions. A second field of applications is quantum control with applications to Nuclear Magnetic Resonance and medical image processing. A third and more recent one is the control of micro-swimmers, i.e. swimming robots where the fluid-structure coupling has a very low Reynolds number.

There is also a form of transfer to other mathematical fields: some problems in dynamical systems are being solved thanks to control theory techniques.

## 3. Research Program

### 3.1. Control Problems

McTAO's major field of expertise is control theory in the large sense. Let us give an overview of this field.

**Modelling.** Our effort is directed toward efficient methods for the control of real (physical) *systems*, based on a *model* of the system to be controlled. Choosing accurate models yet simple enough to allow control design is in itself a key issue. The typical continuous-time model is of the form  $dx/dt = f(x, u)$  where  $x$  is the *state*, ideally finite dimensional, and  $u$  the *control*; the control is left free to be a function of time, or a function of the state, or obtained as the solution of another dynamical system that takes  $x$  as an input. Modelling amounts to deciding the nature and dimension of  $x$ , as well as the dynamics (roughly speaking the function  $f$ ). Connected to modeling is identification of parameters when a finite number of parameters are left free in " $f$ ".

**Controllability, path planning.** Controllability is a property of a control system (in fact of a model) that two states in the state space can be connected by a trajectory generated by some control, here taken as an explicit function of time. Deciding on local or global controllability is still a difficult open question in general. In most cases, controllability can be decided by linear approximation, or non-controllability by "physical" first integrals that the control does not affect. For some critically actuated systems, it is still difficult to decide local or global controllability, and the general problem is anyway still open. Path planning is the problem of constructing the control that actually steers one state to another.

**Optimal control.** In optimal control, one wants to find, among the controls that satisfy some constraints at initial and final time (for instance given initial and final state as in path planning), the ones that minimize some criterion. This is important in many control engineering problems, because minimizing a cost is often very relevant. Mathematically speaking, optimal control is the modern branch of the calculus of variations, rather well established and mature [71], [47], [34], but with a lot of hard open questions. In the end, in order to actually compute these controls, ad-hoc numerical schemes have to be derived for effective computations of the optimal solutions. See more about our research program in optimal control in section 3.2.

**Feedback control.** In the above two paragraphs, the control is an explicit function of time. To address in particular the stability issues (sensitivity to errors in the model or the initial conditions for example), the control has to be taken as a function of the (measured) state, or part of it. This is known as closed-loop control; it must be combined with optimal control in many real problems. On the problem of stabilization, there is longstanding research record from members of the team, in particular on the construction of “Control Lyapunov Functions”, see [62], [73].

**Classification of control systems** One may perform various classes of transformations acting on systems, or rather on models... The simpler ones come from point-to-point transformations (changes of variables) on the state and control, and more intricate ones consist in embedding an extraneous dynamical system into the model, these are dynamic feedback transformations, they change the dimension of the state. In most problems, choosing the proper coordinates, or the right quantities that describe a phenomenon, sheds light on a path to the solution; these proper choices may sometimes be found from an understanding of the modelled phenomena, or it can come from the study of the geometry of the equations and the transformation acting on them. This justifies the investigations of these transformations on models for themselves. These topics are central in control theory; they are present in the team, see for instance the classification aspect in [52] or —although this research has not been active very recently— the study [70] of dynamic feedback and the so-called “flatness” property [65].

## 3.2. Optimal Control and its Geometry

Let us detail our research program concerning optimal control. Relying on Hamiltonian dynamics is now prevalent, instead of the Lagrangian formalism in classical calculus of variations. The two points of view run parallel when computing geodesics and shortest path in Riemannian Geometry for instance, in that there is a clear one-to-one correspondance between the solutions of the geodesic equation in the tangent bundle and the solution of the Pontryagin Maximum Principle in the cotangent bundle. In most optimal control problems, on the contrary, due to the differential constraints (velocities of feasible trajectories do not cover all directions in the state space), the Lagrangian formalism becomes more involved, while the Pontryagin Maximum Principle keeps the same form, its solutions still live in the cotangent bundle, their projections are the extremals, and a minimizing curve must be the projection of such a solution.

**Cut and conjugate loci.** The cut locus —made of the points where the extremals lose optimality— is obviously crucial in optimal control, but usually out of reach (even in low dimensions), and anyway does not have an analytic characterization because it is a non-local object. Fortunately, conjugate points —where the extremals lose *local* optimality— can be effectively computed with high accuracy for many control systems. Elaborating on the seminal work of the Russian and French schools (see [76], [35], [36] and [53] among others), efficient algorithms were designed to treat the smooth case. This was the starting point of a series of papers of members of the team culminating in the outcome of the *cotcot* software [46], followed by the *Hampath* [55] code. Over the years, these codes have allowed for the computation of conjugate loci in a wealth of situations including applications to space mechanics, quantum control, and more recently swimming at low Reynolds number. With in mind the two-dimensional analytic Riemannian framework, a heuristic approach to the global issue of determining cut points is to search for singularities of the conjugate loci; this line is however very delicate to follow on problems stemming from applications in three or more dimensions (see e.g. [56] and [43]). In all these situations, the fundamental object underlying the analysis is the curvature

tensor. In Hamiltonian terms, one considers the dynamics of subspaces (spanned by Jacobi fields) in the Lagrangian Grassmannian [33]. This point of view withstands generalizations far beyond the smooth case: In  $L^1$ -minimization, for instance, discontinuous curves in the Grassmannian have to be considered (instantaneous rotations of Lagrangian subspaces still obeying symplectic rules [60]). The cut locus is a central object in Riemannian geometry, control and optimal transport. This is the motivation for a series of conferences on “The cut locus: A bridge over differential geometry, optimal control, and transport”, co-organized by team members and Japanese colleagues, the next one should take place in Nice in 2020.

