RESEARCH CENTRE

Grenoble - Rhône-Alpes

2021

ACTIVITY REPORT

Project-Team

ELAN

modELing the Appearance of Nonlinear phenomena

IN COLLABORATION WITH: Laboratoire Jean Kuntzmann (LJK)

DOMAIN

Applied Mathematics, Computation and Simulation

THEME

Numerical schemes and simulations

Contents

Project-Team ELAN

Creation of the Project-Team: 2019 August 01

Keywords

Computer sciences and digital sciences

A2.5. – Software engineering

A5.5.4. – Animation

A6.1.1. – Continuous Modeling (PDE, ODE)

A6.1.4. – Multiscale modeling

A6.1.5. – Multiphysics modeling

A6.2.1. – Numerical analysis of PDE and ODE

A6.2.5. – Numerical Linear Algebra

A6.2.6. – Optimization

A6.2.7. – High performance computing

A6.2.8. – Computational geometry and meshes

A6.3.1. – Inverse problems

A6.5. – Mathematical modeling for physical sciences

A6.5.1. – Solid mechanics

A6.5.2. – Fluid mechanics

A6.5.3. – Transport

A9.2. – Machine learning

Other research topics and application domains

B1.1.2. – Molecular and cellular biology

B3.3.1. – Earth and subsoil

B5.5. – Materials

B9.2.2. – Cinema, Television

B9.5.3. – Physics

B9.5.5. – Mechanics

1 Team members, visitors, external collaborators

Research Scientists

- Florence Descoubes [Team leader, Inria, Senior Researcher, HDR]
- Thibaut Metivet [Inria, Researcher]
- Victor Romero Gramegna [Inria, ISFP, from Sep 2021]

PhD Students

- Raphael Charrondiere [Univ Grenoble Alpes, until Oct 2021]
- Emile Hohnadel [École Normale Supérieure de Lyon, from Sep 2021]
- Jean Jouve [École normale supérieure de Rennes, from Sep 2021]
- Mickael Ly [Inria, until Sep 2021]
- Nicolas Parent [Inria, from Oct 2020 to Sep 2021]
- Abdullah Haroon Rasheed [Inria, until Oct 2021]
- Alexandre Teixeira Da Silva [Inria, from Feb 2021]

Technical Staff

- Laurence Boissieux [Inria, Engineer]
- Victor Romero Gramegna [Inria, Engineer, until Aug 2021]

Interns and Apprentices

- Emile Hohnadel [École Normale Supérieure de Lyon, until Mar 2021]
- Jean Jouve [École normale supérieure de Rennes, from Feb 2021 until Jul 2021]

Administrative Assistant

- Julia Di Toro [Inria, until Aug 2022]

2 Overall objectives

ELAN is a young research team of Inria and Laboratoire Jean Kuntzmann (UMR 5224), with an original positioning across Computer Graphics and Computational Mechanics. The team is focussed on the design of predictive, robust, efficient, and controllable numerical models for capturing the shape and motion of visually rich mechanical phenomena, such as the buckling of an elastic ribbon, the flowing of sand, or the entangling of large fiber assemblies. Target applications encompass the digital entertainment industry (e.g., feature film animation, special effects), as well as virtual prototyping for the mechanical engineering industry (e.g., aircraft manufacturing, cosmetology); though very different, these two application fields require predictive and scalable models for capturing complex mechanical phenomena at the macroscopic scale. An orthogonal objective is the improvement of our understanding of natural physical and biological processes involving slender structures and frictional contact, through active collaborations with soft matter physicists. To achieve its goals, the team strives to master as finely as possible the entire modeling pipeline, involving a pluridisciplinary combination of scientific skills across Mechanics and Physics, Applied Mathematics, and Computer Science.

3 Research program

3.1 Discrete modeling of slender elastic structures

For the last 15 years, we have investigated new discrete models for solving the Kirchhoff dynamic equations for thin elastic rods [18, 19, 22]. All our models share a curvature-based spatial discretization, allowing them to capture inextensibility of the rod intrinsically, without the need for adding any kinematic constraint. Moreover, elastic forces boil down to linear terms in the dynamic equations, making them well-suited for implicit integration. Interestingly, our discretization methodology can be interpreted from two different points-of-views. From the finite-elements point-of-view, our strain-based discrete schemes can be seen as discontinuous Galerkin methods of zero and first orders. From the multibody system dynamics point of view, our discrete models can be interpreted as deformable Lagrangian systems in finite dimension, for which a dedicated community has started to grow recently [44]. We note that adopting the multibody system dynamics point of view helped us formulate a linear-time integration scheme [17], which had only be investigated in the case of multibody rigid bodies dynamics so far.

High-order spatial discretization schemes for rods, ribbons and shells Our goal is to investigate similar high-order modeling strategies for surfaces, in particular for the case of inextensible ribbons and shells. Elastic ribbons have been scarcely studied in the past, but they are nowadays drawing more and more the attention from physicists [32, 41]. Their numerical modeling remains an open challenge. In contrast to ribbons, a huge litterature exists for shells, both from a theoretical and numerical viewpoints (see, e.g., [36, 23]). However, no real consensus has been obtained so far about a unified nonlinear shell theory able to support large displacements. In [21] we have started building an inextensible shell patch by taking as degrees of freedom the curvatures of its mid-surface, expressed in the local frame. As in the super-helix model, we show that when taking curvatures uniform over the element, each term of the equations of motion may be computed in closed-form; besides, the geometry of the element corresponds to a cylinder patch at each time step. Compared to the 1D (rod) case however, some difficulties arise in the 2D (plate/shell) case, where compatibility conditions are to be treated carefully.

Numerical continuation of rod equilibria in the presence of unilateral constraints In Alejandro Blumentals' PhD thesis [20], we have adopted an optimal control point of view on the static problem of thin elastic rods, and we have shown that direct discretization methods [1] are particularly well-suited for dealing with scenarios involving both bilateral and unilateral constraints (such as contact). We would like to investigate how our formulations extend to continuation problems, where the goal is to follow a certain branch of equilibria when the rod is subject to some varying constraints (such as one fixed end being applied a constant rotation). To the best of our knowledge, classical continuation methods used for rods [33] are not able to deal with non-persistent or sliding contact.

3.2 Discrete and continuous modeling of frictional contact

Most popular approaches in Computer Graphics and Mechanical Engineering consist in assuming that the objects in contact are locally compliant, allowing them to slightly penetrate each other. This is the principle of penalty-based methods (or molecular dynamics), which consists in adding mutual repulsive forces of the form $k\,f(\delta)$, where δ is the penetration depth detected at current time step [24, 40]. Though simple to implement and computationally efficient, the penalty-based method often fails to prevent excessive penetration of the contacting objects, which may prove fatal in the case of thin objects as those may just end up traversing each other. One solution might be to set the stiffness factor k to a large enough value, however this causes the introduction of parasitical high frequencies and calls for very small integration steps [16]. Penalty-based approaches are thus generally not satisfying for ensuring robust contact handling.

In the same vein, the friction law between solid objects, or within a yield-stress fluid (used to model foam, sand, or cement, which, unlike water, cannot flow beyond a certain threshold), is commonly modeled using a regularized friction law (sometimes even with simple viscous forces), for the sake of

[1] Within this optimal control framework, our previous curvature-based methods can actually be interpreted as a special case of direct single shooting methods.

simplicity and numerical tractability (see e.g., [43, 35]). Such a model cannot capture the threshold effect that characterizes friction between contacting solids or within a yield-stress fluid. The nonsmooth transition between sticking and sliding is however responsible for significant visual features, such as the complex patterns resting on the outer surface of hair, the stable formation of sand piles, or typical stick-slip instabilities occurring during motion.

