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ACTIVITY REPORT

Project-Team CAGE

Control and Geometry

IN COLLABORATION WITH: Laboratoire Jacques-Louis Lions (LJLL)

DOMAIN

Applied Mathematics, Computation and Simulation

THEME

Optimization and control of dynamic systems

Contents

Project-Team CAGE

Creation of the Project-Team: 2018 August 01

Keywords

Computer sciences and digital sciences

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B2. – Health

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B5.11. – Quantum systems

1 Team members, visitors, external collaborators

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- Andrey Agrachev [Inria, Advanced Research Position, from Oct 2021]
- Barbara Gris [CNRS, Researcher]
- Kevin Le Balc'h [Inria, from Sep 2021, Starting Faculty Position]

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- Jean Michel Coron [Université de Paris, Professor]
- Emmanuel Trélat [Université d'Orléans, Associate Professor, HDR]

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- Eugenio Pozzoli [Inria, until Oct 2021]
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2 Overall objectives

CAGE's activities take place in the field of mathematical control theory, with applications in three main directions: geometric models for vision, control of quantum mechanical systems, and control of systems with uncertain dynamics.

The relations between control theory and geometry of vision rely on the notion of sub-Riemannian structure, a geometric framework which is used to measure distances in nonholonomic contexts and which has a natural and powerful control theoretical interpretation. We recall that nonholonomicity refers to the property of a velocity constraint that cannot be recast as a state constraint. In the language of differential geometry, a sub-Riemannian structure is a (possibly rank-varying) Lie bracket generating distribution endowed with a smoothly varying norm.

Sub-Riemannian geometry, and in particular the theory of associated (hypoelliptic) diffusive processes, plays a crucial role in the neurogeometrical model of the primary visual cortex due to Petitot, Citti and Sarti, based on the functional architecture first described by Hubel and Wiesel. Such a model can be used as a powerful paradigm for bio-inspired image processing, as already illustrated in the recent literature (including by members of our team). Our contributions to this field are based not only on this approach, but also on another geometric and sub-Riemannian framework for vision, based on pattern matching in the group of diffeomorphisms. In this case admissible diffeomorphisms correspond to deformations which are generated by vector fields satisfying a set of nonholonomic constraints. A sub-Riemannian metric on the infinite-dimensional group of diffeomorphisms is induced by a length on the tangent distribution of admissible velocities. Nonholonomic constraints can be especially useful to describe distortions of sets of interconnected objects (e.g., motions of organs in medical imaging).

Control theory is one of the components of the forthcoming quantum revolution [1], since manipulation of quantum mechanical systems is ubiquitous in applications such as quantum computation, quantum cryptography, and quantum sensing (in particular, imaging by nuclear magnetic resonance). The efficiency of the control action has a dramatic impact on the quality of the coherence and the robustness of the required manipulation. Minimal time constraints and interaction of time scales are important factors for characterizing the efficiency of a quantum control strategy. Time scales analysis is important for evaluation approaches based on adiabatic approximation theory, which is well-known to improve the robustness of the control strategy. CAGE works for the improvement of evaluation and design tools for efficient quantum control paradigms, especially for what concerns quantum systems evolving in infinite-dimensional Hilbert spaces.

Simultaneous control of a continuum of systems with slightly different dynamics is a typical problem in quantum mechanics and also a special case of the third applicative axis to which CAGE is contributing: control of systems with uncertain dynamics. The slightly different dynamics can indeed be seen as uncertainties in the system to be controlled, and simultaneous control rephrased in terms of a robustness task. Robustification, i.e., offsetting uncertainties by suitably designing the control strategy, is a widespread task in automatic control theory, showing up in many applicative domains such as electric circuits or aerospace motion planning. If dynamics are not only subject to static uncertainty, but may also change as time goes, the problem of controlling the system can be recast within the theory of switched and hybrid systems, both in a deterministic and in a probabilistic setting. Our contributions to this research field concern both stabilization (either asymptotic or in finite time) and optimal control, where redundancies and probabilistic tools can be introduced to offset uncertainties.

[1] As anticipated by the recent launch of the FET Flagship on Quantum Technologies

3 Research program

3.1 Research domain

The activities of CAGE are part of the research in the wide area of control theory. This nowadays mature discipline is still the subject of intensive research because of its crucial role in a vast array of applications.

More specifically, our contributions are in the area of **mathematical control theory**, which is to say that we are interested in the analytical and geometrical aspects of control applications. In this approach, a control system is modeled by a system of equations (of many possible types: ordinary differential equations, partial differential equations, stochastic differential equations, difference equations,...), possibly not explicitly known in all its components, which are studied in order to establish qualitative and quantitative properties concerning the actuation of the system through the control.

Motion planning is, in this respect, a cornerstone property: it denotes the design and validation of algorithms for identifying a control law steering the system from a given initial state to (or close to) a target one. Initial and target positions can be replaced by sets of admissible initial and final states as, for instance, in the motion planning task towards a desired periodic solution. Many specifications can be added to the pure motion planning task, such as robustness to external or endogenous disturbances, obstacle avoidance or penalization criteria. A more abstract notion is that of **controllability**, which denotes the property of a system for which any two states can be connected by a trajectory corresponding to an admissible control law. In mathematical terms, this translates into the surjectivity of the so-called **end-point map**, which associates with a control and an initial state the final point of the corresponding trajectory. The analytical and topological properties of endpoint maps are therefore crucial in analyzing the properties of control systems.

One of the most important additional objective which can be associated with a motion planning task is **optimal control**, which corresponds to the minimization of a cost (or, equivalently, the maximization of a gain) [164]. Optimal control theory is clearly deeply interconnected with calculus of variations, even if the non-interchangeable nature of the time-variable results in some important specific features, such as the occurrence of **abnormal extremals** [125]. Research in optimal control encompasses different aspects, from numerical methods to dynamic programming and non-smooth analysis, from regularity of minimizers to high order optimality conditions and curvature-like invariants.

Another domain of control theory with countless applications is **stabilization**. The goal in this case is to make the system converge towards an equilibrium or some more general safety region. The main difference with respect to motion planning is that here the control law is constructed in feedback form. One of the most important properties in this context is that of **robustness**, i.e., the performance of the stabilization protocol in presence of disturbances or modeling uncertainties. A powerful framework which has been developed to take into account uncertainties and exogenous non-autonomous disturbances is that of hybrid and switched systems [146, 126, 155]. The central tool in the stability analysis of control systems is that of **control Lyapunov function**. Other relevant techniques are based on algebraic criteria or dynamical systems. One of the most important stability property which is studied in the context of control system is **input-to-state stability** [151], which measures how sensitive the system is to an external excitation.

One of the areas where control applications have nowadays the most impressive developments is in the field of **biomedicine and neurosciences**. Improvements both in modeling and in the capability of finely actuating biological systems have concurred in increasing the popularity of these subjects. Notable advances concern, in particular, identification and control for biochemical networks [144] and models for neural activity [113]. Therapy analysis from the point of view of optimal control has also attracted a great attention [148].

Biological models are not the only one in which stochastic processes play an important role. Stock-markets and energy grids are two major examples where optimal control techniques are applied in the non-deterministic setting. Sophisticated mathematical tools have been developed since several decades to allow for such extensions. Many theoretical advances have also been required for dealing with complex systems whose description is based on **distributed parameters** representation and **partial differential equations**. Functional analysis, in particular, is a crucial tool to tackle the control of such systems [161].

Let us conclude this section by mentioning another challenging application domain for control theory: the decision by the European Union to fund a flagship devoted to the development of quantum technologies is a symptom of the role that quantum applications are going to play in tomorrow's society. **Quantum control** is one of the bricks of quantum engineering, and presents many peculiarities with respect to standard control theory, as a consequence of the specific properties of the systems described by the laws of quantum physics. Particularly important for technological applications is the capability of inducing and reproducing coherent state superpositions and entanglement in a fast, reliable, and efficient way [114].

3.2 Scientific foundations

At the core of the scientific activity of the team is the **geometric control** approach, that is, a distinctive viewpoint issued in particular from (elementary) differential geometry, to tackle questions of controllability, observability, optimal control... [75, 118]. The emphasis of such a geometric approach to control theory is put on intrinsic properties of the systems and it is particularly well adapted to study nonlinear and nonholonomic phenomena.

One of the features of the geometric control approach is its capability of exploiting **symmetries and intrinsic structures** of control systems. Symmetries and intrinsic structures can be used to characterize minimizing trajectories, prove regularity properties and describe invariants. An egregious example is given by mechanical systems, which inherently exhibit Lagrangian/Hamiltonian structures which are naturally expressed using the language of symplectic geometry [100]. The geometric theory of quantum control, in particular, exploits the rich geometric structure encoded in the Schrödinger equation to engineer adapted control schemes and to characterize their qualitative properties. The Lie–Galerkin technique that we proposed starting from 2009 [103] builds on this premises in order to provide powerful tests for the controllability of quantum systems defined on infinite-dimensional Hilbert spaces.

Although the focus of geometric control theory is on qualitative properties, its impact can also be disruptive when it is used in combination with quantitative analytical tools, in which case it can dramatically improve the computational efficiency. This is the case in particular in optimal control. Classical optimal control techniques (in particular, Pontryagin Maximum Principle, conjugate point theory, associated numerical methods) can be significantly improved by combining them with powerful modern techniques of geometric optimal control, of the theory of numerical continuation, or of dynamical system theory [159, 147]. Geometric optimal control allows the development of general techniques, applying to wide classes of nonlinear optimal control problems, that can be used to characterize the behavior of optimal trajectories and in particular to establish regularity properties for them and for the cost function. Hence, geometric optimal control can be used to obtain powerful optimal syntheses results and to provide deep geometric insights into many applied problems. Numerical optimal control methods with geometric insight are in particular important to handle subtle situations such as rigid optimal paths and, more generally, optimal syntheses exhibiting abnormal minimizers.

Optimal control is not the only area where the geometric approach has a great impact. Let us mention, for instance, motion planning, where different geometric approaches have been developed: those based on the **Lie algebra** associated with the control system [139, 129], those based on the differentiation of nonlinear flows such as the **return method** [107, 108], and those exploiting the **differential flatness** of the system [112].

Geometric control theory is not only a powerful framework to investigate control systems, but also a useful tool to model and study phenomena that are not *a priori* control-related. Two occurrences of this property play an important role in the activities of CAGE:

- geometric control theory as a tool to investigate properties of mathematical structures;
- geometric control theory as a modeling tool for neurophysical phenomena and for synthesizing biomimetic algorithms based on such models.

Examples of the first type, concern, for instance, hypoelliptic heat kernels [71] or shape optimization [83]. Examples of the second type are inactivation principles in human motricity [86] or neurogeometrical models for image representation of the primary visual cortex in mammals [97].