**Riemann and Finsler geometry.** Studying the distance and minimising geodesics in Riemannian Geometry or Finsler Geometry is a particular case of optimal control, simpler because there are no differential constraints; it is studied in the team for the following two reasons. On the one hand, after some transformations, like averaging (see section 3.2) or reduction, some more difficult optimal control problems lead to a Riemann or Finsler geometry problem. On the other hand, optimal control, mostly the Hamiltonian setting, brings a fresh viewpoint on problems in Riemann and Finsler geometry. On Riemannian ellipsoids of revolution, the optimal control approach allowed to decide on the convexity of the injectivity domain, which, associated with non-negativity of the Ma-Trudinger-Wang curvature tensor, ensures continuity of the optimal transport on the ambient Riemannian manifold [64], [63]. The analysis in the oblate geometry [44] was completed in [59] in the prolate one, including a preliminary analysis of non-focal domains associated with conjugate loci. Averaging in systems coming from space mechanics control (see sections 3.2 and 4.1) with  $L^2$ -minimization yields a Riemannian metric, thoroughly computed in [42] together with its geodesic flow; in reduced dimension, its conjugate and cut loci were computed in [45] with Japanese Riemannian geometers. Averaging the same systems for minimum time yields a Finsler Metric, as noted in [41]. In [51], the geodesic convexity properties of these two types of metrics were compared. When perturbations (other than the control) are considered, they introduce a “drift”, *i.e.* the Finsler metric is no longer symmetric.

**Sub-Riemannian Geometry.** Optimal control problems that pertain to sub-Riemannian Geometry bear all the difficulties of optimal control, like the role of singular/abnormal trajectories, while having some useful structure. They lead to many open problems, like smoothness of minimisers, see the recent monograph [69] for an introduction. Let us detail one open question related to these singular trajectories: the Sard conjecture in sub-Riemannian geometry. Given a totally non-holonomic distribution on a smooth manifold, the Sard Conjecture is concerned with the size of the set of points that can be reached by singular horizontal paths starting from a given point. In the setting of rank-two distributions in dimension three, the Sard conjecture is that this set should be a subset of the so-called Martinet surface, indeed small both in measure and in dimension. In [39], it has been proved that the conjecture holds in the case where the Martinet surface is smooth. Moreover, the case of singular real-analytic Martinet surfaces was also addressed. In this case, it was shown that the Sard Conjecture holds true under an assumption of non-transversality of the distribution on the singular set of the Martinet surface. It is, of course, very interesting to get rid of the remaining technical assumption, or to go to higher dimension. Note that any Sard-type result has strong consequences on the regularity of sub-Riemannian distance functions and in turn on optimal transport problems in the sub-Riemannian setting.

**Small controls and conservative systems, averaging.** Using averaging techniques to study small perturbations of integrable Hamiltonian systems is as old an idea as celestial mechanics. It is very subtle in the case of multiple periods but more elementary in the single period case, here it boils down to taking the average of the perturbation along each periodic orbit [37], [75]. This line of research stemmed out of applications to space engineering (see section 4.1): the control of the super-integrable Keplerian motion of a spacecraft orbiting around the Earth is an example of a slow-fast controlled system. Since weak propulsion is used, the control itself acts as a perturbation, among other perturbations of similar magnitudes: higher order terms of the Earth potential (including  $J_2$  effect, first), potential of more distant celestial bodies (such as the Sun and the Moon), atmospheric drag, or even radiation pressure. Properly qualifying the convergence properties (when the small parameter goes to zero) is important and is made difficult by the presence of control. In [41], convergence is seen as convergence to a differential inclusion; this applies to minimum time; a contribution of this work is to



put forward the metric character of the averaged system by yielding a Finsler metric (see section 3.2). Proving convergence of the extremals (solutions of the Pontryagin Maximum Principle) is more intricate. In [58], standard averaging ([37], [75]) is performed on the minimum time extremal flow after carefully identifying slow variables of the system thanks to a symplectic reduction. This alternative approach allows to retrieve the previous metric approximation, and to partly address the question of convergence. Under suitable assumptions on a given geodesic of the averaged system (disconjugacy conditions, namely), one proves existence of a family of quasi-extremals for the original system that converge towards the geodesic when the small perturbation parameter goes to zero. This needs to be improved, but convergence of all extremals to extremals of an “averaged Pontryagin Maximum Principle” certainly fails. In particular, one cannot hope for  $C^1$ -regularity on the value function when the small parameter goes to zero as swallowtail-like singularities due to the structure of local minima in the problem are expected. (A preliminary analysis has been made in [57].)

**Optimality of periodic solutions/periodic controls.** When seeking to minimize a cost with the constraint that the controls and/or part of the states are periodic (and with other initial and final conditions), the notion of conjugate points is more difficult than with straightforward fixed initial point. In [48], for the problem of optimizing the efficiency of the displacement of some micro-swimmers (see section 4.3) with periodic deformations, we used the sufficient optimality conditions established by R. Vinter’s group [80], [66] for systems with non unique minimizers due to the existence of a group of symmetry (always present with a periodic minimizer-candidate control). This takes place in a long term collaboration with P. Bettiol (Univ. Bretagne Ouest) on second order sufficient optimality conditions for periodic solutions, or in the presence of higher dimensional symmetry groups, following [80], [66]. Another question relevant to locomotion is the following. Observing animals (or humans), or numerically solving the optimal control problem associated with driftless micro-swimmers for various initial and final conditions, we remark that the optimal strategies of deformation seem to be periodic, at least asymptotically for large distances. This observation is the starting point for characterizing dynamics for which some optimal solutions are periodic, and asymptotically attract other solutions as the final time grows large; this is reminiscent of the “turnpike theorem” (classical, recently applied to nonlinear situations in [79]).

**Software.** These applications (but also the development of theory where numerical experiments can be very enlightening) require many algorithmic and numerical developments that are an important side of the team activity. The software *HamPath* (see section 6.1) is maintained by former members of the team in close collaboration with McTAO. We also use direct discretization approaches (such as the *Bocop* solver developed by COMMANDS) in parallel. Apart from this, we develop on-demand algorithms and pieces of software, for instance we have to interact with a production software developed by Thales Alenia Space. A strong asset of the team is the interplay of its expertise in geometric control theory with applications and algorithms (see sections 4.1 to 4.3) on one hand, and with optimal transport, and more recently Hamiltonian dynamics, on the other. In 2019, the ADT ct (Control Toolbox) has started with a first sprint in “AMDT mode” with Sophia SED during spring 2019. In addition to McTAO, researchers from the CAGE team (Inria Paris) and the APO team (CNRS Toulouse) are involved. The idea is to put together the efforts on BOCOP and HamPath to go towards a reference toolbox in optimal control. After the first sprint cycle (24 months being planned on the whole action), some starting points have been addressed including: continuous integration for BOCOP and HamPath, refresh on collaborative development tools, first steps of software refactoring, first test of a high-end interface (through scripting, notebooks, or an *ad hoc* GUI). The next sprint is planned during spring 2020.

