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ACTIVITY REPORT

Project-Team

EDGE

Extended formulations and Decomposition for Generic optimization problems

IN COLLABORATION WITH: Institut de Mathématiques de Bordeaux (IMB)

DOMAIN

Applied Mathematics, Computation and Simulation

THEME

Optimization, machine learning and statistical methods

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Project-Team EDGE

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B6.5. – Information systems

B7. – Transport and logistics

B9.5.2. – Mathematics

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2 Overall objectives

Nowadays, **integer programming based methods are able to solve effectively a large number of classic combinatorial optimization problems** (travelling-salesman, knapsack, facility location problems). Dramatic improvements on benchmarks were obtained due to better linear programming solvers, better (re)formulations, new polyhedral results, new efficient algorithms, fine-tuned implementations, and more powerful computers.

Practitioners are now trying to tackle more advanced problems where the degree of **complexity of the systems to optimize has increased substantially**. In particular, numerous factors should be taken into account in the decision process: ecological impact, new regulations, presence of commercial partners or aggressive competitors, limitation on previously abundant resources, and so forth. Many problems also come with uncertain parameters (energy production, consumer behavior, weather, disasters, resource breakdowns...), and have to be solved in a dynamic environment, where solutions have to be modified/reoptimized on-the-fly to account for last-minute changes.

In project EDGE, our main objective is to **propose new mathematical models, algorithms, and efficient implementations of these algorithms** to deal with families of integer programming problems arising from these complex systems, which are **out-of-reach for current state-of-the-art optimization methods**. Although universal algorithms able to deal with all types of constraints cannot be achieved in practice, we will seek results that are as broadly applicable as possible by designing so-called *generic* methods, *i.e.*, methods that address abstract mathematical models that can be later specialized for a large number of problems. We will mainly rely on two families of mathematical tools: decomposition techniques and extended formulations.

Decomposition methods are valuable tools to address complex optimization problems. Beyond the obvious advantages of the divide-and-conquer paradigm, they offer a way to produce stronger formulations, effective algorithms applicable to a wide range of problems, and they allow the use of right mathematical tools for each subsystem (also called *subproblem*). Several issues have to be addressed when one wants to tackle modern integrated real-life problems by means of decomposition methods. An important limitation of these methods is that they are generally efficient only for problems with a specific structure (typically independent subproblems connected by few common constraints). Many problems we address in this new project either do not have this decomposable structure, or the structure is broken by so-called *non-robust cuts*, which are applied during the solution process. In this case, new decomposition methods are needed to deal effectively with these connecting constraints. Another issue stems from the large number of different subsystems. A methodological challenge is to find a good trade-off between the quantity of information passed from one subproblem to another and the difficulty to integrate the common information into them.

Extended formulations are also a focus of our team. The past ten years have seen much progress in this field, which aims at reformulating effectively a problem/polyhedron with the help of (exponentially many) additional variables. In particular, network-flow-based reformulations have received an increasing interest from the community. A considerable difficulty to overcome when dealing with such a reformulation is to handle its size. One of the most promising paths is to study so-called aggregation and disaggregation techniques, where the level of detail of the formulation is modified iteratively. Similar approaches are popular in other fields of applied mathematics, where detailed information is only needed in some specific parts of the model. In integer programming, determining the suitable level of precision needed for a given subsystem is a major challenge, since the combinatorial structure of the subproblem preclude techniques based on derivatives. Although preliminary results indicate that one can achieve considerable improvements for some specific problems, there is a lack of a theoretical framework that would allow to develop these techniques for a larger class of polyedral structures.

We use our methodological tools to address **robust optimization**, which is an increasingly popular approach to handle the uncertainty arising in mixed-integer linear optimization problems. This optimization paradigm describes the variability of the uncertain parameters through bounded uncertainty sets (and thus replacing the expectation, used in stochastic optimization, with the worst-case objective realization). In particular, we plan to study robust problems with integer recourse. From a theoretical point of view, these problems are known to be Σ_2^p-complete in general (and thus simply verifying that a solution is feasible is an NP-hard problem). We will work on determining the frontier between tractable and non-tractable problems by studying the structure of specific subsets of robust problems characterized by: their deterministic counterpart; their uncertainty set; and the difficulty of optimizing the recourse subproblem. Our first steps in this direction show that decomposition methods and extended formulations can be instrumental in achieving this objective.

State-of-the-art optimization methods often being too hard to use, or too cumbersome to adapt to a new problem, their use is typically limited to a small community of experts. Keeping this in mind, we additionally concentrate on developing **high-level modeling tools and solvers** with the aim of facilitating the adaptation of our algorithms by colleagues and O.R. scientists. Doing so will maximize the impact of our research in academia as well as in the industry.

3 Research program

3.1 Decomposition methods

Dantzig-Wolfe decomposition [36] is a well-established approach for solving hard and/or large-scale combinatorial optimization problems. Problems that are originally formulated as Mixed-Integer Programming problems (MIPs) are reformulated by defining a (generally exponential) number of variables which correspond to the solutions of subproblem(s) obtained by subsets of variables and constraints of the original MIP formulation. To solve the linear approximation of the Dantzig-Wolfe reformulation, the column generation procedure is employed, which iteratively generates missing variables by solving the subproblem(s). Cut generation is then employed to strengthen the linear approximation, and branching is performed to find an optimal solution to the problem. The overall approach is called the branch-cut-and-price (BCP) method [40]. Recent advances, including those developed by our previous project team ReAlOpt, helped significantly improve the efficiency of general-purpose BCP algorithms. To use these algorithms one just has to specify the master problem, and a formulation or an algorithm to solve the column generation subproblem. The improvements include stabilization [63], pre-processing, diving heuristics [71], and strong branching, among others. Generic BCP codes implementing these improvements made the branch-cut-and-price approach more accessible to users.

A major limitation of BCP algorithms is their inefficiency when they face constraints that break the decomposable structure of the problem. Prominent examples are synchronisation and precedence constraints [41]. Moreover, it became clear recently that many efficient BCP approaches are "non-robust", *i.e.*, column and cut generation are interdependent. Thus, for every subproblem type, dedicated cut-generation approaches should be considered. Few such approaches are known in the literature [38],[61]. Their development for different families of cutting planes and different subproblem types is a very promising research direction. **Devising BCP algorithms for problems that do not have the decomposability property** is a difficult and high-risk task. However, in the case of success, the impact on the practice of Operations Research would be significant.

Our numerical experience shows that generic BCP approaches [46],[47] (in which the subproblems are solved as generic MIPs) are rarely competitive with the traditional branch-and-cut method used in widely available and highly efficient MIP solvers. On the other hand, very efficient BCP approaches specialized for particular problems exist in the scientific literature. In such approaches, subproblems are of a certain type and are solved by fast specialized algorithms. These ad-hoc BCP approaches are however rarely used in practice due to the complexity of their implementation [61, 69]. An important goal of our project is to propose innovative methods that have the efficiency of the most advanced specialized algorithms in the literature, yet can be applied to many practical cases. This can be done by developing BCP algorithms which are not completely generic (*i.e.*, they cannot be applied to any model obtained by Dantzig-Wolfe reformulation) but can be used for large classes of combinatorial optimization problems.

There is a clear need for **efficient and reusable semi-generic approaches**, that can be applied to a large number of problems inside a specific class. We have identified several types of subproblems that are often encountered in practical applications as substructures: resource-constrained paths in graphs [52] and hyper-graphs [33], constrained network flows, and decision diagrams [25]. Developing efficient algorithms that can be applied to the broadest possible class of such subproblems is a hard task. We will seek the right trade-off between generality and efficiency.

Recently, we have developed the first **"semi-generic" BCP solver, VRPSolver,** which relies on solving resource constrained shortest path subproblems [62]. We have shown that this solver can be successfully applied to many vehicle routing variants and some packing problems. An open question is whether there are other classes of problems which can be efficiently solved by "semi-specialized" BCP solvers. Scheduling and network design problems are good candidates for such classes. Capabilities of current BCP solvers however need to be extended to efficiently handle these problems. One of the drawbacks of our solver is the absence of good embedded heuristics to quickly find feasible solutions. Diving heuristics [71] can in principle be used, but their speed and performance are generally not satisfactory.

