

RESEARCH CENTRE

**Inria Paris Centre
at Sorbonne University**

IN PARTNERSHIP WITH:

CNRS, Sorbonne Université

2023

ACTIVITY REPORT

Project-Team

ANGE

**Numerical Analysis, Geophysics and
Environment**

IN COLLABORATION WITH: Laboratoire Jacques-Louis Lions (LJLL)

DOMAIN

Digital Health, Biology and Earth

THEME

**Earth, Environmental and Energy
Sciences**

Inria

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Project-Team ANGE

Creation of the Project-Team: 2014 January 01

Keywords

Computer sciences and digital sciences

- A6. – Modeling, simulation and control
- A6.1. – Methods in mathematical modeling
 - A6.1.1. – Continuous Modeling (PDE, ODE)
 - A6.1.4. – Multiscale modeling
 - A6.1.5. – Multiphysics modeling
 - A6.2. – Scientific computing, Numerical Analysis & Optimization
 - A6.2.1. – Numerical analysis of PDE and ODE
 - A6.2.6. – Optimization
 - A6.3. – Computation-data interaction
 - A6.3.2. – Data assimilation
 - A6.3.4. – Model reduction
 - A6.3.5. – Uncertainty Quantification

Other research topics and application domains

- B3. – Environment and planet
 - B3.3. – Geosciences
 - B3.3.2. – Water: sea & ocean, lake & river
 - B3.3.3. – Nearshore
 - B3.4. – Risks
 - B3.4.1. – Natural risks
 - B3.4.3. – Pollution
- B4. – Energy
 - B4.3. – Renewable energy production
 - B4.3.1. – Biofuels
 - B4.3.2. – Hydro-energy

1 Team members, visitors, external collaborators

Research Scientists

- Julien Salomon [Team leader, INRIA, Senior Researcher, HDR]
- Jacques Sainte-Marie [INRIA, HDR]

Faculty Members

- Nina Aguillon [SORBONNE UNIVERSITE, Associate Professor]
- Emmanuel Audusse [Université Sorbonne Paris Nord, Associate Professor Delegation, from Sep 2023, HDR]
- Bernard Di Martino [UNIV CORSE, Associate Professor Delegation, HDR]
- Julien Guillod [SORBONNE UNIVERSITE, Associate Professor Delegation, until Aug 2023]

Post-Doctoral Fellow

- Louis Thiry [INRIA, Post-Doctoral Fellow, from Oct 2023]

PhD Students

- Nelly Boulos Al Makary [UNIV PARIS XIII, ATER, until Aug 2023]
- Juliette Dubois [INRIA, until Oct 2023]
- Chourouk El Hassanieh [Sorbonne-Université, from Sep 2023]
- Chourouk El Hassanieh [UNIV LIBANAISE, until Aug 2023]
- Allan Gouvenaux [CEA]
- Antoine Leblond [SORBONNE UNIVERSITE, until Aug 2023]
- Léon Migus [SORBONNE UNIVERSITE, until Sep 2023]
- Lucas Perrin [INRIA]
- Djahou Norbert Tognon [INRIA]

Technical Staff

- Sibylle Techene [CNRS, Engineer, until Aug 2023]

Interns and Apprentices

- Dana Zilberberg [INRIA, Intern, from May 2023 until Sep 2023]

Administrative Assistants

- Laurence Bourcier [INRIA]
- Julien Guieu [INRIA]

Visiting Scientist

- Marie-Odile Bristeau [Retired from INRIA, external collaborator of ANGE]

2 Overall objectives

2.1 Presentation

Among all aspects of geosciences, we mainly focus on gravity driven flows arising in many situations such as

- hazardous flows (flooding, rogue waves, landslides...),
- sustainable energies (hydrodynamics-biology coupling, biofuel production, marine energies...),
- risk management and land-use planning (morphodynamic evolutions, early warning systems...)

There exists a strong demand from scientists and engineers in fluid mechanics for models and numerical tools able to simulate not only the water depth and the velocity field but also the distribution and evolution of external quantities such as pollutants or biological species and the interaction between flows and structures (seashores, erosion processes...). The key point of the researches carried out within ANGE is to answer this demand by the development of efficient, robust and validated models and numerical tools.

2.2 Scientific challenges

Due to the variety of applications with a wide range of spatial scales, reduced-size models like the shallow water equations are generally required. From the modelling point of view, the main issue is to describe the behaviour of the flow with a reduced-size model taking into account several physical processes such as non-hydrostatic terms, biological species evolution, topography and structure interactions within the flow. The mathematical analysis of the resulting model do not enter the field of hyperbolic equations anymore and new strategies have to be proposed. Moreover, efficient numerical resolutions of reduced-size models require particular attention due to the different time scales of the processes and in order to recover physical properties such as positivity, conservativity, entropy dissipation and equilibria.

The models can remain subject to uncertainties that originate from incomplete description of the physical processes and from uncertain parameters. Further development of the models may rely on the assimilation of observational data and the uncertainty quantification of the resulting analyses or forecasts.

3 Research program

3.1 Overview

The research activities carried out within the ANGE team strongly couple the development of methodological tools with applications to real-life problems and the transfer of numerical codes. The main purpose is to obtain new models adapted to the physical phenomena at stake, identify the main properties that reflect the physical meaning of the models (uniqueness, conservativity, entropy dissipation, ...), propose effective numerical methods to approximate their solution in complex configurations (multi-dimensional, unstructured meshes, well-balanced, ...) and to assess the results with data in the purpose of potentially correcting the models.

The difficulties arising in gravity driven flow studies are threefold.

- Models and equations encountered in fluid mechanics (typically the free surface Navier-Stokes equations) are complex to analyze and solve.
- The underlying phenomena often take place over large domains with very heterogeneous length scales (size of the domain, mean depth, wave length, ...) and distinct time scales, *e.g.* coastal erosion, propagation of a tsunami, ...
- These problems are multi-physics with strong couplings and nonlinearities.

3.2 Modelling and analysis

Hazardous flows are complex physical phenomena that can hardly be represented by shallow water type systems of partial differential equations (PDEs). In this domain, the research program is devoted to the derivation and analysis of reduced complexity models compared to the Navier-Stokes equations, but relaxing the shallow water assumptions. The main purpose is then to obtain models well-adapted to the physical phenomena at stake.

Even if the resulting models do not strictly belong to the family of hyperbolic systems, they exhibit hyperbolic features: the analysis and discretisation techniques we intend to develop have connections with those used for hyperbolic conservation laws. It is worth noticing that the need for robust and efficient numerical procedures is reinforced by the smallness of dissipative effects in geophysical models which therefore generate singular solutions and instabilities.

On the one hand, the derivation of the Saint-Venant system from the Navier-Stokes equations is based on two approximations (the so-called shallow water assumptions), namely

- the horizontal fluid velocity is well approximated by its mean value along the vertical direction,
- the pressure is hydrostatic or equivalently the vertical acceleration of the fluid can be neglected compared to the gravitational effects.

As a consequence the objective is to get rid of these two assumptions, one after the other, in order to obtain models accurately approximating the incompressible Euler or Navier-Stokes equations.

On the other hand, many applications require the coupling with non-hydrodynamic equations, as in the case of micro-algae production or erosion processes. These new equations comprise non-hyperbolic features and a special analysis is needed.

Multilayer approach As for the first shallow water assumption, *multi-layer* systems were proposed to describe the flow as a superposition of Saint-Venant type systems [28, 31, 32]. Even if this approach has provided interesting results, layers are considered separate and non-miscible fluids, which implies strong limitations. That is why we proposed a slightly different approach [29, 30] based on a Galerkin type decomposition along the vertical axis of all variables and leading, both for the model and its discretisation, to more accurate results.

A kinetic representation of our multilayer model allows to derive robust numerical schemes endowed with crucial properties such as: consistency, conservativity, positivity, preservation of equilibria, ... It is one of the major achievements of the team but it needs to be analyzed and extended in several directions namely:

- The convergence of the multilayer system towards the hydrostatic Euler system as the number of layers goes to infinity is a critical point. It is not fully satisfactory to have only formal estimates of the convergence and sharp estimates would provide an optimal number of layers.
- The introduction of several source terms due for instance to the Coriolis force or extra terms from changes of coordinates seems necessary. Their inclusion should lead to substantial modifications of the numerical scheme.
- Its hyperbolicity has not yet been proven and conversely the possible loss of hyperbolicity cannot be characterised. Similarly, the hyperbolic feature is essential in the propagation and generation of waves.

Non-hydrostatic models The hydrostatic assumption consists in neglecting the vertical acceleration of the fluid. It is considered valid for a large class of geophysical flows but is restrictive in various situations where the dispersive effects (like wave propagation) cannot be neglected. For instance, when a wave reaches the coast, bathymetry variations give a vertical acceleration to the fluid that strongly modifies the wave characteristics and especially its height.

Processing an asymptotic expansion (w.r.t. the aspect ratio for shallow water flows) into the Navier-Stokes equations, we obtain at the leading order the Saint-Venant system. Going one step further leads to a vertically averaged version of the Euler/Navier-Stokes equations involving some non-hydrostatic terms. This model has several advantages:

- it admits an energy balance law (that is not the case for most dispersive models available in the literature),
- it reduces to the Saint-Venant system when the non-hydrostatic pressure term vanishes,
- it consists in a set of conservation laws with source terms,
- it does not contain high order derivatives.

