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ACTIVITY REPORT

Project-Team
CAGE

Control and Geometry

IN COLLABORATION WITH: Laboratoire Jacques-Louis Lions (LJLL)

DOMAIN

Applied Mathematics, Computation and Simulation

THEME

Optimization and control of dynamic systems

Contents

Project-Team CAGE

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Keywords

Computer sciences and digital sciences

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Other research topics and application domains

B2. – Health

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B5.11. – Quantum systems

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2 Overall objectives

CAGE's activities take place in the field of mathematical control theory, with applications in several directions: control of quantum mechanical systems, stability and stabilization, in particular in presence of uncertain dynamics, optimal control, and geometric models for vision. Although control theory is nowadays a mature discipline, it is still the subject of intensive research because of its crucial role in a vast array of applications. Our focus is on the analytical and geometrical aspects of control applications.

At the core of the scientific activity of the team is the **geometric control** approach, that is, a distinctive viewpoint issued in particular from (elementary) differential geometry, to tackle questions of controllability, motion planning, stability, and optimal control. The emphasis of such a geometric approach is in intrinsic properties, and it is particularly well adapted to study nonlinear and nonholonomic phenomena [77, 53]. The geometric control approach has historically been associated with the development of finite-dimensional control theory. However, its impact in the study of distributed parameter control systems and, in particular, systems of **controlled partial differential equations** has been growing in the last decades, complementing analytical and numerical approaches by providing dynamical, qualitative, and intrinsic insight [69]. CAGE has the ambition to be at the core of this development.

One of the features of the geometric control approach is its capability of exploiting **symmetries and intrinsic structures** of control systems. Symmetries and intrinsic structures (e.g., Lagrangian or Hamiltonian structures) can be used to characterize minimizing trajectories, prove regularity properties, and describe invariants. The geometric theory of **quantum control**, in particular, exploits the rich geometric structure encoded in the Schrödinger equation to design adapted control schemes and to characterize their qualitative properties.

3 Research program

3.1 Research domain

Our contributions are in the area of **mathematical control theory**, which is to say that we are interested in the analytical and geometrical aspects of control applications. In this approach, a control system is modeled by a system of equations (of many possible types: ordinary differential equations, partial differential equations, stochastic differential equations, difference equations,...), possibly not explicitly known in all its components, which are studied in order to establish qualitative and quantitative properties concerning the actuation of the system through the control.

Motion planning is, in this respect, a cornerstone property: it denotes the design and validation of algorithms for identifying a control law steering the system from a given initial state to (or close to) a target one. Initial and target positions can be replaced by sets of admissible initial and final states as, for instance, in the motion planning task towards a desired periodic solution. Many specifications can be added to the pure motion planning task, such as robustness to external or endogenous disturbances, obstacle avoidance or penalization criteria. A more abstract notion is that of **controllability**, which denotes the property of a system for which any two states can be connected by a trajectory corresponding to an admissible control law. In mathematical terms, this translates into the surjectivity of the so-called **end-point map**, which associates with a control and an initial state the final point of the corresponding trajectory. The analytical and topological properties of endpoint maps are therefore crucial in analyzing the properties of control systems.

One of the most important additional objective which can be associated with a motion planning task is **optimal control**, which corresponds to the minimization of a cost (or, equivalently, the maximization of a gain) [105]. Optimal control theory is clearly deeply interconnected with calculus of variations, even if the non-interchangeable nature of the time-variable results in some important specific features, such as the occurrence of **abnormal extremals** [81]. Research in optimal control encompasses different aspects, from numerical methods to dynamic programming and non-smooth analysis, from regularity of minimizers to high order optimality conditions and curvature-like invariants.

Another domain of control theory with countless applications is **stabilization**. The goal in this case is to make the system converge towards an equilibrium or some more general safety region. The main difference with respect to motion planning is that here the control law is constructed in feedback form. One of the most important properties in this context is that of **robustness**, i.e., the performance of the stabilization protocol in presence of disturbances or modeling uncertainties. A powerful framework which has been developed to take into account uncertainties and exogenous non-autonomous disturbances is that of hybrid and switched systems [92, 82, 98]. The central tool in the stability analysis of control systems is that of **control Lyapunov function**. Other relevant techniques are based on algebraic criteria or dynamical systems. One of the most important stability property which is studied in the context of control system is **input-to-state stability** [96], which measures how sensitive the system is to an external excitation.

One of the areas where control applications have nowadays the most impressive developments is in the field of **biomedicine and neurosciences**. Improvements both in modeling and in the capability of finely actuating biological systems have concurred in increasing the popularity of these subjects. Notable advances concern, in particular, identification and control for biochemical networks [90] and models for neural activity [74]. Therapy analysis from the point of view of optimal control has also attracted a great attention [94].

Biological models are not the only one in which stochastic processes play an important role. Stock-markets and energy grids are two major examples where optimal control techniques are applied in the non-deterministic setting. Sophisticated mathematical tools have been developed since several decades to allow for such extensions. Many theoretical advances have also been required for dealing with complex systems whose description is based on **distributed parameters** representation and **partial differential equations**. Functional analysis, in particular, is a crucial tool to tackle the control of such systems [102].

Let us conclude this section by mentioning another challenging application domain for control theory: the decision by the European Union to fund a flagship devoted to the development of quantum technologies is a symptom of the role that quantum applications are going to play in tomorrow's society. **Quantum control** is one of the bricks of quantum engineering, and presents many peculiarities with respect to standard control theory, as a consequence of the specific properties of the systems described by the laws of quantum physics. Particularly important for technological applications is the capability of inducing and reproducing coherent state superpositions and entanglement in a fast, reliable, and efficient way [75].

3.2 Scientific foundations

At the core of the scientific activity of the team is the **geometric control** approach. One of the features of the geometric control approach is its capability of exploiting **symmetries and intrinsic structures** of control systems. Symmetries and intrinsic structures can be used to characterize minimizing trajectories, prove regularity properties and describe invariants. An egregious example is given by mechanical systems, which inherently exhibit Lagrangian/Hamiltonian structures which are naturally expressed using the language of symplectic geometry [65]. The geometric theory of quantum control, in particular, exploits the rich geometric structure encoded in the Schrödinger equation to engineer adapted control schemes and to characterize their qualitative properties. The Lie–Galerkin technique that we proposed starting in [66] builds on this premises in order to provide powerful tests for the controllability of quantum systems defined on infinite-dimensional Hilbert spaces.

Although the focus of geometric control theory is on qualitative properties, its impact can also be disruptive when it is used in combination with quantitative analytical tools, in which case it can dramatically improve the computational efficiency. This is the case in particular in optimal control. Classical optimal control techniques (in particular, Pontryagin Maximum Principle, conjugate point theory, associated numerical methods) can be significantly improved by combining them with powerful modern techniques of geometric optimal control, of the theory of numerical continuation, or of dynamical system theory [101, 93]. Geometric optimal control allows the development of general techniques, applying to wide classes of nonlinear optimal control problems, that can be used to characterize the behavior of optimal trajectories and in particular to establish regularity properties for them and for the cost function. Hence, geometric optimal control can be used to obtain powerful optimal syntheses results and to provide deep geometric insights into many applied problems. Numerical optimal control methods with geometric insight are in particular important to handle subtle situations such as rigid optimal paths and, more generally, optimal syntheses exhibiting abnormal minimizers.

