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2021
ACTIVITY REPORT

Project-Team MATHRISK

Mathematical Risk handling

IN COLLABORATION WITH: Centre d'Enseignement et de Recherche en Mathématiques et Calcul Scientifique (CERMICS)

DOMAIN

Applied Mathematics, Computation and Simulation

THEME

Stochastic approaches

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Project-Team MATHRISK

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A6.1. – Methods in mathematical modeling

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A6.2.1. – Numerical analysis of PDE and ODE

A6.2.2. – Numerical probability

A6.2.3. – Probabilistic methods

A6.4.2. – Stochastic control

Other research topics and application domains

B3.1. – Sustainable development

B9.6.3. – Economy, Finance

B9.11. – Risk management

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2 Overall objectives

The Inria project team **MathRisk** team was created in 2013. It is the follow-up of the MathFi project team founded in 2000. MathFi was focused on financial mathematics, in particular on computational methods for pricing and hedging increasingly complex financial products. The 2007 global financial crisis and its "aftermath crisis" has abruptly highlighted the critical importance of a better understanding and management of risk. The project **MathRisk** has been reoriented towards mathematical handling of risk, and addresses broad research topics embracing risk measurement and risk management, modeling and optimization in quantitative finance, but also in other related domains where risk control is paramount. The project team **MathRisk** aims both at producing mathematical tools and models in these domains, and developing collaborations with various institutions involved in risk control. Quantitative finance remains for the project an important source of mathematical problems and applications. Indeed, the pressure of new legislation leads to a massive reorientation of research priorities, and the interest of analysts shifted to risk control preoccupation.

The scientific issues related to quantitative finance we consider include systemic risk and contagion modeling, robust finance, market frictions, counterparty and liquidity risk, assets dependence modeling, market micro-structure modeling and price impact. In this context, models must take into account the multidimensional feature and various market imperfections. They are much more demanding mathematically and numerically, and require the development of risk measures taking into account incompleteness issues, model uncertainties, interplay between information and performance and various defaults.

Besides, financial institutions, submitted to more stringent regulatory legislations such as FRTB or XVA computation, are facing practical implementation challenges which still need to be solved. Research focused on numerical efficiency remains strongly needed in this context, renewing the interest for the numerical platform Premia () that Mathrisk is developing in collaboration with a consortium of financial institutions.

While these themes arise naturally in the world of quantitative finance, a number of these issues and mathematical tools are also relevant to the treatment of risk in other areas as economy, social insurance and sustainable development, of fundamental importance in today's society. In these contexts, the management of risk appears at different time scales, from high frequency data to long term life insurance management, raising challenging renewed modeling and numerical issues.

The **MathRisk** project is strongly involved in the development of new mathematical methods and numerical algorithms. Mathematical tools include stochastic modeling, stochastic analysis, in particular stochastic (partial) differential equations and various aspects of stochastic control and optimal stopping of these equations, nonlinear expectations, Malliavin calculus, stochastic optimization, dynamic game theory, random graphs, martingale optimal transport (especially in relation to numerical considerations), long time behavior of Markov processes (with applications to Monte-Carlo methods) and generally advanced numerical methods for effective solutions.

3 Research program

3.1 Risk management: modeling and optimization

3.1.1 Contagion modeling and systemic risk

After the recent financial crisis, systemic risk has emerged as one of the major research topics in mathematical finance. Interconnected systems are subject to contagion in time of distress. The scope is to understand and model how the bankruptcy of a bank (or a large company) may or not induce other bankruptcies. By contrast with the traditional approach in risk management, the focus is no longer on modeling the risks faced by a single financial institution, but on modeling the complex interrelations between financial institutions and the mechanisms of distress propagation among these.

The mathematical modeling of default contagion, by which an economic shock causing initial losses and default of a few institutions is amplified due to complex linkages, leading to large scale defaults, can be addressed by various techniques, such as network approaches (see in particular R. Cont et al. [57] and A. Minca [100]) or mean field interaction models (Garnier-Papanicolaou-Yang [87]).

We have contributed in the last years to the research on the control of contagion in financial systems in the framework of random graph models : In [58, 101], [5], A. Sulem with A. Minca and H. Amini consider a financial network described as a weighted directed graph, in which nodes represent financial institutions and edges the exposures between them. The distress propagation is modeled as an epidemics on this graph. They study the optimal intervention of a lender of last resort who seeks to make equity infusions in a banking system prone to insolvency and to bank runs, under complete and incomplete information of the failure cluster, in order to minimize the contagion effects. The paper [5] provides in particular important insight on the relation between the value of a financial system, connectivity and optimal intervention.

The results show that up to a certain connectivity, the value of the financial system increases with connectivity. However, this is no longer the case if connectivity becomes too large. The natural question remains how to create incentives for the banks to attain an optimal level of connectivity. This is studied in [72], where network formation for a large set of financial institutions represented as nodes is investigated. Linkages are source of income, and at the same time they bear the risk of contagion, which is endogeneous and depends on the strategies of all nodes in the system. The optimal connectivity of the nodes results from a game. Existence of an equilibrium in the system and stability properties is studied. The results suggest that financial stability is best described in terms of the mechanism of network formation than in terms of simple statistics of the network topology like the average connectivity.

3.1.2 Liquidity risk and Market Microstructure

Liquidity risk is the risk arising from the difficulty of selling (or buying) an asset. Usually, assets are quoted on a market with a Limit Order Book (LOB) that registers all the waiting limit buy and sell orders for this asset. The bid (resp. ask) price is the most expensive (resp. cheapest) waiting buy or sell order. If a trader wants to sell a single asset, he will sell it at the bid price, but if he wants to sell a large quantity of assets, he will have to sell them at a lower price in order to match further waiting buy orders. This creates an extra cost, and raises important issues. From a short-term perspective (from few minutes to some days), it may be interesting to split the selling order and to focus on finding optimal selling strategies. This requires to model the market microstructure, i.e. how the market reacts in a short time-scale to execution orders. From a long-term perspective (typically, one month or more), one has to understand how this cost modifies portfolio managing strategies (especially delta-hedging or optimal investment strategies). At this time-scale, there is no need to model precisely the market microstructure, but one has to specify how the liquidity costs aggregate.

For rather liquid assets, liquidity risk is usually taken into account via price impact models which describe how a (large) trader influences the asset prices. Then, one is typically interested in the optimal execution problem: how to buy/sell a given amount of assets optimally within a given deadline. This issue is directly related to the existence of statistical arbitrage or Price Manipulation Strategies (PMS). Most of price impact models deal with single assets. A. Alfonsi, F. Klöck and A. Schied [56] have proposed a multi-assets price impact model that extends previous works. Price impact models are usually relevant when trading at an intermediary frequency (say every hour). At a lower frequency, price impact is usually ignored while at a high frequency (every minute or second), one has to take into account the other traders and the price jumps, tick by tick. Midpoint price models are thus usually preferred at this time scale. With P. Blanc, Alfonsi [3] has proposed a model that makes a bridge between these two types of model: they have considered an (Obizhaeva and Wang) price impact model, in which the flow of market orders generated by the other traders is given by an exogeneous process. They have shown that Price Manipulation Strategies exist when the flow of order is a compound Poisson process. However, modeling this flow by a mutually exciting Hawkes process with a particular parametrization allows them to exclude these PMS. Besides, the optimal execution strategy is explicit in this model. A practical implementation is given in [47].

3.1.3 Dependence modeling

- Calibration of stochastic and local volatility models. The volatility is a key concept in modern mathematical finance, and an indicator of market stability. Risk management and associated instruments depend strongly on the volatility, and volatility modeling is a crucial issue in the finance industry. Of particular importance is the assets *dependence* modeling.

By Gyongy's theorem, a local and stochastic volatility model is calibrated to the market prices of all call options with positive maturities and strikes if its local volatility function is equal to the ratio of the Dupire local volatility function over the root conditional mean square of the stochastic volatility factor given the spot value. This leads to a SDE nonlinear in the sense of McKean. Particle methods based on a kernel approximation of the conditional expectation, as presented by Guyon and Henry-Labordère [90], provide an efficient calibration procedure even if some calibration errors may appear when the range of the stochastic volatility factor is very large. But so far, no existence result is available for the SDE nonlinear in the sense of McKean. In the particular case when the local volatility function is equal to the inverse of the root conditional mean square of the stochastic volatility factor multiplied by the spot value given this value and the interest rate is zero, the solution to the SDE is a fake Brownian motion. When the stochastic volatility factor is a constant (over time) random variable taking finitely many values and the range of its square is not too large, B. Jourdain and A. Zhou proved existence to the associated Fokker-Planck equation [97]. Thanks to results obtained by Figalli in [83], they deduced existence of a new class of fake Brownian motions. They extended these results to the special case of the LSV model called Regime Switching Local Volatility, when the stochastic volatility factor is a jump process taking finitely many values and with jump intensities depending on the spot level.

- Interest rates modeling. Affine term structure models have been popularized by Dai and Singleton [73], Duffie, Filipovic and Schachermayer [74]. They consider vector affine diffusions (the coordinates are usually called factors) and assume that the short interest rate is a linear combination of these factors. A model of this kind is the Linear Gaussian Model (LGM) that considers a vector Ornstein-Uhlenbeck diffusions for the factors, see El Karoui and Lacoste [82]. A. Alfonsi et al. [44] have proposed an extension of this model, when the instantaneous covariation between the factors is given by a Wishart process. Doing so, the model keeps its affine structure and tractability while generating smiles for option prices. A price expansion around the LGM is obtained for Caplet and Swaption prices.

3.1.4 Robust finance

- Numerical Methods for Martingale Optimal Transport problems.