Multi-physics modelling The coupling of hydrodynamic equations with other equations in order to model interactions between complex systems represents an important part of the team research. More precisely, three multi-physics systems are investigated. More details about the industrial impact of these studies are presented in the following section.

- To estimate the risk for infrastructures in coastal zones or close to a river, the resolution of the shallow water equations with moving bathymetry is necessary. The first step consisted in the study of an additional equation largely used in engineering science: The Exner equation. The analysis enabled to exhibit drawbacks of the coupled model such as the lack of energy conservation or the strong variations of the solution from small perturbations. A new formulation is proposed to avoid these drawbacks. The new model consists in a coupling between conservation laws and an elliptic equation, like the Euler/Poisson system, suggesting to use well-known strategies for the analysis and the numerical resolution. In addition, the new formulation is derived from classical complex rheology models and allowed physical phenomena like threshold laws.
- Interaction between flows and floating structures is the challenge at the scale of the shallow water equations. This study requires a better understanding of the energy exchanges between the flow and the structure. The mathematical model of floating structures is very hard to solve numerically due to the non-penetration condition at the interface between the flow and the structure. It leads to infinite potential wave speeds that could not be solved with classical free surface numerical schemes. A relaxation model was derived to overcome this difficulty. It represents the interaction with the floating structure with a free surface model-type.
- If the interactions between hydrodynamics and biology phenomena are known through laboratory experiments, it is more difficult to predict the evolution, especially for the biological quantities, in a real and heterogeneous system. The objective is to model and reproduce the hydrodynamics modifications due to forcing term variations (in time and space). We are typically interested in phenomena such as eutrophication, development of harmful bacteria (cyanobacteria) and upwelling phenomena.

Data assimilation and inverse modelling In environmental applications, the most accurate numerical models remain subject to uncertainties that originate from their parameters and shortcomings in their physical formulations. It is often desirable to quantify the resulting uncertainties in a model forecast. The propagation of the uncertainties may require the generation of ensembles of simulations that ideally sample from the probability density function of the forecast variables. Classical approaches rely on multiple models and on Monte Carlo simulations. The applied perturbations need to be calibrated for the ensemble of simulations to properly sample the uncertainties. Calibrations involve ensemble scores that compare the consistency between the ensemble simulations and the observational data. The computational requirements are so high that designing fast surrogate models or metamodels is often required.

In order to reduce the uncertainties, the fixed or mobile observations of various origins and accuracies can be merged with the simulation results. The uncertainties in the observations and their representativeness also need to be quantified in the process. The assimilation strategy can be formulated in terms of state estimation or parameter estimation (also called inverse modelling). Different algorithms are employed for static and dynamic models, for analyses and forecasts. A challenging question lies in the optimization of the observational network for the assimilation to be the most efficient at a given observational cost.

3.3 Numerical analysis

Non-hydrostatic scheme The main challenge in the study of the non-hydrostatic model is to design a robust and efficient numerical scheme endowed with properties such as: positivity, wet/dry interfaces treatment, consistency. It must be noticed that even if the non-hydrostatic model looks like an extension of the Saint-Venant system, most of the known techniques used in the hydrostatic case are not efficient as we recover strong difficulties encountered in incompressible fluid mechanics due to the extra pressure term. These difficulties are reinforced by the absence of viscous/dissipative terms.

Space decomposition and adaptive scheme In the quest for a better balance between accuracy and efficiency, a strategy consists in the adaptation of models. Indeed, the systems of partial differential equations we consider result from a hierarchy of simplifying assumptions. However, some of these hypotheses may turn out to be irrelevant locally. The adaptation of models thus consists in determining areas where a simplified model (*e.g.* shallow water type) is valid and where it is not. In the latter case, we may go back to the “parent” model (*e.g.* Euler) in the corresponding area. This implies to know how to handle the coupling between the aforementioned models from both theoretical and numerical points of view. In particular, the numerical treatment of transmission conditions is a key point. It requires the estimation of characteristic values (Riemann invariant) which have to be determined according to the regime (torrential or fluvial).

Asymptotic-Preserving scheme for source terms Hydrodynamic models comprise advection and sources terms. The conservation of the balance between source terms, typically viscosity and friction, has a significant impact since the overall flow is generally a perturbation around an equilibrium. The design of numerical schemes able to preserve such balances is a challenge from both theoretical and industrial points of view. The concept of Asymptotic-Preserving (AP) methods is of great interest in order to overcome these issues.

Another difficulty occurs when a term, typically related to the pressure, becomes very large compared to the order of magnitude of the velocity. At this regime, namely the so-called *low Froude* (shallow water) or *low Mach* (Euler) regimes, the difference between the speed of the gravity waves and the physical velocity makes classical numerical schemes inefficient: firstly because of the error of truncation which is inversely proportional to the small parameters, secondly because of the time step governed by the largest speed of the gravity wave. AP methods made a breakthrough in the numerical resolution of asymptotic perturbations of partial-differential equations concerning the first point. The second one can be fixed using partially implicit scheme.

Multi-physics models Coupling problems also arise within the fluid when it contains pollutants, density variations or biological species. For most situations, the interactions are small enough to use a splitting strategy and the classical numerical scheme for each sub-model, whether it be hydrodynamic or non-hydrodynamic.

The sediment transport raises interesting issues from a numerical aspect. This is an example of coupling between the flow and another phenomenon, namely the deformation of the bottom of the basin that can be carried out either by bed load where the sediment has its own velocity or suspended load in which the particles are mostly driven by the flow. This phenomenon involves different time scales and nonlinear retroactions; hence the need for accurate mechanical models and very robust numerical methods. In collaboration with industrial partners (EDF-LNHE), the team already works on the improvement of numerical methods for existing (mostly empirical) models but our aim is also to propose new (quite) simple models that contain important features and satisfy some basic mechanical requirements. The extension of our 3D models to the transport of weighted particles can also be here of great interest.

Optimisation Numerical simulations are a very useful tool for the design of new processes, for instance in renewable energy or water decontamination. The optimisation of the process according to a well-defined objective such as the production of energy or the evaluation of a pollutant concentration is the logical upcoming challenge in order to propose competitive solutions in industrial context. First of all,

the set of parameters that have a significant impact on the result and on which we can act in practice is identified. Then the optimal parameters can be obtained using the numerical codes produced by the team to estimate the performance for a given set of parameters with an additional loop such as gradient descent or Monte Carlo method. The optimisation is used in practice to determine the best profile for turbine pales, the best location for water turbine implantation, in particular for a farm.

4 Application domains

4.1 Overview

Sustainable development and environment preservation have a growing importance and scientists have to address difficult issues such as: management of water resources, renewable energy production, bio/geo-chemistry of oceans, resilience of society w.r.t. hazardous flows, urban pollutions, ...

As mentioned above, the main issue is to propose models of reduced complexity, suitable for scientific computing and endowed with stability properties (continuous and/or discrete). In addition, models and their numerical approximations have to be confronted with experimental data, as analytical solutions are hardly accessible for these problems/models. A. Mangeney (IPGP) and N. Goutal (EDF) may provide useful data.

4.2 Geophysical flows

Reduced models like the shallow water equations are particularly well-adapted to the modelling of geophysical flows since they are characterized by large time or/and space scales. For long time simulations, the preservation of equilibria is essential as global solutions are a perturbation around them. The analysis and the numerical preservation of non-trivial equilibria, more precisely when the velocity does not vanish, are still a challenge. In the fields of oceanography and meteorology, the numerical preservation of the so-called geostrophic state, which is the balance between the gravity field and the Coriolis force, can significantly improve the forecasts. In addition, data assimilation is required to improve the simulations and correct the dissipative effect of the numerical scheme.

The sediment transport modelling is of major interest in terms of applications, in particular to estimate the sustainability of facilities with silt or scour, such as canals and bridges. Dredging or filling-up operations are expensive and generally not efficient in the long term. The objective is to determine a configuration almost stable for the facilities. In addition, it is also important to determine the impact of major events like emptying dam which is aimed at evacuating the sediments in the dam reservoir and requires a large discharge. However, the downstream impact should be measured in terms of turbidity, river morphology and flood.

4.3 Hydrological disasters

It is a violent, sudden and destructive flow. Between 1996 and 2005, nearly 80% of natural disasters in the world have meteorological or hydrological origins. The main interest of their study is to predict the areas in which they may occur most probably and to prevent damages by means of suitable amenities. In France, floods are the most recurring natural disasters and produce the worst damages. For example, it can be a cause or a consequence of a dam break. The large surface they cover and the long period they can last require the use of reduced models like the shallow water equations. In urban areas, the flow can be largely impacted by the debris, in particular cars, and this requires fluid/structure interactions be well understood. Moreover, underground flows, in particular in sewers, can accelerate and amplify the flow. To take them into account, the model and the numerical resolution should be able to treat the transition between free surface and underground flows.

Tsunamis are another hydrological disaster largely studied. Even if the propagation of the wave is globally well described by the shallow water model in oceans, it is no longer the case close to the epicenter and in the coastal zone where the bathymetry leads to vertical accretions and produces substantial dispersive effects. The non-hydrostatic terms have to be considered and an efficient numerical resolution should be induced.

