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2022 ACTIVITY REPORT

Project-Team MCTAO

Mathematics for Control, Transport and Applications

IN COLLABORATION WITH: Laboratoire Jean-Alexandre Dieudonné (JAD)

DOMAIN

Applied Mathematics, Computation and Simulation

THEME

Optimization and control of dynamic systems

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Project-Team MCTAO

Creation of the Project-Team: 2013 January 01

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1 Team members, visitors, external collaborators

Research Scientists

- Jean-Baptiste Pomet [Team leader, INRIA, Senior Researcher, HDR]
- Lamberto Dell Elce [INRIA, Researcher]
- Ludovic Sacchelli [INRIA, Researcher]

Faculty Members

- Bernard Bonnard [Université Bourgogne-Franche-Comté, Professor, HDR]
- Jean-Baptiste Caillau [Université Côte d'Azur, Professor, HDR]

PhD Students

- Adel Malik Annabi [Université Côte d'Azur, from Oct 2022]
- Sandrine Gayrard [SEGULA Technologies, Université Bourgogne-Franche-Comté]
- Alesia Herasimenka [Université Côte d'Azur]
- Frank de Veld [INRIA, from Dec 2022, Supported by Thales Alenia Space and Région PACA]

Interns and Apprentices

- Hugo Blain [INRIA, Intern, from Jun 2022 until Jul 2022, Polytech Nice, 3A]
- Tommy Calendini [INRIA, Intern, from Jun 2022 until Sep 2022, Polytech Nice, 4A]
- Paul Chambroux [INRIA, Intern, from Feb 2022 until Aug 2022, stage M2, Sorbonne Université]
- Azouaou Ouyoucef [INRIA, Intern, from Feb 2022 until Aug 2022, stage M2, Sorbonne Université]
- Lucas Palazzolo [INRIA, Intern, from Jun 2022 until Jul 2022, stage M1, Université de Strasbourg]
- Théo Rolin [INRIA, Intern, from Jun 2022 until Aug 2022, Polytech Nice, 4A]

Administrative Assistant

- Claire Senica [INRIA]

External Collaborators

- Olivier Cots [INP Toulouse, ENSEEIHT]
- Thierry Dargent [Thales Alenia Space, Cannes]
- Joseph Gergaud [INP Toulouse, ENSEEIHT, HDR]
- Jérémy Rouot [Université de Bretagne Occidentale, Brest]

2 Overall objectives

Our goal is to develop methods in geometric control theory for nonlinear systems, mostly finite dimensional, and to transfer our expertise through real applications of these methods. The methodological developments range from feedback control and observers to optimal control, extending to fields like sub-Riemannian geometry. Optimal control leads to developments in Hamiltonian dynamics, and also requires sophisticated numerics, to which the team contributes too. Dynamical systems and modeling are also part of the background of the team.

Our primary domain of industrial applications in the past years has been space engineering, in particular using optimal control and stabilization techniques for mission design with low thrust propulsion: orbit transfer or rendez-vous problems in the gravity field of a single body (typically satellites around the earth), interplanetary missions and multi body problems, or control design of solar sails, where propulsion is drastically constrained.

The team also has continued involvement with applications regarding human bio-mechanics (muscle stimulation), and various modeling and control questions in biology (Lotka-Volterra models, bacterial growth, microbiome models, networks of chemical reaction...). The list is not exhaustive; past domains of application include swimming al low Reynolds number (micro-swimmers) and control of quantum systems for Magnetic Resonance Imaging.

3 Research program

3.1 Control Problems

McTAO's major field of expertise is control theory in the broad sense. Let us give an overview of this field.

Modelling. Our effort is directed toward efficient methods for the control of real (physical) *systems*, based on a *model* of the system to be controlled. Choosing accurate models yet simple enough to allow control design is in itself a key issue. The typical continuous-time model is of the form $\mathrm{d}x/\mathrm{d}t = f(x,u)$ where x is the *state*, ideally finite dimensional, and u the *control*; the control is left free to be a function of time, or a function of the state, or obtained as the solution of another dynamical system that takes x as an input. Modelling amounts to deciding the nature and dimension of x, as well as the dynamics (roughly speaking the function f). Connected to modeling is identification of parameters when a finite number of parameters are left free in "f".

Controllability, path planning. Controllability is a property of a control system (in fact of a model) that two states in the state space can be connected by a trajectory generated by some control, here taken as an explicit function of time. Deciding on local or global controllability is still a difficult open question in general. In most cases, controllability can be decided by linear approximation, or non-controllability by "physical" first integrals that the control does not affect. For some critically actuated systems, it is still difficult to decide local or global controllability, and the general problem is anyway still open. Path planning is the problem of constructing the control that actually steers one state to another.

Optimal control. In optimal control, one wants to find, among the controls that satisfy some constraints at initial and final time (for instance given initial and final state as in path planning), the ones that minimize some criterion. This is important in many control engineering problems, because minimizing a cost is often very relevant. Mathematically speaking, optimal control is the modern branch of the calculus of variations, rather well established and mature [76, 50, 38], but with a lot of hard open questions. In the end, in order to actually compute these controls, ad-hoc numerical schemes have to be derived for effective computations of the optimal solutions. See more about our research program in optimal control in section 3.2.

Feedback control. In the above two paragraphs, the control is an explicit function of time. To address in particular the stability issues (sensitivity to errors in the model or the initial conditions for example), the control has to be taken as a function of the (measured) state, or part of it. This is known as closed-loop control; it must be combined with optimal control in many real problems. On the problem of stabilization, there is longstanding research record from members of the team, in particular on the construction of "Control Lyapunov Functions", see [65, 78]. It may happen that only part of the state is accessible at any one time, because of physical or engineering constraints. In that case, a popular strategy is to pair feedback methods with dynamic estimation of the state, creating so-called output feedback loops. Simultaneous feedback control and estimation can become a major hurdle for nonlinear systems, see [57, 81].

Classification of control systems. One may perform various classes of transformations acting on systems, or rather on models. The simpler ones come from point-to-point transformations (changes of variables) on the state and control. More intricate ones consist in embedding an extraneous dynamical system into the model. These are dynamic feedback transformations that change the dimension of the state. In most problems, choosing the proper coordinates, or the right quantities that describe a phenomenon, sheds light on a path to the solution; these proper choices may sometimes be found from an understanding of the modelled phenomenons, or it can come from the study of the geometry of the equations and the transformation acting on them. This justifies the investigations of these transformations on models for themselves. These topics are central in control theory; they are present in the team, see for instance the classification aspect in [53] or —although this research has not been active very recently— the study [75] of dynamic feedback and the so-called "flatness" property [68]. Likewise, classification tools such as feedback invariants [44] are still currently in use in the team (see, for instance, [20]).

3.2 Optimal Control and its Geometry

Let us detail our research program concerning optimal control. Relying on Hamiltonian dynamics is now prevalent, instead of the Lagrangian formalism in classical calculus of variations. The two points of view run parallel when computing geodesics and shortest path in Riemannian Geometry for instance, in that there is a clear one-to-one correspondance between the solutions of the geodesic equation in the tangent bundle and the solution of the Pontryagin Maximum Principle in the cotangent bundle. In most optimal control problems, on the contrary, due to the differential constraints (velocities of feasible trajectories do not cover all directions in the state space), the Lagrangian formalism becomes more involved, while the Pontryagin Maximum Principle keeps the same form, its solutions still live in the cotangent bundle, their projections are the extremals, and a minimizing curve must be the projection of such a solution.

