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2023 ACTIVITY REPORT

Project-Team MATHRISK

Mathematical Risk handling

IN COLLABORATION WITH: Centre d'Enseignement et de Recherche en Mathématiques et Calcul Scientifique (CERMICS)

DOMAIN

Applied Mathematics, Computation and Simulation

THEME

Stochastic approaches

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Project-Team MATHRISK

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2 Overall objectives

The Inria project team **MathRisk** team was created in 2013. It is the follow-up of the MathFi project team founded in 2000. MathFi was focused on financial mathematics, in particular on computational methods for pricing and hedging increasingly complex financial products. The 2007 global financial crisis and its "aftermath crisis" has abruptly highlighted the critical importance of a better understanding and management of risk.

The project team MathRisk addresses broad research topics embracing risk management in quantitative finance and insurance and in other related domains as economy and sustainable development. In these contexts, the management of risk appears at different time scales, from high frequency data to long term life insurance management, raising challenging renewed modeling and numerical issues. We aim at both producing advanced mathematical tools, models, algorithms, and software in these domains, and developing collaborations with various institutions involved in risk control. The scientific issues we consider include:

Option pricing and hedging, and risk-management of portfolios in finance and insurance. These remain crucial issues in finance and insurance, with the development of increasingly complex products and various regulatory legislations. Models must take into account the multidimensional features, incompleteness issues, model uncertainties and various market imperfections and defaults. It is also important to understand and capture the joint dynamics of the underlying assets and their volatilities. The insurance activity faces a large class of risk, including financial risk, and is submitted to strict regulatory requirements. We aim at proposing modelling frameworks which catch the main specificity of life insurance contracts.

Systemic risk and contagion modeling. These last years have been shaped by ever more interconnectedness among all aspects of human life. Globalization and economics growth as well as technological progress have led to more complex dependencies worldwide. While these complex networks facilitate physical, capital and informational transmission, they have an inherent potential to create and propagate distress and risk. The financial crisis 2007-2009 has illustrated the significance of network structure on the amplification of initial shocks in the banking system to the level of the global financial system, leading to an economic recession. We are contributing on the issues of systemic risk and financial networks, aiming at developing adequate tools for monitoring financial stability which capture accurately the risks due to a variety of interconnections in the financial system.

(Martingale) Optimal transport. Optimal transport problems arise in a wide range of topics, from economics to physics. In mathematical finance, an additional martingale constraint is considered to take the absence of arbitrage opportunities into account. The minimal and maximal costs provide price bounds robust to model risk, i.e. the risk of using an inadequate model. On the other hand, optimal transport is also useful to analyse mean-field interactions. We are in particular interested in particle approximations of McKean-Vlasov stochastic differential equations (SDEs) and the study of mean-field backward SDEs with applications to systemic risk quantization.

Advanced numerical probability methods and Computational finance. Our project team is very much involved in numerical probability, aiming at pushing numerical methods towards the effective implementation. This numerical orientation is supported by a mathematical expertise which permits a rigorous analysis of the algorithms and provides theoretical support for the study of rates of convergence and the introduction of new tools for the improvement of numerical methods. Financial institutions and insurance companies, submitted to more and more stringent regulatory legislations, such as FRTB or XVA computation, are facing numerical implementation challenges and research focused on numerical efficiency is strongly needed. Overcoming the curse of dimensionality in computational finance is a crucial issue that we address by developing advanced stochastic algorithms and deep learning techniques.

The **MathRisk** project is strongly devoted to the development of new mathematical methods and numerical algorithms. Mathematical tools include stochastic modeling, stochastic analysis, in particular various aspects of stochastic control and optimal stopping with nonlinear expectations, Malliavin calculus, stochastic optimization, random graphs, (martingale) optimal transport, mean-field systems, numerical probability and generally advanced numerical methods for effective solutions. The numerical platform Premia that MathRisk is developing in collaboration with a consortium of financial institutions, focuses on the computational challenges the recent developments in financial mathematics encompass, in particular risk control in large dimensions.

3 Research program

3.1 Systemic risk in financial networks

After the recent financial crisis, systemic risk has emerged as one of the major research topics in mathematical finance. Interconnected systems are subject to contagion in time of distress. The scope is to understand and model how the bankruptcy of a bank (or a large company) may or not induce other bankruptcies. By contrast with the traditional approach in risk management, the focus is no longer on modeling the risks faced by a single financial institution, but on modeling the complex interrelations between financial institutions and the mechanisms of distress propagation among these.

The mathematical modeling of default contagion, by which an economic shock causing initial losses and default of a few institutions is amplified due to complex linkages, leading to large scale defaults, can be addressed by various techniques, such as network approaches or mean field interaction models.

The goal of our project is to develop a model that captures the dynamics of a complex financial network and to provide methods for the control of default contagion, both by a regulator and by the institutions themselves.

We have contributed in the last years to the research on the control of contagion in financial systems in the framework of random graph models (see PhD thesis of R. Chen [75] and Z. Cao [31]).

In [60, 103], [8], we consider a financial network described as a weighted directed graph, in which nodes represent financial institutions and edges the exposures between them. The distress propagation is modeled as an epidemics on this graph. We study the optimal intervention of a lender of last resort who seeks to make equity infusions in a banking system prone to insolvency and to bank runs, under complete and incomplete information of the failure cluster, in order to minimize the contagion effects. The paper [8] provides in particular important insight on the relation between the value of a financial system, connectivity and optimal intervention.

The results show that up to a certain connectivity, the value of the financial system increases with connectivity. However, this is no longer the case if connectivity becomes too large. The natural question remains how to create incentives for the banks to attain an optimal level of connectivity. This is studied in [76], where network formation for a large set of financial institutions represented as nodes is investigated. Linkages are source of income, and at the same time they bear the risk of contagion, which is endogeneous and depends on the strategies of all nodes in the system. The optimal connectivity of the nodes results from a game. Existence of an equilibrium in the system and stability properties is studied. The results suggest that financial stability is best described in terms of the mechanism of network formation than in terms of simple statistics of the network topology like the average connectivity.

In [7], H. Amini (University of Florida), A. Minca (Cornell University) and A. Sulem study Dynamic Contagion Risk Model With Recovery Features. We introduce threshold growth in the classical threshold contagion model, in which nodes have downward jumps when there is a failure of a neighboring node. We are motivated by the application to financial and insurance-reinsurance networks, in which thresholds represent either capital or liquidity. An initial set of nodes fail exogenously and affect the nodes connected to them as they default on financial obligations. If those nodes' capital or liquidity is insufficient to absorb the losses, they will fail in turn. In other terms, if the number of failed neighbors reaches a node's threshold, then this node will fail as well, and so on. Since contagion takes time, there is the potential for the capital to recover before the next failure. It is therefore important to introduce a notion of growth. Choosing the configuration model as underlying graph, we prove fluid limits for the baseline model, as well as extensions to the directed case, state-dependent inter-arrival times and the case of growth driven by upward jumps. We then allow nodes to choose their connectivity by trading off link benefits and contagion risk. Existence of an asymptotic equilibrium is shown as well as convergence of the sequence of equilibria on the finite networks. In particular, these results show that systems with higher overall growth may have higher failure probability in equilibrium.

3.2 Stochastic Control, optimal stopping and non-linear backward stochastic differential equations (BSDEs) with jumps

Option pricing in incomplete and nonlinear financial market models with default. A. Sulem with M.C. Quenez and M. Grigorova have studied option pricing and hedging in nonlinear incomplete financial markets model with default. The underlying market model consists of a risk-free asset and a risky asset driven by a Brownian motion and a compensated default martingale. The portfolio processes follow nonlinear dynamics with a nonlinear driver f, which encodes the imperfections or constraints of the market. A large class of imperfect market models can fit in this framework, including imperfections coming from different borrowing and lending interest rates, taxes on profits from risky investments, or from the trading impact of a large investor seller on the market prices and the default probability. Our market is *incomplete*, in the sense that not every contingent claim can be replicated by a portfolio. In this framework, we address in [13] the problem of pricing and (super)hedging of European options. By using a dynamic programming approach, we provide a dual formulation of the seller's superhedging price as the supremum over a suitable set of equivalent probability measures $Q \in \mathcal{Q}$ of the non-linear $\mathcal{E}^f_Q$-expectation under Q of the payoff. We also provide a characterization of this price as the minimal supersolution of a *constrained* BSDE with default. In [87], we study the superhedging problem for American options with irregular payoffs. We establish a dual formulation of the seller's price in terms of the value of a non-linear mixed optimal control/stopping problem. We also characterize the seller's price process as the minimal supersolution of a *reflected* BSDE with *constraints*. We then prove a duality result for the *buyer's* price in terms of the value of a non-linear optimal control/stopping *game* problem. A crucial step in the proofs is to establish a non-linear optional and a non-linear predictable decomposition for processes which are $\mathcal{E}^f_Q$-strong supermartingales under Q, for all $Q \in \mathcal{Q}$. American option pricing in a non-linear *complete* market model with default is previously studied in [78]. A complete analysis of BSDEs driven by a Brownian motion and a compensated default jump process with intensity process (λ_t) is achieved in [77]. Note that these equations do not correspond to a particular case of BSDEs with Poisson random measure, and are particularly useful in default risk modeling in finance.

Optimal stopping. The theory of optimal stopping in connection with American option pricing has been extensively studied in recent years. Our contributions in this area concern:

(i) *The analysis of the binomial approximation of the American put price in the Black-Scholes model.* We proved that the rate of convergence is, up to a logarithmic factor, of the order $1/n$, where n is the number of discretization time points [99]; (ii) *The American put in the Heston stochastic volatility model.* We have results about existence and uniqueness for the associated variational inequality, in suitable weighted Sobolev spaces, following up on the work of P. Feehan et al. (2011, 2015, 2016) (cf [101]). We also established some qualitative properties of the value function (monotonicity, strict convexity, smoothness) [100]. (iii) *A probabilistic approach to the smoothness of the free boundary in the optimal stopping of a one-dimensional diffusion* (work in progress with T. De Angelis)(University of Torino),

Stochastic control with jumps. The 3rd edition of the book *Applied Stochastic Control of Jump diffusions* (Springer, 2019) by B. Øksendal and A. Sulem [15] contains recent developments within stochastic control and its applications. In particular, there is a new chapter devoted to a comprehensive presentation of financial markets modelled by jump diffusions, one on backward stochastic differential equations and risk measures, and an advanced stochastic control chapter including optimal control of mean-field systems, stochastic differential games and stochastic Hamilton-Jacobi-Bellman equations.