While viscous effects can often be neglected in water flows, they have to be taken into account in situations such as avalanches, debris flows, pyroclastic flows, erosion processes, ... *i.e.* when the fluid rheology becomes more complex. Gravity driven granular flows consist of solid particles commonly mixed with an interstitial lighter fluid (liquid or gas) that may interact with the grains and decrease the intensity of their contacts, thus reducing energy dissipation and favoring propagation. Examples include subaerial or subaqueous rock avalanches (e.g. landslides).

4.4 Biodiversity and culture

Nowadays, simulations of the hydrodynamic regime of a river, a lake or an estuary, are not restricted to the determination of the water depth and the fluid velocity. They have to predict the distribution and evolution of external quantities such as pollutants, biological species or sediment concentration.

The potential of micro-algae as a source of biofuel and as a technological solution for CO₂ fixation is the subject of intense academic and industrial research. Large-scale production of micro-algae has potential for biofuel applications owing to the high productivity that can be attained in high-rate raceway ponds. One of the key challenges in the production of micro-algae is to maximize algae growth with respect to the exogenous energy that must be used (paddlewheel, pumps, ...). There is a large number of parameters that need to be optimized (characteristics of the biological species, raceway shape, stirring provided by the paddlewheel). Consequently our strategy is to develop efficient models and numerical tools to reproduce the flow induced by the paddlewheel and the evolution of the biological species within this flow. Here, mathematical models can greatly help us reduce experimental costs. Owing to the high heterogeneity of raceways due to gradients of temperature, light intensity and nutrient availability through water height, we cannot use depth-averaged models. We adopt instead more accurate multilayer models that have recently been proposed. However, it is clear that many complex physical phenomena have to be added to our model, such as the effect of sunlight on water temperature and density, evaporation and external forcing.

Many problems previously mentioned also arise in larger scale systems like lakes. Hydrodynamics of lakes is mainly governed by geophysical forcing terms: wind, temperature variations, ...

4.5 Sustainable energy

One of the booming lines of business is the field of renewable and decarbonated energies. In particular in the marine realm, several processes have been proposed in order to produce electricity thanks to the recovering of wave, tidal and current energies. We may mention water-turbines, buoys turning variations of the water height into electricity or turbines motioned by currents. Although these processes produce an amount of energy which is less substantial than in thermal or nuclear power plants, they have smaller dimensions and can be set up more easily.

The fluid energy has kinetic and potential parts. The buoys use the potential energy whereas the water-turbines are activated by currents. To become economically relevant, these systems need to be optimized in order to improve their productivity. While for the construction of a harbour, the goal is to minimize swell, in our framework we intend to maximize the wave energy.

This is a complex and original issue which requires a fine model of energy exchanges and efficient numerical tools. In a second step, the optimisation of parameters that can be changed in real-life, such as bottom bathymetry and buoy shape, must be studied. Eventually, physical experiments will be necessary for the validation.

4.6 Urban environment

The urban environment is essentially studied for air and noise pollutions. Air pollution levels and noise pollution levels vary a lot from one street to next. The simulations are therefore carried out at street resolution and take into account the city geometry. The associated numerical models are subject to large uncertainties. Their input parameters, e.g. pollution emissions from road traffic, are also uncertain. Quantifying the simulation uncertainties is challenging because of the high computational costs of the numerical models. An appealing approach in this context is the use of metamodels, from which ensembles of simulations can be generated for uncertainty quantification.

The simulation uncertainties can be reduced by the assimilation of fixed and mobile sensors. High-quality fixed monitoring sensors are deployed in cities, and an increasing number of mobile sensors are added to the observational networks. Even smartphones can be used as noise sensors and dramatically increase the spatial coverage of the observations. The processing and assimilation of the observations raises many questions regarding the quality of the measurements and the design of the network of sensors.

4.7 SmartCity

There is a growing interest for environmental problems at city scale, where a large part of the population is concentrated and where major pollutions can occur. Numerical simulation is well established to study the urban environment, *e.g.* for road traffic modelling. As part of the smartcity movement, an increasing number of sensors collect measurements, at traditional fixed observation stations, but also on mobile devices, like smartphones. They must properly be taken into account given their number but also their potential low quality.

Practical applications include air pollution and noise pollution. These directly relate to road traffic. Data assimilation and uncertainty propagation are key topics in these applications.

5 Social and environmental responsibility

5.1 Footprint of research activities

Only few travels were done last year (including one flight) as a consequence of a will of the team to avoid this type of transportation.

5.2 Impact of research results

Part of ANGE activity is devoted to research on renewable energy. In this way, the team took part to the organization of the EMRSim 22 conference, which devoted to Marine Energy Techniques and Simulation.

6 Highlights of the year

- J. Salomon has been promoted to the DR1 grade.
- 2023 was a year of evaluation for our project-team. This evaluation, coordinated by Inria's Evaluation Committee, is an important step in the team's life cycle. In particular, it enabled us to elaborate collectively on the future of all teams involved in the environmental sciences theme. We have produced a summary of this foresight, which raises more questions than it answers, but which reflects the topic's complexity and the debate that has opened up within the institute itself. This report can be found [here](#).

7 New software, platforms, open data

7.1 New software

7.1.1 Freshkiss

Name: FREe Surface Hydrodynamics using KInetic SchemeS

Keywords: Finite volume methods, Hydrostatic Navier-Stokes equations, Free surface flows

Functional Description: Freshkiss3D is a numerical code solving the 3D hydrostatic and incompressible Navier-Stokes equations with variable density.

Contact: Jacques Sainte Marie

Participants: Fabien Souillé, Emmanuel Audusse, Jacques Sainte Marie, Marie-Odile Bristeau

Partners: UPMC, CEREMA

7.1.2 TSUNAMATHS

Keywords: Modeling, Tsunamis

Functional Description: Tsunamaths is an educational platform aiming at simulating historical tsunamis. Real data and mathematical explanations are provided to enable people to better understand the overall process of tsunamis.

URL: <http://tsunamath.paris.inria.fr/>

Contact: Jacques Sainte Marie

Participants: Emmanuel Audusse, Jacques Sainte Marie, Raouf Hamouda

7.1.3 Verdandi

Keywords: HPC, Model, Software Components, Partial differential equation

Functional Description: Verdandi is a free and open-source (LGPL) library for data assimilation. It includes various such methods for coupling one or several numerical models and observational data. Mainly targeted at large systems arising from the discretization of partial differential equations, the library is devised as generic, which allows for applications in a wide range of problems (biology and medicine, environment, image processing, etc.). Verdandi also includes tools to ease the application of data assimilation, in particular in the management of observations or for a priori uncertainty quantification. Implemented in C++, the library may be used with models implemented in Fortran, C, C++ or Python.

URL: <http://verdandi.gforge.inria.fr/>

Contact: Vivien Mallet

Participants: Dominique Chapelle, Gautier Bureau, Nicolas Claude, Philippe Moireau, Vivien Mallet

7.1.4 Polyphemus

Keyword: Simulation

Functional Description: Polyphemus is a modeling system for air quality. As such, it is designed to yield up-to-date simulations in a reliable framework: data assimilation, ensemble forecast and daily forecasts. Its completeness makes it suitable for use in many applications: photochemistry, aerosols, radionuclides, etc. It is able to handle simulations from local to continental scales, with several physical models. It is divided into three main parts:

libraries that gather data processing tools (SeldonData), physical parameterizations (AtmoData) and post-processing abilities (AtmoPy),

programs for physical pre-processing and chemistry-transport models (Polair3D, Castor, two Gaussian models, a Lagrangian model),

model drivers and observation modules for model coupling, ensemble forecasting and data assimilation.

URL: <http://cerea.enpc.fr/polyphemus/>

Contact: Vivien Mallet

Participants: Sylvain Doré, Vivien Mallet

7.1.5 Urban noise analysis

Keyword: Environment perception

Functional Description: This software processes mobile observations collected by the application Ambiciti (previously known as SoundCity). It can merge simulated noise maps with the mobile observations.

Authors: Raphaël Ventura, Vivien Mallet, Guillaume Chérel

Contact: Vivien Mallet

7.1.6 Freshkiss3D

Keywords: Python, Cython, Navier-Stokes

Functional Description: Tool for the numerical solution of free surface Navier-Stokes equations

Publication: [hal-01393147](#)

Contact: Jacques Sainte Marie

Participants: Cedric Doucet, Apolline El Baz, Jacques Sainte Marie

Partner: UPMC

8 New results

8.1 Numerical methods

8.1.1 Optimal periodic resource allocation in reactive dynamical systems: Application to microalgal production.

Participants: Liu-Di Lu, Julien Salomon.

Coll. with Olivier Bernard In [8], we focus on a periodic resource allocation problem applied to a dynamical system which comes from a biological system. More precisely, we consider a system with N resources and N activities, each activity use the allocated resource to evolve up to a given time $T > 0$ where a control (represented by a given permutation) will be applied on the system to reallocate the resources. The goal is to find the optimal control strategies which optimize the cost or the benefit of the system. This problem can be illustrated by an industrial biological application, namely, the optimization of a mixing strategy to enhance the growth rate in a microalgal raceway system. A mixing device, such as a paddle wheel, is considered to control the rearrangement of the depth of the algae cultures, hence the light perceived at each lap. We prove that if the dynamics of the system is periodic, then the period corresponds to one reallocation whatever the order of the involved permutation matrix is. A nonlinear optimization problem for one reallocation process is then introduced. Since $N!$ permutations need to be tested in the general case, it can be numerically solved only for a limited number of N . To overcome this difficulty, we introduce a second optimization problem which provides a suboptimal solution of the initial problem, but whose solution can be determined explicitly. A sufficient condition to characterize cases where the two problems have the same solution is given. Some numerical experiments are performed to assess the benefit of optimal strategies in various settings.

