RESEARCH CENTRE

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2021

ACTIVITY REPORT

Project-Team

POEMS

Wave propagation: mathematical analysis and simulation

IN COLLABORATION WITH: Propagation des ondes : étude mathématique et simulation (POEMS)

DOMAIN

Applied Mathematics, Computation and Simulation

THEME

Numerical schemes and simulations

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Project-Team POEMS

Creation of the Project-Team: 2019 November 01

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- Maryna Kachanovska [Inria, Researcher]
- Luiz Maltez Faria [Inria, Researcher]
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2 Overall objectives

The propagation of waves is one of the most common physical phenomena in nature. From the human scale (sounds, vibrations, water waves, telecommunications, radar) to the scales of the universe (electromagnetic waves, gravity waves) and of the atoms (spontaneous or stimulated emission, interferences between particles), the emission and the reception of waves are our privileged way to understand the world that surrounds us. The study and the simulation of wave propagation phenomena constitute a very broad and active field of research in various domains of physics and engineering sciences. The variety and the complexity of the underlying problems, their scientific and industrial interest, the existence of a common mathematical structure to these problems from different areas altogether justify a research project in applied mathematics and scientific computing devoted to this topic.

3 Research program

3.1 Expertises

The activity of the team is oriented towards the design, the analysis and the numerical approximation of mathematical models for all types of problems involving wave propagation phenomena, in mechanics, physics and engineering sciences. Let us briefly describe our core business and current expertise, in order to clarify the new challenges that we want to address in the short and long terms.

Typically, our works are based on *boundary value problems* established by physicists to model the propagation of waves in various situations. The basic ingredient is a partial differential equation of the hyperbolic type, whose prototype is the scalar wave equation, or the Helmholtz equation if time-periodic solutions are considered. More generally, we systematically consider both the transient problem, in the time domain, and the time-harmonic problem, in the frequency domain. Let us mention that, even if different waves share a lot of common properties, the transition from the scalar acoustic equation to the vectorial electromagnetism and elastodynamics systems raises a lot of mathematical and numerical difficulties, and requires a specific expertise.

A notable particularity of the problems that we consider is that they are generally set in *unbounded domains*: for instance, for radar applications, it is necessary to simulate the interaction of the electromagnetic waves with the airplane only, without any complex environment perturbing the wave phenomena. This raises an intense research activity, both from a theoretical and a numerical point of view. There exist several approaches which all consist in rewriting the problem (or an approximation of it) in a bounded domain, the new formulation being well-suited for classical mathematical and numerical techniques.

One class of methods consists in applying an appropriate condition on some boundary enclosing the zone of interest. In the frequency domain, one can use a non-local transparent condition, which can be expressed by a convolution with a Green function like in integral equation techniques, or by a modal decomposition when a separation of variables is applicable. But for explicit schemes in the time domain, local radiation conditions at a finite distance are generally preferred (constructed as local approximations at various orders of the exact non-local condition). A second class of methods consists in surrounding the computational domain by so called *Perfectly Matched absorbing Layers* (PML), which are very popular because they are easy to implement. POEMS members have provided several contributions to these two classes of methods for more than twenty-five years. Among them, on can mention the understanding of the instability of PMLs in anisotropic media and in dispersive media, the derivation of transparent boundary conditions in periodic media or the improvement of Fast Multipole techniques for elastodynamic integral equations.

In addition to more classical domains of applied mathematics that we are led to use (variational analysis and functional analysis, interpolation and approximation theory, linear algebra of large systems, etc...), we have acquired a deep expertise in *spectral theory*. Indeed, the analysis of wave phenomena is intimately linked to the study of some associated spectral problems. Acoustic resonance frequencies of a cavity correspond to the eigenvalues of a selfadjoint Laplacian operator, modal solutions in a waveguide correspond to a spectral problem set in the cross section. In these two examples, if the cavity or the cross-section is unbounded, a part of the spectrum is a continuum. Again, POEMS has produced several contributions in this field. In particular, a large number of significant results have been obtained for the existence or non-existence of guided modes in open waveguides and of trapped modes in infinite domains.

To end this far from exhaustive presentation of our main expertise domains, let us mention the *asymptotic techniques* with respect to some small scale appearing in the model: it can be the wavelength compared to the size of the scatterer, or on the contrary, the scale of the scatterer compared to the wavelength, it can be the scale of some microstructure in a composite material or the width of a thin layer or a thin tube. In each case, the objective, in order to avoid the use of costly meshes, is to derive effective simplified models. Our specificity here is that we can combine skills in physics, mathematics and numerics: in particular, we take care of the mathematical properties of the effective model, which are used to ensure the robustness of the numerical method, and also to derive error estimates with respect to the small parameter. There has been a lot of contributions of POEMS to this topic, going from the modeling of electromagnetic coatings to the justification of models for piezoelectric sensors. Let us mention that effective models for small scatterers and thin coatings have been used to improve imaging techniques that we are developing (topological gradient, time reversal or sampling techniques).

3.2 New domains

In order to consider more and more challenging problems (involving non-deterministic, large-scale and more realistic models), we decided recently to enlarge our domain of expertise in three directions.

Firstly, we want to reinforce our activity on *efficient solvers for large-scale wave propagation problems*. Since its inception, POEMS has frequently contributed to the development and the analysis of numerical methods that permit the fast solution of large-scale problems, such as high-order finite element methods, boundary elements methods and domain decomposition methods. Nevertheless, implementing these methods in parallel programming environments and dealing with large-scale benchmarks have generally not been done by the team. We want to continue our activities on these methods and, in a more comprehensive approach, we will incorporate modern algebraic strategies and high-performance computing (HPC) aspects in our methodology. In collaboration with academic or industrial partners, we would like to address industrial-scale benchmarks to assess the performance of our approaches. We believe that taking all these aspects into consideration will allow us to design more efficient wave-specific computational tools for large-scale simulations.

Secondly, up to now, *probabilistic methods* were outside the expertise of POEMS team, restricting us to deterministic approaches for wave propagation problems. We however firmly believe in the importance and usefulness of addressing uncertainty and randomness inherent to many propagation phenomena. Randomness may occur in the description of complex propagation media (for example in the modeling of ultrasound waves in concrete for the simulation of non-destructive testing experiments) or of data uncertainties. To quantify the effect of such uncertainties on the design, behavior, performance or reliability of many systems is then a natural goal in diverse fields of application.

Thirdly and lastly, we wish to develop and strengthen collaborations allowing a *closer interaction between our mathematical, modeling and computing activities and physical experiments*, where the latter may either provide reality checks on existing models or strongly affect the choice of modeling assumptions. Within our typical domain of activities, we can mention four areas for which such considerations are highly relevant. One is musical acoustics, where POEMS has made several well-recognized contributions dealing with the simulation of musical instruments. Another area is inverse problems, whose very purpose is to extract useful information from actual measurements with the help of (propagation) models. This is a core of our partnership with CEA on ultrasonic Non Destructive Testing. A third area is the modelling of effective (acoustic or electromagnetic) metamaterials, where predictions based on homogenized models have to be confirmed by experiments. Finally, a fourth area of expertise is the modeling and simulations of waves in reactive media, where the development of simple mathematical models is of great importance in order to better understand the complex dynamics of reactive flows.

4 Application domains

Our research finds applications in many fields where acoustic, elastic, electromagnetic and aquatic waves are involved. Topics that have given rise to industrial partnerships include aircraft noise reduction (aeroacoustics), ultrasonic non-destructive testing of industrial structures, and seismic wave simulations in the subsoil, for the oil exploration.

Nowadays, the numerical techniques for solving the basic academic problems are well mastered, and significant progress has been made during the last twenty years for handling problems closer to real applications. But several bottlenecks remain, among which one can mention the high-frequency problems for radar applications, the multiscale problems that arise for instance in nanotechnologies or the multi-physics couplings, like in *aeroacoustics*. Moreover, in the recent period, new challenges have emerged, related to new discoveries in physics (like negative index metamaterials) or to the fantastic development of information and communication techniques. For example, the growing development of increasingly connected objects (internet of things) and the forthcoming availability of autonomous vehicles depend crucially on electromagnetic waves, raising important issues about radar performance, sensor reliability, component miniaturization and electromagnetic compatibility. Generally, there are a lot of application domains which could benefit from advanced research on waves phenomena. Enhancing ultrasound-based methods for *detection* and *imaging*, which are already intensively used in e.g. medicine, could permit real-time health monitoring of aircrafts or nuclear plants. Guarding against seismic risks still requires considerable advances in the simulation of elastic waves in large and complex media. And many other applications motivating our research and our prospects could be added to this far-from-comprehensive list.

5 Highlights of the year

5.1 Awards

Emile Parolin received the Best Thesis Award Runner-up, awarded by the Institut Polytechnique de Paris.

6 New software and platforms

We are currently developing the softwares COFFEE (a BEM solver for acoustic and elastic waves) and XLiFE++ (a FEM-BEM C++ library).

6.1 New software

6.1.1 COFFEE

Keywords: Numerical simulations, Wave propagation, Boundary element method

Functional Description: COFFEE is an adapted fast BEM solver to model acoustic and elastic wave propagation (full implementation in Fortran 90). The 3-D acoustic or elastodynamic equations are solved with the boundary element method accelerated by the multi-level fast multipole method or a hierarchical-matrices based representation of the system matrix. The fundamental solutions for the infinite space are used in this implementation. A boundary element-boundary element coupling strategy is also implemented so multi-region problems (strata inside a valley for example) can be solved. In order to accelerate the convergence of the iterative solver, various analytic or algebraic preconditioners are available. Finally, an anisotropic mesh adaptation strategy is used to further reduce the computational times.

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6.1.2 XLiFE++

Name: eXtended Library of Finite Elements in C++

Keywords: Numerical simulations, Finite element modelling, Boundary element method

Functional Description: XLiFE++ is an FEM-BEM C++ library developed by POEMS laboratory and IR-MAR laboratory, that can solve 1D/2D/3D, scalar/vector, transient/stationary/harmonic problems. Description: https://uma.ensta-paris.fr/soft/XLiFE++/

Contact: Eric Lunéville

7 New results

7.1 Wave propagation in metamaterials and dispersive media

Mathematical analysis of metamaterials in time domain

Participants: Christophe Hazard, Patrick Joly, Alex Rosas Martinez.

This topic is the subject of our important collaboration with Maxence Cassier (Institut Fresnel).

A huge effort has been devoted to the writing of a long and very technical paper (64 pages) on the limiting absorption and limiting amplitude (understand long time behaviour) principles for the transmission problem between two half-spaces, one made of vacuum and the other filled by a Drude material. This is the second part of a work whose first part, devoted to the spectral theory of the problem, was published in Communications in PDEs. The approach is based on the use of so-called stationary scattering method that must be adapted to the specificities of the constitutive laws of Drude media, specificities that are also the source of new phenomena such as plasmonic interface waves or interface resonances. Our second paper has just been accepted, also in Communications in PDEs.