Optimal control is not the only area where the geometric approach has a great impact. Let us mention, for instance, motion planning, where different geometric approaches have been developed: those based on the **Lie algebra** associated with the control system [86, 83], those based on the differentiation of nonlinear flows such as the **return method** [70, 71], and those exploiting the **differential flatness** of the system [73].

Geometric control theory is not only a powerful framework to investigate control systems, but also a useful tool to model and study phenomena that are not *a priori* control-related. Two occurrences of this property play an important role in the activities of CAGE:

- geometric control theory as a tool to investigate properties of mathematical structures;
- geometric control theory as a modeling tool for neurophysical phenomena and for synthesizing biomimetic algorithms based on such models.

Examples of the first type, concern, for instance, hypoelliptic heat kernels [52] or shape optimization [56]. Examples of the second type are inactivation principles in human motricity [58] or neurogeometrical models for image representation of the primary visual cortex in mammals [63].

A particularly relevant class of control systems, both from the point of view of theory and applications, is characterized by the linearity of the controlled vector field with respect to the control parameters. When the controls are unconstrained in norm, this means that the admissible velocities form a distribution in the tangent bundle to the state manifold. If the distribution is equipped with a point-dependent quadratic form (encoding the cost of the control), the resulting geometrical structure is said to be **sub-Riemannian**. Sub-Riemannian geometry appears as the underlying geometry of nonlinear control systems: in a similar way as the linearization of a control system provides local informations which are readable using the Euclidean metric scale, sub-Riemannian geometry provides an adapted non-isotropic class of lenses which are often much more informative. As such, its study is fundamental for control design. The importance of sub-Riemannian geometry goes beyond control theory and it is an active field of research both in differential geometry [85], geometric measure theory [54] and hypoelliptic operator theory [59].

4 Application domains

4.1 First axis: Quantum control

Quantum control is one of the bricks of quantum engineering, since manipulation of quantum mechanical systems is ubiquitous in applications such as quantum computation, quantum cryptography, and quantum sensing (in particular, imaging by nuclear magnetic resonance).

Quantum control presents many peculiarities with respect to standard control theory, as a consequence of the specific properties of the systems described by the laws of quantum physics. Particularly important for technological applications is the capability of inducing and reproducing coherent state superpositions and entanglement in a fast, reliable, and efficient way. The efficiency of the control action has a dramatic impact on the quality of the coherence and the robustness of the required manipulation. Minimal time constraints and interaction of time scales are important factors for characterizing the efficiency of a quantum control strategy. CAGE works for the improvement of quantum control paradigms, especially for what concerns quantum systems evolving in infinite-dimensional Hilbert spaces. The controllability of quantum system is a well-established topic when the state space is finite-dimensional [72], thanks to general controllability methods for left-invariant control systems on compact Lie groups [64, 78]. When the state space is infinite-dimensional, it is known that in general the bilinear Schrödinger equation is not exactly controllable [103]. The Lie–Galerkin technique [66] combines finite-dimensional geometric control techniques and the distributed parameter framework in order to provide the most powerful available tests for the approximate controllability of quantum systems defined on infinite-dimensional Hilbert spaces. Another important technique to the development of which we contribute is **adiabatic quantum control**. Adiabatic approximation theory and, in particular, adiabatic evolution [87, 99, 106] is well-known to improve the robustness of the control strategy and is strongly related to time scales analysis. The advantage of the adiabatic control is that it is constructive and produces control laws which are both smooth and robust to parameter uncertainty [107, 80, 62].

4.2 Second axis: Stability and stabilization

A control application with a long history and still very challenging open problems is **stabilization**. For infinite-dimensional systems, in particular nonlinear ones, the richness of the possible functional analytical frameworks makes feedback stabilization a challenging and active domain of research. Of particular interest are the different types of stabilization that may be obtained: exponential, polynomial, finite-time, ... Another important aspect of stabilization concerns control of systems with uncertain dynamics, i.e., with dynamics including possibly non-autonomous parameters whose value and dependence on time cannot be anticipated. **Robustification**, i.e., offsetting uncertainties by suitably designing the control strategy, is a widespread task in automatic control theory, showing up in many applicative domains such as electric circuits or aerospace motion planning. If dynamics are not only subject to static uncertainty, but may also change as time goes, the problem of controlling the system can be recast within the theory of switched and hybrid systems, both in a deterministic and in a probabilistic setting. **Switched and hybrid systems** constitute a broad framework for the description of the heterogeneous systems in which continuous dynamics (typically pertaining to physical quantities) interact with discrete/logical components. The development of the switched and hybrid paradigm has been motivated by a broad range of applications, including automotive and transportation industry [95], energy management [88] and congestion control [84]. Even if both controllability [97] and observability [79] of switched and hybrid systems raise several important research issues, the central role in their study is played by uniform stability and stabilizabilization [82, 98]. Uniformity is considered with respect to all signals in a given class, and it is well-known that stability of switched systems depends not only on the dynamics of each subsystem but also on the properties of the considered class of switching signals. In many situations it is interesting for modeling purposes to specify the features of the switched system by introducing **constrained switching rules**. A typical constraint is that each mode is activated for at least a fixed minimal amount of time, called the dwell-time. Our approach to constrained switching is based on the idea of relating the analytical properties of the classes of constrained switching laws (shift-invariance, compactness, closure under concatenation, ...) to the stability behavior of the corresponding switched systems. One can introduce **probabilistic uncertainties** by endowing the classes of admissible signals with suitable probability measures. The interest of this approach is that probabilistic stability analysis filters out highly 'exceptional' worst-case trajectories. Although less explicitly characterized from a dynamical viewpoint than its deterministic counterpart, the probabilistic notion of uniform exponential stability can be studied using several reformulations of Lyapunov exponents proposed in the literature [57, 68, 104].

4.3 Third axis: Motion planning and optimal control

Geometric optimal control allows the development of general techniques, applying to wide classes of nonlinear optimal control problems, that can be used to characterize the behavior of optimal trajectories and in particular to establish regularity properties for them and for the cost function. Hence, geometric optimal control can be used to obtain powerful optimal syntheses results and to provide deep geometric insights into many applied problems. Geometric optimal control methods are in particular important to handle subtle situations such as rigid optimal paths and, more generally, optimal syntheses exhibiting abnormal minimizers.

Although the focus of geometric control theory is on qualitative properties, its impact can also be disruptive when it is used in combination with quantitative analytical tools, in which case it can dramatically improve the computational efficiency. This is the case in particular in **optimal control**. Classical optimal control techniques (in particular, Pontryagin Maximum Principle, conjugate point theory, associated numerical methods) can be significantly improved by combining them with powerful modern techniques of geometric optimal control, of the theory of numerical continuation, or of dynamical system theory [101, 93]. Applications of optimal control theory considered by CAGE concern, in particular, motion planning problems for aerospace (atmospheric re-entry, orbit transfer, low cost interplanetary space missions, ...) [60, 100].