The search for a realistic, robust and stable frictional contact method encouraged us to depart from those, and instead to focus on rigid contact models coupled to the exact nonsmooth Coulomb law for friction (and respectively, to the exact nonsmooth Drucker-Prager law in the case of a fluid), which better integrate the effects of frictional contact at the macroscopic scale. This motivation was the sense of the hiring of F. Bertails-Descoubes in 2007 in the Inria/LJK BIPOP team, specialized in nonsmooth mechanics and related convex optimization methods. In the line of F. Bertails-Descoubes's work performed in the BIPOP team, the ELAN team keeps on including some active research on the finding of robust frictional contact algorithms specialized for slender deformable structures.

Optimized algorithms for large nodal systems in frictional contact In the fiber assembly case, the resulting mass matrix M is block-diagonal, so that the Delassus operator can be computed in an efficient way by leveraging sparse-block computations [26]. This justifies solving the reduced discrete frictional contact problem where primary unknowns are forces, as usually advocated in nonsmooth mechanics [38]. For cloth however, where primal variables (nodal velocities of the cloth mesh) are all interconnected via elasticity through implicit forces, the method developed above is computationally inefficient. Indeed, the matrix M (only block-sparse, but not block-diagonal) is costly to invert for large systems and its inverse is dense. Recently, we have leveraged the fact that generalized velocities of the system are 3D velocities, which simplifies the discrete contact problem when contacts occur at the nodes. Combined with a multiresolution strategy, we have devised an algorithm able to capture exact Coulomb friction constraints at contact, while retaining computational efficiency [39]. This work also supports cloth self-contact and cloth multilayering. How to enrich the interaction model with, e.g., cohesion, remains an open question. The experimental validation of our frictional contact model is also one of our goals in the medium run.

Continuum modeling of granular and fibrous media Though we have recently made progress on the continuum formulation and solving of granular materials in Gilles Daviet's PhD thesis [25, 28, 27], we are still far from a continuum description of a macroscopic dry fibrous medium such as hair. One key ingredient that we have not been considering in our previous models is the influence of air inside divided materials. Typically, air plays a considerable role in hair motion. To advance in that direction, we have started to look at a diphasic fluid representation of granular matter, where a Newtonian fluid and the solid phase are fully coupled, while the nonsmooth Drucker-Prager rheology for the solid phase is enforced implicitly [29]. This first approach could be a starting point for modeling immersed granulars in a liquid, or ash clouds, for instance.

A long path then remains to be achieved, if one wants to take into account long fibers instead of isotropic grains in the solid phase. How to couple the fiber elasticity with our current formulation remains a challenging problem.

3.3 Inverse design of slender elastic structures [ERC GEM]

With the considerable advance of automatic image-based capture in Computer Vision and Computer Graphics these latest years, it becomes now affordable to acquire quickly and precisely the full 3D geometry of many mechanical objects featuring intricate shapes. Yet, while more and more geometrical data get collected and shared among the communities, there is currently very little study about how to infer the underlying mechanical properties of the captured objects merely from their geometrical configurations.

An important challenge consists in developing a non-invasive method for inferring the mechanical properties of complex objects from a minimal set of geometrical poses, in order to predict their dynamics. In contrast to classical inverse reconstruction methods, our claim is that 1/ the mere geometrical shape of physical objects reveals a lot about their underlying mechanical properties and 2/ this property can be fully leveraged for a wide range of objects featuring rich geometrical configurations, such as slender structures subject to contact and friction (e.g., folded cloth or twined filaments).

In addition to significant advances in fast image-based measurement of diverse mechanical materials stemming from physics, biology, or manufacturing, this research is expected in the long run to ease considerably the design of physically realistic virtual worlds, as well as to boost the creation of dynamic human doubles.

To achieve this goal, we shall develop an original inverse modeling strategy based upon the following research topics:

Design of well-suited discrete models for slender structures We believe that the quality of the upstream, reference physics-based model is essential to the effective connection between geometry and mechanics. Typically, such a model should properly account for the nonlinearities due to large displacements of the structures, as well as to the nonsmooth effects typical of contact and friction.

It should also be parameterized and discretized in such a way that inversion gets simplified mathematically, possibly avoiding the huge cost of large and nonconvex optimization. In that sense, unlike concurrent methods which impose inverse methods to be compatible with a generic physics-based model, we instead advocate the design of specific physics-based models which are tailored for the inversion process.

More precisely, from our experience on fiber modeling, we believe that reduced Lagrangian models, based on a minimal set of coordinates and physical parameters (as opposed to maximal coordinates models such as mass-springs), are particularly well-suited for inversion and physical interpretation of geometrical data [31, 30]. Furthermore, choosing a high-order coordinate system (e.g., curvatures instead of angles) allows for a precise handling of curved boundaries and contact geometry, as well as the simplification of constitutive laws (which are transformed into a linear equation in the case of rods). We are currently investigating high-order discretization schemes for elastic ribbons and developable shells [21].

Static inversion of physical objects from geometrical poses We believe that pure static inversion may by itself reveal many insights regarding a range of parameters such as the undeformed configuration of the object, some material parameters or contact forces.

The typical settings that we consider is composed of, on the one hand, a reference mechanical model of the object of interest, and on the other hand a single or a series of complete geometrical poses corresponding each to a static equilibrium. The core challenge consists in analyzing theoretically and practically the amount of information that can be gained from one or several geometrical poses, and to understand how the fundamental under-determinacy of the inverse problem can be reduced, for each unknown quantity (parameter or force) at play. Both the equilibrium condition and the stability criterion of the equilibrium are leveraged towards this goal. On the theoretical side, we have recently shown that a given 3D curve always matches the centerline of an isotropic suspended Kirchhoff rod at equilibrium under gravity, and that the natural configuration of the rod is unique once material parameters (mass, Young modulus) are fixed [1]. On the practical side, we have recently devised a robust algorithm to find a valid natural configuration for a discrete shell to match a given surface under gravity and frictional contact forces [4]. Unlike rods however, shells can have multiple inverse (natural) configurations. Choosing among the multiple solutions based on some selection criteria is an open challenge. Another open issue, in all cases, is the theoretical characterization of material parameters allowing the equilibrium to be stable.

Dynamic inversion of physical objects from geometrical poses To refine the solution subspaces searched for in the static case and estimate dynamic parameters (e.g., some damping coefficients), a dynamic inversion process accounting for the motion of the object of interest is necessary.

In contrast to the static case where we can afford to rely on exact geometrical poses, our analysis in the dynamic case will have to take into account the imperfect quality of input data with possible missing parts or outliers. One interesting challenge will be to combine our high-order discretized physics-based model together with the acquisition process in order to refine both the parameter estimation and the geometrical acquisition.

Experimental validation with respect to real data The goal will be to confront the theories developed above to real experiments. Compared to the statics, the dynamic case will be particularly involving as it will be highly dependent on the quality of input data as well as the accuracy of the motion predicted by our physics-based simulators. Such experiments will not only serve to refine our direct and inverse models, but will also be leveraged to improve the 3D geometrical acquisition of moving objects. Besides, once validation will be performed, we shall work on the setting up of new non-invasive measurement protocols to acquire physical parameters of slender structures from a minimal amount of geometrical configurations.