3.3 Volatility Modeling

J. Guyon and co-authors have investigated the modeling of the volatility of financial markets [25, 24, 26]. In particular, the (mostly) path-dependent nature of volatility has been shown in [25], an article that has been downloaded 7,000+ times on SSRN. Path-dependent volatility (PDV) provides a new paradigm of volatility modeling, which can be mixed with stochastic volatility (PDSV) to account for the exogenous part of volatility. In [88], J. Guyon has uncovered a remarkable property of the S&P 500 and VIX markets, which he called inversion of convex ordering. In [24], M. El Amrani and J. Guyon have shown that, contrary to a common belief in the mathematical finance community, the term-structure of the at-the-money skew does not follow a power law. In [26], J. Guyon and S. Mustapha have calibrated neural stochastic differential equations jointly to S&P 500 smiles, VIX futures, and VIX smiles.

3.4 Insurance modeling

Asset Liability Management. Life insurance contracts are popular and involve very large portfolios, for a total amount of trillions of euros in Europe. To manage them in a long run, insurance companies perform Asset and Liability Management (ALM) : it consists in investing the deposit of policyholders in different asset classes such as equity, sovereign bonds, corporate bonds, real estate, while respecting a performance warranty with a profit sharing mechanism for the policyholders. A typical question is how to determine an allocation strategy which maximizes the rewards and satisfies the regulatory constraints. The management of these portfolios is quite involved: the different cash reserves imposed by the regulator, the profit sharing mechanisms, and the way the insurance company determines the crediting rate to its policyholders make the whole dynamics path-dependent and rather intricate. A. Alfonsi et al. have developed in [49] a synthetic model that takes into account the main features of the life insurance business. This model is then used to determine the allocation that minimizes the Solvency Capital Requirement (SCR). In [50], numerical methods based on Multilevel Monte-Carlo algorithms are proposed to calculate the SCR at future dates, which is of practical importance for insurance companies. The standard formula prescribed by the regulator is basically obtained from conditional expected losses given standard shocks that occur in the future.

3.5 (Martingale) Optimal Transport and Mean-field systems

3.5.1 Numerical methods for Optimal transport

Optimal transport problems arise in a wide range of topics, from economics to physics. There exists different methods to solve numerically optimal transport problems. A popular one is the Sinkhorn algorithm which uses an entropy regularization of the cost function and then iterative Bregman projections. Alfonsi et al. [52] have proposed an alternative relaxation that consists in replacing the constraint of matching exactly the marginal laws by constraints of matching some moments. Using Tchakaloff's theorem, it is shown that the optimum is reached by a discrete measure, and the optimal transport is found by using a (stochastic) gradient descent that determines the weights and the points of the discrete measure. The number of points only depends of the number of moments considered, and therefore does not depend on the dimension of the problem. The method has then been developed in [51] in the case of symmetric multimarginal optimal transport problems. These problems arise in quantum chemistry with the Coulomb interaction cost. The problem is in dimension $(\mathbb{R}^3)^M$ where M is the number of electrons, and the method is particularly relevant since the optimal discrete measure weights only $N+2$ points, where N is the number of moments constraint on the distribution of each electron. Numerical examples up to $M = 100$ can be thus investigated while existing methods could not go beyond $M \approx 10$.

3.5.2 Mean-field systems

Mean-field systems and optimal transport. In [73], O.Bencheikh and B. Jourdain prove that the weak error between a stochastic differential equation with nonlinearity in the sense of McKean given by moments and its approximation by the Euler discretization with time-step h of a system of N interacting particles is $\mathcal{O}(N^{-1}+h)$. The challenge was to improve the $\mathcal{O}(N^{-1/2})$ strong rate of convergence in the number of particles. In [74], they prove the same estimation for the Euler discretization of a system interacting particles with mean-field rank based interaction in the drift coefficient. To deal with the initialization error, they investigate in [72] the approximation rate in Wasserstein distance with index $\rho \geq 1$ of a probability measure μ on the real line with finite moment of order ρ by the empirical measure of N deterministic points.
In [97], B. Jourdain and A. Tse propose a generalized version of the central limit theorem for nonlinear functionals of the empirical measure of i.i.d. random variables, provided that the functional satisfies some regularity assumptions for the associated linear functional derivatives of various orders. Using this result to deal with the contribution of the initialization, they check the convergence of fluctuations between the empirical measure of particles in an interacting particle system and its mean-field limiting measure. In [82], R. Flenghi and B. Jourdain pursue their study of the central limit theorem for nonlinear functionals of the empirical measure of random variables by relaxing the i.i.d. assumption to deal with the successive values of an ergodic Markov chain. In [53], A. Alfonsi and B. Jourdain show that any optimal coupling for the quadratic Wasserstein distance $\mathcal{W}_2^2(\mu,\nu)$ between two probability measures μ and ν on $\mathbf{R}^d$ is the composition of a martingale coupling with an optimal transport map. They prove that $\sigma \mapsto \mathcal{W}_2^2(\sigma,\nu)$ is differentiable at μ in both Lions and the geometric senses iff there is a unique optimal coupling between μ and ν and this coupling is given by a map.

3.5.3 Martingale Optimal Transport

In mathematical finance, optimal transport problems with an additional martingale constraint are considered to handle the model risk, i.e. the risk of using an inadequate model. The Martingale Optimal Transport (MOT) problem introduced in [71] provides model-free hedges and bounds on the prices of exotic options. The market prices of liquid call and put options give the marginal distributions of the underlying asset at each traded maturity. Under the simplifying assumption that the risk-free rate is zero, these probability measures are in increasing convex order, since by Strassen's theorem this property is equivalent to the existence of a martingale measure with the right marginal distributions. For an exotic payoff function of the values of the underlying on the time-grid given by these maturities, the model-free upper-bound (resp. lower-bound) for the price consistent with these marginal distributions is given by the following martingale optimal transport problem : maximize (resp. minimize) the integral of the payoff with respect to the martingale measure over all martingale measures with the right marginal distributions. Super-hedging (resp. sub-hedging) strategies are obtained by solving the dual problem. With J. Corbetta, A. Alfonsi and B. Jourdain [5] have studied sampling methods preserving the convex order for two probability measures μ and ν on $\mathbf{R}^d$, with ν dominating μ. Their method is the first generic approach to tackle the martingale optimal transport problem numerically and it can also be applied to several marginals.

Martingale Optimal Transport provides thus bounds for the prices of exotic options that take into account the risk neutral marginal distributions of the underlying assets deduced from the market prices of vanilla options. For these bounds to be robust, the stability of the optimal value with respect to these marginal distributions is needed. Because of the global martingale constraint, stability is far less obvious than in optimal transport (it even fails in multiple dimensions). B. Jourdain has advised the PhD of W. Margheriti devoted to this issue and related problems. He also initiated a collaboration on this topic with M. Beiglböck, one of the founders of MOT theory. In [91], B. Jourdain and W. Margheriti exhibit a new family of martingale couplings between two one-dimensional probability measures μ and ν in the convex order. The integral of $|x-y|$ with respect to each of these couplings is smaller than twice the $\mathcal{W}^1$ distance between μ and ν. Moreover, for $\rho > 1$, replacing $|x-y|$ and $\mathcal{W}_1$ respectively with $|x-y|^\rho$ and $\mathcal{W}_\rho^\rho$ does not lead to a finite multiplicative constant. In [92], they show that a finite constant is recovered when replacing $\mathcal{W}_\rho^\rho$ with the product of $\mathcal{W}_\rho$ times the centred ρ-th moment of the second marginal to the power $\rho-1$ and they study the generalisation of this stability inequality to higher dimension. In

[93], they give a direct construction of the projection in adapted Wasserstein distance onto the set of martingale couplings of a coupling between two probability measures on the real line in the convex order which satisfies the barycentre dispersion assumption. Under this assumption, Wiesel had given a clear algorithmic construction of the projection for finitely supported marginals before getting rid of the finite support condition by a rather messy limiting procedure. In [70], with M. Beiglböck and G. Pammer they establish stability of martingale couplings in dimension one : when approximating in Wasserstein distance the two marginals of a martingale coupling by probability measures in the convex order, it is possible to construct a sequence of martingale couplings between these probability measures converging in adapted Wasserstein distance to the original coupling. In [21], they deduce the stability of the Weak Martingale Optimal Transport Problem with respect to the marginal distributions in dimension one which is important since financial data can give only imprecise information on these marginals. As application, this yields the stability of the superreplication bound for VIX futures and of the stretched Brownian motion. In [27], B. Jourdain et al. prove that, in dimension one, contrary to the minimum and maximum in the convex order, the Wasserstein projections of μ (resp. ν) on the set of probability measures dominated by ν (resp. dominating μ) in the convex order are Lipschitz continuous in (μ, ν) for the Wasserstein distance. The thesis of K. Shao (advisers: B. Jourdain, A. Sulem) focuses so far on optimal couplings for costs $|y - x|^\rho$ in dimension one.

Quantization. In order to exploit the natural links between quantization and convex order in view of numerical methods for (Weak) Martingale Optimal Transport, B. Jourdain has initiated a fruitful collaboration with G. Pagès, one of the leading experts of quantization. For two compactly supported probability measures in the convex order, any stationary quadratic primal quantization of the smaller remains dominated by any dual quantization of the larger. B. Jourdain and G. Pagès prove in [96] that any martingale coupling between the original probability measures can be approximated by a martingale coupling between their quantizations in Wassertein distance with a rate given by the quantization errors but also in the much finer adapted Wassertein distance. In [94], in order to approximate a sequence of more than two probability measures in the convex order by finitely supported probability measures still in the convex order, they propose to alternate transitions according to a martingale Markov kernel mapping a probability measure in the sequence to the next and dual quantization steps. In the case of ARCH models, the noise has to be truncated to enable the dual quantization steps. They exhibit conditions under which the ARCH model with truncated noise is dominated by the original ARCH model in the convex order and also analyse the error of the scheme combining truncation of the noise according to primal quantization with the dual quantization steps. In [95], they prove that for compactly supported one dimensional probability distributions having a log-concave density, L^r-optimal dual quantizers are unique at each level N. In the quadratic $r = 2$ case, they propose an algorithm which computes this unique optimal dual quantizer with geometric rate of convergence.

3.5.4 Martingale Schrödinger problems

Calibration problems in finance can be cast as Schrödinger problems. Due to the no-arbitrage condition, martingale Schrödinger problems must be considered. To jointly calibrate S&P 500 (SPX) and VIX options, J. Guyon has introduced dispersion-constrained martingale Schrödinger problems. In [24], he solved for the first time this longstanding puzzle of quantitative finance that has often been described as the Holy Grail of volatility modeling: build a model that jointly and exactly calibrates to the prices of SPX options, VIX futures, and VIX options. He did so using a nonparametric, discrete-time, minimum-entropy approach. He established a strong duality theorem and characterized the absence of joint SPX/VIX arbitrage. The minimum entropy jointly calibrating model is explicit in terms of the dual Schrödinger portfolio, i.e., the maximizer of the dual problems, should it exist, and is numerically computed using an extension of the Sinkhorn algorithm. Numerical experiments show that the algorithm performs very well in both low and high volatility regimes.

