RESEARCH CENTRE

Inria Saclay Center at Institut Polytechnique de Paris

IN PARTNERSHIP WITH: EDF R&D, Institut Polytechnique de Paris

2022 ACTIVITY REPORT

Project-Team IDEFIX

Inversion of Differential Equations For Imaging and physiX

DOMAIN

Applied Mathematics, Computation and Simulation

THEME Numerical schemes and simulations



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Project-Team IDEFIX

Creation of the Project-Team: 2021 August 01

Keywords

Computer sciences and digital sciences

- A6.1. Methods in mathematical modeling
- A6.1.1. Continuous Modeling (PDE, ODE)
- A6.1.4. Multiscale modeling
- A6.2.1. Numerical analysis of PDE and ODE
- A6.2.6. Optimization
- A6.2.7. High performance computing
- A6.3.1. Inverse problems
- A6.5.4. Waves

Other research topics and application domains

- B1.2.3. Computational neurosciences
- B2.6.1. Brain imaging
- B3.3.1. Earth and subsoil
- B3.3.2. Water: sea & ocean, lake & river

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2 Overall objectives

Inverse problems are encountered in many real life applications and the ones we are interested in are those that can be formulated as parameter identifications in a PDE (system) modeling physical phenomena, primarily wave propagation and diffusion. As opposed to determining the solution of the forward model, identifying the parameters from measurements of this solution usually leads to an unstable and non linear problem that may be not uniquely solvable. A standard method to formulate this inverse problem is to consider it as a minimization of a cost functional that measures data fidelity. The solution to the latter is computationally much more costly than solving the PDE and may even not be realistic for number of applications that require real time answers or for very large scale problems. These considerations motivated the research guidelines items exposed above and that we shall develop further in the following.

At EDF, the need for algorithms to solve inverse problems is present in numerous applications (see Section 4 for instance). The team Signal, Image and Learning at EDF R&D, PRISME has developed solutions mainly based on signal processing methods that do not require fine modeling of the physical phenomena (describing the experiment). This enables fast simplified responses that can usually be satisfactory. The complexification of the measuring devices and environments appealed for more precise assessment of the experiments and therefore for more reliable/precise inversion methods. This was the motivation behind the intense collaborations between EDF R&D and the DEFI project team that lead to six co-supervised PhD thesis and one PostDoc on various themes (Eddy current imaging for pipes, data assimilation for primary cooling loops, sampling methods for concrete like materials, multi-element eddy current 3-D probes, qualitative inversion methods and spectral signatures for ultrasound applications). The joint team aims at pursuing this collaborative effort that has been beneficial to both partners, motivating at the same time fundamental research to establish solid theoretical foundations of promising inversion methods and (non trivial) adaptations of established methods to solve applications of interest for EDF.

3 Research program

Let us describe the outline of the main challenges that we would like to address for solutions to inverse problems, taking as a guideline the example of non destructive testing which is central for EDF applications. A typical experiment would be to probe some defects inside a given structure by sending waves that can propagate inside the domain of interest. The response of the media is recorded by some receivers and forms the data of the inverse problem. We can distinguish two types of inverse problems. In the first type, referred to by "imaging", one is interested by only the location and/or the shape of the defect/inclusion. In the second one, referred to by "identification", one is interested in getting information on the defect physical properties. Both problems (imaging and identification) are non linear and ill-posed (lack of stability with respect to measurements errors if some careful constrains are not added). Moreover, the unique determination of the geometry and/or the coefficients is not guaranteed in general if sufficient measurements are not available. As an example, in the case of anisotropic inclusions, one can show that an appropriate set of data uniquely determine the geometry but not the material properties. These theoretical considerations are usually difficult to address and are not only important in understanding the mathematical properties of the inverse problem, but also guide the choice of appropriate numerical strategies (which information can be stably reconstructed) and also the design of appropriate regularization techniques and improve the measurement techniques. Moreover, uniqueness proofs can be constructive proofs, i.e. they implicitly contain a numerical algorithm to solve the inverse problem, hence their importance for practical applications. The sampling methods introduced below are one example of such algorithms. As a complementary notion to identifiability is the notion of invisibility. The latter topic has attracted a large attention in the inverse problem community due in particular to the recent and rapid development of metamaterials that made plausible the design of cloaking devices based on transformation optics. However, these transformations require the use of non dissipative materials exhibiting singular physical coefficients taking infinite values, which is indeed not realistic. This motivated us to consider a weaker notion of invisibility where one would like to achieve invisibility for only a finite set of measurements and frequencies. This objective is less ambitious and consequently, it is more easily

achievable. On the other hand, it is pertinent from a practical point of view because one always has a finite number of sensors and very often, one has only access to a small number of measurements. In addition to theoretically investigate this issue for some idealized models, we would like to fructify our findings for non destructive testings in waveguides.

An important part of our research activity is dedicated to numerical methods applied to the first type of inverse problems, where only the geometrical information is sought. In its general setting the inverse problem is very challenging and no method can provide a universal satisfactory solution to it (regarding the balance cost-precision-stability). This is why in the majority of the practically employed algorithms, some simplification of the underlying mathematical model is used, according to the specific configuration of the imaging experiment. The most popular ones are geometric optics (the Kirchhoff approximation) for high frequencies and weak scattering (the Born approximation) for small contrasts or small obstacles. They actually give full satisfaction for a wide range of applications as attested by the large success of existing imaging devices (radar, sonar, ultrasound, X-ray tomography, etc.), that rely on one of these approximations.

Generally speaking, the used simplifications result in a linearization of the inverse problem and therefore are usually valid only if the latter is weakly non-linear. The development of these simplified models and the improvement of their efficiency is still a very active research area. With that perspective we are particularly interested in deriving and studying higher order asymptotic models associated with small geometrical parameters such as small obstacles, thin coatings, periodic media, Higher order models usually introduce some non linearity in the inverse problem, but are in principle easier to handle from the numerical point of view than in the case of the exact model. Asymptotic analysis is also a corner stone in our methodology to prove invisibility for finite number of measurements.

A major research axis is dedicated to algorithms that avoid the use of such approximations and that are efficient where classical approaches may fail: i.e. roughly speaking when the non linearity of the inverse problem is sufficiently strong. This type of configuration is motivated by the applications mentioned below, and occurs as soon as the geometry of the unknown media generates non negligible multiple scattering effects (multiply-connected and closely spaces obstacles) or when the used frequency is in the so-called resonant region (wave-length comparable to the size of the sought medium). It is therefore much more difficult to deal with and requires different approaches such as sampling methods. The sampling methods are fast imaging solvers adapted to multi-static data (multiple receiver-transmitter pairs). Even if they do not use any linearization of the forward model, they rely on computing the solutions to a set of linear problems of small size, that can be performed in a completely parallel procedure. Our team is among the leading international groups in the developments of these techniques. We are one of the main contributors in recent advancements in this field and actively acting in its dissemination among the academic and industrial communities. We shall pursue our efforts in developing and promoting these techniques. A closely related subject is the study of so-called Transmission eigenvalues that naturally arise in the analysis of inverse medium problems and particularly in the theory behind sampling methods. These frequencies can be seen as the extension of the notion of resonant frequencies for impenetrable objects to the case of penetrable media. Our developments of sampling methods lead us to discover that transmission eigenvalues can be reconstructed from multi-frequency and multistatic measurements. The spectrum formed by these special frequencies can be related to the materiel properties of the medium and therefore can be used as a signature that characterizes some aspects of this medium (although a complete answer to this statement is far from being available). More specifically we promote the use of these quantities to obtain qualitative information on changes in the probed domain (as in non destructive testing). The study of transmission eigenvalues has become (as for sampling methods) a well identified branch in the inverse problem community that raises many challenges ranging from purely theoretical questions to numerical schemes and significance for applications.