The Martingale Optimal Transport (MOT) problem introduced in [70] has received a recent attention in finance since it gives model-free hedges and bounds on the prices of exotic options. The market prices of liquid call and put options give the marginal distributions of the underlying asset at each traded maturity. Under the simplifying assumption that the risk-free rate is zero, these probability measures are in increasing convex order, since by Strassen's theorem this property is equivalent to the existence of a martingale measure with the right marginal distributions. For an exotic payoff function of the values of the underlying on the time-grid given by these maturities, the model-free upper-bound (resp. lower-bound) for the price consistent with these marginal distributions is given by the following martingale optimal transport problem : maximize (resp. minimize) the integral of the payoff with respect to the martingale measure over all martingale measures with the right marginal distributions. Super-hedging (resp. sub-hedging) strategies are obtained by solving the dual problem. With J. Corbetta, A. Alfonsi and B.

Jourdain [51] have studied sampling methods preserving the convex order for two probability measures μ and ν on $\mathbf{R}^d$, with ν dominating μ.

Their method is the first generic approach to tackle the martingale optimal transport problem numerically and can also be applied to several marginals.

- Robust option pricing in financial markets with imperfections.

A. Sulem, M.C. Quenez and R. Dumitrescu have studied robust pricing in an imperfect financial market with default. The market imperfections are taken into account via the nonlinearity of the wealth dynamics. In this setting, the pricing system is expressed as a nonlinear g-expectation $\mathscr{E}^g$ induced by a nonlinear BSDE with nonlinear driver g and default jump (see [75]). A large class of imperfect market models can fit in this framework, including imperfections coming from different borrowing and lending interest rates, taxes on profits from risky investments, or from the trading impact of a large investor seller on the market prices and the default probability. Pricing and superhedging issues for American and game options in this context and their links with optimal stopping problems and Dynkin games with nonlinear expectation have been studied. These issues have also been addressed in the case of model uncertainty, in particular uncertainty on the default probability. The seller's robust price of a game option has been characterized as the value function of a Dynkin game under $\mathscr{E}^g$ expectation as well as the solution of a nonlinear doubly reflected BSDE in [9]. Existence of robust superhedging strategies has been studied. The buyer's point of view and arbitrage issues have also been studied in this context.

In a Markovian framework, the results of the paper [8] on combined optimal stopping/stochastic control with $\mathscr{E}^g$ expectation allows us to address American nonlinear option pricing when the payoff function is only Borelian and when there is ambiguity both on the drift and the volatility of the underlying asset price process. Robust optimal stopping of dynamic risk measures induced by BSDEs with jumps with model ambiguity is studied in [106].

3.2 Perspectives in Stochastic Analysis

3.2.1 Optimal transport and longtime behavior of Markov processes

The dissipation of general convex entropies for continuous time Markov processes can be described in terms of backward martingales with respect to the tail filtration. The relative entropy is the expected value of a backward submartingale. In the case of (non necessarily reversible) Markov diffusion processes, J. Fontbona and B. Jourdain [85] used Girsanov theory to explicit the Doob-Meyer decomposition of this submartingale. They deduced a stochastic analogue of the well known entropy dissipation formula, which is valid for general convex entropies, including the total variation distance. Under additional regularity assumptions, and using Itô's calculus and ideas of Arnold, Carlen and Ju [59], they obtained a new Bakry-Emery criterion which ensures exponential convergence of the entropy to 0. This criterion is non-intrinsic since it depends on the square root of the diffusion matrix, and cannot be written only in terms of the diffusion matrix itself. They provided examples where the classic Bakry Emery criterion fails, but their non-intrinsic criterion applies without modifying the law of the diffusion process.

With J. Corbetta, A. Alfonsi and B. Jourdain have studied the time derivative of the Wasserstein distance between the marginals of two Markov processes [50]. The Kantorovich duality leads to a natural candidate for this derivative. Up to the sign, it is the sum of the integrals with respect to each of the two marginals of the corresponding generator applied to the corresponding Kantorovich potential. For pure jump processes with bounded intensity of jumps, J. Corbetta, A. Alfonsi and B. Jourdain [49] proved that the evolution of the Wasserstein distance is actually given by this candidate. In dimension one, they showed that this remains true for Piecewise Deterministic Markov Processes. They applied the formula to estimate the exponential decrease rate of the Wasserstein distance between the marginals of two birth and death processes with the same generator in terms of the Wasserstein curvature.

3.2.2 Mean-field systems: modeling and control

- Mean-field limits of systems of interacting particles. In [94], B. Jourdain and his former PhD student J. Reygner have studied a mean-field version of rank-based models of equity markets such as the Atlas model introduced by Fernholz in the framework of Stochastic Portfolio Theory. They obtained an asymptotic description of the market when the number of companies grows to infinity. Then, they discussed the long-term capital distribution, recovering the Pareto-like shape of capital distribution curves usually derived from empirical studies, and providing a new description of the phase transition phenomenon observed by Chatterjee and Pal. They have also studied multitype sticky particle systems which can be obtained as vanishing noise limits of multitype rank-based diffusions (see [96]). Under a uniform strict hyperbolicity assumption on the characteristic fields, they constructed a multitype version of the sticky particle dynamics. In [95], they obtain the optimal rate of convergence as the number of particles grows to infinity of the approximate solutions to the diagonal hyperbolic system based on multitype sticky particles and on easy to compute time discretizations of these dynamics.

In [86], N. Fournier and B. Jourdain are interested in the two-dimensional Keller-Segel partial differential equation. This equation is a model for chemotaxis (and for Newtonian gravitational interaction).

- Mean field control and Stochastic Differential Games (SDGs). To handle situations where controls are chosen by several agents who interact in various ways, one may use the theory of Stochastic Differential Games (SDGs). Forward–Backward SDG and stochastic control under Model Uncertainty are studied in [104] by A. Sulem and B. Øksendal. Also of interest are large population games, where each player interacts with the average effect of the others and individually has negligible effect on the overall population. Such an interaction pattern may be modeled by mean field coupling and this leads to the study of mean-field stochastic control and related SDGs. A. Sulem, Y. Hu and B. Øksendal have studied singular mean field control problems and singular mean field two-players stochastic differential games [91]. Both sufficient and necessary conditions for the optimal controls and for the Nash equilibrium are obtained. Under some assumptions, the optimality conditions for singular mean-field control are reduced to a reflected Skorohod problem. Applications to optimal irreversible investments under uncertainty have been investigated. Predictive mean-field equations as a model for prices influenced by beliefs about the future are studied in [105].

3.2.3 Stochastic control and optimal stopping (games) under nonlinear expectation

M.C. Quenez and A. Sulem have studied optimal stopping with nonlinear expectation $\mathcal{E}^g$ induced by a BSDE with jumps with nonlinear driver g and irregular obstacle/payoff (see [106]). In particular, they characterize the value function as the solution of a reflected BSDE. This property is used in [78] to address American option pricing in markets with imperfections. The Markovian case is treated in [81] when the payoff function is continuous.

In [8], M.C. Quenez, A. Sulem and R. Dumitrescu study a combined optimal control/stopping problem under nonlinear expectation $\mathcal{E}^g$ in a Markovian framework when the terminal reward function is only Borelian. In this case, the value function u associated with this problem is irregular in general. They establish a *weak* dynamic programming principle (DPP), from which they derive that the upper and lower semi-continuous envelopes of u are the sub- and super- *viscosity solution* of an associated nonlinear Hamilton-Jacobi-Bellman variational inequality.

The problem of a generalized Dynkin game problem with nonlinear expectation $\mathcal{E}^g$ is addressed in [79]. Under Mokobodzki's condition, we establish the existence of a value function for this game, and characterize this value as the solution of a doubly reflected BSDE. The results of this work are used in [9] to solve the problem of game option pricing in markets with imperfections.

A generalized mixed game problem when the players have two actions: continuous control and stopping is studied in a Markovian framework in [80]. In this work, dynamic programming principles (DPP) are established: a strong DPP is proved in the case of a regular obstacle and a weak one in the irregular case. Using these DPPs, links with parabolic partial integro-differential Hamilton-Jacobi- Bellman variational inequalities with two obstacles are obtained.

With B. Øksendal and C. Fontana, A. Sulem has contributed on the issues of robust utility maximization [103, 105], and relations between information and performance [84].

3.2.4 Generalized Malliavin calculus

Vlad Bally has extended the stochastic differential calculus built by P. Malliavin which allows one to obtain integration by parts and associated regularity probability laws. In collaboration with L. Caramellino (Tor Vegata University, Roma), V. Bally has developed an abstract version of Malliavin calculus based on a splitting method (see [61]). It concerns random variables with law locally lower bounded by the

Lebesgue measure (the so-called Doeblin's condition). Such random variables may be represented as a sum of a "smooth" random variable plus a rest. Based on this smooth part, he achieves a stochastic calculus which is inspired from Malliavin calculus [6]. An interesting application of such a calculus is to prove convergence for irregular test functions (total variation distance and more generally, distribution distance) in some more or less classical frameworks as the Central Limit Theorem, local versions of the CLT and moreover, general stochastic polynomials [64]. An exciting application concerns the number of roots of trigonometric polynomials with random coefficients [65]. Using Kac Rice lemma in this framework one comes back to a multidimensional CLT and employs Edgeworth expansions of order three for irregular test functions in order to study the mean and the variance of the number of roots. Another application concerns U statistics associated to polynomial functions. The techniques of generalized Malliavin calculus developed in [61] are applied in for the approximation of Markov processes (see [69] and [68]). On the other hand, using the classical Malliavin calculus, V. Bally in collaboration with L. Caramellino and P. Pigato studied some subtle phenomena related to diffusion processes, as short time behavior and estimates of tubes probabilities (see [63, 62]).

3.3 Numerical Probability

Our project team is very much involved in numerical probability, aiming at pushing numerical methods towards the effective implementation. This numerical orientation is supported by a mathematical expertise which permits a rigorous analysis of the algorithms and provides theoretical support for the study of rates of convergence and the introduction of new tools for the improvement of numerical methods. This activity in the MathRisk team is strongly related to the development of the Premia software.