Another challenge is to design a **modelling paradigm** that can produce formulations for "semi-specialized" BCP solvers. In this context, defining a MIP and applying the standard Danzig-Wolfe reformulation technique is cumbersome as the subproblems cannot be easily defined as MIPs. New innovative modelling paradigms should be developed to simplify the problem definition and thus extend the usage of BCP approaches. They should be simple enough to attract non-specialists in the domain, and complex enough to pass the structural information to the BCP solver. This is mandatory to achieve state-of-the-art performance. One can find inspiration in the notion of *global constraints* designed in the field of Constraint Programming.

Benders' decomposition [24] is also becoming increasingly effective for several benchmark problems. Recent methods based on this reformulation approach involve solving many similar subproblems repeatedly, and make use of sophisticated stabilization techniques. We plan to build on the knowledge developed on Dantzig-Wolfe decomposition to develop techniques to improve Benders' decomposition, which do not use any assumption on the specific structure of the subproblem. A considerable challenge is to find effective methods for handling **integer subproblems** in a generic manner.

Another way to improve Benders' decomposition tools is to use **machine learning** techniques. Such algorithms can also be used to parameterize the different parts of the method or discover some information to help the convergence of these algorithms. This is a promising path for solving stochastic programs with a large number of scenarios. Several scientific questions have to be addressed, including the possible adaptation of stabilization techniques that need a complete dual information for proving the convergence.

In line with the challenges highlighted above, one of our first objectives is to improve the genericity of our BCP solver for the resource constrained path structure. For the moment, it supports only models with standard additive resources. We intend also to cover the case when cost and resource consumption of the current arc in the path depends not only on the arc itself, but also on the accumulated resource consumption of previous arcs in the path. This would allow to solve several important classes of problems such as electric vehicle routing problems, and scheduling problems with additive objective functions much more efficiently. This implies improving both the algorithms used and the modelling capabilities of the solver to support more general resources.

In the longer term, we plan to develop semi-generic BCP approaches and solvers for problems which do not contain the resource constrained path structure in graphs. Alternative structures which can be considered are spanning trees, network-flow problems in hyper-graphs, context-free grammars (both mentioned in Section 3.2), and decision diagrams.

Another interesting idea to explore in this direction is developing decomposition approaches for multi-stage sequential decision-making problems under uncertainty. Such problems can be modeled as multi-agent Markov Decision Processes (MDP). We believe that mathematical programming decomposition algorithms in which sub-problems are single-agent MDPs solved by dynamic programming constitute a very promising alternative to currently used approaches for multi-agent MDPs. Moreover, there are currently no approaches for such problems which provide exact solutions and non-trivial valid bounds on the optimal value.

3.2 Extended formulations

Many successful extended formulations are based on **network-flow models**. These models have excellent polyhedral properties, since the constraint matrix of a flow problem is totally unimodular, and thus all extreme-point solutions of the linear program related to this subsystem are integer if the right-hand sides of the constraints are integer. Additional constraints generally break this property, but in many cases, the linear relaxation remains of excellent quality.

Network-flow reformulations can be obtained when all or parts of discrete optimization problems can be **described by regular or algebraic languages**, or by recursive (such as dynamic programming) formulations, on top of which additional constraints such as resource constraints, disjunctions, are specified. Karp and Held [54] provided a systematic approach to building dynamic programming recurrence equations for a large class of optimization problems by characterizing the representation of discrete decision processes by monotone sequential decision processes. Martin et al. [59] characterized discrete optimization problems that can be solved via dynamic programming using directed hypergraphs [21]. The formalism of sequential decision processes allows to build an internal representation of the sequential structure using network-flows in state-graphs or hypergraphs. Additional constraints can be embedded in this representation by increasing the size of the network, for example by discretizing quantities such as time (scheduling), load (vehicle routing), or width (bin-packing) [48]. This has several advantages: 1) enforcing the consideration of constraints using the graph structure (that is convexifying these constraints), 2) easing the modeling of complex constraints, 3) facilitating the consideration of resource-dependent parameters [73].

The main drawback of these formulations is their typically **exponential or pseudo-polynomial number of variables and constraints**. This is different from models that arise from a Dantzig-Wolfe decomposition, which have an exponential number of variables, but a generally small number of constraints. The size of network-flow models is usually far too large to make them solvable by modern integer programming, SAT or constraint programming solvers.

To overcome this limitation, **aggregation and disaggregation techniques**, based either on a scaling of the input parameters or on aggregating nodes of the network, are good candidates. As they generally make use of sub-routines, they allow an efficient hybridization of different optimization paradigms. Several methods sharing common ideas have been introduced in different communities. For solving dynamic programs, techniques based on iterative state-space relaxations [31] have proved their efficiency since the 1980s. The most popular methods are the decremental state-space relaxation [29, 66] and the successive sublimation dynamic programming [51]. In the field of MIP models, the most promising algorithms for solving exponentially-large network-flow formulations are also based on an increasingly refined series of relaxations (see [32],[27],[65]). These methods produce initial models with fewer variables and fewer constraints than the original ones and iteratively refine them after identifying new variables and/or constraints to add. Although these methods have been shown to be efficient for several benchmark problems, only few generic methods have been proposed (see [28] and [37]).

Iterative aggregation/disaggregation techniques have some relationship to other methods. They share similarities with Benders' decomposition or Logic-based Benders' decomposition. Both methods rely initially on a relaxation of the problem. However, in Benders' decomposition constraints are iteratively introduced to discard infeasible solutions, while iterative aggregation/disaggregation-based techniques may also introduce new variables. Similarities with optimization methods using decision diagrams constructed by (iterative) state aggregation procedures can also be highlighted [25]. Decision diagrams are graphs storing possible variable assignments satisfying some constraints and are particularly used in CP/SAT solvers.

A conclusion of the above is that similar aggregation and disaggregation techniques are used under different names in different fields (MIP, DP, CP/SAT), leading to a scattered scientific literature. Further, most of the proposed techniques are problem-dependent (with the notable exception of decision diagrams, which offer a more general setting, although we are not aware of any generic implementation). Our goal is to go in the direction of developing a **unifying formalism** and express the main methods of the literature within this formalism. This generic view aims to bring together methods whose proximity has never really been highlighted. Existing algorithms may benefit from algorithmic components that have proven their effectiveness in other contexts. As an example, existing MIP algorithms could benefit from state-of-the-art approaches developed for decision diagrams.

We will study and design effective methods based on aggregating and disaggregating models. It is necessary to develop a high-level modeling tool to specify these models only implicitly. The use of algebraic languages to produce such higher-level formulations is a direction we are considering to explore. We also believe that a primal-dual solution approach is worth exploring. Such an approach will rely on the local refinement of a coarse representation of the global problem when needed. The idea is to derive primal and dual bounds on the objective value of the problem with a lower computational effort compared to working with the original model. We can distinguish two types of aggregation: 1) a conservative aggregation ensures that every solution of the original model is also a solution of the aggregated model (i.e., the aggregated model is a relaxation of the original model) and 2) a heuristic aggregation ensures that every solution of the aggregated model is also a solution of the original model. After projecting the original model onto an aggregated one, we aim to converge to an optimal solution without reaching the size of the original model. To keep the model tractable, one can make use of primal and dual information from the different aggregated models to fix variables values (using Lagrangian filtering for example). We will focus on finding a systematic way to 1) aggregate an initial model to yield either a relaxation or a restriction and 2) iteratively disaggregate the current model for obtaining better dual bounds. A path worth exploring is to dynamically aggregate the models taking advantage of the information learned in the current phase to build stronger models [50]. We will also study an emerging technique consisting in the joint use of several aggregated models with decision synchronization [60],[56] which can be helpful to derive stronger bounds and to keep tractable network-flow models.

For designing heuristics, the problems generated by dynamic programs / algebraic languages are suited to so-called structured learning techniques, where the objective is to learn how to approximate a hard combinatorial problem (typically a sequencing problem with resource constraints) by means of a simpler problem (typically a shortest-path problem on a graph). More classical learning techniques can also be used to guess the right parameters (lagrangian or surrogate multipliers, or to guess the right decisions to make (state-space modification, relaxation of some constraints, etc.).

In line with the challenges highlighted above, we started to address **automatic heuristic for dynamic programs with side constraints** based on structured-learning techniques. This is a work that had already begun in a collaboration between F. Clautiaux, A. Froger and A. Parmentier (CERMICS). The objective of the project is to determine the best way to apply machine-learning techniques to obtain heuristics via projection, pruning etc. To work on this challenge, we collaborate with Centre Aquitain des Technologies de l'Information et Électroniques (CATIE). Fulin Yan, a student from INSA Rennes, has done an internship with our team on this subject. We are now waiting for a funding answer to propose him a CIFRE PhD thesis.