4.4 Fourth axis: Geometric models for vision and sub-Riemannian geometry

Geometric control theory is not only a powerful framework to investigate control systems, but also a useful tool to model and study phenomena that are not *a priori* control-related. In particular, we use control theory to investigate the properties of sub-Riemannian structures, both for the sake of mathematical understanding and as a modeling tool for image and sound perception and processing . We recall that **sub-Riemannian geometry** is a geometric framework which is used to measure distances in nonholonomic contexts and which has a natural and powerful optimal control interpretation in terms control-linear systems with quadratic cost. Sub-Riemannian geometry, and in particular the theory of their associated (hypoelliptic) diffusive processes, plays a crucial role in the neurogeometrical model of the primary visual cortex due to Petitot, Citti and Sarti, based on the functional architecture first described by Hubel and Wiesel [76, 89, 67, 91]. Such a model can be used as a powerful paradigm for bio-inspired image processing, as already illustrated in the literature [63, 61]. Our contributions to **geometry of vision** are based not only on this approach, but also on another geometric and sub-Riemannian framework for vision, based on pattern matching in the group of diffeomorphisms. In this case admissible diffeomorphisms correspond to deformations which are generated by vector fields satisfying a set of nonholonomic constraints. A sub-Riemannian metric on the infinite-dimensional group of diffeomorphisms is induced by a length on the tangent distribution of admissible velocities [55]. Nonholonomic constraints can be especially useful to describe distortions of sets of interconnected objects (e.g., motions of organs in medical imaging).

5 Highlights of the year

5.1 Awards

Rémi Robin was awarded with the Prix solennel de thèse de la chancellerie de Paris, the Prix de thèse PGMO 2023, and the Prix de thèse SMAI-GAMNI for his PhD thesis defended in 2022 and obtained while being member of the team CAGE.

Daniele Cannarsa and Mario Sigalotti have been awarded the Brockett-Willems Outstanding Paper Award 2023 for the best paper published in Systems & Control Letters during the two-year period from January 2021 through December 2022 (paper written while Daniele Cannarsa was member of the team CAGE).

6 New results

6.1 Quantum control: new results

Let us list here our new results in quantum control theory.

- Achiral molecules can be made temporarily chiral by excitation with electric fields, in the sense that an average over molecular orientations displays a net chiral signal. In [46], we go beyond the assumption of molecular orientations to remain fixed during the excitation process. Treating both rotations and vibrations quantum mechanically, we identify conditions for the creation of chiral vibrational wavepackets – with net chiral signals – in ensembles of achiral molecules which are initially randomly oriented. Based on the analysis of symmetry and controllability, we derive excitation schemes for the creation of chiral wavepackets using a combination of (a) microwave and IR pulses and (b) a static field and a sequence of IR pulses. These protocols leverage quantum rotational dynamics for pump-probe spectroscopy of chiral vibrational dynamics, extending the latter to regions of the electromagnetic spectrum other than the UV.

- In [48], we explore the controllability of a closed multi-input control-affine quantum system. Previous studies have demonstrated that a spectrum connected by conical intersections which do not pile up yields exact controllability in finite dimension and approximate controllability in infinite dimension. Actually, the property that intersections between eigenvalues are conical and that they do not pile up is generic. However, in physical situations, due to symmetry of the system, the spectrum can exhibit intersections that are not conical and possibly pile up. We extend the controllability result to cover this type of situations under the hypothesis that the intersections have at least one conical direction and the piled-up intersections have "rationally unrelated germs". Finally, we provide a testable first-order sufficient condition for controllability. Physically relevant examples are provided.

- In [45], we obtain observability estimates for Schrödinger equations in the plane. More precisely, considering a periodic bounded potential, we prove that the evolution Schrödinger equation is observable from any periodic measurable set, in any small time. We then extend Taüffer's recent result in the two-dimensional case to less regular observable sets and general bounded periodic potentials. The methodology of the proof is based on the use of the Floquet-Bloch transform, Strichartz estimates and semiclassical defect measures for the obtention of observability inequalities for a family of Schrödinger equations posed on the torus.

- In [20], we study, in the semiclassical sense, the global approximate controllability in small time of the quantum density and quantum momentum of the 1-D semiclassical cubic Schrödinger equation with two controls between two states with positive quantum densities. We first control the asymptotic expansions of the zeroth and first order of the physical observables via the Agrachev–Sarychev method. Then we conclude the proof through techniques of semiclassical approximation of the nonlinear Schrödinger equation.

- In [26], we establish some properties of quantum limits on a product manifold, proving for instance that, under appropriate assumptions, the quantum limits on the product of manifolds are absolutely continuous if the quantum limits on each manifolds are absolutely continuous. On a product of Riemannian manifolds satisfying the minimal multiplicity property, we prove that a periodic geodesic can never be charged by a quantum limit.

6.2 Stability and stabilization: new results

Let us list here our new results about stability and stabilization of control and hybrid systems.

- In [23], we study some spectral properties of the scalar dynamical system defined by a linear delay-differential equation with two positive delays. More precisely, the existing links between the delays and the maximal multiplicity of the characteristic roots are explored, as well as the dominancy of such roots compared with the spectrum localization. As a by-product of the analysis, the pole placement issue is revisited with more emphasis on the role of the delays as control parameters in defining a partial pole placement guaranteeing the closed-loop stability with an appropriate decay rate of the corresponding dynamical system.

- In [18], we consider the problem of determining the stability properties, and in particular assessing the exponential stability, of a singularly perturbed linear switching system. One of the challenges of this problem arises from the intricate interplay between the small parameter of singular perturbation and the rate of switching as both tend to zero. Our approach consists in characterizing suitable auxiliary linear systems that provide lower and upper bounds for the asymptotics of the maximal Lyapunov exponent of the linear switching system as the parameter of the singular perturbation tends to zero.

- In [29], we discuss the notion of universality for classes of candidate common Lyapunov functions of linear switched systems. On the one hand, we prove that a family of absolutely homogeneous functions is universal as soon as it approximates arbitrarily well every convex absolutely homogeneous function for the C0 topology of the unit sphere. On the other hand, we prove several obstructions for a class to be universal, showing, in particular, that families of piecewise-polynomial continuous functions whose construction involves at most l polynomials of degree at most m (for given positive integers l,m) cannot be universal.

- One of the central questions in control theory is achieving stability through feedback control. The paper [31] introduces a novel approach that combines Reinforcement Learning (RL) with mathematical analysis to address this challenge, with a specific focus on the Sterile Insect Technique

(SIT) system. The objective is to find a feedback control that stabilizes the mosquito population model. Despite the mathematical complexities and the absence of known solutions for this specific problem, our RL approach identifies a candidate solution for an explicit stabilizing control. This study underscores the synergy between AI and mathematics, opening new avenues for tackling intricate mathematical problems.

- Consider a non-autonomous continuous-time linear system in which the time-dependent matrix determining the dynamics is piecewise constant and takes finitely many values $A_1, \ldots, A_N$. The paper [19] studies the equality cases between the maximal Lyapunov exponent associated with the set of matrices $\{A_1, \ldots, A_N\}$, on the one hand, and the corresponding ones for piecewise deterministic Markov processes with modes $A_1, \ldots, A_N$, on the other hand. A fundamental step in this study consists in establishing a result of independent interest, namely, that any sequence of Markov processes associated with the matrices $A_1, \ldots, A_N$ converges, up to extracting a subsequence, to a Markov process associated with a suitable convex combination of those matrices.

