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ACTIVITY REPORT

Project-Team

MCTAO

Mathematics for Control, Transport and Applications

IN COLLABORATION WITH: Laboratoire Jean-Alexandre Dieudonné (JAD)

DOMAIN

Applied Mathematics, Computation and Simulation

THEME

Optimization and control of dynamic systems

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Project-Team MCTAO

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Computer sciences and digital sciences

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1 Team members, visitors, external collaborators

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2 Overall objectives

2.1 Control, Transport and Dynamics

Our goal is to develop methods in geometric control theory for finite-dimensional nonlinear systems, and to transfer our expertise through real applications of these techniques.

Our primary domain of industrial applications in the past years has been space engineering, namely using optimal control and stabilization techniques for mission design with low thrust propulsion: orbit transfer or rendez-vous problems in the gravity field of a single body (typically sattelites around the earth), interplanetary missions and multi body problems, or control design of solar sails, where propulsion is drastically constrained. The team also has continued involvement with applications regarding control of micro-swimmers, i.e. swimming robots where the fluid-structure coupling has a very low Reynolds number; and quantum control, with applications to Nuclear Magnetic Resonance and medical image processing. Recent focus has been put into transfer to other domains that can benefit from a control theory point of view, such as biology, with problems of optimal control of microbial cells, or muscular electro-stimulation.

Finally, part of the team's core goals is transfer of its expertise to other mathematical fields, where problems in dynamical systems rely on a control theory approach.

3 Research program

3.1 Control Problems

McTAO's major field of expertise is control theory in the broad sense. Let us give an overview of this field.

Modelling. Our effort is directed toward efficient methods for the control of real (physical) *systems*, based on a *model* of the system to be controlled. Choosing accurate models yet simple enough to allow control design is in itself a key issue. The typical continuous-time model is of the form $dx/dt = f(x,u)$ where x is the *state*, ideally finite dimensional, and u the *control*; the control is left free to be a function of time, or a function of the state, or obtained as the solution of another dynamical system that takes x as an input. Modelling amounts to deciding the nature and dimension of x, as well as the dynamics (roughly speaking the function f). Connected to modeling is identification of parameters when a finite number of parameters are left free in "f".

Controllability, path planning. Controllability is a property of a control system (in fact of a model) that two states in the state space can be connected by a trajectory generated by some control, here taken as an explicit function of time. Deciding on local or global controllability is still a difficult open question in general. In most cases, controllability can be decided by linear approximation, or non-controllability by "physical" first integrals that the control does not affect. For some critically actuated systems, it is still difficult to decide local or global controllability, and the general problem is anyway still open. Path planning is the problem of constructing the control that actually steers one state to another.

Optimal control. In optimal control, one wants to find, among the controls that satisfy some contraints at initial and final time (for instance given initial and final state as in path planning), the ones that minimize some criterion. This is important in many control engineering problems, because minimizing a cost is often very relevant. Mathematically speaking, optimal control is the modern branch of the calculus of variations, rather well established and mature [74, 47, 35], but with a lot of hard open questions. In the end, in order to actually compute these controls, ad-hoc numerical schemes have to be derived for effective computations of the optimal solutions. See more about our research program in optimal control in section 3.2.

Feedback control. In the above two paragraphs, the control is an explicit function of time. To address in particular the stability issues (sensitivity to errors in the model or the initial conditions for example), the control has to be taken as a function of the (measured) state, or part of it. This is known as closed-loop control; it must be combined with optimal control in many real problems. On the problem of stabilization, there is longstanding research record from members of the team, in particular on the construction of "Control Lyapunov Functions", see [64, 76]. It may happen that only part of the state is accessible at any one time, because of physical or engineering constraints. In that case, a popular strategy is to pair feedback methods with dynamic estimation of the state, creating so-called output feedback loops. Simultaneous feedback control and estimation can become a major hurdle for nonlinear systems, see [56, 79].

Classification of control systems. One may perform various classes of transformations acting on systems, or rather on models. The simpler ones come from point-to-point transformations (changes of variables) on the state and control. More intricate ones consist in embedding an extraneous dynamical system into the model. These are dynamic feedback transformations that change the dimension of the state. In most problems, choosing the proper coordinates, or the right quantities that describe a phenomenon, sheds light on a path to the solution; these proper choices may sometimes be found from an understanding of the modelled phenomenons, or it can come from the study of the geometry of the equations and the transformation acting on them. This justifies the investigations of these transformations on models for themselves. These topics are central in control theory; they are present in the team, see for instance the classification aspect in [52] or —although this research has not been active very recently— the study [73] of dynamic feedback and the so-called "flatness" property [67].

3.2 Optimal Control and its Geometry

Let us detail our research program concerning optimal control. Relying on Hamiltonian dynamics is now prevalent, instead of the Lagrangian formalism in classical calculus of variations. The two points of view run parallel when computing geodesics and shortest path in Riemannian Geometry for instance, in that there is a clear one-to-one correspondance between the solutions of the geodesic equation in the tangent bundle and the solution of the Pontryagin Maximum Principle in the cotangent bundle. In most optimal control problems, on the contrary, due to the differential constraints (velocities of feasible trajectories do not cover all directions in the state space), the Lagrangian formalism becomes more involved, while the Pontryagin Maximum Principle keeps the same form, its solutions still live in the cotangent bundle, their projections are the extremals, and a minimizing curve must be the projection of such a solution.

Cut and conjugate loci. The cut locus —made of the points where the extremals lose optimality— is obviously crucial in optimal control, but usually out of reach (even in low dimensions), and anyway does not have an analytic characterization because it is a non-local object. Fortunately, conjugate points —where the extremals lose *local* optimality— can be effectively computed with high accuracy for many control systems. Elaborating on the seminal work of the Russian and French schools (see [78, 34, 36] and [53] among others), efficient algorithms were designed to treat the smooth case. This was the starting point of a series of papers of members of the team culminating in the outcome of the *cotcot* software [46], followed by the *Hampath* [57] code. Over the years, these codes have allowed for the computation of conjugate loci in a wealth of situations including applications to space mechanics, quantum control, and more recently swimming at low Reynolds number. With in mind the two-dimensional analytic Riemannian framework, a heuristic approach to the global issue of determining cut points is to search for singularities of the conjugate loci; this line is however very delicate to follow on problems stemming from applications in three or more dimensions (see *e.g.* [58] and [43]). In all these situations, the fundamental object underlying the analysis is the curvature tensor. In Hamiltonian terms, one considers the dynamics of subspaces (spanned by Jacobi fields) in the Lagrangian Grassmannian [33]. This point of view withstands generalizations far beyond the smooth case: In L^1-minimization, for instance, discontinuous curves in the Grassmannian have to be considered (instantaneous rotations of Lagrangian subspaces still obeying symplectic rules [63]). The cut locus is a central object in Riemannian geometry, control and optimal transport. This is the motivation for a series of conferences on "The cut locus: A bridge over differential geometry, optimal control, and transport", co-organized by team members and Japanese colleagues.