8.1.2 Evaluation of tsunami inundation in the plain of Martil (north Morocco): Comparison of four inundation estimation methods.

Participants: Apolline El Baz, Jacques Sainte-Marie.

other participants: Elise Basquin, Alain Rabaute, Maud Thomas, Sara Lafuerza, Abdelmounim El M'Rini, Denis Mercier, Elia D'acremont, Marie-Odile Bristeau, Axel Creach

The Alboran Basin may be subject to tsunami hazards. If such an event were to occur, it is expected that the urbanised and densely populated areas of northern Moroccan coastline would be affected. Precise inundation hazard maps are needed for tsunami risk management in this region. In [6], we argue that the diversity of hazard mapping methods ensures the robustness of the scientific knowledge about the exposure of a territory. Hence, the main objective of this study is to analyse the exposure of the plain of Martil (north of Morocco), by using four hazard mapping methods to create inundation maps for two scenarios of tsunamis generated by extreme submarine mass failure (SMF) in the Alboran Sea, of 0.9 km³ and 3.8 km³ respectively. A digital terrain model of the plain was used to explore four methods of inundation mapping. The static method identified 4.32 km² and 19.83 km² of flooded areas for each scenario using water height values as inundation thresholds. The hybrid and the volumetric methods use the volume of water to determine the inundation extent. For the first scenario, 3.51 km² of the plain were inundated using the hybrid method, and 20.11 km² for the second scenario. The results of the volumetric methods are 2.32 km² and 7.82 km² respectively for the first and second scenario. Finally, the fourth method relies on numerical hydrodynamic modelling of tsunami inundation (Freshkiss3d@code). With this method, 4.55 km² of the plain were flooded in the first scenario, and 24.12 km² for the second. The comparison of the results highlights that the most sensitive areas to tsunami inundation are the lowest topographic ones, being the beaches and the wadis floodplains. This result raises questions on the current coastal development and the preparedness of its population, thus calling for more attention to engage on tsunami risk management related questions.

8.1.3 Diagnostic of the Lévy area for geophysical flow models in view of defining high order stochastic discrete-time schemes

Participants: Pierre-Marie Boulevard.

other participant: Etienne Mémin

In [10], we characterize numerically through two criteria the Lévy area related to unresolved fluctuation velocities associated to a stochastic coarse-scale representation of geophysical fluid flow dynamics. We study in particular whether or not the process associated to the random unresolved velocity components exhibits a Lévy area corresponding to a Wiener process, and if the law of this process can reasonably be approached by a centered Dirac measure. This exploration enables us to answer positively to a conjecture made for the constitution of high-order discrete time evolution schemes for stochastic representation defined from stochastic transport.

8.1.4 Stability of implicit neural networks for long-term forecasting in dynamical systems

Participants: Léon Migus, Julien Salomon.

other participant: Patrick Gallinari

Forecasting physical signals in long time range is among the most challenging tasks in Partial Differential Equations (PDEs) research. To circumvent limitations of traditional solvers, many different Deep Learning methods have been proposed. They are all based on auto-regressive methods and exhibit

stability issues. Drawing inspiration from the stability property of implicit numerical schemes, we introduce in [14] a stable auto-regressive implicit neural network. We develop a theory based on the stability definition of schemes to ensure the stability in forecasting of this network. It leads us to introduce hard constraints on its weights and propagate the dynamics in the latent space. Our experimental results validate our stability property, and show improved results at long-term forecasting for two transports PDEs.

8.1.5 INFINITY: Neural Field Modeling for Reynolds-Averaged Navier-Stokes Equations

Participants: Leon Migus.

other participant: Louis Serrano, Yuan Yin, Jocelyn Ahmed Mazari, Patrick Gallinari

For numerical design, the development of efficient and accurate surrogate models is paramount. They allow us to approximate complex physical phenomena, thereby reducing the computational burden of direct numerical simulations. In [15], we propose INFINITY, a deep learning model that utilizes implicit neural representations (INRs) to address this challenge. Our framework encodes geometric information and physical fields into compact representations and learns a mapping between them to infer the physical fields. We use an airfoil design optimization problem as an example task and we evaluate our approach on the challenging AirFRANS dataset, which closely resembles real-world industrial use-cases. The experimental results demonstrate that our framework achieves state-of-the-art performance by accurately inferring physical fields throughout the volume and surface. Additionally we demonstrate its applicability in contexts such as design exploration and shape optimization: our model can correctly predict drag and lift coefficients while adhering to the equations.

8.2 Modelling

8.2.1 Low-Mach type approximation of the Navier-Stokes system with temperature and salinity for free surface flows.

Participants: Léa Boittin, Marie-Odile Bristeau, Anne Mangeney, Jacques Sainte-Marie, Fabien Souillé.

Coll. with François Bouchut In [9], we are interested in free surface flows where density variations coming e.g. from temperature or salinity differences play a significant role in the hydrodynamic regime. In water, acoustic waves travel much faster than gravity and internal waves, hence the study of models arising from compressible fluid mechanics often requires a decoupling between these waves. Starting from the compressible Navier-Stokes system, we derive the so-called Navier-Stokes-Fourier system in an "incompressible" regime using the low-Mach scaling, hence filtering the acoustic waves, neglecting the density dependency on the fluid pressure but keeping its variations in terms of temperature and salinity. A slightly modified low-Mach asymptotics is proposed to obtain a model with thermo-mechanical compatibility. The case when the density depends only on the temperature is studied first. Then the variations of the fluid density with respect to temperature and salinity are considered, and it seems to be the first time that salinity dependency is considered in this low Mach limit. We give a layer-averaged formulation of the obtained models in an hydrostatic context, allowing to derive numerical schemes endowed with strong stability properties that are presented in a companion paper. Several stability properties of the layer-averaged Navier-Stokes-Fourier system are proved.

8.2.2 Acoustic and gravity waves in the ocean: a new derivation of a linear model from the compressible Euler equation

Participants: Juliette Dubois, Sébastien Imperiale, Anne Mangeney, François Bouchut, Jacques Sainte-Marie.

In [11], we construct an accurate linear model describing the propagation of both acoustic and gravity waves in water. This original model is obtained by the linearization of the compressible Euler equations, written in Lagrangian coordinates. The system is studied in the isentropic case, with a free surface, an arbitrary bathymetry, and vertical variations of the background temperature and density. We show that our model is an extension of some models from the literature to the case of a non-barotropic fluid with a variable sound speed. Other models from the literature are recovered from our model through two asymptotic analyses, one for the incompressible regime and one for the acoustic regime. We also propose a method to write the model in Eulerian coordinates. Our model includes many physical properties, such as the existence of internal gravity waves or the variation of the sound speed with depth.

8.2.3 Numerical Investigations of Non-uniqueness for the Navier–Stokes Initial Value Problem in Borderline Spaces

Participants: Julien Guillod.

Other participants: Vladimír Šverák

In [13], we consider the Cauchy problem for the incompressible Navier–Stokes equations in for a one-parameter family of explicit scale-invariant axi-symmetric initial data, which is smooth away from the origin and invariant under the reflection with respect to the xy -plane. Working in the class of axi-symmetric fields, we calculate numerically scale-invariant solutions of the Cauchy problem in terms of their profile functions, which are smooth. The solutions are necessarily unique for small data, but for large data we observe a breaking of the reflection symmetry of the initial data through a pitchfork-type bifurcation. By a variation of previous results by Jia and Šverák (*Invent Math* 196(1):233-265, 2013) it is known rigorously that if the behavior seen here numerically can be proved, optimal non-uniqueness examples for the Cauchy problem can be established, and two different solutions can exist for datum which is divergence-free, smooth away from the origin, compactly supported, and locally -homogeneous near the origin. In particular, assuming our (finite-dimensional) numerics represents faithfully the behavior of the full (infinite-dimensional) system, the problem of uniqueness of the Leray–Hopf solutions (with non-smooth initial data) has a negative answer and, in addition, the perturbative arguments such those by Kato (*Math Z* 187(4):471-480, 1984) and Koch and Tataru (*Adv Math* 157(1):22-35, 2001), or the weak-strong uniqueness results by Leray, Prodi, Serrin, Ladyzhenskaya and others, already give essentially optimal results. There are no singularities involved in the numerics, as we work only with smooth profile functions. It is conceivable that our calculations could be upgraded to a computer-assisted proof, although this would involve a substantial amount of additional work and calculations, including a much more detailed analysis of the asymptotic expansions of the solutions at large distances.

8.3 Assessments of models by means of experimental data and assimilation

8.3.1 On the Marginal Cost of the Duration of a Wildfire

Participants: Frédéric Allaire, Vivien Mallet.