3.3.1 Simulation of stochastic differential equations

- Weak convergence of the Euler scheme in optimal transport distances. With A. Kohatsu-Higa, A. Alfonsi and B. Jourdain [4] have proved using optimal transport tools that the Wasserstein distance between the time marginals of an elliptic SDE and its Euler discretization with N steps is not larger than $C\sqrt{\log(N)}/N$. The logarithmic factor may is removed when the uniform time-grid is replaced by a grid still counting N points but refined near the origin of times.

- Strong convergence properties of the Ninomiya Victoir scheme and multilevel Monte-Carlo estimators. With their former PhD student, A. Al Gerbi, E. Clément and B. Jourdain [1] have proved strong convergence with order 1/2 of the Ninomiya-Victoir scheme which is known to exhibit order 2 of weak convergence [102]. This study was aimed at analysing the use of this scheme either at each level or only at the finest level of a multilevel Monte Carlo estimator : indeed, the variance of a multilevel Monte Carlo estimator is related to the strong error between the two schemes used in the coarse and fine grids at each level. In [45], they proved that the order of strong convergence of the crude Ninomiya Victoir scheme is improved to 1 when the vector fields corresponding to each Brownian coordinate in the SDE commute, and in [46], they studied the error introduced by discretizing the ordinary differential equations involved in the Ninomiya-Victoir scheme.

- Non-asymptotic error bounds for the multilevel Monte Carlo Euler method. A. Kebaier and B. Jourdain are interested in deriving non-asymptotic error bounds for the multilevel Monte Carlo method. As a first step, they dealt in [92] with the explicit Euler discretization of stochastic differential equations with a constant diffusion coefficient. They obtained Gaussian-type concentration. To do so, they used the Clark-Ocone representation formula and derived bounds for the moment generating functions of the squared difference between a crude Euler scheme and a finer one and of the squared difference of their Malliavin derivatives. The estimation of such differences is much more complicated than the one of a single Euler scheme contribution and explains why they suppose the diffusion coefficient to be constant. This assumption ensures boundedness of the Malliavin derivatives of both the SDE and its Euler scheme.

- Computation of sensibilities of integrals with respect to the invariant measure. In [60], R. Assaraf, B. Jourdain, T. Lelièvre and R. Roux considered the solution to a stochastic differential equation with constant diffusion coefficient and with a drift function which depends smoothly on some real parameter λ, and admitting a unique invariant measure for any value of λ around $\lambda = 0$. Their aim was to compute the derivative with respect to λ of averages with respect to the invariant measure, at $\lambda = 0$. They analyzed a numerical method which consists in simulating the process at $\lambda = 0$ together with its derivative with respect to λ on a long time horizon. They gave sufficient conditions implying uniform-in-time square integrability of this derivative. This allows in particular to compute efficiently the derivative with respect to λ of the mean of an observable through Monte Carlo simulations.

- Approximation of doubly reflected Backward stochastic differential equations. R. Dumitrescu and C. Labart have studied the discrete time approximation scheme for the solution of a doubly reflected Backward Stochastic Differential Equation with jumps, driven by a Brownian motion and an independent compensated Poisson process [77, 76].

- Parametrix methods. V. Bally and A. Kohatsu-Higa have recently proposed an unbiased estimator based on the parametrix method to compute expectations of functions of a given SDE ([67]). This method is very general, and A. Alfonsi, A. Kohastu-Higa and M. Hayashi [53] have applied it to the case of one-dimensional reflected diffusions. In this case, the estimator can be obtained explicitly by using the scheme of Lépingle [99] and is quite simple to implement. It is compared to other simulation methods for reflected SDEs.

3.3.2 Estimation of the parameters of a Wishart process

A. Alfonsi, A. Kebaier and C. Rey [55] have computed the Maximum Likelihood Estimator for the Wishart process and studied its convergence in the ergodic and in some non ergodic cases. In the ergodic case, which is the most relevant for applications, they obtain the standard square-root convergence. In the non ergodic case, the analysis rely on refined results for the Laplace transform of Wishart processes, which are of independent interest.

3.3.3 Optimal stopping and American options

In joint work with A. Bouselmi, D. Lamberton studied the asymptotic behavior of the exercise boundary near maturity for American put options in exponential Lévy models. In [7], they deal with jump-diffusion models, and establish that, in some cases, the behavior differs from the classical Black and Scholes setting. D. Lamberton has also worked on the binomial approximation of the American put. The conjectured rate of convergence is $O(1/n)$ where n is the number of time periods. He was able to derive a $O((\ln n)^{\alpha}/n)$ bound, where the exponent α is related to the asymptotic behavior of the exercise boundary near maturity.

4 Application domains

4.1 Financial Mathematics, Insurance

The domains of application are quantitative finance and insurance with emphasis on risk modeling and control. In particular, Mathrisk focuses on dependence modeling, systemic risk, market microstructure modeling and risk measures.

5 Social and environmental responsibility

Our work aims to contribute to a better management of risk in the banking and insurance systems, in particuluar by the study of systemic risk, asset price modeling, stability of financial markets.

6 Highlights of the year

Despite the pandemic, Mathrisk team has been increasingly active, with several PhD defenses, participation to the main international conferences and intense publication activity.

7 New software and platforms

PREMIA: Numerical Platform for quantitative finance site

7.1 New software

7.1.1 PREMIA

Keywords: Financial products, Computational finance, Option pricing

Scientific Description: Premia is a numerical platform for computational finance. It is designed for option pricing, hedging and financial model calibration. Premia is developed by the MathRisk project team in collaboration with a consortium of financial institutions. The Premia project keeps track of the most recent advances in the field of computational finance in a well-documented way. It focuses on the implementation of numerical analysis techniques for both probabilistic and deterministic numerical methods. An important feature of the platform Premia is the detailed documentation which provides extended references in option pricing. Premia contains various numerical algorithms: deterministic methods (Finite difference and finite element algorithms for partial differential equations, wavelets, Galerkin, sparse grids ...), stochastic algorithms (Monte-Carlo simulations, quantization methods, Malliavin calculus based methods), tree methods, approximation methods (Laplace transforms, Fast Fourier transforms...) These algorithms are implemented for the evaluation of vanilla and exotic options on equities, interest rate, credit, energy and insurance products. Moreover Premia provides a calibration toolbox for Libor Market model and a toolbox for pricing Credit derivatives. The latest developments of the software address evaluation of financial derivative products, risk management and computations of risk measures required by new financial regulation. They include the implementation of advanced numerical algorithms taking into account model dependence, counterparty credit risk, hybrid features, rough volatility and various nonlinear effects. High-dimensional problems are addressed by deep learning techniques using neural network approximations.

Functional Description: Premia is a software designed for option pricing, hedging and financial model calibration, developed by the MathRisk project team in collaboration with a consortium of financial institutions presently composed of Crédit Agricole CIB and NATIXIS. The Premia project keeps track of the most recent advances in computational finance in a well documented way. It focuses on the implementation of numerical techniques, be they probabilistic or deterministic, to solve financial problems. Premia is thus a powerful tool to assist Research and Development professional teams in their day-to-day duty. It is also a useful support for academics who wish to perform tests on new algorithms or pricing methods. Besides being a single entry point for accessible overviews and basic implementations of various numerical methods, the aim of the Premia project is: - to elaborate a powerful testing platform for comparing different numerical methods between each other, - to build a link between professional financial teams and academic researchers, - to provide a useful teaching support for Master and PhD students in mathematical finance. An important feature of the platform Premia is its detailed documentation which provides extended references in computational finance. The project Premia has started in 1999 and is now considered as a standard reference platform for quantitative finance among the academic mathematical finance community.

Release Contributions: The latest developments of the software address evaluation of financial derivative products, risk management and computations of risk measures required by new financial regulation. They include the implementation of advanced numerical algorithms taking into account model dependence, counterparty credit risk, hybrid features, rough volatility and various nonlinear effects. High-dimensional problems are addressed by deep learning techniques using neural network approximations. We also develop our activity on insurance contracts, in particular on the computation of risk measures (Value at Risk, Condition Tail Expectation) of variable annuities contracts like GMWB (guaranteed minimum withdrawal benefit) including taxation and customers mortality modeling.

News of the Year: Release 23 of the Premia software has been delivered to the Consortium in March 2021. It contains the following new implemented algorithms dedicated to Machine Learning and Equity derivatives pricing:

-" Deep optimal stopping", Becker, P. Cheridito, and A. Jentzen. Journal of Machine Learning Research, 20(74), 2019,

- "Chebyshev Interpolation for Parametric Option Pricing", M.Gass, K.Glau, M.Mahlstedt, M.Mair, Finance Stochastics, Volume 21, Number 5, 2018.

- "A new approach for American option pricing: The dynamic Chebyshev method". K. Glau, M.Mahlsted, C.Potz, SIAM Journal on Scientific Computing, Volume 41, Issue 1, 2019

- "The Chebyshev method for the implied volatility". K. Glau, P.Herold, D. B. Madan, C. Pötz, Journal of Computational Finance, Volume 23, Number 3, 2019.

- "Let's Be Rational". P.Jackel, Wilmott, Issue75, 2015

- Approximation of Stochastic Volterra Equations with kernels of completely monotone type. A.Alfonsi , A.Kebaier

- Hybrid scheme for Brownian semistationary processes. M. Bennedsen, A. Lunde, M. S. Pakkanen, Finance Stochastics, Volume 21, 2017.