Riemann and Finsler geometry. Studying the distance and minimising geodesics in Riemannian Geometry or Finsler Geometry is a particular case of optimal control, simpler because there are no differential constraints; it is studied in the team for the following two reasons. On the one hand, after some tranformations, like averaging or reduction, some more difficult optimal control problems lead to a Riemann or Finsler geometry problem. On the other hand, optimal control, mostly the Hamiltonian setting, brings a fresh viewpoint on problems in Riemann and Finsler geometry. On Riemannian ellipsoids of revolution, the optimal control approach allowed to decide on the convexity of the injectivity domain, which, associated with non-negativity of the Ma-Trudinger-Wang curvature tensor, ensures continuity of the optimal transport on the ambient Riemannian manifold [65, 66]. The analysis in the oblate geometry [44] was completed in [62] in the prolate one, including a preliminary analysis of non-focal domains associated with conjugate loci. Averaging in systems coming from space mechanics control with L^2-minimization yields a Riemannian metric, thoroughly computed in [42] together with its geodesic flow; in reduced dimension, its conjugate and cut loci were computed in [45] with Japanese Riemannian geometers. Averaging the same systems for minimum time yields a Finsler Metric, as noted in [41]. In [51], the geodesic convexity properties of these two types of metrics were compared. When perturbations (other than the control) are considered, they introduce a “drift”, *i.e.* the Finsler metric is no longer symmetric.

Sub-Riemannian Geometry. Optimal control problems that pertain to sub-Riemannian Geometry bear all the difficulties of optimal control, like the role of singular/abnormal trajectories, while having some useful structure. They lead to many open problems, see the monograph [72] for an introduction. The sub-Riemannian problem can be encoded by a non-linear control system with no drift, subjected to a quadratic energy minimization objective. This allows the sub-Riemannian problem to serve as rich model spaces for optimal control. The interest of sub-Riemannian geometry can go beyond these aspects however. It was proved by Hormander in 1967 [71] that local controllability of the system (given in terms of Lie-brackets of vector fields) is equivalent to sub-ellipticity of a second order differential operator associated with the vector fields. In this way, sub-Riemannian geometry acts as a bridge between elements of analysis of PDEs and geometric control theory. For instance, many recent works focus on framing properties of sub-elliptic operators in terms of minimizers of the optimal control problem (such as the influence of cut and conjugate points on diffusion asymptotics [39]). This link even allowed to successfully introduce concepts of sub-elliptic diffusions in computer vision algorithms thanks to sub-Riemannian geometric structures identified in mammal visual mechanisms [55].

Small controls and conservative systems, averaging. Using averaging techniques to study small perturbations of integrable Hamiltonian systems is old an idea as celestial mechanics. It is very subtle in the case of multiple periods but more elementary in the single period case, here it boils down to taking the average of the perturbation along each periodic orbit [37, 77]. This line of research stemmed out of applications to space engineering (see Section 4.1): the control of the super-integrable Keplerian motion of a spacecraft orbiting around the Earth is an example of a slow-fast controlled system. Since weak propulsion is used, the control itself acts as a perturbation, among other perturbations of similar magnitudes: higher order terms of the Earth potential (including J_2 effect, first), potential of more distant celestial bodies (such as the Sun and the Moon), atmospheric drag, or even radiation pressure. Properly qualifying the convergence properties (when the small parameter goes to zero) is important and is made difficult by the presence of control. In [41], convergence is seen as convergence to a differential inclusion; this applies to minimum time; a contribution of this work is to put forward the metric character of the averaged system by yielding a Finsler metric (see Section 3.2). Proving convergence of the extremals (solutions of the Pontryagin Maximum Principle) is more intricate. In [61], standard averaging ([37, 77]) is performed on the minimum time extremal flow after carefully identifying slow variables of the system thanks to a symplectic reduction. This alternative approach allows to retrieve the previous metric approximation, and to partly address the question of convergence. Under suitable assumptions on a given geodesic of the averaged system (disconjugacy conditions, namely), one proves existence of a family of quasi-extremals for the original system that converge towards the geodesic when the small perturbation parameter goes to zero. This needs to be improved, but convergence of all extremals to extremals of an “averaged Pontryagin Maximum Principle” certainly fails. In particular, one cannot hope for C^1-regularity on the value function when the small parameter goes to zero as swallowtail-like singularities due to the structure of local minima in the problem are expected. (A preliminary analysis has been made in [59].)

Optimality of periodic solutions/periodic controls. When seeking to minimize a cost with the constraint that the controls and/or part of the states are periodic (and with other initial and final conditions), the notion of conjugate points is more difficult than with straightforward fixed initial point. In [48], for the problem of optimizing the efficiency of the displacement of some micro-swimmers (see Section 4.5) with periodic deformations, we used the sufficient optimality conditions established by R. Vinter's group [83, 68] for systems with non unique minimizers due to the existence of a group of symmetry (always present with a periodic minimizer-candidate control). This takes place in a long term collaboration with P. Bettiol (Univ. Bretagne Ouest) on second order sufficient optimality conditions for periodic solutions, or in the presence of higher dimensional symmetry groups, following [83, 68]. Another question relevant to locomotion is the following. Observing animals (or humans), or numerically solving the optimal control problem associated with driftless micro-swimmers for various initial and final conditions, we remark that the optimal strategies of deformation seem to be periodic, at least asymptotically for large distances. This observation is the starting point for characterizing dynamics for which some optimal solutions are periodic, and asymptotically attract other solutions as the final time grows large; this is reminiscent of the "turnpike theorem" (classical, recently applied to nonlinear situations in [82]).

3.3 Software

Optimal control applications (but also the development of theory where numerical experiments can be very enlightening) require many algorithmic and numerical developments that are an important side of the team activity. We develop on-demand algorithms and pieces of software, for instance we have to interact with a production software developed by Thales Alenia Space. A strong asset of the team is the interplay of its expertise in geometric control theory with applications and algorithms, and the team has a long-lasting commitment to the development of numerical codes for the efficient resolution of optimal control problems. Methods for solving optimal control problems with ordinary differential equations more or less fall into three main categories. Dynamic Programming (or Hamilton Jacobi Bellman method) computes the global optimum but suffers from high computational costs, the so-called *curse of dimensionality*. Indirect methods based on Pontryagin Maximum Principle are extremely fast and accurate but often require more work to be applied, in terms of mathematical analysis and a priori knowledge of the solution; this kind of fine geometrical analysis is one of the strong know-how of McTAO. Direct transcription methods offer a good tradeoff between robustness and accuracy and are widely used for industrial applications. For challenging problems, an effective strategy is to start with a direct method to find a first rough solution, then refine it through an indirect method. Such a combined approach has been for instance used between McTAO, the former COMMANDS team (Inria Saclay), and CNRS team APO (Université Toulouse, CNRS, ENSEEIHT) for the optimization of contrast in medical imaging (MRI), and fuel-effective trajectories for airplanes. This combination of direct and indirect methods has a lot of interest to solve optimal control problems that contain state or control constraints. In the collaborations mentioned above, the interfacing between the two solvers BOCOP and HamPath were done manually by *ad hoc* `python` or `matlab` layers. In collaboration with COMMANDS and colleagues from ENSEEIHT, McTAO leads the ct: control toolbox project whose goal is to interoperate these solvers using a high level common interface. The project is an Inria Sophia ADT[1] (2019-) in AMDT[1] mode supported by Inria Sophia SED.