3.6 Deep learning for large dimensional financial problems

Neural networks and Machine Learning techniques for high dimensional American options. The pricing of American option or its Bermudan approximation amounts to solving a backward dynamic programming equation, in which the main difficulty comes from the conditional expectation involved in the computation of the continuation value.

In [102], B. Lapeyre and J. Lelong study neural networks approximations of conditional expectations. They prove the convergence of the well-known Longstaff and Schwartz algorithm when the standard least-square regression on a finite-dimensional vector space is replaced by a neural network approximation, and illustrate the numerical efficiency of the method on several numerical examples. Its stability with respect to a change of parameters as interest rate and volatility is shown. The numerical study proves that training neural network with only a few chosen points in the grid of parameters permits to price efficiently for a whole range of parameters.

In [84], two efficient techniques, called GPR Tree (GRP-Tree) and GPR Exact Integration (GPR-EI), are proposed to compute the price of American basket options. Both techniques are based on Machine Learning, exploited together with binomial trees or with a closed formula for integration. On the exercise dates, the value of the option is first computed as the maximum between the exercise value and the continuation value and then approximated by means of Gaussian Process Regression. In [86], an efficient method is provided to compute the price of multi-asset American options, based on Machine Learning, Monte Carlo simulations and variance reduction techniques. Numerical tests show that the proposed algorithm is fast and reliable, and can handle American options on very large baskets of assets, overcoming the curse of dimensionality issue.

• *Machine Learning in the Energy and Commodity Market.* Evaluating moving average options is a computational challenge for the energy and commodity market, as the payoff of the option depends on the prices of underlying assets observed on a moving window. An efficient method for pricing Bermudan style moving average options is presented in [85], based on Gaussian Process Regression and Gauss-Hermite quadrature. This method is tested in the Clewlow-Strickland model, the reference framework for modeling prices of energy commodities, the Heston (non-Gaussian) model and the rough-Bergomi model, which involves a double non-Markovian feature, since the whole history of the volatility process impacts the future distribution of the process.

3.7 Advanced numerical probability methods and Computational finance

Our project team is very much involved in numerical probability, aiming at pushing numerical methods towards the effective implementation. This numerical orientation is supported by a mathematical expertise which permits a rigorous analysis of the algorithms and provides theoretical support for the study of rates of convergence and the introduction of new tools for the improvement of numerical methods. This activity in the MathRisk team is strongly related to the development of the Premia software.

3.7.1 Approximation of stochastic differential equations

High order schemes. The approximation of SDEs and more general Markovian processes is a very active field. One important axis of research is the analysis of the weak error, that is the error between the law of the process and the law of its approximation. A standard way to analyse this is to focus on marginal laws, which boils down to the approximation of semigroups. The weak error of standard approximation schemes such as the Euler scheme has been widely studied, as well as higher order approximations such as those obtained with the Richardson-Romberg extrapolation method.

Stochastic Volterra Equations. Stochastic Volterra Equations (SVE) provide a wide family of non-Markovian stochastic processes. They have been introduced in the early 80's by Berger and Mizel and have received a recent attention in mathematical finance to model the volatility : it has been noticed that SVEs with a fractional convolution kernel $G(t) = c_H t^{H-1/2}$ reproduce some important empirical features. The problem of approximating these equations has been tackled by Zhang [109] and Richard et al. [108] who show under suitable conditions a strong convergence rate of $O(n^{-H-})$ for the Euler scheme, where n is the number of time steps. We almost recover the rate for classical SDEs when $H \to 1/2$. However, an important drawback is that the required computation time is proportional to n^2.

Abstract Malliavin calculus and convergence in total variation. In collaboration with L. Caramellino and G. Poly, V. Bally has settled a Malliavin type calculus for a general class of random variables, which are not supposed to be Gaussian (as it is the case in the standard Malliavin calculus). This is an alternative to the Γ-calculus settled by Bakry, Gentile and Ledoux. The main application is the estimate in total variation distance of the error in general convergence theorems. This is done in [67].

Invariance principles. As an application of the above methodology, V. Bally et al. have studied several limit theorems of Central Limit type (see [62] and [66]). In particular they estimate the total variation distance between random polynomials, and prove a universality principle for the variance of the number of roots of trigonometric polynomials with random coefficients [68]).

Analysis of jump type SDEs. V. Bally, L. Caramellino and A. Kohatsu Higa, study the regularity properties of the law of the solutions of jump type SDE's [64]. They use an interpolation criterion (proved in [61]) combined with Malliavin calculus for jump processes. They also use a Gaussian approximation of the solution combined with Malliavin calculus for Gaussian random variables. Another approach to the same regularity property, based on a semigroup method has been developed by Bally and Caramellino in [63]. An application for the Bolzmann equation is given by V. Bally in [61]. In the same line but with different application, the total variation distance between a jump equation and its Gaussian approximation is studied by V. Bally and his PhD student Y. Qin [69] and by V. Bally, V. Rabiet, D. Goreac [68]. A general discussion on the link between total variation distance and integration by parts is done in [67]. Finally V. Bally et al. estimate in [65] the probability that a diffusion process remains in a tube around a smooth function.

3.7.2 Monte-Carlo and Multi-level Monte-Carlo methods

Error bounds of MLMC. In [90], B. Jourdain and A. Kebaier are interested in deriving non-asymptotic error bounds for the multilevel Monte Carlo method. As a first step, they deal with the explicit Euler discretization of stochastic differential equations with a constant diffusion coefficient. As long as the deviation is below an explicit threshold, they check that the multilevel estimator satisfies a Gaussian-type concentration inequality optimal in terms of the variance.

Approximation of conditional expectations. The approximation of conditional expectations and the computation of expectations involving nested conditional expectations are important topics with a broad range of applications. In risk management, such quantities typically occur in the computation of the regulatory capital such as future Value-at-Risk or CVA. A. Alfonsi et al. [50] have developed a Multilevel Monte-Carlo (MLMC) method to calculate the Solvency Capital Ratio of insurance companies at future dates. The main advantage of the method is that it avoids regression issues and has the same computational complexity as a plain Monte-Carlo method (i.e. a computational time in $O(\varepsilon^{-2})$ to reach a precision of order ε). In other contexts, one may be interested in approximating conditional expectations. To do so, the classical method consists in considering a parametrized family $\varphi(\alpha,\cdot)$ of functions, and to minimize the empirical L^2-distance $\frac{1}{M}\sum_{k=1}^{M}(Y_i-\varphi(\alpha,X_i))^2$ between the observations and their prediction. In general, it is assumed to have as many observations as explanatory variables. However, when these variables are sampled, it may be possible to sample K values of Y's for a given X_i and to minimize $\frac{1}{M}\sum_{k=1}^{M}(\frac{1}{K}\sum_{k=1}^{K}Y_i^k-\varphi(\alpha,X_i))^2$. A. Alfonsi, J. Lelong and B. Lapeyre [18] have determined the optimal value of K which minimizes the computation time for a given precision. They show that K is large when the family approximates well the conditional expectation. The computational gain can be important, especially if the computational cost of sampling Y given X is small with respect to the cost of sampling X.

3.8 Remarks

We have focused above on the research program of the last four years. We refer to the previous MathRisk activity report for a description of the research done earlier, in particular on Liquidity and Market Microstructure [54, 48], [4], dependence modelling [98], interest rate modeling [47], Robust option pricing in financial markets with imperfections [77, 107], [12, 11], Mean field control and Stochastic Differential Games [104, 89, 106], Stochastic control and optimal stopping (games) under nonlinear expectation [78, 81, 79, 80], robust utility maximization [105, 106, 83], Generalized Malliavin calculus and numerical probability.

4 Application domains

4.1 Financial Mathematics, Insurance

The domains of application are quantitative finance and insurance with emphasis on risk modeling and control. In particular, the project-team Mathrisk focuses on financial modeling and calibration, systemic risk, option pricing and hedging, portfolio optimization, risk measures.

5 Social and environmental responsibility

Our work aims to contribute to a better management of risk in the banking and insurance systems, in particular by the study of systemic risk, asset price modeling, stability of financial markets.

6 Highlights of the year

6.1 Conference

MathRisk has organized an international conference on numerical methods in finance in June 2023 in Udine (Italy) to celebrate the 25th anniversary of the software Premia and the team MathRisk.

6.2 Evaluation

MathRisk had a very successful evaluation in 2023.

6.3 Research

On March 14, 2023, FIFA changed the format of the 2026 FIFA World Cup based on Julien Guyon's articles

- J. Guyon: Risk of Collusion: Will Groups of 3 Ruin the FIFA World Cup? Journal of Sports Analytics 6(4):259-279, 2020.
- J. Guyon: Why Groups of 3 Will Ruin the World Cup (So Enjoy This One), The New York Times, June 11, 2018.
- J. Guyon: Mondial 2026 : pourquoi les groupes de trois risquent de fausser la Coupe du monde, Le Monde, June 12, 2018 (in French).

7 New software, platforms, open data

7.1 New software

7.1.1 PREMIA

Keywords: Computational finance, Quantum Finance, Monte-Carlo methods, Option pricing, Numerical probability, Machine learning, Numerical algorithm

Scientific Description: Premia is a numerical platform for computational finance. It is designed for option pricing, hedging and financial model calibration. Premia is developed by the MathRisk project team in collaboration with a consortium of financial institutions. The Premia project keeps track of the most recent advances in the field of computational finance in a well-documented way. It focuses on the implementation of numerical analysis techniques for both probabilistic and deterministic numerical methods. An important feature of the platform Premia is the detailed documentation which provides extended references in option pricing. Premia contains various numerical algorithms: deterministic methods (Finite difference and finite element algorithms for partial differential equations, wavelets, Galerkin, sparse grids ...), stochastic algorithms (Monte-Carlo simulations, quantization methods, Malliavin calculus based methods), tree methods, approximation methods (Laplace transforms, Fast Fourier transforms...) These algorithms are implemented for the evaluation of vanilla and exotic options on equities, interest rate, credit, energy and insurance products. Moreover Premia provides a calibration toolbox for Libor Market model and a toolbox for pricing Credit derivatives. The latest developments of the software address evaluation of financial derivative products, risk management and computations of risk measures required by new financial regulation. They include the implementation of advanced numerical algorithms taking into account model dependence, counterparty credit risk, hybrid features, rough volatility and various nonlinear effects. A big effort has been put these last years on the development and implementation of deep learning techniques using neural network approximations, and Machine Learning algorithms in finance, in particular for high-dimensional American option pricing, high-dimensional PDEs, deep hedging. Moreover Quantum computing in Finance is explored, in particular option pricing using quantum computers.