Cut and conjugate loci. The cut locus —made of the points where the extremals lose optimality— is obviously crucial in optimal control, but usually out of reach (even in low dimensions), and anyway does not have an analytic characterization because it is a non-local object. Fortunately, conjugate points —where the extremals lose *local* optimality— can be effectively computed with high accuracy for many control systems. Elaborating on the seminal work of the Russian and French schools (see [80, 37, 39] and [54] among others), efficient algorithms were designed to treat the smooth case. This was the starting point of a series of papers of members of the team culminating in the outcome of the *cotcot* software [49], followed by the *Hampath* [58] code. Over the years, these codes have allowed for the computation of conjugate loci in a wealth of situations including applications to space mechanics, quantum control, and more recently swimming at low Reynolds number. With in mind the two-dimensional analytic Riemannian framework, a heuristic approach to the global issue of determining cut points is to search for singularities of the conjugate loci; this line is however very delicate to follow on problems stemming from applications in three or more dimensions (see *e.g.* [59] and [46]). In all these situations, the fundamental object underlying the analysis is the curvature tensor. In Hamiltonian terms, one considers the dynamics of subspaces (spanned by Jacobi fields) in the Lagrangian Grassmannian [36]. This point of view withstands generalizations far beyond the smooth case: In L^1-minimization, for instance, discontinuous curves in the Grassmannian have to be considered (instantaneous rotations of Lagrangian subspaces still obeying symplectic rules [63]). The cut locus is a central object in Riemannian geometry, control and optimal transport. This is the motivation for a series of conferences on "The cut locus: A bridge over differential geometry, optimal control, and transport", co-organized by team members and Japanese colleagues.

Riemann and Finsler geometry. Studying the distance and minimising geodesics in Riemannian Geometry or Finsler Geometry is a particular case of optimal control, simpler because there are no differential constraints; it is studied in the team for the following two reasons. On the one hand, after some tranformations, like averaging or reduction, some more difficult optimal control problems lead to a Riemann or Finsler geometry problem. On the other hand, optimal control, mostly the Hamiltonian setting, brings a fresh viewpoint on problems in Riemann and Finsler geometry. On Riemannian ellipsoids of revolution, the optimal control approach allowed to decide on the convexity of the injectivity domain, which, associated with non-negativity of the Ma-Trudinger-Wang curvature tensor, ensures continuity of the optimal transport on the ambient Riemannian manifold [66, 67]. The analysis in the oblate geometry [47] was completed in [62] in the prolate one, including a preliminary analysis of non-focal domains associated with conjugate loci. Averaging in systems coming from space mechanics control with L^2-minimization yields a Riemannian metric, thoroughly computed in [45] together with its geodesic flow; in reduced dimension, its conjugate and cut loci were computed in [48] with Japanese Riemannian geometers. Averaging the same systems for minimum time yields a Finsler Metric, as noted in [43]. In [52], the geodesic convexity properties of these two types of metrics were compared. When perturbations (other than the control) are considered, they introduce a "drift", *i.e.* the Finsler metric is no longer symmetric.

Sub-Riemannian Geometry. Optimal control problems that pertain to sub-Riemannian Geometry bear all the difficulties of optimal control, like the role of singular/abnormal trajectories, while having some useful structure. They lead to many open problems, see the monograph [74] for an introduction. The sub-Riemannian problem can be encoded by a non-linear control system with no drift, subjected to a quadratic energy minimization objective. This allows the sub-Riemannian problem to serve as rich model spaces for optimal control. The interest of sub-Riemannian geometry can go beyond these aspects however. It was proved by Hormander in 1967 [72] that local controllability of the system (given in terms of Lie-brackets of vector fields) is equivalent to sub-ellipticity of a second order differential operator associated with the vector fields. In this way, sub-Riemannian geometry acts as a bridge between elements of analysis of PDEs and geometric control theory. For instance, many recent works focus on framing properties of sub-elliptic operators in terms of minimizers of the optimal control problem (such as the influence of cut and conjugate points on diffusion asymptotics [42]). This link even allowed to successfully introduce concepts of sub-elliptic diffusions in computer vision algorithms thanks to sub-Riemannian geometric structures identified in mammal visual mechanisms [55].

Small controls and conservative systems, averaging. Using averaging techniques to study small perturbations of integrable Hamiltonian systems is as old an idea as celestial mechanics. It is very subtle in the case of multiple periods but more elementary in the single period case, here it boils down to taking the average of the perturbation along each periodic orbit [40, 79]. This line of research stemmed out of applications to space engineering (see Section 4.1): the control of the super-integrable Keplerian motion of a spacecraft orbiting around the Earth is an example of a slow-fast controlled system. Since weak propulsion is used, the control itself acts as a perturbation, among other perturbations of similar magnitudes: higher order terms of the Earth potential (including J_2 effect, first), potential of more distant celestial bodies (such as the Sun and the Moon), atmospheric drag, or even radiation pressure. Properly qualifying the convergence properties (when the small parameter goes to zero) is important and is made difficult by the presence of control. In [43], convergence is seen as convergence to a differential inclusion; this applies to minimum time; a contribution of this work is to put forward the metric character of the averaged system by yielding a Finsler metric (see Section 3.2). Proving convergence of the extremals (solutions of the Pontryagin Maximum Principle) is more intricate. In [61], standard averaging ([40, 79]) is performed on the minimum time extremal flow after carefully identifying slow variables of the system thanks to a symplectic reduction. This alternative approach allows to retrieve the previous metric approximation, and to partly address the question of convergence. Under suitable assumptions on a given geodesic of the averaged system (disconjugacy conditions, namely), one proves existence of a family of quasi-extremals for the original system that converge towards the geodesic when the small perturbation parameter goes to zero. This needs to be improved, but convergence of all extremals to extremals of an "averaged Pontryagin Maximum Principle" certainly fails. In particular, one cannot hope for C^1-regularity on the value function when the small parameter goes to zero as swallowtail-like singularities due to the structure of local minima in the problem are expected. (A preliminary analysis has been made in [60].)

Optimality of periodic solutions/periodic controls. When seeking to minimize a cost with the constraint that the controls and/or part of the states are periodic (and with other initial and final conditions), the notion of conjugate points is more difficult than with straightforward fixed initial point. In [51], for the problem of optimizing the efficiency of the displacement of some micro-swimmers with periodic deformations, we used the sufficient optimality conditions established by R. Vinter's group [85, 69] for systems with non unique minimizers due to the existence of a group of symmetry (always present with a periodic minimizer-candidate control). This takes place in a long term collaboration with P. Bettiol (Univ. Bretagne Ouest) on second order sufficient optimality conditions for periodic solutions, or in the presence of higher dimensional symmetry groups, following [85, 69]. Another question relevant to locomotion is the following. Observing animals (or humans), or numerically solving the optimal control problem associated with driftless micro-swimmers for various initial and final conditions, we remark that the optimal strategies of deformation seem to be periodic, at least asymptotically for large distances. This observation is the starting point for characterizing dynamics for which some optimal solutions are periodic, and asymptotically attract other solutions as the final time grows large; this is reminiscent of the "turnpike theorem" (classical, recently applied to nonlinear situations in [83]).