4 Application domains

4.1 Aerospace Engineering

Participants: Jean-Baptiste Caillau, Thierry Dargent, Lamberto Dell'Elce, Alesia Herasimenka, Jean-Baptiste Pomet.

Space engineering is very demanding in terms of safe and high-performance control laws. It is therefore prone to fruitful industrial collaborations. McTAO now has an established expertise in space and celestial mechanics. Our collaborations with industry are mostly on orbit transfer problems with low-thrust propulsion. It can be orbit transfer to put a commercial satellite on station, in which case the dynamics are a Newtonian force field plus perturbations and the small control. There is also, currently, a renewed interest in low-thrust missions such as Lisa Pathfinder (ESA mission towards a Lagrange point of the Sun-Earth system) or BepiColombo (joint ESA-JAXA mission towards Mercury). Such missions look more like a controlled multibody system. In all cases the problem involves long orbit transfers, typically with many revolutions around the primary celestial body. When minimizing time, averaging techniques provide a good approximation. Another important criterion in practice is fuel consumption minimization (crucial because only a finite amount of fuel is onboard a satellite for all its "life"), which amounts to L^1-minimization. Both topics are studied by the team. We have a steady relationship with CNES and

[1] ADT is the name of software development actions supported by the service "SED", by devoting some engineers to these projects. AMDT means that a group of engineers works on the project part time, rather than sending one engineer in the team.

Thales Alenia Space (Cannes), that have financed or co-financed 3 PhDs and 2 post-docs in the Sophia location of the team in the decade and are a source of inspiration even at the methodological level. Team members also have close connections with Airbus-Safran (Les Mureaux) on launchers.

Some of the authoritative papers in the field were written by team members, with an emphasis on the geometric analysis and on algorithms (coupling of shooting and continuation methods). There are also connections with peers more on the applied side, like D. Scheeres (Colorado Center for Astrodynamics Research at Boulder), the group of F. Bernelli (Politecnico Milano), and colleagues from U. Barcelona (A. Farrès, A. Jorba).

A new action has started in Sep. 2020 with the Phd thesis of Alesia Herasimenka (2020-2023) on the control of solar sails. Solar sailing has been actively studied for two decades and recent missions have demonstrated its interest for "zero-fuel" missions. A lot has to be done to understand even the basic properties of such systems, for instance regarding controllability. Depending on the model used for the sail, not all directions of control are available. Some preliminary studies obtain controllability results by analysing the flow around equilibria of the system, these equilibria being changed when the orientation of the sail is updated. We want to provide a comprehensive understanding of controllability thanks to a systematic use of geometric control theory (orbit theorem, Lie algebraic approach) combined with effective numerical computations to check local controllability properties. The PhD thesis has been selected by ESA for a three-year research co-sponsorship.

4.2 Optimal control of microbial cells

Participants: Jean-Baptiste Caillau, Walid Djema *(BIOCORE project-team)*, Laetitia Giraldi *(CALISTO project-team)*, Jean-Luc Gouzé *(BIOCORE project-team)*, Sofya Maslovskaya *(Paderborn University, Germany)*, Jean-Baptiste Pomet, Agustín Yabo.

The growth of microorganisms is fundamentally an optimization problem which consists in dynamically allocating resources to cellular functions so as to maximize growth rate or another fitness criterion. Simple ordinary differential equation models, called self-replicators, have been used to formulate this problem in the framework of optimal and feedback control theory, allowing observations in microbial physiology to be explained. The resulting control problems are very challenging due to the nonlinearity of the models, parameter uncertainty, the coexistence of different time-scales, a dynamically changing environment, and various other physical and chemical constraints. In the framework of the ANR Maximic (PI Hidde de Jong, Inria Grenoble Rhône-Alpes) we aim at developing novel theoretical approaches for addressing these challenges in order to (i) study natural resource allocation strategies in microorganisms and (ii) propose new synthetic control strategies for biotechnological applications. In order to address (i), we develop extended self-replicator models accounting for the cost of regulation and energy metabolism in bacterial cells. We study these models by a combination of analytical and numerical approaches to derive optimal control solutions and a control synthesis, dealing with the bang-bang-singular structure of the solutions. Moreover, we define quasi-optimal feedback control strategies inspired by known regulatory mechanisms in the cell. To test whether bacteria follow the predicted optimal strategies, we quantify dynamic resource allocation in the bacterium Escherichia coli by monitoring, by means of time-lapse fluorescent microscopy, the expression of selected genes in single cells growing in a microfluidics device. In order to address (ii), we build self-replicator models that include a pathway for the production of a metabolite of interest. We also add a mechanism to turn off microbial growth by means of an external input signal, at the profit of the production of the metabolite. We formulate the maximization of the amount of metabolite produced as an optimal control problem, and derive optimal solutions and a control synthesis, as well as quasi-optimal feedback strategies satisfying chemical and physical design constraints. The proposed synthetic control strategies are being tested experimentally by growing E. coli strains capable of producing glycerol from glucose in a mini-bioreactor system. We aim at quantifying the amount of glucose consumed and glycerol produced, in the case of a predefined input signal (open-loop control) and the adaptive regulation of the input signal based on on-line measurements of the growth rate and the expression of fluorescent reporters of selected genes (closed-loop control). Currently, one PhD (A. Yabo) and one postdoc (S. Maslovskaya) are involved in these tasks and jointly supervised by colleagues from McTAO and Biocore teams at Sophia. Preliminary results concern the definition on extended (higher dimensional) models for the bacteria dynamics, check of second order optimality conditions on the resulting optimal control problem, and study of the turnpike phenomenon for these optimization problems.

4.3 Stability of high frequency active circuits

Participants: Sébastien Fueyo *(CentraleSupélec, Gif-sur-Yvette)*, Jean-Baptiste Pomet, Laurent Baratchart *(FACTAS project-team)*.

Let us focus on amplifiers, as active circuits. Nonlinear hyper-frequency amplifiers are ubiquitous in cell phone relays and many other devices. They must be as compact as possible, yielding a more complicated design. Computer Assisted Design tools are extensively used; for a given amplifier design, they provide frequency responses but fail to provide information of the stability of the response for each frequency. This stability is crucial for an unstable response will not be observed in practice; the actual device should not be built before stability is asserted. Predicting stability/instability from "simulations" in the Computer Assisted Design tool is of utmost importance (simulation between quotation marks because these simulations are in fact computations in the frequency domain). Potential transfer to industry is important.

Some techniques do exist, see [80], based on creating some virtual perturbations and treating them as the input of a (linearized) control system to be "simulated" using the same tools. In an ongoing collaboration between McTAO and the project-team FACTAS, we work on the mathematical ground of these methods and in particular of the relation between stability and the property of the identified time-varying infinite dimensional systems. See recent developments in Section 6.5.