A particularly relevant class of control systems, both from the point of view of theory and applications, is characterized by the linearity of the controlled vector field with respect to the control parameters. When the controls are unconstrained in norm, this means that the admissible velocities form a distribution in the tangent bundle to the state manifold. If the distribution is equipped with a point-dependent quadratic form (encoding the cost of the control), the resulting geometrical structure is said to be **sub-Riemannian**. Sub-Riemannian geometry appears as the underlying geometry of nonlinear control systems: in a similar way as the linearization of a control system provides local informations which are readable using the Euclidean metric scale, sub-Riemannian geometry provides an adapted non-isotropic class of lenses which are often much more informative. As such, its study is fundamental for control design. The importance of sub-Riemannian geometry goes beyond control theory and it is an active field of research both in differential geometry [137], geometric measure theory [77] and hypoelliptic operator theory [89].

The geometric control approach has historically been related to the development of finite-dimensional control theory. However, its impact in the analysis of distributed parameter control systems and in particular systems of controlled partial differential equations has been growing in the last decades, complementing analytical and numerical approaches, providing dynamical, qualitative and intrinsic insight [106]. CAGE's ambition is to be at the core of this development in the years to come.

4 Application domains

4.1 First axis: Geometry of vision

A suggestive application of sub-Riemannian geometry and in particular of hypoelliptic diffusion comes from a model of geometry of vision describing the functional architecture of the primary visual cortex V1. In 1958, Hubel and Wiesel (Nobel in 1981) observed that the visual cortex V1 is endowed with the so-called **pinwheel structure**, characterized by neurons grouped into orientation columns, that are sensible both to positions and directions [117]. The mathematical rephrasing of this discovery is that the visual cortex lifts an image from $\mathbf{R}^2$ into the bundle of directions of the plane [104, 143, 145, 116].

A simplified version of the model can be described as follows: neurons of V1 are grouped into orientation columns, each of them being sensitive to visual stimuli at a given point of the retina and for a given direction on it. The retina is modeled by the real plane, i.e., each point is represented by a pair $(x, y) \in \mathbb{R}^2$, while the directions at a given point are modeled by the projective line, i.e. an element θ of the projective line P^1. Hence, the primary visual cortex V1 is modeled by the so called projective tangent bundle $\mathrm{PTR}^2 = \mathbf{R}^2 \times \mathbf{P}^1$. From a neurological point of view, orientation columns are in turn grouped into hypercolumns, each of them being sensitive to stimuli at a given point (x, y) with any direction.

Orientation columns are connected between them in two different ways. The first kind of connections are the vertical (inhibitory) ones, which connect orientation columns belonging to the same hypercolumn and sensible to similar directions. The second kind of connections are the horizontal (excitatory) connections, which connect neurons belonging to different (but not too far) hypercolumns and sensible to the same directions. The resulting metric structure is sub-Riemannian and the model obtained in this way provides a convincing explanation in terms of sub-Riemannian geodesics of gestalt phenomena such as Kanizsa illusory contours.

The sub-Riemannian model for image representation of V1 has a great potential of yielding powerful bio-inspired image processing algorithms [111, 97]. Image inpainting, for instance, can be implemented by reconstructing an incomplete image by activating orientation columns in the missing regions in accordance with sub-Riemannian non-isotropic constraints. The process intrinsically defines an hypoelliptic heat equation on PTR^2 which can be integrated numerically using non-commutative Fourier analysis on a suitable semidiscretization of the group of roto-translations of the plane [92].

We have been working on the model and its software implementation since 2012. This work has been supported by several project, as the ERC starting grant GeCoMethods and the ERC Proof of Concept ARTIV1 of U. Boscain, and the ANR GCM.

A parallel approach that we will pursue and combine with this first one is based on **pattern matching in the group of diffeomorphisms**. We want to extend this approach, already explored in the Riemannian setting [160, 134], to the general sub-Riemannian framework. The paradigm of the approach is the following: consider a distortable object, more or less rigid, discretized into a certain number of points. One may track its distortion by considering the paths drawn by these points. One would however like to know how the object itself (and not its discretized version) has been distorted. The study in [160, 134] shed light on the importance of Riemannian geometry in this kind of problem. In particular, they study the Riemannian submersion obtained by making the group of diffeomorphisms act transitively on the manifold formed by the points of the discretization, minimizing a certain energy so as to take into account the whole object. Settled as such, the problem is Riemannian, but if one considers objects involving connections, or submitted to nonholonomic constraints, like in medical imaging where one tracks the motions of organs, then one comes up with a sub-Riemannian problem. The transitive group is then far bigger, and the aim is to lift curves submitted to these nonholonomic constraints into curves in the set of diffeomorphisms satisfying the corresponding constraints, in a unique way and minimizing an energy (giving rise to a sub-Riemannian structure).

4.2 Second axis: Quantum control

The goal of quantum control is to design efficient protocols for tuning the occupation probabilities of the energy levels of a system. This task is crucial in atomic and molecular physics, with applications ranging from photochemistry to nuclear magnetic resonance and quantum computing. A quantum system may be controlled by exciting it with one or several external fields, such as magnetic or electric fields. The goal of quantum control theory is to adapt the tools originally developed by control theory and to develop new specific strategies that tackle and exploit the features of quantum dynamics (probabilistic nature of wavefunctions and density operators, measure and wavefunction collapse, decoherence, ...). A rich variety of relevant models for controlled quantum dynamics exist, encompassing low-dimensional models (e.g., single-spin systems) and PDEs alike, with deterministic and stochastic components, making it a rich and exciting area of research in control theory.

The controllability of quantum system is a well-established topic when the state space is finite-dimensional [109], thanks to general controllability methods for left-invariant control systems on compact Lie groups [99, 119]. When the state space is infinite-dimensional, it is known that in general the bilinear Schrödinger equation is not exactly controllable [162]. Nevertheless, weaker controllability properties, such as approximate controllability or controllability between eigenstates of the internal Hamiltonian (which are the most relevant physical states), may hold. In certain cases, when the state space is a function space on a 1D manifold, some rather precise description of the set of reachable states has been provided [84]. A similar description for higher-dimensional manifolds seems intractable and at the moment only approximate controllability results are available [135, 141, 120]. The most widely applicable tests for controllability of quantum systems in infinite-dimensional Hilbert spaces are based on the **Lie–Galerkin technique** [103, 91, 94]. They allow, in particular, to show that the controllability property is generic among this class of systems [132].

A family of algorithms which are specific to quantum systems are those based on adiabatic evolution [166, 165, 123]. The basic principle of adiabatic control is that the flow of a slowly varying Hamiltonian can be approximated (up to a phase factor) by a quasi-static evolution, with a precision proportional to the velocity of variation of the Hamiltonian. The advantage of the **adiabatic approach** is that it is constructive and produces control laws which are both smooth and robust to parameter uncertainty. The paradigm is based on the adiabatic perturbation theory developed in mathematical physics [90, 140, 158], where it plays an important role for understanding molecular dynamics. Approximation theory by adiabatic perturbation can be used to describe the evolution of the occupation probabilities of the energy levels of a slowly varying Hamiltonian. Results from the last 15 years, including those by members of our team [69, 96], have highlighted the effectiveness of control techniques based on adiabatic path following.

4.3 Third axis: Stability and uncertain dynamics

Switched and hybrid systems constitute a broad framework for the description of the heterogeneous aspects of systems in which continuous dynamics (typically pertaining to physical quantities) interact with discrete/logical components. The development of the switched and hybrid paradigm has been motivated by a broad range of applications, including automotive and transportation industry [150], energy management [142] and congestion control [133].

Even if both controllability [154] and observability [121] of switched and hybrid systems have attracted much research efforts, the central role in their study is played by the problem of stability and stabilizability. The goal is to determine whether a dynamical or a control system whose evolution is influenced by a time-dependent signal is uniformly stable or can be uniformly stabilized [126, 155]. Uniformity is considered with respect to all signals in a given class. Stability of switched systems lead to several interesting phenomena. For example, even when all the subsystems corresponding to a constant switching law are exponentially stable, the switched systems may have divergent trajectories for certain switching signals [127]. This fact illustrates the fact that stability of switched systems depends not only on the dynamics of each subsystem but also on the properties of the class of switching signals which is considered.

The most common class of switching signals which has been considered in the literature is made of all piecewise constant signals.

In this case uniform stability of the system is equivalent to the existence of a common quadratic Lyapunov function [136]. Moreover, provided that the system has finitely many modes, the Lyapunov function can be taken polyhedral or polynomial [87, 88, 110]. A special role in the switched control literature has been played by common quadratic Lyapunov functions, since their existence can be tested rather efficiently (see the surveys [128, 149] and the references therein). It is known, however, that the existence of a common quadratic Lyapunov function is not necessary for the global uniform exponential stability of a linear switched system with finitely many modes. Moreover, there exists no uniform upper bound on the minimal degree of a common polynomial Lyapunov function [131]. More refined tools rely on multiple and non-monotone Lyapunov functions [98]. Let us also mention linear switched systems technics based on the analysis of the Lie algebra generated by the matrices corresponding to the modes of the system [74].

For systems evolving in the plane, more geometrical tests apply, and yield a complete characterization of the stability [93, 78]. Such a geometric approach also yields sufficient conditions for uniform stability in the linear planar case [95].

In many situations, it is interesting for modeling purposes to specify the features of the switched system by introducing **constrained switching rules**. A typical constraint is that each mode is activated for at least a fixed minimal amount of time, called the dwell-time. Switching rules can also be imposed, for instance, by a timed automata. When constraints apply, the common Lyapunov function approach becomes conservative and new tools have to be developed to give more detailed characterizations of stable and unstable systems.

Our approach to constrained switching is based on the idea of relating the analytical properties of the classes of constrained switching laws (shift-invariance, compactness, closure under concatenation, ...) to the stability behavior of the corresponding switched systems. One can introduce **probabilistic uncertainties** by endowing the classes of admissible signals with suitable probability measures. One then looks at the corresponding Lyapunov exponents, whose existence is established by the multiplicative ergodic theorem. The interest of this approach is that probabilistic stability analysis filters out highly 'exceptional' worst-case trajectories. Although less explicitly characterized from a dynamical viewpoint than its deterministic counterpart, the probabilistic notion of uniform exponential stability can be studied using several reformulations of Lyapunov exponents proposed in the literature [85, 105, 163].

4.4 Joint theoretical core

The theoretical questions raised by the different applicative area will be pooled in a research axis on the transversal aspects of geometric control theory and sub-Riemannian structures.