3.3 Software

Optimal control applications (but also the development of theory where numerical experiments can be very enlightening) require many algorithmic and numerical developments that are an important side of the team activity. We develop on-demand algorithms and pieces of software, for instance we have to interact with a production software developed by Thales Alenia Space. A strong asset of the team is the interplay of its expertise in geometric control theory with applications and algorithms, and the team has a long-lasting commitment to the development of numerical codes for the efficient resolution of optimal control problems. Methods for solving optimal control problems with ordinary differential equations more or less fall into three main categories. Dynamic Programming (or Hamilton Jacobi Bellman method) computes the global optimum but suffers from high computational costs, the so-called *curse of dimensionality*. Indirect methods based on Pontryagin Maximum Principle are extremely fast and accurate but often require more work to be applied, in terms of mathematical analysis and a priori knowledge of the solution; this kind of fine geometrical analysis is one of the strong know-how of McTAO. Direct transcription methods offer a good tradeoff between robustness and accuracy and are widely used for industrial applications. For challenging problems, an effective strategy is to start with a direct method to find a first rough solution, then refine it through an indirect method. Such a combined approach has been for instance used between McTAO, the former COMMANDS team (Inria Saclay), and CNRS team APO (Université Toulouse, CNRS, ENSEEIHT) for the optimization of contrast in medical imaging (MRI), and fuel-effective trajectories for airplanes. This combination of direct and indirect methods has a lot of interest to solve optimal control problems that contain state or control constraints. In the collaborations mentioned above, the interfacing between the two solvers BOCOP and HamPath were done manually by *ad hoc* `python` or `matlab` layers. In collaboration with COMMANDS and colleagues from ENSEEIHT, McTAO leads the ct: control toolbox project whose goal is to interoperate these solvers using a high level common interface. The project is an Inria Sophia ADT[1] (2019-) in AMDT[1] mode supported by Inria Sophia SED. The last sprint session, closing the project, is planned for February 2023.

4 Application domains

4.1 Aerospace Engineering

Participants: Jean-Baptiste Caillau, Thierry Dargent, Lamberto Dell'Elce, Frank de Veld, Alesia Herasimenka, Lucas Palazollo, Jean-Baptiste Pomet.

[1]ADT is the name of software development actions supported by the service "SED", by devoting some engineers to these projects. AMDT means that a group of engineers works on the project part time, rather than sending one engineer in the team.

Space engineering is very demanding in terms of safe and high-performance control laws. It is therefore prone to fruitful industrial collaborations. McTAO now has an established expertise in space and celestial mechanics. Our collaborations with industry are mostly on orbit transfer problems with low-thrust propulsion. It can be orbit transfer to put a commercial satellite on station, in which case the dynamics are a Newtonian force field plus perturbations and the small control. There is also, currently, a renewed interest in low-thrust missions such as Lisa Pathfinder (ESA mission towards a Lagrange point of the Sun-Earth system) or BepiColombo (joint ESA-JAXA mission towards Mercury). Such missions look more like a controlled multibody system. In all cases the problem involves long orbit transfers, typically with many revolutions around the primary celestial body. When minimizing time, averaging techniques provide a good approximation. Another important criterion in practice is fuel consumption minimization (crucial because only a finite amount of fuel is onboard a satellite for all its “life”), which amounts to L^1-minimization. Both topics are studied by the team. We have a steady relationship with CNES and Thales Alenia Space (Cannes), that have financed or co-financed 4 PhDs and 2 post-docs in the decade and are a source of inspiration even at the methodological level. Team members also have connections with Airbus-Safran (Les Mureaux) on launchers.

Some of the authoritative papers in the field were written by team members, with an emphasis on the geometric analysis and on algorithms (coupling of shooting and continuation methods). There are also connections with peers more on the applied side, like D. Scheeres (Colorado Center for Astrodynamics Research at Boulder), the group of F. Bernelli (Politecnico Milano), and colleagues from U. Barcelona (A. Farrès, A. Jorba).

Two new directions have been taken recently. The first one is about the control of solar sails (see Section 7.8), the second one about collision avoidance for spacecrafts (see Section 7.10). Collision avoidance is becoming very important in nowadays space missions due to the growing number of various bodies (garbage, micro-satellites…) orbiting around the earth. A PhD (Frank de Veld) started in December, supported by Thales Alenia Space. Solar sailing has been actively studied for two decades and recent missions have demonstrated its interest for "zero-fuel" missions; it poses delicate control questions due to drastic constraints on the control direction. A PhD (Alesia Herasimenka) is in progress, selected by ESA for a three-year research co-sponsorship.

4.2 Optimal control of microbial cells

Participants: Jean-Baptiste Caillau, Walid Djema *(BIOCORE project-team)*, Jean-Luc Gouzé *(BIOCORE project-team)*, Sofya Maslovskaya *(Paderborn University, Germany)*, Jean-Baptiste Pomet, Agustín Yabo *(INRAE, Montpellier)*.

The growth of microorganisms is fundamentally an optimization problem which consists in dynamically allocating resources to cellular functions so as to maximize growth rate or another fitness criterion. Simple ordinary differential equation models, called self-replicators, have been used to formulate this problem in the framework of optimal and feedback control theory, allowing observations in microbial physiology to be explained. The resulting control problems are very challenging due to the nonlinearity of the models, parameter uncertainty, the coexistence of different time-scales, a dynamically changing environment, and various other physical and chemical constraints. In the framework of the ANR Maximic (PI Hidde de Jong, Inria Grenoble Rhône-Alpes) we aim at developing novel theoretical approaches for addressing these challenges in order to (i) study natural resource allocation strategies in microorganisms and (ii) propose new synthetic control strategies for biotechnological applications. In order to address (i), we develop extended self-replicator models accounting for the cost of regulation and energy metabolism in bacterial cells. We study these models by a combination of analytical and numerical approaches to derive optimal control solutions and a control synthesis, dealing with the bang-bang-singular structure of the solutions. Moreover, we define quasi-optimal feedback control strategies inspired by known regulatory mechanisms in the cell. To test whether bacteria follow the predicted optimal strategies, we quantify dynamic resource allocation in the bacterium Escherichia coli by monitoring, by means of time-lapse fluorescent microscopy, the expression of selected genes in single cells growing in a microfluidics device. In order to address (ii), we build self-replicator models that include a pathway for the production of a metabolite of interest. We also add a mechanism to turn off microbial growth by means of an external input signal, at the profit of the production of the metabolite. We formulate the maximization of the amount of metabolite produced as an optimal control problem, and derive optimal solutions and a control synthesis, as well as quasi-optimal feedback strategies satisfying chemical and physical design constraints. The proposed synthetic control strategies are being tested experimentally by growing E. coli strains capable of producing glycerol from glucose in a mini-bioreactor system. We aim at quantifying the amount of glucose consumed and glycerol produced, in the case of a predefined input signal (open-loop control) and the adaptive regulation of the input signal based on on-line measurements of the growth rate and the expression of fluorescent reporters of selected genes (closed-loop control). Currently, one PhD (A. Yabo) and one postdoc (S. Maslovskaya) are involved in these tasks and jointly supervised by colleagues from McTAO and Biocore teams at Sophia. Preliminary results concern the definition on extended (higher dimensional) models for the bacteria dynamics, check of second order optimality conditions on the resulting optimal control problem, and study of the turnpike phenomenon for these optimization problems.