- Multi-factor approximation of rough volatility models. E.Abi Jaber, O.El Euch, SIAM J. Finan. Math. 10-2, 2019

- The characteristic function of rough Heston models. O. El Euch, M.Rosenbaum, Mathematical Finance, 29-1, 2019

- Reconciling rough volatility with jumps. E.Abi Jaber, QuantMinds International Conference 2020

- A General Valuation Framework for SABR and Stochastic Local Volatility Models. Z. Cui, J. L. Kirkby, D. Nguyen, SIAM J. Finan. Math. 9-2, 2018

- The Wiener-Hopf Factorization for Pricing Options Made Easy. O.Kudryavtsev, P. Luzhetskaya, Engineering Letters, 28-4, 2020

- Stratified approximation for pricing Asian option under Vasicek interest rate model. X. Wei, L. Zhao

URL: `http://www.premia.fr`

Publications: hal-03526905, hal-03013603, hal-03013606, hal-01873346

Contact: Agnes Sulem

Participants: Agnes Sulem, Antonino Zanette, Aurélien Alfonsi, Benjamin Jourdain, Jérôme Lelong, Bernard Lapeyre, Ahmed Kebaier

Partners: Inria, Ecole des Ponts ParisTech, Université Paris-Est

7.2 New platforms

Participants: Premia Team.

PREMIA : Release 23

8 New results

8.1 Control of systemic risk in a dynamic framework

Participants: A. Sulem, H. Amini, , A. Minca, , Z. Cao.

Dynamic Contagion Risk Model With Recovery Features. The goal of the project is to develop a model that captures the dynamics of a complex financial network and to provide methods for the control of default contagion, both by a regulator and by the institutions themselves. This introduces a new class of problems that are very challenging mathematically, as it relies on using mean field games and random graphs theory. Agnès Sulem, Andreea Minca (Cornell University), Hamed Amini (J. Mack Robinson College of Business, Georgia State University) have studied a Dynamic Contagion Risk Model With Recovery Features. They introduce threshold growth in the classical threshold contagion model, in which nodes have downward jumps when there is a failure of a neighboring node. Choosing the configuration model as underlying graph, they prove fluid limits for the baseline model, as well as extensions to the directed case, state-dependent inter-arrival times and the case of growth driven by upward jumps. They then allow nodes to choose their connectivity by trading off link benefits and contagion risk. They define a rational equilibrium concept in which nodes choose their connectivity according to an expected failure probability of any given link, and then impose condition that the expected failure probability coincides with the actual failure probability under the optimal connectivity. Existence of an asymptotic equilibrium is shown as well as convergence of the sequence of equilibria on the finite networks. In particular, these results show that systems with higher overall growth may have higher failure probability in equilibrium [15].

Default cascades in sparse heterogeneous financial networks. Agnès Sulem, Hamed Amini and their PhD student Zhongyang Cao have obtained some limit results for default cascades in sparse heterogeneous financial networks subject to an exogenous macroeconomic shock in [37]. These results are applied to determine the optimal policy for a social planner to target interventions during a financial crisis, with a budget constraint and under partial information of the financial network. In [38], they present a general tractable framework for understanding the joint impact of fire sales and default cascades on systemic risk in complex financial networks. The effect of heterogeneity in network structure and price impact function on the final size of default cascade and fire sales loss is investigated.

8.2 Mean-field Backward Stochastic Differential Equations and systemic risk measures

Participants: A. Sulem, R. Chen, , A. Minca, , R. Dumitrescu.

Agnès Sulem, Rui Chen, Andreea Minca, Roxana Dumitrescu have studied mean-field BSDEs with a generalized mean-field operator which can capture system influence with higher order interactions. Convergence of finite approximations to the mean-field BSDE have been obtained. In the finite system, the mean-field term can incorporate for example an inhomogeneous graph model in which the intensity of bilateral interactions depends on the states of the end nodes by means of a kernel function. This opens the path towards using dynamic risk measures induced by mean-field BSDE as a complementary approach to systemic risk measurement.

8.3 Risk management in finance and insurance

Participants: A. Sulem, M. Grigorova, , M. Quenez.

8.3.1 Option pricing in a non-linear incomplete market model with default

Agnès Sulem has studied with Miryana Grigorova (University of Leeds) and Marie-Claire Quenez (Université Paris Denis Diderot) superhedging prices and the associated superhedging strategies for both European and American options (see [89] and [20] in a non-linear incomplete market model with default. The underlying market model consists of a risk-free asset and a risky asset driven by a Brownian motion and a compensated default martingale. The portfolio processes follow non-linear dynamics with a non-linear driver f. . By using a dynamic programming approach, we provide a dual formulation of the seller's (superhedging) price for the European option involving a suitable set of equivalent probability measures, which we call f-martingale probability measures. We also establish a characterization of the seller's price as the initial value of the minimal supersolution of a constrained BSDE with default. Our results rely on first establishing a non-linear optional and a non-linear predictable decomposition for processes which are $\mathcal{E}^f$-strong supermartingales under Q, for all $Q \in \mathcal{Q}$. We then studied American options with irregular payoff in this market. Both points of view of the seller and of thave he buyer are analyzed. We give a dual representation of the seller's (superhedging) price in terms of the value of a non-linear mixed control/stopping problem, and provide two infinitesimal characterizations of the seller's price process in terms of the minimal supersolution of a constrained reflected BSDE and of an optional reflected BSDE. Under some regularity assumptions on the payoff, we also prove a duality result for the buyer's price in terms of the value of a non-linear control/stopping game problem.

8.3.2 Weather derivatives

Participants: A. Alfonsi, N. Fernandez.

With his PhD student N. Vadillo Fernandez, A. Alfonsi develops a stochastic model for temperature in order to price derivatives on the climate (Heating Degree Day index). They also work on its estimation to historical data on weather.

8.3.3 Insurance products

Participants: A. Alfonsi, A. Cherchali, A. Zanette, L. Goudenège.

Solvency Capital Requirement in Insurance. A. Alfonsi has obtained a grant from AXA Foundation on a Joint Research Initiative with a team of AXA France working on the strategic asset allocation. This team has to make recommendations on the investment over some assets classes as, for example, equity, real estate or bonds. In order to do that, each side of the balance sheet (assets and liabilities) is modeled in order to take into account their own dynamics but also their interactions. Given that the insurance products are long time contracts, the projections of the company's margins have to be done considering long maturities. When doing simulations to assess investment policies, it is necessary to take into account the SCR which is the amount of cash that has to be settled to manage the portfolio. Typically, the computation of the future values of the SCR involve expectations under conditional laws, which is greedy in computation time.

A. Alfonsi and his PhD student A. Cherchali have constructed a model of the ALM management of insurance companies that takes into account the regulatory constraints on life-insurance [48]. They have developed Multilevel Monte-Carlo methods to approximate the SCR (Solvency Capital Requirement) at a future date and more generally, to calculate the worst of P shocks [13].

Pricing and hedging variable annuities of GMWB type in advanced stochastic models. Antonino Zanette with Ludovic Goudenège (Ecole Centrale de Paris) and Andrea Molent (University of Udine) study the valuation of GMWB variable annuity when both stochastic volatility and stochastic interest rate are considered in the Heston Hull-White model. [19].

8.3.4 Deep learning for large dimensional financial problems

Participants: B. Lapeyre, J. Lelong, A. Zanette, L. Goudenège, A. Molent, H. Madmoun.

Neural network regression for Bermudan option pricing. The pricing of Bermudan options amounts to solving a dynamic programming principle, in which the main difficulty, especially in high dimension, comes from the conditional expectation involved in the computation of the continuation value. These conditional expectations are classically computed by regression techniques on a finite dimensional vector space. In [28], Bernard Lapeyre and Jérôme Lelong study neural networks approximations of conditional expectations. They prove the convergence of the well-known Longstaff and Schwartz algorithm when the standard least-square regression is replaced by a neural network approximation. They illustrate the numerical efficiency of neural networks as an alternative to standard regression methods for approximating conditional expectations on several numerical examples.
Machine learning for pricing American options. In [88], L. Goudenège, A. Molent and A. Zanette develop techniques, called GPR Tree and GPR Exact Integration, both based on Machine Learning, to compute prices of American basket options in high-dimension. Both Markovian and non Markovian models are studied, in particular rough Bergomi model, which provides stochastic volatility with memory. In [29], they propose an efficient method to compute the price of multi-asset American options, based on Machine Learning, Monte Carlo simulations and variance reduction technique.
Big data techniques for portfolio optimization. With his PhD student Hachem Madmoun, Bernard Lapeyre analyses the trajectories of asset prices by Fourier transform, and big data techniques such as vae (variational autoencoder) and hmm (hidden Markov chains) for portfolio management.

8.4 Stochastic analysis and probabilistic numerical methods

8.4.1 Optimal transport and mean-field SDEs

Participants: B. Jourdain, A. Tsé, A. Alfonsi, O. Bencheikh, V. Ehrlacher, R. Coyaud.

In [27], Benjamin Jourdain and Alvin Tse propose a generalised version of the central limit theorem for nonlinear functionals of the empirical measure of i.i.d. random variables, provided that the functional satisfies some regularity assumptions for the associated linear functional derivatives of various orders. This generalisation can be applied to Monte-Carlo methods, even when there is a nonlinear dependence on the measure component. As a consequence of this result, they also analyse the convergence of fluctuation between the empirical measure of particles in an interacting particle system and their mean-field limiting measure (as the number of particles goes to infinity), when the dependence on measure is nonlinear.

In [54], A. Alfonsi and B. Jourdain study the structure of optimal couplings for the squared quadratic Wasserstein distance of probability measures with finite second order moments.

In [17], Oumaima Bencheikh and Benjamin Jourdain study the approximation in Wasserstein distance with index $\rho \geq 1$ of a probability measure μ on the real line with finite moment of order ρ by the empirical measure of N deterministic points. The minimal error converges to 0 as $N \to +\infty$. Apart when μ is a Dirac mass and the error vanishes, the order of convergence is not larger than 1. They give a necessary condition and a sufficient condition for the order to be equal to this threshold 1 in terms of the density of the absolutely continuous with respect to the Lebesgue measure part of μ. They also check that for the order to lie in the interval $\left(1/\rho, 1\right)$, the support of μ has to be a bounded interval, and that, when μ is compactly supported, the order is not smaller than $1/\rho$. Last, they give a necessary and sufficient condition in terms of the tails of μ for the order to be equal to some given value in the interval $\left(0, 1/\rho\right)$. In [18], they analyse the rate of convergence of a system of interacting particles with mean-field rank based interaction in the drift coefficient and constant diffusion coefficient.