Functional Description: Premia is a software designed for quantitative finance, developed by the MathRisk project team in collaboration with a consortium of financial institutions presently composed of Crédit Agricole CIB and NATIXIS. The Premia project keeps track of the most recent advances in computational finance and focuses on the implementation of numerical techniques to solve financial problems. An important feature of the platform Premia is its detailed documentation which provides extended references in computational finance. Premia is a powerful tool to assist Research and Development professional teams in their day-to-day duty. It is also a useful support for academics who wish to perform tests on new algorithms or pricing methods. Besides being a single entry point for accessible overviews and basic implementations of various numerical methods, the aim of the Premia project is: - to elaborate a powerful testing platform for comparing different numerical methods between each other, - to build a link between professional financial teams and academic researchers, - to provide a useful teaching support for Master and PhD students in mathematical finance. The project Premia has started in 1999 and is now considered as a standard reference platform for quantitative finance among the academic mathematical finance community.

Release Contributions: A big effort has been put these last years on the development and implementation of deep learning techniques using neural network approximations, and Machine Learning algorithms in finance, in particular for high-dimensional American option pricing, high-dimensional PDEs, deep hedging.The latest developments of the software address also the evaluation of financial derivative products, risk management and computations of risk measures by advanced numerical algorithms taking into account model dependence, counterparty credit risk (computations of XVA), hybrid features, rough stochastic volatility models and various new regulations. Nested Monte Carlo strategies with GPU optimizations, and Chebyshev Interpolation method for Parametric Option Pricing have been implemented. We have also developed our activity on insurance contracts, in particular on the computation of risk measures (Value at Risk, Condition Tail Expectation) of variable annuities contracts like GMWB (guaranteed minimum withdrawal benefit) including taxation and customers mortality modeling.

News of the Year: The new release Premia 25 has been delivered to the Consortium on September 29 2023. It contains the following new implemented algorithms.

I. Machine Learning algorithms and Risk Management:

• Optimal Stopping via Randomized Neural Networks. C.Herrera, F.Krach, P.Ruyssen, J.Teichmann • Deep Learning-Based Least Square Forward-Backward Stochastic Differential Equation Solver for High-Dimensional Derivative Pricing. J.Liang Z.Xu P.Li Quantitative Finance, 21-8, 2021. • The Deep Parametric PDE Method: Application to Option Pricing. K.Glau L.Wunderlich Applied Mathematics and Computation, 432, 2022. • Computing XVA for American basket derivatives by Machine Learning techniques. L.Goudenege A.Molent A.Zanette • Backward Hedging for American

Options with Transaction Costs. L.Goudenge A.Molent A.Zanette • Pricing high-dimensional American options by kernel ridge regression. W.Hu T.Zastawniak Quantitative Finance, 20-5, 2020. • KrigHedge: Gaussian Process Surrogates for Delta Hedging. M.Ludkovski Y.Saporito Applied Mathematical Finance, 28-4, 2021.

II. Advanced numerical methods for Equity Derivatives:

• Option Pricing using Quantum Computers. N. Stamatopoulos, D. J. Egger, Y. Sun, C. Zoufal, R. Iten, N. Shen, S. Woerner, Quantum, 4-291, 2020. • The interpolated drift implicit Euler scheme Multilevel Monte Carlo method for pricing Barrier options and applications to the CIR and CEV models. M. Ben Derouich, A. Kebaier • Hybrid multifactor scheme for stochastic Volterra equations. S. E. Rømer • On the discrete-time simulation of the rough Heston model. A.Richard, X.Tan, F.Yang Siam Journal of Financial Mathematics, 14-1, 2023. • The stochastic collocation Monte Carlo sampler: highly efficient sampling from 'ex- pensive' distributions. L. A. Grzelak, J. A. S. Witteveen, M. Suarez-Taboada, C. W. Oosterlee Quantitative Finance, 19-2, 2019

URL: `http://www.premia.fr`

Publications: hal-03436046, hal-01940715, hal-01873346, hal-03810106, hal-03526905, hal-03013606, hal-02183587

Contact: Agnes Sulem

Participants: Agnes Sulem, Antonino Zanette, Aurélien Alfonsi, Benjamin Jourdain, Jerome Lelong, Bernard Lapeyre, Ahmed Kebaier, Ludovic Goudenège

Partners: Ecole des Ponts ParisTech, Université d'Udine

8 New results

Participants: A. Sulem, A. Alfonsi, B. Jourdain, J. Guyon, V. Bally, D. Lamberton.

8.1 Control of systemic risk in a dynamic framework

Participants: A. Sulem, H. Amini, Z. Cao.

A. Minca

Default cascades in sparse heterogeneous financial networks. A. Sulem, H. Amini, and their PhD student Z. Cao have studied the control of interbank contagion, dynamics and stability of complex financial networks, by using techniques from random graphs and stochastic control. We have obtained limit results for default cascades in sparse heterogeneous financial networks subject to an exogenous macroeconomic shock in [20]. These limit theorems for different system-wide wealth aggregation functions allow us to provide systemic risk measures in relation with the structure and heterogeneity of the financial network. These results are applied to determine the optimal policy for a social planner to target interventions during a financial crisis, with a budget constraint and under partial information of the financial network. Banks can impact each other due to large-scale liquidations of similar assets or non-payment of liabilities. In [59], we present a general tractable framework for understanding the joint impact of fire sales and default cascades on systemic risk in complex financial networks. The effect of heterogeneity in network structure and price impact function on the final size of default cascade and fire sales loss is investigated.

Central Limit Theorems for Price-Mediated Contagion in Stochastic Financial Networks In [55], we provide central limit theorems to analyze the combined effects of fire sales and default cascades on systemic risk within stochastic financial networks. The impact of prices is modeled through a specifically defined inverse demand function. Our study presents various limit theorems that delve into the dynamics of total shares sold and the equilibrium pricing of illiquid assets in a streamlined fire sales context. We show that the equilibrium prices of these assets demonstrate asymptotically Gaussian fluctuations. In our numerical experiments, we demonstrate how our central limit theorems can be applied to construct confidence intervals for the magnitude of contagion and the extent of losses due to fire sales.

Ruin Probabilities for Risk Processes in Stochastic Networks In [56], We study multidimensional CramẂe study multidimensional Cramér-Lundberg risk processes where agents, located on a large sparse network, receive losses from their neighbors. To reduce the dimensionality of the problem, we introduce classification of agents according to an arbitrary countable set of types. The ruin of any agent triggers losses for all of its neighbours. We consider the case when the loss arrival process induced by the ensemble of ruined agents follows a Poisson process with general intensity function that scales with the network size. When the size of the network goes to infinity, we provide explicit ruin probabilities at the end of the loss propagation process for agents of any type. These limiting probabilities depend, in addition to the agents' types and the network structure, on the loss distribution and the loss arrival process. For a more complex risk processes on open networks, when in addition to the internal networked risk processes the agents receive losses from external users, we provide bounds on ruin probabilities.

8.2 Graphon Mean-field Backward Stochastic Differential Equations

8.2.1 Mean-field (Graphon) Backward Stochastic Differential Equations and systemic risk measures

Participants: A. Sulem, R. Chen, A. Minca, R. Dumitrescu, Z. Cao, H. Amini.

Agnès Sulem, Rui Chen, Andreea Minca, Roxana Dumitrescu have studied mean-field BSDEs with a generalized mean-field operator which can capture system influence with higher order interactions such as those occurring in an inhomogeneous random graph.

We interpret the BSDE solution as a dynamic *global* risk measure for a representative bank whose risk attitude is influenced by the system. This influence can come in a wide class of choices, including the average system state or average intensity of system interactions [22].

This opens the path towards using dynamic risk measures induced by mean-field BSDE as a complementary approach to systemic risk measurement.

Extensions to *Graphon BSDEs* with jumps are studied by H. Amini, A. Sulem, and their PhD student Z. Cao in [57]. The use of graphons has emerged recently in order to analyze heterogeneous interaction in mean-field systems and game theory. Existence, uniqueness and stability of solutions under some regularity assumptions are established. We also prove convergence results for interacting mean-field particle systems with inhomogeneous interactions to graphon mean-field BSDE systems.

8.2.2 Stochastic Graphon Mean-field Games and approximate Nash Equilibria

Participants: A. Sulem, Z. Cao, H. Amini.

In [58], we study continuous stochastic games with inhomogeneous mean field interactions on large networks and explore their graphon limits. We consider a model with a continuum of players, where each player's dynamics involve not only mean field interactions but also individual jumps induced by a Poisson random measure. We examine the case of controlled dynamics, with control terms present in the drift, diffusion, and jump components. We introduce the graphon game model based on a graphon controlled stochastic differential equation system with jumps, which can be regarded as the limiting case of a finite game's dynamic system as the number of players goes to infinity. Under some general assumptions, we establish the existence and uniqueness of Markovian graphon equilibria. We then provide convergence results on the state trajectories and their laws, transitioning from finite game systems to graphon systems. We also study approximate equilibria for finite games on large networks, using the graphon equilibrium as a benchmark. The rates of convergence are analyzed under various underlying graphon models and regularity assumptions.

8.2.3 Reinforcement Learning for Graphon Mean-Field Games

Participants: A. Sulem, Z. Cao, H. Amini, K. Shao.

This is an ongoing work in collaboration with Mathieu Laurière (NYU Shangai). We develop theoretical and numerical analysis of extended Graphon Mean Field Games (GMFG) in a discrete-time setting. On the theoretical side, we provide rigorous analysis on the existence of approximated Nash equilibrium of the GMFG system by considering joined state-action distribution, we also refined the proof of existence by categorizing pure policies and mixed policies. On the numerical side, we explore some learning schemes (i.e. reinforcement learning) to study graphon mean field equilibrium.

8.3 Optimal stopping

Participants: D. Lamberton.

D. Lamberton and Tiziano De Angelis (University of Torino) are working on the optimal stopping problem of a one dimensional diffusion in finite horizon. They develop a probabilistic approach to the regularity of the associated free boundary problem.

They derived a probabilistic proof of the differentiability of the free boundary for the optimal stopping problem of a one-dimensional diffusion. They are working on extensions of our results to higher order derivatives.

Some of the results on the American put price in the Heston model that were obtained in joint work with Giulia Terenzi have also been improved. In particular, we have estimates for the time derivative without the Feller condition.

8.4 Martingale Optimal transport

Participants: B. Jourdain, K. Shao, G. Pammer.

For many examples of couples (μ,ν) of probability measures on the real line in the convex order, B. Jourdain and K. Shao observe numerically in [45] that the Hobson and that the Hobson and Neuberger martingale coupling, which maximizes for $\rho = 1$ the integral of $|y-x|^\rho$ with respect to any martingale coupling between μ and ν, is still a maximizer for $\rho \in (0,2)$ and a minimizer for $\rho > 2$. They investigate the theoretical validity of this numerical observation and give rather restrictive sufficient conditions for the property to hold. We also exhibit couples (μ,ν) such that it does not hold. The support of the Hobson and Neuberger coupling is known to satisfy some monotonicity property which we call non-decreasing. B. Jourdain and K. Shao check that the non-decreasing property is preserved for maximizers when $\rho \in (0,1]$. In general, there exist distinct non-decreasing martingale couplings, and they find some decomposition of ν which is in one-to-one correspondence with martingale couplings non-decreasing in a generalized sense.