We recall that sub-Riemannian geometry is a generalization of Riemannian geometry, whose birth dates back to Carathéodory's seminal paper on the foundations of Carnot thermodynamics [101], followed by E. Cartan's address at the International Congress of Mathematicians in Bologna [102]. In the last twenty years, sub-Riemannian geometry has emerged as an independent research domain, with a variety of motivations and ramifications in several parts of pure and applied mathematics. Let us mention geometric analysis, geometric measure theory, stochastic calculus and evolution equations together with applications in mechanics and optimal control (motion planning, robotics, nonholonomic mechanics, quantum control) [79, 80].

One of the main open problems in sub-Riemannian geometry concerns the regularity of length-minimizers [73, 138]. Length-minimizers are solutions to a variational problem with constraints and satisfy a first-order necessary condition resulting from the Pontryagin Maximum Principle (PMP). Solutions of the PMP are either *normal* or *abnormal*. Normal length-minimizer are well-known to be smooth, i.e., C^∞, as it follows by the Hamiltonian nature of the PMP. The question of regularity is then reduced to abnormal length-minimizers. If the sub-Riemannian structure has step 2, then abnormal length-minimizers can be excluded and thus every length-minimizer is smooth. For step 3 structures, the situation is already more complicated and smoothness of length-minimizers is known only for Carnot groups [122, 157]. The question of regularity of length-minimizers is not restricted to the smoothness in the C^∞ sense. A recent result prove that length-minimizers, for sub-Riemannian structures of any step, cannot have corner-like singularities [115]. When the sub-Riemannian structure is analytic, more is known on the size of the set of points where a length-minimizer can lose analyticity [156], regardless of the rank and of the step of the distribution.

An interesting set of recent results in sub-Riemannian geometry concerns the extension to such a setting of the Riemannian notion of sectional curvature. The curvature operator can be introduced in terms of the symplectic invariants of the Jacobi curve [72, 124, 70], a curve in the Lagrange Grassmannian related to the linearization of the Hamiltonian flow. Alternative approaches to curvatures in metric spaces are based either on the associated heat equation and the generalization of the curvature-dimension inequality [81, 82] or on optimal transport and the generalization of Ricci curvature [153, 152, 130, 76].

5 Highlights of the year

5.1 Awards

Barbara Gris was awarded the *GSI Women in Machine Learning & Data Science Prize* at the conference GSI2021

Rémi Robin was part of the winning team of the challenge *mathématiques et enterprises* de l'AMIES "Modélisation du profil longitudinal de la voie à partir d'enregistrement inclinométriques" -

6 New software and platforms

6.1 New software

6.1.1 Stellacode

Keywords: Plasma physics, Stellarator, Inverse problem, Magnetic fusion, Electromagnetics, Shape optimization

Functional Description: The goal of Stellacode is to optimize the Coil Winding Surface of a stellarator. What does this means and what for? A stellarator is a nuclear fusion reactor which produces energy from the fusion of nuclei in a very hot plasma. The confinement of this plasma is a very complicated task and a stellarator is able to achieve it thanks to a complex set of non-planar supraconductor coils lying on the so-called "Coil Winding Surface". Stellacode is able to perform efficiently the shape optimization of this surface, taking into account both plasma confinement figures of merit and engineering costs.

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Contact: Rémi Robin

7 New results

7.1 Geometry of vision and sub-Riemannian geometry: new results

Let us list here our new results in the geometry of vision axis and, more generally, on hypoelliptic diffusion and sub-Riemannian geometry.

- A natural way to model the evolution of an object (growth of a leaf for instance) is to estimate a plausible deforming path between two observations. This interpolation process can generate deceiving results when the set of considered deformations is not relevant to the observed data. To overcome this issue, the framework of deformation modules allows to incorporate in the model structured deformation patterns coming from prior knowledge on the data. The goal of [45] is twofold. First defining new deformation modules incorporating structures coming from the elastic properties of the objects. Second, presenting the IMODAL library allowing to perform registration through structured deformations. This library is modular: adapted priors can be easily defined by the user, several priors can be combined into a global one and various types of data can be considered such as curves, meshes or images.

- Based on a Sub-Riemannian framework, deformation modules provide a way of building large diffeomorphic deformations satisfying a given geometrical structure. This allows to incorporate prior knowledge about object deformations into the model as a means of regularisation. However, diffeomorphic deformations can lead to deceiving results if the deformed object is composed of several shapes which are close to each other but require drastically different deformations. For the related Large Deformation Diffemorphic Metric Mapping, which yields unstructured deformations, this issue was addressed in previous works, introducing object boundary constraints. We develop in [44] a new registration problem, marrying the two frameworks to allow for different constrained deformations in different coupled shapes.

- The reconstruction mechanisms built by the human auditory system during sound reconstruction are still a matter of debate. The purpose of [43] is to refine the auditory cortex model introduced previously, and inspired by the geometrical modelling of vision. The algorithm transforms the degraded sound in an 'image' in the time-frequency domain via a short-time Fourier transform. Such an image is then lifted in the Heisenberg group and it is reconstructed via a Wilson-Cowan differo-integral equation. Numerical experiments on a library of speech recordings are provided, showing the good reconstruction properties of the algorithm.

- Loss of cellular homeostasis has been implicated in the etiology of several neurodegenerative diseases (NDs). However, the molecular mechanisms that underlie this loss remain poorly understood on a systems level in each case. In [41], using a novel computational approach to integrate dimensional RNA-seq and in vivo neuron survival data, we map the temporal dynamics of homeostatic and pathogenic responses in four striatal cell types of Huntington's disease (HD) model mice. This map shows that most pathogenic responses are mitigated and most homeostatic responses are decreased over time, suggesting that neuronal death in HD is primarily driven by the loss of homeostatic responses. Moreover, different cell types may lose similar homeostatic processes, for example, endosome biogenesis and mitochondrial quality control in Drd1-expressing neurons and astrocytes. HD relevance is validated by human stem cell, genomewide association study, and post-mortem brain data. These findings provide a new paradigm and framework for therapeutic discovery in HD and other NDs.

- The reconstruction mechanisms built by the human auditory system during sound reconstruction are still a matter of debate. The purpose of [19] is to propose a mathematical model of sound reconstruction based on the functional architecture of the auditory cortex (A1). The model is inspired by the geometrical modelling of vision, which has undergone a great development in the last ten years. The algorithm transforms the degraded sound in an image in the time-frequency domain via a short-time Fourier transform. Such an image is then lifted in the Heisenberg group (i.e., the celebrated Brockett integrator) and it is reconstructed via a Wilson-Cowan differo-integral equation. Numerical experiments are provided.

- Hörmander's propagation of singularities theorem does not fully describe the propagation of singularities in subelliptic wave equations, due to the existence of doubly characteristic points. In [63], building upon a visionary 1986 conference paper by R. Melrose, we prove that singularities of subelliptic wave equations only propagate along null-bicharacteristics and abnormal extremal lifts of singular curves, which are well-known curves in optimal control theory. We first revisit in depth the ideas sketched by R. Melrose, notably providing a full proof of its main statement. Making more explicit computations, we then explain how sub-Riemannian geometry and abnormal extremals come into play. This result shows that abnormal extremals have an important role in the classical-quantum correspondence between sub-Riemannian geometry and subelliptic operators. See also [47].

- In the note [57] we consider a closed three-dimensional contact sub-Riemannian manifold. The objective is to provide a precise description of the sub-Riemannian geodesics with large initial momenta: we prove that they “spiral around the Reeb orbits”, not only in the phase space but also in the configuration space. Our analysis is based on a normal form along any Reeb orbit due to Melrose.

- In [26] we prove that for the Martinet wave equation with “flat” metric, which a subelliptic wave equation, singularities can propagate at any speed between 0 and 1 along any singular geodesic. This is in strong contrast with the usual propagation of singularities at speed 1 for wave equations with elliptic Laplacian.

- In [27] we establish small-time asymptotic expansions for heat kernels of hypoelliptic Hörmander operators in a neighborhood of the diagonal, generalizing former results obtained in particular by Métivier and by Ben Arous. The coefficients of our expansions are identified in terms of the nilpotentization of the underlying sub-Riemannian structure. Our approach is purely analytic and relies in particular on local and global subelliptic estimates as well as on the local nature of small-time asymptotics of heat kernels. The fact that our expansions are valid not only along the diagonal but in an asymptotic neighborhood of the diagonal is the main novelty, useful in view of deriving Weyl laws for subelliptic Laplacians. In turn, we establish a number of other results on hypoelliptic heat kernels that are interesting in themselves, such as Kac’s principle of not feeling the boundary, asymptotic results for singular perturbations of hypoelliptic operators, global smoothing properties for selfadjoint heat semigroups.

- In the survey paper [62] we report on recent works concerning exact observability (and, by duality, exact controllability) properties of subelliptic wave and Schrödinger-type equations. These results illustrate the slowdown of propagation in directions transverse to the horizontal distribution. The proofs combine sub-Riemannian geometry, semi-classical analysis, spectral theory and non-commutative harmonic analysis.

- It is well-known that observability (and, by duality, controllability) of the elliptic wave equation, i.e., with a Riemannian Laplacian, in time T_0 is almost equivalent to the Geometric Control Condition (GCC), which stipulates that any geodesic ray meets the control set within time T_0. We show in [35] that in the subelliptic setting, GCC is never verified, and that subelliptic wave equations are never observable in finite time. More precisely, given any subelliptic Laplacian $\Delta = -\sum_{i=1}^m X_i^* X_i$ on a manifold M such that $\text{Lie}(X_1,\dots,X_m) = TM$ but $\text{Span}(X_1,\dots,X_m) \neq TM$, we show that for any $T_0 > 0$ and any measurable subset $\omega \subset M$ such that $M\backslash\omega$ has nonempty interior, the wave equation with subelliptic Laplacian Δ is not observable on ω in time T_0. The proof is based on the construction of sequences of solutions of the wave equation concentrating on spiraling geodesics (for the associated sub-Riemannian distance) spending a long time in $M\backslash\omega$. As a counterpart, we prove a positive result of observability for the wave equation in the Heisenberg group, where the observation set is a well-chosen part of the phase space.

- In [36], we study the observability (or, equivalently, the controllability) of some subelliptic evolution equations depending on their step. This sheds light on the speed of propagation of these equations, notably in the “degenerated directions” of the subelliptic structure. First, for any $\gamma \geq 1$, we establish a resolvent estimate for the Baouendi-Grushin-type operator $\Delta_\gamma = \partial_x^2 + |x|^{2\gamma}\partial_y^2$, which has step $\gamma+1$. We then derive consequences for the observability of the Schrödinger type equation $i\partial_t u - (-\Delta_\gamma)^s u = 0$ where $s \in N$. We identify three different cases: depending on the value of the ratio $(\gamma+1)/s$, observability may hold in arbitrarily small time, or only for sufficiently large times, or even fail for any time. As a corollary of our resolvent estimate, we also obtain observability for heat-type equations $\partial_t u + (-\Delta_\gamma)^s u = 0$ and establish a decay rate for the damped wave equation associated with Δ_γ.