- The article [39] deals with the stability of linear periodic difference delay systems, where the value at time t of a solution is a linear combination with periodic coefficients of its values at finitely many delayed instants $t - \tau_1, \ldots, t - \tau_N$. We establish a necessary and sufficient condition for exponential stability of such systems when the coefficients have Hölder-continuous derivative, that generalizes the one obtained for difference delay systems with constant coefficients by Henry and Hale in the 1970s. This condition may be construed as analyticity, in a half plane, of the (operator valued) harmonic transfer function of an associated linear control system.

- In [35], we explicitly compute the maximal Lyapunov exponent for a switched system on $SL_2(\mathbb{R})$. This computation is reduced to the characterization of optimal trajectories for an optimal control problem on the Lie group.

- In [38], we prove the (uniform) global exponential stabilization of the cubic defocusing Schrödinger equation on the torus d-dimensional torus, for d=1, 2 or 3, with a linear damping localized in a subset of the torus satisfying some geometrical assumptions. In particular, this answers an open question of Dehman, Gérard, Lebeau from 2006. Our approach is based on three ingredients. First, we prove the well-posedness of the closed-loop system in Bourgain spaces. Secondly, we derive new Carleman estimates for the nonlinear equation by directly including the cubic term in the conjugated operator. Thirdly, by conjugating with energy estimates and Morawetz multipliers method, we then deduce quantitative observability estimates leading to the uniform exponential decay of the total energy of the system. As a corollary of the global stabilization result, we obtain an upper bound of the minimal time of the global null-controllability of the nonlinear equation by using a stabilization procedure and a local null-controllability result.

- The Sterile Insect Technique or SIT is presently one of the most ecological methods for controlling insect pests responsible for disease transmission or crop destruction worldwide. This technique consists of releasing sterile males into the insect pest population. This approach aims at reducing fertility in the population and, consequently, reduce significantly the native insect population after a few generations. In the work [32], we study the global stabilization of a pest population at extinction equilibrium by the SIT method. We construct explicit feedback laws that stabilize the model and do numerical simulations to show the efficiency of our feedback laws. The different feedback laws are also compared taking into account their possible implementation in field interventions.

- In the article [24], we study the problem of stabilizing the traffic flow on a ring road to a uniform steady-state using autonomous vehicles (AV). Traffic is represented at a microscopic level via a Bando-Follow-the-Leader model capable of reproducing phantom jams. For the single-lane case, a single AV can stabilize an arbitrary large ring road with an arbitrary large number of cars. Moreover, this stabilization is exponentially quick with a decay rate independent of the number of cars and a control gain also independent of the number of cars. On the other side, the stabilization domain and stabilization time depend on the number of cars. Two types of controller algorithms are proposed: a proportional control and a proportional-integral control. In both cases, the measurements used by the controller only depend on the local data around the AV, enabling an easy implementation.

After numerical tests of the single-lane case, a multilane model is described using a safety-incentive mechanism for lane change. Numerical simulations for the multilane ring road suggest that the control strategy is also very efficient in such a setting, even with a single AV.

6.3 Motion planning and optimal control: new results

Let us list here our new results on controllability and motion planning algorithms, including optimal control, optimization beyond the quantum control framework.

- In [27], we address the problem of catching all speed 1 geodesics of a Riemannian manifold with a moving ball: given a compact Riemannian manifold (M, g) and small parameters $\epsilon > 0$ and $v > 0$, is it possible to find $T > 0$ and an absolutely continuous map $x : [0, T] \to M, t \mapsto x(t)$ satisfying $\|\dot{x}\|_\infty \leq v$ and such that any geodesic of (M, g) traveled at speed 1 meets the open ball $B_g(x(t), \epsilon) \subset M$ within time T? Our main motivation comes from the control of the wave equation: our results show that the controllability of the wave equation can sometimes be improved by allowing the domain of control to move adequately, even very slowly. We first prove that, in any Riemannian manifold (M, g) satisfying a geodesic recurrence condition (GRC), our problem has a positive answer for any $\epsilon > 0$ and $v > 0$, and we give examples of Riemannian manifolds (M, g) for which (GRC) is satisfied.
- In the lecture notes [51] we introduce controllability, tabilization and optimal control for systems in finite and infinite dimension.
- In [25], we prove the small-time global null-controllability of forward (resp. backward) semilinear stochastic parabolic equations with globally Lipschitz nonlinearities in the drift and diffusion terms (resp. in the drift term). In particular, we solve the open question posed by S. Tang and X. Zhang, in 2009. We propose a new twist on a classical strategy for controlling linear stochastic systems. By employing a new refined Carleman estimate, we obtain a controllability result in a weighted space for a linear system with source terms. The main novelty here is that the Carleman parameters are made explicit and are then used in a Banach fixed point method. This allows to circumvent the well-known problem of the lack of compactness embeddings for the solutions spaces arising in the study of controllability problems for stochastic PDEs.
- The article [42] deals with the controllability of linear one-dimensional hyperbolic systems. Reformulating the problem in terms of linear difference equations and making use of infinite-dimensional realization theory, we obtain both necessary and sufficient conditions for approximate and exact controllability, expressed in the frequency domain. The results are applied to flows in networks.
- In [36], we consider a linear quadratic (LQ) optimal control problem in both finite and infinite dimensions. We derive an asymptotic expansion of the value function as the fixed time horizon T tends to infinity. The leading term in this expansion, proportional to T, corresponds to the optimal value attained through the classical turnpike theory in the associated static problem. The remaining terms are associated with optimal stabilization problems towards the turnpike.
- In [16], we deal with the global exact controllability to the trajectories of the Boussinesq system posed in 2D or 3D smooth bounded domains. The velocity field of the fluid must satisfy a Navier slip-with-friction boundary condition and a Robin boundary condition is im- posed to the temperature. We assume that one can act on the velocity and the temperature on a small part of the boundary. For the proof, we first transform the boundary control problem into a distributed control problem. Then, we prove a global approximate controllability result by adapting the strategy of Coron et al [J. Eur. Math. Soc., 22 (2020), pp. 1625–1673]; this relies on the controllability properties of the inviscid Boussinesq system and the analysis of appropriate asymptotic boundary layer expansions. Finally, we conclude with a local control- lability result; as in many other cases, this can be established as a consequence of the null controllability of a linearized system through a fixed-point argument. Our contribution can be viewed as an extension of the results in [J. Eur. Math. Soc., 22 (2020), pp. 1625–1673], where thermal effects were not considered. Thus, we prove that the ideas behind the

controllability properties of the Euler system and the well-prepared dissipation technique can be adapted to the present situation. Furthermore, we cover all the classical boundary conditions for the temperature, that is, those of the Robin, Neumann and Dirichlet kinds.

- The article [17] deals with the controllability of finite-dimensional linear difference delay equations, i.e., dynamics for which the state at a given time t is obtained as a linear combination of the control evaluated at time t and of the state evaluated at finitely many previous instants of time $t-\Lambda_1,\dots,t-\Lambda_N$. Based on the realization theory developed by Y.Yamamoto for general infinite-dimensional dynamical systems, we obtain necessary and sufficient conditions, expressed in the frequency domain, for the approximate controllability in finite time in L^q spaces, $q\in[1,+\infty)$. We also provide a necessary condition for L^1 exact controllability, which can be seen as the closure of the L^1 approximate controllability criterion. Furthermore, we provide an explicit upper bound on the minimal times of approximate and exact controllability, given by $d\max\{\Lambda_1,\dots,\Lambda_N\}$, where d is the dimension of the state space.