4.3 Stability of high frequency active circuits

Participants: Laurent Baratchart *(FACTAS project-team)*, Sébastien Fueyo *(Centrale-Supélec, Gif-sur-Yvette)*, Jean-Baptiste Pomet.

Nonlinear hyper-frequency amplifiers are ubiquitous in cell phone relays and many other devices. They must be as compact as possible, yielding a more complicated design. Computer Assisted Design tools are extensively used; for a given amplifier design, they provide frequency responses but fail to provide information of the stability of the response for each frequency. This stability is crucial for an unstable response will not be observed in practice; the actual device should not be built before stability is asserted. Predicting stability/instability from “simulations” in the Computer Assisted Design tool is of utmost importance (simulation between quotation marks because these simulations are in fact computations in the frequency domain). Potential transfer to industry is important.

Some techniques do exist, see [82], based on creating some virtual perturbations and treating them as the input of a (linearized) control system to be “simulated” using the same tools. In an ongoing collaboration between McTAO and the project-team FACTAS, we work on the mathematical ground of these methods and in particular of the relation between stability and the property of the identified time-varying infinite dimensional systems. See recent developments in Section 7.4.

5 Highlights of the year

5.1 Awards

- Agustin Yabo (now postdoc at INRAE Montpellier) received the 2nd ATSI prize of the STIC Doctoral School.
- Alesia Herasimenka was awarded the 2022 Pierre Laffitte prize (first prize), rewarding the best second year PhD students working in innovative topics in the Sophia Antipolis region.
- Jean-Baptiste Caillau was awarded the “Défi Défense” prize during the 2022 “Assises des mathématiques” of CNRS.
- The paper [12] won the best paper award of the IFAC conference “Control Applications of Optimisation” (CAO’2022).
- Alesia Herasimenka was awarded the best poster award at the “Journées SMAI-MODE 2022”.

6 New software and platforms

6.1 New software

6.1.1 ct

Name: control toolbox

Keywords: Optimal control, Ordinary differential equations, Mathematical Optimization, Differential homotopy, Automatic differentiation

Scientific Description: Numerical resolution of optimal control problems

Functional Description: The project gathers and allows to interoperate tools designed to solve numerically optimal control problems on ordinary differential equations. The available approaches include direct methods (based on a transcription of optimal control problems into mathematical programs) as well as indirect ones (based on Pontrjagin maximum principle, like the shooting method). The latter can be coupled to differential continuation. Automatic differentiation (aka Différentiable Programming) plays a crucial a role in all these algorithms. The project strongly leverages on SED Sophia support.

Release Contributions: - bocop refactoring - nutopy library - project gallery

URL: http://ct.gitlabpages.inria.fr/gallery

Contact: Jean-Baptiste Caillau

Participants: Jean-Baptiste Caillau, Pierre Martinon, Olivier Cots, Thibaud Kloczko, Tristan Cabel, Jean-Luc Szpyrka, Erwan Demairy, Julien Wintz, Carlos Zubiaga Pena, Nicolas Niclausse

Partners: Université de Toulouse, CNRS, IRIT, ENSEEIHT

7 New results

7.1 Averaging fast-oscillating optimal control systems with two fast variables

Participants: Jean-Baptiste Caillau, Lamberto Dell'Elce, Jean-Baptiste Pomet.

Research on fast-oscillating optimal control systems is a long-standing topic in McTAO. We investigated in 2021 how trajectories of fast-oscillating control system with a single fast variable converge to their averaged counterpart [64]. During 2022, further insight was gained on the averaging of systems with two fast variables. This study was motivated by the need for understanding how to generate "consistent" averaged trajectories of minimum time control systems with two fast variables (*i.e.*, characterized by moderate drift with respect to their original counterpart). For this purpose, we leveraged on an academic example to provide evidence that existing theorems on double averaging of dynamical systems (namely, the Neishtadt theorem) cannot be directly applied to controlled systems. Hence, we developed a proper near-identity transformation of initial adjoint variables that was sufficient to prevent large drift between averaged and original trajectories in non-resonant regions. Finally, we investigated the impact of resonance crossing on the dynamics of adjoint variables and proposed a way to predict this behavior via resonant averaged forms.

These considerations are of use to generate reliable initial guesses for indirect techniques to solve two-point boundary value problems. This work was presented at the IFAC conference "Control Applications of Optimisation" (CAO'2022) [12].

7.2 Controllability of fast-oscillating system with control constraints

Participants: Jean-Baptiste Caillau, Lamberto Dell'Elce, Alesia Herasimenka, Jean-Baptiste Pomet.

Control of solar sails requires a comprehensive understanding of controllability, that is not a given because forces (control) are drastically constrained. The paper [7] (see Section 7.8) describes effective methods to test controllability, depending on the current orbit and the optical properties of the sail. It is based on a controllability study for general affine control systems where the drift vector fields yields periodic solutions and the controls are constrained in a set that is not a neighborhood of zero. We obtained sufficient conditions for controllability, both local and global, presented in [26, 15]; they are formulated in terms of pushforwards along the flow of the drift, rather than in terms of Lie brackets, and take into account not only the vector fields associated to the system but also the definition of the control set; it turns out that they amount to local controllability of a time-varying linear approximation with constrained controls. These conditions are harder to check in practice than these formulated in terms of the rank of a family of vector fields, and the purpose of [7] is, in part, to give a method, based on convex optimization to assess these conditions in the case of a solar sail. (The approach leverages fine properties of trigonometric polynomials as well as Nesterov's technique of sum of squares relaxation.)

7.3 Small time local controllability for some degenerated two-input systems

Participants: Laetitia Giraldi *(CALISTO project-team)*, Pierre Lissy *(Université Paris Dauphine, Paris)*, Clément Moreau *(Kyoto University, Japan)*, Jean-Baptiste Pomet.

Motivated by the study of planar articulated magnetically actuated swimmers at low Reynolds number (see C. Moreau's PhD [35]), around the straight configuration, where all magnetic moments are aligned, we studied local controllability (STLC) for a more general, but still particular, class of systems with two controls, around an equilibrium where the two control vector fields are co-linear. The manuscript [29], now provisionally accepted for publication in *Control, Optimization and Calculus of Variations*, gives novel necessary conditions for STLC of these systems, based on Chen-Fliess expansions of solutions, in the spirit of [73].

7.4 Stability of linear time-varying time-delay systems

Participants: Laurent Baratchart *(FACTAS project-team)*, Sébastien Fueyo *(Tel-Aviv University)*, Jean-Baptiste Pomet.

Sébastien Fueyo's doctoral work ([34], PhD defended) on testing the stability of amplifiers in a CAD (computer assisted design) process, described also in Section 4.3, led us to revisiting stability of time-varying systems/networks of 1-D hyperbolic PDEs, or linear "delay difference systems" (*i.e.* of the type $z(t) = A_1(t)z(t-\tau_1)+\cdots+A_N(t)z(t-\tau_N)$). In the time-invariant case, a well known necessary and sufficient condition for stability is due to Hale and Henry [71, 70]; it gives, in a sense, a final answer to the question, but it is not so easy to check explicitly this criteria, and there is still a vast literature on more specific sufficient conditions. The article [18], submitted to *Integral Equations and Operator Theory*, proposes a generalization to the periodic time-varying case. In [10] we detail how this and a former sufficient condition, more restrictive but more explicit, stated in [41], apply to various situations.