With V. Ehrlacher and R. Coyaud, Aurelien Alfonsi is working on numerical methods based to approximate the optimal transport in the symmetric case in the multimarginal case. [52] [14].

8.4.2 Martingale optimal transport

Participants: B. Jourdain, W. Margheriti, M. Beigkböck, G. Pammer.

It is known since [93] that two one-dimensional probability measures in the convex order admit a martingale coupling with respect to which the integral of $|x-y|$ is smaller than twice their Wasserstein distance $\mathcal{W}_1$ with index 1. Moreover, replacing $|x-y|$ and $\mathcal{W}_1$ respectively with $|x-y|^\rho$ and $\mathcal{W}_\rho^\rho$ does not lead to a finite multiplicative constant. In [23], B. Jourdain and W. Margheriti show that a finite constant is recovered when replacing $\mathcal{W}_\rho^\rho$ with the product of $\mathcal{W}_\rho$ times the centred ρ-th moment of the second marginal to the power $\rho-1$. Then they study the generalisation of this new stability inequality to higher dimension. In [40], with M. Beiglböck and G. Pammer they establish stability of martingale couplings in dimension one. If π is a martingale coupling with marginals μ,ν. Then, given approximating marginal measures $\tilde{\mu}\approx\mu$, $\tilde{\nu}\approx\nu$ in convex order, they show that there exists an approximating martingale coupling $\tilde{\pi}\approx\pi$ with marginals $\tilde{\mu},\tilde{\nu}$. In mathematical finance, prices of European call / put option yield information on the marginal measures of the arbitrage free pricing measures. The above result asserts that small variations of call / put prices lead only to small variations on the level of arbitrage free pricing measures. While these facts have been anticipated for some time they do not generalize to higher dimensions and the actual proof requires somewhat intricate stability results for the adapted Wasserstein distance. Notably the result has consequences for a several related problems. Specifically, it is relevant for numerical approximations, it leads to a new proof of the monotonicity principle of martingale optimal transport and it implies stability of weak martingale optimal transport as well as optimal Skorokhod embedding. On the mathematical finance side this yields continuity of the robust pricing problem for exotic options and VIX options with respect to market data. These applications will be detailed in two companion papers.

In [42], B. Jourdain and W. Margheriti are interested in martingale rearrangement couplings. As introduced by Wiesel, in order to prove the stability of Martingale Optimal Transport problems, these are projections in adapted Wasserstein distance of couplings between two probability measures on the real line in the convex order onto the set of martingale couplings between these two marginals. In reason of the lack of relative compactness of the set of couplings with given marginals for the adapted Wasserstein topology, the existence of such a projection is not clear at all. Under a barycentre dispersion assumption on the original coupling which is in particular satisfied by the Hoeffding-Fréchet or comonotone coupling, Wiesel gives a clear algorithmic construction of a martingale rearrangement when the marginals are finitely supported and then gets rid of the finite support assumption by relying on a rather messy limiting procedure to overcome the lack of relative compactness. B. Jourdain and W. Margheriti give a direct general construction of a martingale rearrangement coupling under the barycentre dispersion assumption. This martingale rearrangement is obtained from the original coupling by an approach similar to the construction given in [93] of the inverse transform martingale coupling, a member of a family of martingale couplings close to the Hoeffding-Fréchet coupling, but for a slightly different injection in the set of extended couplings introduced by Beiglböck and Juillet and which involve the uniform distribution on $[0,1]$ in addition to the two marginals. They last discuss the stability in adapted Wassertein distance of the inverse transform martingale coupling with respect to the marginal distributions.

While many questions in (robust) finance can be posed in the martingale optimal transport (MOT) framework, others require to consider also non-linear cost functionals. Following the terminology of Gozlan, Roberto, Samson and Tetali for classical optimal transport, this corresponds to *weak* martingale optimal transport (WMOT). In [41], M. Beigkböck, B. Jourdain, W. Margheriti and G. Pammer establish stability of WMOT in dimension one which is important since financial data can give only imprecise information on the underlying marginals. As application, they deduce the stability of the superreplication bound for VIX futures as well as the stability of the stretched Brownian motion and we derive a monotonicity principle for WMOT.

8.4.3 Quantization methods

Participants: B. Jourdain, G. Pagès.

In [25], Benjamin Jourdain and Gilles Pagès establish for dual quantization the counterpart of Kieffer's uniqueness result for compactly supported one dimensional probability distributions having a log-concave density (also called strongly unimodal): for such distributions, Lr-optimal dual quantizers are unique at each level N, the optimal grid being the unique critical point of the quantization error. An example of non-strongly unimodal distribution for which uniqueness of critical points fails is exhibited. In the quadratic r=2 case, they propose an algorithm which computes the unique optimal dual quantizer with geometric rate of convergence in the log-concave case. It provides a counterpart of Lloyd's method I algorithm in a Voronoi framework. Finally semi-closed forms of Lr-optimal dual quantizers are established for power distributions on compacts intervals and truncated exponential distributions.

Quantization provides a very natural way to preserve the convex order when approximating two ordered probability measures by two finitely supported ones. Indeed, when the convex order dominating original probability measure is compactly supported, it is smaller than any of its dual quantizations while the dominated original measure is greater than any of its stationary (and therefore any of its optimal) quadratic primal quantization. Moreover, the quantization errors then correspond to martingale couplings between each original probability measure and its quantization. This enables B. Jourdain and G. Pagès to prove in [26] that any martingale coupling between the original probability measures can be approximated by a martingale coupling between their quantizations in Wassertein distance with a rate given by the quantization errors but also in the much finer adapted Wassertein distance. As a consequence, while the stability of (Weak) Martingale Optimal Transport problems with respect to the marginal distributions has only been established in dimension 1 so far, their value function computed numerically for the quantized marginals converges in any dimension to the value for the original probability measures as the numbers of quantization points go to ∞. In [24], the approximation of martingale ARCH models is studied.

8.4.4 Approximation of stochastic differential equations

Euler-Maruyama discretization

Participants: B. Jourdain, O. Bencheikh, S. Menozzi.

In [71], Oumaima Bencheikh and Benjamin Jourdain are interested in the Euler-Maruyama discretization of a stochastic differential equation in dimension d with constant diffusion coefficient and bounded measurable drift coefficient. In the scheme, a randomization of the time variable is used to get rid of any regularity assumption of the drift in this variable. We prove weak convergence with order 1/2 in total variation distance. When the drift has a spatial divergence in the sense of distributions with ρ-th power integrable with respect to the Lebesgue measure in space uniformly in time for some $\rho \geq d$, the order of convergence at the terminal time improves to 1 up to some logarithmic factor. In dimension d=1, this result is preserved when the spatial derivative of the drift is a measure in space with total mass bounded uniformly in time. The theoretical analysis is confirmed by numerical experiments.

In [43], S. Menozzi and B. Jourdain are interested in the time discretization of stochastic differential equations with additive d-dimensional Brownian noise and $L^q - L^\rho$ drift coefficient when the condition $\frac{d}{\rho} + \frac{2}{q} < 1$, under which Krylov and Röckner proved existence of a unique strong solution, is met. They show weak convergence with order $\frac{1}{2}(1 - (\frac{d}{\rho} + \frac{2}{q}))$ which corresponds to half the distance to the threshold for the Euler scheme with randomized time variable and cutoffed drift coefficient so that its contribution on each time-step does not dominate the Brownian contribution. More precisely, they prove that both the diffusion and this Euler scheme admit transition densities and that the difference between these densities is bounded from above by the time-step to this order multiplied by some centered Gaussian density.

Approximation of Stochastic Volterra Equations

Participants: A. Alfonsi, A. Kebaier.

A. Alfonsi and A. Kebaier have studied the strong error for the approximation of Stochastic Volterra Equations and processes with rough paths in [36]. They are now working on the study of the weak error for some approximation schemes.

High order schemes for the weak error for the CIR and Heston processes

Participants: A. Alfonsi, E. Lombardo.

With E. Lombardo, A. Alfonsi is studying high order schemes for the weak error for the CIR process, based on the construction proposed in a recent paper by A. Alfonsi and V. Bally [12].

Eigen values of Wishart processes.

Participants: B. Jourdain, E. Kahn.

B. Jourdain and his PhD student E. Kahn study stochastic differential equations coming from the eigenvalues of Wishart processes [21], [98].

8.4.5 Abstract Malliavin calculus and convergence in total variation

Participants: V. Bally, L. Caramellino, G. Poly, Y. Qin.

In collaboration with L. Caramellino (University Tor Vergata) and with G. Poly (University of Rennes), V. Bally has settled a Malliavin type calculus for a general class of random variables, which are not supposed to be Gaussian (as it is the case in the standard Malliavin calculus). This is an alternative to the Γ calculus settled by Bakry, Gentile and Ledoux. The main application is the estimate in total variation distance of the error in general convergence theorems. This is done in [66].

In [39], V. Bally and his PhD student Yifen Qin obtain total variation distance result between a jump-equation and its Gaussian approximation by Malliavin calculus techniques.

8.4.6 Boltzmann and McKean-Vlasov equations

Participants: V. Bally, A. alfonsi.

A. Alfonsi and V. Bally propose a new approach to study existence and uniqueness of solutions of the Boltzmann equation in [35].They now work with L. Caramellino to extend their results under more general assumptions on jumps.

8.4.7 Regularity of the low of the solution of jump type equations

Participants: V. Bally, L. Caramellino, A. Kohatsu Higa.

In collaboration with L. Caramellino and A. Kohatsu Higa, V. Bally study the regularity of the solutions of jump type equations. A first result is obtained in [16].