4.4 Magnetic resonance imaging (MRI)

Participants: Bernard Bonnard, Jérémy Rouot, Joseph Gergaud, Olivier Cots.

The starting point of our interest in optimal control for quantum systems was a collaboration with physicist from ICB, University of Burgundy (Dominique Sugny), motivated by an ANR project where we worked on the control of molecular orientation in a dissipative environment using a laser field, and developed optimal control tools, combined with numerical simulations, to analyze the problem for Qubits. This was related to quantum computing rather than MRI. Using this expertise and under the impulse of Prof. S. Glaser and his group (Chemistry, TU München), we investigated Nuclear Magnetic resonance (NMR) for medical imaging (MRI), where the model is the Bloch equation describing the evolution of the Magnetization vector controlled by a magnetic field, but in fine is a specific Qubit model without decoherence. We worked on, and brought strong contributions to, the contrast problem: typically, given two chemical substances that have an importance in medicine, like oxygenated and de-oxygenated blood, find the (time-dependent) magnetic field that will produce the highest difference in brightness between these two species on the image resulting from Nuclear Magnetic Resonance. This has immediate and important industrial applications in medical imaging. Our contacts are with the above mentioned physics academic labs, who are themselves in contact with major companies. The team has produced and is producing important work on this problem. One may find a good overview in [50], a reference book has been published on the topic [54], a very complete numerical study comparing different optimization techniques was performed in [49]. We conduct this project in parallel with S. Glaser team, which validated experimentally the pertinence of the methods, the main achievement being the in vivo experiments realized at the Creatis team of Insa Lyon showing the interest to use optimal control methods implemented in modern softwares in MRI in order to produce a better image in a shorter time. A goal is to arrive to a cartography of the optimal contrast with respect to the relaxation parameters using LMI techniques and numerical simulations with the Hamapth and Bocop code; note that the theoretical study is connected to the problem of understanding the behavior of the extremal solutions of a controlled pair of Bloch equations, and this is an ambitious task. Also, one of the difficulties to go from the obtained results, checkable on experiments, to practical control laws for production is to deal with magnetic field space inhomogeneities.

4.5 Swimming at low-Reynolds number

Participants: Bernard Bonnard, Laetitia Giraldi *(CALISTO project team)*, Clément Moreau *(Kyoto University, Japan)*, Jean-Baptiste Pomet, Jérémy Rouot.

Following the historical reference for low Reynolds number locomotion [75], the study of the swimming strategies of micro-organisms is attracting increasing attention in the recent literature. This is both because of the intrinsic biological interest, and for the possible implications these studies may have on the design of bio-inspired artificial replicas reproducing the functionalities of biological systems. In the case of micro-swimmers, the surrounding fluid is dominated by the viscosity effects of the water and becomes reversible. In this regime, it turns out that the infinite dimensional dynamics of the fluid do not have to be retained as state variables, so that the dynamics of a micro-swimmer can be expressed by ordinary differential equations if its shape has a finite number of degrees of freedom. Assuming this finite dimension, and if the control is the rate of deformation, one obtains a control system that is linear (affine without drift) with respect to the controls, *i.e.* the optimal control problem with a quadratic cost defines a sub-Riemannian structure (see section 3.2). This is the case where the shape is "fully actuated", *i.e.* if all the variables describing the shape are angles, there is an actuator on each of these angles. For artificial micro-swimmers, this is usually unrealistic. For example, (artificial) magneto-elastic micro-swimmers are deformed by an external magnetic field. In this case, the control functions are the external magnetic field. In both cases, questions are controllability (straightforward in the fully actuated case), optimal control, possibly path planning. We collaborate with teams that have physical experiments for both.

- In collaboration with D. Takagi and M. Chyba (University of Hawaii), this approach is currently at the experimental level for copepod-like swimmer at the university of Hawaii: on the one hand, this zooplankton and its locomotion can be observed, and a robot micro swimmer mimicking a copepod has been constructed. The robot is large enough for direct actuation to be possible, and the low Reynolds number is achieved by using a very viscous fluid. This gives possibilities, through an inverse optimization problem, to determine what cost can be optimised by these crustaceans, see [40, 81], and to validate models on the robot.
- For magneto-elastic micro-robots, Y. El-Alaoui's PhD is co-advised with Stéphane Régnier from the robotics lab ISIR, Sorbonne Université. Magneto-elastic micro-robots and their magnetic actuation are actually built at ISIR and the aim of the collaboration is to validate models and improve the existing control laws both in performance and in energy; of course, the micro scale does make things difficult.

The questions about optimality of periodic controls raised in Section 3.2 are related to these applications for periodic deformations, or strokes, playing important role in locomotion.

5 New software and platforms

With a strong support from Inria Sophia SED, we are currently working on `ct: control toolbox` InriaHub project. The objective of this ADT (started in 2019) is to pave the way towards a collaborative set of reference tools to solve optimal control problems numerically. On the basis of the two well established codes BOCOP and HamPath from the optimal control community, we have designed a high-level modular architecture to interoperate the solvers and now offer a high-level common user interface for the two codes. An immediate outcome of the ADT is to integrate state of the art processes into the development of the two solvers (collaborative dev tools, reliable repositories and continuous integration). Initially, the BOCOP project of COMMANDS team aimed at developing an open-source toolbox for solving optimal control problems, with collaborations involving industrial and academic partners. Optimal control (optimization of dynamical systems governed by differential equations) has numerous applications in the fields of transportation, energy, process optimization, and biology. BOCOP implements a direct transcription approach, where the continuous optimal control problem is transformed into a nonlinear program. The reformulation is done by a discretization of the time interval, with an approximation of the dynamics of the system by a generalized Runge Kutta scheme. On the other hand, HamPath is a an open-source package developed to solve optimal control problems by shooting (single or multiple) and pathfollowing methods for Hamiltonian dynamical systems. It was initially developed by members of the APO (Algorithmes Parallèles et Optimisation) team from Institut de Recherche en Informatique de Toulouse (ENSEEIHT / CNRS), jointly with colleagues from the Université de Bourgogne and Inria Sophia. Applying the maximum principle leads to define a set of Hamiltonians and a Boundary Value Problem, itself described by a set of nonlinear equations. HamPath compiles the Fortran codes of these (maximized) Hamiltonians and of the shooting function into a collection of `Matlab` / `Fortran` functions which allow to solve the problem very efficiently. The code makes an intensive use of automatic differentiation, currently thanks to Tapenade. The well known difficulty of this approach is to find a good initial guess, so HamPath embeds a differential path following method, as well as advanced computations such as conjugate point estimation to test second order optimality conditions in optimal control. Thanks to the ongoing ADT, BOCOP has been entirely rewriten in C++ (BOCOP v3) and now has a `python` interface. This allows interoperability with `nutopy`, a `python` port of HamPath. The gallery of the project mirrors interactions of the two solvers and contains a bunch of examples coming from mechanics, agronomics, biology, geometry, *etc.* The project is open to external contributions and aims to be a living repository for algorithms and applications of optimal control.