- In [50], we consider the internal control of linear parabolic equations through on-off shape controls, i.e., controls of the form $M(t)\chi_{\omega(t)}$ with $M(t)\geq 0$ and $\omega(t)$ with a prescribed maximal measure. We establish small-time approximate controllability towards all possible final states allowed by the comparison principle with nonnegative controls. We manage to build controls with constant amplitude $M(t)\equiv\bar{M}$. In contrast, if the moving control set $\omega(t)$ is confined to evolve in some region of the whole domain, we prove that approximate controllability fails to hold for small times. The method of proof is constructive. Using Fenchel-Rockafellar duality and the bathtub principle, the on-off shape control is obtained as the bang-bang solution of an optimal control problem, which we design by relaxing the constraints. Our optimal control approach is outlined in a rather general form for linear constrained control problems, paving the way for generalisations and applications to other PDEs and constraints.

- The goal of [21] is to obtain observability estimates for non-homogeneous elliptic equations in the presence of a potential, posed on a smooth bounded domain Ω in 2d and observed from a non-empty open subset $\omega\subset\Omega$. More precisely, for every real-valued bounded potential V, our main result shows that, when Ω has a finite number of holes, the observability constant of the elliptic operator $-\Delta+V$, with domain $H^2\cap H^1_0(\Omega)$, is of the form $C\exp(C|V|^{1/2}\log^{1/2}(|V|))$ where C is a positive constant depending only on Ω and ω. Our methodology of proof is crucially based on the one recently developed by Logunov, Malinnikova, Nadirashvili, and Nazarov, in the context of the Landis conjecture on exponential decay of solutions to homogeneous elliptic equations in the plane. The main difference and additional difficulty is that the zero set of the solutions to elliptic equations with source term can be very intricate and should be dealt with carefully. As a consequence of these new observability estimates, we obtain new results concerning control of semi-linear elliptic equations in the spirit of Fernández-Cara, Zuazua's open problem concerning small-time global null-controllability of slightly super-linear heat equations.

- In [33], we consider a smooth system of the form $q'=f_0(q)+\sum_{i=1}^k u_i f_i(q)$, $q\in M$, $u_i\in\mathbb{R}$, and study controllability issues on the group of diffeomorphisms of M. It is well-known that the system can arbitrarily well approximate the movement in the direction of any Lie bracket polynomial of $f_1,\dots,f_k$. Any Lie bracket polynomial of $f_1,\dots,f_k$ is good in this sense. Moreover, some combinations of Lie brackets which involve the drift term f_0 are also good but surely not all of them. In this paper we try to characterize good ones and, in particular, all universal good combinations, which are good for any nilpotent truncation of any system.

- In [12], we consider a mechanical system of three ants on the floor, which move according to two independt rules: Rule A - forces the velocity of any given ant to always point at a neighboring ant, and Rule B - forces the velocity of every ant to be parallel to the line defined by the two other ants. We observe that Rule A equips the 6-dimensional configuration space of the ants with a structure of a homogeneous (3,6) distribution, and that Rule B foliates this 6-dimensional configuration space onto 5-dimensional leaves, each of which is equiped with a homogeneous (2,3,5) distribution. The symmetry properties and Bryant-Cartan local invariants of these distributions are determined. In the case of Rule B we study and determine the singular trajectories (abnormal extremals) of the corresponding distributions. We show that these satisfy an interesting system of two ODEs of Fuchsian type.

- In the work [43], we present a general framework which guarantees the existence of optimal domains for isoperimetric problems within the class of $C^{1,1}$-regular domains satisfying a uniform ball condition as long as the desired objective function satisfies certain properties. We then verify that the helicity isoperimetric problem studied by Cantarella, DeTurck, Gluck and Teytel in 2002 satisfies the conditions of our framework and hence establish the existence of optimal domains within the given class of domains. We additionally use the same framework to prove the existence of optimal domains among uniform $C^{1,1}$-domains for a first curl eigenvalue problem which has been studied recently for other classes of domains.

- In [44], we investigate properties of the image and kernel of the Biot-Savart operator in the context of stellarator designs for plasma fusion. We first show that for any given coil winding surface (CWS) the image of the Biot-Savart operator is L^2-dense in the space of square-integrable harmonic fields defined on a plasma domain surrounded by the CWS. Then we show that harmonic fields which are harmonic in a proper neighbourhood of the underlying plasma domain can in fact be approximated in any C^k-norm by elements of the image of the Biot-Savart operator. In the second part of this work we establish an explicit isomorphism between the space of harmonic Neumann fields and the kernel of the Biot-Savart operator which in particular implies that the dimension of the kernel of the Biot-Savart operator coincides with the genus of the coil winding surface and hence turns out to be a homotopy invariant among regular domains in 3-space. Lastly, we provide an iterative scheme which we show converges weakly in $W^{-\frac{1}{2},2}$-topology to elements of the kernel of the Biot-Savart operator.

- In [37], considering a general nonlinear dissipative finite dimensional optimal control problem in fixed time horizon T, we establish a two-term asymptotic expansion of the value function as $T \to +\infty$. The dominating term is T times the optimal value obtained from the optimal static problem within the classical turnpike theory. The second term, of order unity, is interpreted as the sum of two values associated with optimal stabilization problems related to the turnpike.

- The turnpike phenomenon stipulates that the solution of an optimal control problem in large time, remains essentially close to a steady-state of the dynamics, itself being the optimal solution of an associated static optimal control problem. Under general assumptions, it is known that not only the optimal state and the optimal control, but also the adjoint state coming from the application of the Pontryagin maximum principle, are exponentially close to a steady-state, except at the beginning and at the end of the time frame. In such results, the turnpike set is a singleton, which is a steady-state. In the paper [30], we establish a turnpike result for finite-dimensional optimal control problems in which some of the coordinates evolve in a monotone way, and some others are partial steady-states of the dynamics. We prove that the discrepancy between the optimal trajectory and the turnpike set is then linear, but not exponential: we thus speak of a linear turnpike theorem.

- Consider, on the one part, a general nonlinear finite-dimensional optimal control problem and assume that it has a unique solution whose state is denoted by x^*. On the other part, consider the sampled-data control version of it. Under appropriate assumptions, in [41], we prove that the optimal state of the sampled-data problem converges uniformly to x^* as the norm of the corresponding partition tends to zero. Moreover, applying the Pontryagin maximum principle to both problems, we prove that, if x^* has a unique weak extremal lift with a costate p that is normal, then the costate of the sampled-data problem converges uniformly to p. In other words, under a nondegeneracy assumption, control sampling commutes, at the limit of small partitions, with the application of the Pontryagin maximum principle.

- In this paper [34], we prove Morse index theorems for a big class of constrained variational problems on graphs. Such theorems are useful in various physical and geometric applications. Our formulas compute the difference of Morse indices of two Hessians related to two different graphs or two different sets of boundary conditions. Several applications such as the iteration formulas or lower bounds for the index are proved.

- The work [49] tackles the open pit planning problem in an optimal control framework. We study the optimality conditions for the so-called continuous formulation using Pontryagin's Maximum Principle, and introduce a new, semi-continuous formulation that can handle the optimization of a two-dimensional mine profile. Numerical simulations are provided for several test cases, including global optimization for the one-dimensional final open pit, and first results for the two-dimensional sequential open pit. Theses indicate a good consistency between the different approaches, and with the theoretical optimality conditions.