- In [32] we give necessary and sufficient conditions for the controllability of a Schrödinger equation involving a subelliptic operator on a compact manifold. This subelliptic operator is the sub-Laplacian of the manifold that is obtained by taking the quotient of a group of Heisenberg type by one of its discrete subgroups. This class of nilpotent Lie groups is a major example of stratified Lie groups of step 2. The sub-Laplacian involved in these Schrödinger equations is subelliptic, and, contrarily to what happens for the usual elliptic Schrödinger equation for example on flat tori or on negatively curved manifolds, there exists a minimal time of controllability. The main tools used in the proofs are (operator-valued) semi-classical measures constructed by use of representation theory and a notion of semi-classical wave packets that we introduce here in the context of groups of Heisenberg type.

- In [13] we are concerned with stochastic processes on surfaces in three-dimensional contact sub-Riemannian manifolds. Employing the Riemannian approximations to the sub-Riemannian manifold which make use of the Reeb vector field, we obtain a second order partial differential operator on the surface arising as the limit of Laplace-Beltrami operators. The stochastic process associated with the limiting operator moves along the characteristic foliation induced on the surface by the contact distribution. We show that for this stochastic process elliptic characteristic points are inaccessible, while hyperbolic characteristic points are accessible from the separatrices. We illustrate the results with examples and we identify canonical surfaces in the Heisenberg group, and in SU(2) and SL(2,$\mathbb{R}$) equipped with the standard sub-Riemannian contact structures as model cases for this setting. Our techniques further allow us to derive an expression for an intrinsic Gaussian curvature of a surface in a general three-dimensional contact sub-Riemannian manifold.

- Two-dimension almost-Riemannian structures of step 2 are natural generalizations of the Grushin plane. They are generalized Riemannian structures for which the vectors of a local orthonormal frame can become parallel. Under the 2-step assumption the singular set Z, where the structure is not Riemannian, is a 1D embedded submanifold. While approaching the singular set, all Riemannian quantities diverge. A remarkable property of these structure is that the geodesics can cross the singular set without singularities, but the heat and the solution of the Schrödinger equation (with the Laplace-Beltrami operator Δ) cannot. This is due to the fact that (under a natural compactness hypothesis), the Laplace-Beltrami operator is essentially self-adjoint on a connected component of the manifold without the singular set. In the literature such phenomenon is called quantum confinement. In [16] we study the self-adjointness of the curvature Laplacian, namely $-\frac{1}{2}\Delta + cK$, for $c > 0$ (here K is the Gaussian curvature), which originates in coordinate free quantization procedures (as for instance in path-integral or covariant Weyl quantization). We prove that there is no quantum confinement for this type of operators.

7.2 Quantum control: new results

Let us list here our new results in quantum control theory.

- In [64] we present an analytical approach to construct the Lie algebra of finite-dimensional subsystems of the driven asymmetric top rotor. Each rotational level is degenerate due to the isotropy of space, and the degeneracy increases with rotational excitation. For a given rotational excitation, we determine the nested commutators between drift and drive Hamiltonians using a graph representation. We then generate the Lie algebra for subsystems with arbitrary rotational excitation using an inductive argument. See also [48].

- In [17] we discuss which controllability properties of classical Hamiltonian systems are preserved after quantization. We discuss some necessary and some sufficient conditions for small-time controllability of classical systems and quantum systems using the WKB method. In particular, we investigate the conjecture that if the classical system is not small-time controllable, then the corresponding quantum system is not small-time controllable either.

- In [59], we study, in the semiclassical sense, the global approximate controllability in small time of the quantum density and quantum momentum of the 1-D semiclassical cubic Schrödinger equation with two controls between two states with positive quantum densities. We first control the asymptotic expansions of the zeroth and first order of the physical observables via Agrachev-Sarychev's method. Then we conclude the proof through techniques of semiclassical approximation of the nonlinear Schrödinger equation.

- In [12] we study up to which extent we can apply adiabatic control strategies to a quantum control model obtained by rotating wave approximation. In particular, we show that, under suitable assumptions on the asymptotic regime between the parameters characterizing the rotating wave and the adiabatic approximations, the induced flow converges to the one obtained by considering the two approximations separately and by combining them formally in cascade. As a consequence, we propose explicit control laws which can be used to induce desired populations transfers, robustly with respect to parameter dispersions in the controlled Hamiltonian.

- In the physics literature it is common to see the rotating wave approximation and the adiabatic approximation used "in cascade" to justify the use of chirped pulses for two-level quantum systems driven by one external field, in particular when the resonance frequency of the system is not known precisely. Both approximations need relatively long time and are essentially based on averaging theory of dynamical systems. Unfortunately, the two approximations cannot be done independently since, in a sense, the two time scales interact. The purpose of [66] is to study how the cascade of the two approximations can be justified and how large becomes the final time as the fidelity goes to one, while preserving the robustness of the adiabatic strategy. Our main result gives a precise quantification of the uncertainty interval of the resonance frequency for which the strategy works.

- Optimal Control Theory has become a widely used method to improve process performance in quantum technologies by means of highly efficient control of quantum dynamics. Our review [20] aims at providing an introduction to key concepts of optimal control theory, accessible to physicists and engineers working in quantum control or in related fields. The different mathematical results are introduced intuitively, before being rigorously stated. The review describes modern aspects of optimal control theory, with a particular focus on the Pontryagin Maximum Principle, which is the main tool for determining open-loop control laws without experimental feedback. The different steps to solve an optimal control problem are discussed, before moving on to more advanced topics such as the existence of optimal solutions or the definition of the different types of extremals, namely normal, abnormal, and singular. The review covers various quantum control issues and describes their mathematical formulation suitable for optimal control. The optimal solution of different low-dimensional quantum systems is presented in detail, illustrating how the mathematical tools are applied in a practical way.

7.3 Stability and uncertain dynamics: new results

Let us list here our new results about stability and stabilization of control systems, on the properties of systems with uncertain dynamics.

- The paper [30] is devoted to the controllability of a general linear hyperbolic system in one space dimension using boundary controls on one side. Under precise and generic assumptions on the boundary conditions on the other side, we previously established the optimal time for the null and the exact controllability for this system for a generic source term. In this work, we prove the null-controllability for any time greater than the optimal time and for any source term. Similar results for the exact controllability are also discussed.

- Consider a non-autonomous continuous-time linear system in which the time-dependent matrix determining the dynamics is piecewise constant and takes finitely many values $A_1,\dots,A_N$. The paper [56] studies the equality cases between the maximal Lyapunov exponent associated with the set of matrices $\{A_1,\dots,A_N\}$, on the one hand, and the corresponding ones for piecewise deterministic Markov processes with modes $A_1,\dots,A_N$, on the other hand. A fundamental step in this study consists in establishing a result of independent interest, namely, that any sequence of Markov processes associated with the matrices $A_1,\dots,A_N$ converges, up to extracting a subsequence, to a Markov process associated with a suitable convex combination of those matrices. The analogous problem in continuous time has been studied in [56].

- In [55] we recall some general properties of extremal and Barabanov norms and we give a necessary and sufficient condition for a finite-dimensional continuoustime linear switched system to admit a Barabanov norm.

- Adaptive control using the -modification provides an easily implementable way to stabilize systems with uncertain or fluctuating parameters. Motivated by a specific application from neuroscience, we extend in [42] this methodology to nonlinear time-delay systems ruled by globally Lipschitz dynamics. In order to make the result more handy in practice, we provide an explicit construction of a Lyapunov–Krasovskii functional (LKF) with linear bounds and strict dissipation rate based on the knowledge of an LKF with quadratic bounds and point-wise dissipation rate. When applied to a model of neuronal populations involved in Parkinson's disease, the benefits with respect to a pure proportional stabilization scheme are discussed through numerical simulations.

- The paper [38] is concerned with the Proportional Integral (PI) regulation control of the left Neumann trace of a one-dimensional semilinear wave equation. The control input is selected as the right Neumann trace. The control design goes as follows. First, a preliminary (classical) velocity feedback is applied in order to shift all but a finite number of the eivenvalues of the underlying unbounded operator into the open left half-plane. We then leverage on the projection of the system trajectories into an adequate Riesz basis to obtain a truncated model of the system capturing the remaining unstable modes. Local stability of the resulting closed-loop infinite-dimensional system composed of the semilinear wave equation, the preliminary velocity feedback, and the PI controller, is obtained through the study of an adequate Lyapunov function. Finally, an estimate assessing the set point tracking performance of the left Neumann trace is derived.

- In [31] we construct explicit time-varying feedback laws leading to the global (null) stabilization in small time of the viscous Burgers equation with three scalar controls. Our feedback laws use first the quadratic transport term to achieve the small-time global approximate stabilization and then the linear viscous term to get the small-time local stabilization.

- In [15] we give sufficient conditions for Input-to-State Stability in C^1 norm of general quasilinear hyperbolic systems with boundary input disturbances. In particular the derivation of explicit Input-to-State Stability conditions is discussed for the special case of 2×2 systems.

- Because they represent physical systems with propagation delays, hyperbolic systems are well suited for feedforward control. This is especially true when the delay between a disturbance and the output is larger than the control delay. In [14], we address the design of feedforward controllers for a general class of 2×2 hyperbolic systems with a single disturbance input located at one boundary and a single control actuation at the other boundary. The goal is to design a feedforward control that makes the system output insensitive to the measured disturbance input. We show that, for this class of systems, there exists an efficient ideal feedforward controller which is causal and stable. The problem is first stated and studied in the frequency domain for a simple linear system. Then, our main contribution is to show how the theory can be extended, in the time domain, to general nonlinear hyperbolic systems. The method is illustrated with an application to the control of an open channel represented by Saint- Venant equations where the objective is to make the output water level insensitive to the variations of the input flow rate. Finally, we address a more complex application to a cascade of pools where a blind application of perfect feedforward control can lead to detrimental oscillations. A pragmatic way of modifying the control law to solve this problem is proposed and validated with a simulation experiment.

- Self-organization and control around flocks and mills is studied in [54] for second-order swarming systems involving self-propulsion and potential terms. It is shown that through the action of constrained control, is it possible to control any initial configuration to a flock or a mill. The proof builds on an appropriate combination of several arguments: LaSalle invariance principle and Lyapunov-like decreasing functionals, control linearization techniques, and quasi-static deformations. A stability analysis of the second-order system guides the design of feedback laws for the stabilization to flock and mills, which are also assessed computationally.