5.1 New software

5.1.1 ct

Name: control toolbox

Keywords: Optimal control, Ordinary differential equations, Mathematical Optimization, Differential homotopy, Automatic differentiation

Scientific Description: Numerical resolution of optimal control problems

Functional Description: The project gathers and allows to interoperate tools designed to solve numerically optimal control problems on ordinary differential equations. The available approaches include direct methods (based on a transcription of optimal control problems into mathematical programs) as well as indirect ones (based on Pontrjagin maximum principle, like the shooting method). The latter can be coupled to differential continuation. Automatic differentiation (aka Différentiable Programming) plays a crucial a role in all these algorithms.

Release Contributions: - bocop refactoring - nutopy library - project gallery

URL: `http://ct.gitlabpages.inria.fr/gallery`

Contact: Jean-Baptiste Caillau

Participants: Jean-Baptiste Caillau, Pierre Martinon, Olivier Cots, Thibaud Kloczko, Tristan Cabel, Jean-Luc Szpyrka, Erwan Demairy, Julien Wintz, Carlos Zubiaga Pena, Nicolas Niclausse

Partners: Université de Toulouse, CNRS, IRIT, ENSEEIHT

6 New results

6.1 Convergence of time-optimal trajectories of fast oscillating control systems

Participants: Jean-Baptiste Caillau, Lamberto Dell'Elce, Jean-Baptiste Pomet.

This study, started in 2020, is devoted to the understanding of convergence properties of time optimal trajectories of fast oscillating control systems. Specifically, for a control system with one fast periodic variable, with a small parameter measuring the ratio between time derivatives of fast and slow variables, we considered the Hamiltonian equation resulting from applying Pontryagin Maximum Principle for the minimum time problem with fixed initial and final slow variables and free fast variable. One may perform averaging at least under normalization of the adjoint vectors and define a “limit” average system.

When the small parameter approaches 0, we showed that using the right transformations between initial/final conditions in the "real" and average systems led to a reconstruction of the fast variable on interval of times of order $1/\epsilon$ where ϵ is the small parameter. Relying on this, we proposed a procedure to reconstruct very efficiently the solution of the two point boundary problem for nonzero ϵ using only the solution of the averaged optimal control problem. This result was presented at the ECC conference [9] and a more formal paper is in preparation.

6.2 Controllability of solar sails

Participants: Jean-Baptiste Caillau, Lamberto Dell'Elce, Alesia Herasimenka, Jean-Baptiste Pomet.

We studied the local controllability of non-ideal solar sails in planet-centered orbits. Classical approaches fail when considering this control problem because sails cannot generate forces with positive components toward the Sun direction. More precisely, the control set is delimited by a convex cone of revolution with axis toward the Sun and, as such, it is not defined in a neighborhood of the origin, which precludes the use of standard sufficient conditions for controllability. We introduced both a necessary condition certifying the local non-controllability of the system [13] and a sufficient condition guaranteeing controllabilitly under mild assumptions [26] for given optical properties of the sail and orbital elements. These conditions can be verified by solving two semi-infinite programming problems. By evaluating these conditions on the entire space of slow orbital elements, we identified minimal optical properties that provide local controllability of any planet-centered orbit.

6.3 Attitude determination and control system of NiceCUBE

Participants: Lamberto Dell'Elce, Jean-Baptiste Pomet, Samuel Sarrazin.

Université Côte d'Azur is planning to build its first single-unit CubeSat (i.e., a recent standard for satellites with mass of the order of few kilograms and volume of few dm^3), named NiceCube. The objective of the mission is to establish an optical link between a ground station in Nice and the satellite by enlightning it with a laser and by moduling the reflected signal with an occulter onboard NiceCube. To this end, the face of the satellite equipped with retroreflectors should be oriented towards the ground station so that the reflected signal can be received. McTAO part of UCA and is in charge of the attitude determination and control system of NiceCube.

The mission is currently in phase A (i.e., feasibility study). This year, we carried out a detailed analysis of the mission focusing on the agility of attitude maneuvers when establishing the optical link with the ground station (these events last few minutes and occur twice per day in general). We quantified the power consumption of such maneuvers and verified its compatibility with the available resources of NiceCube.

6.4 Optimal allocation of ressources in bacteria

Participants: Agustín Yabo, Jean-Baptiste Caillau, Jean-Luc Gouzé *(BIOCORE project-team)*.

In the framework of the ANR Maximic, we carry on the study of self-replicator models. These models describe the allocation of resources inside the bacteria: the substrate is used to produce precursors that, in turn, can be employed either to produce genetic machinery (and increase the biomass) or metabolic machinery (that will further catalyse the transformation of substrate into precursors). To this internal control and external action that aims, after some genetic engineering on the bacteria (to create a strain that reacts to light stimuli), at producing a new metabolite of interest. Then, while the behaviour of the untouched bacteria tends to be very well mimicked by biomass maximization strategies, maximizing the production of the metabolite of interest induces new biological strategies. This kind of model (and refinements) are studied in [30] and [6]. Key properties of the system are: (i) the Fuller phenomenon as connection between bang and singular arcs requires an infinite number of switchings in finite time; (ii) the turnpike phenomenon. Indeed, for large fixed final times, trajectories of the system are essentially singular and close to the optimal (wrt. a constant static control) equilibrium which is a hyperbolic fixed point of the singular flow. See ct gallery for an example, and the recently defended PhD thesis of A. Yago [15] for a discussion of these results.

6.5 Stability of linear time-varying time-delay systems

Participants: Laurent Baratchart *(FACTAS project-team)*, Sébastien Fueyo *(Centrale-Supélec, Gif-sur-Yvette)*, Gilles Lebeau *(Université Côte d'Azur, Nice)*, Jean-Baptiste Pomet.

Sébastien Fueyo's doctoral work (PhD defended in 2020, see [31]) on testing the stability of amplifier in a CAD (computer assisted design) process, described also in Section 4.3, led us to revisiting stability of time-varying linear "delay difference systems" (continuous time, but strictly difference equations). In the time-invariant case, a well known necessary and sufficient condition for stability is due to Hale and Henry [70, 69]; it gives, in a sense, a final answer to the question, but it is not so easy to check explicitly this criteria, and there is still a vast literature on more specific sufficient conditions. The time-varying case, on the contrary, had seldom been touched.

The article [2] presents a novel sufficient condition for hyperbolic stability in the time-varying case; it has a passivity interpretation that is exactly fit to classical assumptions on the behavior of electronic devices "at high frequency". A more general result is presented in [17], to be submitted soon; it can be viewed as the proper generalization to the time-varying case of the Henry-Hale criteria mentionned above. The criteria is however less easily checkable than the one in [2].

6.6 Small time local controllability with two input, motivated by bio-inspired non-linear systems

Participants: Laetitia Giraldi *(CALISTO project-team since January, 2021)*, Pierre Lissy *(Université Paris Dauphine, Paris)*, Clément Moreau *(Kyoto University, Japan)*, Jean-Baptiste Pomet.