- In the paper [14], we consider a measure-theoretical formulation of the training of NeurODEs in the form of a mean-field optimal control with L^2-regularization of the control. We derive first order optimality conditions for the NeurODE training problem in the form of a mean-field maximum principle, and show that it admits a unique control solution, which is Lipschitz continuous in time. As a consequence of this uniqueness property, the mean-field maximum principle also provides a strong quantitative generalization error for finite sample approximations. Our derivation of the mean-field maximum principle is much simpler than the ones currently available in the literature for mean-field optimal control problems, and is based on a generalized Lagrange multiplier theorem on convex sets of spaces of measures. The latter is also new, and can be considered as a result of independent interest.

6.4 Geometric models for vision and sub-Riemannian geometry: new results

Let us list here our new results in the geometry of vision axis and, more generally, on hypoelliptic diffusion and sub-Riemannian geometry.

- In the article [28], we study the observability (or, equivalently, the controllability) of some subelliptic evolution equations depending on their step. This sheds light on the speed of propagation of these equations, notably in the "degenerated directions" of the subelliptic structure. First, for any $\gamma \geq 1$, we establish a resolvent estimate for the Baouendi-Grushin-type operator $\Delta_\gamma = \partial_x^2 + |x|^{2\gamma}\partial_y^2$, which has step $\gamma + 1$. We then derive consequences for the observability of the Schr ́odinger type equation $i\partial_t u - (-\Delta_\gamma)^s u = 0$ where $s \in \mathbb{N}$. We identify three different cases: depending on the value of the ratio $(\gamma+1)/s$, observability may hold in arbitrarily small time, or only for sufficiently large times, or even fail for any time. As a corollary of our resolvent estimate, we also obtain observability for heat-type equations $\partial_t u + (-\Delta_\gamma)^s u = 0$ and establish a decay rate for the damped wave equation associated with Δ_γ.

- In [40], we consider the evolution of a free quantum particle on the Grushin cylinder, under different type of quantizations. In particular we are interested to understand if the particle can cross the singular set, i.e., the set where the structure is not Riemannian. We consider intrinsic and extrinsic quantizations, where the latter are obtained by embedding the Grushin structure isometrically in $\mathbb{R}^3$ (with singularities). As a byproduct we provide formulas to embed the Grushin cylinder in $\mathbb{R}^3$ that could be useful for other purposes. Such formulas are not global, but permit to study the embedding arbitrarily close to the singular set. We extend these results to the case of α-Grushin cylinders.

- In [47], we establish two results concerning the Quantum Limits (QLs) of some sub-Laplacians. First, under a commutativity assumption on the vector fields involved in the definition of the sub-Laplacian, we prove that it is possible to split any QL into several pieces which can be studied separately, and which come from well-characterized parts of the associated sequence of eigenfunctions. Secondly, building upon this result, we study in detail the QLs of a particular family of sub-Laplacians defined on products of compact quotients of Heisenberg groups. We express the QLs through a disintegration of measure result which follows from a natural spectral decomposition of the sub-Laplacian in which harmonic oscillators appear. Both results are based on the construction of an adequate elliptic operator commuting with the sub-Laplacian, and on the associated joint spectral calculus. They illustrate the fact that, because of the possible high degeneracies in the spectrum, the spectral theory of sub-Laplacians is very rich.

- In [15], we consider surfaces embedded in a 3D contact sub-Riemannian manifold and the problem of the finiteness of the induced distance (i.e., the infimum of the length of horizontal curves that belong to the surface). Recently it has been proved that for a surface having the topology of a sphere embedded in a tight co-orientable structure, the distance is always finite. In this paper we study closed surfaces of genus larger than 1, proving that such surfaces can be embedded in such a way that the induced distance is finite or infinite. We then study the structural stability of the finiteness/not-finiteness of the distance.

- The relative heat content associated with a subset $\Omega \subset M$ of a sub-Riemannian manifold, is defined as the total amount of heat contained in Ω at time t, with uniform initial condition on Ω, allowing the heat to flow outside the domain. In the work [13], we obtain a fourth-order asymptotic expansion in square root of t of the relative heat content associated with relatively compact non-characteristic domains. Compared to the classical heat content that we studied in [Rizzi, Rossi - J. Math. Pur. Appl., 2021], several difficulties emerge due to the absence of Dirichlet conditions at the boundary of the domain. To overcome this lack of information, we combine a rough asymptotic for the temperature function at the boundary, coupled with stochastic completeness of the heat semi-group. Our technique applies to any (possibly rank-varying) sub-Riemannian manifold that is globally doubling and satisfies a global weak Poincaré inequality, including in particular sub-Riemannian structures on compact manifolds and Carnot groups.

- In [22], we study the isoperimetric problem for anisotropic left-invariant perimeter measures on $\mathbb{R}^3$, endowed with the Heisenberg group structure. The perimeter is associated with a left-invariant norm ϕ on the horizontal distribution. We first prove a representation formula for the ϕ-perimeter of regular sets and, assuming some regularity on ϕ and on its dual norm ϕ_*, we deduce a foliation property by sub-Finsler geodesics of C^2-smooth surfaces with constant ϕ-curvature. We then prove that the characteristic set of C^2-smooth surfaces that are locally extremal for the isoperimetric problem is made of isolated points and horizontal curves satisfying a suitable differential equation. Based on such a characterization, we characterize C^2-smooth ϕ-isoperimetric sets as the sub-Finsler analogue of Pansu's bubbles. We also show, under suitable regularity properties on ϕ, that such sub-Finsler candidate isoperimetric sets are indeed C^2-smooth. By an approximation procedure, we finally prove a conditional minimality property for the candidate solutions in the general case (including the case where ϕ is crystalline).

7 Bilateral contracts and grants with industry

Participants: Emmanuel Trélat, Veljko Askovic, Georgy Scholten.

7.1 Bilateral contracts with industry

Contract with MBDA (Palaiseau), 2021–2023. Subject: “Contrôle optimal pour la planification de trajectoires et l'estimation des ensembles accessibles". Pariticpants: V. Askovic (MBDA & CAGE), E. Trélat (coordinator).

7.2 Bilateral grants with industry

Grant by AFOSR (Air Force Office of Scientific Research), 2020–2023. Participants : Mohab Safey El Din (LIP6), E. Trélat.

8 Partnerships and cooperations

8.1 International research visitors

8.1.1 Visits of international scientists

Inria International Chair Andrei Agrachev (SISSA, Trieste, Italy) made two visits to CAGE (16/1–15/3 and 19/9–18/11) in the framework of his Inria International Chair 2020-2024.

Other international visits to the team Riccardo Adami (Politecnico di Torino, Italy), March.

8.2 National initiatives

8.2.1 ANR

- ANR TRECOS, for *New Trends in Control and Stabilization: Constraints and non-local terms*, coordinated by Sylvain Ervedoza, University of Bordeaux. The ANR started in 2021 and runs up to 2025. TRECOS' focus is on control theory for partial differential equations, and in particular models from ecology and biology.