For the identification problem, one would also like to have information on the physical properties of the targets. Of course optimization methods is a tool of choice for these problems. The application of non linear optimization methods for inverse problems has to be supplemented by regularization strategies. While convergence for Hilbertian regularization is well understood from the theoretical point of view, it is still far from being the case for non Hilbertian norms. For instance regularization strategies that promote sparsity belong to the latter class and is of great interest for inverse problems where the coefficients have singularities (point sources, crack like defects, piecewise constant material properties, etc...). Exploring theoretical and numerical issues raised by these regularization is of interest for applications. We plan to invest on these issues together with the use of non standard fidelity functional that may help reducing the number of local minima. Combining deterministic techniques with stochastic ones is also an interesting perspective that has not been sufficiently explored in the literature and that we aim at developing. This would allows us to also investigate feasibility of Bayesian inference for these non linear and computationally involving inverse problems. Exploiting neural networks in the design of solutions to inverse problems is major trend in the inverse problem community as in many other scientific area. Using these techniques to directly solve severe ill posed problems, as inverse scattering problems, does not seem a certifiable route. A more reasonable approach to benefit from the ability of these networks to encode high dimensional complex non linear functional would be to use them for automatically adjusting deterministic optimization parameters such as descent steps and/or regularization parameters (or priors). This also holds true for their use in the sampling methods invoked above and we shall start by exploring this combination first.

From the practical point of view, the major limitation of sampling methods would be the need of a large amount of data to achieve a reasonable accuracy. On the other hand, optimization methods do not suffer from this constrain but they require good initial guess to ensure convergence and reduce the number of iterations. Therefore it seems natural to try to combine the two class of methods in order to calibrate the balance between cost and precision.

Independently from the formulation of the optimization problem, the efficiency of inversion algorithm associated with this formulation greatly depends on the efficiency of the forward solver. Our team has already made significant contributions in acceleration techniques for solutions to the forward problem (waves and diffusion). We developed strong expertise in H-matrix compression and combination with other acceleration techniques such as FFT and fast multipole methods for wave problems. Fructifying this into the solution to large scale inverse problems in link with geophysical application or non destructive testing is promising for obtaining feasible inversion algorithm for the full non linear inverse problem. Domain decomposition technique is yet another expertise that we have developed and would like to explore for accelerating the solution of the forward and inverse problems. For the latter, methodologies where iterations on the inversion parameter and the forward and adjoint problems are combined merit investigations. Several approaches have been proposed in the literature, but the convergence of these schemes and their efficiency are not yet clear in the context of inverse ill-posed problems.

Although a major focus will be given to applications and methodologies that can be of interest for EDF, the contours of the IDEFIX research team include topics that may go beyond that perspective.

In particular we are also interested in applications involving the imaging of biological tissues with the technique of Diffusion Magnetic Resonance Imaging (DMRI). Roughly speaking, DMRI gives a measure of the average distance travelled by water molecules in the imaged medium and can give useful information on cellular structure and structural change when the medium is biological tissue. In particular, we would like to infer from DMRI measurements changes in tissue and cellular structure occurring under various physiological or pathological conditions, as well generally the cell morphology in the region of interest. The main challenges here are: 1) to model correctly the measured signals using diffusive-type evolution equations, 2) to handle numerically the geometical complexity of biological issue; 3) to use the first two to identify physically relevant parameters from the measurements. There are two main groups of approaches to the first two challenges. The first one relies on using random walkers to mimic the diffusion process in a given geometrical configuration. The second one exploits the model given by the Bloch-Torrey partial differential equation, which describes the evolution of the complex transverse water proton magnetization under the influence of diffusion-encoding magnetic field gradients pulses. We primarily work in the simulation and analysis of the Bloch-Torrey partial differential equation in complex geometries, in other words, we follow the second group of approaches. For the third challenge, we are particularly interested in constructing reduced models of the multiple-compartment Bloch-Torrey model using homogenization methods.

Our team will also develop computational tools specifically designed for acoustics. They are motivated by applications that need a high fidelity restitution of the sound distribution. This includes in particular fast simulation of head-related transfer function (HRTF) which is a response that characterizes how an ear receives a sound from a point in space. Obtaining the best way to define HRTF is still a matter of debate. To help addressing this issue we would like to study theoretically and numerically the injectivity of HRTF functionals and the pertinence of their simplifications. The main challenge for the numerical simulation of HRTF is the modeling of the ear tissue and its complex interaction with incoming sound. We also would like to develop quick algorithms for real time ambisonics (three dimensional sound captation/restitution). Fast and robust solutions to this inverse source problem would be of interest for solutions to immersive audio. We are also investing in the development of reduced models for the simulation of complex room acoustics and auralization.

4 Application domains

4.1 Eddy Current Imaging for steam generator and rotating machine

Eddy Current is an approximation of Maxwell system at low frequency. Probe that works in that quasistationary regime are commonly used in non destructive testing. We are interested in inverse problems for these type of measurement in steam generator and more recently for rotating machine.

Steam generators are critical components in nuclear power plants. For a sake of radioactive safety, the water flow (called the primary fluid) which ensures the cooling of the core reactor is separated from the water flow (called the secondary fluid) which is transformed in steam to generate electricity: the heat must be transferred via the steam generator. The primary fluid circulates in tubes with U-shape while the secondary fluid rises up in the steam generator along these tubes.

Without disassembling the steam generator, the lower part of the U-tubes is inaccessible for normal inspections. Therefore, a non-destructive examination procedure, called eddy current testing (ECT), is usually used to detect the presence of deposits. In an ECT, one introduces a probe consisting of coils of wire in the tube that deliver electromagnetic excitation at low frequencies (eddy current regime) and measure induced currents by the external media (ECT signals). The design of robust and reliable PDE based inversion methods to analyze ECT signals is a long term ongoing project with and within EDF R&D. We first developed and analyzed the simplified setting of axisymmetric geometries which allows to model the problem in 2D and to test various options for the direct and inverse algorithms and in particular an algorithm based on the Level Set method. We start extending this work to 3D inversions for various types of probes (with angular resolutions) and various types of defects. And will pursue this in the future in order to treat real data with multiple defects (i.e. cracks, deposit, thin deposit).

Finally related to Eddy Current modeling, we work on low frequency/quasi-stationary Maxwell system in alternators and engines. On those systems EDF is trying to assess from measurements the exact geometry and the physical properties which have potentially (parametric) non linear constitutive laws. A collaboration on this topic with the EDF team that contributes to code CARMEL. Together with the same team and others, we submit a project on an experimental facilities for eddy current where we will contribute on the calibration of the data using inverse problems methodology and extension to geometry not limited to pipes.

4.2 Non destructive testing of concrete-like material

Concrete is a widely used material thanks to its appealing (when reinforced) properties and its manageable cost. However, it is a very complex material: highly heterogeneous, multiscale, evolving over time, etc. Yet, it has to be inspected to ensure that the structure is safe and especially when this structure is part of sensitive infrastructures such as power plants. Using mechanical waves to inspect concrete is widely used in practice but many aspects still constitute very challenging problems due to the complex properties of the material. Several defects are of interest and measurements might vary with respect to the target. Imaging are very difficult with conventional techniques due to potentially low contrast and complicated structure (i.e. reinforcing bar, metallic liners). Sampling methods are able to tackle this type of problem by integrating the information on the a priori structure of the geometry. Other methods such as homogenization could be a valuable approached for identifying this type of defects.

The region between air and the first reinforcement steel bar are of primary importance because it shields the steel from the exterior and therefore prevents corrosion. To inspect this area, engineers rely on surface wave techniques similar to Multi-channel Analysis of Surface Waves (MASW) in geophysics. However, concrete is not a layered material and therefore interpretation of this type of measurements is not easy. Basically this method constructs the dispersion curve of surface waves through data processing

and then uses this information to invert a layered model for the material. This procedure shares similarities with spectral signature identification and it would be interesting to further clarify this link in order to obtain better interpretation (or reformulation) of MASW in this context.

Similarly to ultrasonic waves, electromagnetic waves are generally used to inspect concrete type materials, we will be involved in imaging complex structure with radar type measurements. A more challenging application is to determine the electromagnetic properties of the material and relate them to the hydration of the cement paste. MASW analysis of the measurements are also consider in this setting similarly to mechanical waves.

4.3 Subsurface imaging

Subsurface imaging up to one hundred meters is of primary importance for power plant safety assessment with respect to seismic activity. Issues range from testing the interface between the bedrock and the bottom of dam, to quantitative map and detection of fracture underneath nuclear power plant and imaging of the sea bed to evaluate the feasibility of construction of offshore wind turbine. Earth imaging is a well studies area but primarily at medium to long distance thanks to oil industry and seismology. However subsurface imaging is not very common and has its own difficulties due to the heterogeneous structure of the soil and the higher frequency of the wave needed to have a satisfactory resolution.