7.5 State estimation and stabilisation of non-uniformly observable systems

Participants: Ludovic Sacchelli, Lucas Brivadis *(CentraleSupélec, Gif-sur-Yvette)*, Jean-Paul Gauthier *(Université de Toulon)*, Jean-Luc Gouzé *(BIOCORE project-team)*, Théo Rolin.

As was discussed in the research program, stabilization of the state of a system by means of a feedback control is a classical and important problem in control theory. When only part of the system is known, a usual strategy is to rely on a dynamic algorithm, known as an observer, in order to provide an estimate of the state that can be fed to the controller. Designing a stable closed-loop based on an observer requires that some necessary information on the state can be accessed through this partial measurement. Critically, for nonlinear systems, whether or not it is possible to reliably estimate the state can depend on the control. This fact is known as non-uniform observability and is a root issue for observer design. In a collaboration between L. Sacchelli and J.-L. Gouzé, we supervised T. Rolin's M1 internship on the topic of robust state-estimation methods for non-uniformly observable systems. The goal of the internship was to analyze and reproduce the method proposed in [77] for monitoring batch bioreactors that asymptotically lose observability.

Regarding output feedback control, if singular controls exist for observability, there is no clear definitive answer as to how to achieve stabilization. [24] (now in press) reviews some strategies that showed to be efficient in tackling the difficulties posed by non-uniform observability, and explores the genericity side of the matter, including a proof that this critical situation can be generic in some key classes of systems. [25] explores the issue through the point of view of hybrid systems, a new framing for the group. In the paper, we propose a method to monitor observability of the system online. We use this method as the backbone of a dynamically switching Kalman-type observer and feedback to achieve stabilization of bilinear systems. Allowing Kalman filters in that context was a long term goal but had remained an open problem. This new approach will be explored more thoroughly in future works.

7.6 Geometry and optimal control

Participants: Bernard Bonnard, Joseph Gergaud, Boris Wembe *(ENSEEIHT, Toulouse)*, Olivier Cots, Jérémy Rouot *(Université de Bretagne Occidentale, Brest)*.

This work was started in the frame of Wembe PhD thesis on time-minimum problems in navigation in presence of a current and a vortex, defended at ENSEEIHT Toulouse, November 2021.

Our contribution was to revisit and generalize the Navigation Problem set by Carathéodory and Zermelo of a ship navigating on a river with a linear current and aiming to reach the opposite shore in minimum time. This work is motivated by the displacement of particles in a two dimensional fluid, in presence of a vortex (initially, a singularity in the Helhmoltz-Kirchhoff equations) inducing a strong current hampering local controllability. To define a minimum time Zermelo navigation problem, we consider the particle as the ship of the navigation problem and the control is defined as the heading angle of the ship axis. It turns out that the historical problem and our recent vortex study are two examples of the general case of Zermelo navigation problems on surfaces of revolution that we tackled this year. Our main contributions in this setting are multiple. In [3, 4], we analyze the role of abnormal geodesics in the problem, in particular in relation with cusp singularities of the geodesics and non continuity properties of the value function. In [19] (currently in revision), we relate the existence, in the geodesics dynamic, of separatrices interpreted as Reeb components, with the Morse–Reeb classification of the geodesics. Furthermore, we provide explicit computations of the conjugate and cut loci in case studies such as the averaged Kepler case in space mechanics [19].

7.7 Sub-Riemannian diffusions

Participants: Ludovic Sacchelli, Robert Neel *(Lehigh University, USA)*.

A version of the heat equation on sub-Riemannian manifolds can be obtained by considering a generalized Laplace-Beltrami operator taking into account the non-holonomic constraints of the control structure. It has been known since Varadhan's formula for Riemannian manifolds [84] that the distance between two points can be recovered by taking the log of the heat kernel and pushing time to 0. Combining tools from the study of Brownian motions on manifolds and geometric analysis of sub-Riemannian length minimizing curves, these type of asymptotics were successively extended to more and more general sub-Riemannian structures. In [30], we expanded on previously developed techniques to study bounds on logarithmic derivatives of the heat kernel. They have been long known on compact Riemannian manifolds, and were recently extended to general complete Riemannian manifolds. We further extend these bounds to incomplete Riemannan manifolds under the least restrictive condition available. We also discuss the difficulties that arise when we consider the sub-Riemannian case.

7.8 Control of solar sails

Participants: Jean-Baptiste Caillau, Tommy Calendini, Paul Chambroux, Lamberto Dell'Elce, Alesia Herasimenka, Jean-Baptiste Pomet.

Solar sails offer a propellantless solution to perform interplanetary transfers, planet escapes, and deorbiting maneuvers by leveraging on solar radiation pressure (SRP). Although very few solar-sail missions have been launched to this date, the possibility to use SRP as an inexhaustible source of propulsion has attracted the interest of researchers in the last decades. Various contributions on the control of solar sails were obtained in the framework of A. Herasimenka's thesis.

We proposed an efficient test to conclude controllability of non-ideal solar sails in planet-centered orbits. This requirement is aimed at assessing whether a non-ideal solar sail with given optical parameters is capable of decreasing or increasing all possible functions of the Keplerian integrals of motion over an orbital period. In other words, we verify if a solar sail can change the orbit in any desirable way given its optical properties. This requirement is formulated as a worst-case convex optimization problem characterized by a finite number of design variables and a two-parameter family of inequality constraints. By leveraging on the formalism of squared functional systems and on the peculiar form of Gauss variational equations, we recast the semi-infinite problem into a finite-dimensional convex programming problem with a finite number of linear matrix inequalities (LMI) and an unique well-defined solution [7].

The internship of P. Chambroux was focused on a feasibility study on the use of solar sail to carry out observations of the solar corona by occulting the Sun's disk with the Earth. This can be achieved by placing the sail on an orbit in proximity of the Lagrangian point L_2 of the Earth-Sun system. The circular restricted three-body problem (Earth-Sun system, where Earth serves as occulting body) was used to model the motion of the satellite. Trajectory design was formulated as a periodic optimal control problem aimed at maximizing the duty cycle of the observations, defined as the fraction of orbital period devoted to observations. Charge of the batteries was part of the state variables, and its rate of change is function of the position of the satellite and increases from zero to its maximum value across the penumbra region. Pontryagin maximum principle (PMP) was then applied to deduce necessary conditions of optimality for the aforementioned cost function (duty cycle) and dynamical model. Numerical solution was achieved by using indirect techniques.

Finally, optimal control of solar sails in planet-centered orbits was investigated in the internship of T. Calendini. Maximization of the Hamiltonian requires the solution of a nonlinear equation (that is well defined thanks to the implicit function theorem). This was obtained numerically by augmenting the Hamiltonian system with an auxiliary variable representing the unknown maximizing control and including the associated equation in the constraints (rather that solving numerically for the control at each step).

7.9 Dynamical systems theory applied to space mechanics

Participants: Nicola Baresi *(University of Surrey, UK)*, Lamberto Dell'Elce, Pini Gurfil *(Technion, Israel)*, Gianpaolo Izzo *(Technion, Israel)*, Aaron Rosengren *(University of California San Diego, US)*.