- ANR QUACO, for *QUAntum COntrol: PDE systems and MRI applications*, coordinated by Thomas Chambrion, started in 2017 and finished in June 2023. Other partners: Burgundy University. QUACO contributed to quantum control theory in two directions: improving the comprehension of the dynamical properties of controlled quantum systems in infinite-dimensional state spaces, and improving the efficiency of control algorithms for MRI.

- ANR/DFG CoRoMo for *Efficient quantum control of molecular rotations – time and controllability*, 2023–2025. The grant is co-coordinated by Ugo Boscain (CAGE) and Christiane Koch (Berlin). In this project, we seek to elucidate the role of time in quantum control, using the important benchmark of molecular rotations as testbed. We will leverage controllability analysis to tackle the role of time in quantum control, combining physical intuition from the control of molecular rotations with recent advances of mathematical methods.

- ANR EINSTEIN-PPF for *Contraintes d'Einstein : passé', présent et futur*, coordinated by Philippe Lefloch. Relying on a close collaboration between analysts and geometers, the ANR project is aimed at advancing our knowledge of the analytic and geometric properties of Einstein spacetimes, especially when the metrics under consideration have low regularity.

8.2.2 Other national initatives

- The Inria Exploratory Action "StellaCage" is supporting since Spring 2020 a collaboration between CAGE, Yannick Privat (Inria team TONUS), and the startup Renaissance Fusion, based in Grenoble. StellaCage approaches the problem of designing better stellarators (yielding better confinement, with simpler coils, capable of higher fields) by combining geometrical properties of magnetic field lines from the control perspective with shape optimization techniques.

- The 80 prime project BioSpeech (2023–2024), coordinated by Ugo Boscain, studies a bio-inspired geometric model for speech sound reconstruction. It is a collaboration between mathematicians, automatic control scientists, and linguists.

8.3 Regional initiatives

The Bourse Emergence(s) de la Ville de Paris "Morphométrie sous contrainte pour l'analyse de données biologiques : un nouvel outil pour la communauté scientifique", whose principal investigator is Barbara Gris, runs from 2022 to 2025.

9 Dissemination

9.1 Promoting scientific activities

9.1.1 Scientific events: organisation

- Ugo Boscain, Jean-Michel Coron, Kévin Le Balc'h, Mario Sigalotti, and Emmanuel Trélat are members of the scientific committee of the Groupe de Travail Contrôle. Emmanuel Trélat is the main organizer of this regular seminar.

Member of the organizing and scientific committees

- Ugo Boscain was organizer (together with Domenico D'Alessandro) of the triple session "Geometric Control Theory with Quantum and Classical Applications" at the SIAM Conference on Control and Its Applications, Philadelphia, USA, July.
- Ugo Boscain was organizer (together with Giovanni Marelli) of the CIMPA school "Contemporary Geometry". Windhoeck Namibia, January.
- Ugo Boscain and Mario Sigalotti were organizers (with D. Barilari, D. Prandi, L. Rizzi, Y. Sachkov, A. Sarychev) of the conference "Geometry and Control in Cortona", Palazzone, Cortona Italy, March.
- Barbara Gris was in the dans local organizing committee of the conference of the FoCM society.
- Kévin Le Balc'h was co-organizer of the workhsop "Contrôle, Stabilisation et EDP" in Rennes, June.
- Emmanuel Trélat is in the scientific committee of the next SMAI Mode conference, Lyon.
- Emmanuel Trélat was in the scientific committee of the conference "New Trends and Challenges in Optimization Theory Applied to Space Engineering", l'Aquila, Itay, December.

9.1.2 Journal

Member of the editorial boards

- Ugo Boscain is Associate editor of SIAM Journal on Control and Optimization and he is Corresponding editor of the special section "Control of Quantum Mechanical Systems".
- Ugo Boscain is Managing editor of Journal of Dynamical and Control Systems
- Jean-Michel Coron is Editor-in-chief of Comptes Rendus Mathématique
- Jean-Michel Coron is Associate editor of Journal of Evolution Equations
- Jean-Michel Coron is Associate editor of Asymptotic Analysis
- Jean-Michel Coron is Associate editor of ESAIM: Control, Optimisation and Calculus of Variations
- Jean-Michel Coron is Associate editor of Applied Mathematics Research Express
- Jean-Michel Coron is Associate editor of Advances in Differential Equations
- Jean-Michel Coron is Associate editor of Mathematics of Control, Signals, and Systems
- Jean-Michel Coron is Associate editor of Annales de l'IHP, Analyse non linéaire
- Mario Sigalotti is Associate editor of SIAM Journal on Control and Optimization
- Mario Sigalotti is Associate editor of ESAIM: Control, Optimisation and Calculus of Variations
- Mario Sigalotti is Associate editor of Journal on Dynamical and Control Systems
- Emmanuel Trélat is Editor-in-chief of ESAIM: Control, Optimisation and Calculus of Variations

- Emmanuel Trélat is Associate editor of SIAM Review
- Emmanuel Trélat is Associate editor of Systems & Control Letters
- Emmanuel Trélat is Associate editor of Journal on Dynamical and Control Systems
- Emmanuel Trélat is Associate editor of Bollettino dell'Unione Matematica Italiana
- Emmanuel Trélat is Associate editor of ESAIM: Mathematical Modelling and Numerical Analysis
- Emmanuel Trélat is Editor of BCAM Springer Briefs
- Emmanuel Trélat is Associate editor of IEEE Transactions on Automatic Control
- Emmanuel Trélat is Associate editor of Journal of Optimization Theory and Applications
- Emmanuel Trélat is Associate editor of Mathematical Control & Related Fields
- Emmanuel Trélat is Associate editor of Mathematics of Control, Signals, and Systems
- Emmanuel Trélat is Associate editor of Optimal Control Applications and Methods
- Emmanuel Trélat is Associate editor of Advances in Continuous and Discrete Models: Theory and Modern Applications

9.1.3 Invited talks

- Ugo Boscain was invited speaker at the Journée Colloquium de Mathématiques, Laboratoire de Mathématiques d'Avignon.
- Ugo Boscain was invited speaker at ENS de Lyon, Workshop Defi EQIP 2023.
- Ugo Boscain was invited speaker at the conference "Optimization and Control in Burgundy".
- Barbara Gris was invited speaker at the séminaire de modélisation mathématique en sciences de la vie et santé (LAGA).
- Barbara Gris was invited speaker at the *journée annuelle du groupe thématique SIGMA de la SMAI*.
- Kévin Le Balc'h was invited speaker at the Workshop "Control and Related Fields", University of Sevilla, Spain.
- Kévin Le Balc'h was invited speaker at the Colloquium UNAM, Mexico City, Mexico.
- Kévin Le Balc'h was invited speaker at the Online seminar of Dortmund, Germany.
- Kévin Le Balc'h was invited speaker at at the Workshop EDP Cosy, Toulouse.
- Kévin Le Balc'h was invited speaker at the seminar of the numberical analysis and PDEs team, Sevilla, Spain.
- Mario Sigalotti was invited speaker at the Workshop EDP-COSy, Toulouse.
- Mario Sigalotti was invited speaker at the seminar of the Math department of the Humboldt-Universität zu Berlin, Germany.
- Emmanuel Trélat was plenary speaker at SCINDIS 2023, Wuppertal, Germany.
- Emmanuel Trélat was plenary speaker at SMAI 2023, Guadeloupe.
- Emmanuel Trélat was invited speaker at the conference "Control of Partial Differential Equations in Hauts-de-France", Valenciennes.
- Emmanuel Trélat was invited speaker at the Workshop EDP-COSy, Toulouse.