We additionally started to lay the scientific foundation for a generic framework that embed various techniques from dynamic programming based on regular languages, resource-constrained shortest path, and decision diagrams. We are comparing the different elements coming from the different fields (SAT / MIP / heuristics), determining the common parts and analyzing the main differences between them. Our objective is to develop a common formalism and express the most important methods of the literature using this formalism. This is a work in progress between F. Clautiaux, B. Detienne and A. Froger. We hired Luis Lopes Marques as a PhD student starting from October 2022 to work on this subject. This thesis is financed by our ANR project AD-LIB.

In the near future, we will design **exact convergent methods based on aggregation/disaggregation techniques** (e.g., use of problem-dependent or primal/dual information) relying on the generic framework cited above. We will focus on finding a systematic way 1) to aggregate an initial model to yield either a relaxation or a restriction and 2) to iteratively disaggregate the current model for obtaining better dual bounds. This is part of our ANR project AD-LIB.

We will also study **matheuristics for problems based on time-indexed models**. This work will focus on the ability of the method to provide the best possible solution within a given time limit. This is a challenge when solving operational problems where the decision-making time is limited. The key area of research will be (i) how to disaggregate in a way that a good feasible solution can be provided as fast as possible (search strategy) and (ii) how to modify infeasible solutions provided by an aggregated model in order to make them feasible without impairing their cost. This is also a part of the ANR project AD-LIB.

Upon sucessful completion of the above objectives, we may explore many extensions and generalizations to our framework. Among them, we may cite new mechanisms to **integrate several different state-space relaxations** in the same solution process and extensions to non-linear constraints and objective / dynamic programs based on different semi-rings than (max, +) in $\mathbb{R}^n$. In the longer term, we wish to explore the links with polyedral theory, and develop extensions of the framework from regular languages to algebraic languages (and to extend our algorithms from graph-based algorithms to hypergraph-based algorithms). Polyhedral theory can be used to improve the performance of our algorithms for solving dynamic programs with side constraints, and to provide the equivalent of the branch-and-cut algorithm for dynamic programs.

3.3 Structure analysis and problem specific studies

Polyhedral analysis [53] remains a part of our project. When integer linear programming methods are involved, information on the structure of the polyhedron representing the feasible region is key for solving hard problems effectively. The most spectacular success of this kind of methods can be found in Concorde [35], a software dedicated to the traveling salesman problem. Network design has also been the subject of many contributions (see e.g. [42]). Since we aim to propose methods that can apply to broad classes of problems, we will focus on families of constraints/problems that occur in a large number of practical problems (including capacity, disjunction, precedence or set-covering constraints). There are two promising directions that we can follow in order to study these structures.

In the case where the linear relaxation of an integer programming formulation has an excellent quality, but requests a large computational time, being able to compute good dual solutions rapidly allows to provide useful **dual bounds for combinatorial optimization problems**. This technique was used for the classical cutting-stock problem under the name *dual-feasible functions* (see [20]). These functions lead to bounds of linear complexity that have an excellent behaviour on average. This concept has already been extended to several packing and location problems (see [55], [64]). We plan to explore further extensions to common problem/constraint types.

Many hard problems on graphs become easy when the graph has a special structure [43], typically perfect graphs such as trees, interval or triangulated graphs (see for example [57, 70]). In most problems, the graphs do not belong to an easy class. However, one can compute effective relaxations by exhibiting subgraphs of suitable structure in these general graphs. Unfortunately for many classes the problem of **finding a subgraph belonging to this class maximizing a certain criterion** is NP-hard. We plan to study methods for finding efficient models to compute such relaxations.

In line with the challenges highlighted above, our first objective is to work on **polyhedral analysis and computational studies of formulations for the "best" interval subgraph of a general graph**. We have identified several families of formulations, among them strong formulations based on an exponential number of variables and/or constraints. Pricing algorithms and separation problems have to be studied in order to efficiently solve these formulations. The same work can be undertaken with chordal graphs, which offer a stronger relaxation (since they are a superclass of interval graphs), for a higher computational cost.

In the longer term, we plan to work on the extension of our work on so-called **dual feasible functions** to more complex cases. The theory of superadditive and dual-feasible functions have mainly been studied in the case of a single knapsack constraint. Extensions to the intersection of the knapsack polytope with polytopes related to highly structured constraints (such as precedences, or disjunctions) is already a hard challenge.

More generally, we plan to work on several aspects related to polyedral theory: study of extended formulations generated by algebraic grammars. A possible objective is to use implicit formulations expressed in different state spaces to derive efficient cuts for a base formulation ; determining whether considering the structure of the problems will be a stronger tool to derive dual-feasible functions than learning to guess the best dual values from the input parameters (for instance, using structured learning to compute a smaller dual polytope from an initial one) ; studying decomposition schemes based on graph decomposition techniques, such as path or tree-decompositions.

4 Application domains

Although we aim to develop generic methodologies, the types of problems solved will play an important role in the choice of paradigms employed, and in the methods used to address these problems. We will mainly focus on two classes of problems: those that possess several levels of decisions, and those that have some uncertain parameters.

These types of problems are prelavent in certain **application fields**, including **energy**, and **supply-chain**. In both fields, our objectives are in line with aspirations of modern societies (reducing the pollution, improving the sustainability of human activities, including a better and more robust design of supply-chains). Our optimization tools will be useful in accompanying the profound shifts that are needed in these sectors, by taking into account the interconnection between the different problems faced by the companies.

In **energy**, the main challenges we face are related to the uncertainty of both production and consumption. The uncertainty in production grows with the development of renewable energy production, while uncertainty in consumption remains driven by weather. This leads to large-scale robust and/or stochastic optimization problems, which push our methodologies to their limits. A typical problem is to ensure the technical feasibility of some energy transition scenarios where the percentage of nuclear power in the energetic mix decreases, leading to an increasing level of uncertainty.

Supply Chain and logistics are also fertile playgrounds for our research. In particular, successful applications in routing, production planning, inventory control, warehouse optimization, or network design are numerous. Besides, new technologies such as the internet of things or physical internet bring new core optimization problems and the need for new relevant mathematical models and solutions approaches. The main challenges are to deal with integrated problems, including location decisions, inventory, routing, packing, employee timetabling, among others. Current methods tend to take tactical and operational decisions independently despite the fact that they are interrelated problems. Supply chains also have to be robust to uncertain parameters (from regular variations of the system parameters to disaster management).

4.1 Integrated problems

Most practical optimization problems are complex, i.e., they involve different types of decisions to be made. Such problems are commonly called *integrated*. They often involve different time scales related to strategic, tactical and operational decisions. A classical approach to solve such problems is their disintegration into independent problems or stages, where the solution of a stage is the input for next one. We prefer the term "disintegration" here instead of "decomposition" to avoid confusion with decomposition approaches presented above. The disintegration approach usually results in highly sub-optimal solutions. There is large potential for improving the quality of solutions if **two and more decision stages** are considered together.

Recently there has been an increase in interest for solving integrated optimization problems. Such problems may involve for example integration of production and outbound distribution (production-distribution problem [45]), facility location and vehicle routing (location-routing problem [72]), inventory management and vehicle routing (inventory routing problem [34]), and different levels of distribution (two-echelon routing [58] and cross-docking based distribution).

As we point out above, integrated problems are common in practice but difficult to solve, since they involve synchronization between stages, and different time scales in different stages. Moreover, different classes of problems are usually encountered in different stages. This means that different exact approaches are usually applied to solve such classes of problems. It is often not known how one can efficiently combine these approaches to solve an integrated problem. Thus, heuristic algorithms are used in a vast majority of cases. However, we believe that advances in efficiency and ease of use of exact algorithms and decomposition algorithms in particular allow us to think of applying them here.

Exact approaches are usually limited to small instances of integrated problems, while real-life instances are tackled by heuristic approaches. Nevertheless, for estimating the quality of these heuristics we need approaches that obtain lower bounds of a good quality, even for large scale instances. We think that BCP algorithms are good candidates to obtain such bounds. The column generation approach has however several drawbacks when applied to large-scale integrated problems. When solving pure academic problems, the bottleneck of BCP algorithms is usually the solution of the pricing problem. On the contrary, when dealing with integrated real-life problems, the size of the master problem may become too large and the solution of the restricted master LP becomes a bottleneck. One possible approach to tackle this problem is a dynamic aggregation of constraints in the master LP [30]. Another approach could be to use machine learning techniques to choose “good” columns from a large pool generated by the pricing problem. The goal here is to help the column generation algorithm converge faster in the case of a “heavy” master problem or to find “compatible” columns to obtain better primal solutions.