Two applications of dynamical systems theory to space mechanics were tackled.

First, motivated by upcoming missions towards remote planetary moons that will fly towards chaotic dynamical environments which are significantly perturbed by the oblateness of the host planet, we introduced a new time-periodic set of equations of motion that is based on the analytical solution of the zonal equatorial problem. Such a system, which referred to as the Zonal Hill Problem, remains populated by resonant periodic orbits and families of two-dimensional quasi-periodic invariant tori that we calculated by means of homotopy continuation procedures. The resulting periodic and quasi-periodic trajectories were investigated for the trajectory design of future planetary moons explorers [2].

Second, we proposed a new analytical solution for the first-order, short-periodic perturbations due to planetary oblateness [8]. Our approach was based on the Milankovitch vectorial elements and is free of all the mathematical singularities. Being a non-canonical set, our derivation followed the scheme used by Kozai in his oblateness solution. We adopted the mean longitude as the fast variable and presented a compact power-series solution in eccentricity for its short-periodic perturbations that relies on Hansen's coefficients. We also used a numerical averaging algorithm based on the fast-Fourier transform to further validate our new mean-to-osculating and inverse transformations. This technique constitutes a new approach for deriving short-periodic corrections and exhibits performance that are comparable to other existing and well-established theories, with the advantage that it can be potentially extended to modeling non-conservative orbit perturbations.

7.10 Collision avoidance

Participants: Lamberto Dell'Elce, Frank de Veld, Lucas Palazzolo, Jean-Baptiste Pomet.

On the one hand, we have been defining a PhD topic with Thales Alenia Space on collision avoidance during low thrust orbit transfer, that is now funded by Thales Alenia Space together with a grant from Région Provence Alpes Côte d'Azur. The PhD candidate is Frank de Veld, his PhD started in December, 2022.

On the other hand, Lucas Palazzolo's internship (June-July plus a part-time project in the winter-spring) was about "Control Barrier functions" (CBF), after [56] (not chosen to be a reference, rather a current conference paper to study); he conducted numerical studies to improve techniques in that paper or apply them to specific test cases. This is also aimed at safe collision avoidance, but rather in low relative velocity; it is not directly linked to the PhD topic mentioned just above.

7.11 Attitude determination and control system of NiceCUBE

Participants: Lamberto Dell'Elce, Azouaou Ouyoucef, Jean-Baptiste Pomet.

Université Côte d'Azur is planning to build its first single-unit CubeSat (i.e., a recent standard for satellites with mass of the order of few kilograms and volume of few dm^3), named NiceCube. The objective of the mission is to establish an optical link between a ground station in Nice and the satellite by enlightening it with a laser and by moduling the reflected signal with an occulter onboard NiceCube. To this end, the face of the satellite equipped with retroreflectors should be oriented towards the ground station so that the reflected signal can be received. McTAO part of UCA and is in charge of the attitude determination and control system of NiceCube.

The mission is currently in phase A (i.e., feasibility study). This year, we completed the development of the attitude determination component of the numerical simulator that we implemented to simulate the mission.

7.12 Optimal allocation of resources in bacteria

Participants: Agustín Yabo, Jean-Baptiste Caillau, Jean-Luc Gouzé *(BIOCORE project-team)*, Mohab Safey El Din *(Sorbonne Université)*.

In the framework of the ANR Maximic, we carry on the study of self-replicator models. These models describe the allocation of resources inside the bacteria: the substrate is used to produce precursors that, in turn, can be employed either to produce genetic machinery (and increase the biomass) or metabolic machinery (that will further catalyse the transformation of substrate into precursors). To this internal control and external action that aims, after some genetic engineering on the bacteria (to create a strain that reacts to light stimuli), at producing a new metabolite of interest. Then, while the behaviour of the untouched bacteria tends to be very well mimicked by biomass maximization strategies, maximizing the production of the metabolite of interest induces new biological strategies. This kind of model (and refinements) are studied in [33] and [9]. Key properties of the system are: (i) the Fuller phenomenon as connection between bang and singular arcs requires an infinite number of switchings in finite time; (ii) the turnpike phenomenon. Indeed, for large fixed final times, trajectories of the system are essentially singular and close to the optimal (wrt. a constant static control) equilibrium which is a hyperbolic fixed point of the singular flow. See ct gallery for an example, and the recently defended PhD thesis of A. Yago [86] for a discussion of these results. In collaboration with M. Safey El Din, stability properties of the system have been established thanks to a consistency check of a system of polynomial inequalities. (To appear in *MathS In Action*.)

7.13 Biomechanics: control of muscular force response using FES with application to the conception of a smart electrostimulator

Participants: Bernard Bonnard, Sandrine Gayrard, Jérémy Rouot *(Université de Bretagne Occidentale, Brest)*, Toufik Bakir *(Université de Bourgogne Franche Comté, Dijon)*.

This topic started in McTAO in 2017 with a collaboration between B. Bonnard and T. Bakir (ImVia-UBFC), and J. Rouot in the frame of Cifre PhD thesis of S. Gayrard with Segula Technologies.

The problem of control of muscular force is posed in terms of optimization of the train pulses of a Functional Electro-Stimulation (FES) signal to produce the muscular contraction. Based on preliminary experimental studies, the dynamical model that was chosen for muscular control is known as *Ding et al. force-fatigue model* in the literature. It is a refinement of the historical Hill model (Medicine Nobel Prize in 1922) that takes into account the variations of the fatigue variable. From the control methodology point of view, this required some developments on optimal control for sample control systems. This is by itself already a rich topic. In 2020, this project took the industrial transfer direction with a Cifre PhD funding in partnership with Segula Technologies (see Section 8) whose goal is the design of a smart electrostimulator for force reinforcement or rehabilitation in the framework of S. Gayrard's PhD.

This year, our contribution to the project was double. From the theoretical point of view, we have derived a finite dimensional approximation of the forced dynamics based on the *Ding et al.* model to provide fast optimizing schemes aiming to track a reference force or maximize the force (see [1]). On the other hand, S. Gayrard is also finalizing a prototype of the smart electrostimulator, which is a major objective of the collaboration with Segula. It is currently being tested on a cohort of human subjects. See S. Gayrard 2022 Preprint for a succint presentation of this work at ECC 2023 Conference [28].

7.14 Feedback invariants and optimal control with biological applications and numerical calculations

Participants: Bernard Bonnard, Jérémy Rouot *(Université de Bretagne Occidentale, Brest)*, Cristiana J. Silva *(Instituto Universitário de Lisboa, Lisbon)*.

The starting point of the study was the problem of controlled Lotka Volterra dynamics motivated by curing microbiote infection by a pathogenic agent. This lead to complicated optimal control problems in the frame of permanent or sampled-data controls, in relation with medical constraints. The problem can be set as a time-minimal control problem with terminal manifold of codimension one. Our contributions are presented in the series of works [20, 23, 21, 22]. In relation with previous work regarding the optimal control of chemical networks, the time minimal synthesis has been described near the terminal manifold up to codimension two situations in the jets spaces. We have compared both permanent controls and sampled-data control in relation with numerical issues. Finally, we have obtained feedback invariants to classify the geodesic dynamics associated to rays of abnormal geodesics which are related to shifted equilibria of the free Lotka-dynamics and which can be calculated using only linear computations.