4 Application domains

4.1 Mechanical Engineering

Many physicists and mathematicians have strived for centuries to understand the principles governing those complex mechanical phenomena, providing a number of continuous models for slender structures, granular matter, and frictional contact. In the XX$^{\text{th}}$ century, industrial applications such as process automatization and new ways of transportation have boosted the fields of Mechanical Engineering and Computer-Aided Design, where material strength, reliability of mechanisms, and safety, standed for the main priorities. Instead, large displacements of structures, buckling, tearing, or entanglement, and even dynamics, were long considered as undesirable behaviors, thus restraining the search for corresponding numerical models.

Only recently, the engineering industry has shown some new and growing interest into the modeling of dynamic phenomena prone to large displacements, contact and friction. For instance, the cosmetology industry is more and more interested in understanding the nonlinear deformation of hair and skin, with the help of simulation. Likewise, auto and aircraft manufacturers are facing new challenges involving buckling or entanglement of thin structures such as carbon or optical fibers; they clearly lack predictive, robust and efficient numerical tools for simulating and optimizing their new manufacturing process, which share many common features with the large-scale simulation scenarii traditionally studied in Computer Graphics applications.

4.2 Computer Graphics

In contrast, Computer Graphics, which has emerged in the 60's with the advent of modern computers, was from the very beginning eager to capture such peculiar phenomena, with the sole aim to produce spectacular images and create astonishing stories. At the origin, Computer Graphics thus drastically departed from other scientific fields. Everyday-life phenomena such as cloth buckling, paper tearing, or hair fluttering in the wind, mostly ignored by other scientists at that time, became actual topics of interest, involving a large set of new research directions to be explored, both in terms of modelling and simulation. Nowadays, although the image production still remains the core activity of the Computer Graphics community, more and more research studies are directed through the virtual and real prototyping of mechanical systems, notably driven by a myriad of new applications in the virtual try on industry (e.g., hairstyling and garment fitting). Furthermore, the advent of additive fabrication is currently boosting research in the free design of new mechanisms or systems for various applications, from architecture design and fabrication of metamaterials to the creation of new locomotion modes in robotics. Some obvious common interests and approaches are thus emerging between Computer Graphics and Mechanical Engineering, yet the two communities remain desperately compartmentalized.

4.3 Soft Matter Physics

From the physics-based viewpoint, since a few decades a new generation of physicists became interested again in the understanding of such visually fascinating phenomena, and started investigating the tight links between geometry and elasticity [2]. Common objects such as folded or torn paper, twined

[2] In France this new trend was particularly stimulated by the work of Yves Pomeau, who convinced many young scientists to study the nonlinear physics of common objects such as paper, plants, or hair [34].

plants, coiled honey threads, or human hair have thus regained some popularity among the community in Nonlinear Physics [3]. In consequence, phenomena of interest have become remarkably close to those of Computer Graphics, since scientists in both places share the common goal to model complex and integrated mechanical phenomena at the macroscopic scale. Of course, the goals and employed methodologies differ substantially from one community to the other, but showcase some evident complementarity: while computer scientists are eager to learn and understand new physical models, physicists get more and more interested in the numerical tools, in which they perceive not only a means to confirm predictions afterwards, but also a support for testing new hypothesis and exploring scenarios that would be too cumbersome or even impossible to investigate experimentally. Besides, numerical exploration starts becoming a valuable tool for getting insights into the search for analytic solutions, thus fully participating to the modeling stage and physical understanding. However, physicists may be limited to a blind usage of numerical black boxes, which may furthermore not be dedicated to their specific needs. According to us, promoting a science of modeling in numerical physics would thus be a promising and rich avenue for the two research fields. Unfortunately, very scarce cooperation currently exists between the two communities, and large networks of collaboration still need to be set up.

5 Social and environmental responsibility

5.1 Footprint of research activities

The ELAN team is environment-sensitive. Since its creation in 2017, 100% of its research staff moves daily from home to the lab using soft transportation means (biking, public transportation). Intercontinental missions are limited while train is the preferred mode of transportation in Europe.

5.2 Impact of research results

A large part of the research conducted in the team is of fundamental level. Direct applications lie in numerical arts, cloth design, sports, and environmental studies, all of these being of limited negative impact for the environment. Collaborations with industry leading specially harmful activities to the environment are avoided.

6 Highlights of the year

6.1 Sustained experimental work in ELAN

From its beginning, ELAN has integrated experimental work into its modelling pipeline. Experiments are particularly interesting for validating numerical models carefully, as well as for investigating new physical behaviours (collective effects in granular and fibrous media, for instance). This original coupling of experiments together with numerics will be sustained thanks to the hiring of Victor Romero in 2021 as a permanent researcher of the team. In particular, the ELANFAB experimental platform (see Section 7.5.1), which made it possible to conduct important scientific studies within ELAN and/or in collaboration with other Inria groups (MORPHÉO, ANIMA) and external partners (Sorbonne Université, IIT Delhi, Yale), will continue to be developed and maintained in the long run.

6.2 Fish In Silico with Hydrodynamics and Social Forces - FISHSIF

The FISHSIF project has received a three-year funding from the ANR (Agence Nationale pour la Recherche). The goal of this project is to introduce dynamical cognition models within full hydrodynamic simulations in order to understand the role played by social or flow interactions in the organisation and behaviour of schools of fish. The project will be led in a collaboration between the ELAN team, the Laboratoire Interdisciplianire de Physique (LIPhy, UGA/CNRS) and the Laboratoire de Psychologie et NeuroCognition (LPNC, UGA/CNRS).

[3] It is however amusing to observe that research in these areas is quite successful in obtaining the IG Nobel prize [15, 37], thus still being considered as an exotic research topic by physicists.

6.3 Semi-plenary Talk at WCCM-ECCOMAS

In January 2021, F. Bertails-Descoubes was **semi-plenary speaker** at WCCM-ECCOMAS 2020, the 14th World Congress in Computational Mechanics.

7 New software and platforms

7.1 Software development policy

Our research work involves a large number of software developments, as each new numerical model gives rise to the development of one or several home-made simulation softwares. On the one hand, our peculiar research activity has led to the design of effective and competitive softwares addressing some dedicated problems (such as specialised fiber or frictional contact models), some of which having been transfered to industry in the past. On the other hand, in the team we participate to the development and maintainance of some more generic platforms (such as **Feel++**), mostly for academic usage.

7.2 Software distribution policy

In the ELAN team we chose to favor the free distribution of our source codes accompanying our major scientific publications, for replicability and dissemination purposes. To accomodate both free dissemination and industrial transfer, our current policy, publicly displayed here, is to find the right balance between free and proprietary licensing agreements, through **dual licensing**.

7.3 Experimental set-ups

In addition to software development, our recent activity includes the development of experimental set-ups, initially realised both in the Amiqual4Home “Atelier Numérique” in Montbonnot and in the experimental laboratory of Institut Jean le Rond d’Alembert in Paris, and now conducted in the new experimentation place of Inria GRA (formerly the "Halle Robotique"). The goal of our current platform **ElanFab** (see Section 7.5.1) is twofold: first, validate our simulators on some controlled experiments; second, discover some new physical phenomena worth investigating, with the help of the predictions brought by our simulators.