Other participants: Antoine Belgodere, Jean-Baptiste Filippi, Florian Guéniot

Avoiding catastrophic wildfires is a natural rationale for fighting fires in their early stage. Beside this benefit, may a marginal decrease in the duration of smaller wildfires be worthwhile? The present article addresses this topic by estimating the marginal damage of the duration of forest fires. In [7], we perform two sets of wildfire simulations in Corsica, and estimate the damage based on the type of land use in burned areas. Results suggest that the marginal cost of the duration of fires rises by a factor of 4 during the first 400 minutes. The two reasons appear to be the increase in the marginal burned area (a physical mechanism) and the increase in the value of the marginal burned area, due to the ignition points being located in low-value places (a human mechanism). Using a conservative calibration, our results

corroborate the principle of early initial attack already in use in countries with sufficient fire fighting forces, but subject to debate because of its cost.

8.4 Other contribution

8.4.1 A Parareal algorithm for a highly oscillating Vlasov-Poisson system with reduced models for the coarse solving

Participants: Julien Salomon.

Other participants: Laura Grigori, Sever A Hirstoaga

In [12], we introduce a new strategy for solving highly oscillatory two-dimensional Vlasov-Poisson systems by means of a specific version of the parareal algorithm. The novelty consists in using reduced models, obtained from the two-scale convergence theory, for the coarse solving. The reduced models are useful to approximate the original Vlasov-Poisson model at a low computational cost since they are free of high oscillations. Both models are numerically solved in a particle-in-cell framework. We illustrate this strategy with numerical experiments based on long time simulations of a charged beam in a focusing channel and under the influence of a rapidly oscillating external electric field. On the basis of computing times, we provide an analysis of the efficiency of the parareal algorithm in terms of speedup.

9 Bilateral contracts and grants with industry

Participants: Yohan Penel.

Yohan Penel, who left the team in 2022, supervised the PhD thesis of Giuseppe Parasiliti about the Physical, mathematical and numerical modelling of a gas flow for the transportation of liquefied natural gas. This work is the result of a close collaboration with the corporation GTT, which has already collaborated with ANGE in the last years, through the Carnot institute SMILE. The defens

Participants: Jacques Sainte-Marie.

Jacques Sainte-Marie has a contract with Eaux de Paris about Hydraulic modeling, calibration and diagnosis. (2020-2023, with S. Labbé, Laboratoire D'Alembert and LPSM)

10 Partnerships and cooperations

10.1 International initiatives

10.1.1 Associate Teams in the framework of an Inria International Lab or in the framework of an Inria International Program

Title: Intelligence Artificielle, Données et Modèles pour Comprendre les Océans et le Changement Climatique (OCEANIA)

Program:

Duration: 11.2020-10.2024

Local supervisor: Julien Salomon

Partners:

- ANGE, BIOCORE, TAU Inria teams, France
- Universidad de Chile (Center of Mathematical Modeling), Chile
- Pontificia Universidad Católica de Chile, Chile
- Fondation TARA Océan, France,
- GO-SEE CNRS Federation, France
- Université de Nantes (ComBi team), France

Inria contact: Julien Salomon

Summary: There is strong scientific evidence of the negative effects of climate change on the ocean. These changes will have a drastic impact on nearly all life forms in the ocean, as well as additional consequences for food security and ecosystems in coastal communities as well as inland. Despite these impacts, scientific data and infrastructure are still lacking to better understand and quantify the consequences of these disturbances on the marine ecosystem. There is a need not only to collect more data, but also to develop and apply state-of-the-art mechanisms capable of transforming this data into real knowledge, policy, and action. This is where artificial intelligence, machine learning and modeling tools are needed. OceanIA, this ambitious interdisciplinary Inria Challenge, aims to develop new artificial intelligence and mathematical modeling tools to contribute to the understanding of the structure, functioning, and underlying mechanisms and dynamics of the Ocean and its role in regulating and sustaining the biosphere and fighting climate change. OceanIA is also an opportunity to structure Inria's contributions around a global scientific challenge in the convergence of artificial intelligence, biodiversity and climate change.

10.1.2 STIC/MATH/CLIMAT AmSud projects

Title: Mathematical analysis of neural networks for solving partial differential equations and inverse problems

Program: MATH-AmSud

Duration: January 1, 2023 – December 31, 2024

Local supervisor: Julien Salomon

Partners:

- Inria Chile
- Escuela Politecnica Nacional
- Pont. Cath. Univ. of Chile

Inria contact: Julien Salomon

Summary: This program aims at investigating techniques that combine neural network and PDE modeling. In a first work, we consider Holocene dust transport simulation using PINN.

10.2 International research visitors

- May-June 2023: visit of F. Kwok (Université Laval, Canada). Work on time parallelization and preconditioning.
- June 2023: visit of G. Ciaramella (MOX, Polytechnico de Milano). Work about greedy approaches for identification.
- July 2023: visit of H. Carillon (Universidad de Chile (Center of Mathematical Modeling), Chile). Work on topography reconstruction.
- Oct. 2023: visit of G. Barennechea (U. Strathclyde, Ecosse) NS simulation incompressible with free surface.
- Dec. 2023: visit of S. Hörschemeyer (RWTH Aachen, Allemagne).

10.3 National initiatives

Projet Emergence (2021-2023)

Participants: Julien Guillod.

- Project acronym: Emergence
- Project title: Etudes numériques d'équations fluides
- Coordinator: Julien Guillod (SU)
- Funding: 28 000 euros.

Projet Emergence (2023-2025)

Participants: Nathalie Ayi.

- Project acronym: Emergence
- Project title: Numerical studies of STOChastic Kinetic partial differential equations (STOCK)
- Coordinator: Nathalie Ayi (SU)
- Funding: 15 000 euros.

Bourses PEPS JCJC (2023)

Participants: Nathalie Ayi.

- Project title: Étude numérique et théorique d'équations cinétiques en présence de stochasticité
- Coordinator: Nathalie Ayi (SU)
- Funding: 4500 euros.

Tremplin Jeunes Chercheurs et Jeunes Chercheuses de SU (2022-2023)

Participants: Nina Aguillon.

- Project title: Antidiffusive advection scheme in the NEMO ocean global circulation model
- Coordinator: Nina Aguillon (SU)
- Funding: 10 000 euros.

ANR ALLOWAPP (2019-2024)

Participants: Julien Salomon.

- Project acronym: ALLOWAPP
- Project title: Algorithmes pour l'optimisation à grande échelle de problèmes de propagation d'ondes
- Coordinator: Laurence Halpern (Université Paris-Nord)
- Funding: 317 891 euros.

The goal of the ALLOWAPP project is the design of space-time parallel algorithms for large-scale optimization problems associated with wave propagation phenomena. Such problems appear in seismology, geophysics, but also in various applications from data assimilation. The large amount of data and the volume of computations required for the accurate numerical solution of wave propagation problems, within an optimization loop, requires the use of massively parallel computers. Time-parallel methods have experienced a great development in the last ten years, and for parabolic problems an almost perfect efficiency for a large number of processors has been achieved (scalability). It is quite different for wave propagation problems. In this project, we propose to develop robust, efficient and scalable methods for space-time parallelization of these optimization problems.

ANR GeoFun (2020-2024)

Participants: Nina Aguillon.

- Project acronym: GeoFun
- Project title: Ecoulements géophysiques avec des modèles unifiés
- Coordinator: Martin Parisot (INRIA Bordeaux Sud-Ouest)
- Funding: 524 880 euros.

The GeoFun project aims to improve the modeling and simulation of geophysical flows involving at least two different processes. Numerical simulation of watersheds and estimation of water resources is the main application of the project's achievements. In this context, a free surface flow (rivers, lakes) is the upper part of a groundwater flow (water table). Our vision of river transport is often naive, because we think first of rivers, lakes and floods, but in reality, 80 % of the water of the continents is underground. Sometimes, the porous substratum is covered by an impermeable rock stratum, which confines the flow as in pipes, except for some points where springs and resurgences appear.

ANR SingFlows (2019-2023)

Participants: Julien Guillod.

- Project acronym: SingFlows
- Project title: Ecoulements avec singularités : couches limites, filaments de vortex, interaction vague-structure
- Coordinator: David Gerard-Varet (Institut de mathématiques de Jussieu - Paris Rive Gauche)

- Funding: 263 628 euros.

The objective of SingFlows is to develop mathematical and numerical tools for the analysis of three problems in fluid dynamics: the behaviour of anisotropic flows (boundary layers, shallow water flows), the dynamics of vortical structures, and the evolution of fixed or floating structures in water waves. Our will to unify these different problems is natural, because they share many mathematical features. The underlying keypoint is that they are described by singular solutions of Euler or Navier-Stokes equations. The word singular refers here: - either to a lack of smoothness: it applies for instance to vortex filaments, which are Dirac masses along curves, or to the contact line between water and the floating structure, - or to a singular dependence of the solution with respect to a parameter, typically the Reynolds number (like in boundary layers). The connection between the two points of view is usually made by viscous regularization of the non-smooth structure, or conversely by taking the vanishing limit of the parameter. More generally, the three problems considered in SingFlows involve flows with very small scales. A relevant description then requires the derivation of reduced models.

ANR Saphir (2022-2024)

Participants: Jacques Sainte-Marie, Bernard Di Martino.

- Project acronym: Saphir
- Project title: Sensor Augmented weather Prediction at high Resolution
- Coordinator: J-F. Muzy (Université de Toulouse Paul Sabatier)
- Funding: 296 000 euros.

ANR DEEPNUM (2022-2026)

Participants: Julien Salomon, Jacques Sainte-Marie.