See Section 9.1.1 for an upcomming project on the topic with C. Silva.

7.15 ct: control toolbox

Participants: Jean-Baptiste Caillau, Olivier Cots, Pierre Martinon *(CAGE project-team, on leave)*.

The ADT `ct: control toolbox` had one sprint in 2022. The focus was on to initiate new developments in Julia to take advantage of the powerful features of the language. Julia is indeed a perfect match for our needs in scientific computing for numerical optimal control; the language has a high level of abstraction well suited for mathematical descriptions, but still makes no compromise when it comes to performance thanks to efficient just-in-time compilation. Moreover, it currently has several efficient backends for AD / DP (automatic differentiation / differentiable programming), including ForwarDiff, Zygote of Enzyme: this is a crucial step for our project, both for direct and indirect methods. (Some examples of the project gallery require up to five levels of nested automatic differentiation.)

8 Bilateral contracts and grants with industry

Intelligent muscle electrostimulator, Segula Technologies

Participants: Toufik Bakir *(Université de Bourgogne Franche Comté, Dijon)*, Bernard Bonnard, Sandrine Gayrard.

In the framework of a **CIFRE grant**, a contract (title: “Réalisation d’un prototype d’électrostimulateur intelligent”) between Segula Technologies and Université de Bourgogne is partially funding (together with ANRT) Sandrine Gayrard’s PhD.

This is completed by an **additionnal collaboration contract** between Segula and SAYENS (representing Université de Bourgogne), aiming at constructing the prototype of the smart electrostimulator.

- PI: Bernard Bonnard and Toufik Bakir (ImViA, Université de Bourgogne Franche Comté, Dijon).
- Period: 2020–2023.

Optimal Control of Solar Sails, ESA (European Space Agency)

Participants: Jean-Baptiste Caillau, Lamberto Dell'Elce, Alesia Herasimenka, Jean-Baptiste Pomet.

Three year contract starting in 2021 between the team and the European Space agency. Its purpose is to support the environment of Alesia Herasimenka's PhD on this topic.

- Partners: McTAO and ESA.
- Total amount: 24k€.
- Period: 2021–2024.
- Inria reference: 16016.

Méthodes de contrôle pour l'évitement de collisions entre satellites, Thales Alenia Space

Participants: Thierry Dargent, Lamberto Dell'Elce, Frank de Veld, Jean-Baptiste Pomet.

Thales Alenia Space co-funded the thesis of Frank de Veld named "Méthodes de Contrôle pour l'évitement de collisions entre satellites".

- Partners: McTAO and Thales Alenia Space.
- Period: 2022–2025
- Total amount: 75k€
- Inria reference: 022-0674

9 Partnerships and cooperations

9.1 International initiatives

9.1.1 Participation in other International Programs

B. Bonnard is consultant for the Portuguese project "Mathematical model of multiscale control systems : applications to human disease. P.I. Cristiana Silva (Instituto Universitário de Lisboa, Portugal). 36 months strating January 8th 2023.

9.2 National initiatives

9.2.1 ANR

Maximic: optimal control of microbial cells by natural and synthetic strategies. Started 2017, duration: 6 years. J.-B. Caillau and J.-B. Pomet are participants. More information and news on the site of this project.

9.2.2 Others

- PGMO grant (2020-2022) on "Sampled-data control". B. Bonnard (P.I.). Total amount 6k€.
- PGMO grant (2020-2022) on "Extremal determinants". Participants are Y. Chitour (Université Paris Saclay), J.-B. Caillau, P. Freitas (University of Lisbon), Y. Privat (Université de Strasbourg). Total amount 7k€.
- Labex CIMI grant on ”Singular control and numerical optimisation in Julia”. Participants are J.-B. Caillau, J. Gergaud (PI) and O. Cots (both Université de Toulouse). Total amount 8k€.
- "Recherche en réseaux" Université de Bourgogne grant on "Design of an asymmetric copepod-microswimmer for 2d-motion". Participants are T. Bakir (P.I., Université de Bourgogne), B. Bonnard. Total amount 5k€.
- McTAO project-team participates in the GdR MOA, a CNRS network on Mathematics of Optimization and Applications.

9.3 Regional initiatives

Grant from Région SUD – Provence Alpes Côte d'Azur “Emplois jeunes Chercheurs”, 2022-2025, that co-funds Frank de Veld's PhD. See also the contract with Thales Alenia Space in Section 8. Total amount: 54k€.

10 Dissemination

10.1 Promoting scientific activities

10.1.1 Scientific events: organisation

The McTAO project team maintains a recurring seminar on topics of control theory, optimization and space mechanics (main organizer: L. Sacchelli). The seminar has monthly periodicity and has hosted 11 sessions in 2022:

27/01/22 Ariadna Farrés (NASA, University of Maryland): station keeping in space mechanics.

04/03/22 Mario Siaglotti (Inria, CAGE): ensemble quantum control.

25/03/22 Jean-Paul A. Gauthier (LIS, université de Toulon): time-optimal quantum control.

25/05/22 Dario Prandi (CNRS, L2S, CentraleSupélec): control of optical illusions.

20/06/22 Michael Orieux (Universitat Politécnica de Catalunya): optimal control of neural dynamics.

01/07/22 Nicolas Leclère (Université de Liège): orbital propagation.

07/09/22 Robert W. Neel (Lehigh University): sub-Riemannian diffusions.

20/10/22 Sébastien Fueyo (Tel-Aviv University): controllability of difference equations.

27/10/22 Rémi Robin (Inria, QUANTIC): shape optimization of stellarators.

17/11/22 Slava G. Turyshev (JPL Caltech): solar sails.

14/12/22 Eric Busvelle (LIS, Université de Toulon): observability.

Member of the organizing committees J.-B. Caillau was member of the organizing committee (with A. Habbal and S. Vaiter) of the 2022 GdR MOA days (held in Nice).

10.1.2 Scientific events: selection

Member of the conference program committees J.-B. Caillau was member of the Program Committee of PGMO days 2022 (held at EDF research center, Saclay).

10.1.3 Journal

Member of the editorial boards

- B. Bonnard is member of the editorial board of Pacific Journal of Mathematics for Industry.
- J.-B. Caillau is member of the editorial board of ESAIM: Mathematical Modelling and Numerical Analysis (ESAIM: M2AN).

Reviewer - reviewing activities

All team members take part in a continued effort to offer reviews in various journals of importance to the community.

10.1.4 Invited talks

J.-B. Caillau was invited at Lab. J. A. Dieudonné colloquium (CNRS, Nice).

10.1.5 Research administration

J.-B. Caillau is

- member of the Scientific Committee of the PGMO program of FMJH;
- member of the Scientific Committee of 3IA Côte d'Azur;
- member of the Commission de Développement Technologique of Inria Sophia.

J.-B. Pomet is

- member of the steering committee of Academy of excellence on complex systems, Université Côte d'Azur, and of the executive bureau of the "EUR SPECTRUM" (graduate school),
- an elected member of Inria's permanent Evaluation Committee.