### 3.3. Optimal Transport

Given two measures, and calling transport maps the maps that transport the first measure into the second one, the Monge-Kantorovich problem of Optimal Transport is the search of the minimum of some cost on the set of transport maps. The cost of a map usually comes from some point to point cost and the transport measure. This topic attracted renewed attention in the last decade, and has ongoing applications of many types. Matching optimal transport with geometric control theory is one originality of our team. Let us sketch an important

class of open problems. In collaboration with R. McCann [68], we worked towards identifying the costs that admit unique optimizers in the Monge-Kantorovich problem of optimal transport between arbitrary probability densities. For smooth costs and densities on compact manifolds, the only known examples for which the optimal solution is always unique require at least one of the two underlying spaces to be homeomorphic to a sphere. We have introduced a multivalued dynamics induced by the transportation cost between the target and source space, for which the presence or absence of a sufficiently large set of periodic trajectories plays a role in determining whether or not optimal transport is necessarily unique. This insight allows us to construct smooth costs on a pair of compact manifolds with arbitrary topology, so that the optimal transport between any pair of probability densities is unique. We investigated further this problem of uniquely minimizing costs and obtained in collaboration with Abbas Moameni [10] a result of density of uniquely minimizing costs in the  $C^0$ -topology. The results in higher topology should be the subject of some further research.

## 4. Application Domains

### 4.1. Aerospace Engineering

**Participants:** Bernard Bonnard, Jean-Baptiste Caillau, Thierry d'Argent, Lamberto Dell'Elce, Jean-Baptiste Pomet, Jérémy Rouot.

Space engineering is very demanding in terms of safe and high-performance control laws. It is therefore prone to fruitful industrial collaborations. McTAO now has an established expertise in space and celestial mechanics. Our collaborations with industry are mostly on orbit transfer problems with low-thrust propulsion. It can be orbit transfer to put a commercial satellite on station, in which case the dynamics are a Newtonian force field plus perturbations and the small control. There is also, currently, a renewed interest in low-thrust missions such as Lisa Pathfinder (ESA mission towards a Lagrange point of the Sun-Earth system) or BepiColombo (joint ESA-JAXA mission towards Mercury). Such missions look more like a controlled multibody system. In all cases the problem involves long orbit transfers, typically with many revolutions around the primary celestial body. When minimizing time, averaging techniques provide a good approximation. Another important criterion in practice is fuel consumption minimization (crucial because only a finite amount of fuel is onboard a satellite for all its "life"), which amounts to  $L^1$ -minimization. Both topics are studied by the team. We have a steady relationships with CNES and Thales Alenia Space (Cannes), that have financed or co-financed 3 PhDs and 2 post-docs in the Sophia location of the team in the decade and are a source of inspiration even at the methodological level. Team members also have close connections with Airbus-Safran (Les Mureaux) on launchers. Some of the authoritative papers in the field were written by team members, with an emphasis on the geometric analysis and on algorithms (coupling of shooting and continuation methods). There are also connections with peers more on the applied side, like D. Scheeres (Colorado Center for Astrodynamics Research at Boulder), the group of F. Bernelli (Politecnico Milano), and colleagues from U. Barcelona (A. Farrès, A. Jorba).

### 4.2. Magnetic resonance imaging (MRI)

**Participants:** Bernard Bonnard, Alice Nolot [Professeur Classes Préparatoires Troyes], Jérémy Rouot, Joseph Gergaud, Olivier Cots [ENSEEIH, Toulouse], Stephen Glaser [TU München, Germany], Dominique Sugny [Univ. de Bourgogne].

The starting point of our interest in optimal control for quantum systems was a collaboration with physicist from **ICB**, University of Burgundy (Dominique Sugny), motivated by an ANR project where we worked on the control of molecular orientation in a dissipative environment using a laser field, and developed optimal control tools, combined with numerical simulations, to analyze the problem for Qubits. This was related to quantum computing rather than MRI. Using this expertise and under the impulse of Prof. S. Glaser and his group (Chemistry, TU München), we investigated Nuclear Magnetic resonance (NMR) for medical imaging (MRI), where the model is the Bloch equation describing the evolution of the Magnetization vector controlled by a magnetic field, but in fine is a specific Qubit model without decoherence. We worked on, and brought strong

contributions to, the contrast problem: typically, given two chemical substances that have an importance in medicine, like oxygenated and de-oxygenated blood, find the (time-dependent) magnetic field that will produce the highest difference in brightness between these two species on the image resulting from Nuclear Magnetic Resonance. This has immediate and important industrial applications in medical imaging. Our contacts are with the above mentioned physics academic labs, who are themselves in contact with major companies. The team has produced and is producing important work on this problem. One may find a good overview in [50], a reference book has been published on the topic [54], a very complete numerical study comparing different optimization techniques was performed in [49]. We conduct this project in parallel with S. Glaser team, which validated experimentally the pertinence of the methods, the main achievement being the in vivo experiments realized at the Creatis team of Insa Lyon showing the interest to use optimal control methods implemented in modern softwares in MRI in order to produce a better image in a shorter time. A goal is to arrive to a cartography of the optimal contrast with respect to the relaxation parameters using LMI techniques and numerical simulations with the Hamaph and Bocop code; note that the theoretical study is connected to the problem of understanding the behavior of the extremal solutions of a controlled pair of Bloch equations, and this is an ambitious task. Also, one of the difficulties to go from the obtained results, checkable on experiments, to practical control laws for production is to deal with magnetic field space inhomogeneities.

### 4.3. Swimming at low-Reynolds number

**Participants:** Bernard Bonnard, Yacine El Alaoui-Faris, Laetitia Giraldi, Clément Moreau, Alice Nolot, Jean-Baptiste Pomet, Jérémy Rouot.