Dykes are structures that are difficult to inspect but are of primary importance for the safety of power plant and MASW is usually applied for that purpose. Using data processing techniques dispersion curves are extracted from recorded surface waves. Engineers then use semi-analytic model of dispersion curves for stratified media to obtain an subsurface image. Extending this type of methods to non-stratified media is challenging and it would be interesting to see how it compares with classical optimization based inversion methods.

As for concrete imaging mechanical, electromagnetic waves and conductivity are the various physics used to probe the medium. Both shared heterogeneous physical properties, embedded in potentially complex geometries and seek unknown of several types and are applications less investigated that medical, seismic or metallic imaging. As research on measurements techniques are carried by the same group in EDF R&D PRISME we will seek for synergy between this two fields.

4.4 Applications of concepts related to invisibility for finite set of measurements

As mentioned above, in the team we have developed different approaches to construct obstacles which are invisible for imaging techniques with a finite number of measurements. In what we did, obstacles are invisible at a fixed frequency. It would be important to study more the sensitivity of the invisibility results with respect to the frequency. In the construction algorithms, can we add constraints so that invisibility remains robust to the frequency? In our studies, we have mainly focused our attention on acoustics in waveguides. It would be interesting to investigate electromagnetic and elasticity problems. Moreover, it would be very instructive to investigate how the invisible objects we design theoretically and numerically behave in practice. To proceed, we contemplate to work with experimentalists at the Acoustic Department of the University of Le Mans with whom we are in contact.

Until now, we have only constructed obstacles which are invisible in time-harmonic regime. It would be interesting to study what can be done for time dependent problems. Maybe the first question is as follows: imagine that a time dependent source term is given (a pulse), how to design the geometry so that the signal passes through the structure as if they were no defect? For the applications, one can think to the optimal design of a stent to fix a damaged coronary artery. It is known that an inappropriate shape and material for the stent can produce scattered waves which are harmful for the cardiac muscle.

In the physical community, spectacular advances have recently been made in the development of a new field called "wave front shaping". For a given unknown complex scatterer, the goal is to find the best entrance signal to obtain a given physical property (focalisation of the energy, good transmission, ...). We emphasize that in this context, optimization techniques cannot be used because the scatterer is complex and a priori unknown. This point of view is different from the one considered up to know in the team. Indeed, in our case, the entrance signal is given and we look for the scatterer to have invisibility. It would be interesting to study the connections existing between the two approaches.

Another area which would benefit from the expertise of the team is the design of barriers that would isolate critical buildings of nuclear power plant from seismic solicitation. This topic is directly related to the partial invisibility exposed above. Indeed, for this application, complete cloaking is not necessary as it is enough to transfer the seismic solicitation to other area in space and to reduce the maximal solicitation on specific modes of the structure (either by spreading the solicitation in time or by transferring the solicitation to a more robust mode of vibration). This boils down to design structures ensuring zero transmission of energy between different given modes, a question we also consider in our works dealing with waves propagation in waveguides. This problematic has not yet been planned by EDF in the short term perspectives but we believe it is worth exploring as there already exist realistic experiments of such cloaking constructions formed by stilts embedded in the soil.

4.5 Other potential applications of interest for EDF

One way to measure flow rate in stationary environment is to use ultrasound measurements. Ultrasound propagates faster in the direction of the flow and slower in the opposite direction. This principle has been used to measure flow rate in pipes with relatively clear water. It is also used for water carrying heterogeneities, like bubbles, sand, stone... but with an experimental approach, with few understanding of the limits of the method. Being able to model the propagation in such an heterogeneous medium and to adapt the analysis of the data would not only improve current measurement techniques but also give tools to know a priori the method limits. There are issue in hydraulic power plants to assess the presence of obstacles or objects in moving fluid using SONAR type measurements. Depending on the application it is not clear yet if the fluid motion could be neglect and it will be the purpose of further research. These applications would be a promising first step to open our expertise towards fluid related problems, which might have large perspectives in our collaboration with EDF. Similarly to non destructive testing of concrete, other approaches rely on electromagnetic or passive measurements (sources of sound locations) in order to assess the flow rate.

4.6 Diffusion MRI for the classification of neuron populations

Sensing the microstructural characteristics of human brain tissue with clinical MRI scanners has been an area of heated debate in the diffusion MRI community. We have recently presented evidence that, if we focus on the cortex, specifically in the insula and anterior cingulate cortex (ACC), the unique characteristics of the cellular population in these gyri allow using clinical-grade scanners to sense the presence of Von Economo neurons (VENs). VENs, uniquely localized in the insula and ACC, are large neurons thought to play an important role in goal-directed behaviors and emotional regulation. However, there is a lack of tools enabling studies on VENs population characteristics and their link to brain function and behavior. We plan to attack a new frontier in diffusion MRI for microstructure quantification by focusing on specific areas in the cortex, rather than the heretofore much studied white matter areas. We will benefit from a collaboration with the Stanford Cognitive and Systems Neuroscience Laboratory lead by Dr. Vinod Menon and Dr. Wassermann of the Inria Parietal Team.

5 New software and platforms

5.1 New software

5.1.1 ECIP

Name: Eddy Current Imaging for Pipes

Keywords: Inverse problem, Partial differential equation, HPC, Domain decomposition

Functional Description: This software identifies deposit on pipes from measurements of eddy current probes. It is based on finite elements and domain decomposition through the softwares HPDDM, PETSc and FreeFEM, for the resolution of the PDE model of the eddy current measurements. It uses an iterative algorithm to identify the deposit properties.

Contact: Lorenzo Audibert

Partner: Edf

5.1.2 CASTOR

Keyword: C++

Functional Description: The objective of the castor library is to propose high-level semantics, inspired by the Matlab language, allowing fast software prototyping in a low-level compiled language. It is nothing more than a matrix management layer using the tools of the standard C++ library, in different storage formats (full, sparse and hierarchical). Indeed, the use of IDEs 1 such as Xcode, Visual studio, Eclipse, etc. allows today to execute compiled code (C, C++, fortran, etc.) with the same flexibility as interpreted languages (Matlab, Python, Julia, etc.).

A header-only template library for matrix management has been developed based on the standard C++ library, notably the std::vector class. Many tools and algorithms are provided to simplify the development of scientific computing programs. Particular attention has been paid to semantics, for a simplicity of use "à la matlab", but written in C++. This high-level semantic/low-level language coupling makes it possible to gain efficiency in the prototyping phase, while ensuring performance for applications. In addition, direct access to data allows users to optimize the most critical parts of their code in native C++. Finally, complete documentation is available, as well as continuous integration unit tests. All of this makes it possible to meet the needs of teaching, academic issues and industrial applications at the same time.

The castor library provides tools to :

create and manipulate dense, sparse and hierarchical matrices make linear algebra computations based on optimized BLAS library make graphical representations based on VTK library These tools are used by applicative projects :

finite and boundary element method using Galerkin approximation analytical solutions for scattering problems

URL: https://leprojetcastor.gitlab.labos.polytechnique.fr/castor/#

Contact: Matthieu Aussal

5.1.3 SpinDoctor

Name: SpinDoctor Diffusion MRI Simulation Toolbox

Keywords: MRI, Simulation, Finite element modelling

Functional Description: SpinDoctor can be used

1. to solve the Bloch-Torrey PDE to obtain the dMRI signal (the toolbox provides a way of robustly fitting the dMRI signal to obtain the fitted Apparent Diffusion Coefficient (ADC)), 2. to solve the diffusion equation of the H-ADC model to obtain the ADC, 3. a short-time approximation formula for the ADC is also included in the toolbox for comparison with the simulated ADC.

URL: https://github.com/SpinDoctorMRI/

Contact: Jing Rebecca Li

6 New results

6.1 Fast data driven imaging methods

Participants: Lorenzo Audibert, Houssem Haddar, Nouha Jenhani, Hadrien Montanelli, Fabien Pourre, Jean-Marie Henault.