10.2 Teaching - Supervision - Juries

10.2.1 Teaching

- Engineering school and University: J.-B. Caillau has a full teaching duty of Professor at L3, M1 and M2 level at Polytech Nice Sophia - Université Côte d'Azur. He was director of the Applied math. & modelling department of the school.
- J.-B. Caillau gave a lecture on "Optimisation for machine learning" at SEAMS-CIMPA on School on modern trends in signal and data processing at University of the Philippines Diliman.
- Engineering school: L. Dell'Elce, A. Herasimenka and L. Sacchelli each took part as teaching assistants at L3 and M1 levels at Polytech Nice Sophia, Université Côte d'Azur.

10.2.2 Supervision

PhD students.

- Adel Malik Annabi, "Observability and observer synthesis for neural fields equations", Université Côte d'Azur, co-supervised by J.-B. Pomet, L. Sacchelli and D. Prandi (CentraleSupélec). Started October, 2022.
- Frank de Veld, "Méthodes de Contrôle pour l'évitement de collisions entre satellites", Inria, co-supervised by J.-B. Pomet, L. Dell'Elce and Thierry Dargent. Started December 2022.
- Sandrin Gayrard, "Réalisation d'un prototype d'électrostimulateur intelligent", Université de Bourgogne Franche Comté, co-supervised by B. Bonnard and T. Bakir (Université de Bourgogne Franche Comté). Started January, 2020.
- Alesia Herasimenka, "Optimal control of solar sails", Université Côte d'Azur, co-supervised by L. Dell'Elce and J.-B. Caillau. Started October, 2020.

Interns.

- Hugo Blain, 3rd year Polytech Nice. Supervised by J.-B. Caillau. June to July 2022.
- Tommy Calendini, 4th year Polytech Nice. Co-supervised by J.-B. Caillau and A. Herasimenka on the topic of "Optimal control of a solar sail". June to September 2022.
- Paul Chambroux, Master's thesis (M2) Sorbonne Université. Co-supervised by L. Dell'Elce and A. Herasimenka on the topic of "Sun Occultation Mission with Solar Sails". February to August 2022.
- Azouaou Ouyoucef, Master's thesis (M2) Sorbonne Université. Co-supervised by L. Dell'Elce and J.-B. Pomet on the topic of "Study of the attitude control system of the NICEcube nanosatellite". February to August 2022.
- Lucas Palazzolo, M1, Université de Strasbourg. Co-supervised by J.-B. Caillau, L. Dell'Elce, and J.-B. Pomet on the topic of "Collision avoidance via control barrier functions". June to July 2022.
- Théo Rolin, 4th year Polytech Nice. Co-supervised by L. Sacchelli and J.-L. Gouzé (Inria Biocore) on the topic of "Observability and observers for mathematical models of bacterial growth". June to August 2022.

10.2.3 Juries

- J.-B. Caillau was reviewer and sat at the jury of Yves Fotso Fotso PhD thesis (Université Dschang).
- J.-B. Caillau sat at the jury of Vincent Vadez PhD thesis (Université Côte d'Azur).
- J.-B. Caillau was reviewer and sat at the jury of Emiliano Molina PhD thesis (Sorbonne Université).
- J.-B. Caillau sat at the jury of Salma Chabbar PhD thesis (Université de Rabat).
- J.-B. Caillau was reviewer and sat at the jury of Étienne Bertin PhD thesis (Université Paris Saclay).
- J.-B. Pomet sat at the jury of Timothée Schmoderer PhD thesis (INSA Rouen).
- L. Sacchelli sat at the jury of Fazia Harrache PhD thesis (Université de Toulon).

10.3 Popularization

10.3.1 Interventions

Jean-Baptiste Caillau participated to the national program "Regards de géomètre" (2 meetings at Vence Toreille elementary school).

11 Scientific production

11.1 Publications of the year

International journals

[1] T. Bakir, B. Bonnard, S. Gayrard and J. Rouot. 'Finite Dimensional Approximation to Muscular Response in Force-Fatigue Dynamics using Functional Electrical Stimulation'. In: *Automatica* (Oct. 2022). DOI: 10.1016/j.automatica.2022.110464. URL: https://hal.inria.fr/hal-03154450.

[2] N. Baresi and L. Dell'Elce. 'Periodic and Quasi-Periodic Orbits near Close Planetary Moons'. In: *Journal of Guidance, Control, and Dynamics* (2023), pp. 1–15. DOI: 10.2514/1.G007221. URL: https://hal.inria.fr/hal-03937917.

[3] B. Bonnard, O. Cots, J. Gergaud and B. Wembe. 'Abnormal Geodesics in 2D-Zermelo Navigation Problems in the Case of Revolution and the Fan Shape of the Small Time Balls'. In: *Systems and Control Letters* 161 (Mar. 2022), p. 105140. DOI: 10.1016/j.sysconle.2022.105140. URL: https://hal.science/hal-02437507.

[4] B. Bonnard, J. Rouot and B. Wembe. 'Accessibility Properties of Abnormal Geodesics in Optimal Control Illustrated by Two Case Studies.' In: *Mathematical Control and Related Fields* (Nov. 2022). DOI: 10.3934/mcrf.2022052. URL: https://hal.inria.fr/hal-03310348.

[5] J.-B. Caillau, W. Djema, J.-L. Gouzé, S. Maslovskaya and J.-B. Pomet. 'Turnpike property in optimal microbial metabolite production'. In: *Journal of Optimization Theory and Applications* (8th Apr. 2022). DOI: 10.1007/s10957-022-02023-0. URL: https://hal.archives-ouvertes.fr/hal-03328686.

[6] J.-B. Caillau, J. Fejoz, M. Orieux and R. Roussarie. 'On singularities of minimum time control-affine systems'. In: *SIAM Journal on Control and Optimization* 60.2 (2022), pp. 1143–1162. DOI: 10.1137/20M1366861. URL: https://hal.inria.fr/hal-01718345.

[7] A. Herasimenka, L. Dell'Elce, J.-B. Caillau and J.-B. Pomet. 'Controllability Properties of Solar Sails'. In: *Journal of Guidance, Control, and Dynamics* (2023). URL: https://hal.archives-ouvertes.fr/hal-03331300.

[8] P. Izzo, L. Dell'Elce, P. Gurfil and A. J. Rosengren. 'Nonsingular vectorial reformulation of the short-period corrections in Kozai's oblateness solution'. In: *Celestial Mechanics and Dynamical Astronomy* 134.2 (Apr. 2022), p. 12. DOI: 10.1007/s10569-022-10067-7. URL: https://hal.inria.fr/hal-03937910.

[9] A. G. Yabo, J.-B. Caillau, J.-L. Gouzé, H. de Jong and F. Mairet. 'Dynamical analysis and optimization of a generalized resource allocation model of microbial growth'. In: *SIAM Journal on Applied Dynamical Systems* 21.1 (2022), pp. 137–165. DOI: 10.1137/21M141097X. URL: https://hal.inria.fr/hal-03251044.

International peer-reviewed conferences

[10] L. Baratchart, S. Fueyo and J.-B. Pomet. 'Exponential stability of periodic difference delay systems and 1-D hyperbolic PDEs of conservation laws'. In: TDS 2022 - 17th IFAC Workshop on Time Delay Systems. Vol. 55. IFAC-PapersOnLine 36. Montréal, Canada: Elsevier, 2022, pp. 228–233. DOI: 10.1016/j.ifacol.2022.11.362. URL: https://hal.archives-ouvertes.fr/hal-03792038.

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