Following the historical reference for low Reynolds number locomotion [72], the study of the swimming strategies of micro-organisms is attracting increasing attention in the recent literature. This is both because of the intrinsic biological interest, and for the possible implications these studies may have on the design of bio-inspired artificial replicas reproducing the functionalities of biological systems. In the case of micro-swimmers, the surrounding fluid is dominated by the viscosity effects of the water and becomes reversible. In this regime, it turns out that the infinite dimensional dynamics of the fluid do not have to be retained as state variables, so that the dynamics of a micro-swimmer can be expressed by ordinary differential equations if its shape has a finite number of degrees of freedom. Assuming this finite dimension, and if the control is the rate of deformation, one obtains a control system that is linear (affine without drift) with respect to the controls, *i.e.* the optimal control problem with a quadratic cost defines a sub-Riemannian structure (see section 3.2). This is the case where the shape is “fully actuated”, *i.e.* if all the variables describing the shape are angles, there is an actuator on each of these angles. For artificial micro-swimmers, this is usually unrealistic, hence (artificial) magneto-elastic micro-swimmers, that are magnetized in order to be deformed by an external magnetic field. In this case, the control functions are the external magnetic field. In both cases, questions are controllability (straightforward in the fully actuated case), optimal control, possibly path planning. We collaborate with teams that have physical experiments for both.

- In collaboration with D. Takagi and M. Chyba (Univ of Hawaii), this approach is currently at the experimental level for copepod-like swimmer at the university of Hawaii: on the one hand, this zooplankton and its locomotion can be observed, and a robot micro swimmer mimicking a copepod has been constructed, but in fact large enough for direct actuation to be possible, and the low Reynolds number is achieved by using a more viscous fluid. This gives possibilities, through an inverse optimization problem, to determine what cost can be optimised by these crustaceans, see [40], [78], and to validate models on the robot.
- For magneto-elastic micro-robots, Y. El-Alaoui’s PhD is co-advised with Stéphane Régnier from the robotics lab ISIR, Univ. Paris 6. Magneto-elastic micro-robots and their magnetic actuation are actually built at ISIR and the aim of the collaboration is to validate models and improve the existing control laws both in performance and in energy; of course, the micro scale does make things difficult.

The questions about optimality of periodic controls raised in section 3.2 are related to these applications for periodic deformations, or strokes, play an important role in locomotion.

## 4.4. Stability of high frequency amplifiers

**Participants:** Sébastien Fueyo, Gilles Lebeau, Jean-Baptiste Pomet, Laurent Baratchart [FACTAS project-team].

Nonlinear hyper-frequency amplifiers are ubiquitous in cell phone relays and many other devices. They must be as compact as possible, yielding a more complicated design. Computer Assisted Design tools are extensively used; for a given amplifier design, they provide frequency responses but fail to provide information of the stability of the response for each frequency. This stability is crucial for an unstable response will not be observed in practice; the actual device should not be built before stability is asserted. Predicting stability/instability from “simulations” in the Computer Assisted Design tool is of utmost importance (simulation between quotation marks because these simulations are in fact computations in the frequency domain). Potential transfer is important.

Some techniques do exist, see [77], based on creating some virtual perturbations and treating them as the input of a (linearized) control system to be “simulated” using the same tools. In an ongoing collaboration between McTAO and the project-team FACTAS, we work on the mathematical ground of these methods and in particular of the relation between stability and the property of the identified time-varying infinite dimensional systems. See recent developments in Section 7.14.

## 4.5. Optimal control of microbial cells

**Participants:** Jean-Baptiste Caillaud, Walid Djema [BIOCORE project-team], Laetitia Giraldi, Jean-Luc Gouzé [BIOCORE project-team], Sofya Maslovskaya, Jean-Baptiste Pomet, Agustín Yabo.

The growth of microorganisms is fundamentally an optimization problem which consists in dynamically allocating resources to cellular functions so as to maximize growth rate or another fitness criterion. Simple ordinary differential equation models, called self-replicators, have been used to formulate this problem in the framework of optimal and feedback control theory, allowing observations in microbial physiology to be explained. The resulting control problems are very challenging due to the nonlinearity of the models, parameter uncertainty, the coexistence of different time-scales, a dynamically changing environment, and various other physical and chemical constraints. In the framework of the ANR Maximic (PI Hidde de Jong, Inria Grenoble Rhône-Alpes) we aim at developing novel theoretical approaches for addressing these challenges in order to (i) study natural resource allocation strategies in microorganisms and (ii) propose new synthetic control strategies for biotechnological applications. In order to address (i), we develop extended self-replicator models accounting for the cost of regulation and energy metabolism in bacterial cells. We study these models by a combination of analytical and numerical approaches to derive optimal control solutions and a control synthesis, dealing with the bang-bang-singular structure of the solutions. Moreover, we define quasi-optimal feedback control strategies inspired by known regulatory mechanisms in the cell. To test whether bacteria follow the predicted optimal strategies, we quantify dynamic resource allocation in the bacterium *Escherichia coli* by monitoring, by means of time-lapse fluorescent microscopy, the expression of selected genes in single cells growing in a microfluidics device. In order to address (ii), we build self-replicator models that include a pathway for the production of a metabolite of interest. We also add a mechanism to turn off microbial growth by means of an external input signal, at the profit of the production of the metabolite. We formulate the maximization of the amount of metabolite produced as an optimal control problem, and derive optimal solutions and a control synthesis, as well as quasi-optimal feedback strategies satisfying chemical and physical design constraints. The proposed synthetic control strategies are being tested experimentally by growing *E. coli* strains capable of producing glycerol from glucose in a mini-bioreactor system. We aim at quantifying the amount of glucose consumed and glycerol produced, in the case of a predefined input signal (open-loop control) and the adaptive regulation of the input signal based on on-line measurements of the growth rate and the expression of fluorescent reporters of selected genes (closed-loop control). Currently, one PhD (A. Yabo) and one postdoc (S. Maslovskaya) are involved in these tasks and jointly supervised by colleagues from McTAO and Biocore teams at Sophia. Preliminary results concern the definition on extended (higher dimensional) models for the bacteria dynamics, check of second order optimality conditions on the resulting optimal control problem, and study of the turnpike phenomenon for these optimization problems.