In [44], they complete the analysis of the Martingale Wasserstein Inequality started by checking that this inequality fails in dimension $d \geq 2$ when the integrability parameter ρ belongs to $[1,2)$ while a stronger Maximal Martingale Wasserstein Inequality holds whatever the dimension d when $\rho \geq 2$.

While many questions in robust finance can be posed in the martingale optimal transport framework or its weak extension, others like the subreplication price of VIX futures, the robust pricing of American options or the construction of shadow couplings necessitate additional information to be incorporated into the optimization problem beyond that of the underlying asset. In [43], B. Jourdain and G. , B. Jourdain and G. Pammer take into account this extra information by introducing an additional parameter to the weak martingale optimal transport problem. They prove the stability of the resulting problem with respect to the risk neutral marginal distributions of the underlying asset. Finally, they deduce stability of the three previously mentioned motivating examples.

8.5 Convex order

Participants: B. Jourdain, G. Pagès.

In [29], B. Jourdain and G. Pagès are interested in comparing solutions to stochastic Volterra equations for the convex order on the space of continuous $\mathbb{R}^d$-valued paths and for the monotonic convex order when $d = 1$. Even if in general these solutions are neither semi-martingales nor Markov processes, they are able to exhibit conditions on their coefficients enabling the comparison. The approach consists in first comparing their Euler schemes and then taking the limit as the time step vanishes. They consider two types of Euler schemes depending on the way the Volterra kernels are discretized. The conditions ensuring the comparison are slightly weaker for the first scheme than for the second one and this is the other way round for convergence. Moreover, they extend the integrability needed on the starting values in the existence and convergence results in the literature to be able to only assume finite first order moments, which is the natural framework for convex ordering.

In [42], B. Jourdain and G. Pagès are interested in the propagation of convexity by the strong solution to a one-dimensional Brownian stochastic diffential equation with coefficients Lipschitz in the spatial variable uniformly in the time variable and in the convex ordering between the solutions of two such equations. They prove that while these properties hold without further assumptions for convex functions of the processes at one instant only, an assumption almost amounting to spatial convexity of the diffusion coefficient is needed for the extension to convex functions at two instants. Under this spatial convexity of the diffusion coefficients, the two properties even hold for convex functionals of the whole path. For directionally convex functionals, the spatial convexity of the diffusion coefficient is no longer needed. The method of proof consists in first establishing the results for time discretization schemes of Euler type and then transfering them to their limiting Brownian diffusions. They thus exhibit approximations which avoid *convexity arbitrages* by preserving convexity propagation and comparison and can be computed by Monte Carlo simulation.

8.6 Insurance modeling

Participants: B. Jourdain, H. Andrès.

In the spirit of Guyon and Lekeufack (2023) [25] who are interested in the dependence of volatility indices (e.g. the VIX) on the paths of the associated equity indices (e.g. the S&P 500), H. Andrès, A. Boumezoued and B. Jourdain study in [37] how implied volatility can be predicted using the past trajectory of the underlying asset price. The empirical study reveals that a large part of the movements of the at-the-money (ATM) implied volatility for up to two years maturities can be explained using the past returns and their squares. Moreover, this feedback effect gets weaker when the maturity increases and that up to four years of the past evolution of the underlying price should be used for the prediction. Building on this new stylized fact, H. Andrès, A. Boumezoued and B. Jourdain fit to historical data a parsimonious version of the SSVI parameterization (Gatheral and Jacquier, 2014) of the implied volatility surface relying on only four parameters and show that the two parameters ruling the ATM implied volatility as a function of the maturity exhibit a path-dependent behavior with respect to the underlying asset price. By adding this feedback effect to the path-dependent volatility model of Guyon and Lekeufack for the underlying asset price and by specifying a hidden semi-Markov diffusion model for the residuals of these two parameters and the two other parameters, they are able to simulate highly realistic paths of implied volatility surfaces that are arbitrage-free.

8.7 Stochastic modeling of the Temperature and Electricity for pricing quanto

Participants: A. Alfonsi, N. Vadillo Fernandez.

With N. Vadillo Fernandez, A. Alfonsi has proposed in [35] a joint model for temperature and electricity spot price in order to quantify the risk of derivatives such as quanto that deal with the fluctuations of climate (Heating Degree Day index) and electricity. We present an estimation method for this model and give analytic formula for the average payoff and for a static quadratic hedging strategy based on HDD and Electricity spot options.

8.8 Volatility Modeling

Participants: J. Guyon.

J. Guyon and J. Lekeufack [25] learn from data that volatility is mostly path-dependent: up to 90% of the variance of the implied volatility of equity indexes is explained endogenously by past index returns, and up to 65% for (noisy estimates of) future daily realized volatility. The path-dependency that we uncover is remarkably simple: a linear combination of a weighted sum of past daily returns and the square root of a weighted sum of past daily squared returns with different time-shifted power-law weights capturing both short and long memory. This simple model, which is homogeneous in volatility, is shown to consistently outperform existing models across equity indexes and train/test sets for both implied and realized volatility. It suggests a simple continuous-time path-dependent volatility (PDV) model that may be fed historical or risk-neutral parameters. The weights can be approximated by superpositions of exponential kernels to produce Markovian models. In particular, J. Guyon and J. Lekeufack propose a 4-factor Markovian PDV model which captures all the important stylized facts of volatility, produces very realistic price and (rough-like) volatility paths, and jointly fits SPX and VIX smiles remarkably well. They thus show that a continuous-time Markovian parametric stochastic volatility (actually, PDV) model can practically solve the joint SPX/VIX smile calibration problem.

Using two years of S&P 500, Eurostoxx 50, and DAX data, M. El Amrani and J. Guyon [24], empirically investigate the term-structure of the at-the-money-forward (ATM) skew of equity indexes. While a power law (2 parameters) captures the term-structure well away from short maturities, the power law fit deteriorates considerably when short maturities are included. By contrast, 3-parameter shapes that look like power laws but do not blow up at vanishing maturity, such as time-shifted or capped power laws, are shown to fit well regardless of whether short maturities are included or not. Their study suggests that the term-structure of equity ATM skew has a power-law shape for maturities above 1 month but has a different behavior, and in particular may not blow up, for shorter maturities. The 3-parameter shapes are derived from non-Markovian variance curve models using the Bergomi-Guyon expansion. A simple 4-parameter term-structure similarly derived from the (Markovian) two-factor Bergomi model is also considered and provides even better fits. The extrapolated zero-maturity skew, far from being infinite, is distributed around a typical value of 1.5 (in absolute value).

J. Guyon and S. Mustapha [26] calibrate neural stochastic differential equations jointly to S&P 500 smiles, VIX futures, and VIX smiles. Drifts and volatilities are modeled as neural networks. Minimizing a suitable loss allows them to fit market data for multiple S&P 500 and VIX maturities. A one-factor Markovian stochastic local volatility model is shown to fit both smiles and VIX futures within bid-ask spreads. The joint calibration actually makes it a pure path-dependent volatility model, confirming the findings in [Guyon, 2022, The VIX Future in Bergomi Models: Fast Approximation Formulas and Joint Calibration with S&P 500 Skew].

8.9 Pricing and calibration of path-dependent volatility models

Participants: J. Guyon, G Gazzani.

G. Gazzani and J. Guyon consider a stochastic volatility model where the dynamics of the volatility process are described by a linear combination of a (exponentially) weighted sum of past daily returns and the square root of a weighted sum of past daily squared returns in the spirit of [25].They discuss the influence of an additional parameter that allows to reproduce the implied volatility smiles of SPX an VIX options within a 4-factor Markovian model (4FPDV). The empirical nature of this class of path-dependent volatility models (PDVs) comes with computational challenges, especially in relation to VIX options pricing and calibration. To address these challenges, they propose an accurate neural network approximation of the VIX leveraging on the markovianity of the 4FPDV. This approximation is subsequently used to tackle the joint calibration problem of SPX and VIX options. They additionally discuss a local volatility extension of the 4FPDV, in order to exactly calibrate market smiles. A preprint will be posted in Q1 2024.

8.10 Numerical probability

Participants: A. Alfonsi, B. Jourdain, A. Kebaier, V. Bally, O. Bencheikh, B. Jourdain, J. Lelong, A. Zanette, L. Goudenège, A. Molent.

8.10.1 Approximations of Stochastic Differential Equations (SDEs)

High order schemes for the weak error for the CIR and Heston processes. A. Alfonsi and E. Lombardo have developed in [19] high order schemes for the weak error for the CIR process, based on the construction proposed in a recent paper by A. Alfonsi and V. Bally. We keep on this analysis to extend these results to the Heston model.

Approximation of Stochastic Volterra Equations (SVE). In [34], A. Alfonsi studies the stochastic invariance in a convex domain of SVEs. He also provides a second order approximation scheme for SVEs with multiexponential kernels which stay in some convex domain, and this is used for the multi-exponential Heston model. A. Alfonsi and A. Kebaier study the weak error for the approximation of Stochastic Volterra Equations and processes with rough paths.

8.10.2 Central limit theorem for the stratified resampling mechanism

In [41], R. Flenghi and B. Jourdain prove the joint convergence in distribution of q variables modulo one obtained as partial sums of a sequence of i.i.d. square integrable random variables multiplied by a common factor given by some function of an empirical mean of the same sequence. The limit is uniformy distributed over $[0,1]^q$. To deal with the coupling introduced by the common factor, we assume that the joint distribution of the random variables has a non zero component absolutely continuous with respect to the Lebesgue measure, so that the convergence in the central limit theorem for this sequence holds in total variation distance. While this result provides a generalization of Benford's law to a data adapted mantissa, the main motivation is the derivation of a central limit theorem for the stratified resampling mechanism.

The stratified resampling mechanism is one of the resampling schemes commonly used in the resampling steps of particle filters. In [40], R. Flenghi and B. Jourdain prove a central limit theorem for this mechanism under the assumption that the initial positions are independent and identically distributed and the weights proportional to a positive function of the positions such that the image of their common distribution by this function has a non zero component absolutely continuous with respect to the Lebesgue measure. This result relies on the convergence in distribution of the fractional part of partial sums of the normalized weights to some random variable uniformly distributed on $[0,1]$, which is established in hal-04338337. Under the conjecture that a similar convergence in distribution remains valid at the next steps of a particle filter which alternates selections according to the stratified resampling mechanism and mutations according to Markov kernels, they provide an inductive formula for the asymptotic variance of the resampled population after n steps. They perform numerical experiments which support the validity of this formula.