One part of C. Moreau's PhD [32] was on controllability properties of planar articulated magnetically actuated swimmers. The sinularities in controllability around the straight configuration, where all magnetic moments are aligned, motivated research on necessary conditions for local controllability of a more general, but still particular, class of systems with two controls. Technical results based on the study of the Chen-Fliess series associated to these systems were obtained. The manuscript [25] is under revision and possible generalization are currently being explored.

6.7 Optimal control techniques for 2-D Zermelo navigation problems

6.7.1 Zermelo problems on revolution surfaces and the "vortex problem"

Participants: Bernard Bonnard, Joseph Gergaud, Boris Wembe *(ENSEEIHT, Toulouse)*, Olivier Cots.

B. Bonnard took an active part in the research around the doctoral work of B. Wembe (PhD defended in November [84]) on time-minimum problems in navigation in presence of a current and a vortex. As explained in [4], this work is motivated by the displacement of particles in a two dimensional fluid, in presence of a vortex (initially, a singularity in the Helhmoltz-Kirchhoff equations). To define a minimum time Zermelo navigation problem, we consider the particle as the ship of the navigation problem and the control is defined as the heading angle of the ship axis. We have two cases, that coexist in different regions: strong versus weak current. In the weak case, the time minimal problem defines a 2-D Randers metric (a specific Finsler metric) in a portion of the the plane, while in the other case one cannot consider the problem as locally metric. Because of the singularity, we have a non trivial extension of the classical case. In a subsequent work [18], we analyze the effect of the same vortex singularity on more general surfaces of revolutions. Some salient points are: integrability of the extremal flow of the Pontryagin Maximum Principle, due to the rotational symmetry, absence of conjugate points (where an extremal curve ceases to be optimal for the C^1-topology), as verified numerically, so that optimality boils down to computing cut points, presence of Reeb component in the foliation arising from integrability of the flow, importance of the abnormal curves. A classification of small "balls" (level sufaces of the minumum time to a given point) is given in [3]. Finally, [20] comes back on the geometric issues raised by this study, and parallels it with the optimal control problem for networks of chemical reactions (McKeithan network) aiming at maximizing one species with a terminal condition that is always a co-dimension one manifold.

6.7.2 Zermelo-Markov-Dubins problem and navigation with trailers

Participants: Jean-Baptiste Caillau, Jean-Baptiste Pomet, Ludovic Sacchelli.

Parallel efforts have been dedicated to the particular case of ship navigation with trailers. The work is motivated by the optimization of turns and maneuvers of marine vessels towing a set of long and fragile underwater cables for seismic data acquisition. In a variant of the Zermelo-Markov-Dubins problem, we consider a model where the towed cables are represented with articulated trailers. Prior work had been achieved on a simpler trailer model [60]. Recently, in [10] and during internships, we theoretically and numerically studied the properties and optimality of trajectories of with more complex trailer models. These results serve as a basis for meetings with the subsurface exploration company Sercel on an expected first collaboration in 2022.

6.8 Muscular Electrostimulation Project

Participants: Bernard Bonnard, Sandrine Gayrard, Jérémy Rouot, Toufik Bakir *(Université de Bourgogne Franche Comté, Dijon)*.

This topic started in McTAO in 2017 with a collaboration between B. Bonnard and T. Bakir (ImVia-UBFC). Based on preliminary experimental studies, the chosen model to muscular control integrates the fatigue variables and is known as *Ding et al force-fatigue model* in the literature. It is a refinement of the historical Hill model (Medecine Nobel Prize in 1922) that takes into account the variations of the fatigue variable. The problem is the one of optimizing the train pulses of the Functional Electro-Stimulation (FES) signal to produce the muscular contraction. From the control methodology point of view, this required some developments on optimal control for sample control systems. This is by itself a rich topic, on which a workshop was organised in September in Brest on this occasion, see Section 9.1.1. Results on applications of sampled geometric optimal control to the isometric case are presented in [28] and [1], with preliminary work on extension to the non isometric case in [29] (non isometric means that movements and deformations are taken into account).

The attempt to an industrial transfer started last fall (2020) with a contract (CIFRE PhD funding) with SEGULA Technologies (see Section 7) whose goal is the construction of an electro-stimulator prototype, in the framework of S. Gayrard's PhD. Preliminary results lead to construct a nonlinear observer and the optimized pulses trains are computed using MPC methods [38]. Serious effort is put in identifying the dynamics; in [16], a detailed study of the muscle response is presented and used to build the necessary finite-dimensional dynamic model to perform closed-loop control. We have hope to improve over state-of-the-art isometric stimulators and be able to tackle the non-isometric case, which is richer in applications.

6.9 ct: control toolbox

Participants: Jean-Baptiste Caillau, Olivier Cots, Pierre Martinon *(CAGE project-team, on leave)*.

The ADT `ct: control toolbox` had two sprints in 2021. The focus was on `python` as a high level backend to interoperate existing solvers. We also initiated new developments in Julia to take advantage of the powerful features of the language. Julia is indeed a perfect match for our needs in scientific computing for numerical optimal control; the language has a high level of abstraction well suited for mathematical descriptions, but still makes no compromise when it comes to performance thanks to efficient just-in-time compilation. Moreover, it currently has several efficient backends for AD / DP (automatic differentiation / differentiable programming), including ForwarDiff and Zygote: this is a crucial step for our project, both for direct and indirect methods. (We added to the project gallery some examples stemming from Riemannian geometry in the gallery that require up to four levels of automatic differentiation.)

7 Bilateral contracts and grants with industry

Bilateral grants with industry

Participants: Bernard Bonnard, Sandrine Gayrard, Toufik Bakir *(Université de Bourgogne Franche Comté, Dijon)*.

CIFRE PhD grant. Co-financed by Segula Technologies on the topic "Réalisation d'un prototype d'électrostimulateur intelligent". The PhD etudent on this topic is Sandrine Gayrard.
PI: Bernard Bonnard and Toufik Bakir (IMvia, Université de Bourgogne Franche Comté, Dijon).
Period: 2020–2023.

Participants: Jean-Baptiste Caillau, Lamberto Dell'Elce, Alesia Herasimenka, Jean-Baptiste Pomet.

ESA contract "Optimal Control of Solar Sails". Three year contract starting in 2021 between the team and the European Space agency. Its purpose is to support the environment of Alesia Herasimenka's PhD on this topic.
Partners: McTAO and ESA.
Total amount: 24k€.

Period: 2021–2024.
Inria reference: 16016.

8 Partnerships and cooperations

8.1 National initiatives

8.1.1 ANR

Maximic: optimal control of microbial cells by natural and synthetic strategies. Started 2017, duration: 6 years. J.-B. Caillau, L. Giraldi, J.-B. Pomet, S. Maslovkaya and A. Yabo are participants. More information and news on the site of this project.

8.1.2 Others

PGMO grant (2019-2021) on "Sampled-Data Control Systems and Applications" (PI B. Bonnard).