## 5. Highlights of the Year

### 5.1. Highlights of the Year

#### 5.1.1. Awards

Laetita Giraldi has been awarded the price "My Innovation is" by SATT Sud & Corse for her research project on the control of swimming microbots.

## 6. New Software and Platforms

### 6.1. Hampath

**KEYWORDS:** Optimal control - Second order conditions - Differential homotopy - Ordinary differential equations

**FUNCTIONAL DESCRIPTION:** Hampath is a software developed to solve optimal control problems by a combination of Hamiltonian et path following methods. Hampath includes shooting and computation of conjugate points. It is an evolution of the software cotcot ([apo.enseeiht.fr/cotcot](http://apo.enseeiht.fr/cotcot)). It has a Fortran kernel, uses Tapenade ([www-sop.inria.fr/tropics/tapenade.html](http://www-sop.inria.fr/tropics/tapenade.html)) for automatic differentiation and has a Matlab interface.

- Participants: Jean-Baptiste Caillau, Joseph Gergaud and Olivier Cots
- Contact: Jean-Baptiste Caillau
- URL: <http://www.hampath.org>

## 7. New Results

### 7.1. Analysis of singularities in minimum time control problems

**Participants:** Jean-Baptiste Caillau, Jacques Féjoz [Université Paris-Dauphine & Observatoire de Paris], Michaël Orioux [SISSA], Robert Roussarie [Université de Bourgogne-Franche Comté].

An important class of problems is affine control problems with control on the disk (or the Euclidean ball, in higher dimensions). Such problems show up for instance in space mechanics and have been quite extensively studied from the mathematical (geometric) and numerical point of view. Still, even for the simplest cost, namely time minimization, the analysis of singularities occurring was more or less open. Building on previous results of the team and on recent studies of Agrachev and his collaborators, we give a detailed account of the behaviour of minimum time extremals crossing the so-called singular locus (typically a switching surface). The result is twofold. First, we show that there the set of initial conditions of the Hamiltonian flow can be stratified, and that the flow is smooth on each stratum, one of them being the codimension stratum leading to the singular locus. This generalizes in higher codimension the known case of switching conditions of codimension one encountered, for instance, in  $L^1$ -minimization (consumption minimization, in aerospace applications). We give a clear geometric interpretation of this first result in terms of normally hyperbolic invariant manifold. Secondly, we provide a model for the singularity on the flow when strata are crossed, proving that it is of logarithmic type. This paves the way for *ad hoc* numerical methods to treat this kind of extremal flow. The crucial tool for the analysis is a combination of blow-up and normal form techniques for dynamical systems.

### 7.2. The Sard Conjecture in sub-Riemannian Geometry

**Participants:** Ludovic Rifford, André Belotto Da Silva [Univ. Aix-Marseille], Adam Parusinski [Univ. Côte d'Azur].

In a work in progress, we address the Sard conjecture for sub-Riemannian structures on analytic manifolds and related problems. We present a description of singular horizontal curves of a totally nonholonomic analytic distribution in term of the projections of the orbits of some integrable and isotropic subanalytic distribution in the cotangent bundle. In the generic smooth case, we obtain an extension of an important result by Chitour, Jean and Trélat by showing that singular curves are the projection of a Hamiltonian singular vector field. As a by-product of our first result, we obtain a proof of the so-called minimal rank Sard conjecture in some analytic cases. It establishes that from a given point the set of points accessible through singular horizontal curves of minimal rank, which corresponds to the rank of the distribution, has Lebesgue measure zero under additional technical assumptions.

### 7.3. Local controllability of magnetic micro-swimmers and more general classes of control systems

**Participants:** Laetitia Giraldi, Pierre Lissy [Univ. Paris Dauphine], Clément Moreau, Jean-Baptiste Pomet.

As a part of Clément Moreau’s PhD, we gave fine results on local controllability of magnetized micro swimmers actuated by an external magnetic field. We had shown that the “two-link” magnetic swimmer had some local controllability around its straight configuration but that it was not Small Time Locally Controllable” (STLC) in the classical sense that asks that points close to the initial condition can be reached using “small” controls.

We derived in [30] some necessary conditions for STLC of affine control systems with two scalar controls, around an equilibrium where not only the drift vector field vanishes but one of the two control vector fields vanishes too; we state various necessary conditions (involving the value at the equilibrium of some iterated Lie brackets of the system vector fields), where the “smallness” of the controls is intended in the  $L^\infty$  (classical) or  $W^{1,\infty}$  (less classical, used in recent work by K. Beauchard and F. Marbach).

We also arrived to local controllability results in higher dimension than the “two-link” micro-robots, see [9]. This relies on the following remark: classical STLC does not hold, but STLC is concerned with small controls, hence with variations around the zero control... but, due to one of the control fields vanishing, the system also rests at the equilibrium for (infinitely many) nonzero constant values of the control. It is proved that there is one nonzero value of the control such that STLC holds when considered *around this constant control* and not around the zero control. In other terms, classical STLC holds after a constant feedback transformation.

### 7.4. Time-optimal deorbiting maneuvers of solar sails

**Participants:** Jean-Baptiste Caillaud, Lamberto Dell’Elce, Jean-Baptiste Pomet.

Increasing interest in optimal low-thrust orbital transfers was triggered in the last decade by technological progress in electric propulsion and by the ambition of efficiently leveraging on orbital perturbations to enhance the maneuverability of small satellites. This work was aimed at investigating time optimal propellantless deorbiting maneuvers in low-Earth orbit using solar sails. The solution of this problem was achieved by doubly averaging the optimal control Hamiltonian with respect to both satellite and Sun longitudes. Initial conditions for the osculating trajectory were inferred via a near-identity transformation that approximates the quasi-periodic oscillations of both state and adjoint variables [61]. The outcomes of the study were presented at the 4th KePASSA meeting in Logrono [18].

### 7.5. Long-term evolution of quasi-satellite orbits

**Participants:** Lamberto Dell’Elce, Nicola Baresi [Univ. of Surrey, UK], Josué Cardoso Dos Santos [Sao Paulo State Univ., Brazil], Yasuhiro Kawakatsu [JAXA, Japan].