8.10.3 Abstract Malliavin calculus and convergence in total variation

In [69], V. Bally and his PhD student Yifen Qin obtain total variation distance result between a jump-equation and its Gaussian approximation by Malliavin calculus techniques.

They approximate the invarient measure of a Markov process, solution of a stochastic equation with jumps by using a Euler scheme with decreasing step introduced by D Lamberton and G Pages in the early 2000 in the case of diffusion processes driven by Brownian motion. The novelty here is that They deal with jump processes. Under appropiate non degeneracy hypothesis, they have estimated the error in total variation distance and also proved convergence of the density functions.

A. Alfonsi, V. Bally and A Kohatzu Higa (Ritzumikan University) are working on a continuation of the above mentioned work on the approximation of the invarient measure for some non linear stochastic differential equations of Mc Kean Vlasov and Bozmann type.

8.10.4 Numerical approximation of American/Bermudean options

A. Alfonsi, J. Lelong and A. Kebaier are working on a numerical method to price American options based on the dual representation introduced by Rogers (2002).

8.10.5 Sewing Lemma

A. Alfonsi and V. Bally have proposed a new approach based on the sewing lemma on the Wasserstein Space to study existence and uniqueness of solutions of the Boltzmann equation [16]. They are now working with L. Caramellino (Roma Univ) to extend their results by using the stochastic sewing lemma recently proposed by Khoa Lê (2020).

8.11 Deep learning for large dimensional financial problems

Participants: A. Zanette, L. Goudenège, A. Molent, A. Kebaier, J Ben Mohamed, E. Kaabar, C Ovo.

We pursue the development of Machine Learning an Deep learnig techniques in particular for McKean-Vlasov models of singular stochastic volatility, robust utility maximization, and high-dimensional optimal stopping problems. The corresponding algorithms are implemented in the Premia software.

8.12 Quantum Computing in Finance

Participants: A. Zanette, L. Goudenège, A Espa, A. Molent, A. Sulem.

We have started to construct a pricing framework using the Qiskit framework. Comparison of efficiency with other techniques has been done.

9 Bilateral contracts and grants with industry

9.1 Bilateral contracts with industry

- Consortium PREMIA, Crédit Agricole Corporate Investment Bank (CA - CIB) - INRIA
- CIFRE agreement AXA Climate/ENPC PhD thesis of Nerea Vadillo Fernandez. Supervisor: A. Alfonsi

9.2 Bilateral grants with industry

- Chair Ecole Polytechnique-Ecole des Ponts ParisTech-Sorbonne Université-Société Générale "Financial Risks" of the Risk fondation.

  Participants: Aurélien Alfonsi, Benjamin Jourdain.

  Postdoctoral grant : G.Szulda
- Chair Ecole des Ponts ParisTech - Université Paris-Cité - BNP Paribas "Futures of Quantitative Finance"

  Participants: Julien Guyon.
- Institut Europlace de Finance Louis Bachelier and Labex Louis Bachelier grant : "Multi-Agent Reinforcement Learning in Large Financial Networks with Heterogeneous Interactions" from November 2023.

  Participants: Agnès Sulem, , Hamed Amini.

10 Partnerships and cooperations

Participants: Antonino Zanette, Benjamin Jourdain, Agnès Sulem.

10.1 International research visitors

International visits to the team

- Hamed Amini, Associate Professor, University of Florida, Research stay.
- Xiao Wei, Associate Professor, China Institute for Actuarial Science(CIAS), Beijing Research stay in connection with Premia.
- Gudmund Pammer, Postdoctoral fellow, ETH Zurich, lecture.
- Antonino Zanette, rofessor, University of Udine, Reserach stays in connectiion with Premia.

10.1.1 Visits to international teams

Research stays abroad

- A. Zanette visited Prof.Lucia Caramellino, Department of Mathematics ,University of Roma Tor Vergata to work on pricing issues in the Sabr model.
- Z. Cao visited Prof. Hamed Amini, Univ. of Florida, June 2023.
- K. Shao visited Prof Mathieu Laurière, NYU Shangai, October 23, 2023 - December 23, 2023.

10.2 National initiatives

- FMSP (Fondation Sciences Mathématiques de Paris) PhD grants :
  - Cofund MathInParis program: K. Shao (2021 - Present)(INRIA)
  - DIM Math Innov: Z. Cao (2020- Present)(INRIA)
  - DIM Math Innov: Y. Qin (2020- Present)(UGE)
- Labex Bezout

11 Dissemination

Participants: Project-team MathRisk.

11.1 Promoting scientific activities

- A. Alfonsi

  Co-organizer of the Mathrisk seminar "Méthodes stochastiques et finance"

  Co-organizer of the Bachelier (Mathematical Finance) seminar (IHP, Paris).
- V. Bally

  Organizer of the seminar of the LAMA laboratory, Université Gustave Eiffel.
- A. Sulem

  Co-organizer of the seminar INRIA-MathRisk /Université Paris Diderot LPSM "Numerical probability and mathematical finance"

11.1.1 Scientific events: organisation

- The members of MathRisk with Prof. Antonino Zanette (Univ of Udine) organized an international conference on numerical methods in finance, 14-16 June 2023, in Udine (Italy) to celebrate the 25th anniversary of the software Premia and the team MathRisk.
- J. Guyon organized the minisymposium "Volatility modeling in finance" during the 2023 ICIAM Conference, Tokyo, Aug 2023.
- J. Guyon organized the minisymposium "Recent Advances in Volatility modeling" during the 2023 SIAM Conference on Financial Mathematics & Engineering, Philadelphia, June 2023.
- A. Sulem and A. Zanette organized the Premia meeting for the delivery of the 25th release of the software to the Consortium. Talks by A. Zanette (Univ Udine), A. Molente (Univ Udine), A. Kbaier (Univ Evry), L. Goudenege (CNRS), 30 September 2023, INRIA Paris.
- A. Sulem organized the joint seminar MathRisk/LPSM 19 October 2023, INRIA Paris. Talks by Gudmund PAMMER (ETH Zurich), Mehdi TALBI (LPSM), Robert DENKERT (HU Berlin), A. ALFONSI (CERMICS/ENPC).

Member of the organizing committees

- B. Jourdain : Member of the organizing committee of the 14th international conference on Monte Carlo methods and applications, Sorbonne University, 26-30 June 2023
- J. Guyon: member of the scientific committee of the 2023 SIAM Conference of Financial Mathematics.

11.1.2 Journal editorship

Member of the editorial boards

- A. Alfonsi

  Member of the editorial board of the Book Series "Mathématiques et Applications" of Springer.

- J. Guyon

  Associate editor of

  - Finance and Stochastics
  - Quantitative Finance
  - SIAM Journal on Financial Mathematics
  - Journal of Dynamics and Games

- B. Jourdain

  Associate editor of

  - ESAIM : Proceedings and Surveys
  - Stochastic Processes and their Applications (SPA)
  - Stochastic and Partial Differential Equations : Analysis and Computations

- D. Lamberton

  Associate editor of

  - Mathematical Finance,
  - ESAIM Probability & Statistics

- A. Sulem

  Associate editor of

  - *Mathematics*, (Financial Mathematics Section)
  - *Journal of Mathematical Analysis and Applications* (JMAA)
  - *SIAM Journal on Financial Mathematics* (SIFIN)

Reviewer - reviewing activities

- J. Guyon : Reviewer for Finance and Stochastics, Quantitative Finance.
- B. Jourdain : Reviewer for *Mathematical Reviews*
- A. Sulem: Reviewer for *Mathematical Reviews*

11.1.3 Seminars and Conferences

- A. Alfonsi
  - 19 10 2023: "Nonnegativity preserving convolution kernels. Application to Stochastic Volterra Equations in closed convex domains and their approximation.", Séminaire commun MathRisk-LPSM
  - 09 11 2023: "How many inner simulations to compute conditional expectations with least-square Monte Carlo?", Séminaire LPSM.
  - 20 12 2023: "How many inner simulations to compute conditional expectations with least-square Monte Carlo?", Séminaire de la chaire Futures of quantitative finance.
- V. Bally
  - International Conference on Malliavin Calculus and Related Topics June 12, 2023 - June 16, 2023 Esch-sur-Alzette, Luxembourg Exposé: Construction of Boltzmann and Mc-Kean Vlasov Type Flows
  - A Random Walk in the Land of Stochastic Analysis and Numerical Probability, Conference in honor of Denis Talay 3-7 January 2022 - CIRM, Marseille. Exposé: Construction of Boltzmnn and Mc-Kean Vlasov Type Flows
- Z. Cao
  - MathRisk seminar, ENPC/CERMICS, January 30th 2023
  - SIAM Conference on Financial Mathematics and Engineering, Philadelphia, USA, June 6-9, 2023
  - Conference on stochastic control and financial engineering, Princeton, June 2023
  - SPA Conference, Lisbon , July 2023
- B. Jourdain
  - Workshop Mean Field interaction with singular kernels and their approximations, IHP Paris, 18-22 December 2023: Weak and strong error analysis for systems of particles with mean-field rank-based interaction in the drift
  - Talks in Financial and Insurance Mathematics, ETH Zürich, 12 October 2023: Convexity propagation and convex ordering of one-dimensional stochastic differential equations
  - Workshop SDEs with Low-regularity coefficients: Theory and Numerics, Torino, 21-22 September 2023: Convergence rate of the Euler-Maruyama scheme applied to diffusion processes with Lq-Lp drift coefficient and additive noise
  - Workshop A Random Walk in the Land of Stochastic Analysis and Numerical Probability, Marseille, 4-8 September 2023: Convergence rate of the Euler-Maruyama scheme applied to diffusion processes with Lq-Lp drift coefficient and additive noise
  - Workshop Stochastic processes, metastability and applications, Nancy, 31 May-2 June 2023: Central limit theorem for nonlinear functionals of empirical measures and fluctuations of mean-field interacting particle systems
  - Hong Kong - Singapore joint Seminar Series in Financial Mathematics/Engineering, 25 May 2023: Approximation of martingale couplings on the real line and stability in robust finance
- J. Guyon
  - QuantMinds International, London, Nov 2023, Minicourse
  - Research in Options 2023, Rio de Janeiro, Dec 2023, minicourse
  - Workshop "Frontiers in Stochastic Modelling for Finance", Palermo, October 2023.
  - Workshop "Stochastics around Finance", Kanazawa, August 2023.