The second aspect of our research on this topic corresponds the PhD thesis of Alex Rosas Martinez who started in November 2020.

This concerns more precisely the dissipative version of generalized Lorentz media and more precisely the large time behaviour of the associated Cauchy problem. It appears that the electromagnetic energy decays in t^{-q} with $q > 0$ depending on the regularity of the initial data : one speaks of polynomial stability (as opposed to exponential stability for instance). Two methods have been developed. The first one is based on "frequency dependent" Lyapunov estimates. This approach has the advantage to lead to polynomial stability with "little" effort but requires some "strong dissipativity" assumptions. The alternative spectral approach allows us to get rid of these assumptions and leads to sharp estimates, but is technically more involved, in particular because non self-adjoint spectral theory has to be used. Our results generalize and give a new light (in particular from the physical point of view) to previous results of the literature by S. Nicaise et al., for a restricted class of models, or by Figotin et al., for a somewhat larger class of models but without any energy decay estimate.

Wave Propagation in Plasmas

Participants: Patrick Ciarlet, Maryna Kachanovska, Etienne Peillon.

This work is a continuation of the research done in collaboration with B. Després et al. on the degenerate elliptic equations describing plasma heating and is a part of the PhD thesis of E. Peillon. Plasma heating is modelled by the Maxwell equations with variable coefficients, which, in the simplest 2D setting can be reduced to the 2D Helmholtz equation, where the coefficient the principal part of the operator changes its sign smoothly along an interface. Such problems are naturally well-posed in a certain weighted Sobolev space; however, the corresponding solutions cannot contribute to the plasma heating, due to their high regularity.

It is possible to demonstrate that plasma heating is induced by singular solutions, which are square integrable but do not longer lie in this weighted Sobolev space. Such solutions can be represented as a product of an L^2-function varying in tangential direction of the interface and a singular function depending on the normal direction. The numerical method suggested in previous works is based on exploiting the plasma heating property, however, leads to the variational formulation which requires high regularity of the tangential component of the singular solution along the interface where the variable coefficient changes its sign.

We were able to alter the existing variational formulation in the way that requires a lower regularity of this tangential component of the solution; this allows in particular to be able to use lower order finite elements in the numerical simulation while preserving the general order of convergence of the scheme. Moreover, our numerical experiments indicate that the original regularity assumption for the tangential component is likely to be too restrictive, and does not seem to hold even in the cases when the coefficients of the problem are analytic in both variables. We are currently investigating this phenomenon, both numerically and theoretically.

Optimal control-based numerical method for problems with sign-changing coefficient

Participants: Patrick Ciarlet, David Lassounon, Mahran Rihani.

One considers the equation $\mathrm{div}(\sigma\nabla u) = f$ in Ω (plus boundary conditions), where the diffusivity is piecewise constant, with stricly positive values in part of the domain, and stricly negative values elsewhere. When the problem is well-posed in $H^1(\Omega)$, meshing rules have been designed in the past, to ensure convergence of the discrete solution towards the exact solution (the so-called T-conform meshes). We have begun to investigate some techniques that allow in principle to compute solutions of problems with sign-changing coefficients without having to comply with those meshing rules. To that aim, ideas borrowed from control theory are used to compute the numerical solution iteratively. The results are very promising, and in particular optimal, monotonic, convergence rates are recovered for families of meshes that are not T-conform.

Towards non-local interface models

Participants: Patrick Ciarlet.

A collaboration with Juan Pablo Borthagaray (DMEL, Universidad de la República, Montevideo, Uruguay). Consider the equation $\mathrm{div}(\sigma\nabla u) = f$ in Ω (plus boundary conditions), where the diffusivity is piecewise constant, and equals σ_i in Ω_i ($i = \{1,2\}$), with $\overline{\Omega_1} \cup \overline{\Omega_2} = \overline{\Omega}$ and $\Omega_1 \cap \Omega_2 = \emptyset$. If σ_1 and σ_2 have different sign, well-posedness in $H^1(\Omega)$ may not hold. This occurs when the ratio σ_2/σ_1 belongs to the so-called *critical interval*. When the interface has a corner, we have observed that this critical interval is shrunk if one replaces the standard H^1-bilinear forms by corresponding H^s-forms ($s \in (0,1)$). However, the cost of computing the nonlocal interactions may be prohibitive in applications. Thus, our long term goal is to confine the non-local model to a neighborhood of the interface, while keeping the standard local model in the rest of the domain. A first step in this direction consists in considering the numerical solution of the fractional Laplacian of index $s \in (1/2,1)$, whose solution a priori belongs to the fractional order Sobolev space $\widetilde{H}^s(\Omega)$. Under suitable assumptions on the data, its solution is also in $H^1(\Omega)$, and we showed that one can derive error estimates in $H^1(\Omega)$-norm if one uses the standard Lagrange finite element to discretize the problem.

Computation of plasmon resonances localized at corners using frequency-dependent complex scaling

Participants: Anne-Sophie Bonnet-Ben Dhia, Christophe Hazard, Florian Monteghetti.

A smooth metallic particle supports surface plasmonic modes for a discrete sequence of negative permittivity values. For a subwavelenth particle, in the quasi-static approximation, these values are solution of the so-called plasmonic eigenvalue problem. This work investigates the case where the particle has corners. It is well-known that a metallic particle with a non-smooth boundary can exhibit, in some range of permittivity values, strongly-oscillating surface waves whose phase velocities vanish as they reach the corners. This range of permittivity corresponds to the essential spectrum of the former spectral problem.

We are interested by the existence of corner resonances, which are analogous to scattering resonances in the sense that the local behavior at each corner plays the role of the behavior at infinity. Resonant values of the permittivities are sought as eigenvalues of the plasmonic eigenvalue problem with an appropriate complex scaling applied at the corners. The finite element discretization requires a very specific mesh in the vicinity of the surface of the particle, to avoid spurious eigenvalues. Numerical results obtained for an elliptic particle with one corner show that the complex scaling deforms the essential spectrum (associated with the corner) so as to unveil both embedded plasmonic eigenvalues and complex plasmonic resonances.

Maxwell's equations in presence of a conical tip with negative electromagnetic constants

Participants: Anne-Sophie Bonnet-Ben Dhia, Mahran Rihani.

This is the continuation of many works on transmission problems involving negative materials done in collaboration with Lucas Chesnel from INRIA team IDEFIX.

In the PhD of Mahran Rihani, we were interested in the analysis of time-harmonic Maxwell's equations in presence of a conical tip of a material with negative dielectric constants. When these constants belong to some critical range, the electromagnetic field exhibits strongly oscillating singularities at the tip, and the problem is no longer well-posed in the classical framework. Previous results using the T-coercivity approach are not applicable.

Our work is inspired by what is known (thanks to ou rprevious works) for the 2D scalar case with critical coefficients: it has been proved hat well-posedness in the classical $H1$ framework is lost. This well-posedness can be recovered by working in an appropriate weighted Sobolev spaces and adding in the space one singular fonction, the so-called outgoing propagating singularity.

For the 3D tip, we established similar results, firstly for the scalar problem: a main difference with the 2D case is the possible existence of several outgoing singularities (the smallest the apex angle, the largest the number of outgoing singularities). These singularities are the eigenfunctions of a Laplace Beltrami spectral problem on the sphere, with sign-changing coefficients. The discreteness of the corresponding spectrum can be proved, for a smooth conical tip, using T-coercivity techniques on the sphere. The so-called critical interval of coefficients, for which propagating singularities exist, has been completely determined for the case of a circular conical tip.

Then we have established a new functional framework for the study of Maxwell's equations in presence of a conical tip of a negative material, when either the dielectric permittivity or the magnetic permeability, or both, belong to critical intervals. The space of electric fields for instance is obtained by adding to some weighted Sobolev space (included in $L2$) the gradients of outgoing singularities (which do not belong to $L2$). The formulation is proved to be Fredholm, the proof requiring establishing new vector potential decomposition results for non $L2$ vector fields.

Generalized normal modes of a metallic nanoparticle

Participants: Anne-Sophie Bonnet-Ben Dhia, Christophe Hazard. .

In the context of a collaboration with Matias Ruiz (University of Edinburgh) who spent 3 months at POEMS (from october to december, 2021), we have started a work concerning theoretical and numerical aspects of the so-called *plasmonic eigenvalue problem.* This problem can be formulated as follows, considering time-harmonic electromagnetic scattering (at a fixed frequency) by a metallic particle of given (possibly complex) permittivity ε in vacuum. Thanks to integral equation methods or Dirichlet-to-Neumann map techniques, such a problem can be reduced to a problem set on the particle itself. In the absence of incident wave, the reduced problem can be seen as a *non selfadjoint* eigenvalue problem where ε plays the role of an eigenvalue. The eigenfunctions associated to a possible eigenvalue are called *generalized normal modes.* The questions we are interested in are the following. What can be said about the essential / discrete spectrum? Do the generalized normal modes form a complete family (Riesz basis ?) of the natural energy space? How can we compute these eigenfunctions and associated modes?

In order to deal first with a simple situation for which a dispersion equation can be derived, we have considered the two-dimensional Helmholtz equation in a half uniform waveguide, where the end of the waveguide plays the role of a rectangular metallic particle. The problem has been implemented using the finite element library XLiFE++. Some promising numerical results have already been obtained. They show in particular two categories of modes: bulk modes and plasmonic modes (localized near the interface between the metallic particle and vacuum). A remedy for the occurrence of spurious modes has been tested.

7.2 Methods for unbounded domains, Perfectly Matched Layers and Half Space Matching method

On the Halfspace Matching Method for real wavenumber

Participants: Anne-Sophie Bonnet-Ben Dhia, Sonia Fliss.

We developed for several years a new method for the solution of 2D scattering problems with complex backgrounds, providing an alternative to Perfectly Matched Layers (PML) or other artificial boundary conditions. This method is called the Half-Space Matching (HSM) method. Based on half-plane representations for the solution, the scattering problem is rewritten as a system coupling (1) a standard finite element discretisation localised around the scatterer and (2) integral equations whose unknowns are traces of the solution on the boundaries of a finite number of overlapping half-planes contained in the domain. While satisfactory numerical results have been obtained for real wavenumbers, well-posedness and equivalence of this HSM formulation to the original scattering problem were established only for complex wavenumbers. Our new results, obtained in collaboration with Simon Chandle-Wilde (Reading University) concern the case of a real wavenumber and a homogeneous background. We proved that the HSM formulation is equivalent to the original scattering problem, and so is well-posed, provided the traces satisfy radiation conditions at infinity analogous to the standard Sommerfeld radiation condition.

As a key component of our argument we show that, if the trace on the boundary of a half-plane satisfies our new radiation condition, then the corresponding solution to the half-plane Dirichlet problem satisfies the Sommerfeld radiation condition in a slightly smaller half-plane. We expect that this last result will be of independent interest, in particular in studies of rough surface scattering.