6.1.1 Ultrasonic imaging in highly heterogeneous backgrounds

H. Haddar, F. Pourahmadian

This work formally investigates the differential evolution indicators as a tool for ultrasonic tracking of elastic transformation and fracturing in randomly heterogeneous solids. Within the framework of periodic sensing, it is assumed that the background at time to contains (i) a multiply connected set of viscoelastic, anisotropic, and piece-wise homogeneous inclusions, and (ii) a union of possibly disjoint fractures and pores. The support, material properties, and interfacial condition of scatterers in (i) and (ii) are unknown, while elastic constants of the matrix are provided. The domain undergoes progressive variations of arbitrary chemo-mechanical origins such that its geometric configuration and elastic properties at future times are distinct. At every sensing step t0, t1, ..., multi-modal incidents are generated by a set of boundary excitations, and the resulting scattered fields are captured over the observation surface. The test data are then used to construct a sequence of wavefront densities by solving the spectral scattering equation. The incident fields affiliated with distinct pairs of obtained wavefronts are analyzed over the stationary and evolving scatterers for a suit of geometric and elastic evolution scenarios entailing both interfacial and volumetric transformations. The main theorem establishes the invariance of pertinent incident fields at the loci of static fractures and inclusions between a given pair of time steps, while certifying variation of the same fields over the modified regions. These results furnish a basis for theoretical justification of differential evolution indicators for imaging in complex composites which, in turn, enable the exclusive tomography of evolution in a background endowed with many unknown features [23].

6.1.2 Sampling method in linear elasticity for concrete: gravel honeycomb

L. Audibert, H. Haddar, JM Henault, F. Pourre

We consider the propagation of elastic waves at a given wavenumber in a penetrable medium with different levels of complexity ranging from an unbounded homogeneous medium to a partially bounded medium with or without a microstructure mimicking aggregates in concrete. In those various configurations we studied the Linear Sampling Method for imaging gravel honeycomb defects. The numerical solver is based on FreeFEM together with PETSc to perform parallel computations. Different source modeling has been investigated for point sources. The numerical results show satisfactory reconstruction in some idealistic setting. This is preparatory work before handling experimental data on mockup.

6.1.3 Differential imaging in periodic media

Y. Boukari, H. Haddar, N. Jenhani

We consider a problem of nondestructive testing of an infinite periodic penetrable layer using acoustic waves. This is an important problem with growing interest since periodic structures are part of many fascinating modern technological designs with applications in (bio)engineering and material sciences. In many sophisticated devises the periodic structure is complicated or difficult to model mathematically, hence evaluating its Green's function which is the fundamental tool of many imaging methods, is computationally expensive or even impossible. On the other hand, when looking for flows in such complex media, the option of reconstructing everything, i.e. both periodic structure and the defects, may not be viable. We proposed in an earlier work an approach which provides a criteria to reconstruct the support of local anomalies without knowing explicitly or reconstructing the periodic healthy background. We provide a theoretical justification of this method that avoids assuming that the local perturbation is also periodic. Our theoretical framework uses functional spaces with continuous dependence with respect to the Floquet-Bloch variable. The corner stone of the analysis is the justification of the Generalized Linear Sampling Method in this setting for a single Floquet-Bloch mode [16].

6.1.4 Time-vs. frequency-domain inverse elastic scattering: Theory and experiment

H. Haddar, X. Liu, F. Pourahmadian, J. Song

This study formally adapts the time-domain linear sampling method (TLSM) for ultrasonic imaging of stationary and evolving fractures in safety-critical components. The TLSM indicator is then applied to the laboratory test data and the obtained reconstructions are compared to their frequency-domain counterparts. The results highlight the unique capability of the time-domain imaging functional for

high-fidelity tracking of evolving damage, and its relative robustness to sparse and reduced-aperture data at moderate noise levels. A comparative analysis of the TLSM images against the multifrequency LSM maps further reveals that thanks to the full-waveform inversion in time and space, the TLSM generates images of remarkably higher quality with the same dataset [22].

6.1.5 The linear sampling method for random sources

J. Garnier, H. Haddar, H. Montanelli

We present an extension of the linear sampling method for solving the sound-soft inverse acoustic scattering problem with randomly distributed point sources. The theoretical justification of our sampling method is based on the Helmholtz–Kirchhoff identity, the cross-correlation between measurements, and the volume and imaginary near-field operators, which we introduce and analyze. Implementations in MATLAB using boundary elements, the SVD, Tikhonov regularization, and Morozov's discrepancy principle are also discussed. We demonstrate the robustness and accuracy of our algorithms with several numerical experiments in two dimensions [21].

6.2 Transmission eigenvalues, Invisibility, Stability

Participants: Lucas Chesnel, Houssem Haddar, Amal Labidi.

6.2.1 Acoustic passive cloaking using thin outer resonators

L. Chesnel, J. Heleine, S.A. Nazarov

We consider the propagation of acoustic waves in a 2D waveguide unbounded in one direction and containing a compact obstacle. The wavenumber is fixed so that only one mode can propagate. The goal of this work is to propose a method to cloak the obstacle. More precisely, we add to the geometry thin outer resonators of width ϵ and we explain how to choose their positions as well as their lengths to get a transmission coefficient approximately equal to one as if there were no obstacle. In the process we also investigate several related problems. In particular, we explain how to get zero transmission and how to design phase shifters. The approach is based on asymptotic analysis in presence of thin resonators. An essential point is that we work around resonance lengths of the resonators. This allows us to obtain effects of order one with geometrical perturbations of width ϵ . Various numerical experiments illustrate the theory. [7]

6.2.2 Inverse Scattering Theory and Transmission Eigenvalues

F. Cakoni, D. Colton, H. Haddar

In the first edition of this book, we discussed methods for determining the support of an inhomogeneous medium from measured far field data as well as an extensive study of the central role played by the transmission eigenvalue problem in the mathematical development of these methods. In particular, we introduced the generalized linear sampling method (GLSM) and showed that this method provides a mathematical explanation of why it is permissible to use Tikhonov regularization to obtain an approximate solution of the far field equation associated with the linear sampling method. In the six years since the first edition of our book appeared, there has been considerable progress in both the development of GLSM as well as the theory of transmission eigenvalues. In this second edition, in addition to correcting typos in the first edition, we have added several highlights taken from these new developments. In particular, we have included new chapters on (1) the use of modified background media in the non-destructive testing of materials and in particular methods for determining the modified transmission eigenvalues that arise in such applications from measured far field data, (2) a study of a subset of transmission eigenvalues, called nonscattering wave numbers, through the use of techniques taken from the theory of free boundary value problems for elliptic partial differential equations, and (3) the duality between scattering poles and transmission eigenvalues which, in addition to their intrinsic mathematical interest, leads to new methods for the numerical computation of scattering poles [13].

6.2.3 Inside-Outside Duality for Modified Transmission Eigenvalues

H. Haddar, M. Khenissi, M. Mansouri

We introduce a new spectrum associated with the scattering from penetrable objects at a fixed frequency by employing the modified background technique. This spectrum can be used as qualitative indicator on the material properties of the medium. We prove that, as opposed to the case of classical transmission eigenvalues, the inside-outside duality method allows to reconstruct this spectrum from full aperture far field measurements at a fixed frequency. We derive a necessary and sufficient condition that characterizes the modified transmission eigenvalues using the phase of a modified far field operator constructed form measured data. The main ingredients in the analysis are the factorization of the modified far field operator and the asymptotic expansion of the solution in the neighborhood of the eigenvalues. We also prove that one can obtain an approximation of the associated incident waves. We then propose a numerical implementation of the method and validated its efficiency on some synthetic two dimensional examples [9].

6.2.4 Stability estimate for an inverse problem for the time harmonic magnetic schrödinger operator from the near and far field pattern

M. Bellassoued, H. Haddar, A. Labidi

We derive conditional stability estimates for inverse scattering problems related to time harmonic magnetic Schrödinger equation. We prove logarithmic type estimates for retrieving the magnetic (up to a gradient) and electric potentials from near field or far field maps. Our approach combines techniques from similar results obtained in the literature for inhomogeneous inverse scattering problems based on the use of geometrical optics solutions. [5].