Column generation-based algorithms may also prove useful for obtaining good feasible solutions for integrated problems. A potential directon is to develop matheuristics, *i.e.*, heuristic methods that make use of sophisticated algorithms initially designed for exact methods. The goal is to benefit from the two worlds: exact methods to exploit the special structures of some subproblems, and heuristics to produce solutions in a small amount of time. **Column generation-based matheuristics** have already shown good results for some integrated problems [19]. A common approach is to use heuristics and/or column generation to obtain interesting columns and then solve the restricted master problem as a MIP with a time limit. Then the set of columns can possibly be updated and the restricted master is resolved. However, **generic frameworks for such approaches are absent**, although they would be highly welcome for a wide class of problems.

Our initial efforts will be concentrated on one or two well-chosen problems of this type. The inventory routing problem (IRP) is probably the most "academic" and simple to state example of integrated problems which stays very hard to solve for both exact and heuristic methods[39]. In this problem, the decision maker in charge of the delivery is also in charge of the stock level of his/her customer. IRP is widely encountered in practice. The integration of electric vehicles in transportation is also a timely topic. Planning the charging operations of electric vehicles is necessary to account for a wide variety of costs (e.g., time-dependent energy costs), charging infrastructure constraints (e.g., grid restrictions, limited number of chargers), battery degradation, and the potential integration of Vehicle-to-Grid technologies. Taking this variety of constraints into account introduces complex coupling decisions between charging and routing, which leads to integrated problems.

We plan to design a comprehensive modelling and solution framework for **planning the charging**

activities of a fleet of electric vehicles considering time-dependent costs and the potential integration of Vehicle-to-Grid technologies. Particular attention is directed towards decreasing the maximum power of energy required in the planning interval. This will be a joint work with O. Jabali (Politecnico di Milano, Italy) that has already began with a master student from Politecnico di Milano visiting our research team for 4 months last year and studying the case of a fleet of electric buses. Our objectives are to design advanced mathematical methods to produce high-quality solutions in a time efficient manner as well as to propose an exact solution method to solve electric routing problems (E-VRPs) with limited charging infrastructure constraints (based on a previous work [44]) using a Branch-and-cut-and-price algorithm. We will investigate the **trade-offs that exist between driving time and energy consumption** (e.g., existence of alternative paths, speed reduction) either by refining the abstraction of the road network on which E-VRPs are defined or by working directly with the road network.

In the long term, our objective is to be able to cope with an increasing amount of integrated problems, with a focus on **supply-chain applications** (scheduling, location, inventory, routing, etc.). Our work on this topic will benefit from our findings on new decomposition methods. Once our main objectives are achieved, we will explore methodological tools to take **different sources of uncertainty** into account (e.g., energy consumption of electric vehicles, time-dependent energy costs) using robust optimization or stochastic programming paradigms.

4.2 Robust optimization

In most decision making problems the data used in the mathematical model is subject to some form of uncertainty. This uncertainty can be caused by measurement errors, variability or the time duration of the processes under study, or simple lack of access to reliable data. Incomplete information can also come from the presence of **competitors or adversarial individuals** whose policies are not known to the decision maker. Recent events have also shown that the capability to resist to **major disasters** is an issue for important organizations and critical infrastructure.

There are two commonly accepted paradigms that are used to incorporate uncertainty in mathematical programming: stochastic optimization (including chance-constraints), and **robust optimization**. In stochastic optimization, one assumes that a probability distribution is known for the uncertain parameters and optimizes a statistical risk measure such as the expected value or the conditional value-at-risk. In this new project, our main tool for dealing with uncertainty will be robust optimization. Unlike stochastic programming, which requires exact knowledge of the probability distributions, robust optimization only describes the variability of the uncertain parameters through bounded uncertainty sets. Further, replacing the risk measures employed in stochastic optimization, with the worst-case realization, robust optimization is more adapted to applications where the decision-making process involves high risks or adversarial participants.

In the last 20 years, the robust optimization field has seen significant progress. Static robust optimization problems, which assume that all decisions are here-and-now, have received considerable attention. They have been formulated and studied with polyhedral, conic and ellipsoidal uncertainty sets. From a complexity viewpoint, it has been shown that these problems are barely harder than their deterministic counterparts in most practical cases. From the numerical viewpoint, mixed-integer linear (or second-order conic) reformulations have been proposed and can handle large-scale problems.

Despite these rather encouraging results, static models suffer from over-conservatism. Indeed, in these models, no recourse action can be taken to change or adapt the solution once the uncertain values have been revealed. This substantially limits the applicability of the robust optimization paradigm since in most applications that involve temporal decision-making, it is possible to adapt the solution to (at least partially) mitigate the effects of uncertainty.

Acknowledging the importance of adjustable models, the scientific community has started to address solution methods for these problems. While it is theoretically well-known that even two-stage linear programs are NP-hard, approximate (affine decision rules) [23], or exact (row-and-column generation algorithms) [22],[74] solution methods for problems featuring continuous recourse variables have been proposed in the literature. These two sets of algorithmic tools have made it possible to solve exactly or approximately a large number of adjustable robust optimization problems with continuous recourse. Several studies have also sought to understand the quality of the bounds provided by affine decision rules, as well as proposed extensions to piece-wise affine functions.

The problems we wish to investigate in this project are adjustable robust optimization problems where **recourse decisions are integer**. The situation is significantly more complex for this setting than for the case of continuous recourse. The two aforementioned approaches do not apply: affine decision rules provide continuous recourse actions by construction, while the separation problem in the row-and-column generation algorithm is based on linear programming duality.

Having no theoretical guarantees regarding their computational complexity, there are some numerical algorithms that aim to solve adjustable robust optimization problems with integer recourse approximately. They are all based on simplifying the type of feasible recourse actions, either by partitioning the uncertainty set or by imposing that the recourse decisions should be chosen among a predetermined finite set of solutions (finite/K-adaptability). Unfortunately, **these methods hardly scale up** and the recent results [26],[49] are limited to small problem instances, which can be partially explained by the fact that they rely on big-M reformulations, known to yield poor continuous relaxations.

In the following years, we plan to develop exact and approximate solution methods for adjustable robust optimization problems with integer recourse. On the one hand, we will study relatively general row-and-column generation algorithms that are based on stronger continuous relaxations than what was done in the literature (aforementioned network formulations and decision diagram-based approaches seem to be promising leads in achieving this goal). On the other hand, we will focus on classes of specific robust models (special cases), which will be studied both from a theoretical and a numerical perspective. Some our efforts will be spent on improving the efficiency of existing methods such as K-adaptability since these approaches could benefit from the considerable numerical experience of the team.

We developed a first approach to dealing with binary recourse decisions in the context of adjustable robust optimization problems with binary recourse in [1]. The main idea of this work is to **convexify the recourse feasible region** using a Dantzig-Wolfe reformulation. Promising results were presented for the special case where the coupling constraints are deterministic and satisfy additional technical assumptions. In this case, the inner maximization and minimization problems can be interchanged, resulting in a large-scale deterministic equivalent model. We plan to extend this technique to the more general case and to understand how to relax the aforementioned technical assumption.

In the near term, we plan to carry out several projects to respond to the challenges we outline above. We plan to study solution approaches for complex problems arising in **future 5G networks**. This topic is in discussion with the Orange company. We will also address **complex network-design and location problems** (topic of ANR project DE-SIDE, in collaboration with Sobolev Institute and Kedge Business School). We will determine easy and hard cases for robust multi-stage network-design problems, depending on the set of constraints, the definition of the uncertainty and the possible second-stage actions, with a focus on integer recourse.

We will also **adress primal and dual bounds for two- and multi-stage adjustable robust optimization problems** (topic of ANR-JCJC project DROI which was granted to Ayse N. Arslan). Building upon the results of [1] to deal with constraint uncertainty in adjustable robust optimization, using Lagrange and Fenchel duality which will provide dual bounds. Several issues need to be resolved investigating this approach, such as the optimization of the dual (infinite size) problems and how to close the duality gap. We additionally propose to study improved decision rules for these problems.

In the long term, our objectives in relation with this challenge are as follows. First, we want to **design efficient approximate approaches to solve multi-stage adjustable robust optimization problems**. Exact methods seem to be out of reach for the current state-of-the-art. Even solving special cases of such problems would be a major step. Second, we seek to **bridge the gap between multi-stage integer robust**

optimization and combinatorial games. Some similar concepts are used in both types of problems (typically min-max-min problems have to be solved). This may allow to disseminate our techniques to other fields as well as adapt results obtained for those problems to the context of adjustable robust optimization.