The Half-Space Matching method for elastodynamic scattering problems in unbounded domains

Participants: Eliane Bécache, Anne-Sophie Bonnet-Ben Dhia, Sonia Fliss.

In collaboration with Antoine Tonnoir from INSA of Rouenm, we have extended the Half-Space Matching (HSM) method, first introduced for scalar problems, to elastodynamics, to solve time-harmonic 2D scattering problems, in locally perturbed infinite anisotropic homogeneous media. The HSM formulation couples a variational formulation around the perturbations with Fourier integral representations of the outgoing solution in four overlapping half-spaces. These integral representations involve outgoing plane waves, selected according to their group velocity, and evanescent waves. Numerically, the HSM method consists in a finite element discretization of the HSM formulation, together with an approximation of the Fourier integrals. We have performed numerical results, validating the method for different materials, isotropic and anisotropic, and we have compared them to results obtained with the Perfectly Matched Layers (PML) method. For materials for which PMLs are unstable in the time domain, these results highlight the robustness of the HSM method, contrary to the PML method which is very sensitive to the choice of the parameters.

Radial PML for anisotropic media

Participants: Maryna Kachanovska, Markus Wess.

This work is done in collaboration with M. Halla (MPI for Solar System Research, Göttingen). We study the question of stability of radial PMLs for the simplest case of scalar anisotropic wave equation. Our preliminary numerical experiments on wave scattering in such media indicate time-domain stability of the radial PMLs in the case when geometrical configuration of the PMLs is well-chosen (i.e. the PML is sufficiently far away from the scatterer); however, for some configurations radial PMLs fail. Our analysis in the simplified case (based on the analysis of the fundamental solution for the non-truncated perfectly matched layers, and the construction of the singular sequence for the underlying sesquilinear form) confirm existence of such instability, but for the moment it is unclear why it does not appear in other configurations.

Stability and Convergence of Perfectly Matched Layers in Dispersive Waveguides

Participants: Eliane Bécache, Maryna Kachanovska, Markus Wess.

During the post-doc of Markus Wess, we have worked on the extension of the stability and convergence analysis of the generalized perfectly matched layers (GPMLs) for dispersive media, proposed by Bécache, Joly, Vinoles. The idea is to extend the techniques developed for non-dispersive waveguides to the dispersive case. Like in the non-dispersive case, we obtain an explicit representation of the solution in the Laplace domain. The point is to estimate the inverse Laplace integral. One of the difficulties compared to the non-dispersive case is the appearance of various propagation regimes (also in the time domain): there exist backward propagating waves, forward propagating waves, evanescent waves depending on the frequency. We demonstrate that surprisingly, despite the complexity of the physical phenomena, the

GPMLs for dispersive media converge as fast as the classical PMLs for non-dispersive media. A paper is in progress.

The complex-scaled Half-Space Matching Method

Participants: Anne-Sophie Bonnet-Ben Dhia, Christophe Hazard, Sonia Fliss.

We developed for some years a method that we call the Half-Space Matching (HSM) method, to solve scattering problems in unbounded domains, when classical approaches are either not applicable or too expensive. This method is based on an explicit expression of the "outgoing" solution of the problem in half-spaces, by using Fourier, generalized Fourier or Floquet transforms when the background is respectively homogeneous (possibly anisotropic), stratified or periodic. The domain exterior to a bounded region enclosing the scatterers is recovered by a finite number of halfspaces (at least 3). The unknowns of the formulation are the restriction of the solution to the bounded region and the traces of the solution on the infinite boundaries of the halfspaces. The system of equations is derived by writing compatibility conditions between the different representations of the solution. Although the HSM method works in the non-dissipative case, the theoretical and the numerical analysis of the method has been done only in the dissipative case. In the present work done in collaboration with Simon Chandler-Wilde and Karl-Michaël Perfekt form Reading University, we propose a new formulation of the method which is well-suited for the theoretical and numerical analysis of the non dissipative case. In the spirit of PMLs, the idea is to replace the system of equations on the traces by similar equations on exponentially decaying analytical extensions of the traces. In the simple case of the Helmholtz equation, we have proved that this formulation is of Fredholm type and is well-posed. Besides the interest for the theory, this new formulation is also well-suited for numerical purposes. Indeed one can show that the error due to the truncation of the infinite boundaries of the half-spaces decays exponentially with the distance of truncation, which was not the case for the standard method. The analysis requires the study of double-layer potential integral operators on intersecting infinite lines, and their analytic continuations. The effectiveness of the method is validated by preliminary numerical results.

PML-BIE methods for unbounded interfaces

Participants: Alexis Anne, Anne-Sophie Bonnet-Ben Dhia, Luiz Faria.

In order to handle infinite interfaces in a boundary integral equation context, a few options are available. For simple geometries, one can construct a problem specific Greens function which incorporates the imposed boundary condition on all but a bounded portion of the interface, thus reducing the problem again to integrals over bounded curves/surfaces. This has the advantage of being conceptually simple provided such problem-specific Greens function can be efficiently computed. Unfortunately, for all but the simplest geometries, the representation of the problem specific Greens function involves challenging integrals which must be approximated numerically.

An alternative approach consists of utilizing the free-space Greens function — readily available for many PDEs of physical relevance — in conjunction with a truncation technique. For non-dissipative problems, the slow (algebraic) decay of the Greens function makes the choice of truncation technique an important aspect which needs to be considered in order to reduce the errors associated with the domain's truncation. An easy-to-implement solution, the so-called windowed Green function approach, has been proposed and validated in several configurations.

We are currently investigating the interest of using instead a complex-scaled Green function, which amounts to combine the method of perfectly-matched-layers (PMLs) and boundary integral equations.

We have applied this idea to the 2D linear time-harmonic water wave problem, in finite or infinite depth, writing a complex-scaled integral equation on the free surface.

The method works well and numerical results show that the error decays exponentially with respect to the distance of truncation. Let us mention that because the water waves are surface waves, the windowed Green function approach does not work for this problem.

Evaluation of oscillatory integrals in the Halfspace Matching Method

Participants: Amond Allouko, Anne-Sophie Bonnet-Ben Dhia, Sonia Fliss.

This work concerns the Half Space Matching method described just above. A main ingredient of this method is a half-space formula for the outgoing scattered field, as a function of its trace on the boundary of the half-space. For homogeneous isotropic or anisotropic backgrounds, this formula is obtained thanks to a partial Fourier transform and gives raise to oscillatory integrals, hard to compute with standard quadrature formulas. We proposed two ways for improving both the accuracy and the efficiency of the evaluation of this half-space formula. When it is evaluated at a point located far enough from the boundary of the half-space, a simple far-field formula can be used. For other points, a deformation of the Fourier path in the complex plane allows to get rid of the oscillations and of the singularity at the branch point. Numerical experiments for the simple 2D isotropic case confirm that these two ideas drammatically reduce the cost of the method.

7.3 Fast solution of boundary integral equations

General-purpose kernel regularization of boundary integral equations via density interpolation

Participants: Luiz Faria, Marc Bonnet.

This research is done in collaboration with Carlos Pérez-Arancibia (PUC, Chile).

We develop a general high-order kernel regularization technique applicable to all four integral operators of Calderón calculus associated with linear elliptic PDEs in two and three spatial dimensions. The proposed technique relies on interpolating the density function around the kernel singularity in terms of solutions of the underlying homogeneous PDE, so as to recast singular and nearly singular integrals in terms of bounded (or more regular) integrands. We developed a simple interpolation strategy which, unlike previous approaches, does not entail explicit computation of high-order derivatives of the density function along the surface. Furthermore, the proposed approach is kernel- and dimension-independent in the sense that the sought density interpolant is constructed as a linear combination of point-source fields, given by the same Green's function used in the integral equation formulation, thus making the procedure applicable, in principle, to any PDE with known Green's function. In the initial work [23], we have focused on Nyström methods for the (scalar) Laplace and Helmholtz equations and the (vector) elastostatic and time-harmonic elastodynamic equations. The method's accuracy, flexibility, efficiency, and compatibility with fast solvers was demonstrated by means of a variety of large-scale three-dimensional numerical examples.

Boundary Element - Finite Element coupling for transient fluid-structure interaction

Participants: Marc Bonnet, Stéphanie Chaillat, Alice Nassor.

This study is done in collaboration with Bruno Leblé (Naval Group). It aims at developing computational strategies for modelling the impact of a far-field underwater explosion shock wave on a structure, in deep water. An iterative transient acoustic-elastic coupling is developed to solve the problem. Two complementary methods are used: the Finite Element Method (FEM), that offers a wide range of tools to compute the structure response; and the Boundary Element Method (BEM), more suitable to deal with large surrounding acoustic fluid domains. We concentrate on developing a transient FEM-BEM coupling algorithm with a fast convergence. Since the fast transient BEM is based on a fast multipole-accelerated Laplace-domain BEM (implemented in the in-house code COFFEE), extended to the time domain by the Convolution Quadrature Method (CQM), we have proposed a proof of convergence of global-in-time iterative solution procedures for the coupled transient fluid-structure problem. This proof itself relies on solvability results for the coupled acoustic-elastic problems, some of which we established in the course of this study, and the initial-boundary value problems involved in coupling iterations. We have also implemented a 2D version of this algorithm and assessed its sensitivity to various parameters.

Improvements of hierarchical matrix based Boundary Element Methods for visco-elastodynamic problems

Participants: Laura Bagur, Stéphanie Chaillat, Patrick Ciarlet, Sara Touhami.

It is well known in the literature that standard $\mathcal{H}$-matrix based methods, although very efficient tools for asymptotically smooth kernels, are not optimal for oscillatory kernels. In a previous work, we have shown that the method is already an efficient tool and should be used in the mechanical engineering community due to its straightforward implementation compared to $\mathcal{H}^2$-matrix, or directional, approaches.

We are currently investigating two possible improvements of this approach. Since in practice, not all materials are purely elastic it is important to be able to consider visco-elastic cases. In this context, we have studied the effect of the introduction of a complex wavenumber on the accuracy and efficiency of hierarchical matrix ($\mathcal{H}$-matrix) based fast methods for solving dense linear systems arising from the discretization of the elastodynamic Green's tensors. We have shown that more compression can be achieved when a lot of damping is introduced while for a small amount of damping (of the order of what is typically observed in real soils), no more compression can be obtained. Hopefully, such configurations with a large amount of damping are encountered in the context of the solution of transient purely elastic problems with the convolution quadrature method. We have proposed a new admissibility condition to improve the efficiency of such methods.

Then, since $\mathcal{H}$-matrices are an automatic tool to remove redundant informations, we are studying its efficiency in the context of realistic sedimentary basins with high velocity contrasts. Fast multipole accelerated Boundary Element Methods have been shown to be inefficient in this context due to the need to use an over refined mesh for one of the two connected domains. We have shown that the use of $\mathcal{H}$-matrices even though the method is known to be less efficient than fast multipole methods, enable to consider large-scale configurations of sedimentary basins with high velocity contrasts between the different layers of the soil. The capabilities of the methods have been demonstrated on a lot of configurations with semi-analytical or reference solutions.