The Martian Moons eXploration mission is currently under development at the Japan space agency (JAXA) and will be the first spacecraft mission to retrieve pristine samples from the surface of Phobos. In preparation for the sampling operations, MMX will collect observations of Phobos from stable retrograde relative trajectories, which are referred to as quasi-satellite orbits (QSOs). This study, started in 2018 in collaboration with JAXA, investigates the navigability of mid- and high-altitude QSOs in terms of relative orbit element. Our developments are based on the Yamanaka-Ankersen solution of the Tschauner-Hempel equations and capture the effects of the secondary's gravity and orbital eccentricity on the shape and orientation of near-equatorial retrograde relative orbits. The analytic solution that we obtained by averaging the equations of motion with respect to the longitude of the satellite is suitable to gain insight into the long-term evolution of QSOs. These results were recently published in [38].

## 7.6. Non-singular analytical solution of perturbed satellite motion using Milankovitch elements

**Participants:** Lamberto Dell'Elce, Pini Gurfil [Technion, Israel], Gianpaolo Izzo [Technion, Israel], Aaron J. Rosengren [Univ. of Arizona, US].

In the brief span of time after the launch of Sputnik, a whole succession of analyses was devoted to the problem poised by the drag-free motion of an artificial satellite about an oblate planet, employing almost every known perturbation method. Although in a sense, the problem is a classic one that also occurred among the natural satellites, it was necessary in the applications of artificial satellite motion to obtain a more general, detailed, and accurate solution. In this study, we developed a new formulation of the mean-to-osculating conversion for first-order oblateness perturbations based on the Milankovitch elements [74] that corrects the critical-inclination deficiency. We use the direct method of Kozai [67], and present an explicit analytical short-period correction in vector form that is valid for all orbits with nonzero angular momentum. Preliminary results were presented at the International Symposium of Space Flight Mechanics (ISSFM) [19].

## 7.7. Sub-Riemannian Geometry and Micro-Swimmers and Extensions to Control in Hydrodynamics

**Participants:** Bernard Bonnard, Piernicola Bettiol [Univ. Bretagne Ouest], Alice Nolot [Univ. de Bourgogne Franche Comté], Jérémy Rouot.

We pursue our study concerning the 1-copepod swimmer using techniques from SR-geometry and numerical simulations, see [40] for previously obtained results. Following Takagi model, a 2d-swimmer is currently analyzed whose aim is to perform a 2d-motion where the copepod swimmer can change its orientation. Preliminary study concerning this problem was analysed during the internship of A. Lenc in relation with the motion planning of a car. Under the impulse of O. Cots and B. Wembe an extension of the project is the developments of the geometric optimal control techniques in hydrodynamics. In particular we studied a Zermelo navigation problem in a current with a vortex [25] (submitted to ESAIM-COCV). An interesting and new phenomenon detected in optimal control is the existence for the geodesic flow of a Reeb foliation.

## 7.8. Swimming at low Reynolds number an optimal control problem

**Participants:** François Alouges [École Polytechnique], Luca Berti, Antonio Desimone [SISSA Trieste], Yacine El Alaoui-Faris, Laetitia Giraldo, Yizhar Or [Technion, Israel], Christophe Prud'Homme [Univ. de Strasbourg], Jean-Baptiste Pomet, Stéphane Régnier [Sorbonne Université], Oren Wiezel [Technion, Israel].

This part is devoted to study the displacement of micro-swimmers. We attack this problem using numerical tools and optimal control theory. Micro-scale swimmers move in the realm of negligible inertia, dominated by viscous drag forces, the fluid is governed by the Stokes equation. We study two types of models. First, deriving from the PDE system, in [5] we use Feel++, a finite elements library in order to simulate the motion of a one-hinged swimmer, which obeys to the scallop theorem. Then, we address the flagellar microswimmers. In [31] we formulate the leading order dynamics of a  $2D$  slender multi-link microswimmer assuming small-amplitude undulations about its straight configuration. The energy optimal stroke to achieve a given prescribed displacement in a given time period is obtained as the largest eigenvalue solution of a constrained optimal control problem. We prove that the optimal stroke is an ellipse lying within a two-dimensional plane in the  $(N - 1)$  dimensional space of shape variables, where  $N$  can be arbitrarily large. If the number of shape variables is small, we can consider the same problem when the prescribed displacement in one time period is large, and not attainable with small variations of the joint angles. The fully non-linear optimal control problem is solved numerically for the cases  $N = 3$  and  $N = 5$  showing that, as the prescribed displacement becomes small, the optimal solutions obtained using the small-amplitude assumption are recovered. We also show that, when the prescribed displacements become large, the picture is different. Finally, in [28] we present an automated procedure for the design of optimal actuation for flagellar magnetic microswimmers based on numerical optimization. Using this method, a new magnetic actuation method is provided which allows these devices to swim significantly faster compared to the usual sinusoidal actuation. This leads to a novel swimming strategy which shows that a faster propulsion is obtained when the swimmer is allowed to go out-of-plane. This approach is experimentally validated on a scaled-up flexible swimmer.

## 7.9. Periodic body deformations are optimal for locomotion

**Participants:** Laetitia Giraldi, Frédéric Jean.

A periodic cycle of body's deformation is a common strategy for locomotion (see for instance birds, fishes, humans). The aim of this work (see [29]) is to establish that the auto-propulsion of deformable object is optimally achieved using periodic strategies of body's deformations. This property is proved for a simple model using optimal control theory framework.

## 7.10. Optimal Control of Chemical Networks by Temperature Control

**Participants:** Bernard Bonnard, Jérémy Rouot.

The objective of the project is to develop previous results obtained at the end of the 90's by B. Bonnard and his collaborators to control the production of batch reactors by temperature control in relation with the Shell Company. These results were derived to analyze the simple (but relevant for applications) irreversible reaction scheme  $A \rightarrow B \rightarrow C$ . More complicated weakly reversible scheme like the McKeithan network are currently under investigation taking into account the bridge phenomenon detected in [7], where complicated optimal policies with two singular arcs can occur. Preliminary results are presented in [3] where also the geometric techniques are described. See also the article in the 58th IEEE-CDC Nice conference [11].