5 Social and environmental responsibility

5.1 Impact of research results

Our work [15] on a stochastic generation and transmission expansion planning problem studied at RTE will help the company in their future strategic studies (e.g., studies similar to [68, 67]). Specifically, we show how to incorporate the French legislation[1] that imposes that the number of hours with energy not served should be inexpectation less than or equal to three per year in the mathematical formulation of the expansion planning problem. This work was part of the PhD thesis of Xavier Blanchot.

6 Highlights of the year

Our team has recruited a **new permanent researcher, Ayse Nur Arslan**, who will help develop our activities on robust optimization. Dr. Arslan has obtained ANR "young researcher" funding on her research project concerning primal and dual bounds for adjustable robust optimization problems. This project will finance a PhD thesis and a postdoctoral researcher during one year.

We obtained an **ANR project in "Artificial Intelligence"** to fund our activities on aggregation/disaggregation techniques for extended formulations. This project is realized with collaborators in Toulouse LAAS and Toulouse Business School. This project allowed us to hire a PhD student to develop our research plan in this area.

7 New software and platforms

7.1 New software

7.1.1 BaPCod

Name: A generic Branch-And-Price-And-Cut Code

Keywords: Column Generation, Branch-and-Price, Branch-and-Cut, Mixed Integer Programming, Mathematical Optimization, Benders Decomposition, Dantzig-Wolfe Decomposition, Extended Formulation

Functional Description: BaPCod is a prototype code that solves Mixed Integer Programs (MIP) by application of reformulation and decomposition techniques. The reformulated problem is solved using a branch-and-price-and-cut (column generation) algorithms, Benders approaches, network flow and dynamic programming algorithms. These methods can be combined in several hybrid algorithms to produce exact or approximate solutions (primal solutions with a bound on the deviation to the optimum).

Release Contributions: Bug fixes and enhancements. Better support for compact MIP models. Debug solution support. More cutting planes statistics. Apple M1 support. Experimental CLP solver support. Compiled RCSP library is now included.

News of the Year: First public release.

URL: `https://bapcod.math.u-bordeaux.fr/`

[1] Article D141-12-6 from the French Energy Code

Publication: hal-03340548

Contact: Ruslan Sadykov

Participants: Artur Alves Pessoa, Boris Detienne, Eduardo Uchoa Barboza, Franck Labat, François Clautiaux, François Vanderbeck, Halil Sen, Issam Tahiri, Michael Poss, Pierre Pesneau, Romain Leguay, Ruslan Sadykov

Partners: Université de Bordeaux, CNRS, IPB, Universidade Federal Fluminense

7.1.2 VRPSolver

Name: VRPSolver

Keywords: Column Generation, Vehicle routing, Numerical solver

Scientific Description: Major advances were recently obtained in the exact solution of Vehicle Routing Problems (VRPs). Sophisticated Branch-Cut-and-Price (BCP) algorithms for some of the most classical VRP variants now solve many instances with up to a few hundreds of customers. However , adapting and reimplementing those successful algorithms for other variants can be a very demanding task. This work proposes a BCP solver for a generic model that encompasses a wide class of VRPs. It incorporates the key elements found in the best recent VRP algorithms: ng-path relaxation, rank-1 cuts with limited memory, and route enumeration, all generalized through the new concept of "packing set". This concept is also used to derive a new branch rule based on accumulated resource consumption and to generalize the Ryan and Foster branch rule. Extensive experiments on several variants show that the generic solver has an excellent overall performance, in many problems being better than the best existing specific algorithms. Even some non-VRPs, like bin packing, vector packing and generalized assignment, can be modeled and effectively solved.

Functional Description: This solver allows one to model and solve to optimality many combinatorial optimization problems, belonging to the class of vehicle routing, scheduling, packing and network design problems. The problem is formulated using variables, linear objective function, linear and integrality constraints, definition of graphs, resources, and mapping between graph arcs and variables. A complex Branch-Cut-and-Price algorithm is used to solve the model. A new concept of elementarity and packing sets is used to pass an additional information to the solver, so that several state-of-the-art Branch-Cut-and-Price components can be used to improve radically the efficiency of the solver. The interface of the solver is implemented in Julia using JuMP package. To simplify the installation and usage, the solver is distributed as a docker image. The solver can be used only for academic purposes.

Release Contributions: Version 0.4.1a allows the users to continue to use the software, it removes the current date check.

News of the Year: New version is released

URL: `https://vrpsolver.math.u-bordeaux.fr/`

Publication: hal-02178171v2

Contact: Ruslan Sadykov

Participants: Ruslan Sadykov, Eduardo Uchoa Barboza, Artur Alves Pessoa, Eduardo Queiroga, Teobaldo Bulhões, Laurent Facq

Partners: Universidade Federal Fluminense, Universidade Federal da Paraiba

8 New results

8.1 Decomposition methods and extended formulations

Our team had several important contributions around the "decomposition methods" axis and its objectives in the past year.

Novel algorithms based on Benders' decomposition for stochastic programs and a combination of Benders' and Dantzig-Wolfe decomposition were developed in [15] and [4], respectively. In [15], a new finitely-convergent exact algorithm to solve two-stage stochastic linear programs is proposed. Based on the multicut Benders reformulation of such problems, with one subproblem for each scenario, this method relies on a partition of the subproblems into batches. The key idea is to solve only a small proportion of the subproblems at most iterations by detecting that a first-stage candidate solution cannot be proven optimal as soon as possible. Additionally, a general framework to stabilize the algorithm is developed. In [4] the nuclear unit outage problem for the French electricity company EDF is treated. This problem is quite challenging given the specific operating constraints of nuclear units, the stochasticity of both the demand and non-nuclear units availability, and the scale of the instances. To tackle these difficulties a combined decomposition approach is proposed. The operating constraints of the nuclear units are built into a Dantzig-Wolfe pricing subproblem whose solutions define the columns of a demand covering formulation. The scenarios of demand and non-nuclear units availability are handled in a Benders decomposition. The approach is shown to scale up to the real-life instances of the French nuclear fleet.

We also continued our progress in the development of a semi-generic branch-and-price solver that is capable of outperforming state-of-the-art specialized algorithms. In [9] we study a plethora of vehicle routing variants. Specifically, we propose graph-based models for several vehicle routing problems with intermediate stops: the capacitated multi-trip vehicle routing problem with time windows, the multi-depot vehicle routing problem with inter-depot routes, the arc routing problem with intermediate facilities under capacity and length restrictions and the green vehicle routing problem. In these models, the set of feasible routes is represented by a set of resource constrained paths in one or several graphs. Intermediate stops are supported by the possibility to define negative resource consumption for some arcs. The models that we propose are then solved by VRPSolver, which implements a generic branch-cut-and-price exact algorithm. Thus, a simple parameterization enables us to use several state-of-the-art algorithmic components: automatic stabilization by dual price smoothing, limited-memory rank-1 cuts, reduced cost-based arc elimination, enumeration of elementary routes, and hierarchical strong branching.

Our algorithmic developments were accompanied by many applied contributions on problems ranging from bin packing variants to nuclear outage planning. In [8], we introduce and motivate several variants of the bin packing problem where bins are assigned to time slots, and minimum and maximum lags are required between some pairs of items. We suggest two integer programming formulations for the general problem: a compact one and a stronger formulation with an exponential number of variables and constraints. We propose a branch-cut-and-price approach that exploits the latter formulation. For this purpose, we devise separation algorithms based on a mathematical characterization of feasible assignments for two important special cases of the problem: when the number of possible bins available at each period is infinite and when this number is limited to one and time lags are nonnegative. Computational experiments are reported for instances inspired from a real-case application of chemical treatment planning in vineyards, as well as for academic instances for special cases of the problem. The experimental results show the efficiency of our branch-cut-and-price approach, as it outperforms the compact formulation on newly proposed instances and is able to obtain improved lower and upper bounds for academic instances. In [3], we study the prize-collecting job sequencing problem with one common and multiple secondary resources. In this problem, a set of jobs is given, each with a profit, multiple time windows for its execution, and a duration during which it requires the main resource. Each job also requires a preassigned secondary resource before, during, and after its use of the main resource. The goal is to select and schedule the subset of jobs that maximize the total profit. We present a new mixed integer linear programming formulation of the problem and a branch-cut-and-price algorithm as an exact solution method. We also introduce a heuristic algorithm to tackle larger instances. Extensive numerical experiments show that our exact algorithm can solve to optimality literature instances with up to 500 jobs for a particular dataset and up to 250 jobs for another dataset with different characteristics. Our heuristic builds high-quality solutions in a small computational time. It computes new best-known solutions for most of the larger instances.