8.4.8 Optimal stopping and free boundary problems

Participant: D. Lamberton.

Damien Lamberton has worked on American options in stochastic volatility models (especially the Heston model). He has improved some results on the regularity of the value function and on the exercise boundary.

9 Bilateral contracts and grants with industry

9.1 Bilateral contracts with industry

- Consortium PREMIA, Natixis - INRIA
- Consortium PREMIA, Crédit Agricole Corporate Investment Bank (CA - CIB) - INRIA
- AXA Joint Research Initiative on Numerical methods for the ALM, from September 2017 to August 2020. PhD grant of Adel Cherchali, Supervisor: A. Alfonsi.
- CIFRE agreement Milliman company/Ecole des Ponts (Milliman),

  PhD thesis of Sophian Mehalla on "Interest rate risk modeling for insurance companies", Supervisor: Bernard Lapeyre.
- CIFRE agreement Brahham gardens/Ecole des Ponts

  PhD thesis of Hachem Madmoun: "Gestion de portefeuilles utilisant des techniques de (big) data" site
- Collaboration with IRT Systemx

  PhD grant of Adrien Touboul on "Uncertainty computation in a graph of physical simulations", Supervisors: Bernard Lapeyre and Julien Reygner.
- CIFRE agreement AXA Climate/ENPC PhD thesis of Nerea Vadillo Fernandez. Supervisor: Aurélien Alfonsi

9.2 Bilateral grants with industry

Chair X-ENPC-SU-Société Générale "Financial Risks" of the Risk fondation.

Participants: Aurélien Alfonsi, Benjamin Jourdain, Bernard Lapeyre.

10 Partnerships and cooperations

Participants: A. Sulem, V. Bally.

10.1 International research visitors

10.1.1 Visits of international scientists

Research stay of Hamed Amini, Associate Professor, Department of Risk Management and Insurance, Mack Robinson College of Business, Georgia State University, June-August 2021.
Collaboration with A. Sulem and PhD student Z. Cao on systemic risk in financial networks.

10.2 International research collaboration

Collaborations with L. Caramellino (Tor Vegata University, Roma), A. Kohatsu Higa (Ritsumeikan University, Japan), A. Minca (Cornell University), M. Grigorova (University of Leeds)), H. Amini (Georgia State University).

10.3 National initiatives

- Labex Bezout
  web site

10.3.1 Competitivity Clusters

Pôle Finance Innovation

11 Dissemination

Participant: Mathrisk Team.

11.1 Promoting scientific activities

11.1.1 Scientific events: organisation

- A. Alfonsi
  - Co-organizer of the working group seminar of MathRisk "Méthodes stochastiques et finance". seminar
  - Co-organizer of the Bachelier Finance seminar, IHP Paris seminar
- V. Bally: Organizer of the seminar of the LAMA laboratory, Université Gustave Eiffel.
- A. Sulem : Co-organizer of the seminar INRIA-MathRisk / Université Paris Diderot LPSM "Numerical probability and mathematical finance". seminaire

11.1.2 Journal

Member of the editorial boards

- B. Jourdain: Associate editor of
  - ESAIM : Proceedings and Surveys
  - Stochastic Processes and their Applications (SPA)
  - Stochastic and Partial Differential Equations : Analysis and Computations
- D. Lamberton: Associate editor of
  - Mathematical Finance
  - ESAIM Probability & Statistics
- A. Sulem : Associate editor of
  - Journal of Mathematical Analysis and Applications JMAA
  - SIAM Journal on Financial Mathematics SIFIN
  - Mathematics, (Financial Mathematics Section) url

Reviewer - reviewing activities

- B. Jourdain : Reviewer for *Mathematical Reviews*
- A. Sulem: Reviewer for *Mathematical Reviews*

11.1.3 Invited talks

- V. Bally
  - Conference of Numerical Probability in honor of Gilles Pagès' 60th birthday 26-28 May 2021 Paris : "Construction of Mac-Kean and Boltzmann type flows: the sewing lemma approach".
  - Journées des probabilités 2021 Guidel, 21-25 June 2021. "Integration by parts and convergence in distribution norms in the CLT"
  - Distances for stochastic processes and applications, 8 October 2021, UFR des sciences de Versailles, "Total variation distance between a jump-equation and its Gaussian approximation"
  - Random excursions with Jean Bertoin. July 5 - 9, 2021 - Sorbonne Université, Paris. " Integration by parts and convergence in distribution norms in the CLT"
- B. Jourdain:
  - Bachelier seminar, 22 October 2021 : Approximation of real martingale couplings in the weak adapted topology
  - Journées de probabilités 2021, Guidel, 21-25 June 2021 : Approximation of real martingale couplings in the weak adapted topology
  - SIAM Conference on Financial Mathematics and Engineering, 3 June 2021 : Weak and strong error analysis for the mean-field Atlas model
  - Conference of numerical probability in honor of Gilles PagËs' 60th birthday, 28 May 2021 : quantization and preservation of the convex order
  - Seminar Probability and financial mathematics LaMME, 29 april 2021 : Approximation of real martingale couplings in the weak adapted topology

  - Mathrisk/LPSM seminar, 15 april 2021 : Approximation of real martingale couplings in the weak adapted topology

- D. Lamberton
  - On the American put in the Heston model, Probability Seminar, Manchester University, May 2021 (online),
  - “One World Optimal Stopping and Related Topics” seminars, September 2021 (online)

- A. Sulem
  - Séminaire Bachelier, IHP Paris, January 2021 (online) seminaire
  - World seminar of the Bachelier Finance Society and SIAM Activity Group on Financial Mathematics and Engineering, January 2021 , online
  - **Plenary speaker** : EPCO 2021 – “Portuguese Meeting on Optimal Control”, June 2021, Lisbon, (online), conference
  - AMaMef conference, Padova, June 2021 (online) conference
  - "Stochastic-Tage", Mannhneim, October 2021 (online) conference
  - Control and Optimisation Seminars, Mathematics Department, Imperial College, London, October 2021 (online) conference
  - Seminar on stochastic methods and Finance ENPC - INRIA - UPEMLV seminar ENPC December 2021

11.1.4 Research administration

- A. Alfonsi
  - Director of CERMICS laboratory until August,
  - Deputy director of CERMICS since September.
  - In charge of the Master “Finance and Data” at the Ecole des Ponts.

- V. Bally
  - Member of the LAMA committee
  - Responsible of the Master 2, option finance.

- A. Sulem
  - Member of the Nominating Committee of the Bachelier Finance Society FBS
  - Member of the Scientific Committee of AMIES
  - Corresponding member of the Operational Committee for the assesment of Legal and Ethical risks (COERLE) at INRIA Paris research center
  - Member of the Committee for INRIA international Chairs

11.2 Teaching - Supervision - Juries

11.2.1 Teaching

- Licence

  - A. Alfonsi: "Probability theory", first year course at Ecole des Ponts.
  - V. Bally : "Analyse Hilbertienne", Course L3, UPEMLV
  - B. Jourdain : course "Mathematical tools for engineers", 1st year ENPC.
  - B. Jourdain course "Mathematical finance", 2nd year ENPC
  - D. Lamberton: "Intégration et probabilités", L3 course, Université Gustave Eiffel.

- Master
  - Aurélien Alfonsi

  – "Données Haute Fréquence en finance", Master program, UPEMLV.
  – "Mesures de risque", Master course of UPEMLV and Sorbonne Université.
  – Professeur chargé de cours at Ecole Polytechnique.

  - Vlad Bally

  – "Taux d'Intêret", M2 Finance. Université Gustave Eiffel
  – "Calcul de Malliavin et applications en finance", M2 Finance, UGE
  – "Analyse du risque" M2 Actuariat, UGE
  – "Processus Stochastiques" M2 Recherche, UGE

  - Benjamin Jourdain

  – course "Monte-Carlo Markov chain methods and particle algorithms", Research Master Probabilités et Modèles Aléatoires, Sorbonne Université

  - B. Jourdain, B. Lapeyre

  – course "Monte-Carlo methods", 3rd year ENPC and Research Master Mathématiques et Application, University Gustave Eiffel

  - J.-F. Delmas, B.Jourdain

  – course "Jump processes with applications to energy markets", 3rd year ENPC and Research Master Mathématiques et Application, University Gustave Eiffel

  - D. Lamberton

  – "Calcul stochastique pour la finance", master 1 course, Université Gustave Eiffel
  – "Arbitrage, volatilité et gestion de portefeuille", master 2 course, Université Gustave Eiffel

  - B. Lapeyre

  – Monte-Carlo methods in quantitative finance, Master of Mathematics, University of Luxembourg,

  - A. Sulem

  – "PDE methods in Finance", Master of Mathematics, University of Luxembourg, 22 h lectures and responsible of the module "Numerical Methods in Finance".