Modelling the sound radiated by a turbulent flow

Participants: Stéphanie Chaillat, Jean-François Mercier.

This was the subject of the PhD study of Nicolas Trafny, done in collaboration with Gilles Serre (Naval Group) and Benjamin Cotté (IMSIA). The aim is to develop an optimized method to determine numerically the 3D Green's function of the Helmholtz equation in presence of an obstacle of arbitrary shape, satisfying the Neumann boundary condition at the boundary surfaces. This so-called rigid Green's function is useful to solve the Lighthill's equation, giving the hydrodynamic noise radiated by a ship. First an integral equation is derived, expressing the rigid Green's function versus the free space Green's function. Then a Boundary Element Method (BEM) implemented in the code COFFEE is used to compute the rigid Green's functions. In order to consider realistic geometries in a reasonable amount of time, fast BEMs are used: fast multipole accelerated BEM and hierarchical matrix based BEM. The efficiency of these two approaches is tested on simple geometries for which analytic solutions can be determined (sphere, cylinder, half plane) and the methods are also compared on realistic geometries of interest for the industrial partner (NACA profiles, boat propeller). A second PhD is scheduled, extending the study to elastic geometries.

Asymptotic based methods for very high frequency problems.

Participant: Eric Lunéville.

This research is developed in collaboration with Marc Lenoir and Daniel Bouche (CEA).

It has recently been realized that the combination of integral and asymptotic methods was a remarkable and necessary tool to solve scattering problems, in the case where the frequency is high and the geometry must be finely taken into account.

In order to implement the high-frequency approximations that we are developing as part of these hybrid HF/BF methods, we have introduced new geometric tools into the XLiFE++ library, in particular splines and B-Splines approximations as well as parameterizations to access quantities such as curvature, curvilinear abscissa, etc. We have also started to interface the OpenCascad library to the XLiFE++ library, which will eventually allow us to manage more complex geometric situations (cylinder and sphere intersection for example). In parallel, we have completed the implementation of 2D HF approximations in the shadow-light transition zone based on the Fock function. Diffraction by a 2D corner is in progress.

7.4 Domain decomposition methods

Non-overlapping DDM with PML transmission conditions and multidirectionnal sweeping preconditioners for Helmholtz problems

Participant: Modave Axel.

In collaboration with Ruiyang Dai (UCLouvain), Christophe Geuzaine (Liège U.) and Anthony Royer (Liège U.), we have worked on non-overlapping domain decomposition methods (DDM) for Helmholtz problems with transmission conditions based on non-reflecting boundary treatments. It is well-known that the convergence rate of non-overlapping DDM applied to the parallel finite-element solution of large-scale time-harmonic wave problems strongly depends on the transmission conditions enforced at the interfaces between the subdomains.

Transmission operators based on perfectly matched layers (PML) have proved to be well-suited for configurations with layered domain partitions. Unfortunately, the extension of the PML-based DDM for more general partitions with cross-points (where more than two subdomains meet) is rather tricky and requires some care. We have proposed a non-overlapping DDM with PML transmission conditions for checkerboard decompositions that takes cross-points into account. In such decompositions, each subdomain is surrounded by PMLs associated to edges and corners. The continuity of Dirichlet traces at the interfaces between a subdomain and PMLs is enforced with Lagrange multipliers. This coupling strategy offers the benefit of naturally computing Neumann traces, which allows to use the PMLs as discrete operators approximating the exact Dirichlet-to-Neumann maps. We considered two possible Lagrange multiplier finite element spaces. We have studied the behavior of the corresponding DDM on several numerical examples.

In parallel, we have explored a family of generalized sweeping preconditionners for Helmholtz problems with non-overlapping checkerboard partition of the computational domain. The domain decomposition procedure relies on high-order transmission conditions and cross-point treatments, which cannot scale without an efficient preconditioning technique when the number of subdomains increases. With the proposed approach, existing sweeping preconditioners, such as the symmetric Gauss-Seidel and parallel double sweep preconditioners, can be applied to checkerboard partitions with different sweeping directions (e.g. horizontal and diagonal). Several directions can be combined thanks to the flexible version of GMRES, allowing for the rapid transfer of information in the different zones of the computational domain, then accelerating the convergence of the final iterative solution procedure. We have proposed several two-dimensional finite element results to study and to compare the sweeping preconditioners, and to illustrate the performance on cases of increasing complexity.

Non-overlapping Domain Decomposition Method (DDM) using non-local transmission operators for wave propagation problems.

Participants: Patrick Joly, Emile Parolin.

A chapter of this research has been closed with the PhD defense of É. Parolin abd the end of the ANR Project NonlocalDD leaded by X. Claeys (Paris 6 and Alpines). Our research in this domain has recently received several regognitions : Patrick Joly has been invited as a plenary speaker at DDM20 (in the framework of the series of international conferences dedicated to Domain Decomposition Methods) and Émile Parolin has received a Prize from the Institut Polytechnique de Paris for his thesis. In addition, three articles on the subject - included a review article - have been accepted for publication.

Our work will know a new orientation in the framework of the RAPID Project Hybox2. This project, which is headed by T. Abboud and regroups IMACS, Ariane Group and POEMS, started in September. One particular subject of attention will concern the quasi-localization of the communication operator between subdomains in the presence of junction points (see also the previous activity report).

7.5 Inverse Problems, Invisibility and Optimization

Imaging junctions of waveguides

Participants: Laurent Bourgeois, Fliss Sonia, Fritsch Jean-François, Hazard Christophe.

A new activity recently started concerning forward and inverse scattering in junctions of waveguides. It corresponds to the PHD of Jean-François Fritsch and is a collaboration with the CEA-List, in particular Arnaud Recoquillay. Firstly, we have considered the junction between several closed waveguides. It is well-known that defects such as cracks often occur in weld bead of metallic pipes, which can be seen as junctions of waveguides. This explains why it is necessary to adapt Non Destructive Testing procedures to that kind of configuration. Forward scattering problems in junctions of closed waveguides are quite standard and can be solved with classical finite element methods coupled with transparent boundary conditions using Dirichlet to Neumann maps. However, solving inverse scattering problems for such geometries is less standard. In order to cope with those problems, we use a modal version of the classical Colton-Kirsch Linear Sampling Method. The main issue stems from the fact the LSM relies on the fundamental solution, which does not have a closed-form expression in a junction of waveguides, contrary to the case of a homogeneous waveguide. In order to cope with this problem, the main ingredient we introduce is the so-called reference fields, which are the responses of the junction without any defect to the guided modes considered as incident waves. We also use the symmetry property of the fundamental solution. The reference fields enable us to adapt the LSM by paying a reasonable computational cost, in the sense that it is not necessary to actually compute the fundamental solution for each sampling point of the grid. We have shown the feasibility of our method with the help of two-dimensional acoustic numerical experiments, for instance in the case of a junction of three waveguides.

Secondly, we have considered the more challenging case of a closed waveguide which is embedded in a surrounding infinite medium. A typical situation is the case of a steel cable which is partially embedded into concrete or some fluid. It may happen that some defects within the embedded part of the cable or at the interface between the cable and the surrounding medium have to be retrieved from measurements located on the only accessible part of the cable, that is its free part. In a first attempt we have simplified the problem by considering a two-dimensional acoustic problem. Despite such simplification, both the forward and the inverse scattering problems are challenging. We address the forward problem by using Perfectly Matched Layers in the transverse direction, which has the effect of closing the waveguide but of introducing a non-selfadjoint eigenvalue problem in the transverse direction. At the continuous level, we use the Kondratiev approach to establish well-posedness of the forward problem and to specify the asymptotic behaviour of solutions at infinity. In order to compute a numerical solution, we also introduced transparent boundary conditions in the infinite direction of the waveguide, that is a Dirichlet to Neumann map with an overlap. This latter amounts to a transparent thick boundary condition. Using again the Kondratiev approach, we estimated the error introduced by these transparent boundary conditions. We addressed the inverse problem with the help of the modal Linear Sampling Method by using the same ingredient as introduced in the previous case of the junction of closed waveguides. However a new difficulty arises since an open waveguide is characterized by radiation losses, which can be seen as lost information from the point of view of the inverse problem. In the case of a steel cable embedded into a concrete medium or into a fluid, this is all the more difficult as the speed in the core is larger than the speed in the sheath. As a consequence, once we have replaced the infinite medium by PMLs, the modes are either leaky modes or PML modes, both of them being evanescent. In this sense, the inverse problem is more challenging than in the case of closed waveguides, which benefit from a finite number of propagating modes. However, the numerical experiments that we have made show that the LSM is efficient to retrieve defects provided they are sufficiently close to the interface between the closed and the open waveguide. We are currently investigate the more involved case of an elastic waveguide which is embbeded in water, which correspond to a solid-fluid interaction problem.

Computation of the interior transmission eigenvalues in presence of strongly oscillating singularities

Participants: Anne-Sophie Bonnet-Ben Dhia, Christophe Hazard. .

In the context of time-harmonic scattering by a bounded penetrable scatterer, interior transmission eigenvalues correspond, when they are real, to discrete frequencies for which there exists an incident wave which does not scatter. At such frequencies, inversion algorithms such as the linear sampling method fail. Real interior transmission eigenvalues are a part of a larger spectrum made of complex values, which has been largely studied in the case where the difference between the parameters in the scatterer and outside does not change sign on the boundary. In collaboration with Lucas Chesnel (INRIA team IDEFIX), we obtained some years ago some results for a 2D configuration where such sign-change occurs. The main idea was that, due to very strong singularities that can occur at the boundary, the problem may lose Fredholmness in the natural H^1 framework. Using Kondratiev theory, we proposed a new functional framework where the Fredholm property is restored. This is very similar (while more intricate) to what happens for the plasmonic eigenvalue problem in presence of a corner of negative material.

This explains why we decided, in collaboration with Lucas Chesnel and Florian Monteghetti (ISAE), to extend the numerical method we used for plasmonic eigenvalues to interior transmission eigenvalues. It has been already checked that a naive finite element computation does not converge, and that the convergence is restored by using some complex scaling near the singular point.

Mixed formulations of the Tikhonov regularization

Participant: Laurent Bourgeois.

Work done in collaboration with Jérémi Dardé, from IMT Toulouse.

For a quite long time now, we have developed the notion of mixed formulations of the Tikhonov regularization in collaboration with Jérémi Dardé (IMT Toulouse). This notion has connections with the old concept of quasi-reversibility introduced by Lattès and Lions (67) and is intended to regularize ill-posed PDE problems. Essentially, our mixed formulations enable us to adapt this concept to a friendly Finite Element Method context using simple conforming elements. We adapted the notion of mixed formulations of the Tikhonov regularization to the context of data assimilation problems, in particular by applying the Morozov's discrepancy principle to select the regularization parameter as a function of the amplitude of noise. A specific problem in data assimilation problems is that the underlying operator has not a dense range, which required to extend some well-known results to that situation. A dual problem was introduced in order to compute the regularized solution corresponding to such regularization parameter, possibly by enforcing the solution to satisfy some constraints.