Some of these applied contributions were on integrated problems including the location-routing problem and two-echelon capacitated vehicle routing.

In [2], a distribution network design problem under uncertainty is treated. This problem is defined as the two-echelon stochastic multi-period capacitated location-routing problem (2E-SM-CLRP). It considers a network partitioned into two capacitated distribution echelons: each echelon involves a specific location-assignment-transportation schema that must cope with the future demand. It aims to decide the number and location of warehousing/storage platforms (WPs) and distribution/fulfillment platforms (DPs), and on the capacity allocated from first echelon to second echelon platforms. In the second echelon, the goal is to construct vehicle routes that visit ship-to locations (SLs) from operating distribution platforms under a stochastic and time-varying demand and varying costs. This problem is modeled as a two-stage stochastic program with integer recourse, where the first-stage includes location and capacity decisions to be fixed at each period over the planning horizon, while routing decisions of the second echelon are determined in the recourse problem. We propose a logic-based Benders decomposition approach to solve this model. In the proposed approach, the location and capacity decisions are taken by solving the Benders master problem. After these first-stage decisions are fixed, the resulting sub-problem is a capacitated vehicle-routing problem with capacitated multiple depots (CVRP-CMD) that is solved by a branch-cut-and-price algorithm. Computational experiments show that instances of realistic size can be solved optimally within a reasonable time and provide relevant managerial insights on the impact of the stochastic and multi-period settings on the 2E-CLRP.

In [7], we study a two-echelon capacitated vehicle routing problem with time windows, in which delivery of freight from depots to customers is performed using intermediate facilities called satellites. We consider the variant of the problem with precedence constraints for unloading and loading freight at satellites. In this variant allows for storage and consolidation of freight at satellites. Thus, the total transportation cost may decrease in comparison with the alternative variant with exact freight synchronization at satellites. We suggest a mixed integer programming formulation for the problem with an exponential number of route variables and an exponential number of precedence constraints that link first-echelon and second-echelon routes. Routes at the second echelon connecting satellites and clients may consist of multiple trips and visit several satellites. A branch-cut-and-price algorithm is proposed to solve efficiently the problem. This is the first exact algorithm in the literature for the multi-trip variant of the problem. We also present a postprocessing procedure to check whether the solution can be transformed to avoid freight consolidation and storage without increasing its transportation cost. Experimental results reveal that our algorithm can be used to solve these instances significantly faster than another recent approach proposed in the literature.

In [6] we present theoretical foundations of pseudo-polynomial arc flow formulations, by showing a relation between their network and Dynamic Programming (DP) representations. This relation allows a better understanding of the strength of these formulations, through a link with models obtained by Dantzig-Wolfe reformulation. The relation with DP also allows a new perspective to relate state-space relaxation methods for DP with arc flow models. We also present a dual point of view to contrast the linear relaxation of arc flow models with that of models based on paths and cycles. To conclude, we review the main solution methods and applications of arc flow models based on DP in several domains such as cutting, packing, scheduling, and routing.

8.2 Robust optimization

We also made some methodological and applied progress in the field of robust optimization.

In [1], we study a class of two-stage robust binary optimization problems with objective uncertainty, where recourse decisions are restricted to be mixed-binary. For these problems, we present a deterministic equivalent formulation through the convexification of the recourse-feasible region. We then explore this formulation under the lens of a relaxation, showing that the specific relaxation we propose can be solved by using the branch-and-price algorithm. We present conditions under which this relaxation is exact and describe alternative exact solution methods when this is not the case. Despite the two-stage nature of the problem, we provide NP-completeness results based on our reformulations. Finally, we present various applications in which the methodology we propose can be applied. Our computational results show that our methodology is able to produce better solutions in less computational time compared with the approximate K-adaptability approach, as well as to solve bigger instances than those previously managed in the literature.

In [5], we consider the problem of minimizing the weighted number of tardy jobs on one machine. It is a classical and intensively studied scheduling problem. In this paper, we develop a two-stage robust approach, where exact weights are known after accepting to perform the jobs, and before sequencing them on the machine. This assumption allows diverse recourse decisions to be taken in order to better adapt one's mid-term plan. The contribution of this paper is twofold: first, we introduce a new scheduling problem and model it as a min-max-min optimization problem with mixed-integer recourse by extending existing models proposed for the deterministic case. Second, we take advantage of the special structure of the problem to propose two solution approaches based on results from the recent robust optimization literature: namely the finite adaptability (Bertsimas and Caramanis, 2010) and a convexification-based approach (Arslan and Detienne, 2022). We also study the additional cost of the solutions if the sequence of jobs has to be decided before the uncertainty is revealed. Computational experiments are reported to analyze the effectiveness of our approaches.

9 Bilateral contracts and grants with industry

9.1 Bilateral contracts with industry

Participants: Xavier Blanchot, Francois Clautiaux, Boris Detienne, Aurélien Froger.

We had a contract with RTE to develop strategies inspired from stochastic gradient methods to speed-up Benders' decomposition. We also worked on a specific bilevel stochastic expansion planning problem. The PhD thesis of Xavier Blanchot was part of this contract. The methods developed during the PhD thesis of Xavier are now used by RTE for strategic studies.

Participants: Francois Clautiaux, Ruslan Sadykov.

We also had a short contract (one year) with Renault on a specific scheduling problem, which involves optimizing the configuration of the machines. The problem comes from the practical application of scheduling tests for car prototypes.

Participants: Boris Detienne, Pierre Pesneau, John Jairo Quiroga Orozco.

We have a collaboration with Orange (the contract is still being negotiated between Orange and University of Bordeaux), around the PhD thesis of John Jairo Quiroga Orozco which started in October 2022. The project focuses on models and algorithms for the design of robust 5G networks.

10 Partnerships and cooperations

10.1 International initiatives

10.1.1 Participation in other International Programs

ANR DE-SIDE is an international ANR project. It is coordinated by François Clautiaux. This project is conducted in collaboration with Kedge Business School and Sobolev Institute (Russia). The main objective of the DE-SIDE project is to provide new mathematical models and optimization approaches for design of spatio-temporal networks in a stochastic and dynamic environment. Optimization approaches and mathematical modelling will concern strategic, tactical and operational levels. More specifically, such combinatorial NP-hard problems as Facility Location and Vehicle Routing problems will be considered jointly through optimizing the following decisions: (1) optimum number of facilities, (2) optimal facility location, (3) optimal relocation of facilities according to dynamic evolving parameters, (4) allocation of clusters to each facility, over the time period where the information about spatio-temporal parameters is incomplete or inexact. Various formulations of routing problems among the facilities of the network will be considered under different optimality criteria.

10.2 International research visitors

10.2.1 Visits of international scientists

Prof. Eduardo Uchoa obtained **Inria International Chair** for 2022-2026. During these years, he should spend 12 months in Bordeaux with the team. In 2022, Edurado spent 7 months in total in Bordeaux: 1 month in March and 6 months in June-November.

During the stay of E. Uchoa with the team, he allocated most of his time to a joint book project with Ruslan Sadykov and Artur Pessoa entitled “Optimizing with Column Generation”. First five sections of the book were written in 2022. The first version of the book is expected to be completed in 2023. Secondly, Eduardo worked on papers [9, 17] in collaboration with team members. Eduardo also gave

- an invited tutorial at the Computational Combinatorial Optimization doctoral school in September 2022,
- an invited talk at the Combinatorial Optimization for Telecommunication Networks seminar of the French Group on Polyhedra and Comtinatorial Optimization in September 2022,
- an invited talk in Bonn Workshop on Combinatorial Optimization in October 2022.

Finally, Eduardo taught two courses in the Operations Research master program of University of Bordeaux.

10.2.2 Visits to international teams

Research stays abroad

- A. Froger

Visited institution: Politecnico di Milano

Country: Italy

Dates: 2022/04/01 - 2022/06/28

Context of the visit: The aim of this visit was to start a collaboration with Ola Jabali on the development of optimization models and efficient solution algorithms for the daily planning of charging activities related to a fleet of electric freight vehicles taking into account a wide variety of costs (e.g., energy time-dependent costs), charging infrastructure constraints (e.g., grid restrictions, limited number of chargers), battery degradation, and considering the potential integration of Vehicle-to-Grid technologies. Taking this variety of constraints into account introduces complex coupling decisions between charging and routing. Planning the charging operations of the vehicles becomes necessary.