7.4 New software

7.4.1 Feel++

Keywords: High order finite elements, Discontinuous Galerkin, High-Performance Computing

Functional Description: Feel++ is a high-performance C++ library for the resolution of general variational formulations, including continuous and discontinuous Galerkin methods, finite element or spectral element methods, reduced basis formulations, etc. It features a high-level domain specific embedded language (DSEL) for Galerkin methods, space dimension-agnostic computation kernels and seamless and automatic parallelism. It also includes applicative toolboxes to solve physics problems in fluid mechanics, solid mechanics, thermal conduction, and the corresponding multi-physics coupling.

Release Contributions: - Enable adaptive remeshing - Optimisation of automatic parallelism (for export and degrees of freedom management) - Add automatic computation of simulation statistics

URL: `http://www.feelpp.org`

Contact: Thibaut Metivet

Partners: Université de Strasbourg, UGA, Inria

7.4.2 ProjectiveFriction

Name: Projective Dynamics with Dry Frictional Contact

Keywords: Physical simulation, Frictional contact, Real time

Functional Description: Simulation based on the Projective Dynamics method that accurately reproduce dry frictional contact while keeping the high performance brought by the Projective Dynamics method. This is possible thanks to the iterative solve of an approximated discrete frictional contact problem at the local step of Projective Dynamics.

Publication: hal-02563307

Contact: Florence Bertails Descoubes

Participants: Florence Bertails Descoubes, Laurence Boissieux, Mickael Ly, Jean Jouve

7.4.3 Sand6

Keywords: Granular matter, Frictional contact, Drücker-Prager rheology

Scientific Description: sand6 is a software to simulate the dynamics of granular matter using a continuum approach accounting for non-smooth flow rules. It is based on the nonsmooth Material Point Method described in [DBD16a] and is currently maintained and developed in the team for various aspects related to the modeling of frictional contact in continuous systems.

Functional Description: Simulation of granular matter as a continuum media

Release Contributions: This software contains a C++ implementation of the algorithms described in the 2016 ACM SIGGRAPH paper entitled "A Semi-Implicit Material Point Method for the Continuum Simulation of Granular Materials" by Gilles Daviet and Florence Bertails-Descoubes.

Publication: hal-01310189

Contact: Florence Bertails Descoubes

Participants: Gilles Daviet, Florence Bertails Descoubes, Thibaut Metivet

7.4.4 MERCI

Name: Energy Minimisation of Curvature-based numerical models for Inextensible Ribbons

Keywords: Thin elastic rod, Thin elastic ribbon, Physical simulation

Scientific Description: MERCI is a C++/lua software for computing the statics of thin elastic ribbons discretised with curvature-based elements. It is based on the super-ribbon model described in [Charrondière et al. 2020, Charrondière et al. 2022], and relies on the free [IPOPT](https://coin-or.github.io/Ipopt/) optimisation software (coinor project) for the static solver. The ribbon can be clamped at one or both ends, and even closed. Contact is treated by contraints with planes. Once the setup is defined, the equilibrium of the ribbon under the specified boundary conditions, external forces, and constraints, is computed. MERCI can be used as a C++ library, or via its lua interface.

Reference code of the PhD thesis:

Raphaël Charrondière, "Modélisation numérique de rubans par éléments en courbures", 2021, Université Grenoble Alpes, https://hal.inria.fr/tel-03545017v2

and of the following papers:

R. Charrondière, F. Bertails-Descoubes, S. Neukirch, V. Romero, "Numerical modeling of inextensible elastic ribbons with curvature-based elements", Computer Methods in Applied Mechanics and Engineering 364, June 2020, p. 1–32, [doi:10.1016/j.cma.2020.112922], [hal-02515877].

R. Charrondière, S. Neukirch, F. Bertails-Descoubes, "MERCI: Mixed curvature-based elements for computing equilibria of thin elastic ribbons", to appear in 2022.

Functional Description: MERCI is a C++/lua software for computing the statics of thin elastic ribbons discretised with curvature-based elements. It is based on the super-ribbon model described in [Charrondière et al. 2020, Charrondière et al. 2022], and relies on the free [IPOPT](https://coin-or.github.io/Ipopt/) optimisation software (coinor project) for the static solver. The ribbon can be clamped at one or both ends, and even closed. Contact is treated by contraints with planes. Once the setup is defined, the equilibrium of the ribbon under the specified boundary conditions, external forces, and constraints, is computed. MERCI can be used as a C++ library, or via its lua interface.

Reference code of the PhD thesis:

Raphaël Charrondière, "Modélisation numérique de rubans par éléments en courbures", 2021, Université Grenoble Alpes, https://hal.inria.fr/tel-03545017v2

and of the following papers:

R. Charrondière, F. Bertails-Descoubes, S. Neukirch, V. Romero, "Numerical modeling of inextensible elastic ribbons with curvature-based elements", Computer Methods in Applied Mechanics and Engineering 364, June 2020, p. 1–32, [doi:10.1016/j.cma.2020.112922], [hal-02515877].

R. Charrondière, S. Neukirch, F. Bertails-Descoubes, "MERCI: Mixed curvature-based elements for computing equilibria of thin elastic ribbons", to appear in 2022.

Publication: hal-02515877

Authors: Raphael Charrondiere, Florence Descoubes, Sébastien Neukirch

Contact: Florence Bertails Descoubes

Partners: Sorbonne Université, UGA

7.4.5 MECHE

Name: Modeling the Entanglement between fibers

Keywords: Frictional contact, Thin elastic rod

Functional Description: Software plateform for computing the dynamics of fiber assemblies sub- ject to frictional contact (derived from DynamicHair3D). Includes in particular the reference implementation of the super-helix model [BAC+06] and nonsmooth frictional contact algorithms [DBDB11].

Release Contributions: - Unclamped thin elastic rods - New collision detection - Mixed Hertz and constraint-based software

Authors: Florence Descoubes, Emile Hohnadel

Contact: Florence Bertails Descoubes

7.4.6 Cloth

Keyword: Thin elastic shell

Functional Description: Direct and inverse simulation of thin elastic shells subject to frictional contact. Reference implementation of the ACM Siggraph 2018 paper: M. Ly, R. Casati, F. Bertails-Descoubes, M. Skouras, L. Boissieux, Inverse Elastic Shell Design with Contact and Friction, ACM Transactions on Graphics 37, 6, November 2018, p. 1–16, [doi:10.1145/3272127.3275036], [hal:hal-01883655].

Contact: Florence Bertails Descoubes

7.4.7 ElanFab

Keywords: Experimental mechanics, Experimental design, Thin elastic ribbon, Thin elastic rod, Thin elastic shell, Frictional contact

Functional Description: Experimental platform of the ELAN team. Fabrication in silicone of thin elastic rods with controlled radius, stiffness, and natural curliness. 3D reconstruction of their sus- pended shape, using 2 cameras and a mirror view. Accompanying software for image processing.

Author: Victor Romero Gramegna

Contact: Victor Romero Gramegna

7.5 New platforms

7.5.1 ElanFab: experimental platform

Participants: Victor Romero.

Keywords: Experimental mechanics, Experimental design, Thin elastic ribbon, Thin elastic rod, Thin elastic shell, Frictional contact

Functional Description: Experimental platform of the ELAN team. Fabrication of silicone-made thin elastic rods with controlled radius, stiffness, and natural curliness. 3D reconstruction of their suspended shape using two cameras and a mirror view. Mechanical characterisation of complex materials. Experimental exploration of elastic instabilities within slender objects. Accompanying software for image processing (7.4.7).

Author: Victor Romero Gramegna

Contact: Victor Romero Gramegna

8 New results

8.1 Estimation of friction coefficients in cloth

Participants: Haroon Rasheed, Victor Romero, Florence Bertails-Descoubes.