Adaptive eigenspace bases for topology optimization

Participant: Marc Bonnet.

Work done in collaboration with Wilkins Aquino, Duke University, USA.

We investigated the use of adaptive eigenspace bases (AEBs) to reduce design dimension in topology optimization (TO). The concept of AEB, recently shown to be effective for solving medium imaging problems, is here applied to define approximation spaces for density design fields describing optimal topologies. This approach yields low-dimensional approximation spaces and implicitly regularized designs. The known ability of AEBs to approximate piecewise constant fields is especially useful for TO. Performance of the AEBs has been compared against conventional TO implementations in problems for static linear elasticity, showing comparable structural solutions, computational cost benefits, and consistent design dimension reduction.

Asymptotic model for elastodynamic scattering by a small surface-breaking defect

Participant: Marc Bonnet.

Work done in collaboration with Marc Deschamps and Eric Ducasse, I2M, Bordeaux.

We establish a leading-order asymptotic model for the scattering of elastodynamic fields by small surface-breaking defects in elastic solids. The asymptotic form of the representation formula of the scattered field is written in terms of the elastodynamic Green's tensor, which is in fact available in semi-analytical form for some geometrical configurations that are of practical interest in ultrasonic NDT configurations. A rigorous proof of the resulting leading asymptotic approximation is obtained. Preliminary numerical examples have been performed on cylindrical elastic pipes with small indentations on the outer surface.

Shape optimization problems involving slow viscous fluids

Participant: Marc Bonnet.

Work done in collaboration with Shravan Veerapaneni and his group, University of Michigan, USA.

This collaboration addresses the design and implementation of computational methods for solving shape optimization problems involving slow viscous fluids modelled by the Stokes equations. We have developed a new boundary integral equation (BIE) approach for finding optimal shapes of peristaltic pumps that transport viscous fluids, as well as dedicated formulas for computing the shape derivatives of the relevant cost functionals and constraints, expressed in boundary-only form. By employing these formulas in conjunction with a BIE approach for solving forward and adjoint problems, we completely avoid the issue of volume remeshing when updating the pump shape as the optimization proceeds. This is especially useful when the fluid carries objects (e.g. particles, deformable vesicles) whose motion is not known beforehand. Significant cost savings are achieved as a result, and we demonstrate the performance on several numerical examples. We also investigate the optimization of the slip velocity (modeling cilia beating) and the shape of self-propelled micro-swimmers achieving self-propelling at least energy expense. Potential applications pertain to the understanding and optimization of various processes involving biological fluids and organisms moving in them.

7.6 Asymptotic analysis, homogenization and effective models

Effective wave motions in periodic media at finite frequencies

Participant: Marc Bonnet.

Work done in collaboration with Bojan Guzina, University of Minnesota, USA.

We investigated effective time-harmonic (anti-plane elastic) wave motions in periodic media containing fractures at finite frequencies and wavenumbers, when the latter are close to apexes of the Brillouin zone. Mean motions are defined via inner products with eigenfunctions of the wave equation in Bloch-wave form, and their governing equation is found from asymptotic expansions with respect to frequency-wavenumber perturbations. We considered cases with isolated, repeated or nearby apex eigenvalues. This approach in particular yields approximations of forced wave motions, and of the dispersion surfaces

Propagation of ultrasounds in complex biological media

Participants: Laure Giovangigli, Quentin Goepfert.

This is a joint work with Josselin Garnier (X-CMAP) and Pierre Millien (Institut Langevin). This project aim at modelling and studying the propagation and diffusion of ultrasounds in complex multi-scale media in order to obtain quantitive images of physical parameters of these media.

The propagation of ultrasounds in biological tissues or composite materials is a complex multi-scale phenomenon : the scattered wave is produced by small (compared to the wavelength) inhomogeneities randomly distributed throughout the media, but the time of flight is affected by the slow (compared to the wavelength) variations of the medium. To capture both of those effects, the medium is described as a slowly varying or homogeneous by parts medium in which lie randomly distributed small scatterers. In order to characterize the response of this medium to an incident plane wave, we perform an asymptotic expansion of the scattered wave using stochastic homogenisation techniques. The difficulties lie in the slow varying background and the transmission conditions at the boundary of the medium or between the different components. We then derive quantitative error estimates given that the random distribution of inhomogeneities in the different components verify mixing conditions. Finally we present numerical simulations to illustrate and validate our results. This constitutes the first part of this project.

Modelling a thin layer of randomly distributed nano-particles

Participants: Sonia Fliss, Laure Giovangigli, Amandine Boucart.

This is a joint work with Bruno Stupfel from CEA-CESTA.We study the time-harmonic scattering by a heterogeneous object covered with a thin layer of randomly distributed nanoparticles. The size of the particles, their distance between each other and the layer's thickness are all of the same order but small compared to the wavelength of the incident wave. Solving numerically Maxwell's equation in this context is very costly. To circumvent this, we propose, via a multi-scale asymptotic expansion of the solution, an effective model where the layer of particles is replaced by an equivalent boundary condition. The coefficients that appear in this equivalent boundary condition depend on the solutions to corrector problems of Laplace type defined on unbounded random domains. Under the assumption that the particles are distributed given a stationary and mixing random point process, we prove that those problems admit a unique solution in the proper spaces with either homogeneous Dirichlet (for $d \geq 2$) or Neumann boundary conditions (for $d = 3$) on the inclusions. We then establish quantitative error estimates for the effective model and present numerical simulations that illustrate our theoretical results.

Stability and Accuracy of the Time-Domain Foldy-Lax Model

Participant: Maryna Kachanovska.

The Foldy-Lax model is an asymptotic model used to compute the solution to the problem of scattering by small obstacles. While this subject had been fairly well-studied in the frequency-domain, its time-domain analysis is still in its infancy stage. Necessity of such analysis is justified by the following observation: the time-domain counterparts of the frequency-domain Foldy-Lax models can exhibit instabilities. We show that this is in particular the case for the model derived earlier in [Cassier, Hazard, 2014] whose time-domain counterpart admits exponentially growing solutions when the obstacles are located close to each other.

A stabilization of the Foldy-Lax model was obtained by reconsidering it as a Galerkin discretization of the single-layer boundary integral equation with a specula basis consisting of functions which are constant on each of the obstacles. Let us remark that the model of [Cassier, Hazard, 2014] fits into this framework, however, the respective Galerkin matrix is perturbed; this perturbation is responsible for occurrence of time-domain instabilities. We have proven the convergence of this new asymptotic method when the size of the obstacles tends to zero. An interesting side effect of our analysis is the proof of the superconvergence of the approximation of the scattered field, compared to the approximation of its Foldy-Lax density. This idea can be extended to analyze other problems: transmission problems, sound-hard scattering etc.

We are currently summarizing our results in a manuscript.

Enriched homogenized model in the presence of boundaries for wave equations

Participant: Clément Bénéteau, Sonia Fliss.

We study the wave equation in presence of a periodic medium when the period is small compared to the wavelength. The classical homogenization theory enables to derive an effective model which provides an approximation of the solution. However, it is well known that these models are not accurate near the boundaries. In this work, we propose an enriched asymptotic expansion which enables to derive high order effective models at any order, when the geometry of the periodic medium is simple — absence of corners and its boundary (or the interface with other media) must lie in a direction of periodicity. For the model at order 1, the volume equation is the same than the classical one, but the boundary/transmission conditions is modified. Let us mention that the model of order 2 is particularly relevant when one is interested in the long time behaviour of the solution of the time-dependent wave equation. Indeed, it is well-known that the classical homogenized model does not capture the long time dispersion of the exact solution. In several works, homogenized models involving differential operators of high order (at least 4), are proposed for the wave equation in infinite domains. Dealing with boundaries and proposing boundary conditions for these models were open questions. Our approach enables to propose appropriate and accurate boundary conditions for these models. The analysis of such model and its implementation is under investigation. This work is the fruit of a long time collaboration with Xavier Claeys (LJLL, Sorbonne University) and a recent one with Timothée Pouchon (EPFL). Clement Beneteau has defended his PhD thesis in January 2021.

A stable and unified model for Faraday cages

Participants: Eric Lunéville, Jean-François Mercier.

In collaboration with Agnès Maurel (Langevin Institut), Kim Pham (IMSIA) and B. Delourme (LAGA, Paris 13), we study effective transmission conditions capable of reproducing the effect of a periodic array of Dirichlet wires on wave propagation, in particular when the array delimits a Faraday cage capable of resonating. In the literature, different transmission conditions are obtained by homogenization, the validity of which depends on the frequency. In practice, it is difficult to deal with such piecewise conditions, especially if the problem is in the time domain. By using an homogenization process at higher orders, we have developed a new interface effective model, involving jump conditions for the fields whose interface parameters are determined thanks to matched asymptotic expansions. We have demonstrated the validity of this new model, called unified because it is valid at all frequencies and therefore easy to use in time. To illustrate the accuracy of the model, a formulation guaranteeing the stability of a numerical scheme based on finite elements has been implemented.

7.7 Waves in quasi 1D or 2D domains

Heat and momentum losses effect on hydrogen detonations

Participants: Luiz Faria.

This is work in collaboration with Josue Melguizo-Gavilanes and Ferando Veiga-Lopez (institute PPrime) in the context of the ANR-JCJC FASTD.

The steady propagation of hydrogen-fueled detonation waves with momentum and heat losses is analyzed including detailed kinetics by means of the detonation velocity - friction coefficient ($D-c_f$) curves. We show that for undiluted H_2-O_2 mixtures the heat losses ($c_q = \alpha c_f$ with α a similarity factor) yield strong changes on the $D-c_f$ steady solutions, moving the critical point, $c_f^\star$, towards faster detonations propagating in smoother tubes (i.e., lower c_f), which also limits their propagation at low velocities. Thus we found that for a high enough sim. factor $\alpha > \alpha^\star$ no detonations may propagate at the *choking* regime. For $\alpha < \alpha^\star$, we found several solutions given a fixed D, confirming the existence of set-valued solutions only for *choking* detonations with a realistic chemistry model and a popular mixture. We also assessed the effect of the mixture composition (nitrogen and argon were added) on the $D-c_f$ curves choosing different similarity factors: (i) nitrogen strongly reduces $c_f^\star$ and moves it to faster D; no detonations were found at the *choking* regime for very low α, limiting the appearance of the set-values; (ii) argon presents a peculiar non-linear effect on the $c_f^\star$ but always moves it to slower D; the set-valued regions widen and higher α are required to avoid them. We finally relate our results to realistic configurations, comparing them to previous works: good agreements were found. We asses the uncertainties that appear due to the models applied; the detailed chemical schemes used strongly modify the predictions.