- Project acronym: DEEPNUM
- Project title: Algorithmes pour l'optimisation à grande échelle de problèmes de propagation d'ondes
- Coordinator: Julien Salomon
- Funding: 493 799,20 euros.

The project aims at developing the interplay between Deep Neural Networks (DNNs) and Differential Equations (DEs), with the goal of modeling complex dynamical systems arising from the observation of natural phenomena. Two application domains are targeted, environment and healthcare. We address three fundamental questions: how to adapt and apply numerical analysis theory to DNNs for analyzing them, providing theoretical guaranties and improving their robustness, how to combine simulation and data based models into hybrid systems, how could DNNs help solving DEs and complement numerical solvers. In addition, we evaluate our methods on simulation and real world data in the environment and health domains. DeepNuM gathers partners with complementary skills: DEs and Environment (INRIA-ANGE), Machine Learning and DNNs (Sorbonne – MLIA), DEs and Biophysics (INRIA-EPIONE).

ANR MEGA (2023-2028)

Participants: Bernard Di Martino, Jacques Sainte-Marie, Nina Aguillon.

- Project acronym: MEGA
- Project title: Giant submarine landslides in gas hydrate provinces: a comparison of the Nile and Amazon deep-sea fans
- Coordinator: Sébastien Mingeon
- Funding: 533,348 euros.

Giant submarine landslides (10-2000 km³) are found in the thick Quaternary sediment succession of passive continental margins. Their ages coincide with periods of sea-level fall and rise, but it is unclear how such vast failures can be triggered on low seafloor slopes (<2°) in the absence of a triggering factor such as seismicity. Key hypotheses involve excess pore pressures linked to reductions in gas-hydrate stability, driven by changes either in climate or in subsurface fluid flow. The MEGA project wants to explore such hypotheses through the first modelling of linked changes in gas hydrate and slope stability in response to ocean pressure and temperature changes, using an innovative comparison of the Nile and Amazon deep-sea fans that experience different forms of climate forcing over glacial-interglacial timescales. As such megaslides have never triggered in historical times, MEGA will provide input for the first modelling of their tsunamogenic consequences on coastal zones.

GdR MathGeoPhy (2022–2027)

Participants: Emmanuel Audusse, Bernard di Martino, Nicole Goutal, Martin Parisot, Jacques Sainte-Marie.

The MathGeoPhy interdisciplinary research group was created in January 2022, for five years. It is funded by the French National Center for Scientific Research (CNRS), with the mission of animating the French scientific community around the theme of mathematics in interaction with the geophysics of fluid and solid envelopes. The members of the GdR are interested in mathematical modeling, scientific computing and the development of new numerical methods applied in particular to :

- offshore and coastal ocean dynamics, gravity waves, coastal erosion problems
- micro-macro approaches, granular and complex flows
- fluvial and torrential hydrodynamics, extreme events and environmental risks, landslides, avalanches, volcanic eruptions, glaciology, etc.

GdR EOL-EMR (2021–2026)

Participants: Julien Salomon, Jacques Sainte-Marie.

The objectives of this project are the following:

- To promote the dissemination of existing knowledge and expertise within and across disciplines.
- The GDR EMR is a forum for the exchange of expertise and know-how within and across disciplines.
- To promote the implementation of collaborations, between partners of the GDR and with the industrial fabric.

The GDR is an entry and orientation point. It provides a forum for the exchange of information concerning industrial needs and the skills of the academic community; and enables the bringing together of players. Valuing the national scientific community The GDR EMR gives visibility to the community, in particular through the development of a mapping of the actors and themes available on the web platform

11 Dissemination

11.1 Promoting scientific activities

11.1.1 Scientific events: organisation

Member of the organizing committees

- JG took part of the organization of the conference [FoCM 2023](#) at Paris,
- JS co-organized the conference "[Workshop on Assimilation, Control and Computational Speedup](#)", Villetaneuse (France), 6-7 juin 2023.
- JS co-organizes the bi-monthly seminar "Rencontres INRIA-JLL en analyse numérique et calcul scientifique"
- JG co-organizes the monthly seminar "Analyse non-linéaire et EDP"
- JG co-organizes the monthly seminar "[InfoMath](#)"
- NAg co-organizes the yearly "Journée interne du Laboratoire J-L. Lions"
- NAg co-organizes [Hyp 2023](#), Bordeaux

Reviewer The team members served as referees for the following journals:

- JS: SIAM SISC, IEEE TAC,
- JG: Journal of Functional Analysis, Journal of Mathematical Fluid Mechanics, Journal of Differential Equations, Inventiones
- BDM: M2AN
- NAyi: Journal of Differential Equations, Kinetic and Related Models
- EA: Computer & Fluids, Calcolo, CMS
- NAgui: Journal of Advances in Modelling Earth Systems
- LM: NeurIPS 2023, ICLR 2024
- EG: M2AN, Siam SISC
- JSM: Nonlinear dynamics, M2AN, Acta Applicandae Mathematicae

11.1.2 Journal

JS is editor in chief of MATAPLI (national journal of the applied maths community). EA is responsible for its section "Du côté des écoles d'ingénieurs".

11.1.3 Invited talks

- JG: Prague summer school on Stochastic in Fluids Institute of Mathematics of the Academy of Sciences of Czech Republic 21/08-25/08
- NAgui: séminaire RWTH, Aachen, 18/07
- NAgui: Séminaire de calcul scientifique et modélisation IMB, Bordeaux 23/03
- EG: NumHyp23: Numerical Methods for Hyperbolic Problems Bordeaux 26-30/06/2023
- NAyi: Round mean field 2 Rome, Italie 13-14/06/2023
- NAyi: Classical and Quantum Mechanical Models of Many-Particle Systems Oberwolfach, Allemagne 04-08/09/2023
- NAyi: Séminaire du Laboratoire de Mathématiques de Bretagne Atlantique Brest 14/4
- NAyi: Groupe de travail Analyse, Modélisation & Simulation Paris Cité 21/05
- NAyi: Séminaire Denis Poisson Orléans 25/05
- NAyi: Séminaire Analyse Numérique et EDP Paris Saclay 15/06
- NAyi: Séminaire Modélisation, Analyse, Calcul Toulouse 17/10/23
- NAyi: Journées internes du laboratoire Jacques Louis Lions, 22/11/23
- NAyi: Séminaire de l'équipe EDP Analyse Numérique Nice 14/12/23

11.1.4 Leadership within the scientific community

- EA is adjoint director of GDR MathGeoPhy (2022-)
- JS is member of the board of AMIES (2018-2023)

11.1.5 Research administration

- NAyi was in the selection committee of an assistant professor (Maître de Conférences) in Orsay and ENS Lyon
- JS is member of Scientific Job commission (CES, 2018-)
- JS is an elected member of social administration council of INRIA (CSA de l'INRIA, 2023-)
- JS is an elected member of the evaluation commission (2023-)
- JSM is scientific director adjoint of INRIA (2019-)
- JG is member of administration council of IHP (2021-2023)
- JSM is belongs to External advisory board - ERC Synergy 2020-2024
- JSM Co-advisor of PEPR 'agroécologie et numérique' 2022-2028
- JSM is responsible for the 'Numérique et environnement' program (2022-)
- NAyi is a member of the LJLL laboratoire council (2020 -)
- EA is a member of CR and CAC at USPN (2020-2024)
- NAgui is member of Scientific Job commission (CES, 2023-)

11.2 Faculty administration

- JG belongs to Bonus allocation committee at (Comission d'attribution des primes à)
- NAyi Substitute on the Bonus Committee at (Comission d'attribution des primes à)
- JG belongd to Equality and anti-discrimination referent for UFR 929, SU (Réfèrent égalité et lutte contre les discriminations pour l'UFR 929, SU)
- NAg is member of the integration cycle department council (conseil de département du cycle d'intégration)
- NAg is member of CAPSULE (center for pedagogical support and experimentation - centre d'accompagnement à la pédagogie et support à l'expérimentation)
- NAyi belongs to University national council (CNU section 26)
- JG is member of the faculty council (UFR 929), SU
- NAgui is member of the faculty council (UFR 929), SU
- NAyi is member of the scientific committee of the faculty council (UFR 929), SU

11.3 Teaching - Supervision - Juries

11.3.1 Teaching

EG is the president of commission française pour l'enseignement des mathématiques (CFEM)

Teaching activities of ANGE are summarized in the following.