- In [18], we study sufficient conditions for the emergence of asymptotic consensus and flocking in a certain class of non-linear generalised Cucker-Smale systems subject to multiplicative communication failures. Our approach is based on the combination of strict Lyapunov design together with the formulation of a suitable persistence condition for multi-agent systems. The latter can be interpreted as a lower bound on the algebraic connectivity of the time-average of the interaction graph generated by the communication weights, and provides quantitative decay estimates for the variance functional along the solutions of the system.

- In [51], we investigate the asymptotic formation of consensus for several classes of time-dependent cooperative graphon dynamics. After motivating the use of this type of macroscopic model to describe multi-agent systems, we adapt the classical notion of scrambling coefficient to this setting, and leverage it to establish sufficient conditions ensuring the exponential convergence to consensus with respect to the L^∞-norm topology. We then shift our attention to consensus formation expressed in terms of the L^2-norm, and prove three different consensus result for symmetric, balanced and strongly connected topologies, which involve a suitable generalisation of the notion of algebraic connectivity to this infinite-dimensional framework. We then show that, just as in the finite-dimensional setting, the notion of algebraic connectivity that we propose encodes information about the connectivity properties of the underlying interaction topology. We finally use the corresponding results to shed some light on the relation between L^2-and L^∞-consensus formation, and illustrate our contributions by a series of numerical simulations.

- In [58] we study the so-called water tank system. In this system, the behavior of water contained in a 1-D tank is modelled by Saint-Venant equations, with a scalar distributed control. It is well-known that the linearized systems around uniform steady-states are not controllable, the uncontrollable part being of infinite dimension. Here we will focus on the linearized systems around non-uniform steady states, corresponding to a constant acceleration of the tank. We prove that these systems are controllable in Sobolev spaces, using the moments method and perturbative spectral estimates. Then, for steady states corresponding to small enough accelerations, we design an explicit Proportional Integral feedback law (obtained thanks to a well-chosen dynamic extension of the system) that stabilizes these systems exponentially with arbitrarily large decay rate. Our design relies on feedback equivalence/backstepping.

- In [53] we consider finite and infinite-dimensional first-order consensus systems with time-constant interaction coefficients. For symmetric coefficients, convergence to consensus is classically established by proving, for instance, that the usual variance is an exponentially decreasing Lyapunov function. We investigate here the convergence to consensus in the non-symmetric case: we identify a positive weight which allows to define a weighted mean corresponding to the consensus, and obtain exponential convergence towards consensus. Moreover, we compute the sharp exponential decay rate.

- In [24] we study asymptotic stability of continuous-time systems with mode-dependent guaranteed dwell time. These systems are reformulated as special cases of a general class of mixed (discrete-continuous) linear switching systems on graphs, in which some modes correspond to discrete actions and some others correspond to continuous-time evolutions. Each discrete action has its own positive weight which accounts for its time-duration. We develop a theory of stability for the mixed systems; in particular, we prove the existence of an invariant Lyapunov norm for mixed systems on graphs and study its structure in various cases, including discrete-time systems for which discrete actions have inhomogeneous time durations. This allows us to adapt recent methods for the joint spectral radius computation (Gripenberg's algorithm and the Invariant Polytope Algorithm) to compute the Lyapunov exponent of mixed systems on graphs.

- In [49] we study the numerical approximation of the stabilization of the semidiscrete linearized Boussinesq system around an unstable stationary state. The stabilization is achieved by internal feedback controls applied on the velocity and the temperature equations, localized in an arbitrary open subset. The goal is to study the approximation by penalization of the free divergence condition in the semidiscrete case. More precisely, considering infinite time horizon LQR optimal control problem, we establish convergence results for the optimal controls, optimal solutions and Riccati operators when the penalization parameter goes to zero. We then propose a numerical validation of these results in a two-dimensional setting.

- In [61] we study the exponential stability in the H^2 norm of the nonlinear Saint-Venant (or shallow water) equations with arbitrary friction and slope using a single Proportional-Integral (PI) control at one end of the channel. Using a good but simple Lyapunov function we find a simple and explicit condition on the gain the PI control to ensure the exponential stability of any steady-states. This condition is independent of the slope, the friction coefficient, the length of the river, the inflow disturbance and, more surprisingly, can be made independent of the steady- state considered. When the inflow disturbance is time-dependent and no steady-state exist, we still have the Input-to-State stability of the system, and we show that changing slightly the PI control enables to recover the exponential stability of slowly varying trajectories.

- In [29] we are interested in the boundary stabilization in finite time of one-dimensional linear hyperbolic balance laws with coefficients depending on time and space. We extend the so called "backstepping method" by introducing appropriate time-dependent integral transformations in order to map our initial system to a new one which has desired stability properties. The kernels of the integral transformations involved are solutions to nonstandard multi-dimensional hyperbolic PDEs, where the time dependence introduces several new difficulties in the treatment of their well-posedness. This work generalizes previous results of the literature, where only time-independent systems were considered.

- Given a discrete-time linear switched system associated with a finite set of matrices, we consider the measures of its asymptotic behavior given by, on the one hand, its deterministic joint spectral radius and, on the other hand, its probabilistic joint spectral radius for Markov random switching signals with given transition matrix and corresponding invariant probability. In [25], we investigate the cases of equality between the two measures.

- The general context of [37] is the feedback control of an infinite-dimensional system so that the closed-loop system satisfies a fading-memory property and achieves the setpoint tracking of a given reference signal. More specifically , this paper is concerned with the Proportional Integral (PI) regulation control of the left Neumann trace of a one-dimensional reaction-diffusion equation with a delayed right Dirichlet boundary control. In this setting, the studied reaction-diffusion equation might be either open-loop stable or unstable. The proposed control strategy goes as follows. First, a finite-dimensional truncated model that captures the unstable dynamics of the original infinite-dimensional system is obtained via spectral decomposition. The truncated model is then augmented by an integral component on the tracking error of the left Neumann trace. After resorting to the Artstein transformation to handle the control input delay, the PI controller is designed by pole shifting. Stability of the resulting closed-loop infinite-dimensional system, consisting of the original reaction-diffusion equation with the PI controller, is then established thanks to an adequate Lyapunov function. In the case of a time-varying reference input and a time-varying distributed disturbance, our stability result takes the form of an exponential Input-to-State Stability (ISS) estimate with fading memory. Finally, another exponential ISS estimate with fading memory is established for the tracking performance of the reference signal by the system output. In particular, these results assess the setpoint regulation of the left Neumann trace in the presence of distributed perturbations that converge to a steady-state value and with a time-derivative that converges to zero. Numerical simulations are carried out to illustrate the efficiency of our control strategy.

- In [33] we deal with infinite-dimensional nonlinear forward complete dynamical systems which are subject to external disturbances. We first extend the well-known Datko lemma to the framework of the considered class of systems. Thanks to this generalization, we provide characterizations of the uniform (with respect to disturbances) local, semi-global, and global exponential stability, through the existence of coercive and non-coercive Lyapunov functionals. The importance of the obtained results is underlined through some applications concerning 1) exponential stability of nonlinear retarded systems with piecewise constant delays, 2) exponential stability preservation under sampling for semilinear control switching systems, and 3) the link between input-to-state stability and exponential stability of semilinear switching systems.

7.4 Controllability: new results

Let us list here our new results on controllability beyond the quantum control framework.

- The work [60] studies the reachable space of infinite dimensional control systems which are null controllable in any positive time, the typical example being the heat equation controlled from the boundary or from an arbitrary open set. The focus is on the robustness of the reachable space with respect to linear or nonlinear perturbations of the generator. More precisely, our first main results asserts that this space is invariant under perturbations which are small (in an appropriate sense). A second main result asserts the invariance of the reachable space with respect to perturbations which are compact (again in an appropriate sense), provided that a Hautus type condition is satisfied. Moreover, our methods give precise information on the behavior of the reachable space when the generator is perturbed by a class of nonlinear operators. When applied to the classical heat equation, our results provide detailed information on the reachable space when the generator is perturbed by a small potential or by a class of non local operators, and in particular in one space dimension, we deduce from our analysis that the reachable space for perturbations of the 1-d heat equation is a space of holomorphic functions. We also show how our approach leads to reachability results for a class of semi-linear parabolic equations.
- In [23] we show that a bilinear control system is approximately controllable if and only if it is controllable in $\mathbb{R}^n \setminus \{0\}$. We approach this problem by looking at the foliation made by the orbits of the system, and by showing that there does not exist a codimension-one foliation in $\mathbb{R}^n \setminus \{0\}$ with dense leaves that are everywhere transversal to the radial direction. The proposed geometric approach allows to extend the results to homogeneous systems that are angularly controllable.
- Under a regularity assumption we prove in [21] that reachability in fixed time for nonlinear control systems is robust under control sampling.
- Our goal in [34] is to relate the observation (or control) of the wave equation on observation domains which evolve in time with some dynamical properties of the geodesic flow. In comparison to the case of static domains of observation, we show that the observability of the wave equation in any dimension of space can be improved by allowing the domain of observation to move.
- In [39] we consider the controllability problem for finite-dimensional linear autonomous control systems with nonnegative controls. Despite the Kalman condition, the unilateral nonnegativity control constraint may cause a positive minimal controllability time. When this happens, we prove that, if the matrix of the system has a real eigenvalue, then there is a minimal time control in the space of Radon measures, which consists of a finite sum of Dirac impulses. When all eigenvalues are real, this control is unique and the number of impulses is less than half the dimension of the space. We also focus on the control system corresponding to a finite-difference spatial discretization of the one-dimensional heat equation with Dirichlet boundary controls, and we provide numerical simulations.

7.5 Optimal control and optimization: new results

Let us list here our new results in optimal control theory beyond quantum control and the sub-Riemannian framework.

- We are interested in the design of stellarators, devices for the production of controlled nuclear fusion reactions alternative to tokamaks. The confinement of the plasma is entirely achieved by a helical magnetic field created by the complex arrangement of coils fed by high currents around a toroidal domain. Such coils describe a surface called "coil winding surface" (CWS). In [65], we model the design of the CWS as a shape optimization problem, so that the cost functional reflects both optimal plasma confinement properties, through a least square discrepancy, and also manufacturability, thanks to geometrical terms involving the lateral surface or the curvature of the CWS. We completely analyze the resulting problem: on the one hand, we establish the existence of an optimal shape, prove the shape differentiability of the criterion, and provide the expression of the differential in a workable form. On the other hand, we propose a numerical method and perform simulations of optimal stellarator shapes. We discuss the efficiency of our approach with respect to the literature in this area.

- The outbreak of COVID-19 resulted in high death tolls all over the world. The aim of [40] is to show how a simple SEIR model was used to make quick predictions for New Jersey in early March 2020 and call for action based on data from China and Italy. A more refined model, which accounts for social distancing, testing, contact tracing and quarantining, is then proposed to identify containment measures to minimize the economic cost of the pandemic. The latter is obtained taking into account all the involved costs including reduced economic activities due to lockdown and quarantining as well as the cost for hospitalization and deaths. The proposed model allows one to find optimal strategies as combinations of implementing various non-pharmaceutical interventions and study different scenarios and likely initial conditions.