11.2.2 Supervision

- PhD defended
  - Rafaël Coyaud, “Study of approximations of optimal transport problems and application to physics”, Supervisor: Aurélien Alfonsi and Virgine Ehrlacher, defended on 25 January 2021, ENPC. Presently: Quant Qube (Hedge fund)
  - Ezechiel Kahn, "Functional inequalities for random matrices models", supervisors: B. Jourdain and D. Chafai, defended on 5 July 2021, ENPC Presently: Adjoint Négociations multilatérales OCDE, UE et relations bilatérales» au bureau « Crédits-export et garanties à l’international » de la Direction générale du Trésor du ministère de l’économie et des finances et de la relance.
  - Adrien Touboul, "Model of margin, margin sensitivity analysis and uncertainty quantification in graphs of functions in complex industrial systems", defended on 21 July, ENPC, Supervisor: Bernard Lapeyre Presently: Senior Machine Learning Engineer at Gense Technologies Limited, Hong Kong.
  - Sophian Mehalla, "Taux d’intérêt pour l’assurance : approximations et calibrages de modèles", CIFRE agreement Milliman company/Ecole des Ponts (, Supervisor: B. Lapeyre, defended on 18 October 2021, ENPC. Presently: Consultant R&D Milliman (Assurance)
  - Adel Cherchali, “Numerical methods for the ALM”, (Méthodes numériques pour la gestion actif/passif) funded by Fondation AXA, defended on 18 January 2021, ENPC. Supervisor: Aurélien Alfonsi, Presently: Quantitative Researcher, Milliman
- PhD in progress :
  - Anas Bentaleb : Mathematical techniques for expected exposure evaluation, Supervisor: B. Lapeyre, started in February 2018
  - Zhonguyan Cao, "Dynamics and Stability of Complex Financial networks", Université Paris-Dauphine, Supervisor: Agnès Sulem, Dim Mathinnov doctoral allocation, started October 2020
  - Hachem Madmoun "Gestion de portefeuilles utilisant des techniques de (big) data", supervisor: B. Lapeyre, started in September 2018
  - Edoardo Lombardo (International PhD student, co-advisor: Lucia Caramellino), “High order numerical approximation for some singular stochastic processes and related PDEs”, started in November 2020.
  - Nerea Vadillo Fernandez (CIFRE AXA Climate), “Risk valuation for weather derivatives in index-based insurance”, started in November 2020. Supervisor: A. Alfonsi
  - Yfen Qin, "Regularity properties for non linear problems", Dim-Mathinnov doctoral allocation, supervisor: V. Bally, , started in October 2020.
  - Kexin Shao, INRIA MathinParis Paris-Dauphine, "Martingale optimal transport and financial applications", Started October 2021 Supervisors: B. Jourdain and A. Sulem
  - Hervé Andrès (Cifre ENPC/Milliman) "Dependence modelling in economic scenario generation for insurance", Supervisor: B. Jourdain, started in June 2021
  - Roberta Flenghi, ENPC, "Central limit theorems for nonlinear functionals of the empirical measure of correlated random variables", Supervisor: B. Jourdain, started in January 2021
- Internships
  - Youssef Ben Slimane (May- July), INRIA; Internship ENSTA , adviser: A. Kebaier "Artificial neural network for option pricing with and without asymptotic correction". H. Funahashi, Quantitative Finance, 2020, "Deep learning for limit order books". J.A.Sirignano Quantitative Finance, 19-4, 2019.

- Mourad Jemal (May - July), NRIA; Internship ENSTA , adviser: A. Kebaier, Deep learning techniques in option pricin
- Charles Meynard (May-August), adviser: Ludovic Goudenège.
  "Numerical methods in finance and insurance"

11.2.3 Juries

- Aurélien Alfonsi
  - Reviewer of the PhD thesis of Houzhi Li “Etude de méthodes numériques pour certaines équations différentielles stochastiques en finance et modélisation de la distribution du capital dans le marché financier”.
  - Reviewer of the PhD thesis of Enzo Miller “Contrôle linéaire-quadratique stochastique non markovien: dynamique de Volterra, volatilité rugueuse et équations avec retard”,25/05/2021
  - Reviewer of the PhD thesis of Elizabeth Zuniga “Processus de Volterra et applications en finance”.
  - Reviewer and president of the jury of the PhD thesis of Guillaume Szulda “Processus de branchement et modélisation des structures à terme multiples”.
  - Member of the recruiting committee for an Assistant Professor position at Paris Diderot University.
- Benjamin Jourdain
  - Member of the jury for the PhD of Thi Bao Tram Ngo, defended on July 9, UniversitÈ Paris Nord,
  - Reviewer of the PhD of Arthur Macherey, defended on June 28, Ecole Centrale de Nantes,
  - Member of committee for recrutment of a Professor in financial mathematics and probability, Université d’Evry
- Agnès Sulem
  - Member of committee for recrutment of a Professor in financial mathematics and probability, Université d’Evry
  - Member of committee for the attribution of Prix INRIA -Académie des Sciences
  - Member of the jury for the PhD thesis of Enzo Miller “Contrôle linéaire-quadratique stochastique non markovien: dynamique de Volterra, volatilité rugueuse et équations avec retard”,25/05/2021
  - Member of the jury for the PhD thesis of Linda Chamakh, *Quantification des incertitudes en gestion d’actifs: méthodes à noyaux et fluctuations statistiques*, Ecole Polytechnique, 31 August 2021
  - Member and president of the jury of the PhD thesis of Maxime Grangereau, *Contrôle optimal de flexibilités énergétiques en contexte incertain*, 23 March 2021, Ecole Polytechnique
  - Member and president of the jury of the PhD thesis ofHouzi Li, *Etude de méthodes numériques pour certaines équations différentielles stochastiques en finance et modélisation de la distribution du capital dans le marché financier*, 30 March 2021, Université de Paris,

12 Scientific production

12.1 Major publications

[1] A. Al Gerbi, B. Jourdain and E. Clément. 'Ninomiya-Victoir scheme: strong convergence, antithetic version and application to multilevel estimators'. In: *Monte Carlo Method and Applications* 22.3 (July 2016). https://arxiv.org/abs/1508.06492, pp. 197–228. URL: https://hal-enpc.archives-ouvertes.fr/hal-01188675.

[2] A. Alfonsi. *Affine Diffusions and Related Processes: Simulation, Theory and Applications.* 2015. DOI: 10.1007/978-3-319-05221-2. URL: https://hal-enpc.archives-ouvertes.fr/hal-03127212.

[3] A. Alfonsi and P. Blanc. 'Dynamic optimal execution in a mixed-market-impact Hawkes price model'. In: *Finance and Stochastics* (Jan. 2016). https://arxiv.org/abs/1404.0648. DOI: 10.1007/s00780-015-0282-y. URL: https://hal-enpc.archives-ouvertes.fr/hal-00971369.

[4] A. Alfonsi, B. Jourdain and A. Kohatsu-Higa. 'Optimal transport bounds between the time-marginals of a multidimensional diffusion and its Euler scheme'. In: *Electronic Journal of Probability* (2015). https://arxiv.org/abs/1405.7007. URL: https://hal-enpc.archives-ouvertes.fr/hal-00997301.

[5] H. Amini, A. Minca and A. Sulem. 'Control of interbank contagion under partial information'. In: *SIAM Journal on Financial Mathematics* 6.1 (Dec. 2015), p. 24. URL: https://hal.inria.fr/hal-01027540.

[6] V. Bally and L. Caramellino. 'Convergence and regularity of probability laws by using an interpolation method'. In: *Annals of Probability* 45.2 (2017), pp. 1110–1159. URL: https://hal-upec-upem.archives-ouvertes.fr/hal-01109276.

[7] A. Bouselmi and D. Lamberton. 'The critical price of the American put near maturity in the jump diffusion model'. In: *SIAM Journal on Financial Mathematics* 7.1 (May 2016). https://arxiv.org/abs/1406.6615, pp. 236–272. DOI: 10.1137/140965910. URL: https://hal-upec-upem.archives-ouvertes.fr/hal-00979936.

[8] R. Dumitrescu, M.-C. Quenez and A. Sulem. 'A Weak Dynamic Programming Principle for Combined Optimal Stopping/Stochastic Control with $\mathcal{E}^f$-Expectations'. In: *SIAM Journal on Control and Optimization* 54.4 (2016), pp. 2090–2115. DOI: 10.1137/15M1027012. URL: https://hal.inria.fr/hal-01370425.

[9] R. Dumitrescu, M.-C. Quenez and A. Sulem. 'Game Options in an Imperfect Market with Default'. In: *SIAM Journal on Financial Mathematics* 8.1 (Jan. 2017), pp. 532–559. DOI: 10.1137/16M1109102. URL: https://hal.inria.fr/hal-01614758.

[10] B. Jourdain. *Probabilités et statistique.* seconde édition. Ellipses, 2016. URL: https://hal.archives-ouvertes.fr/hal-03133840.

[11] B. Øksendal and A. Sulem. *Applied Stochastic Control of Jump Diffusions.* 3rd edition. Springer, Universitext, 2019, p. 436. DOI: 10.1007/978-3-030-02781-0. URL: https://hal.archives-ouvertes.fr/hal-02411121.

12.2 Publications of the year

International journals

[12] A. Alfonsi and V. Bally. 'A generic construction for high order approximation schemes of semigroups using random grids'. In: *Numerische Mathematik* (2021). DOI: 10.1007/s00211-021-01219-2. URL: https://hal-enpc.archives-ouvertes.fr/hal-02406433.

[13] A. Alfonsi, A. Cherchali and J. A. Infante Acevedo. 'Multilevel Monte-Carlo for computing the SCR with the standard formula and other stress tests'. In: *Insurance: Mathematics and Economics* (2021). DOI: 10.1016/j.insmatheco.2021.05.005. URL: https://hal.archives-ouvertes.fr/hal-03026795.

[14] A. Alfonsi, R. Coyaud and V. Ehrlacher. 'Constrained overdamped Langevin dynamics for symmetric multimarginal optimal transportation'. In: *Mathematical Models and Methods in Applied Sciences* (2021). URL: https://hal.archives-ouvertes.fr/hal-03131763.

[15] H. Amini, A. Minca and A. Sulem. 'A dynamic contagion risk model with recovery features'. In: *Mathematics of Operations Research* (24th Nov. 2021). DOI: 10.1287/moor.2021.1174. URL: https://hal.inria.fr/hal-02421342.

[16] V. Bally and L. Caramellino. 'Transfer of regularity for Markov semigroups'. In: *Journal of Stochastic Analysis* 2.3 (2021), Article 13. URL: https://hal-upec-upem.archives-ouvertes.fr/hal-02429530.

[17] O. Bencheikh and B. Jourdain. 'Approximation rate in Wasserstein distance of probability measures on the real line by deterministic empirical measures'. In: *Journal of Approximation Theory* 274.105684 (2022). DOI: 10.1016/j.jat.2021.105684. URL: https://hal.inria.fr/hal-03081116.