Mathematical modelling of thin coaxial cables

Participants: Patrick Joly, Akram Beni Hamad.

This topic is the subject of a long term collaboration with Sébastien Imperiale (M3disim).

We are interested in the electromagnetic wave propagation in a network of thin coaxial cables (made of a dielectric material which surrounds a metallic inner wire) with heterogeneous cross section. The first goal, achieved in the PhD thesis of G. Beck few years ago, was to reduce 3D Maxwell's equations to a quantum graph in which, along each edge, one is reduced to compute the electrical potential and current by solving 1D wave equations coupled by vertex conditions. We obtained various effective 1D like models.

Since two years, in the framework of the PhD thesis of Akram Beni Hamad, we work on the numerical validation and quantitative evaluation of these models.

In a first step we have proposed, analysed and implemented efficient numerical methods for solving the 1D approximate problems. However, In order to achieve the 1D/3D comparison, a major challenge is to design numerical methods for solving 3D Maxwell's equations dedicated to taking into account the specificity of thin electric cables. A naive discretization procedure based on a leap-frog explicit scheme can be really costly because of the thinness of the cable. In the case have then proposed an original approach consisting in adapting Nédélec's edge elements to elongated prismatic meshes and proposing a hybrid time discretization procedure which is explicit in the longitudinal directions and implicit in the transverse ones. In particular, the resulting CFL stability condition is not affected by the small thickness of the cable.

However the above is only effective for perfectly cylindrical cables : its naive extension do deformed cables generates longitudinal / transverse recoupling that destroys the efficiency of the method. Thus, In the presence of deformations, the method needs to be modified. In order to preserve the longitudinal / transverse decoupling, we propose a hybrid method combining a conforming discretization in the longitudinal variable and a discontinuous Galerkin method in the transverse ones. This method is designed in order to coincide with the previous one in the cylindrical parts of the cable. The analysis ans implementation of this method are currently under way.

Finally, in the framework of a recent collaboration with G. Beck, A. Beni Hamad is deveploping adapted numerical methods for higher order effective models.

7.8 From periodic to random media

Guided modes in a hexagonal periodic graph-like domain: the zigzag and the armchair cases

Participant: Sonia Fliss.

In collaboration with Bérangère Delourme (LAGA, Paris 13), we have studied the spectrum of periodic operators in thin graph-like domains: more precisely Neumann Laplacian defined in periodic media which are close to quantum graphs. Moreover, we exhibit situations where the introduction of lineic defects into the geometry of the domain leads to the appearance of guided modes. We dealt with rectangular lattices few years ago and more recently we are studying hexagonal lattices. In this last case, we have shown that the dispersion curves have conical singularities called Dirac points. Their presence is linked to the invariance by rotation, symmetry and conjugation of the model. We have also observed that the direction of the line defect leads to very different properties of the guided modes. Finally, we have also proven the stability of the guided modes when the position of the edge varies (keeping the same direction). We want now to (1) open gaps (around the Dirac points) in the spectrum of the periodic operator by breaking one of the invariance of the problem (2) study the effect on guided waves.

Discrete honeycombs, rational edges and edge states

Participant: Sonia Fliss.

This work is done in collaboration with C.L. Fefferman (Princeton University) and M.I. Weinstein (Columbia University). We Consider the tight binding model of graphene terminated along a *rational edge*, *i.e.* an arbitrary line in a direction of periodicity of the structure. We present a comprehensive rigorous study of zero energy / flat band edge states; all zigzag-type edges support zero energy / flat bands and armchair-type edges support no zero energy / flat bands. We also perform a careful numerical investigations showing very strong evidence for the existence of non-zero energy (dispersive / non-flat) edge states. We are investigating now the existence of states which are bounded and oscillatory parallel to an irrational edge and which decay into the bulk. The idea is to construct an « edge » state for irrational termination as the limit of a sequence of edge state wave-packets (superpositions of edge states) of rationally terminated structures.

Wave propagation in quasi periodic media

Participants: Sonia Fliss, Patrick Joly, Pierre Amenoagbadji.

This is the subject of the PhD thesis of P. Amenoagbadji. We refer the reader to the previous activity report for a general presentation of the thematic and the related scientific context.

Our main objective is to develop original numerical methods for the solution of the time- harmonic wave equation in quasi-periodic media, in the spirit of the methods that we have developed previously for periodic media.

We began with the 1D case, in the presence of absorption. More precisely, we consider the 1D Helmholtz equation whose coefficients are quasi-periodic functions, namely the traces along a particular line of a periodic functions in higher dimensions (such functions are no longer periodic in general). Accordingly, the idea is to interpret the solution of the 1D Helmholtz equation as the trace along the same line of the solution of an augmented PDE in higher dimensions, with periodic coefficients. This allows us to extend the ideas of the DtN based methods previously developed at POEMS for periodic media. However the associated mathematical and numerical analysis of the method is more difficult because the augmented PDE is anisotropically degenerate in the sense that the principal part of the equation is no longer elliptic. The associated numerical method, based on the solution of Dirichlet cell problems and a Ricatti equation, has been implemented, an article is being written.

The continuation of this work separates in two directions :

- The 1D non absorbing case. The natural idea is to pass to the limit is the previous method when the absorption goes to 0. Understanding the limit process is related to the spectral analysis of a 1D differential operator along the real line with quasi-periodic coefficients. Most often, this spectrum could have an absolutely continuous part, a singular continuous part or even a point part. For now, we can conclude easily on the limiting absorption principle when the spectral variable (i.e. the square of the frequency) is not in the spectrum. When it is in the point part of the spectrum, the limiting absorption principle cannot hold in a classical setting. In all the other cases, the question is still open. Even if from a theoretical point of view, the answer of the limiting absorption principle is not clear, we can use a numerical procedure thanks to the method described above. The first difficulty is that we have shown that the Dirichlet cell problems are not well-posed for intervals of frequencies. The solution is to solve Robin cell problems instead and extend our method to construct DtN transparent boundary conditions or RtR boundary conditions. The second difficulty concerns the solution of the Ricatti equations. This work is in progress.
- The 2D periodic half-space. We are studying the propagation of waves in presence of a 2D periodic half-space. When the interface lies in a direction of periodicity of the half-space, it suffices (and this is what we have done few years ago) to apply the Floquet Bloch Transform in the direction of the interface, and then solve a family of waveguide problems. Dealing with an interface which

does not lie in any direction of periodicity was an open question until few months ago. This problem does not concern, properly speaking a quasi-periodic medium bur the problem can be seen as quasi-periodic in the coordinate along the boundary. That is why we can use a similar but more subtle approach than above, to interpret the solution of our problem as the trace on a half-plane of the solution of a 3D PDE with periodic coefficients. The theoretical justification and the implementation of the method are in progress.

8 Bilateral contracts and grants with industry

8.1 Bilateral Contracts with Industry

- Contract with DGA and Naval Group on *transient fluid-structure coupling caused by remote underwater explosions, including cavitation effects*
  Participants: M. Bonnet, S. Chaillat, A. Nassor
  Start: 10/2020. End: 09/2023. Administrator: CNRS
- Contract and CIFRE PhD with CEA on *Modelling of thin layers of randomly distributed nanoparticles for electromagnetic waves*
  Participants: A. Boucart, S. Fliss, L. Giovangigli
  Start: 10/2019. End: 09/2022. Administrator: ENSTA
- Contract and CIFRE PhD with Naval Group on *flow noise prediction*
  Participants: J-F Mercier, B. Cotté, N. Trafny
  Start: 04/2018. End: 03/2021. Administrator: ENSTA

9 Partnerships and cooperations

9.1 National initiatives

ANR

- ANR project MODULATE *(Modeling lOng-perioD groUnd motions, and assessment of their effects on Large-scale infrAsTructurEs)*
  Partners: ENSTA (UME), Inria POEMS, CentraleSupelec, BRGM, GDS
  Start: 11/2018. End: 10/2021. Administrator: ENSTA
  Participant of POEMS: S. Chaillat
  Coordinator: K. Meza Fajardo (BRGM)
- ANR JCJC project FAsTD *(Flame Acceleration and Transition to Detonation in Narrow Channels)*
  Partners: INRIA (POEMS), CNRS (Institut Pprime)
  Start: 12/2020. End: 12/2024. Administrator: CNRS
  Participant of POEMS: L. Faria
  Coordinator: J. Melguizo Gavilanes (Institut Pprime)
- ANR JCJC project WavesDG *(Wave-specific Discontinuous Galerkin Finite Element Methods for Time-Harmonic Problems)*
  Partners: POEMS (CNRS, INRIA, ENSTA), Atlantis (INRIA), LAUM (U. Le Mans), U. Liège
  Start: 10/2021. End: 12/2025. Administrator: CNRS
  Coordinator: A. Modave (POEMS, CNRS)
  Other participant of POEMS: P. Ciarlet (POEMS, ENSTA)

DGA

- Contract on *boundary element methods and high-frequency problems*
  Participants: E. Lunéville, M. Lenoir, N. Kielbasiewicz
  Start: 10/2018. End: 09/2021. Administrator: ENSTA
  In partnership with F. Alouges and M. Aussal (CMAP, Ecole Polytechnique)

- Projet RAPID HyBOX *(Hybridization toolbox for complex materials and metamaterials)*
  Partners: IMACS, ARIANEGROUP, ENSTA
  Start: 10/2020. End: 09/2023. Administrator: ENSTA
  Participants of POEMS: Patrick Joly, Sonia Fliss, Maryna Kachanovska, Axel Modave, Pierre Marchand

- DGA provides partial funding for several PhD students:

  – D. Chicaud on *domain decomposition methods for time-harmonic electromagnetic wave problems with complex media* (Start: 10/2018)

  – A. Nassor on *transient fluid-structure coupling caused by remote underwater explosions, including cavitation effects* (Start: 10/2020)

10 Dissemination

10.1 Promoting scientific activities

10.1.1 Scientific events: organisation

- Most of the permanent members of POEMS are involved in the organization of the 15th International Conference on Mathematical and Numerical Aspects of Wave Propagation (organizing committee chaired by Sonia Fliss and Christophe Hazard). The conference was initially scheduled for July 26-30, 2021. This particular edition will be an opportunity to celebrate a double anniversary: the 15th occurrence of the conference, which coincides with its 30 years birthday! Unfortunately, due to the Covid situation, we have decided to postpone it to July 25-29, 2022. The conference will be held at ENSTA-Paris. Our wish is that this scientific meeting will also be a human meeting, taking advantage of the structure of ENSTA-Paris to receive on site in "full board" all participants who wish it. Website: www.waves2022.fr.

- S. Chaillat is a co-animator of the topic *"Modeling and simulation"* of the GDR Ondes (gdr-ondes.cnrs.fr). Within this framework, she co-organizes a webinar series.

- J.-F. Mercier is a co-animator of the topic *"Effective dynamics of microstructured media"* of the GDR MecaWave (mecawave.cnrs.fr).