- In [52] we study a family of optimal control problems in which one aims at minimizing a cost that mixes a quadratic control penalization and the variance of the system, both for finitely many agents and for the mean-field dynamics as their number goes to infinity. While solutions of the discrete problem always exist in a unique and explicit form, the behavior of their macroscopic counterparts is very sensitive to the magnitude of the time horizon and penalization parameter. When one minimizes the final variance, there always exists a Lipschitz-in-space optimal controls for the infinite dimensional problem, which can be obtained as a suitable extension of the optimal controls for the finite-dimensional problems. The same holds true for variance maximizations whenever the time horizon is sufficiently small. On the contrary, for large final times (or equivalently for small penalizations of the control cost), it can be proven that there does not exist Lipschitz-regular optimal controls for the macroscopic problem.

- Magnetic confinement devices for nuclear fusion can be large and expensive. Compact stellarators are promising candidates for construction, but introduce new difficulties: confinement in smaller volumes requires higher magnetic field, which calls for higher coil-currents and ultimately causes higher Laplace forces on the coils-if everything else remains the same. This motivates the inclusion of force reduction in stellarator coil optimization. In [67] we consider a coil winding surface, we prove that there is a natural and rigorous way to define the Laplace force (despite the magnetic field discontinuity across the current-sheet), we provide examples of cost associated (peak force, surface-integral of the force squared) and discuss easy generalizations to parallel and normal force-components, as these will be subject to different engineering constraints. Such costs can then be easily added to the figure of merit in any multi-objective stellarator coil optimization code. We demonstrate this for a generalization of the REGCOIL code, which we rewrote in python, and provide numerical examples for the NCSX (now QUASAR) design. We present results for various definitions of the cost function, including peak force reductions by up to 40%, and outline future work for further reduction.

- In [50] we consider a measure-theoretical formulation of the training of NeurODEs in the form of a mean-field optimal control with L^2-regularization of the control. We derive first order optimality conditions for the NeurODE training problem in the form of a mean-field maximum principle, and show that it admits a unique control solution, which is Lipschitz continuous in time. As a consequence of this uniqueness property, the mean-field maximum principle also provides a strong quantitative generalization error for finite sample approximations. Our derivation of the mean-field maximum principle is much simpler than the ones currently available in the literature for mean-field optimal control problems, and is based on a generalized Lagrange multiplier theorem on convex sets of spaces of measures. The latter is also new, and can be considered as a result of independent interest.

- Our aim in [11] is to present a new model which encompasses pace optimization and motor control effort for a runner on a fixed distance. We see that for long races, the long term behaviour is well approximated by a turnpike problem. We provide numerical simulations quite consistent with this approximation which leads to a simplified problem. We are also able to estimate the effect of slopes and ramps.

- In [22] we revisit and extend the Riccati theory, unifying continuous-time linear-quadratic optimal permanent and sampled-data control problems, in finite and infinite time horizons. In a nutshell, we prove that: – when the time horizon T tends to $+\infty$, one passes from the Sampled-Data Difference Riccati Equation (SD-DRE) to the Sampled-Data Algebraic Riccati Equation (SD-ARE), and from the Permanent Differential Riccati Equation (P-DRE) to the Permanent Algebraic Riccati Equation (P-ARE); – when the maximal step of the time partition Δ tends to 0, one passes from (SD-DRE) to (P-DRE), and from (SD-ARE) to (P-ARE). Our notations and analysis provide a unified framework in order to settle all corresponding results.
- The turnpike phenomenon stipulates that the solution of an optimal control problem in large time, remains essentially close to a steady-state of the dynamics, itself being the optimal solution of an associated static optimal control problem. Under general assumptions, it is known that not only the optimal state and the optimal control, but also the adjoint state coming from the application of the Pontryagin maximum principle, are exponentially close to a steady-state, except at the beginning and at the end of the time frame. In such results, the turnpike set is a singleton, which is a steady-state. In [68] we establish a turnpike result for finite-dimensional optimal control problems in which some of the coordinates evolve in a monotone way, and some others are partial steady-states of the dynamics. We prove that the discrepancy between the optimal trajectory and the turnpike set is then linear, but not exponential: we thus speak of a linear turnpike theorem. See also [46] for shape turnpike results.

Let us also mention the publication [28], in honor of Enrique Zuazua, at the occasion of his sixtieth birthday.

8 Bilateral contracts and grants with industry

8.1 Bilateral contracts with industry

Contract CIFRE with ArianeGroup (les Mureaux), 2019–2021, funding the thesis of A. Nayet. Participants : M. Cerf (ArianeGroup), E. Trélat (coordinator).

Contract with MBDA (Palaiseau), 2021–2023. Subject: “Controôle optimal pour la planification de trajectoires et l’estimation des ensembles accessibles". Pariticpants: V. Askovic (MBDA & CAGE), E. Tre‘lat (coordinator).

8.2 Bilateral grants with industry

Grant by AFOSR (Air Force Office of Scientific Research), 2020–2023. Participants : Mohab Safey El Din (LIP6), E. Trélat.

9 Partnerships and cooperations

9.1 International research visitors

9.1.1 Visits of international scientists

Inria International Chair

IIC AGRACHEV Andrei

Name of the chair: Sub-Riemannan geometry and applications

Institution of origin: SISSA

Country: Italy

Dates: From Sep 01 2020 to Nov 30 2021

Title: Sub-Riemannan geometry and applications

9.2 European initiatives

9.2.1 FP7 & H2020 projects

Program: H2020-EU.1.3.1. - Fostering new skills by means of excellent initial training of researchers
Call for proposal: MSCA-ITN-2017 - Innovative Training Networks
Project acronym: QUSCO
Project title: Quantum-enhanced Sensing via Quantum Control
Duration: From November 2017 to October 2021.
Coordinator: Christiane Koch
Coordinator for the participant Inria: Ugo Boscain
Abstract: Quantum technologies aim to exploit quantum coherence and entanglement, the two essential elements of quantum physics. Successful implementation of quantum technologies faces the challenge to preserve the relevant nonclassical features at the level of device operation. It is thus deeply linked to the ability to control open quantum systems. The currently closest to market quantum technologies are quantum communication and quantum sensing. The latter holds the promise of reaching unprecedented sensitivity, with the potential to revolutionize medical imaging or structure determination in biology or the controlled construction of novel quantum materials. Quantum control manipulates dynamical processes at the atomic or molecular scale by means of specially tailored external electromagnetic fields. The purpose of QuSCo is to demonstrate the enabling capability of quantum control for quantum sensing and quantum measurement, advancing this field by systematic use of quantum control methods. QuSCo will establish quantum control as a vital part for progress in quantum technologies. QuSCo will expose its students, at the same time, to fundamental questions of quantum mechanics and practical issues of specific applications. Albeit challenging, this reflects our view of the best possible training that the field of quantum technologies can offer. Training in scientific skills is based on the demonstrated tradition of excellence in research of the consortium. It will be complemented by training in communication and commercialization. The latter builds on strong industry participation whereas the former existing expertise on visualization and gamification and combines it with more traditional means of outreach to realize target audience specific public engagement strategies.

9.3 National initiatives

The Inria Exploratory Action "StellaCage" is supporting since Spring 2020 a collaboration between CAGE, Yannick Privat (Inria team TONUS), and the startup Renaissance Fusion, based in Grenoble (Francesco Volpe, CEO & Chris Smiet, CSO).

StellaCage approaches the problem of designing better stellarators (yielding better confinement, with simpler coils, capable of higher fields) by combining geometrical properties of magnetic field lines from the control perspective with shape optimization techniques.

9.3.1 ANR

- ANR SRGI, for *Sub-Riemannian Geometry and Interactions*, coordinated by **Emmanuel Trélat**, started in 2015 and run until March 2021. Other partners: Toulon University and Grenoble University. SRGI deals with sub-Riemannian geometry, hypoelliptic diffusion and geometric control.

- ANR TRECOS, for *New Trends in Control and Stabilization: Constraints and non-local terms*, coordinated by Sylvain Ervedoza, University of Bordeaux. The ANR started in 2021 and runs up to 2024. TRECOS' focus is on control theory for partial differential equations, and in particular models from ecology and biology.

- ANR QUACO, for *QUAntum COntrol: PDE systems and MRI applications*, coordinated by Thomas Chambrion, started in 2017 and will run until mid 2022. Other partners: Burgundy University. QUACO aims at contributing to quantum control theory in two directions: improving the comprehension of the dynamical properties of controlled quantum systems in infinite-dimensional state spaces, and improve the efficiency of control algorithms for MRI.

9.4 Regional initiatives

Barbara Gris is the PI of a *Bourse Emergence(s)* by the *Ville de Paris*.

10 Dissemination

10.1 Promoting scientific activities

10.1.1 Scientific events: organisation

Barbara Gris was the organizer of the *Journée thématique MIA* “Registering Medical Images” at Henri Poincaré Institute(IHP), October 21, 2021.

Ugo Boscain organized (with Dario Prandi and Alessandro Sarti) the session “Sub-Riemannian geometry and neuromathematics” at GSI2021, Paris, July 2021.

Jean-Michel Coron organized (with Tatsien Li and Zhiqiang Wang) two LIASFMA (Laboratoire International Associé Sino-Français de Mathématiques Appliquées) International Graduate School on Applied Mathematics, online, the first one in April 2021 and the second one in November-December 2021.

Ugo Boscain, Eugenio Pozzoli, and Mario Sigalotti were the organizers of the workshop “Quantum day: analysis and control” at LJLL, September 13, 2021.

Mario Sigalotti was one of the organizers of the “Padua Paris Sub-Riemannian seminar”, Padua, Italy, September 6-7, 2021.