PGMO grant (2020-2022) on "Extremal determinants". Participants are Y. Chitour (Université Paris Saclay), J.-B. Caillau, P. Freitas (University of Lisbon), Y. Privat (Université de Strasbourg)

The McTAO team participates in the GdR MOA, a CNRS network on Mathematics of Optimization and Applications.

J.-B. Caillau is associate researcher of the CNRS team Parallel Algorithms & Optimization at ENSEEIHT, Univ. Toulouse.

8.2 Regional initiatives

UCAJEDI, académie d'excellence "Systèmes complexes" grant (2021) for organization of a workshop on "observability defects in nonlinear systems" (P.I. L. Sacchelli).

9 Dissemination

9.1 Promoting scientific activities

9.1.1 Scientific events: organisation

General chair, scientific chair

B. Bonnard co-organized a workshop on sampled-data-control optimal control, September 8 to 10, 2021, at Université de Bretagne Occidentale, Brest, France.
J.-B. Caillau co-organized the SEME / MSGI *AI & companies week* in Sophia.
L. Sacchelli was the principal organizer of Mini-workshop : Défauts d'observabilité des systèmes non linéaires. December 1-2 2021, Inria Sophia Antipolis. The event was made possible thanks to a grant from UCA JEDI Académie d'Excellence "Systèmes complexes".

Member of the organizing committees

J.-B. Caillau was member of the Organizing Committee of 2021 Journées de la Statistique (Nice), of the Organizing Committee of the CIMPA school Optimal Control and Applications in Engineering, held at Arba Minch University (Ethiopia), and of the Scientific Committee of PGMO days 2021.

9.1.2 Journal

Member of the editorial boards

B. Bonnard is member of the editorial board of Pacific Journal of Mathematics for Industry.

J.-B. Caillau is member of the editorial board of ESAIM: Mathematical Modelling and Numerical Analysis (ESAIM: M2AN).

Reviewer - reviewing activities

All team members take part in a continued effort to offer reviews in various journals of importance to the community.

9.1.3 Invited talks

J.-B. Caillau was plenary speaker at the Dynamic Control and Optimization conference held in honor of Andrey Sarychev in Aveiro, Portugal (2021).

9.1.4 Research administration

J.-B. Caillau was

- member of the Scientific Committee of the PGMO program of FMJH,
- member of the Scientific Committee of 3IA Côte d'Azur.

J.-B. Pomet was

- a member of the steering committee of academy of excellence on complex systems, Université Côte d'Azur,
- an elected member of Inria's permanent Evaluation Committee.

9.2 Teaching - Supervision - Juries

9.2.1 Teaching

- Engineering school and University: J.-B. Caillau has a full teaching duty of Professor at L3, M1 and M2 level at Polytech Nice Sophia - Université Côte d'Azur. He was director of the Applied math. & modelling department of the school.
- Engineering school: L. Dell'Elce was teaching assistant of the M1 course "*Mecanique des milieux continus*" at Polytech Nice Sophia, Université Côte d'Azur.
- Engineering school: A. Herasimenka was teaching assistant at L3 and M1 level at Polytech Nice Sophia, Université Côte d'Azur.
- Engineering school: A. Yabo was teaching assistant at L3 and M1 level at Polytech Nice Sophia, Université Côte d'Azur.

9.2.2 Supervision

- PhD : Agustin Yabo, "Control of biological processes" [15], Université Côte d'Azur, co-supervised by J.-B. Caillau and J.-L. Gouzé (Biocore team), defended December 9, 2021.
- PhD in progress : Sandrine Gayrard, "Réalisation d'un prototype d'électrostimulateur intelligent", Université de Bourgogne Franche Comté, co-supervised by B. Bonnard and Toufik Bakir, started January, 2020.

- PhD in progress : Alesia Herasimenka, “Optimal control of solar sails”, Université Côte d’Azur, co-supervised by L. Dell’Elce and J.-B. Caillau, started October, 2020.

- Master’s thesis (M2): Samuel Sarrazin, ELISA Aerospace École d’ingénieurs, co-supervised by L. Dell’Elce and J.-B. Pomet, March to August.

- Master, 1st year internship (M1): Sacha Psalmon and Baptiste Schall, Polytech Nice Sophia, co-supervised by J.-B. Caillau and J.-L. Gouzé (BIOCORE), June to September.
- Master, 1st year internship (M1): Philippe Nègre and Mourad Sadak, “Contrôle optimal appliqué à la navigation maritime”, Polytech Nice Sophia, co-supervised by J.-B. Caillau and J.-B. Pomet, June to September.

9.2.3 Juries

J.-B. Caillau reviewed and sat at the jury of Victor Cantu PhD thesis (Université Toulouse)

9.3 Popularization

9.3.1 Internal or external Inria responsibilities

J.-B. Caillau is member of MASTIC and of Terra Numerica.
L. dell’Elce is member of MASTIC.

10 Scientific production

10.1 Publications of the year

International journals

[1] T. Bakir, B. Bonnard, L. Bourdin and J. Rouot. ‘Direct and Indirect Methods to Optimize the Muscular Force Response to a Pulse Train of Electrical Stimulation’. In: *ESAIM: Proceedings and Surveys* 71 (1st Sept. 2021), pp. 1–10. DOI: 10.1051/proc/202171101. URL: https://hal.inria.fr/hal-02053566.

[2] L. Baratchart, S. Fueyo, G. Lebeau and J.-B. Pomet. ‘Sufficient Stability Conditions for Time-varying Networks of Telegrapher’s Equations or Difference Delay Equations’. In: *SIAM Journal on Mathematical Analysis* 53 (1st Apr. 2021), pp. 1831–1856. DOI: 10.1137/19M1301795. URL: https://hal.inria.fr/hal-02385548.

[3] B. Bonnard, O. Cots, J. Gergaud and B. Wembe. ‘Abnormal Geodesics in 2D-Zermelo Navigation Problems in the Case of Revolution and the Fan Shape of the Small Time Balls’. In: *Systems and Control Letters* (2022). URL: https://hal.archives-ouvertes.fr/hal-02437507.

[4] B. Bonnard, O. Cots and B. Wembe. ‘A Zermelo navigation problem with a vortex singularity’. In: *ESAIM: Control, Optimisation and Calculus of Variations* 27.S (1st Mar. 2021), S10. DOI: 10.1051/cocv/2020058. URL: https://hal.archives-ouvertes.fr/hal-02296046.

[5] J.-B. Caillau, J. Fejoz, M. Orieux and R. Roussarie. ‘On singularities of minimum time control-affine systems’. In: *SIAM Journal on Control and Optimization* (2021). URL: https://hal.inria.fr/hal-01718345.

[6] A. G. Yabo, J.-B. Caillau, J.-L. Gouzé, H. De Jong and F. Mairet. ‘Dynamical analysis and optimization of a generalized resource allocation model of microbial growth’. In: *SIAM Journal on Applied Dynamical Systems* 21.1 (2022), pp. 137–165. DOI: 10.1137/21M141097X. URL: https://hal.inria.fr/hal-03251044.