This leads to integrated problems with both routing and scheduling decisions. During the visit, a joint routing and charge scheduling problem has been defined and contextualized with respect to the existing literature. Mathematical formulations of the problem have been introduced with first solution methods. The collaboration is still ongoing after the end .

Mobility program/type of mobility: Research stay funded following a call for proposals from the Department of Electronics, Information and Bioengineering.

10.3 National initiatives

Dr. Arslan has obtained funding for her ANR-JCJC project DROI which concentrates on primal and dual bounds for adjustable robust optimization problems. Being a "young researcher" fund, this project will help Dr. Arslan build collaborations on national and international level. The team of experts that will help Dr. Arslan conduct this project include team member Boris Detienne, Michael Poss (CR-CNRS-LIRMM), Merve Bodur (Assistant Professor-U. Toronto) and Jérémy Omer (Mdc, INSA-Rennes).

ANR AD-LIB is coordinated by François Clautiaux. This projet is conducted with the LAAS labratory in Toulouse and Toulouse Business School. We consider general aggregation/disaggregation techniques to address optimization problems that are expressed with the help of sequential decision processes. Our main goals are threefold: a generic formalism that encompasses the aforementioned techniques ; more efficient algorithms to control the aggregation procedures ; open-source codes that leverage and integrate these algorithms to efficiently solve hard combinatorial problems in different application fields. We will jointly study two types of approaches, MIP and SAT, to reach our goals. MIP-based methods are useful to obtain proven optimal solutions, and to produce theoretical guarantees, whereas SAT solvers are strong to detect infeasible solutions and learn clauses to exclude these solutions. Their combination with CP through lazy clause generation is one of the best tools to solve highly combinatorial and non-linear problems. Aggregation/disaggregation techniques generally make use of many sub- routines, which allows an efficient hybridization of the different optimization paradigms. We also expect a deeper cross-fertilization between these different sets of techniques and the different communities.

10.4 Regional initiatives

Our team also leads a projet titled *Solution approaches for the Inventory Routing Problem* (November 2020 - October 2023), coordinated by R. Sadykov, and conducted in collaboration with Atoptima start-up. This project is funded by Région Nouvelle Aquitaine and Inria. The recent progress made in solving vehicle routing problems allows us to tackle more complex variants such as the planning of routes over a multi-period horizon combined with the management of inventory levels at the customer sites. This problem, known in the literature as the Inventory Routing Problem (IRP), is not yet within the reach of exact mathematical optimization methods. It combines three levels of decisions to be made for each period: (i) which customer to serve, (ii) how much to deliver or pick up, (iii) which routes to use. It becomes even more complex with the arrival of multi-level and multi-modal logistics solutions: intermediate depots are delivered via large trucks, while the last mile is delivered via light and non-polluting vehicles from these intermediate depots. Finally, it is necessary to be able not only to optimize a tactical schedule, but also to re-optimize this schedule in real time in the light of the hazards of the solution's deployment. The real applications for this optimization model are multiple throughout urban logistics: whether it is in the collection of recycled waste, the delivery of gas stations, the collection of milk in farms, or in maintenance problems with a prescribed time between two services, as well as sales representatives found with this same type of characteristics. It is important to focus on producing new optimization approaches capable of handling this level of complexity by advancing the state-of-the-art.

11 Dissemination

11.1 Promoting scientific activities

11.1.1 Scientific events: organisation

2022 Organization of a workshop "Pricing algorithms". This workshop was organized with Antoine Jeanjean, Benoît Rottembourg and Luce Brotcorne. It involved researchers and industrial practionners of pricing algorithms.

2022 Organization of the conference "Dataquitaine". In this conference, researchers, students, and companies can share their latest results on machine learning, optimization, and artificial intelligence.

11.1.2 Scientific events: selection

Member of the conference program committees

P. Pesneau was part of the program committees of the International Network Optimization Conference (INOC 2022) and of the International Symposium on Combinatorial Optimization (ISCO 2022).

Ruslan Sadykov was part of the scientific committee of the One-day workshop on the Inventory Routing Problem.

Reviewer

Boris Detienne was part of the jury of scientific prize of the ROADEF/EURO Challenge 2020 (awarded in 2022).

Ruslan Sadykov was part of the jury of the PGMO Ph.D. thesis prize 2022.

11.1.3 Journal

Member of the editorial boards

F. Clautiaux is a member of the editorial board of Open Journal on Mathematical Optimization.

Ruslan Sadykov is a member of the editorial board of EURO Journal on Computational Optimization.

Reviewer - reviewing activities

F. Clautiaux has been reviewer for Computers & Operations Research, Discrete Applied Mathematics, European Journal of Operational Research, Mathematical Programming Computation, and for IPCO conference.

A. Froger has been reviewer for Computers & Operations Research, Operations Research, Transportation Research Part C: Emerging technologies, Transportation Research Part E: Logistics and Transportation Review, and Transportation Science.

P. Pesneau has been reviewer for Networks and European Journal of Operational Research.

Ruslan Sadykov has been reviewer for European Journal of Operations Research (2 articles), Transportation Science (3 articles), Integer Programming and Combinatorial Optimization Conference, International Journal on Production Research (2 articles), Discrete Applied Mathematics, Computers & Operations Research (2 articles), IISE Transactions, RAIRO - Operations Research, Computers & Industrial Engineering, Ad Hoc Networks.

Ruslan Sadykov has been awarded 2022 Transportation Science Meritorious Service Award for the exceptional service in the review process.

11.1.4 Invited talks

F. Clautiaux. Compiègne - Prospective en recherche opérationnelle et optimisation, Séminaire invité pour les 40 ans du laboratoire HeuDiaSyC.

F. Clautiaux. Tours - Problèmes de packing en 3D, Séminaire invité pour la journée scientifique annuelle de l'équipe ROOT.

B. Detienne. Lille - Decomposition approaches for robust optimization with mixed-integer recourse, Séminaire invité pour la journée scientifique de l'équipe Inria INOCS "INOCS Days".

R. Sadykov. Tutorial "The power of non-robust cuts in branch-cut-and-price algorithms" at the 23ème Congrès Annuel de la Société Française de Recherche Opérationnelle et d'Aide à la Décision ROADEF 2022. [11]

R. Sadykov. "New route formulations for the Split-Delivery VRP and the IRP" at the One-day workshop on the Inventory Routing Problem [12].

R. Sadykov. "Non-Robust Strong Knapsack Cuts for Capacitated Location-Routing and Related Problems" 1ère Journée commune ROADEF/AIRO [13].

11.1.5 Leadership within the scientific community

F. Clautiaux: **President of ROADEF** (up to february). ROADEF is the French Operations Research society (500 members).

F. Clautiaux: Member of the board of DOMEX IA/Data Science in Nouvelle Aquitaine. This entity aims at developping company activies in AI, data science and operations research in Nouvelle Aquitaine.

F. Clautiaux: **Scientific board member of GDR Recherche Opérationnelle**.

A. Arslan: **Scientific board member of GDR Recherche Opérationnelle** and **responsible for young resercher-oriented activities**.

11.1.6 Scientific expertise

F. Clautiaux is a scientific expert for Région Nouvelle Aquitaine.

B. Detienne is a scientific expert for the European Science Foundation.

11.2 Teaching - Supervision - Juries

11.2.1 Teaching

F. Clautiaux is the head of the master program in Applied Mathematics (2 years, 5 master programs, around 200 students in total).

B. Detienne is the head of the master program in operations research at university of Bordeaux (2 years, 35 students).

P. Pesneau is the head of the Master of Engineering in Mathematical Optimization (CMI OPTIM) of the University of Bordeaux (5 years, 25 students).

A. Froger: Optimisation (L2, Université de Bordeaux), Challenges algorithmiques, Groupe de travail applicatif (L3, Université de Bordeaux), Graphes et algorithmes, Rémédiation Programmation linéaire, Rémédiation Optimisation non linéaire, Travaux d'étude et de recherche (M1, Université de Bordeaux), Gestion des opérations et planification de la production (M2, Université de Bordeaux).

F. Clautiaux: Integer programming (M2, Univ. Bordeaux), industrial project in optimization (M2, Univ. Bordeaux), optimization project (L1, Univ. Bordeaux), algorithms for combinatorial optimization (M1, Univ. Bordeaux).