10.1.2 Journal

Member of the editorial boards

- Ugo Boscain is Associate editor of SIAM Journal of Control and Optimization
- Ugo Boscain is Managing editor of Journal of Dynamical and Control Systems
- Jean-Michel Coron is Editor-in-chief of Comptes Rendus Mathématique
- Jean-Michel Coron is Associate editor of Journal of Evolution Equations
- Jean-Michel Coron is Associate editor of Asymptotic Analysis
- Jean-Michel Coron is Associate editor of ESAIM: Control, Optimisation and Calculus of Variations
- Jean-Michel Coron is Associate editor of Applied Mathematics Research Express
- Jean-Michel Coron is Associate editor of Advances in Differential Equations
- Jean-Michel Coron is Associate editor of Mathematics of Control, Signals, and Systems
- Jean-Michel Coron is Associate editor of Annales de l’IHP, Analyse non linéaire
- Mario Sigalotti is Associate editor of ESAIM: Control, Optimisation and Calculus of Variations
- Mario Sigalotti is Associate editor of Journal on Dynamical and Control Systems
- Emmanuel Trélat is Editor-in-chief of ESAIM: Control, Optimisation and Calculus of Variations
- Emmanuel Trélat is Associate editor of SIAM Review
- Emmanuel Trélat is Associate editor of Systems & Control Letters

- Emmanuel Trélat is Associate editor of Journal on Dynamical and Control Systems
- Emmanuel Trélat is Associate editor of Bollettino dell'Unione Matematica Italiana
- Emmanuel Trélat is Associate editor of ESAIM: Mathematical Modelling and Numerical Analysis
- Emmanuel Trélat is Editor of BCAM Springer Briefs
- Emmanuel Trélat is Associate editor of IEEE Transactions on Automatic Control
- Emmanuel Trélat is Associate editor of Journal of Optimization Theory and Applications
- Emmanuel Trélat is Associate editor of Mathematical Control & Related Fields
- Emmanuel Trélat is Associate editor of Mathematics of Control, Signals, and Systems
- Emmanuel Trélat is Associate editor of Optimal Control Applications and Methods
- Emmanuel Trélat is Associate editor of Advances in Continuous and Discrete Models: Theory and Modern Applications

10.1.3 Invited talks

- Ugo Boscain and Daniele Cannarsa were invited speakers at the "Padua Paris Sub-Riemannian seminar", Padua, Italy, September 2021.
- Ugo Boscain was invited speaker at the conference "Stochastic Differential Geometry and Mathematical Physics", Rennes (online talk), June 2021.
- Ugo Boscain was invited speaker at the *Séminaire de Neuromathématiques Ehess, Collège de France* (online), April 2021.
- Ugo Boscain was invited speaker at the conference "Microlocal and Global Analysis, Interactions with Geometry", Potsdam, Germany (online talk), February 2021.
- Ugo Boscain was invited speaker at the International conference on Dynamic Control and Optimization DCO2021, Aveiro, Portugal (online talk), February 2021.
- Ugo Boscain was invited speaker at the 41th Winter school on Geometry and Physics, Brno, Czech republic (online talk), January 2021.
- Kévin Le Balc'h was invited speaker at the *Séminaire du LJLL*, December 2021.
- Mario Sigalotti and Emmanuel Trélat were invited speakers at the conference "Analysis, Control, and Numerics for PDE Models of Interest to Physical and Life Sciences", Levico Terme, Italy (online talks), September, 2021
- Mario Sigalotti was invited speaker at the conference "3 days on Evolution PDEs", Urbino, Italy, September 2021.
- Mario Sigalotti was invited speaker at the conference "Analyse et contrôle des systèmes d'EDP", Bordeaux, November 2021.
- Emmanuel Trélat was invited speaker at the Colloquium du Laboratoire J.A. Dieudonneé, Univ. Nice, June 2021.
- Emmanuel Trélat was invited speaker at the Workshop on Nonlinear Analysis and Control Theory, in honor of Enrique Zuazua, online, November 2021.
- Emmanuel Trélat was invited speaker at the conference "Kinetic and mean field problems", Ferrara, Italy (online talk), October 2021.

- Emmanuel Trélat was invited speaker at the SIAM Conference on Control and Its Applications (CT21), online, July 2021.
- Emmanuel Trélat was invited speaker at the INdAM Workshop "Analysis and Numerics of Design, Control and Inverse Problems", Rome, July 2021.
- Emmanuel Trélat was invited speaker at the conference "Stochastic Differential Geometry and Mathematical Physics", Rennes, June 2021.
- Emmanuel Trélat was invited speaker at the miniworkshop "Mathematics of Dissipation – Dynamics, Data and Control", Oberwolfach, Germany, May 2021.
- Emmanuel Trélat was invited speaker at the online seminar in EDP and applied mathematics, Brazil, October 2021.
- Emmanuel Trélat was invited speaker at the Asia Pacific Analysis and PDE Seminar (online), February 2021.
- Emmanuel Trélat was invited speaker at the seminar at Université Clermont Auvergne, October 2021.
- Emmanuel Trélat was invited speaker at the seminar at Université de Brest, September 2021.
- Emmanuel Trélat was invited speaker at the seminar at Chongqing University (online talk), Mars 2021.

10.1.4 Research administration

Emmanuel Trélat is Head of the Laboratoire Jacques-Louis Lions (LJLL).

10.2 Teaching - Supervision - Juries

10.2.1 Teaching

- Ugo Boscain and Mario Sigalotti thought "Geometric control theory" to M2 students at Sorbonne Université
- Ugo Boscain thought "Automatic control with applications in robotics and in quantum engineering" at Ecole Polytechnique.
- Ugo Boscain thought "MODAL of applied mathematics. Contrôle de modèles dynamiques" at Ecole Polytechnique
- Ugo Boscain thought "Sub-Riemannian geometry" to PhD students at SISSA, Trieste, Italy
- Kévin Le Balc'h thought "Analyse numérique" to L3 students at Sorbonne Université.
- Mario Sigalotti thought "Équations d'évolution, stabilité et contrôle" to M1 students at Sorbonne Université
- Emmanuel Trélat thought "Contrôle en dimension finie et infinie" to M2 students at Sorbonne Université
- Emmanuel Trélat thought "Optimisation numérique et sciences des données" to M1 students at Sorbonne Université

10.2.2 Supervision

- PhD: Daniele Cannarsa, “Surfaces in three-dimensionnal contact sub-Riemannian manifolds and controllability of nonlinear ODEs”, October 2021. Supervisors: Davide Barilari (Padova, Italy) and Ugo Boscain.
- PhD: Gontran Lance, “Shape turnpike, numerical control and optimal design for PDES”, started in September November 2021. Supervisors: Emmanuel Trélat and Enrique Zuazua (Erlangen, Germany).
- PhD: Cyril Letrouit, “Équations sous-elliptiques : contrôle, singularités et théorie spectrale”, October 2021. Supervisors: Emmanuel Trélat and Yves Colin de Verdière (Grenoble).
- PhD: Eugenio Pozzoli, “Some problems of evolution and control in quantum mechanics”, October 2021. Supervisors: Ugo Boscain and Mario Sigalotti.
- PhD: Justine Dorsz, “Contrôlabilité et limite de champs moyen”, resigned in Febrary 2021. Supervisors: Olivier Glass and Emmanuel Trélat.
- PhD in progress: Emilio Molina, “Application of optimal control techniques to natural resources management”, started in September 2018, supervisors: Pierre Martinon, Héctor Ramírez, and Mario Sigalotti.
- PhD in progress: Aymeric Nayet, “Améliorations d’un solveur de contrôle optimal pour de nouvelles missions Ariane”, started in September 2019. Supervisor: Emmanuel Trélat.
- PhD in progress: Rémi Robin, “Orbit spaces of Lie groups and applications to quantum control”, started in September 2019, supervisors: Ugo Boscain and Mario Sigalotti.
- PhD in progress: Veljko Askovic, “Planification de trajectoires par HJB & PMP”, started in 2020. Supervisors: Emmanuel Trélat and Hasnaa Zidani (INSA, Rouen).
- PhD in progress: Liangying Chen, “Sensitivity, Verification and Conjugate Times in Stochastic Optimal Control”, started in 2021. Supervisors: Emmanuel Trélat and Xu Zhang.
- PhD in progress: Robin Roussel, “Magnetic field lines and confinement in stellarators: a Hamiltonian perspective”, started in 2021. Supervisors: Ugo Boscain and Mario Sigalotti.

10.2.3 Juries

- Ugo Boscain was referee and member of the jury of the PhD thesis of Irigo Zibo, INSA Rouen.
- Emmanuel Trélat was referee and member of the jury of the PhD thesis of B. Wembe, Université de Toulouse.
- Emmanuel Trélat was president of the jury of the PhD thesis of E. Pozzoli, Sorbonne Universite‘.
- Emmanuel Trélat was referee and member of the jury of the PhD thesis of T. Rossi, Université de Grenoble.
- Emmanuel Trélat was referee and member of the jury of the PhD thesis of Y. Song, Hong-Kong University.
- Emmanuel Trélat was member of the jury of the PhD thesis of L. Zonca, ENS Ulm.
- Emmanuel Trélat was referee and member of the jury of the PhD thesis of M. Dus, Université de Toulouse.
- Emmanuel Trélat was referee and member of the jury of the PhD thesis of E. Blazquez, ISAE, Toulouse.
- Emmanuel Trélat was member of the jury of the PhD thesis of K. Kassab, Sorbonne Universite‘.

10.3 Popularization

10.3.1 Internal or external Inria responsibilities

Emmanuel Trélat is member of the Comité d'Honneur du Comité International des Jeux Mathématiques

10.3.2 Articles and contents

The work [11] by Amandine Aftalion and Emmaneul Trélat has been popularized in the article *Voici la course parfaite* by Muriel Valin, Epsiloon, December 2021.

10.3.3 Interventions

Emmanuel Trélat gave a general public lecture at *Les forums re'gionaux du savoir*, Rouen, October 2021.

11 Scientific production

11.1 Major publications

[1] D. Barilari, U. Boscain, D. Cannarsa and K. Habermann. 'Stochastic processes on surfaces in three-dimensional contact sub-Riemannian manifolds'. In: *Annales de l'Institut Henri Poincaré (B) Probabilités et Statistiques* (2021). 25 pages, 2 figures. DOI: 10.1214/20-AIHP1124. URL: https://hal.archives-ouvertes.fr/hal-02557862.

[2] D. Barilari, Y. Chitour, F. Jean, D. Prandi and M. Sigalotti. 'On the regularity of abnormal minimizers for rank 2 sub-Riemannian structures'. In: *Journal de Mathématiques Pures et Appliquées* 133 (2020), pp. 118–138. DOI: 10.1016/j.matpur.2019.04.008. URL: https://hal.archives-ouvertes.fr/hal-01757343.

[3] M. Bertalmio, L. Calatroni, V. Franceschi, B. Franceschiello and D. Prandi. 'Cortical-inspired Wilson-Cowan-type equations for orientation-dependent contrast perception modelling'. In: *Journal of Mathematical Imaging and Vision* (June 2020). DOI: 10.1007/s10851-020-00960-x. URL: https://hal.archives-ouvertes.fr/hal-02316989.

[4] R. Bonalli, B. Hérissé and E. Trélat. 'Optimal Control of Endo-Atmospheric Launch Vehicle Systems: Geometric and Computational Issues'. In: *IEEE Transactions on Automatic Control* 65.6 (2020), pp. 2418–2433. DOI: 10.1109/tac.2019.2929099. URL: https://hal.archives-ouvertes.fr/hal-01626869.