International peer-reviewed conferences

[7] N. Augier and A. G. Yabo. 'Time-optimal control of piecewise affine bistable gene-regulatory networks: preliminary results'. In: 7th IFAC Conference on Analysis and Design of Hybrid Systems (ADHS 2021). Brussels, Belgium, 7th July 2021. URL: https://hal.inria.fr/hal-03099681.

[8] M. Barlaud, A. Chambolle and J.-B. Caillau. 'Classification and feature selection using a primal-dual method and projection on structured constraints'. In: ICPR 2020. ICPR Proceedings. Milan, Italy, 2021, pp. 6538–6545. URL: https://hal.archives-ouvertes.fr/hal-03039073.

[9] L. Dell'Elce, J.-B. Caillau and J.-B. Pomet. 'On the convergence of time-optimal maneuvers of fast-oscillating control systems'. In: European Control Conference 2021. Virtual, France, 29th June 2021. URL: https://hal.inria.fr/hal-03391620.

[10] L. Sacchelli, J.-B. Caillau, T. Combot and J.-B. Pomet. 'Zermelo-Markov-Dubins with two trailers'. In: 7th IFAC Workshop on Lagrangian and Hamiltonian Methods for Nonlinear Control. Vol. 54. IFAC-PapersOnLine 19. Berlin, Germany, 11th Oct. 2021, pp. 249–245. DOI: 10.1016/j.ifacol.2021.11.086. URL: https://hal.inria.fr/hal-03211710.

[11] A. G. Yabo, J.-B. Caillau and J.-L. Gouzé. 'Hierarchical MPC applied to bacterial resource allocation and metabolite synthesis'. In: 2021 IEEE 60th Conference on Decision and Control (CDC). Conference on Decision and Control (CDC). Austin, United States, 13th Dec. 2021. URL: https://hal.archives-ouvertes.fr/hal-03189960.

Conferences without proceedings

[12] J.-B. Caillau, J.-L. Gouzé and A. G. Yabo. 'Optimal bacterial resource allocation'. In: International Conference on Dynamic Control and Optimization - DCO 2021. Aveiro, Portugal, Feb. 2021. URL: https://hal.inria.fr/hal-03528000.

[13] A. Herasimenka, L. Dell'Elce, J.-B. Caillau and J.-B. Pomet. 'Impact of Optical Properties on the Controllability of Solar Sails'. In: ICATT 2021 - International Conference on Astrodynamics Tools and Techniques. Sopot, Poland, 2021. URL: https://hal.archives-ouvertes.fr/hal-03331300.

Edition (books, proceedings, special issue of a journal)

[14] D. Auroux, J.-B. Caillau, R. Duvigneau, A. Habbal, C. Malot, O. Pantz, L. Pronzato, L. Rifford, R. Ruelle and C. Soresi, eds. *French-German-Swiss'2019 conference on optimization proceedings*. Vol. 71. ESAIM: Proceedings and Surveys, Aug. 2021. URL: https://hal.inria.fr/hal-03409764.

Doctoral dissertations and habilitation theses

[15] A. G. Yabo. 'Optimal resource allocation in bacterial growth: theoretical study and applications to metabolite production'. Université Côte d'Azur, 9th Dec. 2021. URL: https://hal.archives-ouvertes.fr/tel-03520563.

Reports & preprints

[16] T. Bakir, B. Bonnard, S. Gayrard and J. Rouot. *Finite Dimensional Approximation to Muscular Response in Force-Fatigue Dynamics using Functional Electrical Stimulation*. 12th Mar. 2021. URL: https://hal.inria.fr/hal-03154450.

[17] L. Baratchart, S. Fueyo and J.-B. Pomet. *Exponential stability of linear periodic difference-delay equations*. 2021. URL: https://hal.inria.fr/hal-03500720.

[18] B. Bonnard, O. Cots and B. Wembe. *Zermelo Navigation Problems on Surfaces of Revolution and Hamiltonian Dynamics*. 7th May 2021. URL: https://hal.archives-ouvertes.fr/hal-03209491.

[19] B. Bonnard and J. Rouot. *Toward Geometric Time Minimal Control without Legendre Condition and with Multiple Singular Extremals for Chemical Networks. An Extended Version.* INRIA Sophia Antipolis - Méditerranée; LMBA, 2021. URL: https://hal.inria.fr/hal-03327127.

[20] B. Bonnard, J. Rouot and B. Wembe. *Accessibility, Abnormal Geodesics in Optimal Control. A Geometric Approach from Singularity Theory using Two Cases Studies'.* 30th July 2021. URL: https://hal.inria.fr/hal-03310348.

[21] L. Brivadis and L. Sacchelli. *A switching technique for output feedback stabilization at an unobservable target.* 25th Mar. 2021. URL: https://hal.archives-ouvertes.fr/hal-03180479.

[22] L. Brivadis and L. Sacchelli. *Approximate observability and back and forth observer of a PDE model of crystallisation process.* 19th Mar. 2021. URL: https://hal.archives-ouvertes.fr/hal-03175017.

[23] J.-B. Caillau, W. Djema, J.-L. Gouzé, S. Maslovskaya and J.-B. Pomet. *Turnpike property in optimal microbial metabolite production.* 30th Aug. 2021. URL: https://hal.archives-ouvertes.fr/hal-03328686.

[24] L. Dell'Elce, J.-B. Caillau and J.-B. Pomet. *Considerations on Two-Phase Averaging of Time-Optimal Control Systems.* 11th Jan. 2022. URL: https://hal.inria.fr/hal-01793704.

[25] L. Giraldi, P. Lissy, C. Moreau and J.-B. Pomet. *Necessary conditions for local controllability of a particular class of systems with two scalar controls.* 19th Aug. 2021. URL: https://hal.archives-ouvertes.fr/hal-02178973.

[26] A. Herasimenka, J.-B. Caillau, L. Dell'Elce and J.-B. Pomet. *Controllability Test for Systems with Constrained Control. Application to Solar Sailing.* 24th Nov. 2021. URL: https://hal.inria.fr/hal-03185532.

[27] A. G. Yabo, J.-B. Caillau and J.-L. Gouzé. *Optimal allocation of bacterial resources in fed-batch reactors.* 9th Nov. 2021. URL: https://hal.inria.fr/hal-03421358.

10.2 Cited publications

[28] T. Bakir, B. Bonnard, L. Bourdin and J. Rouot. 'Pontryagin-Type Conditions for Optimal Muscular Force Response to Functional Electric Stimulations'. In: *Journal of Optimization Theory and Applications* 184.2 (Feb. 2020), pp. 581–602. DOI: 10.1007/s10957-019-01599-4. URL: https://hal.inria.fr/hal-01854551.

[29] B. Bonnard and J. Rouot. 'Geometric optimal techniques to control the muscular force response to functional electrical stimulation using a non-isometric force-fatigue model'. In: *Journal of Geometric Mechanics* (2020). DOI: 10.3934/jgm.2020032. URL: https://hal.inria.fr/hal-02611095.

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