Our objective was to estimate friction coefficients in fabric, for which no reliable experimental process exists yet. Our idea was to leverage our accurate simulator for cloth frictional contact [39], possibly complemented by deep learning techniques, in order to considerably alleviate this task and build non-invasive measurement protocols. The PhD thesis of Abdullah-Haroon Rasheed (2017 - 2021), co-advised by F. Bertails-Descoubes and Stefanie Wuhrer and Jean-Sébastien Franco from the MORPHEO team (Computer Vision), made several important advances on this topic. Thanks to a pluridisciplinary collaboration encompassing Physical Modelling, Computer Vision, Machine Learning, and Experimental Physics (in collaboration with Victor Romero and Arnaud Lazarus, Sorbonne Université), we have built a new non-invasive protocol for estimating material properties of cloth and friction during dynamic interaction, including cloth-solid and cloth-cloth interaction. The method relies on a neural network trained only on simulated data (yielded by our cloth simulator **Argus**), after a careful validation of the simulator. From this trained network, we were able to predict on a real experiment both the material class of the cloth sample as well as the friction coefficient between the cloth sample and the substrate (either smooth or cloth-like), with a good level of prediction. This work was published in a major conference venue in Computer Vision [42] (selected for an oral presentation) and the cloth-to-cloth extension was published in the journal IEEE PAMI [9]. The latter paper, where we have evidenced the influence of the predictibility of the simulator on the accuracy of the network, also marked a turning point in our research interests, as now we consider the *physical validation* of numerical models as a major research axis in the ELAN team.

8.2 Willmore flow simulation with diffusion-redistanciation numerical schemes

Participant: Thibaut Metivet.

In collaboration with Arnaud Sengers (Université Claude Bernard, Emmanuel Maitre (Laboratoire Jean Kuntzmann, Grenoble INP) and Mourad Ismail (Laboratoire Interdisciplinaire de Physique, UGA), we have proposed original diffusion-redistanciation numerical schemes to compute the static shapes of elastic membranes with bending stiffness under constant area and volume constraints. This numerical method relies on an implicit representation of the surface which is used as an initial condition for diffusion-like equations. This allows to circumvent the usual difficulties pertaining to the high geometrical order and non-linearities of the bending energy and to benefit from the robustness of discretised diffusion operators. The resulting numerical schemes provide very a good stability behaviour thanks to their inherent diffusive nature and demonstrate a convergence order close to the optimal one, which is a nice achievement in regards of the low-order geometrical discretisation used. We have implemented the schemes within the finite-element library Feel++ and provided efficient and parallel solvers for the resolution of the diffusion equation and the redistanciation of the implicit surface representation. We have validated our method using comparative benchmarks computed with standard approaches. This work has led to a recent publication in Computational Physics [8] and has been presented at the Numerical Analysis Seminar from Laboratoire J.A. Dieudonné and Inria Cote d'Azur.

8.3 Validation of simulators for slender elastic structures and frictional contact

Participants: Victor Romero, Mickaël Ly, Haroon Rasheed, Raphaël Charrondière, Florence Bertails-Descoubes.

In collaboration with Arnaud Lazarus and Sébastien Neukirch (Sorbonne Université, Institut Jean le Rond d'Alembert), we have set up a new framework for validating simulators of slender elastic structures (rods and plates) and frictional contact. To this aim we leverage and enrich a set of protocols from the Soft Matter Physics community, initially devised for measuring elasticity and frictional properties of slender elastic structures. These retained tests, that we experimentally validate, are characterized by scaling laws which only depend on a few dimensionless parameters, making them ideal for benchmarking robustly a large diversity of codes across different physical regimes, without having to worry about scales or dimensions. We have passed a number of popular codes of Computer Graphics through our benchmarks by defining a rigorous, consistent, and as fair as possible methodology. Our results show that while some popular simulators for plates/shells and frictional contact fail even on the simplest scenarios, more recent ones, as well as well-known codes for rods, generally perform well and sometimes even better than some reference commercial tools of Mechanical Engineering. This long-term study led to an original publication at ACM Siggraph 2021 [10] and multiple invited talks in both Computer Science events (Colloque Sciences & Games 2021) and Physics events (Rencontres du Non-Linéaire 2021), see Section 10.1.2 .

8.4 Validation of granular flow simulations against experimental column collapses

Participants: Thibaut Métivet, Florence Bertails-Descoubes.

In collaboration with Gauthier Rousseau, formerly post-doc in the team (and PhD student at EPFL), and with Hugo Rousseau (INRAE) and Gilles Daviet (formerly PhD student in the team), we have performed some thorough comparisons between the predictions of our numerical solver **Sand6** for granular flows 7.4.3, and collapse experiments conducted in a narrow channel (in collaboration with EPFL). We have shown that our nonsmooth simulator, which relies on a constant friction coefficient corresponding to the yield angle of a granular heap, is able to reproduce with high fidelity various experimental granular collapses over inclined erodible beds. Our results, obtained for two different granular materials and for various bed inclinations, suggest that a simple constant friction rheology choice remains reasonable for capturing a large variety of unsteady granular flows at low inertial number. We will submit this original study for publication in Mechanics in 2022.

8.5 Lateral Indentation of a Thin Elastic Film.

Participant: Victor Romero.

In collaboration with Enrique Cerda, Eugenio Hamm, from Universidad de Santiago de Chile, and Miguel Trejo from Universidad de Buenos Aires, we published a paper [11] where we present an experimental setup for testing thin-film materials by studying the lateral indentation of a narrow opening cut into a film, triggering a cascade of buckling events. We showed that the force response F is dominated by bending and stretching effects for small displacements and slowly varies with indenter displacement $F \sim d^{2/5}$, to finally reach a wrinkled state that results in a robust nonlinear asymptotic relation, $F \sim d^4$. We present experiments with films of various thicknesses and material properties, and numerical simulations to confirm our analysis defining an order parameter that accounts for the different response regimes observed in experiments and simulations.

9 Partnerships and cooperations

Participants: Thibaut Métivet, Victor Romero, Florence Bertails-Descoubes.

9.1 International Collaborations

- Long-term collaboration with Rahul Narain (IIT Delhi, India).
- Long-term collaboration with Enrique Cerda (Universidad de Santiago, Chile).
- Long-term collaboration with Eugenio Hamm (Universidad de Santiago, Chile).
- Long-term collaboration with Miguel Trejo (Universidad de Buenos Aires, Argentina).
- Starting collaboration with Theodore Kim (Yale, USA).
- Starting collaboration with Tomohiko Sano (Dept of Mechanical Engineering, Keio University, Sigma Lab (Japan)).