6.3 Propagation of waves in waveguides

Participants: Lucas Chesnel.

6.3.1 Acoustic waveguide with a dissipative inclusion

L. Chesnel, J. Heleine, S.A. Nazarov, J. Taskinen

We consider the propagation of acoustic waves in a waveguide containing a penetrable dissipative inclusion. We prove that as soon as the dissipation, characterized by some coefficient η , is non zero, the scattering solutions are uniquely defined. Additionally, we give an asymptotic expansion of the corresponding scattering matrix when $\eta \rightarrow 0^+$ (small dissipation) and when $\eta \rightarrow +\infty$ (large dissipation). Surprisingly, at the limit $\eta \rightarrow +\infty$, we show that no energy is absorbed by the inclusion. This is due to a skin-effect phenomenon and can be explained by the fact that the field no longer penetrates into the highly dissipative inclusion. These results guarantee that in monomode regime, the amplitude of the reflection coefficient has a global minimum with respect to η . The situation where this minimum is zero, that is when the device acts as a perfect absorber, is particularly interesting for certain applications. However it does not happen in general. In this work, we show how to perturb the geometry of the waveguide to create 2D perfect absorbers in monomode regime. Asymptotic expansions are justified by error estimates and theoretical results are supported by numerical illustrations. [18]

6.3.2 Design of an acoustic energy distributor using thin resonant slits

L. Chesnel, S.A. Nazarov

We consider the propagation of time harmonic acoustic waves in a device with three channels. The wave number is chosen such that only the piston mode can propagate. The main goal of this work is to present a geometry which can serve as an energy distributor. More precisely, the geometry is first designed so that for an incident wave coming from one channel, the energy is almost completely transmitted in the two other channels. Additionally, tuning a bit two geometrical parameters, we can control the ratio of energy transmitted in the two channels. The approach is based on asymptotic analysis for thin slits around resonance lengths. We also provide numerical results to illustrate the theory.

6.3.3 Design of a mode converter using thin resonant slits

L. Chesnel, J. Heleine, S.A. Nazarov

The goal of this work is to design an acoustic mode converter. More precisely, the wave number is chosen so that two modes can propagate. We explain how to construct geometries such that the energy of the modes is completely transmitted and additionally the mode 1 is converted into the mode 2 and conversely. To proceed, we work in a symmetric waveguide made of two branches connected by two thin ligaments whose lengths and positions are carefully tuned. The approach is based on asymptotic analysis for thin ligaments around resonance lengths. We also provide numerical results to illustrate the theory. [8]

6.4 Quantum periodic waveguides

Participants: Lucas Chesnel.

6.4.1 Spectrum of the Dirichlet Laplacian in a thin cubic lattice

L. Chesnel, S.A. Nazarov

We give a description of the lower part of the spectrum of the Dirichlet Laplacian in an unbounded 3D periodic lattice made of thin bars (of width $\varepsilon \ll 1$) which have a square cross section. This spectrum coincides with the union of segments which all go to $+\infty$ as ε tends to zero due to the Dirichlet boundary condition. We show that the first spectral segment is extremely tight, of length $O(e^{-\delta/\varepsilon})$, $\delta > 0$, while the length of the next spectral segments is $O(\varepsilon)$. To establish these results, we need to study in detail the properties of the Dirichlet Laplacian A^{Ω} in the geometry Ω obtained by zooming at the junction regions of the initial periodic lattice. This problem has its own interest and playing with symmetries together with max-min arguments as well as a well-chosen Friedrichs inequality, we prove that A^{Ω} has a unique eigenvalue in its discrete spectrum, which generates the first spectral segment. Additionally we show that there is no threshold resonance for A^{Ω} , that is no non trivial bounded solution at the threshold frequency for A^{Ω} . This implies that the correct 1D model of the lattice for the next spectral segments is a graph with Dirichlet conditions at the vertices. We also present numerics to complement the analysis. [19]

6.5 Analysis of negative metamaterials

Participants: Lucas Chesnel.

6.5.1 Maxwell's equations with hypersingularities at a conical plasmonic tip

A.-S. Bonnet-Ben Dhia, L. Chesnel, M. Rihani

In this work, we are interested in the analysis of time-harmonic Maxwell's equations in presence of a conical tip of a material with negative dielectric constants. When these constants belong to some critical range, the electromagnetic field exhibits strongly oscillating singularities at the tip which have infinite energy. Consequently Maxwell's equations are not well-posed in the classical L2 framework. The goal of the present work is to provide an appropriate functional setting for 3D Maxwell's equations when the dielectric permittivity (but not the magnetic permeability) takes critical values. Following what has been done for the 2D scalar case, the idea is to work in weighted Sobolev spaces, adding to the space the so-called outgoing propagating singularities. The analysis requires new results of scalar and vector potential representations of singular fields. The outgoing behaviour is selected via the limiting absorption principle. [6]

6.6 Diffusion MRI

Participants: J.-R. Li, C. Fang, Z. Yang.

6.6.1 Fourier representation of the diffusion MRI signal using layer potentials

Chengran Fang, Demian Wassermann, Jing-Rebecca Li

The diffusion magnetic resonance imaging signal arising from biological tissues can be numerically simulated by solving the Bloch-Torrey partial differential equation. Numerical simulations can facilitate the investigation of the relationship between the diffusion MRI signals and cellular structures. With the rapid advance of available computing power, the diffusion MRI community has begun to employ numerical simulations for model formulation and validation, as well as for imaging sequence optimization. Existing simulation frameworks use the finite difference method, the finite element method, or the Matrix Formalism method to solve the Bloch-Torrey partial differential equation. We propose a new method based on the efficient evaluation of layer potentials. In this paper, the mathematical framework and the numerical implementation of the new method are described. We demonstrate the convergence of our method via numerical experiments and analyze the errors linked to various model and simulation parameters. Since our method provides a Fourier-type representation of the diffusion MRI signal, it can potentially facilitate new physical and biological signal interpretations in the future.[20]

6.6.2 Asymptotic models of the diffusion MRI signal accounting for geometrical deformations

Zheyi Yang, Imen Mekkaoui, Jan Hesthaven, Jing-Rebecca Li

The complex transverse water proton magnetization subject to diffusion-encoding magnetic field gradient pulses can be modeled by the Bloch-Torrey partial differential equation (PDE). The associated diffusion MRI signal is the spatial integral of the solution of the Bloch-Torrey PDE. In addition to the signal, the time-dependent apparent diffusion coefficient (ADC) can be obtained from the solution of another partial differential equation, called the HADC model, which was obtained using homogenization techniques. In this paper, we analyze the Bloch-Torrey PDE and the HADC model in the context of geometrical deformations starting from a canonical configuration. To be more concrete, we focused on two analytically defined deformations: bending and twisting. We derived asymptotic models of the diffusion MRI signal and the ADC where the asymptotic parameter indicates the extent of the geometrical deformation. We compute numerically the first three terms of the asymptotic models and illustrate the effects of the deformations by comparing the diffusion MRI signal and the ADC from the canonical configuration with those of the deformed configuration. The purpose of this work is to relate the diffusion MRI signal more directly with tissue geometrical parameters.[24]

6.6.3 Three-dimensional micro-structurally informed in silico myocardium—Towards virtual imaging trials in cardiac diffusion weighted MRI

Mojtaba Lashgari , Nishant Ravikumar , Irvin Teh , Jing-Rebecca Li , David Buckley, Jurgen Schneider, Alejandro Frangi

In silico tissue models (viz. numerical phantoms) provide a mechanism for evaluating quantitative models of magnetic resonance imaging. This includes the validation and sensitivity analysis of imaging biomarkers and tissue microstructure parameters. This study proposes a novel method to generate a realistic numerical phantom of myocardial microstructure. The proposed method extends previous studies by accounting for the variability of the cardiomyocyte shape, water exchange between the cardiomyocytes (intercalated discs), disorder class of myocardial microstructure, and four sheetlet orientations. In the first stage of the method, cardiomyocytes and sheetlets are generated by considering the shape variability and intercalated discs in cardiomyocyte—cardiomyocyte connections. Sheetlets are then aggregated and oriented in the directions of interest. The morphometric study demonstrates no significant difference between the distribution of volume, length, and primary and secondary axes of the numerical and real (literature) cardiomyocyte data. Moreover, structural correlation analysis validates that the in-silico tissue is in the same class of disorderliness as the real tissue. Additionally, the absolute angle differences between the simulated helical angle (HA) and input HA (reference value) of the cardiomyocytes demonstrate a good agreement with the absolute angle difference between the measured HA using experimental cardiac diffusion tensor imaging (cDTI) and histology (reference value). Furthermore, the angular distance between eigenvectors and sheetlet angles of the input and simulated cDTI is much smaller than those between measured angles using structural tensor imaging (as a gold standard) and experimental cDTI. Combined with the qualitative results, these results confirm that the proposed method can generate richer numerical phantoms for the myocardium than previous studies.[10]

6.7 Modelling and HPC for wave propagation problems

Participants: Jing Rebecca Li, Marcella Bonazzoli, Houssem Haddar, Hadrien Montanelli.