  - 10th International Congress on Industrial and Applied Mathematics (ICIAM 2023), Tokyo, August 2023.
  - 7th International Conference on Mathematics in Finance, Kruger National Park, South Africa, July 2023.
  - BNP Paribas, Paris, Seminar of the BNP Paribas chair "Futures of Quantitative Finance", June 2023.
  - MathSport International 2023, Budapest, June 2023.
  - MathRisk Conference on Numerical Methods in Finance, Udine, June 2023.
  - SIAM Conference on Financial Mathematics and Engineering, Philadelphia, June 2023.
  - Rough volatility workshop 2023, Isle of Skye, May 2023.
  - Quantitative Finance, conference in honor of Michael Demspter's 85th birthday, Cambridge, UK, April 2023.
  - Quantitative Finance Workshop 2023, Gaeta, April 2023.
  - Bank of America, New York, Invited seminar, April 2023.
  - Barclays Capital, New York, Invited seminar, March 2023.
  - Columbia University, New York, Columbia Mathematical Finance Seminar Series, March 2023.
  - Bloomberg, New York, Keynote speaker at BBQ (Bloomberg Quant Seminar), March 2023.
  - Imperial College London, Finance and Stochastics Seminar, March 2023.
  - BNP Paribas, Paris, Kickoff event of the BNP Paribas chair "Futures of Quantitative Finance", March 2023.
  - Capital Fund Management, Paris, EconoPhysiX seminar, February 2023.
  - Capital Fund Management, Paris, Seminar, January 2023.

- D. Lamberton
  - Régularité de la frontière d'exercice : une approche probabiliste. Groupe de travail modélisation stochastique et finance, Ecole des Ponts/Université Gustave Eiffel, January 16th, 2023.
  - Regularity results in optimal stopping: a probabilistic approach. A Random Walk in the Land of Stochastic Analysis and Numerical Probability, in honor of Denis Talay. Luminy, September 2023.

- K. Shao
  - Ceremade Young researchers' days 2023, Paris, France. (June 1-2, 2023)
  - MathRisk Conference on Numerical Methods in Finance, Udine, Italy. (June 14-16, 2023)
  - Conference: New Monge Problems and Applications, Champs-sur-Marne, France. (September 14-15, 2023)

11.1.4 Scientific expertise

- A. Alfonsi

  Member of the council of the Bachelier Finance Society

- A. Sulem

  Member of the Nominating Committee of the Bachelier Finance Society

11.1.5 Research administration

- A. Alfonsi
  - Deputy director of the CERMICS.
  - In charge of the Master “Finance and Data” at Ecole des Ponts.
- V. Bally
  - Responsible of the Master 2, option finance, Université Gustave Eiffel
  - Member of the LAMA committee, UGE.
- B. Jourdain

  Deputy head of the Labex Bézout.
- A. Sulem
  - Member of the Scientific Committee of AMIES(Agence pour les Mathématiques en Interaction avec l’Entreprise et la Société)
  - Member of the Committee for INRIA international Chairs

11.2 Teaching - Supervision - Juries

11.2.1 Teaching

- A. Alfonsi
  - “Probabilités”, first year course at the Ecole des Ponts.
  - “Données Haute Fréquence en finance”, lecture for the Master at UPEMLV.
  - “Mesures de risque”, Master course of UPEMLV and Sorbonne Université.
  - Professeur chargé de cours at Ecole Polytechnique.
- V. Bally
  - Course "Taux d’Intêret" M2 Finance.
  - Course "Calcul de Malliavin et applications en finance" M2 Finance
  - Course "Analyse du risque" M2 Actuariat,
  - Course "Calcul Stochastiques" M2 Recherche
  - Course "Probabilités approfondies" M1
- J. Guyon
  - course "Probability Theory", 1st year ENPC
  - course "Volatility Modeling", Master MFD, 3rd year ENPC - UGE
  - course "Advanced Computational Methods in Finance", Master of Financial Engineering, Baruch College, City University of New York
  - J. Guyon, B. Liang : course "Nonlinear Option Pricing", Master MAFN, Columbia University
  - J. Guyon, F. Meunier : Project of the ENPC course TDLOG : Live probability calculator for the draws of the European football cups
- B. Jourdain
  - course "Mathematical finance", 2nd year ENPC
  - course "Monte-Carlo methods", 3rd year ENPC and Research Master MathÈmatiques et Application, university Gustave Eiffel

  - course "Machine Learning 1", MSC Data Science for Business, X-HEC
  - course "Monte-Carlo Markov chain methods and particle algorithms", Research Master ProbabilitÈs et ModËles AlÈatoires, Sorbonne UniversitÈ

- D. Lamberton
  - "Arbitrage, volatilité et gestion de portefeuille", master 2 course, Université Gustave Eiffel.
  - "Intégration et probabilités", L3 course, Université Gustave Eiffel.
  - "Modélisation et probabilités", L2 course and exercises, Université Gustave Eiffel.

- A. Sulem
  - Master of Mathematics, Université du Luxembourg, Responsible of the course on "Numerical Methods in Finance", and lectures (22 hours)

11.2.2 Supervision

- Postdoral fellows
  - Guido Gazzani (from May 2023); ENPC, Advisor: J. Guyon
  - Guillaume Szulda, ENPC, Advisor: A. Alfonsi (From January 2023 to December 2024)

- PhD defended
  - Zhonguyan Cao, "Dynamics and Stability of Complex Financial networks", Université Paris-Dauphine, Supervisor: Agnès Sulem, Dim Mathinnov doctoral allocation, defended September 20th 2023, Université Paris-Dauphine
  - Roberta Flenghi "Central limit theorems for nonlinear functionals of the empirical measure and for stratified resampling", Supervisor: B. Jourdain, defended on December 20 2023
  - Yfen Qin, "Regularity properties for non linear problems", Dim-Mathinnov doctoral allocation, supervisor: V. Bally, defended June 23 2023, Université Gustave Eiffel

- PhD in progress
  - Elise Devey (started October 2023), "Graphon Mean-Field Games and Renewable Energy Systems", Supervisor: Agnès Sulem, INRIA doctoral grant
  - Hervé Andrës (started in June 2021) "Dependence modelling in economic scenario generation for insurance", supervised by B. Jourdain
  - Faten Ben Said (CIFRE EDF, co-advisor: Julien Reygner), “Caractérisation et prise en compte des dépendances statistiques dans le cadre d’applications de dynamique sédimentaire”, started in March 2023, supervised by A. Alfonsi
  - Kexin Shao (started in october 2021) "Martingale optimal transport and financial applications", supervised by B. Jourdain and A. Sulem
  - Edoardo Lombardo, “High order numerical approximation for some singular stochastic processes and related PDEs”, started in November 2020, International PhD, advisors: Aurélien Alfonsi and Lucia Caramellino (Tor Vegata Roma University),
  - Nerea Vadillo Fernandez (CIFRE AXA Climate), “Risk valuation for weather derivatives in index-based insurance”, started in November 2020, supervised by A. Alfonsi

- Internship
  - Bryan Khan, UGE, "Applications of neural networks for optimal stopping problems", Master 2 report, Summer 2023, Advisor: D. Lamberton
  - Ben Mohamed Jihed, Ecole Polytechnique de Tunisie, 01/03/2023 to 31/08/2023, "Etude de la régularisation de modèles de McKean-Vlasov de volatilité stochastique locale singulière par des techniques de noyau régularisant et par Machine et Deep learning."" Advisor: A. Kbaier

- Eya Kaabar, Ecole Polytechnique de Tunisie, 01/03/2023 to 31/08/2023 "Développement d'une approche Machine Learning pour la maximisation robuste d'utilité en finance quantitative et implémentation dans le logiciel Premia". Advisor: A. Kbaier
- Adriano Todisco (Master MFD internship, April-August 2023), "The quintic Ornstein-Uhlenbeck volatility model that jointly calibrates SPX & VIX smiles", Advisor: J. Guyon
- Caleb Ovo ENSTA Paris, Development and implementation in the software Premia of numerical methods for pricing and hedging financial derivatives presented in [1908.01602 - Solving high-dimensional optimal stopping problems using deep learning], [2202.02717 - Learning the random variables in Monte Carlo simulations with stochastic gradient descent: Machine learning for parametric PDEs and financial derivative pricing], [2210.04645 - Optimal Stopping with Trees] et [Quantitative Finance - Empirical deep hedging]. Advisor: L. Goudenege
- Alessio Espa, Sorbonne Université, Development of quantum computing quantique in finance; Construction of a pricing framework using Qiskit; Comparison of efficiency with other techniques. Advisor: L. Goudenege

11.2.3 Juries

- A. Alfonsi
  - Member of the jury of the PhD thesis of Zhongyuan Cao "Systemic Risk, Complex Financial Networks and Graphon Mean Field Interacting Systems".
  - Jury president of the PhD thesis of Wanqing Wang "Approximation et simulation des équations différentielles stochastiques rétrogrades réfléchies, applications en finance".
  - Jury president of the PhD thesis of Mouna Ben Derouich "Improved Multi Level Monte Carlo Methods for Pricing Barrier Options in Finance".
- J. Guyon PhD thesis examination of Long Zhao (Columbia University, April 2023): Martingale Schrodinger bridges and optimal semistatic portfolios.
- B. Jourdain
  - Reviewer for the Habilitation of Zhenjie Ren, defended on January 4, University Paris-Dauphine,
  - Reviewer for the PhD thesis of Thomas Cavallazzi, defended on June 23, University Rennes 1,
  - PhD of Yoan Tardy, defended on june 29, Sorbonne university,
  - PhD of Arnaud Descours, defended on October 20, University Clermont Auvergne, President of the jury
- A. Sulem
  - Reviewer for the PhD thesis of Jeremy Chichportich, defended on March 10th 2023, LPSM, Sorbonne Université.

11.3 Popularization

11.3.1 Articles and contents

- J. Guyon: Un PSG-Bayern Munich en huitièmes de finale de la Ligue des champions ? Probable, mais…, Le Monde, December 18, 2023

11.3.2 Interventions

- J. Guyon: Live intervention (2 h 30 min) during the draws of the football European Cups on La chaine L'Equipe on December 18. football cup
- J. Guyon gave a one-hour interview to Risk magazine in August 2023, "Podcast: Julien Guyon on volatility modelling and World Cup draws". In this edition of Quantcast, Risk's quantitative podcast, Julien discusses volatility modeling and option pricing as well as World Cup draws and formats. Available at volatility modeling

12 Scientific production

12.1 Major publications

[1] A. Al Gerbi, B. Jourdain and E. Clément. 'Ninomiya-Victoir scheme: strong convergence, antithetic version and application to multilevel estimators'. In: *Monte Carlo Method and Applications* 22.3 (July 2016). https://arxiv.org/abs/1508.06492, pp. 197–228. URL: https://hal-enpc.archives-ouvertes.fr/hal-01188675.

[2] A. Alfonsi. *Affine Diffusions and Related Processes: Simulation, Theory and Applications*. 2015. DOI: 10.1007/978-3-319-05221-2. URL: https://hal-enpc.archives-ouvertes.fr/hal-03127212.

[3] A. Alfonsi and V. Bally. 'A generic construction for high order approximation schemes of semigroups using random grids'. In: *Numerische Mathematik* (2021). DOI: 10.1007/s00211-021-01219-2. URL: https://hal-enpc.archives-ouvertes.fr/hal-02406433.