- Emmanuel Trélat was invited speaker at the seminar MBDA, Le Plessis-Robinson.
- Emmanuel Trélat was invited speaker at Texas A& M, College Station.
- Emmanuel Trélat was invited speaker at the franco-corean weminar.
- Emmanuel Trélat was invited speaker at ENS Ker Lann.
- Emmanuel Trélat was invited speaker at Fe‘de‘ration Charles Hermite, Nancy.
- Emmanuel Trélat was invited speaker at the seminar of the Institut Jean Le Rond d'Alembert.
- Emmanuel Trélat was invited speaker at the seminar PDE, LJK, Grenoble.

9.1.4 Leadership within the scientific community

- Ugo Boscain is Délégueé Scientifique at INSMI in charge of interdisciplinarity and member of the *Comité de pilotage* of the *Mission pour les initiatives transverses et interdisciplinaires (MITI)*.
- Emmanuel Trélat is Head of the Laboratoire Jacques-Louis Lions (LJLL).

9.1.5 Scientific expertise

- Emmanuel Trélat is member of the conseil scientifique de la Fédération de Mathématiques de CentraleSupelec.
- Emmanuel Trélat is member of the Advisory Board of the Department of Data Science, FAU (Erlangen), Germany.

9.1.6 Research administration

- Kévin Le Balc'h is SMAI correspondent for the Laboratoire Jacques-Louis Lions.
- Emmanuel Trélat is member of the Bureau de comité des équipes-projets, Inria Paris center.

9.2 Teaching - Supervision - Juries

9.2.1 Teaching

- Ugo Boscain thought " Geometric Control Theory" to PhD students at SISSA, Trieste, Italy.
- Ugo Boscain thought "Complements on sub-Riemannian geometry" at the CIMPA PhD school "Contemporary Geometry", Windhoeck Namibia.
- Ugo Boscain and Mario Sigalotti thought "Geometric control theory" at the M2 Mathématiques de la Modélisation, Sorbonne Université.
- Barbara Gris was in charge of the supervision of projects for l3 students,Sorbonne Université.
- Kévin Le Balc'h thought *Encadrement de leçons d'agrégation externe de mathématiques* to M2 students at Sorbonne Université.
- Kévin Le Balc'h thought "Approximation des EDP elliptiques" to M1 students at Sorbonne Université.
- Kévin Le Balc'h thought "Agrégation (analyse, probabilités)" to M2 students at Sorbonne Université.
- Kévin Le Balc'h thought "Analyse numérique" to L3 students at Sorbonne Université.
- Kévin Le Balc'h was the tutor of a M2 student at Sorbonne Université.
- Emmanuel Trélat thought "Contrôle en dimension finie et infinie" to M2 students at Sorbonne Université
- Emmanuel Trélat thought "Optimisation numérique et sciences des données" to M1 students at Sorbonne Université

9.2.2 Supervision

- PhD: Veljko Askovic, "Aerial vehicle guidance problem by the Pontryagin Maximum Principle and Hamilton Jacobi Bellman approach", December 2023. Supervisors: Emmanuel Trélat and Hasnaa Zidani (INSA, Rouen).
- PhD in progress: Kala Agbo Bidi, "Robust pest control strategies". Supervisors: Luis Almeida and Jean-Michel Coron.
- PhD in progress: Liangying Chen, "Sensitivity, Verification and Conjugate Times in Stochastic Optimal Control", started in 2021. Supervisors: Emmanuel Trélat and Xu Zhang (Chengdu, China).
- PhD in progress: Ruikang Liang, "The quantum speed limit in Quantum Control", started in 2022. Supervisors: Ugo Boscain and Mario Sigalotti.
- PhD in progress: Xiangyu Ma, "A bio-inspired geometric model for speech sound reconstruction", started in 2023. Supervisors: Ugo Boscain, Dario Prandi, and Giuseppina Turco.
- PhD in progress: Rayane Mouhli, "L'ontogénèse par grandes déformations", started in 2023. Supervisors: Barbara Gris and Irène Kaltenmark.
- PhD in progress: Robin Roussel, "Magnetic field lines and confinement in stellarators: a Hamiltonian perspective", started in 2021. Supervisors: Ugo Boscain and Mario Sigalotti.
- PhD in progress: Lucia Tessarolo, "Sub- Riemannian geometry and pinwheels", started in 2023. Supervisor: Ugo Boscain.
- Kévin Le Balc'h was member of the *comités de suivi* of the PhD theses of Cristobal Loboya and Ivan Hasenohr.

9.2.3 Juries

- Mario Sigalotti was member of the PhD jury of Gautier Roman, Sorbonne Universite'.
- Mario Sigalotti and Emmanuel Trélat were members of the PhD jury of Veljko Askovic, Sorbonne Universite'.
- Emmanuel Trélat was president of the HDR jury of Ihab Haidar, Univiversité de Cergy.
- Emmanuel Trélat was member of the HDR jury of C. J. Silva, University of Aveiro, Portugal.
- Emmanuel Trélat was member of the HDR jury of Amaury Hayat, Université Paris-Dauphine.
- Emmanuel Trélat was member of the PhD jury of R. Pre'bet, Sorbonne Universite'.
- Emmanuel Trélat was referee and member of the PhD jury of R. Loyer, Université du Littoral.
- Emmanuel Trélat was member of the PhD jury of M. Harakeh, Université d'Orle'ans.
- Emmanuel Trélat was referee and member of the PhD jury of A. Bouali, Université d'Avignon.
- Emmanuel Trélat was referee and member of the PhD jury of A. Herasimenka, Université Côte d'Azur.
- Emmanuel Trélat was referee and member of the PhD jury of L. Mascolo, Politecnico di Torino, Italy.
- Emmanuel Trélat was referee and member of the PhD jury of H. Me'nou, Ecole des Mines de Paris.

10 Scientific production

10.1 Major publications

[1] D. Barilari, Y. Chitour, F. Jean, D. Prandi and M. Sigalotti. 'On the regularity of abnormal minimizers for rank 2 sub-Riemannian structures'. In: *Journal de Mathématiques Pures et Appliquées* 133 (2020), pp. 118–138. DOI: 10.1016/j.matpur.2019.04.008. URL: https://hal.archives-ouvertes.fr/hal-01757343.

[2] R. Bonalli, B. Hérissé and E. Trélat. 'Optimal Control of Endo-Atmospheric Launch Vehicle Systems: Geometric and Computational Issues'. In: *IEEE Transactions on Automatic Control* 65.6 (2020), pp. 2418–2433. DOI: 10.1109/tac.2019.2929099. URL: https://hal.archives-ouvertes.fr/hal-01626869.

[3] Y. Colin de Verdière, L. Hillairet and E. Trélat. *Spectral asymptotics for sub-Riemannian Laplacians*. 5th Dec. 2022. URL: https://hal.archives-ouvertes.fr/hal-03885610.

[4] J.-M. Coron, A. Hayat, S. Xiang and C. Zhang. 'Stabilization of the linearized water tank system'. In: *Archive for Rational Mechanics and Analysis* 244.3 (2022), pp. 1019–1097. URL: https://hal.archives-ouvertes.fr/hal-03161523.

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