6.7.1 Influence of the partition of unity on SORAS preconditioner

M. Bonazzoli, X. Claeys, F. Nataf, P.-H. Tournier

We have investigated numerically the influence of the choice of the partition of unity on the convergence of the Symmetrized Optimized Restricted Additive Schwarz (SORAS) preconditioner for the heterogeneous reaction-convection-diffusion equation. Previously, we had analyzed the convergence of this overlapping domain decomposition preconditioner for generic non self-adjoint or indefinite problems, like the reaction-convection-diffusion equation. In the numerical experiments, we had noticed that the number of iterations for convergence of preconditioned GMRES appeared not to vary significantly when increasing the overlap width. In this work we show that actually this is due to the particular choice of the partition of unity for the preconditioner, and we study the dependence on the overlap and on the number of subdomains for two kinds of partition of unity. Our numerical investigation shows that the second kind of partition of unity, which is non-zero in the interior of the whole overlapping region, generally improves the iteration counts obtained with the first kind of partition of unity, whose gradient is zero on the subdomain interfaces. Moreover, the first kind of partition of unity, which would be the natural choice for ORAS solver instead, yields for SORAS preconditioner iterations counts that do not vary significantly when increasing the overlap width. A proceedings on this topic has been submitted [14].

6.7.2 Computing weakly singular and near-singular integrals in high-order boundary elements

M. Aussal, H. Haddar and H. Montanelli

We proposed algorithms for computing weakly singular and near-singular integrals arising when solving the 3D Helmholtz equation with high-order boundary elements. These are based on the computation of the preimage of the singularity on the reference element using Newton's method, singularity subtraction with high-order Taylor-like asymptotic expansions, the continuation approach, and transplanted Gauss quadrature. We demonstrated the accuracy of our method with several numerical experiments, including the scattering by two nearby half-spheres or consphere geometries [11].

6.8 Optimization based inversion methods

Participants: Lorenzo Audibert, Marcella Bonazzoli, Mohamed Aziz Boukraa, Houssem Haddar, Tuan Anh Vu, Xiaoli Liu, Denis Vautrin.

6.8.1 Convergence analysis of multi-step one-shot methods for linear inverse problems

M. Bonazzoli, H. Haddar, T. A. Vu

In this work we are interested in general linear inverse problems where the corresponding forward problem is solved iteratively using fixed point methods. Then one-shot methods, which iterate at the same time on the forward problem solution and on the inverse problem unknown, can be applied. We analyze two variants of the so-called multi-step one-shot methods and establish sufficient conditions on the descent step for their convergence, by studying the eigenvalues of the block matrix of the coupled iterations. Several numerical experiments are provided to illustrate the convergence of these methods in comparison with the classical usual and shifted gradient descent. In particular, we observe that very few inner iterations on the forward problem are enough to guarantee good convergence of the inversion algorithm. The details are available in the research report [15].

6.8.2 A combination of Kohn-Vogelius and DDM methods for a geometrical inverse problem

S. Chaabane, H. Haddar, R. Jerbi We consider the inverse geometrical problem of identifying the discontinuity curve of an electrical conductivity from boundary measurements. This standard inverse problem is used as a model to introduce and study a combined inversion algorithm coupling a gradient descent on the Kohn-Vogelius cost functional with a domain decomposition method that includes the unknown curve in the domain partitioning. We prove the local convergence of the method in a simplified case and numerically show its efficiency for some two dimensional experiments [17].

6.8.3 Application of shape optimization methods to a concrete-rock interface for a hydroelectric dam

L. Audibert, M. Bonazzoli, M. A. Boukraa, H. Haddar, D. Vautrin

The objective is to reconstruct the interface between the bottom of a dam, made of concrete, and the bedrock on which it was built. The conditions of data acquisition are complicated: we have access just to a relatively small part of the dam, the characteristics of the inspected structure are not known precisely, and a limited number of sensors is available. We are currently studying a shape optimization method to reconstruct the geometry of the concrete-rock interface.

6.8.4 Acceleration method for shape optimization in inverse scattering

L. Audibert, H. Haddar, X. Liu

To find the shape of an object from scattering data one could rely to optimisation based method and more precisely to gradient-based method. This could be applied to shape in order to retrieve an unknown geometry. an option is to use the adjoint method to compute the gradient efficiently. Yet it could be very costly if the minimization requires to many iteration. We investigate the extension of the famous Nesterov acceleration from convex analysis to shape optimization. First we show that the extension is natural for parametric model of shapes then we propose and motivate an extension to levelset parametrisation of shapes. This extension is not justified theoretically but we made an extensive numerical study that show as expected a reduction in iteration count and more surprisingly an increase in reconstruction quality. We will pursue this project by considering other imaging situation and by a theoretical analyses of our algorithm [].

7 Bilateral contracts and grants with industry

7.1 Bilateral contracts with industry

Participants: Houssem Haddar, Lorenzo Audibert.

- Grant associated with one long internships with EDF R&D on solving the Helmholtz equation in 2D in curvilinear coordinates and with the thin film model. The student is Paul Invernizzi.
- Grant from France Relance associated with one PostDoc with EDF R&D on imaging the interface between dam and Bedrock. The postdoc is Mohammed Aziz Boukraa
- Grant from France Relance associated with the mise a disposition of a researcher from EDF R&D on all the theme of IDEFIX. The researcher is Lorenzo Audibert.
- Grant from ANRT associated with one CIFRE phd with EDF R&D on imaging cracks and multiple defects with eddy current using inverse problems. The phd candidate is Morgane Mathevet.

7.2 Bilateral Grants with Industry

Participants: Marcella Bonazzoli, Houssem Haddar, Hadrien Montanelli, Lucas Chesnel.

- Grant from DGA in the framework of the CIEDS, with the objective of extending sampling methods to passive imaging and imaging in a cluttered media (2021-2024). It partly served financing the Postdoc work of H. Montanelli (2022).
- M. Bonazzoli and L. Chesnel are members of ElectroMATH project (Electromagnetic wave propagation in complex media and configurations, 2022-2026), granted by CIEDS (IP Paris-AID), coordinated by P. Ciarlet and A. Modave.

8 Partnerships and cooperations

8.1 International initiatives

8.1.1 Associate Teams in the framework of an Inria International Lab or in the framework of an Inria International Program

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ISP-EDP
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Participants: Lorenzo Audibert, Marcella Bonazzoli, Houssem Haddar, Tuan Anh Vu.

Title: Identification of singular parameters in partial differential equations

Duration: 2022 -> 2024

Inria Coordinator: Houssem Haddar

Partner Coordinator: Slim Chaabane

Partners:

- Faculté des Sciences de Sfax, Sfax University (Tunisie)
- ENIT, Tunis El-manar University (Tunisie)

Website: link

Summary: The goal of this associate team is to contribute to the analysis of inverse problems where the sought parameters lack regularity. A typical example is the inverse geometrical problem where the geometry to recover from given data represents the discontinuity set for some physical coefficients in a PDE model. This problem arises in a variety of applications like geophysics (e.g. the parameter being the sound velocity), non destructive testing (e.g. the parameter being the crack's impedance, dielectric properties of deposits), medical imaging (e.g. the parameter being the conductivity), etc... For this type of problems, a classical formulation of the inverse problem as an optimisation problem would be faced in general with the lack of differentiability of the state variable with respect to the discontinuity location. We explore two main strategies to address this issue. The first one is based on the design of a suitable misfit functional that would be differentiable although the state variable is not. This is the case for example of the Kohn-Vogelius cost function for selfadjoint operators as it has been previously established by the team members. The second strategy would be to develop optimization free inversion procedures that avoid the derivative of the state variable. This is the case for instance of sampling methods that have been developed for cracks by the team members.