9.2 European initiatives

9.2.1 FP7 & H2020 projects

GEM

Title: from GEometry to Motion, inverse modeling of complex mechanical structures
Program: H2020
Type: ERC Starting Grant
Duration: September 2015 - February 2022
Coordinator: Inria
Principal Investigator: Florence Bertails-Descoubes

With the considerable advance of automatic image-based capture in Computer Vision and Computer Graphics these latest years, it becomes now affordable to acquire quickly and precisely the full 3D geometry of many mechanical objects featuring intricate shapes. Yet, while more and more geometrical data get collected and shared among the communities, there is currently very little study about how to infer the underlying mechanical properties of the captured objects merely from their geometrical configurations. The GEM challenge consists in developing a non-invasive method for inferring the mechanical properties of complex objects from a minimal set of geometrical poses, in order to predict their dynamics. In contrast to classical inverse reconstruction methods, my proposal is built upon the claim that 1/ the mere geometrical shape of physical objects reveals a lot about their underlying mechanical properties and 2/ this property can be fully leveraged for a wide range of objects featuring rich geometrical configurations, such as slender structures subject to frictional contact (e.g., folded cloth or twined filaments). To achieve this goal, we shall develop an original inverse modeling strategy based upon a/ the design of reduced and high-order discrete models for slender mechanical structures including rods, plates and shells, b/ a compact and well-posed mathematical formulation of our nonsmooth inverse problems, both in the static and dynamic cases, c/ the design of robust and efficient numerical tools for solving such complex problems, and d/ a thorough experimental validation of our methods relying on the most recent capturing tools. In addition to significant advances in fast image-based measurement of diverse mechanical materials stemming from physics, biology, or manufacturing, this research is expected in the long run to ease considerably the design of physically realistic virtual worlds, as well as to boost the creation of dynamic human doubles.

9.3 National Collaborations

- Long-term collaboration with Arnaud Lazarus and Sébastien Neukirch (Institut Jean le Rond d'Alembert, Sorbonne Université), within the GEM project.
- Long-term collaboration with Christophe Prud'homme and Vincent Chabannes (Université de Strasbourg and Centre de modélisation et de simulation de Strasbourg).
- Collaboration with Olivier Pouliquen and Joël Marthelot (IUSTI, Aix-Marseille University).

9.4 Regional collaborations

- Collaboration with Philippe Peyla, Aurélie Dupont (LIPhy, UGA/CNRS) and Christian Graff (LPNC, UGA/CNRS) within the FISHSIF project.
- Collaboration with Mélina Skouras (Inria/LJK - ANIMA team) within the GEM project.
- Collaboration with Stefanie Wuhrer and Jean-Sébastien Franco (Inria/LJK - MORPHÉO team) within the GEM project.

10 Dissemination

10.1 Promoting scientific activities

Member of the conference program committees

- Florence Bertails-Descoubes was member of the **ACM Siggraph** Technical Paper Program Committee 2021.

10.1.1 Journal

Member of the editorial boards

- Since 2021, Florence Bertails-Descoubes is Associate Editor of ACM Transactions on Graphics.

Reviewer - reviewing activities

- Transactions on Graphics (TOG, ACM)
- Journal of the Mechanics and Physics of Solids (JMPS, Elsevier)
- Transactions on visualization and computer graphics (TVCG,IEEE)
- Siggraph (ACM)

10.1.2 Invited talks

- June 2021: Invited seminar at Laboratoire J.A. Dieudonné, Université Côte d'Azur (Thibaut Metivet).
- April 2021: Invited talk at the hybrid Physics / Video Game SxG Colloquium, Paris (Florence Bertails-Descoubes).
- March 2021: Invited keynote at Mini-colloquium on non-linear phenomena, Paris (Florence Bertails-Descoubes).
- January 2021: Semi-plenary talk at WCCM-ECCOMAS 2020 (Florence Bertails-Descoubes).

10.1.3 Research administration

Hiring Committees

- Florence Bertails-Descoubes was member of the jury for the Maître de Conférences Concours 27/MCF/1878 at Université de Strasbourg in 2021.

Ph.D. Award Committees

- Florence Bertails-Descoubes was member of the international ACM Siggraph Ph.D. award in 2021.
- Florence Bertails-Descoubes was member of the national GdR IG-RV Ph.D. award in 2021.

10.2 Teaching - Supervision - Juries

10.2.1 Teaching

Licence

- Raphaël Charrondière: CM Validation d'algorithmes, 27h, L3 Miage, Université Grenoble Alpes.
- Raphaël Charrondière: TD Validation d'algorithmes, 18h, L3 Miage, Université Grenoble Alpes.
- Thibaut Metivet: Analyse, 33h, ENSIMAG 1A, Grenoble INP.
- Thibaut Metivet: Méthodes Numériques de Base, 16.5h, ENSIMAG 1A, Grenoble INP.
- Victor Romero: Structures en pratique, 27h, L1, Sorbonne Université.

Master

- Raphaël Charrondière: Complexité Algorithmique Des Problèmes, 15h, M1, Université Grenoble Alpes.
- Florence Bertails-Descoubes and Mickaël Ly: “Numerical Mechanics: From Lagrangian mechanics to simulation tools for computer graphics“, 19h+14h, M2 Special Course, École Normale Supérieure de Lyon.
- Victor Romero: Supervision de projets de fin Master 1, 24h, M1, Sorbonne Université.

Doctorate

- Florence Bertails-Descoubes and Thibaut Metivet: Non-smooth methods for frictional contact, 9h+9h, THREAD PhD European Training Program, Centrale-Supelec Paris.

10.2.2 Supervision

Ph.D.

- Mickaël Ly: 01/10/2017-30/09/2021, **defended** the 28/09/2021, title: *Static inverse modelling of cloth*, [13], supervised by Florence Bertails-Descoubes and Mélina Skouras (ANIMA, Inria GRA)
- Raphaël Charrondière: 01/09/2018-31/10/2021, **defended** the 14/10/2021, title: *Modélisation numérique de rubans par éléments en courbures*, [12], supervised by Florence Bertails-Descoubes and Sébastien Neukirch (Sorbonne Université)
- Abdullah-Haroon Rasheed: 01/11/2017-31/10/2021, **defended** the 09/12/2021, title: *Inverse Dynamic Modeling of Cloth - Deep Learning using Physics based Simulations*, [14], supervised by Florence Bertails-Descoubes and Stefanie Wuhrer (MORPHÉO, Inria GRA) and Jean-Sébastien Franco (MORPHÉO, Inria GRA)
- Alexandre Teixeira: 01/02/2021-, supervised by Florence Bertails-Descoubesand Thibaut Metivet and Mélina Skouras (ANIMA, Inria GRA)
- Emile Hohnadel: 01/09/2021-, supervised by Florence Bertails-Descoubes and Thibaut Metivet
- Jean Jouve: 01/09/2021-, supervised by Florence Bertails-Descoubes and Victor Romero
- Nicolas Parent: 01/10/2020-30/09/2021, supervised by Florence Bertails-Descoubes and Thibaut Metivet and Mélina Skouras (ANIMA, Inria GRA)

Internship

- Emile Hohnadel: 15/09/2020-19/03/2021, supervised by Florence Bertails-Descoubes and Thibaut Metivet
- Jean Jouve: 08/02/2021-02/07/2021, supervised by Florence Bertails-Descoubes

10.2.3 Juries

Habilitation à Diriger des Recherches (HdR) Committees

- Florence Bertails-Descoubes was member (Examinatrice, President) of the HdR committee of Damien Durville (Title: *Contribution à la modélisation et à la simulation du comportement mécanique de milieux enchevêtrés*), Université Paris Saclay.

Ph.D. Committees

- Florence Bertails-Descoubes was member (Examinatrice) of the Ph.D. Thesis committee of Roméo Antier (15 December 2021), Université de Paris, Spécialité Physique (Title: *Des plis et des ailes: De l'étude d'un pli simple au renforcement d'ailes battantes bio-inspirées*, Ph.D. advisors: R. Godoy-Diana and B. Thiria).
- Victor Romero was invited member of the Ph.D. Thesis committee of Haroon Rasheed (09 December 2021), Université Grenoble Alpes, Spécialité Mathématiques et Informatique (Title: *Modélisation dynamique inverse de tissus - Apprentissage profond à l'aide de simulations basées sur la physique*, Ph.D. advisors: F. Bertails-Descoubes, S. Wuhrer, and J. S. Franco).