[4] A. Alfonsi and P. Blanc. 'Dynamic optimal execution in a mixed-market-impact Hawkes price model'. In: *Finance and Stochastics* (Jan. 2016). https://arxiv.org/abs/1404.0648. DOI: 10.1007/s00780-015-0282-y. URL: https://hal-enpc.archives-ouvertes.fr/hal-00971369.

[5] A. Alfonsi, J. Corbetta and B. Jourdain. 'Sampling of probability measures in the convex order by Wasserstein projection'. In: *Annales de l'Institut Henri Poincaré (B) Probabilités et Statistiques* 56.3 (2020), pp. 1706–1729. DOI: 10.1214/19-AIHP1014. URL: https://hal.archives-ouvertes.fr/hal-01589581.

[6] A. Alfonsi, B. Jourdain and A. Kohatsu-Higa. 'Optimal transport bounds between the time-marginals of a multidimensional diffusion and its Euler scheme'. In: *Electronic Journal of Probability* (2015). https://arxiv.org/abs/1405.7007. URL: https://hal-enpc.archives-ouvertes.fr/hal-00997301.

[7] H. Amini, A. Minca and A. Sulem. 'A dynamic contagion risk model with recovery features'. In: *Mathematics of Operations Research* (24th Nov. 2021). DOI: 10.1287/moor.2021.1174. URL: https://hal.inria.fr/hal-02421342.

[8] H. Amini, A. Minca and A. Sulem. 'Control of interbank contagion under partial information'. In: *SIAM Journal on Financial Mathematics* 6.1 (Dec. 2015), p. 24. URL: https://hal.inria.fr/hal-01027540.

[9] V. Bally and L. Caramellino. 'Convergence and regularity of probability laws by using an interpolation method'. In: *Annals of Probability* 45.2 (2017), pp. 1110–1159. URL: https://hal-upec-upem.archives-ouvertes.fr/hal-01109276.

[10] A. Bouselmi and D. Lamberton. 'The critical price of the American put near maturity in the jump diffusion model'. In: *SIAM Journal on Financial Mathematics* 7.1 (May 2016). https://arxiv.org/abs/1406.6615, pp. 236–272. DOI: 10.1137/140965910. URL: https://hal-upec-upem.archives-ouvertes.fr/hal-00979936.

[11] R. Dumitrescu, M.-C. Quenez and A. Sulem. 'A Weak Dynamic Programming Principle for Combined Optimal Stopping/Stochastic Control with E^f-Expectations'. In: *SIAM Journal on Control and Optimization* 54.4 (2016), pp. 2090–2115. DOI: 10.1137/15M1027012. URL: https://hal.inria.fr/hal-01370425.

[12] R. Dumitrescu, M.-C. Quenez and A. Sulem. 'Game Options in an Imperfect Market with Default'. In: *SIAM Journal on Financial Mathematics* 8.1 (Jan. 2017), pp. 532–559. DOI: 10.1137/16M1109102. URL: https://hal.inria.fr/hal-01614758.

[13] M. Grigorova, M.-C. Quenez and A. Sulem. 'European options in a non-linear incomplete market model with default'. In: *SIAM Journal on Financial Mathematics* 11.3 (2nd Sept. 2020), pp. 849–880. DOI: 10.1137/20M1318018. URL: https://hal.archives-ouvertes.fr/hal-02025833.

[14] B. Jourdain. *Probabilités et statistique*. seconde édition. Ellipses, 2016. URL: https://hal.archives-ouvertes.fr/hal-03133840.

[15] B. Øksendal and A. Sulem. *Applied Stochastic Control of Jump Diffusions*. 3rd edition. Springer, Universitext, 2019, p. 436. DOI: 10.1007/978-3-030-02781-0. URL: https://hal.archives-ouvertes.fr/hal-02411121.

12.2 Publications of the year

International journals

[16] A. Alfonsi and V. Bally. 'Construction of Boltzmann and McKean Vlasov type flows (the sewing lemma approach)'. In: *The Annals of Applied Probability* 33.5 (1st Oct. 2023). DOI: 10.1214/22-AAP1894. URL: https://enpc.hal.science/hal-03241604.

[17] A. Alfonsi and A. Kebaier. 'Approximation of Stochastic Volterra Equations with kernels of completely monotone type'. In: *Mathematics of Computation* 93.346 (2nd Nov. 2023), pp. 643–677. DOI: 10.1090/mcom/3911. URL: https://enpc.hal.science/hal-03526905.

[18] A. Alfonsi, B. Lapeyre and J. Lelong. 'How many inner simulations to compute conditional expectations with least-square Monte Carlo?' In: *Methodology and Computing in Applied Probability* 25.3 (20th June 2023), p. 71. DOI: 10.1007/s11009-023-10038-x. URL: https://hal.science/hal-03770051.

[19] A. Alfonsi and E. Lombardo. 'High order approximations of the Cox-Ingersoll-Ross process semigroup using random grids'. In: *IMA Journal of Numerical Analysis* (24th Aug. 2023). DOI: 10.1093/imanum/drad059. URL: https://enpc.hal.science/hal-03791594.

[20] H. Amini, Z. Cao and A. Sulem. 'Limit Theorems for Default Contagion and Systemic Risk'. In: *Mathematics of Operations Research* (27th Dec. 2023). DOI: 10.1287/moor.2021.0283. URL: https://inria.hal.science/hal-03429191.

[21] M. Beiglböck, B. Jourdain, W. Margheriti and G. Pammer. 'Stability of the Weak Martingale Optimal Transport Problem'. In: *The Annals of Applied Probability* 33.6B (1st Dec. 2023). DOI: 10.1214/23-AAP1950. URL: https://hal.science/hal-03344429.

[22] R. Chen, R. Dumitrescu, A. Minca and A. Sulem. 'Mean-field BSDEs with jumps and dual representation for global risk measures'. In: *Probability, Uncertainty and Quantitative Risk* 8.1 (2023), pp. 33–52. DOI: 10.3934/puqr.2023002. URL: https://inria.hal.science/hal-02421316.

[23] J. Guyon. 'Dispersion-constrained martingale Schrödinger problems and the exact joint S&P 500/VIX smile calibration puzzle'. In: *Finance and Stochastics* 28.1 (17th Nov. 2023), pp. 27–79. DOI: 10.1007/s00780-023-00524-y. URL: https://hal.science/hal-04373379.

[24] J. Guyon and M. El Amrani. 'Does the Term-Structure of the At-the-Money Skew Really Follow a Power Law?' In: *Risk* (1st Aug. 2023). URL: https://hal.science/hal-04373382.

[25] J. Guyon and J. Lekeufack. 'Volatility is (mostly) path-dependent'. In: *Quantitative Finance* 23.9 (19th July 2023), pp. 1221–1258. DOI: 10.1080/14697688.2023.2221281. URL: https://hal.science/hal-04373380.

[26] J. Guyon and S. Mustapha. 'Neural Joint S&P 500/VIX Smile Calibration'. In: *Risk Magazine* (1st Dec. 2023). DOI: 10.2139/ssrn.4309576. URL: https://hal.science/hal-03932780.

[27] B. Jourdain, W. Margheriti and G. Pammer. 'Lipschitz continuity of the Wasserstein projections in the convex order on the line'. In: *Electronic Communications in Probability* 28.none (1st Jan. 2023). DOI: 10.1214/23-ECP525. URL: https://hal.science/hal-03768703.

[28] B. Jourdain and S. Menozzi. 'Convergence Rate of the Euler-Maruyama Scheme Applied to Diffusion Processes with L Q – L ρ Drift Coefficient and Additive Noise'. In: *The Annals of Applied Probability* (2023). URL: https://hal.science/hal-03223426.

[29] B. Jourdain and G. Pagès. 'Convex ordering for stochastic Volterra equations and their Euler schemes'. In: *Finance and Stochastics* (2023). URL: https://hal.science/hal-03862241.

Edition (books, proceedings, special issue of a journal)

[30] H. Amini, Z. Cao and A. Sulem, eds. *The Default Cascade Process in Stochastic Financial Networks*. ICAIF ’23: 4th ACM International Conference on AI in Finance. ACM, 2023, pp. 227–234. DOI: 10.2139/ssrn.4020598. URL: https://hal.science/hal-03830139.

Doctoral dissertations and habilitation theses

[31] Z. Cao. ‘Systemic risk, complex financial networks and graphon mean field interacting systems’. Université Paris sciences et lettres, 22nd Sept. 2023. URL: https://theses.hal.science/tel-04298637.

[32] R. Flenghi. ‘Central limit theorem over non-linear functionals of empirical measures and for the stratified resampling mechanism’. Ecole des Ponts, 20th Dec. 2023. URL: https://hal.science/tel-04360933.

[33] Y. Qin. ‘Approximation of the stochastic differential equation with jumps and convergence in total variation distance’. Université Gustave Eiffel, 23rd June 2023. URL: https://hal.science/tel-04231403.

Reports & preprints

[34] A. Alfonsi. *Nonnegativity preserving convolution kernels. Application to Stochastic Volterra Equations in closed convex domains and their approximation*. 15th Feb. 2023. URL: https://enpc.hal.science/hal-03991952.

[35] A. Alfonsi and N. Vadillo. *Risk valuation of quanto derivatives on temperature and electricity*. 2023. DOI: 10.48550/arXiv.2310.07692. URL: https://hal.science/hal-04358505.

[36] H. Amini, Z. Cao, A. Minca and A. Sulem. *Ruin Probabilities for Risk Processes in Stochastic Networks*. 13th Feb. 2023. DOI: 10.48550/arXiv.2302.06668. URL: https://hal.science/hal-04393897.

[37] H. Andrès, A. Boumezoued and B. Jourdain. *Implied volatility (also) is path-dependent*. 22nd Dec. 2023. URL: https://hal.science/hal-04362544.

[38] H. Andrès, A. Boumezoued and B. Jourdain. *Signature-based validation of real-world economic scenarios*. 8th Feb. 2023. URL: https://hal.science/hal-03740740.

[39] V. Bally and Y. Qin. *Approximation for the invariant measure with applications for jump processes (convergence in total variation distance)*. 31st May 2023. URL: https://hal.science/hal-04112474.

[40] R. Flenghi and B. Jourdain. *Central limit theorem for the stratified resampling mechanism*. 4th Aug. 2023. URL: https://hal.science/hal-04338384.

[41] R. Flenghi and B. Jourdain. *Convergence to the uniform distribution of vectors of partial sumsmodulo one with a common factor*. 4th Aug. 2023. URL: https://hal.science/hal-04338337.

[42] B. Jourdain and G. Pagès. *Convex ordering of solutions to one-dimensional SDEs*. 15th Dec. 2023. URL: https://hal.science/hal-04349619.

[43] B. Jourdain and G. Pammer. *An extension of martingale transport and stability in robust finance*. 19th Apr. 2023. URL: https://hal.science/hal-04075283.

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