[18] O. Bencheikh and B. Jourdain. 'Weak and strong error analysis for mean-field rank based particle approximations of one dimensional viscous scalar conservation law'. In: *Annals of Applied Probability* (2022). URL: https://hal.archives-ouvertes.fr/hal-02332760.

[19] L. Goudenège, A. Molent and A. Zanette. 'Gaussian process regression for pricing variable annuities with stochastic volatility and interest rate'. In: *Decisions in Economics and Finance* 44.1 (1st May 2021), p. 26. DOI: 10.1007/s10203-020-00287-7. URL: https://hal.archives-ouvertes.fr/hal-03013603.

[20] M. Grigorova, M.-C. Quenez and A. Sulem. 'American options in a non-linear incomplete market model with default'. In: *Stochastic Processes and their Applications* 142 (2021). DOI: 10.1016/j.spa.2021.09.004. URL: https://hal.archives-ouvertes.fr/hal-02025835.

[21] B. Jourdain and E. Kahn. 'Strong solutions to a beta-Wishart particle system'. In: *Journal of Theoretical Probability* (2022). DOI: 10.1007/s10959-021-01109-1. URL: https://hal.archives-ouvertes.fr/hal-02512855.

[22] B. Jourdain, T. Lelièvre and P.-A. Zitt. 'Convergence of metadynamics: discussion of the adiabatic hypothesis'. In: *Annals of Applied Probability* 31.5 (Oct. 2021), pp. 2441–2477. DOI: 10.1214/20-AAP1652. URL: https://hal.archives-ouvertes.fr/hal-02104961.

[23] B. Jourdain and W. Margheriti. 'Martingale Wasserstein inequality for probability measures in the convex order'. In: *Bernoulli* (2022). URL: https://hal.archives-ouvertes.fr/hal-03021483.

[24] B. Jourdain and G. Pagès. 'Convex order, quantization and monotone approximations of ARCH models'. In: *Journal of Theoretical Probability* (2022). DOI: 10.1007/s10959-021-01141-1. URL: https://hal.archives-ouvertes.fr/hal-02304190.

[25] B. Jourdain and G. Pagès. 'Optimal dual quantizers of 1D log-concave distributions: uniqueness and Lloyd like algorithm'. In: *Journal of Approximation Theory* 267.105581 (2021). URL: https://hal.archives-ouvertes.fr/hal-02975674.

[26] B. Jourdain and G. Pagès. 'Quantization and martingale couplings'. In: *ALEA : Latin American Journal of Probability and Mathematical Statistics* 19 (2022). URL: https://hal.archives-ouvertes.fr/hal-03083022.

[27] B. Jourdain and A. Tse. 'Central limit theorem over non-linear functionals of empirical measures with applications to the mean-field fluctuation of interacting particle systems'. In: *Electronic Journal of Probability* 26.154 (2021). URL: https://hal.archives-ouvertes.fr/hal-02467706.

[28] B. Lapeyre and J. Lelong. 'Neural network regression for Bermudan option pricing'. In: *Monte Carlo Methods and Applications* 27.3 (Sept. 2021), pp. 227–247. DOI: 10.1515/mcma-2021-2091. URL: https://hal.univ-grenoble-alpes.fr/hal-02183587.

Scientific book chapters

[29] L. Goudenège, A. Molent and A. Zanette. 'Variance Reduction Applied to Machine Learning for Pricing Bermudan/American Options in High Dimension'. In: *Applications of Lévy Processes.* Nova Science Publishers, 27th Aug. 2021. URL: https://hal.inria.fr/hal-03524108.

Doctoral dissertations and habilitation theses

[30] A. Cherchali. 'Numerical methods for the ALM'. Université Paris-Est, 18th Jan. 2021. URL: https://pastel.archives-ouvertes.fr/tel-03606038.

[31] R. Coyaud. 'Study of approximations of optimal transport problems and application to physics'. Université Paris-Est, 25th Jan. 2021. URL: https://pastel.archives-ouvertes.fr/tel-03529782.

[32] E. Kahn. 'Covariance matrices : diffusions, free probability, and deep learning'. Ecole des Ponts ParisTech, 5th July 2021. URL: https://hal.archives-ouvertes.fr/tel-03531875.

[33] S. Mehalla. 'Interest rates for insurance : models calibrations and approximations.' École des Ponts ParisTech, 18th Oct. 2021. URL: https://pastel.archives-ouvertes.fr/tel-03541696.

[34] A. Touboul. 'Model of margin, margin sensitivity analysis and uncertainty quantification in graphs of functions in complex industrial systems'. École des Ponts ParisTech, 21st July 2021. URL: https://pastel.archives-ouvertes.fr/tel-03435011.

Reports & preprints

[35] A. Alfonsi and V. Bally. *Construction of Boltzmann and McKean Vlasov type flows (the sewing lemma approach).* 28th May 2021. URL: https://hal-enpc.archives-ouvertes.fr/hal-03241604.

[36] A. Alfonsi and A. Kebaier. *Approximation of Stochastic Volterra Equations with kernels of completely monotone type.* 14th Jan. 2022. URL: https://hal-enpc.archives-ouvertes.fr/hal-03526905.

[37] H. Amini, Z. Cao and A. Sulem. *Limit Theorems for Default Contagion and Systemic Risk.* 15th Nov. 2021. URL: https://hal.inria.fr/hal-03429191.

[38] H. Amini, Z. Cao and A. Sulem. *Fire Sales, Default Cascades and Complex Financial Networks.* 16th Nov. 2021. DOI: 10.2139/ssrn.3935450. URL: https://hal.inria.fr/hal-03425599.

[39] V. Bally and Y. Qin. *Total variation distance between a jump-equation and its Gaussian approximation.* 22nd Sept. 2021. URL: https://hal.archives-ouvertes.fr/hal-03351643.

[40] M. Beiglböck, B. Jourdain, W. Margheriti and G. Pammer. *Approximation of martingale couplings on the line in the weak adapted topology.* 8th Jan. 2021. URL: https://hal.archives-ouvertes.fr/hal-03103430.

[41] M. Beiglböck, B. Jourdain, W. Margheriti and G. Pammer. *Stability of the Weak Martingale Optimal Transport Problem.* 15th Sept. 2021. URL: https://hal.archives-ouvertes.fr/hal-03344429.

[42] B. Jourdain and W. Margheriti. *One dimensional martingale rearrangement couplings.* 1st Feb. 2021. URL: https://hal.archives-ouvertes.fr/hal-03126853.

[43] B. Jourdain and S. Menozzi. *Convergence Rate of the Euler-Maruyama Scheme Applied to Diffusion Processes with* $L Q - L \rho$ *Drift Coefficient and Additive Noise.* 10th May 2021. URL: https://hal.archives-ouvertes.fr/hal-03223426.

12.3 Cited publications

[44] A. Ahdida, A. Alfonsi and E. Palidda. 'Smile with the Gaussian term structure model'. In: *Journal of Computational Finance* (2017). https://arxiv.org/abs/1412.7412. URL: https://hal.archives-ouvertes.fr/hal-01098554.

[45] A. Al Gerbi, B. Jourdain and E. Clément. 'Asymptotics for the normalized error of the Ninomiya-Victoir scheme'. In: *Stochastic Processes and their Applications* 128.6 (2018), pp. 1889–1928. URL: https://hal-enpc.archives-ouvertes.fr/hal-01259915.

[46] A. Al Gerbi, B. Jourdain and E. Clément. 'Ninomiya-Victoir scheme : Multilevel Monte-Carlo estimators and discretization of the involved Ordinary Differential Equations'. In: *ESAIM: Proceedings and Surveys*. Thematic cycle on Monte-Carlo techniques 59 (Nov. 2017). https://arxiv.org/abs/1612.07017, pp. 1–14. URL: https://hal.archives-ouvertes.fr/hal-01421337.

[47] A. Alfonsi and P. Blanc. 'Extension and calibration of a Hawkes-based optimal execution model'. In: *Market microstructure and liquidity* (Aug. 2016). https://arxiv.org/abs/1506.08740. DOI: 10.1142/S2382626616500052. URL: https://hal-enpc.archives-ouvertes.fr/hal-01169686.

[48] A. Alfonsi, A. Cherchali and J. A. I. Acevedo. 'A full and synthetic model for Asset-Liability Management in life insurance, and analysis of the SCR with the standard formula'. In: *European Actuarial Journal* (2020). DOI: 10.1007/s13385-020-00240-3. URL: https://hal-enpc.archives-ouvertes.fr/hal-02406439.

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[50] A. Alfonsi, J. Corbetta and B. Jourdain. 'Sampling of one-dimensional probability measures in the convex order and computation of robust option price bounds'. In: *International Journal of Theoretical and Applied Finance* 22.3 (2019). This paper is an updated version of a part of the paper https://hal.archives-ouvertes.fr/hal-01589581 (or https://arxiv.org/pdf/1709.05287.pdf). DOI: 10.1142/S021902491950002X. URL: https://hal-enpc.archives-ouvertes.fr/hal-01963507.

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[52] A. Alfonsi, R. Coyaud, V. Ehrlacher and D. Lombardi. 'Approximation of Optimal Transport problems with marginal moments constraints'. In: *Mathematics of Computation* (2020). DOI: 10.1090/mcom/3568. URL: https://hal.archives-ouvertes.fr/hal-02128374.

[53] A. Alfonsi, M. Hayashi and A. Kohatsu-Higa. 'Parametrix Methods for One-Dimensional Reflected SDEs'. In: *Modern Problems of Stochastic Analysis and StatisticsSelected Contributions In Honor of Valentin Konakov*. Vol. Springer Proceedings in Mathematics & Statistics. 208. Springer, Nov. 2017. URL: https://hal-enpc.archives-ouvertes.fr/hal-01670011.

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