11 Scientific production

11.1 Major publications

[1] F. Bertails-Descoubes, A. Derouet-Jourdan, V. Romero and A. Lazarus. 'Inverse design of an isotropic suspended Kirchhoff rod: theoretical and numerical results on the uniqueness of the natural shape'. In: *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences* 474.2212 (Apr. 2018), pp. 1–26. DOI: 10.1098/rspa.2017.0837. URL: https://hal.inria.fr/hal-01827887.

[2] R. Charrondière, F. Bertails-Descoubes, S. Neukirch and V. Romero. 'Numerical modeling of inextensible elastic ribbons with curvature-based elements'. In: *Computer Methods in Applied Mechanics and Engineering* 364 (1st June 2020), pp. 1–32. DOI: 10.1016/j.cma.2020.112922. URL: https://hal.inria.fr/hal-02515877.

[3] J. Li, G. Daviet, R. Narain, F. Bertails-Descoubes, M. Overby, G. Brown and L. Boissieux. 'An Implicit Frictional Contact Solver for Adaptive Cloth Simulation'. In: *ACM Transactions on Graphics*. Proceedings SIGGRAPH 2018 37.4 (Aug. 2018), pp. 1–15. DOI: 10.1145/3197517.3201308. URL: https://hal.inria.fr/hal-01834705.

[4] M. Ly, R. Casati, F. Bertails-Descoubes, M. Skouras and L. Boissieux. 'Inverse Elastic Shell Design with Contact and Friction'. In: *ACM Transactions on Graphics* 37.6 (Nov. 2018), pp. 1–16. DOI: 10.1145/3272127.3275036. URL: https://hal.inria.fr/hal-01883655.

[5] T. Metivet, A. Sengers, M. Ismail and E. Maitre. 'Diffusion-redistanciation schemes for 2D and 3D constrained Willmore flow: application to the equilibrium shapes of vesicles'. In: *Journal of Computational Physics* 436 (2021), p. 110288. DOI: 10.1016/j.jcp.2021.110288. URL: https://hal.archives-ouvertes.fr/hal-02905870.

[6] A.-H. Rasheed, V. Romero, F. Bertails-Descoubes, S. Wuhrer, J.-S. Franco and A. Lazarus. 'A Visual Approach to Measure Cloth-Body and Cloth-Cloth Friction'. In: *IEEE Transactions on Pattern Analysis and Machine Intelligence* (July 2021). DOI: 10.1109/TPAMI.2021.3097547. URL: https://hal.inria.fr/hal-03285624.

[7] V. Romero, M. Ly, A.-H. Rasheed, R. Charrondière, A. Lazarus, S. Neukirch and F. Bertails-Descoubes. 'Physical validation of simulators in Computer Graphics: A new framework dedicated to slender elastic structures and frictional contact'. In: *ACM Transactions on Graphics* 40.4 (1st Aug. 2021), Article 66: 1–19. DOI: 10.1145/3450626.3459931. URL: https://hal.inria.fr/hal-03217459.

11.2 Publications of the year

International journals

[8] T. Metivet, A. Sengers, M. Ismail and E. Maitre. 'Diffusion-redistanciation schemes for 2D and 3D constrained Willmore flow: application to the equilibrium shapes of vesicles'. In: *Journal of Computational Physics* 436 (2021), p. 110288. DOI: 10.1016/j.jcp.2021.110288. URL: https://hal.archives-ouvertes.fr/hal-02905870.

[9] A.-H. Rasheed, V. Romero, F. Bertails-Descoubes, S. Wuhrer, J.-S. Franco and A. Lazarus. 'A Visual Approach to Measure Cloth-Body and Cloth-Cloth Friction'. In: *IEEE Transactions on Pattern Analysis and Machine Intelligence* (July 2021). DOI: 10.1109/TPAMI.2021.3097547. URL: https://hal.inria.fr/hal-03285624.

[10] V. Romero, M. Ly, A.-H. Rasheed, R. Charrondière, A. Lazarus, S. Neukirch and F. Bertails-Descoubes. 'Physical validation of simulators in Computer Graphics: A new framework dedicated to slender elastic structures and frictional contact'. In: *ACM Transactions on Graphics* 40.4 (1st Aug. 2021), Article 66: 1–19. DOI: 10.1145/3450626.3459931. URL: https://hal.inria.fr/hal-03217459.

[11] M. Trejo, V. Romero, E. Hamm and E. CERDA. 'Lateral indentation of a thin elastic film'. In: *Soft Matter* 18.4 (2022), pp. 762–767. DOI: 10.1039/d1sm01348c. URL: https://hal.inria.fr/hal-03512390.

Doctoral dissertations and habilitation theses

[12] R. Charrondiere. 'Numerical modeling of ribbons by curved elements'. Université Grenoble Alpes, 14th Oct. 2021. URL: https://hal.inria.fr/tel-03545017.

[13] M. Ly. 'Static inverse modelling of cloth'. Université Grenoble Alpes [2020-....], 28th Sept. 2021. URL: https://tel.archives-ouvertes.fr/tel-03516830.

[14] A.-H. Rasheed. 'Inverse Dynamic Modeling of Cloth - Deep Learning using Physics based Simulations'. Inria; Universite Grenoble Alpes, 9th Dec. 2021. URL: https://hal.inria.fr/tel-03501532.

11.3 Cited publications

[15] B. Audoly and S. Neukirch. 'Fragmentation of Rods by Cascading Cracks: Why Spaghetti Does Not Break in Half'. In: *Physical Review Letters* 95.9 (2005), p. 095505.

[16] D. Baraff. 'Analytical methods for dynamic simulation of non-penetrating rigid bodies'. In: *Computer Graphics Proceedings (Proc. ACM SIGGRAPH'89)*. New York, NY, USA: ACM, 1989, pp. 223–232.

[17] F. Bertails. 'Linear Time Super-Helices'. In: *Computer Graphics Forum (Proc. Eurographics'09)* 28.2 (Apr. 2009). URL: http://www-ljk.imag.fr/Publications/Basilic/com.lmc.publi.PUBLI_Article@1203901df78_1d3cdaa/.

[18] F. Bertails, B. Audoly, M.-P. Cani, B. Querleux, F. Leroy and J.-L. Lévêque. 'Modélisation de coiffures naturelles à partir des propriétés physiques du cheveu'. In: *Journées Francophones d'Informatique Graphique (AFIG)*. AFIG / EG-France. Strasbourg, France, Nov. 2005.

[19] F. Bertails-Descoubes. 'Super-Clothoids'. In: *Computer Graphics Forum (Proc. Eurographics'12)* 31.2pt2 (2012), pp. 509–518. DOI: 10.1111/j.1467-8659.2012.03030.x. URL: http://www.inrialpes.fr/bipop/people/bertails/Papiers/superClothoids.html.

[20] A. Blumentals. 'Numerical modelling of thin elastic solids in contact'. PhD thesis. Université de Grenoble Alpes, July 2017.

[21] A. Blumentals, F. Bertails-Descoubes and R. Casati. 'Dynamics of a developable shell with uniform curvatures'. In: *The 4th Joint International Conference on Multibody System Dynamics*. Montréal, Canada, May 2016. URL: https://hal.inria.fr/hal-01311559.