8.2 International research visitors

8.2.1 Visits of international scientists

Other international visits to the team

• The team welcomed the post-doc Marta De León Contreras from Norway during one week.

8.3 National initiatives

Participants: Lorenzo Audibert, Marcella Bonazzoli, Houssem Haddar.

- Funding from France Relance of 80% of 24 months postdoc of Mohammed Aziz Boukraa (Sept 2022-Sept 2024) with EDF R&D on imaging the interface between dam and bedrocks using seismic waves.
- Funding from France Relance of 80% of 24 months detachement of Lorenzo Audibert (fev 2022-fev 2024) with EDF R&D on all research themes of IDEFIX.

Action exploratoire OptiGPR3D

Participants: Lorenzo Audibert, Marcella Bonazzoli, Houssem Haddar, Frédéric Taillade.

Title: Action exploratoire OptiGPR3D (*Optimal direct and inverse modeling for 3D GPR imaging in complex environments*)

Partner Institution(s): IDEFIX (Inria, EDF, ENSTA Paris), POEMS (CNRS, Inria, ENSTA Paris)

Duration: Start: 05/2022, 4 years

Coordinators: Marcella Bonazzoli (IDEFIX, Inria), Pierre Marchand (POEMS, Inria)

Administrator: Inria

8.4 Regional initiatives

• A PhD funding from DIM, a regional program promoting collaboration between academia and small industrial partners. It served financing the PhD of D. Lerévérand in collaboration with the association Le collectif with the goal of providing numerical solutions for realistic audio applicable to the project Le Cartable Connecté.

9 Dissemination

Participants: Lorenzo Audibert, Marcella Bonazzoli, Lucas Chesnel, Houssem Haddar, Jing Rebecca Li.

9.1 Promoting scientific activities

9.1.1 Scientific events: organisation

Member of the organizing committees

- L. Audibert is a member of the scientific comitee of EDF-INRIA-CEA summer school (since 2020).
- L. Audibert was a member of the evaluating committe 46 of the ANR for the aapg 2022.
- M. Bonazzoli co-organized the IP Paris *Semaine d'Etude Mathématiques–Entreprises*, a week-long workshop where PhD students and postdocs in mathematics work on problems presented by companies (BNP Paribas, CEA, CGG, CLARINS, EDF, Goldman Sachs). One problem was presented by L. Audibert and a colleague at EDF.
- M. Bonazzoli (with T. Chaumont-Frelet, J. Heleine, P. Marchand) organized a francophone young researchers conference on wave propagation, in Sophia Antipolis (France) (3ème Rencontre JCJC Ondes 2022).
- M. Bonazzoli organizes the group seminar of IDEFIX team.
- L. Chesnel co-organizes the seminar common to the three teams IDEFIX-MEDISIM-POEMS.
- H. Haddar co-organized with S. Chaabane a minisymposium at Picof Conference

9.1.2 Scientific events: selection

Reviewer

• L. Chesnel and H. Haddar are members of the scientific committee of the Waves conference

9.1.3 Journal

Member of the editorial boards

• H. Haddar is member of the editorial board of Siam J. of Mathematical Analysis, Siam J. of Scientific Computing and Inverse Problems.

9.1.4 Invited talks

- M. Bonazzoli: DD27, 27th International Domain Decomposition Conference 2022, Prague, Czech Republic, Jul. 2022.
- M. Bonazzoli: Solvers for frequency-domain wave problems and applications ICMS Workshop, Glasgow, UK, Jun. 2022.
- M. Bonazzoli: ECCOMAS 2022, 8th European Congress on Computational Methods in Applied Sciences and Engineering, Oslo, Norway, Jun. 2022.
- L. Chesnel: Conference Picof 2022, Inverse problems, control and shape optimisation, Caen, Oct. 2022.
- L. Chesnel: Conference Aspect'22 (Asymptotic Analysis and Spectral Theory), Oldenburg, Sep. 2022.
- L. Chesnel: Journées EDP de l'IECL, Nancy, Mar. 2022.

- L. Chesnel: Conference on Mathematics of Wave Phenomena 2022, Karlsruhe, Feb. 2022.
- H. Haddar: Plenary speaker at the conference on Mathematics of Wae Phenomena, 2022, KIT, Germany.
- H. Haddar: Plenary speaker at the ICAAM confrence, 2022, Monastir, Tunis.
- H. Haddar: Key lecturer at the SIMPA summer school on inverse problems in Tirana, 2022, Albania.

9.1.5 Leadership within the scientific community

• J.-R. Li is a member of the SIAM Committee on Programs and Conferences.

9.1.6 Research administration

- M. Bonazzoli is the International partnerships Scientific Correspondent for Inria Saclay.
- M. Bonazzoli took part in Jul. 2022 to the prize committee for Prix Junior Maryam Mirzakhani awarded by Fondation Mathématique Jacques-Hadamard (FMJH) to two young female students for a mathematics project.
- M. Bonazzoli is a volunteer member of Opération Postes (newsletter and website, which gathers
 detailed information about the French competitive selections for permanent positions in Mathematics and Informatics, supported by the French academic societies SMAI, SMF, SFdS, and SIF).

9.2 Teaching - Supervision - Juries

9.2.1 Teaching

- Master: L. Chesnel, Analyse variationnelle des équations aux dérivées partielles, for students in the second year of Ecole Polytechnique, 2x5 TDs of 2h each.
- Master: L. Chesnel, Modal Modélisation mathématique par la démarche expérimentale, for students in the second year of Ecole Polytechnique, creation and supervision of a project for two students.
- Master: L. Chesnel, cosupervision of a psc project of five students, for students in the second year of Ecole Polytechnique, 2h meetings every 2 weeks.
- Bachelor: L. Chesnel, Numerical Methods for ODEs, for students in the third year of the Bachelor of Ecole Polytechnique, 2x5 TDs of 2h each.
- Master: J.-R. Li, M2 internship supervision of Anh Tu Tran.
- Master: M. Bonazzoli, Mathematics for data science, 1st year of Computer Science Master, Université Paris-Saclay, 21 hours (lessons and TD).
- Master: M. Bonazzoli, La méthode des éléments finis, 2nd year of Engineer School, ENSTA Paris, 12 TD hours.
- Bachelor: M. Bonazzoli, Fonctions de variable complexe, 1st year of Engineer School, ENSTA Paris, 12 TD hours.
- Licence: H. Haddar, Complex analysis and Elementary tools of analysis for partial differential equations, for students in the first year of Ensta ParisTech curriculum. 37 equivalent TD hours. 2021-present.

9.2.2 Supervision

- PhD in progress: T.A. Vu, One-shot inversion methods and domain decomposition (2020-), M. Bonazzoli and H. Haddar.
- PhD in progress: F. Pourre, Construction and analysis of spectral signatures for defects in complex media(2021-), L. Audibert and H. Haddar.
- PhD in progress: M. Mathevet, (2022-), Imaging cracks and deposits in steam generator using eddy currents measurements and inverse problems. L. Audibert and H. Haddar.
- PhD in progress: A. Parigaux, (2022-), Construction of transparent conditions for electromagnetic waveguides, analysis and applications. (2022-), L. Chesnel and A.S. Bonnet Ben Dhia.
- Postdoc in progress: M.A. Boukraa, Inverse problem methods for interface imaging: application to a concrete-rock interface for a hydroelectric dam (2022-), supervisors: M. Bonazzoli, D. Vautrin, collaborators: L. Audibert, H. Haddar, F. Taillade.
- PhD in progress: C. Fang, Neuron modeling, Bloch-Torrey equation, and their application to brain microstructure imaging using diffusion MRI (2019-), J.-R. Li and D. Wassermann.
- PhD in progress: Z. Yang, Reduced model of diffusion MRI in the brain white matter (2020-), J.-R. Li.
- PhD in progress: D. Lerévérand, Forward and inverse modeling of HRTF (2019-), M. Aussal and H. Haddar.
- PhD in progress: N. Jenhani, Identifcation of defects in periodic layers using differential imaging, (2020-) H. Haddar and Y. Boukari.
- PhD in progress: A. Labidi, Inverse problems for Magnetic Schrodinger operator. (2020-) H. Haddar and M. Bellassoued.