- A.-S. Bonnet-Ben Dhia is a member of the Organization Committee of the Scientific Committee of the hybrid conference on *"Herglotz-Nevanlinna Functions and their Applications to Dispersive Systems and Composite Materials"* which will be held in May 2022 in CIRM.

- POEMS organizes, under the responsability of M. Kachanovska, a monthly seminar. One occurrence each semester is co-organized with two other inria teams, IDEFIX and M3DISIM.

10.1.2 Journal

Member of the editorial boards

- A. S. Bonnet-Ben Dhia is a member of the Editorial Board of the SIAM journal of applied mathematics.

- M. Bonnet is a member in the Editorial Board of Computational Mechanics (Comput. Mech.), Engineering Analysis with Boundary Elements (EABE), J. of Integral Equations and Applications (JIE), J. Optimization Theory and Applications (JOTA), and Inverse Problems.

- P. Ciarlet is a member in the Editorial Board of ESAIM:M2AN *(Mathematical Modeling and Numerical Analysis)*.

- P. Joly is a member of the Editorial Board of the Book series *"Scientific Computing"* of Springer,

Reviewer - reviewing activities

- The team members regularly review papers for many international journals.

10.1.3 Research administration

- E. Bécache is a deputy chair of the Doctoral School EDMH.
- M. Bonnet is a member of the COMEVAL, a committee of the Ministry of Ecological and Inclusive Transition (MEIT) similar to a CNRS National Committee section and tasked with the recruiting and career overseeing of the cadre of junior and senior scientists managed by MEIT.
- A.-S. Bonnet-Ben Dhia is deputy-chair of the *Applied Mathematics Department* (UMA) at ENSTA Paris. She is a member of the Scientific Council of the Doctoral School EDMH, and of the BCEP (Bureau du Comité des Equipes Projets) at INRIA Saclay from 2018. She is a member of the exterior scientific councils of Institut Fresnel and Laboratoire de Mécanique et d'Acoustique, both in Marseille.
- A.-S. Bonnet-Ben Dhia and S. Chaillat are members of the Academic Council of IP Paris (Institut Polytechnique de Paris).
- S. Chaillat has been a member of the section 09 *(Solid mechanics, materials and structures, biomechanics, acoustics)* of the CoCNRS from January 2020 to September 2021. She is a member of the Scientific Council of CNRS from October 2021.
- P. Ciarlet is coordinator of the *Mathematics in Computational Science and Engineering Program* of the Mathematics Hadamard Labex (LMH).
- M. Kachanovska is a member of the INRIA Scientific Committee for PhD and Postdoctoral Positions, from 2017.
- A. Modave is a member of the scientific committee of the mesocenter of IP Paris (Institut Polytechnique de Paris).

10.2 Teaching - Supervision - Juries

10.2.1 Administration

Permanent members of POEMS are involved in the management of the engineering program at ENSTA Paris and the master program in applied mathematics at IP Paris and Université Paris-Saclay.

- L. Bourgeois: coordinator of the 2nd year Maths Program at ENSTA; co-head of the M1 Applied Mathematics common to IP Paris and Université Paris-Saclay;
- P. Ciarlet: coordinator at Institut Polytechnique de Paris of the Mathematics and Applications Master's Program;
- S. Fliss: coordinator of the 3nd year ENSTA programs on modelling and simulation; co-head of the M2 AMS *(Analyse, Modélisation et Simulation)* common to IP Paris and Université Paris-Saclay;
- L. Giovangigli: coordinator of the 3nd year ENSTA programs on finance and mathematics for life sciences.

10.2.2 Courses taught

All permanent members of POEMS, as well as most PhD students and post-docs, are involved in teaching activities. A large fraction of these activities is included in the curriculum of the engineering school ENSTA Paris that hosts POEMS team. The 3rd year of this curriculum is coupled with various research masters, in particular the master *Analysis, Modelization and Simulation* (denoted below M2 AMS) common to Institut Polytechnique de Paris and Université Paris-Saclay.

Teaching activities of the permanent members of POEMS

- Eliane Bécache
  - *Introduction à la discrétisation des équations aux dérivées partielles*, ENSTA (1st year)
  - *Analyse et approximation par éléments finis d'EDP*, ENSTA (2nd year) and Master Applied Math (M1)
  - *Equations intégrales de frontière*, ENSTA (3rd year) and Master AMS (M2)
- Marc Bonnet
  - *Calcul scientifique à haute perforance*, ENSTA (2nd year) and Master Applied Math (M1)
- Anne-Sophie Bonnet-Ben Dhia
  - *Fonctions de variable complexe*, ENSTA (1st year)
  - *Théorie spectrale des opérateurs autoadjoints*, ENSTA (2nd year) and Master Applied Math (M1)
  - *Méthodes variationnelles pour l'analyse et la résolution de problèmes non coercifs*, ENSTA (3rd year) and Master AMS (M2)
  - *Problèmes de diffraction en domaines non bornés*, ENSTA (3rd year) and Master AMS (M2)
- Laurent Bourgeois
  - *Outils élémentaires pour l'analyse des équations aux dérivées partielles*, ENSTA (1st year)
  - *Fonctions de variable complexe*, ENSTA (1st year)
  - *Problémes inverses pour des systémes gouvernés par des EDPs*, ENSTA (3rd year) and Master AMS (M2)
- Stéphanie Chaillat
  - *Techniques numériques et algorithmiques pour les équations intégrales*, ENSTA (3rd year) and Master AMS (M2)
  - *Éléments finis et éléments de frontière : parallélisation, couplage et performance*, ENSTA (3rd year) and Master AMS (M2)
- Colin Chambeyron
  - *Remise à niveau en maths*, Licence (1st year), Paris-Dauphine University
  - *Outils mathématiques*, Licence (L1), Paris-Dauphine University
  - *Analyse - Optimisation*, Licence (L1), Paris-Dauphine University
  - *Algèbre linéaire*, Licence (L2), Paris-Dauphine University
- Patrick Ciarlet
  - *Méthodes variationnelles pour l'analyse et la résolution de problèmes non coercifs*, ENSTA (3rd year) and Master AMS (M2)
  - *Modèles mathématiques et leur discrétisation en électromagnétisme*, ENSTA (3rd year) and Master AMS (M2)
- Luiz Faria
  - *Programmation scientifique en C++*, ENSTA (2nd year) and Master Applied Math (M1)
  - *Projet de simulation numérique*, ENSTA (2nd year) and Master Applied Math (M1)
  - *Calcul scientifique à haute perforance*, ENSTA (2nd year) and Master Applied Math (M1)

  - *Éléments finis et éléments de frontière : parallélisation, couplage et performance*, ENSTA (3rd year) and Master AMS (M2)
- Sonia Fliss
  - *La méthode des éléments finis*, ENSTA (2nd year) and Master Applied Math (M1)
  - *Introduction à la discrétisation des équations aux dérivées partielles*, ENSTA (1st year)
  - *Homogénéisation périodique*, ENSTA (3rd year), ENSTA (3rd year) and Master AMS (M2)
- Laure Giovangigli
  - *Introduction aux probabilités et aux statistiques*, ENSTA (1st year)
  - *Martingales et algorithmes stochastiques*, ENSTA (2nd year)
  - *Calcul stochastique*, ENSTA (3rd year) and Master MMMEF (M2)
  - *Introduction à l'imagerie médicale*, ENSTA (3rd year) and Master AMS and MSV (M2)
  - *Homogénéisation stochastique*, ENSTA (3rd year) and Master AMS and MSV (M2)
- Christophe Hazard
  - *Outils élémentaires d'analyse pour les équations aux dérivées partielles*, ENSTA (1st year)
  - *Théorie spectrale des opérateurs autoadjoints*, ENSTA (2nd year) and Master Applied Math (M1)
- Patrick Joly
  - *Introduction à la discrétisation des équations aux dérivées partielles*, ENSTA (1st year)
  - *Analyse fonctionnelle*, ENSTA (2nd year) and Master AMS (M2)
  - *Techniques de discrétisation avancées pour les problèmes d'évolution*, ENSTA (3rd year) and Master AMS (M2)
- Maryna Kachanovska
  - *Analyse fonctionnelle*, ENSTA (2nd year) and Master Applied Math (M1)
  - *Modèles mathématiques et leur discrétisation en électromagnétisme*, ENSTA (3rd year) and Master AMS (M2)
  - *Equations intégrales de frontière*, ENSTA (3rd year) and Master AMS (M2)
- Nicolas Kielbasiewicz
  - *Programmation scientifique en C++*, ENSTA (2nd year) and Master Applied Math (M1)
  - *Projet de simulation numérique*, ENSTA (2nd year) and Master Applied Math (M1)
  - *Calcul scientifique parallèle*, ENSTA (3rd year) and Master AMS (M2)
- Eric Lunéville
  - *Introduction au calcul scientifique*, ENSTA (2nd year).
  - *Programmation scientifique en C++*, ENSTA (2nd year) and Master Applied Math (M1)
  - *Projet de simulation numérique*, ENSTA (2nd year) and Master Applied Math (M1)
  - *Problèmes de diffraction en domaines non bornés*, ENSTA (3rd year) and Master AMS (M2)
- Pierre Marchand
  - *Introduction à MATLAB*, ENSTA (1st year)
- Jean-François Mercier

  - *Outils élémentaires d'analyse pour les équations aux dérivées partielles*, ENSTA (1st year)
  - *Fonctions de variable complexe*, ENSTA (1st year)
  - *Théorie spectrale des opérateurs autoadjoints*, ENSTA (2nd year) and Master Applied Math (M1)

- Axel Modave
  - *Calcul scientifique à haute performance*, ENSTA (2rd year) and Master Applied Math (M1)
  - *Calcul scientifique parallèle*, ENSTA (3rd year) and Master AMS (M2)
  - *Éléments finis et éléments de frontière : parallélisation, couplage et performance*, ENSTA (3rd year) and Master AMS (M2)