[5] U. Boscain, E. Pozzoli and M. Sigalotti. 'Classical and quantum controllability of a rotating 3D symmetric molecule'. In: *SIAM Journal on Control and Optimization* (2020). URL: https://hal.inria.fr/hal-02421593.

[6] J.-M. Coron, F. Marbach and F. Sueur. 'Small-time global exact controllability of the Navier-Stokes equation with Navier slip-with-friction boundary conditions'. In: *Journal of the European Mathematical Society* 22.5 (May 2020), pp. 1625–1673. DOI: 10.4171/JEMS/952. URL: https://hal.archives-ouvertes.fr/hal-01422161.

[7] J.-M. Coron and H.-M. Nguyen. 'Finite-time stabilization in optimal time of homogeneous quasilinear hyperbolic systems in one dimensional space'. In: *ESAIM: Control, Optimisation and Calculus of Variations* 26 (2020), p. 119. DOI: 10.1051/cocv/2020061. URL: https://hal.archives-ouvertes.fr/hal-03080852.

[8] J. Lohéac, E. Trélat and E. Zuazua. 'Nonnegative control of finite-dimensional linear systems'. In: *Annales de l'Institut Henri Poincaré (C) Non Linear Analysis* 38 (2021), pp. 301–346. DOI: 10.1016/j.anihpc.2020.07.004. URL: https://hal.archives-ouvertes.fr/hal-02335968.

[9] O. Öktem, B. Gris and C. Chen. 'Image reconstruction through metamorphosis'. In: *Inverse Problems* 36 (2020). DOI: 10.1088/1361-6420/ab5832. URL: https://hal.archives-ouvertes.fr/hal-01773633.

[10] R. Robin, N. Augier, U. Boscain and M. Sigalotti. 'On the compatibility of the adiabatic and rotating wave approximations for robust population transfer in qubits'. working paper or preprint. Mar. 2020. URL: https://hal.archives-ouvertes.fr/hal-02504532.

11.2 Publications of the year

International journals

[11] A. Aftalion and E. Trélat. 'Pace and motor control optimization for a runner'. In: *Journal of Mathematical Biology* 83.1 (2021), Paper No. 9. URL: https://hal.archives-ouvertes.fr/hal-02613182.

[12] N. Augier, U. Boscain and M. Sigalotti. 'Effective adiabatic control of a decoupled Hamiltonian obtained by rotating wave approximation'. In: *Automatica* 136 (2022). URL: https://hal.inria.fr/hal-02562363.

[13] D. Barilari, U. Boscain, D. Cannarsa and K. Habermann. 'Stochastic processes on surfaces in three-dimensional contact sub-Riemannian manifolds'. In: *Annales de l'Institut Henri Poincaré (B) Probabilités et Statistiques* (2021). DOI: 10.1214/20-AIHP1124. URL: https://hal.archives-ouvertes.fr/hal-02557862.

[14] G. Bastin, J.-M. Coron and A. Hayat. 'Feedforward boundary control of 2×2 nonlinear hyperbolic systems with application to Saint-Venant equations'. In: *European Journal of Control* (2021), pp. 41–53. URL: https://hal.inria.fr/hal-03070482.

[15] G. Bastin, J.-M. Coron and A. Hayat. 'Input-to-State Stability in sup norms for hyperbolic systems with boundary disturbances'. In: *Nonlinear Analysis: Theory, Methods and Applications* 208 (2021). URL: https://hal.archives-ouvertes.fr/hal-02552991.

[16] I. Beschastnyi, U. Boscain and E. Pozzoli. 'Quantum confinement for the curvature Laplacian $=\Delta + cK$ on 2D-almost-Riemannian manifolds'. In: *Potential Analysis* (6th Aug. 2021). URL: https://hal.inria.fr/hal-02996832.

[17] I. Beschastnyi, U. Boscain and M. Sigalotti. 'An obstruction to small-time controllability of the bilinear Schrödinger equation'. In: *Journal of Mathematical Physics* 62 (2021). URL: https://hal.inria.fr/hal-02385244.

[18] B. Bonnet and E. Flayac. 'Consensus and Flocking under Communication Failures for a Class of Cucker-Smale Systems'. In: *Systems and Control Letters* 152 (1st June 2021), p. 104930. URL: https://hal.archives-ouvertes.fr/hal-02087941.

[19] U. Boscain, D. Prandi, L. Sacchelli and G. Turco. 'A bio-inspired geometric model for sound reconstruction'. In: *Journal of Mathematical Neuroscience* 11.1 (4th Jan. 2021), p. 2. DOI: 10.1186/s13408-020-00099-4. URL: https://hal.archives-ouvertes.fr/hal-02531537.

[20] U. Boscain, M. Sigalotti and D. Sugny. 'Introduction to the Pontryagin Maximum Principle for Quantum Optimal Control'. In: *PRX Quantum* 2.3 (2021), p. 030203. DOI: 10.1103/PRXQuantum.2.030203. URL: https://hal.inria.fr/hal-02972049.

[21] L. Bourdin and E. Trélat. 'Robustness under control sampling of reachability in fixed time for nonlinear control systems'. In: *Mathematics of Control, Signals, and Systems* 33.3 (2021), pp. 515–551. URL: https://hal.archives-ouvertes.fr/hal-02873100.

[22] L. Bourdin and E. Trélat. 'Unified Riccati theory for optimal permanent and sampled-data control problems in finite and infinite time horizons'. In: *SIAM Journal on Control and Optimization* 59.2 (2021), pp. 489–508. DOI: 10.1137/20M1318535. URL: https://hal.archives-ouvertes.fr/hal-02473706.

[23] D. Cannarsa and M. Sigalotti. 'Approximately controllable finite-dimensional bilinear systems are controllable'. In: *Systems and Control Letters* 157.3 (Nov. 2021), p. 105028. DOI: 10.1016/j.sysconle.2021.105028. URL: https://hal.archives-ouvertes.fr/hal-03213531.

[24] Y. Chitour, N. Guglielmi, V. Y. Protasov and M. Sigalotti. 'Switching systems with dwell time: computing the maximal Lyapunov exponent'. In: *Nonlinear Analysis: Hybrid Systems* (2021). DOI: 10.1016/j.nahs.2021.101021. URL: https://hal.inria.fr/hal-02423619.

[25] Y. Chitour, G. Mazanti and M. Sigalotti. 'On the gap between deterministic and probabilistic joint spectral radii for discrete-time linear systems'. In: *Linear Algebra and its Applications* 613 (15th Mar. 2021), pp. 24–45. DOI: 10.1016/j.laa.2020.12.013. URL: https://hal.archives-ouvertes.fr/hal-01961003.

[26] Y. Colin De Verdière and C. Letrouit. 'Propagation of well-prepared states along Martinet singular geodesics'. In: *Journal of Spectral Theory* (2021). URL: https://hal.archives-ouvertes.fr/hal-03220110.

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[35] C. Letrouit. 'Subelliptic wave equations are never observable'. In: *Analysis & PDE* (2021). URL: https://hal.archives-ouvertes.fr/hal-02466229.

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[37] H. Lhachemi, C. Prieur and E. Trélat. 'PI Regulation of a Reaction-Diffusion Equation with Delayed Boundary Control'. In: *IEEE Transactions on Automatic Control* 66.4 (Apr. 2021), pp. 1573–1587. DOI: 10.1109/TAC.2020.2996598. URL: https://hal.archives-ouvertes.fr/hal-02294321.

[38] H. Lhachemi, C. Prieur and E. Trélat. 'Proportional Integral regulation control of a one-dimensional semilinear wave equation'. In: *SIAM Journal on Control and Optimization* 60.1 (2022), pp. 1–21. URL: https://hal.archives-ouvertes.fr/hal-02872341.

[39] J. Lohéac, E. Trélat and E. Zuazua. 'Nonnegative control of finite-dimensional linear systems'. In: *Annales de l'Institut Henri Poincaré (C) Non Linear Analysis* 38.2 (Mar. 2021), pp. 301–346. DOI: 10.1016/j.anihpc.2020.07.004. URL: https://hal.archives-ouvertes.fr/hal-02335968.

[40] S. T. Mcquade, R. Weightman, N. J. Merrill, A. Yadav, E. Trélat, S. R. Allred and B. Piccoli. 'Control of COVID-19 outbreak using an extended SEIR model'. In: *Mathematical Models and Methods in Applied Sciences* 31.12 (Nov. 2021), pp. 2399–2424. DOI: 10.1142/S0218202521500512. URL: https://hal.archives-ouvertes.fr/hal-03524027.

[41] L. Mégret, B. Gris, S. Sasidharan Nair, J. Cevost, M. Wertz, J. Aaronson, J. Rosinski, T. Vogt, H. Wilkinson, M. Heiman and C. Neri. 'Shape deformation analysis reveals the temporal dynamics of cell-type-specific homeostatic and pathogenic responses to mutant huntingtin'. In: *eLife* 10 (2021), e64984. DOI: 10.7554/eLife.64984. URL: https://hal.sorbonne-universite.fr/hal-03151218.

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[43] R. Asswad, U. Boscain, G. Turco, D. Prandi and L. Sacchelli. 'An auditory cortex model for sound processing'. In: *Geometric Science of Information*. GSI 2021: Geometric Science of Information. Vol. 12829. Lecture Notes in Computer Science. Paris, France, 14th July 2021. DOI: 10.1007/978-3-030-80209-7_7. URL: https://hal.archives-ouvertes.fr/hal-03160486.

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[45] L. Lacroix, B. Charlier, A. Trouvé and B. Gris. 'IMODAL: creating learnable user-defined deformation models'. In: *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition (CVPR)*. CVPR 2021 - IEEE/CVF Conference on Computer Vision and Pattern Recognition. Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition (CVPR). Virtual event, United States, 19th June 2021, pp. 12905–12913. URL: https://hal.archives-ouvertes.fr/hal-03251752.

Doctoral dissertations and habilitation theses

[46] G. Lance. 'Shape turnpike, numerical control and optimal design for PDEs'. Sorbonne Université, 26th Nov. 2021. URL: https://tel.archives-ouvertes.fr/tel-03486151.

[47] C. Letrouit. 'Subelliptic equations : control, singularities and spectral theory'. Sorbonne Université, 4th Oct. 2021. URL: https://tel.archives-ouvertes.fr/tel-03482007.

[48] E. Pozzoli. 'Some problems of evolution and control in quantum mechanics'. Sorbonne Université, 20th Oct. 2021. URL: https://tel.archives-ouvertes.fr/tel-03482146.

Reports & preprints

[49] M. Azaiez and K. Le Balc'h. *Semidiscrete approximation of the penalty approach to the stabilization of the Boussinesq system by localized feedback control*. 23rd Nov. 2021. URL: https://hal.archives-ouvertes.fr/hal-03442394.

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