9.2.3 Internal or external Inria responsibilities

- M. Bonazzoli supervised the "science popularization doctoral mission" of E. Courtoux (Inria Saclay research center offers to PhD students complementary funding for science popularization missions).
- J.-R. Li is a member of the INRIA Commission d'Evaluation.
- J.-R. Li is a member of the INRIA Saclay's Mentoring Committee.
- J.-R. Li is a member of the INRIA Saclay's GT Comité de Centre.
- H. Haddar is member of the Bureau du Comité de Projets of Inria Saclay center.

9.2.4 Articles and contents

• J.-R. Li published "Modélisation du signal d'IRM de diffusion par une équation aux dérivées partielles" for Journées mathématiques X-UPS 2022.

9.2.5 Interventions

- M. Bonazzoli was a volunteer at Inria stand at *Fête de la Science* (Université Paris-Saclay), Oct. 2022.
- M. Bonazzoli participated to several speed-meetings with high/middle school students (Rendezvous des Jeunes Mathématiciennes et Informaticiennes, Inria Saclay, Feb. 2022; Pôle Diversité et Réussite de l'École Polytechnique, Feb. 2022) to answer their questions about the studies and career as a mathematician.
- L. Chesnel helped a team of students of Ensta to participate to the International Physicists' Tournament.

10 Scientific production

10.1 Major publications

- L. Audibert, H. Girardon, H. Haddar and P. Jolivet. 'Inversion of Eddy-Current Signals Using a Level-Set Method and Block Krylov Solvers'. In: *SIAM Journal on Scientific Computing* (2023). URL: https://hal.science/hal-03043491.
- [2] M. Bonazzoli, H. Haddar and T. A. Vu. Convergence analysis of multi-step one-shot methods for linear inverse problems. RR-9477. Inria Saclay; ENSTA ParisTech, July 2022. URL: https://hal.in ria.fr/hal-03727759.
- [3] F. Cakoni, D. Colton and H. Haddar. *Inverse Scattering Theory and Transmission Eigenvalues: Second Edition*. Vol. 98. CBMS-NSF, 7th Dec. 2022. URL: https://hal.inria.fr/hal-03899457.
- [4] C. Fang, D. Wassermann and J.-R. Li. Fourier representation of the diffusion MRI signal using layer potentials. 2022. URL: https://hal.science/hal-03940100.

10.2 Publications of the year

International journals

- [5] M. Bellassoued, H. haddar and A. Labidi. 'Stability estimate for an inverse problem for the time harmonic magnetic schrödinger operator from the near and far field pattern'. In: *SIAM Journal on Mathematical Analysis* (2022). URL: https://hal.inria.fr/hal-03876858.
- [6] A.-S. Bonnet-Ben Dhia, L. Chesnel and M. Rihani. 'Maxwell's equations with hypersingularities at a conical plasmonic tip'. In: *Journal de Mathématiques Pures et Appliquées* 161 (7th Mar. 2022), pp. 70–110. DOI: 10.1016/j.matpur.2022.03.001. URL: https://hal.science/hal-02969739.
- [7] L. Chesnel, J. Heleine and S. A. Nazarov. 'Acoustic passive cloaking using thin outer resonators'. In: Zeitschrift für Angewandte Mathematik und Physik 73.3 (June 2022). URL: https://hal.science /hal-03216053.
- [8] L. Chesnel, J. Heleine and S. A. Nazarov. 'Design of a mode converter using thin resonant ligaments'. In: *Communications in Mathematical Sciences* 20.2 (2022), pp. 425–445. URL: https://hal.science/hal-03141128.
- H. haddar, M. Khenissi and M. Mansouri. 'Inside-Outside Duality for Modified Transmission Eigenvalues'. In: *Inverse Problems and Imaging* (20th Dec. 2022). URL: https://hal.inria.fr/h al-03876859.
- [10] M. Lashgari, N. Ravikumar, I. Teh, J.-R. Li, D. Buckley, J. Schneider and A. Frangi. 'Three-dimensional micro-structurally informed in silico myocardium—Towards virtual imaging trials in cardiac diffusion weighted MRI'. In: *Medical Image Analysis* 82 (Nov. 2022), p. 102592. DOI: 10.1016/j.media .2022.102592.URL: https://hal.science/hal-03912441.
- [11] H. Montanelli, M. Aussal and H. Haddar. 'Computing weakly singular and near-singular integrals over curved boundary elements'. In: *SIAM Journal on Scientific Computing* 44.6 (2022), A3728– A3753. DOI: 10.1137/21M1462027. URL: https://hal.science/hal-03518653.

Conferences without proceedings

[12] L. Baratchart, H. Haddar and C. Villalobos Guillén. 'Inverse problem for the Helmholtz equation and singular sources in the divergence form'. In: WAVES 2022. Palaiseau, France, 2022. URL: https: //hal.inria.fr/hal-03899189.

Scientific books

[13] F. Cakoni, D. Colton and H. Haddar. Inverse Scattering Theory and Transmission Eigenvalues: Second Edition. Vol. 98. CBMS-NSF, 7th Dec. 2022. URL: https://hal.inria.fr/hal-03899457.

Reports & preprints

- [14] M. Bonazzoli, X. Claeys, F. Nataf and P.-H. Tournier. *How does the partition of unity influence SORAS preconditioner*? 2nd Dec. 2022. URL: https://hal.science/hal-03882577.
- [15] M. Bonazzoli, H. Haddar and T. A. Vu. Convergence analysis of multi-step one-shot methods for linear inverse problems. RR-9477. Inria Saclay; ENSTA ParisTech, July 2022. URL: https://hal.in ria.fr/hal-03727759.
- [16] Y. Boukari, H. Haddar and N. Jenhani. *Analysis of sampling methods for imaging a periodic layer and its defects*. 28th Nov. 2022. URL: https://hal.science/hal-03876852.
- [17] S. Chaabane, H. Haddar and R. Jerbi. A combination of Kohn-Vogelius and DDM methods for a geometrical inverse problem. 16th Jan. 2023. URL: https://hal.inria.fr/hal-03940961.
- [18] L. Chesnel, J. Heleine, S. A. Nazarov and J. Taskinen. *Acoustic waveguide with a dissipative inclusion*. 1st July 2022. URL: https://hal.science/hal-03711258.
- [19] L. Chesnel and S. A. Nazarov. *Spectrum of the Dirichlet Laplacian in a thin cubic lattice*. 16th Jan. 2023. URL: https://hal.science/hal-03940303.
- [20] C. Fang, D. Wassermann and J.-R. Li. Fourier representation of the diffusion MRI signal using layer potentials. 2022. URL: https://hal.science/hal-03940100.
- [21] J. Garnier, H. Haddar and H. Montanelli. *The linear sampling method for random sources*. 28th Oct. 2022. URL: https://hal.science/hal-03832969.
- [22] X. Liu, J. Song, F. Pourahmadian and H. Haddar. *Time-vs. frequency-domain inverse elastic scattering: Theory and experiment.* 28th Nov. 2022. URL: https://hal.inria.fr/hal-03876860.
- [23] F. Pourahmadian and H. Haddar. *Ultrasonic imaging in highly heterogeneous backgrounds*. 28th Nov. 2022. URL: https://hal.inria.fr/hal-03876861.
- [24] Z. Yang, I. Mekkaoui, J. Hesthaven and J.-R. Li. *Asymptotic models of the diffusion MRI signal accounting for geometrical deformations*. 2022. URL: https://hal.science/hal-03939649.

Other scientific publications

[25] H. Boujlida, H. haddar and M. Khenissi. 'ASYMPTOTIC EXPANSION OF TRANSMISSION EIGEN-VALUES FOR ANISOTROPIC THIN LAYERS'. In: *Applicable Analysis* (2023). URL: https://hal.sc ience/hal-03876855.