## 7.11. Muscular Isometric Force Contraction by Electric Stimulation

**Participants:** Bernard Bonnard, Jérémy Rouot, Toufik Bakir [Univ. de Bourgogne Franche Comté].

This project started two years ago under the impulse of T. Bakir (ImVia-UBFC) who defended his HDR on the subject (November, 2018). The problem is the one of optimizing the train pulses of the FES signal to produce the muscular contraction. It is based on the Hill model refined by Ding et al to take into account the variations of the fatigue variable. Preliminary closed loop results were obtained using an MPC-method where the state variable is estimated with a non linear observer [2]. The problem can be stated in the optimal sampled-control data framework; with the collaboration of L. Bourdin (Maths Dept Limoges), Pontryagin-type necessary conditions were derived and partially numerically implemented [1]. The project is supported by a PEPS 1 AMIES and a PGMO Project. A CIFRE Thesis is planned to start in January, 2020 (Phd Student: Quentin Arnaud) in the Company **SEGULA**, supervised by T. Bakir and co-Supervised by B. Bonnard. See section 8.1.



## 7.12. Selection of microalgae

**Participants:** Walid Djema, Laetitia Giraldi, Olivier Bernard [BIOCORE project-team].

We investigate a minimal-time control problem in a chemostat continuous photo-bioreactor model that describes the dynamics of two distinct microalgae populations. Our objective is to optimize the time of separation between two species of microalgae by controlling the dilution rate. We focus on Droop’s model. Using Pontryagin’s principle, we develop a dilution-based control strategy that steers the model trajectories to a suitable target in minimal time. Our study reveals that the optimal solution has a turpike property [14] [27]. A numerical optimal-synthesis, based on direct optimal control tools, is performed in [15] and it shows that the optimal solution is of type bang-singular.

## 7.13. Extensions of the Zermelo-Markov-Dubins problem in optimal control

**Participants:** Ahmed Dieng, Jean-Baptiste Caillaud, Jean-Baptiste Pomet, Sofya Maslovskaya.

Motivated by a collaboration with CGG in 2018, we continued investigating minimum time problems for simplified kinematic models of a marine vessel towing some equipments, with various curvature constraints. These are extensions (by adding the towed equipment) of the so-called Zermelo-Markov-Dubins problem. In [12], we describe the problem with the simplest possible model of the trailer, and show that the Hamiltonian system resulting from Pontryagin Principle for minimum time is integrable in that case, both for the “regular” flow and for the flow giving singular extremals; without giving an explicit analytic solution, this drastically simplifies the computation of optimal solutions; a description of the marine application solution is also given.

Ahmed Dieng’s internship was the opportunity to test more complex models numerically and to have a qualitative approach of the results from [12].

Note that the collaboration with CGG (we had a short bilateral contract with this company in 2018) did not continue mainly because they stopped this activity and more generally marine acquisition. This recent [press release](#) details the transactions.

## 7.14. Stability of nonlinear high frequency amplifiers and stability of linear time-varying time-delay systems

**Participants:** Laurent Baratchart [FACTAS project-team], Sébastien Fueyo, Jean-Baptiste Pomet, Gilles Lebeau.

These amplifiers contain on the one hand nonlinear active components and on the other hand lines, that induce some sort of delays and make the system infinite-dimensional: they are, for each choice of a periodic input, a nonlinear infinite dimensional dynamical system. The Computer Aided Design tools mentioned in Section 4.4 provide a periodic solution under this periodic forcing and may also give the frequency response of the linearized system along this trajectory with some artificial “small” excitation. The goal is to deduce stability from these data.

It is an opportunity to build theoretical basis and justification to a stability analysis through harmonic identification; the latter is one of the specialties of FACTAS, we collaborate on the infinite-dimensional non-linear stability analysis for periodic solutions and how it works with the results of harmonic identification. This is the topic of Sébastien Fueyo’s PhD.

On academic examples of simple circuits, we have given full justification (with some possible obstructions) to the prediction of stability through transfer function identification. The theoretical interest is that the spectrum of the operator that gives stability is not as elementary as predicted in the literature, but stability can be predicted nonetheless. Publication in progress on this point, a preliminary version was presented in [17].

It was also the opportunity to re-visit stability of time-delay time-varying linear system. A new sufficient condition can be found in [22], and a more general result is the purpose of a publication to come. These results are important to the domain of linear time-delay systems because the time-varying case has seldom been touched.

## 8. Bilateral Contracts and Grants with Industry

### 8.1. Bilateral Grants with Industry

- A grant “PEPS AMIES”, title: “Conception d’un électrostimulateur intelligent”, was obtained, co-financed by **AMIES** and **SEGULA**.  
PI: Bernard Bonnard.  
Start: December 2018. Duration: 2 years.
- A grant CIFRE co-financed by and **SEGULA**, title: “Réalisation d’un prototype d’électrostimulateur intelligent”, was obtained.  
PI: Bernard Bonnard and T. Bakir (IMvia).  
Start: January 2020. Duration: 3 years.

## 9. Partnerships and Cooperations

### 9.1. National Initiatives

#### 9.1.1. ANR

**Sub-Riemannian Geometry and Interactions (SRGI)**. Started 2015 (decision ANR-15-CE40-0018), duration: 4 years. L. Rifford is a member.

**Intéactions Systèmes Dynamiques Équations d’Évolution et Contrôle (ISDEEC)**. Started 2016 (decision ANR-16-CE40-0013), duration: 4 years. L. Rifford is a member.

**Maximic: optimal control of microbial cells by natural and synthetic strategies**. Started 2017, duration: 4 years. J.-B. Caillaud, L. Giraldu, J.-B. Pomet are members.

#### 9.1.2. Others

**Défi Infniti CNRS project, Control and Optimality of Magnetic Microrobots**, (PI L. Giraldu). Started 2017, duration: 2 years. This project involves colleagues from Paris Sorbonne Université S. Régnier and from University of Strasbourg C. Prud’Homme’s.

**PGMO** grant (2017-2019) on “Algebro-geometric techniques with applications to global optimal control for Magnetic Resonance Imaging (MRI)”. B. Bonnard, A. Nolot and J. Rouot participate in this project, the PI is O. Cots, from ENSEIHHT, Toulouse.

**PGMO** grant (2019-2021) on "Sampled-Data Control Systems and Applications" (PI B. Bonnard).

The McTAO team participates in the **GdR MOA**, a CNRS network on Mathematics of Optimization and Applications.