- - Participants:** Julien Salomon.
 - Méthodes numériques pour des modèles incluant des EDP, 45,M2, Université d'Abomey-Calavi, Bénin CM
- - Participants:** Jacques Sainte-Marie.
 - Modélisation des écoulements gravitaires 40H, M1, Univ. Paris-Diderot et IPGP
 - Méthodes numériques en géosciences 50H, M2H, Univ. Paris-Diderot et IPGP
 - Hyperbolic models for complex flows 25H, M2, SU
- - Participants:** Nathalie Ayi.
 - Approximation des EDPs 36H M1 CM
 - EDO 18H L2 TD
 - Algèbre linéaire 10H L2 Colles
 - " Responsabilité du parcours Bi-DI Info-Math L2-L3", 12H, L2-L3, SU, responsabilité

- **Participants:** Juliette Dubois.
 - Structures mathématiques, 22H, L3, Polytech Sorbonne TD
- **Participants:** Nina Aguillon.
 - Directrice des études de L2 mathématiques, 64H, L2, , responsabilité
 - Topologie et calcul différentiel 1, 36H, L2, , TD
 - Modèles hyperboliques d'écoulements complexes dans le domaine de l'environnement, 10H, M2, SU, CM
 - Formation des nouveaux moniteurs en mathématiques 8H, D, SU, responsabilité
 - Atelier de Recherche Encadrée 30H, L1, SU, TD
- **Participants:** Bernard Di Martino.
 - Calcul différentiel 54H, L3, Université de Corse, CM,TD
 - Pratiques d'Algèbre 18H, L1, Université de Corse, TP
 - Pratique d'Analyse 18H, L1, Université de Corse, TP
 - Pratique d'Algèbre 18H, L2, Université de Corse, TP
 - Pratique d'Analyse 18H, L2, Université de Corse, TP
- **Participants:** Emmanuel Audusse.
 - Optimisation, 30H, ING2, USPN, TD-TP
 - Calcul scientifique, 30H, L2, USPN, CM-TD-TP
 - TP analyse numérique avec Matlab, 30H, L3 (ING1), USPN
- **Participants:** Julien Guillod.
 - Programmation Python pour les mathématiques, 45H, L2, SU, TP/TD
 - Fondements des méthodes numériques, 48H, M1, SU, TD
- **Participants:** Nelly Boulos.
 - Mathématiques pour les études scientifiques, 36H, L1, SU, TP

- - Participants:** Lucas Perrin.
 - Méthodes numériques : algèbre matricielle et fonctions d'une variable réelle, 40H, L2, Univ. Paris-Dauphine, TP+TD
 - Analyse de Fourier, 28H, L3, Polytech Sorbonne, TD
- - Participants:** Antoine Leblond.
 - Analyse numérique, 72H, L3, TP, TD+TP
- - Participants:** Léon Migus.
 - Informatique 2 (fortran), 40H, M1, Polytech Sorbonne, TP
- - Participants:** Chourouk El Hassanieh.
 - Mathématiques pour les études scientifiques 2, 38H, L1, SU, TD

11.3.2 Supervision

The format is here supervisor, grade, student, institution, period, title.

- JS, PhD, Lucas Perrin, SU, 2021-2024, Parallélisation en temps et assimilation de données
- JS (with Patrick Gallinari), PhD, Léon Migus, SU, 2020-2023, Deep Neural Networks and Differential Equations
- JS, PhD, Norbert Tognon, SU, 2022-2025, Techniques d'accélération pour le contrôle optimal
- JS (with Patrick Armand), PhD, Allan Gouvenaux, CEA, 2023-2026, Méta-modélisation pour la simulation rapide de phénomènes de transport
- BDM, JSM, JG, EG (with Samer Israwi, Libanese university), PhD, Chourouk El Hassanieh, Inria 2019-2023, Analysis and numerical approximation of some mathematical models free-surface flows
- JSM (with Sébastien Impériale), PhD, Juliette Dubois, Inria 2020-2023, Modélisation et approximation numérique de la propagation des ondes acoustique et des ondes de gravité dans les fluides à surface libre
- YP (with Nora Aissiouene and Pierre-Yves Lagrée), PhD, Giuseppe Parasiliti, SU, 2020-2023, Physical, mathematical and numerical modelling of a gas flow for the transportation of liquified natural gas
- JG, ALD, PhD, Antoine Leblond, SU, 09.2020-11.2023, Well-posedness and long-time behaviour of the Stokes-transport equation
- JSM (with Etienne Mémin), Post-doc, Pierre-Marie Boulevard, Inria 2021-2023, Location uncertainties in free surface flows models - Numerical analysis and implementation in Freshkiss3d
- EA, F. Benkhaldoun, PhD, Laila Baroukh, USPN, 2021-2024, Simulation numérique pour des écoulements partiellement congestionnés avec rhéologie complexe
- BDM, Master, Dana Zilberberg, SU, Ecole des ponts, 2023, On the Characterization of Hyberbolicity of Free-Surface Euler Equations

11.3.3 Juries

- JS, 23.11.2023, PhD, Referee, Rishabh BHATT, LJK, Univ. Grenoble Parallel In Time Algorithms for Data Assimilation
- JS, 01.12.2023, PhD, Jury, Advisor, Leon Migus, LIP6-ISIR-LJLL (SU), Réseaux de neurones profonds et équations aux dérivées partielles
- JS, 12.2023, PhD, Jury, Yohan Poirier, ECN, Contribution à l'accélération d'un code de calcul des interactions vagues/structures basée sur la théorie potentielle instationnaire des écoulements à surface libre
- JS, 12.2023, PhD, Jury, Vuong, LAGA, Sorbonne-Paris-Nord, Contrôle optimal et décomposition de domaines
- JG, 12.2023, PhD, Jury, Chourouk El Hassanieh, SU, Analysis and numerical approximation of some mathematical models free-surface flows
- EG, 12.2023, PhD, Jury, Chourouk El Hassanieh, SU, Analysis and numerical approximation of some mathematical models free-surface flows
- BDM, 12.2023, PhD, Jury, Chourouk El Hassanieh, SU, Analysis and numerical approximation of some mathematical models free-surface flows
- JSM, 12.2023, PhD, Jury, Advisor, Chourouk El Hassanieh, SU, Analysis and numerical approximation of some mathematical models free-surface flows
- JSM, 12.2023, PhD, Jury, Advisor, Juliette Dubois, SU, Acoustic-gravity waves in free surface flows: modeling: analysis and simulation towards tsunami early warning systems
- NAyi, 09.2023, PhD, Jury, Corentin Le Bihan, ENS Lyon, Effets de bord et comportement en temps long en théorie cinétique collisionnelle

11.4 Popularization

- JSM organized [Journée de la FSMP : "A l'eau les maths"](#)
- JS co-organized a round-table discussion at the forum emploi-maths ("Pollution, des mesures aux solutions")

11.4.1 Interventions

- Julien Salomon
 - décembre 2023 : Accueil de Collégiens
 - janvier 2023 : Intervention "Chiche" au lycée Charlemagne
- Nina Aguillon
 - 2018- : co-organisation de Mathematic Park (public licence et prépa)
 - janvier 2023 : présentation métier dans un lycée
 - (with N. Ayi) mars 2023 : organisation d'une journée filles, maths et info, une équation lumineuse à SU: accueil de 100 lycéennes, programme 2 conférences scientifiques, rencontre avec des étudiantes, visite du campus, speed meeting métier (20 intervenantes), théâtre forum
 - mai 2023 : exposé lors de la journée "maths en mouvement" de la FSMP
- Jacques Sainte-Marie
 - octobre 2022 : Ambassadeur pour la fête de la science 2022

- 2022 : MOOC 'impact environnemental du numérique'
- Nathalie Ayi
 - Sept 2022 - ... : Création du podcast Tête-à-tête Chercheuse(s) et réseaux sociaux associés
 - Mai 2022 - ... : Participation à l'exposition "Mathématiques, Informatique ... avec elles" par l'association Femmes et Mathématiques
 - 2023- ... : ambassadrice de la Maison Poincaré
 - mai 2023 : exposé grand public lors de l'évènement "LIHES célèbre les mathématiciennes"
 - mai 2023 : exposé grand public lors du salon de la culture et jeux mathématiques
 - octobre 2023 : exposé grand public lors de la cérémonie de remise des prix des Olympiades de Mathématiques
 - novembre 2023 : exposé en ligne aux étudiantes du lycée Nelson Mandela, Pibrac
 - novembre 2023 : exposé dans le lycée dans le cadre des "Villon conférences", Beaugency
 - novembre 2023 : exposé et atelier mathématiques dans le collège Henri Becquerel, Avoine
 - décembre 2023 : exposé dans le lycée Saint-Barthélémy, Nice
- Juliette Dubois
 - décembre 2022 : Intervention devant des lycéens et lycéennes, Lycée Racine Paris
- Julien Guillod
 - mai 2023 : stage de formation pour les enseignantes et enseignants du secondaire

12 Scientific production

12.1 Major publications

- [1] E. Audusse, M.-O. Bristeau, M. Pelanti and J. Sainte-Marie. 'Approximation of the hydrostatic Navier-Stokes system for density stratified flows by a multilayer model. Kinetic interpretation and numerical validation'. In: *J. Comput. Phys.* 230 (2011), pp. 3453–3478. DOI: [10.1016/j.jcp.2011.01.042](https://doi.org/10.1016/j.jcp.2011.01.042). URL: <http://dx.doi.org/10.1016/j.jcp.2011.01.042>.
- [2] E. Audusse, M.-O. Bristeau, B. Perthame and J. Sainte-Marie. 'A multilayer Saint-Venant system with mass exchanges for Shallow Water flows. Derivation and numerical validation'. In: *ESAIM Math. Model. Numer. Anal.* 45 (2011), pp. 169–200. DOI: [10.1051/m2an/2010036](https://doi.org/10.1051/m2an/2010036). URL: <http://dx.doi.org/10.1051/m2an/2010036>.
- [3] M.-O. Bristeau, A. Mangeney, J. Sainte-Marie and N. Seguin. 'An energy-consistent depth-averaged Euler system: derivation and properties'. In: *Discrete and Continuous Dynamical Systems - Series B* 20.4 (2015), p. 28.
- [4] J. Ledoux, S. Riffo and J. Salomon. 'Analysis of the Blade Element Momentum Theory'. working paper or preprint. Apr. 2020. URL: <https://hal.archives-ouvertes.fr/hal-02550763>.
- [5] J. Sainte-Marie. 'Vertically averaged models for the free surface Euler system. Derivation and kinetic interpretation'. In: *Math. Models Methods Appl. Sci. (M3AS)* 21.3 (2011), pp. 459–490. DOI: [10.1142/S0218202511005118](https://doi.org/10.1142/S0218202511005118). URL: <http://dx.doi.org/10.1142/S0218202511005118>.