B. Detienne: Optimisation continue sous contraintes (M1, Université de Bordeaux), Optimisation dans l'incertain (M2, Université de Bordeaux).

P. Pesneau: Programmation pour le calcul scientifique (L2, Université de Bordeaux), Recherche opérationnelle, (L3, INP Bordeaux), Introduction à la programmation en variables entières, Remédiation C++, Travaux d'étude et de recherche (M1, Université de Bordeaux), Integer Programming (M2, Université de Bordeaux).

R. Sadykov: Introduction à la Programmation par Contraintes (M2, Université de Bordeaux).

A. Arslan: Optimisation dans l'incertain (M2, Université de Bordeaux).

11.2.2 Supervision

X. Blanchot defended his PhD thesis "Solving large-scale stochastic optimization programs: application to investment problems for power systems" on December 15, 2022, under the supervision of F. Clautiaux and A.Froger.

R. Sadykov and A. Froger supervise one postdoctoral researcher: E. Queiroga.

F. Clautiaux and A. Froger supervise one PhD student: Luis Lopes Marques (starting from October 2022).

Fulin Yan (a student intern from INSA Rennes) defended his master thesis "Combining optimization techniques and graph neural networks for solving discrete optimization problems" in September 2022, under the supervision of F. Clautiaux and A. Froger. The work was also supervised by a member of CATIE (Centre Aquitain des Technologies de l'Information et Électroniques).

F. Clautiaux and B. Detienne supervise one PhD student: Komlanvi Parfait Ametana (starting from October 2021).

B. Detienne and P. Pesneau supervise one PhD student: John Jairo Quiroga Oruzco (starting from October 2022).

B. Detienne and G. Stauffer supervise one PhD student: Mickael Gaury (starting from October 2021.

R. Sadykov supervises three Ph.D. students: Isaac Balster (starting from November 2020), Daniil Khachay (starting from September 2020), and Sylvain Lichau (starting from October 2022).

11.2.3 Juries

F. Clautiaux: Luis Alberto Salazar Zendeja (Inria Lille, rapporteur), Bingqian Liu (Paris Saclay, rapporteur), Marvin Stanczak (Toulouse, examinateur), HDR : Hassene Aissi (Paris Dauphine, examinateur)

11.3 Popularization

11.3.1 Education

A. Froger: Lectures at the AI4Industry workshop to introduce the field of Operations Research to engineering students of the Nouvelle-Aquitaine region.

A. Arslan and B. Detienne: Winter School of GDR-RO on Robust Optimization. This school targeted at PhD students in France focused on teaching robust optimization techniques to students coming from diverse fields of operations research. Dr. Arslan et Dr. Detienne have each presented lectures, problem sessions and lab sessions during the event.

11.3.2 Interventions

F. Clautiaux: webinar for Banque Publique d'Investissement (BPI). In the context of BPI France - Université, Inria Academy. One hour webinar on "Operations optimization in a incertain context".

12 Scientific production

12.1 Publications of the year

International journals

[1] A. N. Arslan and B. Detienne. 'Decomposition-based approaches for a class of two-stage robust binary optimization problems'. In: *INFORMS Journal on Computing* 34.2 (Mar. 2022). DOI: 10.1287/ijoc.2021.1061. URL: https://hal.inria.fr/hal-02190059.

[2] I. Ben Mohamed, W. Klibi, R. Sadykov, H. Şen and F. Vanderbeck. 'The Two-Echelon Stochastic Multi-period Capacitated Location-Routing Problem'. In: *European Journal of Operational Research* (16th July 2022). DOI: 10.1016/j.ejor.2022.07.022. URL: https://hal.inria.fr/hal-02987266.

[3] A. Froger and R. Sadykov. 'New exact and heuristic algorithms to solve the prize-collecting job sequencing problem with one common and multiple secondary resources'. In: *European Journal of Operational Research* (2022). DOI: 10.1016/j.ejor.2022.07.012. URL: https://hal.inria.fr/hal-03287769.

[4] R. Griset, P. Bendotti, B. Detienne, M. Porcheron, H. Şen and F. Vanderbeck. 'Combining Dantzig-Wolfe and Benders decompositions to solve a large-scale nuclear outage planning problem'. In: *European Journal of Operational Research* 298.3 (May 2022), pp. 1067–1083. DOI: 10.1016/j.ejor.2021.07.018. URL: https://hal.inria.fr/hal-03521369.

[5] H. Lefebvre, F. Clautiaux and B. Detienne. 'A two-stage robust approach for minimizing the weighted number of tardy jobs with objective uncertainty'. In: *Journal of Scheduling* (2023). URL: https://hal.archives-ouvertes.fr/hal-02905849.

[6] V. L. de Lima, C. Alves, F. Clautiaux, M. Iori and J. M. Valério de Carvalho. 'Arc flow formulations based on dynamic programming: Theoretical foundations and applications'. In: *European Journal of Operational Research* 296.1 (Jan. 2022), pp. 3–21. DOI: 10.1016/j.ejor.2021.04.024. URL: https://hal.inria.fr/hal-03478786.

[7] G. Marques, R. Sadykov, J.-C. Deschamps and R. Dupas. 'A branch-cut-and-price approach for the single-trip and multi-trip two-echelon vehicle routing problem with time windows'. In: *Transportation Science* 56.6 (16th Mar. 2022), pp. 1598–1617. DOI: 10.1287/trsc.2022.1136. URL: https://hal.inria.fr/hal-03139799.

[8] O. Rivera Letelier, F. Clautiaux and R. Sadykov. 'Bin Packing Problem with Time Lags'. In: *INFORMS Journal on Computing* 34.4 (16th Mar. 2022), pp. 2249–2270. DOI: 10.1287/ijoc.2022.1165. URL: https://hal.inria.fr/hal-02986895.

[9] M. Roboredo, R. Sadykov and E. Uchoa. 'Solving Vehicle Routing Problems With Intermediate Stops Using VRPSolver Models'. In: *Networks* (7th Dec. 2022). DOI: 10.1002/net.22137. URL: https://hal.archives-ouvertes.fr/hal-03899372.

International peer-reviewed conferences

[10] I. Balster, T. Bulhões, P. Munari and R. Sadykov. 'A new branch-cut-and-price algorithm for the split delivery vehicle routing with time windows'. In: 23ème congrès annuel de la Société Française de Recherche Opérationnelle et d'Aide à la Décision. Villeurbanne - Lyon, France, 23rd Feb. 2022. URL: https://hal.archives-ouvertes.fr/hal-03595220.

Conferences without proceedings

[11] R. Sadykov. 'Tutorial: The power of non-robust cuts in branch-cut-and-price algorithms'. In: ROADEF 2022 - 23ème édition du congrès annuel de la Société Française de Recherche Opérationnelle et d'Aide à la Décision. Lyon, France, 23rd Feb. 2022. URL: https://hal.archives-ouvertes.fr/hal-03899389.

[12] R. Sadykov, I. Balster, T. Bulhões and P. Munari. 'New route formulations for the Split-Delivery Vehicle Routing Problems'. In: One-day workshop on the Inventory Routing Problem. Champs-sur-Marne, France, 20th May 2022. URL: https://hal.archives-ouvertes.fr/hal-03899404.

[13] R. Sadykov, P. Liguori, A. R. Mahjoub, G. Marques and E. Uchoa. 'Non-Robust Strong Knapsack Cuts for Capacitated Location-Routing and Related Problems'. In: Première journée commune ROADEF-AIRO. Virtual, France, 30th Nov. 2022. URL: https://hal.archives-ouvertes.fr/hal-03899418.

Reports & preprints

[14] I. Balster, T. Bulhões, P. Munari, A. A. Pessoa and R. Sadykov. *A new family of route formulations for split delivery vehicle routing problems*. Inria Centre at the University of Bordeaux, 30th Nov. 2022. URL: https://hal.archives-ouvertes.fr/hal-03900628.

[15] X. Blanchot, F. Clautiaux, B. Detienne, A. Froger and M. Ruiz. *The Benders by batch algorithm: design and stabilization of an enhanced algorithm to solve multicut Benders reformulation of two-stage stochastic programs*. 15th Dec. 2022. URL: https://hal.archives-ouvertes.fr/hal-03286135.

[16] D. Khachai, O. Battaïa, M. Khachay and R. Sadykov. *An Exact Approach for the Precedence Constrained Generalized Traveling Salesman Problem*. 20th Nov. 2022. URL: https://hal.archives-ouvertes.fr/hal-03906983.

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