J.-B. Caillaud is associate researcher of the CNRS team **Parallel Algorithms & Optimization** at ENSEIHT, Univ. Toulouse.

### 9.2. International Research Visitors

#### 9.2.1. Visits of International Scientists

Prof. Sorin Sabau (Tokai University) visited Inria during two weeks in May 2019. He gave a talk on "The calculus of variations on Finsler manifolds".

### 9.2.1.1. Research Stays Abroad

- Bernard Bonnard visited the University of Hawaii at Manoa, Mars 2019 (1 month, host: M. Chyba).

## 10. Dissemination

### 10.1. Promoting Scientific Activities

#### 10.1.1. Scientific Events: Organisation

##### 10.1.1.1. General Chair, Scientific Chair

- J.-B. Caillau has been chair (with Didier Auroux, Castor team) of the 19th French-German-Swiss conference, held in Nice from September 17 to 19, 2019. The conference gathered around 150 researchers in optimization. More information on [fgs-2019.sciencesconf.org](http://fgs-2019.sciencesconf.org)
- L. Giraldi together with M. Chaves (Biocore) organized an invited session at the Conf. Decision and Control (Nice, France).

##### 10.1.1.2. Member of the Organizing Committees

J.-B. Caillau has been member of the organizing committee of **PGMO days** held in Paris Saclay from December 3 to 4, 2019.

#### 10.1.2. Journal

##### 10.1.2.1. Reviewer - Reviewing Activities

The team members are regular reviewers for the leading journal in control (SIAM J. Control, ESAIM COCV...) and, more generally, for journals of pure and applied mathematics.

#### 10.1.3. Invited Talks

- L. Rifford gave a plenary talk at the *2nd International Conference of Mathematics in Erbil* (Iraq), .
- L. Rifford was a keynote speaker at the *1st International MIPAnet Conference on Science and Mathematics* (Parapat, Indonesia), .
- L. Rifford gave a talk during the workshop *Real and Complex Singularities in Cargèse* at the IESC (Cargèse, France). .
- B. Bonnard gave the seminar “*Techniques Géométriques pour le Contrôle Optimal des Réacteurs Chimiques*” at the Department of Mathematics of the Université de Genève, October 2019.
- L. Dell’Elce was a keynote speaker at the *4th International Workshop on Key topics in orbit Propagation Applied to Space Situational Awareness* (Logrono, Spain), .
- L. Dell’Elce gave the seminar “*Multi-Phase Averaging of Time-Optimal Low-Thrust Transfers*” at the Surrey Space Center in the University of Surrey, Octobre 19.
- J.-B. Caillau gave an invited talk at the *Learning week* if Air France, in Sophia Antipolis, December 2019.

#### 10.1.4. Scientific Expertise

L. Giraldi is reviewer for DFG the Deutsche Forschungsgemeinschaft (German Research Foundation).

### 10.1.5. Research Administration

J.-B. Caillau is

- member of the Scientific Council of CNRS GdR Calcul Scientifique
- member of the Scientific Council of Programme Gaspard Monge pour l'Optimisation (PGMO)
- member of the Scientific Council of Institut de Mécanique Céleste et de Calcul des Ephémérides (Observatoire de Paris)
- member of the Scientific Council of 3IA

Jean-Baptiste Pomet is

- a member of the steering committee of the Center for Planetary Origin (C4PO),
- a member of the scientific council of Académie 2 "Complex system", both for Université Côte d'Azur (UCA),
- an elected member of Commission d'Évaluation (Inria permanent evaluation committee).

## 10.2. Teaching - Supervision - Juries

### 10.2.1. Teaching

Master in Astrophysics Université Côte Azur (MAUCA): Lamberto Dell'Elce, Build a Nanosatellite (Attitude Determination and Control System), 6 hours TH, niveau M1, Université Côte Azur, France.

Engineering school: J.-B. Caillau has a full teaching duty of Professor at L3, M1 and M2 level of the Applied Math. department of Polytech Nice Sophia. (He is the head of the department since September 2018.)

Licence : L. Giraldi, Colles de mathématiques, 90h (2h en MPSI et 2h en MP par semaine), MPSI-MP, Lycée internationale de Valbonne, France.

Master : L. Giraldi, Natation à bas nombre de Reynolds, 6h, Master 2 Recherche, Université de Srasbourg, France.

### 10.2.2. Supervision

PhD in progress : Sébastien Fueyo, "Testing stability of nonlinear amplifier by frequency-domain methods", started October, 2016, co-supervised by J.-B. Pomet and L. Baratchart (FACTAS team).

PhD in progress : Yacine El alaoui-faris, "modeling magneto-elastic micro-robot from theory to experiment", started October, 2017, co-supervised by L. Giraldi, J.-B. Pomet and Stephane Régner (Univ. Paris Sorbonne).

PhD in progress : Clément Moreau, "Contrôlabilité de systèmes en dimension finie ou infinie issus du vivant", started September, 2017, co-supervised by L. Giraldi, Pierre Lissy and J.-B. Pomet.

PhD in progress : Agustín Yabo, "Optimal control of microbial cells", started October, 2018, co-supervised by J.-L. Gouzé (Biocore team) and J.-B. Caillau.

### 10.2.3. Juries

L. Giraldi is member of the jury of agrégation de mathématiques.

L. Giraldi was examiner of the PhD Thesis of Fatima Tani (supervised by A. Rapaport).

J.-B. Pomet sat in the jury for Armand Koenig's PhD (Université côte d'Azur).

J.-B. Caillau sat in the PhD jury of Isabelle Santos (Toulouse), HDR jury of Max Cerf (Paris). He is member of the jury of agrégation de mathématiques.

## 10.3. Popularization

### 10.3.1. Internal or external Inria responsibilities

J.-B. Caillaud belongs to the MASTIC initiative at Inria Sophia.

### 10.3.2. Interventions

Lamberto Dell'Elce is involved in the PoBot challenge promoted by MEDITES. Specifically he supervises a class in the College Emile Roux in Cannes.

J.-B. Caillaud has given several talks at high-school level in Nice and Sophia on application of mathematics in social choice.

## 11. Bibliography

### Publications of the year

#### Articles in International Peer-Reviewed Journals

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