10.2.3 Supervision

- PhD: Clément Bénéteau, *"Asymptotic analysis of time harmonic Maxwell equations in presence of metamaterials"*, January 2021, S. Fliss and X. Claeys
- PhD: Hajer Methenni, *"Mathematical modelling and numerical method for the simulation of ultrasound structural health monitoring of composite plates"*, March 2021, S. Fliss and S. Impériale
- PhD: Nicolas Trafny, *"Development of semi-analytical models to predict the noise produced by turbulence-edges interactions"*, November 2021, J.-F. Mercier and B. Cotté
- PhD: Damien Chicaud, *"Méthodes de décomposition de domaine pour la résolution de problèmes harmoniques d'ondes électromagnétiques en milieux complexes"*, December 2021, P. Ciarlet and A. Modave
- PhD in progress: Mahran Rihani, *"Équations de Maxwell en présence de méta-matériaux"*, November 2018, A.-S. Bonnet-Ben Dhia and L. Chesnel
- PhD in progress: Akram Beni Hamad, *"Propagation d'ondes électromagnétiques dans les cables coaxiaux"*, Septembre 2019, S. Imperiale, P. Joly and M. Khenissi
- PhD in progress: Jean-François Fritsch, *"Imagerie dans les guides d'ondes enfouis"*, Octobre 2019, L. Bourgeois and C. Hazard
- PhD in progress: Amandine Boucart *"Modélisation d'une couche mince de nanoparticules réparties aléatoirement pour les ondes électromagnétiques*, Octobre 2019, S. Fliss and L. Giovangigli
- PhD in progress: Amond Allouko, *"Approche semi-analytique hybride utilisant les guides d'ondes et la méthode des éléments finis pour le contrôle de santé intégrée de plaques composites"*, September 2020, A.-S. Bonnet and A. Lhemery
- PhD in progress: Laura Bagur, *"Three dimensional modeling of seismic and aseismic slip using Fast Boundary Element Methods"*, September 2020, S. Chaillat, J.-F. Samblat and I. Stéfanou
- PhD in progress: Pierre Amenoagbadji, *"Propagation des ondes dans des milieux quasi-périodiques"*, Octobre 2020, S. Fliss and P. Joly
- PhD in progress: Etienne Peillon, *"Justification et analyse mathématique de modèles de métamatériaux hyperboliques"*, Octobre 2020, P. Ciarlet and M. Kachanovska
- PhD in progress: Alice Nassor, *"Transient fluid-structure coupling caused by remote underwater explosions, including cavitation effects"*, Octobre 2020, S. Chaillat and M. Bonnet
- PhD in progress: Alejandro Rosas Martinez Luis, *"Dispersive electromagnetic media: mathematical and numerical analysis"*, November 2020, M. Cassier and P. Joly
- PhD in progress: Quentin Goepfert, *"Inverse problems in ultrasonic imaging"*, October 2021, J. Garnier, L. Giovangigli and P. Millien

10.3 Popularization

10.3.1 Interventions

- Luiz Faria has given a conference *"La simulation numérique au service des phénomènes physiques"* in *"Unithé ou Café"* at INRIA Saclay in June 2021.

11 Scientific production

11.1 Publications of the year

International journals

[1] J. Albella Martínez, S. Imperiale, P. Joly and J. Rodríguez. 'Numerical Analysis of a Method for Solving 2D Linear Isotropic Elastodynamics with Free Boundary Condition using Potentials and Finite Elements'. In: *Mathematics of Computation* (2021). DOI: 10.1007/s10915-018-0768-9. URL: https://hal.inria.fr/hal-02345808.

[2] E. Bécache, S. Fliss, M. Kachanovska and M. Kazakova. 'On a surprising instability result of Perfectly Matched Layers for Maxwell's equations in 3D media with diagonal anisotropy'. In: *Comptes Rendus. Mathématique* (2021). URL: https://hal.archives-ouvertes.fr/hal-02873620.

[3] E. Bécache and M. Kachanovska. 'Stability and Convergence Analysis of Time-domain Perfectly Matched Layers for The Wave Equation in Waveguides'. In: *SIAM Journal on Numerical Analysis* (2021). URL: https://hal.archives-ouvertes.fr/hal-02536375.

[4] A. Bera, A.-S. Bonnet-Ben Dhia and L. Chesnel. 'A continuation method for building invisible obstacles in waveguides'. In: *Quarterly Journal of Mechanics and Applied Mathematics* (26th Feb. 2021). URL: https://hal.archives-ouvertes.fr/hal-02573706.

[5] H. Bériot and A. Modave. 'An automatic PML for acoustic finite element simulations in convex domains of general shape'. In: *International Journal for Numerical Methods in Engineering* 122.5 (2021), pp. 1239–1261. DOI: 10.1002/nme.6560. URL: https://hal.archives-ouvertes.fr/hal-02738261.

[6] M. Bonnet. 'On the justification of topological derivative for wave-based qualitative imaging of finite-sized defects in bounded media'. In: *Engineering Computations* (2021). DOI: 10.1108/EC-08-2021-0471. URL: https://hal.archives-ouvertes.fr/hal-03319821.

[7] A.-S. Bonnet-Ben Dhia, M.-O. Bristeau, E. Godlewski, S. Imperiale, A. Mangeney and J. Sainte-Marie. 'Pseudo-compressibility, dispersive model and acoustic waves in shallow water flows'. In: *SEMA SIMAI Springer Series* (21st May 2021), pp. 209–250. URL: https://hal.inria.fr/hal-02493518.

[8] A.-S. Bonnet-Ben Dhia, S. N. Chandler-Wilde, S. Fliss, C. Hazard, K.-M. Perfekt and Y. Tjandrawidjaja. 'The Complex-Scaled Half-Space Matching Method'. In: *SIAM Journal on Mathematical Analysis* (2021). URL: https://hal.inria.fr/hal-03087232.

[9] A.-S. Bonnet-Ben Dhia, C. Hazard and F. Monteghetti. 'Complex-scaling method for the complex plasmonic resonances of planar subwavelength particles with corners'. In: *Journal of Computational Physics* 440 (1st Sept. 2021). DOI: 10.1016/j.jcp.2021.110433. URL: https://hal.archives-ouvertes.fr/hal-02923259.

[10] L. Bourgeois, J.-F. Fritsch and A. Recoquillay. 'Imaging junctions of waveguides'. In: *Inverse Problems and Imaging* (Feb. 2021). DOI: 10.3934/ipi.2020065. URL: https://hal.inria.fr/hal-02567182.

[11] C. Carvalho, P. Ciarlet and C. Scheid. 'Limiting amplitude principle and resonances in plasmonic structures with corners: numerical investigation'. In: *Computer Methods in Applied Mechanics and Engineering* (21st Oct. 2021). URL: https://hal.archives-ouvertes.fr/hal-03160574.

[12] S. Chaillat, M. Darbas and F. Le Louër. 'Analytical preconditioners for Neumann elastodynamic Boundary Element Methods'. In: *SN Partial Differential Equations and Applications* 2.22 (2nd Mar. 2021). URL: https://hal.archives-ouvertes.fr/hal-02512652.

[13] D. Chicaud, P. Ciarlet and A. Modave. 'Analysis of variational formulations and low-regularity solutions for time-harmonic electromagnetic problems in complex anisotropic media'. In: *SIAM Journal on Mathematical Analysis* 53.3 (4th May 2021), pp. 2691–2717. DOI: 10.1137/20M1344111. URL: https://hal.archives-ouvertes.fr/hal-02651682.

[14] P. Ciarlet and M. Kachanovska. 'A mathematical study of a hyperbolic metamaterial in free space'. In: *SIAM Journal on Mathematical Analysis* (2021). URL: https://hal.archives-ouvertes.fr/hal-03164619.

[15] A. Dansou, S. Mouhoubi, C. Chazallon and M. Bonnet. 'Modelling of the fatigue cracking resistance of grid reinforced asphalt concrete by coupling fast BEM and FEM'. In: *Road Materials and Pavement Design* (2022). DOI: 10.1080/14680629.2022.2029755. URL: https://hal.archives-ouvertes.fr/hal-03524856.

[16] B. Delourme, E. Lunéville, J.-J. Marigo, A. Maurel, J.-F. Mercier and K. Pham. 'A stable, unified model for resonant Faraday cages'. In: *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences* 477.2245 (Jan. 2021), p. 20200668. DOI: 10.1098/rspa.2020.0668. URL: https://hal.archives-ouvertes.fr/hal-03370501.

[17] S. Fliss, P. Joly and V. Lescarret. 'A DtN approach to the mathematical and numerical analysis in waveguides with periodic outlets at infinity'. In: *Pure and Applied Analysis* (Feb. 2021). URL: https://hal.inria.fr/hal-02444754.

[18] H. Guo, H. Zhu, R. Liu, M. Bonnet and S. Veerapaneni. 'Optimal Ciliary Locomotion of Axisymmetric Microswimmers'. In: *Journal of Fluid Mechanics* 927 (2021), A22. DOI: 10.1017/jfm.2021.744. URL: https://hal.archives-ouvertes.fr/hal-03185015.

[19] H. Guo, H. Zhu, R. Liu, M. Bonnet and S. Veerapaneni. 'Optimal slip velocities of micro-swimmers with arbitrary axisymmetric shapes'. In: *Journal of Fluid Mechanics* 910 (2021), A26. DOI: 10.1017/jfm.2020.969. URL: https://hal.archives-ouvertes.fr/hal-03099172.

[20] B. B. Guzina and M. Bonnet. 'Effective wave motion in periodic discontinua near spectral singularities at finite frequencies and wavenumbers'. In: *Wave Motion* 103 (2021), p. 102729. DOI: 10.1016/j.wavemoti.2021.102729. URL: https://hal.archives-ouvertes.fr/hal-03184997.

[21] P. Joly and M. Kachanovska. 'Local transparent boundary conditions for wave propagation in fractal trees (I). Method and numerical implementation'. In: *SIAM Journal on Scientific Computing* (2021). URL: https://hal.archives-ouvertes.fr/hal-02462264.

[22] P. Joly and M. Kachanovska. 'Local transparent boundary conditions for wave propagation in fractal trees (ii): error and complexity analysis'. In: *SIAM Journal on Numerical Analysis* (2021). URL: https://hal.archives-ouvertes.fr/hal-02909750.

[23] L. Maltez Faria, C. Pérez-Arancibia and M. Bonnet. 'General-purpose kernel regularization of boundary integral equations via density interpolation'. In: *Computer Methods in Applied Mechanics and Engineering* 378 (2021), p. 113703. DOI: 10.1016/j.cma.2021.113703. URL: https://hal.archives-ouvertes.fr/hal-02964015.

[24] K. Pham, J.-F. Mercier, D. Fuster, J.-J. Marigo and A. Maurel. 'Scattering of acoustic waves by a nonlinear resonant bubbly screen'. In: *Journal of Fluid Mechanics* 906 (10th Jan. 2021), A19. DOI: 10.1017/jfm.2020.799. URL: https://hal.archives-ouvertes.fr/hal-03083513.

[25] C. Sanders, M. Bonnet and W. Aquino. 'An Adaptive Eigenfunction Basis Strategy to Reduce Design Dimension in Topology Optimization'. In: *International Journal for Numerical Methods in Engineering* 122 (2021), pp. 7452–7481. DOI: 10.1002/nme.6837. URL: https://hal.archives-ouvertes.fr/hal-02865859.

Doctoral dissertations and habilitation theses

[26] C. Beneteau. 'Enriched homogenized models in presence of boundaries : analysis and numerical treatment'. Institut Polytechnique de Paris, 20th Jan. 2021. URL: https://tel.archives-ouvertes.fr/tel-03160431.

[27] H. Methenni. 'Mathematical modelling and numerical method for the simulation of ultrasound structural health monitoring of laminated composite plates'. Institut Polytechnique de Paris, 26th Mar. 2021. URL: https://tel.archives-ouvertes.fr/tel-03191607.

Reports & preprints

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