[22] R. Casati and F. Bertails-Descoubes. 'Super Space Clothoids'. In: *ACM Transactions on Graphics (Proc. ACM SIGGRAPH'13)* 32.4 (July 2013), 48:1–48:12. DOI: 10.1145/2461912.2461962. URL: http://doi.acm.org/10.1145/2461912.2461962.

[23] D. Chapelle and K. Bathe. *The Finite Element Analysis of Shells - Fundamentals - Second Edition*. Computational Fluid and Solid Mechanics. Springer, 2011, p. 410. DOI: 10.1007/978-3-642-16408-8.

[24] P. Cundall. 'A computer model for simulating progressive large scale movements of blocky rock systems. In Proceedings of the Symposium of the International Society of Rock Mechanics'. In: *Proceedings of the Symposium of the International Society of Rock Mechanics*. Vol. 1. 1971, pp. 132–150.

[25] G. Daviet. 'Modèles et algorithmes pour la simulation du contact frottant dans les matériaux complexes : application aux milieux fibreux et granulaires'. PhD thesis. Grenoble Alpes Universités, Dec. 2016.

[26] G. Daviet, F. Bertails-Descoubes and L. Boissieux. 'A hybrid iterative solver for robustly capturing Coulomb friction in hair dynamics'. In: *ACM Transactions on Graphics (Proc. ACM SIGGRAPH Asia'11)* 30 (6 2011), 139:1–139:12. URL: http://www.inrialpes.fr/bipop/people/bertails/Papiers/hybridIterativeSolverHairDynamicsSiggraphAsia2011.html.

[27] G. Daviet and F. Bertails-Descoubes. 'A semi-implicit material point method for the continuum simulation of granular materials'. In: *ACM Transactions on Graphics*. SIGGRAPH '16 Technical Papers 35.4 (July 2016), p. 13. DOI: 10.1145/2897824.2925877. URL: https://hal.inria.fr/hal-01310189.

[28] G. Daviet and F. Bertails-Descoubes. 'Nonsmooth simulation of dense granular flows with pressure-dependent yield stress'. In: *Journal of Non-Newtonian Fluid Mechanics* 234 (Apr. 2016), pp. 15–35. DOI: 10.1016/j.jnnfm.2016.04.006. URL: https://hal.inria.fr/hal-01236488.

[29] G. Daviet and F. Bertails-Descoubes. 'Simulation of Drucker–Prager granular flows inside Newtonian fluids'. working paper or preprint. Feb. 2017. URL: https://hal.inria.fr/hal-01458951.

[30] A. Derouet-Jourdan, F. Bertails-Descoubes, G. Daviet and J. Thollot. 'Inverse Dynamic Hair Modeling with Frictional Contact'. In: *ACM Trans. Graph.* 32.6 (Nov. 2013), 159:1–159:10. DOI: 10.1145/2508363.2508398. URL: http://doi.acm.org/10.1145/2508363.2508398.

[31] A. Derouet-Jourdan, F. Bertails-Descoubes and J. Thollot. 'Stable Inverse Dynamic Curves'. In: *ACM Transactions on Graphics (Proc. ACM SIGGRAPH Asia'10)* 29 (6 Dec. 2010), 137:1–137:10. DOI: http://doi.acm.org/10.1145/1882261.1866159. URL: http://doi.acm.org/10.1145/1882261.1866159.

[32] M. A. Dias and B. Audoly. '"Wunderlich, Meet Kirchhoff": A General and Unified Description of Elastic Ribbons and Thin Rods'. In: *Journal of Elasticity* 119.1 (Apr. 2015), pp. 49–66. DOI: 10.1007/s10659-014-9487-0. URL: https://doi.org/10.1007/s10659-014-9487-0.

[33] E. J. Doedel, R. C. Pfaffenroth, A. R. Chambodut, T. F. Fairgrieve, Y. A. Kuznetsov, B. E. Oldeman, B. Sandstede and X. Wang. *AUTO 2000: Continuation and Bifurcation Software for Ordinary Differential Equations (with HomCont)*. Mar. 2006.

[34] ESPCI. *Rencontre en l'honneur de Yves Pomeau, octobre 2016*. https://www.sfpnet.fr/rencontre-celebrant-la-medaille-boltzmann-d-yves-pomeau. ESPCI, 2016. URL: https://www.sfpnet.fr/rencontre-celebrant-la-medaille-boltzmann-d-yves-pomeau.

[35] I. Frigaard and C. Nouar. 'On the usage of viscosity regularisation methods for visco-plastic fluid flow computation'. In: *Journal of Non-Newtonian Fluid Mechanics* 127.1 (Apr. 2005), pp. 1–26. DOI: 10.1016/j.jnnfm.2005.01.003.

[36] A. L. Gol'Denveizer. *Theory of Elastic Thin Shells*. Pergamon Press, 1961.

[37] R. E. Goldstein, P. B. Warren and R. C. Ball. 'Shape of a Ponytail and the Statistical Physics of Hair Fiber Bundles'. In: *Phys. Rev. Lett.* 108 (7 Feb. 2012), p. 078101. DOI: 10.1103/PhysRevLett.108.078101. URL: https://link.aps.org/doi/10.1103/PhysRevLett.108.078101.

[38] M. Jean. 'The Non Smooth Contact Dynamics Method'. In: *Computer Methods in Applied Mechanics and Engineering* 177 (1999). Ed. by J. Martins and A. Klarbring. Special issue on computational modeling of contact and friction, J.A.C. Martins and A. Klarbring, editors, pp. 235–257.

[39] J. Li, G. Daviet, R. Narain, F. Bertails-Descoubes, M. Overby, G. Brown and L. Boissieux. 'An Implicit Frictional Contact Solver for Adaptive Cloth Simulation'. In: *ACM Transactions on Graphics*. Proceedings SIGGRAPH 2018 37.4 (Aug. 2018), pp. 1–15. DOI: 10.1145/3197517.3201308. URL: https://hal.inria.fr/hal-01834705.

[40] M. Moore and J. Wilhelms. 'Collision detection and response for computer animation'. In: *Computer Graphics Proceedings (Proc. ACM SIGGRAPH'88)*. 1988, pp. 289–298.

[41] D. Moulton, P. Grandgeorge and S. Neukirch. 'Stable elastic knots with no self-contact'. In: *Journal of the Mechanics and Physics of Solids* 116 (2018), pp. 33–53. DOI: 10.1016/j.jmps.2018.03.019. URL: http://www.sciencedirect.com/science/article/pii/S0022509617310104.

[42] A.-H. Rasheed, V. Romero, F. Bertails-Descoubes, S. Wuhrer, J.-S. Franco and A. Lazarus. 'Learning to Measure the Static Friction Coefficient in Cloth Contact'. In: *CVPR 2020 - IEEE Conference on Computer Vision and Pattern Recognition*. Seattle, United States: IEEE, June 2020, pp. 9909–9918. DOI: 10.1109/CVPR42600.2020.00993. URL: https://hal.inria.fr/hal-02511646.

[43] J. Spillmann and M. Teschner. 'An Adaptive Contact Model for the Robust Simulation of Knots'. In: *Computer Graphics Forum* 27.2 (2008). Proc. Eurographics'08.

[44] H. Sugiyama, J. Gertsmayr and A. Mikkola. 'Flexible Multibody Dynamics — Essential for Accurate Modeling in Multibody System Dynamics'. In: *Journal of Computational Nonlinear Dynamics* 9 (1 Nov. 2013).