12.2 Publications of the year

International journals

- [6] E. Basquin, A. El Baz, J. Sainte-Marie, A. Rabaute, M. Thomas, S. Lafuerza, A. El M'Rini, D. Mercier, E. D'acremont, M.-O. Bristeau and A. Creach. 'Evaluation of tsunami inundation in the plain of Martil (north Morocco): Comparison of four inundation estimation methods'. In: *Natural Hazards Research* (June 2023). DOI: [10.1016/j.nhres.2023.06.002](https://doi.org/10.1016/j.nhres.2023.06.002). URL: <https://hal.science/hal-04134599>.

- [7] A. Belgodere, F. Allaire, J.-B. Filippi, V. Mallet and F. Guéniot. ‘On the Marginal Cost of the Duration of a Wildfire’. In: *Journal of Forest Economics* 38.3 (2023), pp. 265–292. DOI: [10.1561/112.00000565](https://doi.org/10.1561/112.00000565). URL: <https://hal.science/hal-04256909>.
- [8] O. Bernard, L.-d. Lu and J. Salomon. ‘Optimal periodic resource allocation in reactive dynamical systems: Application to microalgal production’. In: *International Journal of Robust and Nonlinear Control* 33.9 (June 2023), pp. 4989–5010. DOI: [10.1002/rnc.6171](https://doi.org/10.1002/rnc.6171). URL: <https://hal.science/hal-03170481>.
- [9] L. Boittin, M.-O. Bristeau, F. Bouchut, A. Mangeney, J. Sainte-Marie and F. Souillé. ‘Low-Mach type approximation of the Navier-Stokes system with temperature and salinity for free surface flows’. In: *Communications in Mathematical Sciences* 21.1 (Jan. 2023), pp. 151–172. DOI: [10.4310/CMS.2023.v21.n1.a7](https://doi.org/10.4310/CMS.2023.v21.n1.a7). URL: <https://inria.hal.science/hal-02510711>.
- [10] P.-M. Boulevard and E. Mémin. ‘Diagnostic of the Lévy area for geophysical flow models in view of defining high order stochastic discrete-time schemes’. In: *Foundations of Data Science* (2023), pp. 1–25. DOI: [10.3934/fods.2023011](https://doi.org/10.3934/fods.2023011). URL: <https://inria.hal.science/hal-04241686>.
- [11] J. Dubois, S. Imperiale, A. Mangeney, F. Bouchut and J. Sainte-Marie. ‘Acoustic and gravity waves in the ocean: a new derivation of a linear model from the compressible Euler equation’. In: *Journal of Fluid Mechanics* 970 (Sept. 2023), A28. DOI: [10.1017/jfm.2023.595](https://doi.org/10.1017/jfm.2023.595). URL: <https://hal.science/hal-03880423>.
- [12] L. Grigori, S. A. Hirstoaga and J. Salomon. ‘A Parareal algorithm for a highly oscillating Vlasov-Poisson system with reduced models for the coarse solving’. In: *Computers & Mathematics with Applications* 130 (2023), pp. 137–148. URL: <https://hal.science/hal-03903280>.
- [13] J. Guillod and V. Šverák. ‘Numerical Investigations of Non-uniqueness for the Navier-Stokes Initial Value Problem in Borderline Spaces’. In: *Journal of Mathematical Fluid Mechanics* 25.3 (Aug. 2023), p. 46. DOI: [10.1007/s00021-023-00789-5](https://doi.org/10.1007/s00021-023-00789-5). URL: <https://hal.science/hal-04112126>.

Conferences without proceedings

- [14] L. Migus, J. Salomon and P. Gallinari. ‘Stability of implicit neural networks for long-term forecasting in dynamical systems’. In: ICLR 2023 Workshop on Physics for Machine Learning. Kigali, Rwanda, 1st May 2023. URL: <https://hal.science/hal-04132587>.
- [15] L. Serrano, L. Migus, Y. Yin, J. A. Mazari and P. Gallinari. ‘INFINITY: Neural Field Modeling for Reynolds-Averaged Navier-Stokes Equations’. In: Workshop on Synergy of Scientific and Machine Learning Modeling (ICML 2023). Honolulu, HI, United States, 28th July 2023. URL: <https://hal.science/hal-04171439>.

Doctoral dissertations and habilitation theses

- [16] N. Boulos Al Makary. ‘Stationary solutions and well-balanced schemes for the shallow water model with two velocities and topography’. Sorbonne Paris Nord; Inria Paris, Équipe ANGE, 2022. URL: <https://hal.science/tel-03990996>.
- [17] C. El Hassanieh. ‘Analysis and numerical approximation of some mathematical models of free-surface flows’. Sorbonne Université; Université Libanaise; Inria Paris, Équipe ANGE, 7th Dec. 2023. URL: <https://theses.hal.science/tel-04356497>.
- [18] A. Leblond. ‘Well-posedness and long-time behaviour of the Stokes-transport equation’. Sorbonne Université, 10th Nov. 2023. URL: <https://theses.hal.science/tel-04356556>.
- [19] L. Migus. ‘Deep neural networks and partial differential equations’. Sorbonne Université, 1st Dec. 2023. URL: <https://theses.hal.science/tel-04336969>.

Reports & preprints

- [20] E. Audusse, V. Dubos, N. Gaveau and Y. Penel. *Energy stable and linearly well-balanced numerical schemes for the non-linear Shallow Water equations with Coriolis force*. 29th July 2023. URL: <https://hal.science/hal-03509990>.

- [21] N. Ayi and N. Pouradier Duteil. *Graph Limit for Interacting Particle Systems on Weighted Random Graphs*. 18th July 2023. URL: <https://hal.science/hal-04164951>.
- [22] A.-L. Dalibard, J. Guilloid and A. Leblond. *Long-time behavior of the Stokes-transport system in a channel*. 1st June 2023. URL: <https://hal.science/hal-04114516>.
- [23] B. Di Martino, C. El Hassanieh, E. Godlewski, J. Guilloid and J. Sainte-Marie. *Hyperbolicity of a semi-Lagrangian formulation of the hydrostatic free-surface Euler system*. 29th Aug. 2023. URL: <https://hal.science/hal-04190892>.
- [24] C. El Hassanieh, S. Israwi, H. Kalisch, D. Mitsotakis and A. Senthilkumar. *Mechanical balance laws for two-dimensional Boussinesq systems*. 22nd Jan. 2024. URL: <https://hal.science/hal-04307057>.
- [25] C. El Hassanieh, M. Rigal and J. Sainte-Marie. *Implicit kinetic schemes for the Saint-Venant system*. 28th Mar. 2023. URL: <https://hal.science/hal-04048832>.
- [26] E. Franck, I. Lannabi, Y. Nasser, L. Navoret, G. Parasiliti and G. Steimer. *Hyperbolic reduced model for Vlasov-Poisson equation with Fokker-Planck collision*. 4th Apr. 2023. URL: <https://hal.science/hal-04099697>.
- [27] F. Hecht, S.-M. Kaber, L. Perrin, A. Plagne and J. Salomon. *Parallel approximation of the exponential of Hermitian matrices*. 28th June 2023. URL: <https://hal.science/hal-03948509>.

12.3 Cited publications

- [28] E. Audusse. 'A multilayer Saint-Venant model : Derivation and numerical validation'. In: *Discrete Contin. Dyn. Syst. Ser. B* 5.2 (2005), pp. 189–214.
- [29] E. Audusse, M.-O. Bristeau, M. Pelanti and J. Sainte-Marie. 'Approximation of the hydrostatic Navier-Stokes system for density stratified flows by a multilayer model. Kinetic interpretation and numerical validation'. In: *J. Comput. Phys.* 230 (2011), pp. 3453–3478. DOI: [10.1016/j.jcp.2011.01.042](https://doi.org/10.1016/j.jcp.2011.01.042). URL: <http://dx.doi.org/10.1016/j.jcp.2011.01.042>.
- [30] E. Audusse, M.-O. Bristeau, B. Perthame and J. Sainte-Marie. 'A multilayer Saint-Venant system with mass exchanges for Shallow Water flows. Derivation and numerical validation'. In: *ESAIM Math. Model. Numer. Anal.* 45 (2011), pp. 169–200. DOI: [10.1051/m2an/2010036](https://doi.org/10.1051/m2an/2010036). URL: <http://dx.doi.org/10.1051/m2an/2010036>.
- [31] F. Bouchut and V. Zeitlin. 'A robust well-balanced scheme for multi-layer shallow water equations'. In: *Discrete Contin. Dyn. Syst. Ser. B* 13 (2010), pp. 739–758.
- [32] M. Castro, J. García-Rodríguez, J. González-Vida, J. Macías, C. Parés and M. Vázquez-Cendón. 'Numerical simulation of two-layer shallow water flows through channels with irregular geometry'. In: *J. Comput. Phys.* 195.1 (2004), pp. 202–235.