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2023
ACTIVITY REPORT

Project-Team

MCTAO

Mathematics for Control, Transport and Applications

IN COLLABORATION WITH: Laboratoire Jean-Alexandre Dieudonné (JAD)

DOMAIN

Applied Mathematics, Computation and Simulation

THEME

Optimization and control of dynamic systems

Contents

Project-Team MCTAO

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Keywords

Computer sciences and digital sciences

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1 Team members, visitors, external collaborators

Research Scientists

- Jean-Baptiste Pomet [Team leader, INRIA, Senior Researcher, HDR]
- Ivan Beschastnyi [INRIA, Researcher, from Dec 2023]
- Lamberto Dell'Elce [INRIA, Researcher]
- Ludovic Sacchelli [INRIA, Researcher]

Faculty Member

- Jean-Baptiste Caillau [UNIV COTE D'AZUR, Professor, HDR]

PhD Students

- Adel Malik Annabi [UNIV COTE D'AZUR]
- Antonin Bavoil [CNRS, from Oct 2023]
- Frank De Veld [INRIA]
- Sandrine Gayrard [SEGULA, until Sep 2023]
- Alesia Herasimenka [UNIV COTE D'AZUR, until Nov 2023]

Interns and Apprentices

- Antonin Bavoil [CNRS, Intern, from Mar 2023 until Sep 2023]
- Martin Fleurial [INRIA, Intern, from Mar 2023 until Aug 2023]

Administrative Assistant

- Claire Senica [INRIA]

External Collaborators

- Bernard Bonnard [Univ. de Bourgogne, founding member of the team, HDR]
- Olivier Cots [TOULOUSE INP, HDR]
- Thierry Dargent [Thales Alenia Space, Cannes]
- Joseph Gergaud [TOULOUSE INP, HDR]
- Jérémy Rouot [Université de Bretagne Occidentale, Brest]

2 Overall objectives

Our goal is to develop methods in geometric control theory for nonlinear systems, mostly finite dimensional, and to transfer our expertise through real applications of these methods. The methodological developments range from feedback control and observers to optimal control, extending to fields like sub-Riemannian geometry. Optimal control leads to developments in Hamiltonian dynamics, and also requires sophisticated numerics, to which the team contributes too. Dynamical systems and modeling are also part of the background of the team.

Our primary domain of industrial applications in the past years has been space engineering, in particular using optimal control and stabilization techniques for mission design: orbit transfer or rendezvous problems in the gravity field of a single body (typically satellites around the earth), interplanetary missions and multi body problems, or control design of solar sails, where propulsion is drastically constrained.

The team also has continued involvement with applications regarding human bio-mechanics (muscle stimulation), and various modeling and control questions in biology (Lotka-Volterra models, bacterial growth, microbiome models, networks of chemical reaction...) The list is not exhaustive; past domains of application include swimming at low Reynolds number (micro-swimmers) and control of quantum systems for Magnetic Resonance Imaging.

3 Research program

3.1 Control Problems

McTAO's major field of expertise is control theory in the broad sense. Let us give an overview of this field.

Modeling. Our effort is directed toward efficient methods for the control of real (physical) *systems*, based on a *model* of the system to be controlled. Choosing accurate models yet simple enough to allow control design is in itself a key issue. The typical continuous-time model is of the form $\mathrm{d}x/\mathrm{d}t = f(x,u)$ where x is the *state*, ideally finite dimensional, and u the *control*; the control is left free to be a function of time, or a function of the state, or obtained as the solution of another dynamical system that takes x as an input. Modeling amounts to deciding the nature and dimension of x, as well as the dynamics (roughly speaking the function f). Connected to modeling is identification of parameters when a finite number of parameters are left free in "f".

Controllability, path planning. Controllability is a property of a control system (in fact of a model) that two states in the state space can be connected by a trajectory generated by some control, here taken as an explicit function of time. Deciding on local or global controllability is still a difficult open question in general. In most cases, controllability can be decided by linear approximation, or non-controllability by "physical" first integrals that the control does not affect. For some critically actuated systems, it is still difficult to decide local or global controllability, and the general problem is anyway still open. Path planning is the problem of constructing the control that actually steers one state to another.

Optimal control. In optimal control, one wants to find, among the controls that satisfy some constraints at initial and final time (for instance given initial and final state as in path planning), the ones that minimize some criterion. This is important in many control engineering problems, because minimizing a cost is often very relevant. Mathematically speaking, optimal control is the modern branch of the calculus of variations, rather well established and mature [78, 45, 31], but still displaying important and hard open questions. In the end, in order to actually compute these controls, ad-hoc numerical schemes have to be derived for effective computations of the optimal solutions. See more about our research program in optimal control in section 3.2.

Feedback control. In the above two paragraphs, the control is an explicit function of time. To address in particular the stability issues (sensitivity to errors in the model or the initial conditions for example), the control has to be taken as a function of the (measured) state, or part of it. This is known as closed-loop control; it must be combined with optimal control in many real problems. On the problem of stabilization, there is longstanding research record from members of the team, in particular on the construction of "Control Lyapunov Functions", see [65, 79]. It may happen that only part of the state is accessible at any one time, because of physical or engineering constraints. In that case, a popular strategy is to pair feedback methods with dynamic estimation of the state, creating so-called output feedback loops. Simultaneous feedback control and estimation can become a major hurdle for nonlinear systems, see [55, 82].

Classification of control systems. One may perform various classes of transformations acting on systems, or rather on models. The simpler ones come from point-to-point transformations (changes of variables) on the state and control. More intricate ones consist in embedding an extraneous dynamical system into the model. These are dynamic feedback transformations that change the dimension of the state. In most problems, choosing the proper coordinates, or the right quantities that describe a phenomenon, sheds light on a path to the solution; these proper choices may sometimes be found from an understanding of the modeled phenomenons, or it can come from the study of the geometry of the equations and the transformation acting on them. This justifies the investigations of these transformations on models for themselves. These topics are central in control theory; they are present in the team, see for instance the classification aspect in [49] or —although this research has not been active very recently— the study [77] of dynamic feedback and the so-called "flatness" property [68]. Likewise, classification tools such as feedback invariants [39] are still currently in use in the team (see, for instance, [22]).

3.2 Optimal Control and its Geometry

Let us detail our research program concerning optimal control. Relying on Hamiltonian dynamics is now prevalent, instead of the Lagrangian formalism in classical calculus of variations. The two points of view run parallel when computing geodesics and shortest path in Riemannian Geometry for instance, in that there is a clear one-to-one correspondance between the solutions of the geodesic equation in the tangent bundle and the solution of the Pontryagin Maximum Principle in the cotangent bundle. In most optimal control problems, on the contrary, due to the differential constraints (velocities of feasible trajectories do not cover all directions in the state space), the Lagrangian formalism becomes more involved, while the Pontryagin Maximum Principle keeps the same form, its solutions still live in the cotangent bundle, their projections are the extremals, and a minimizing curve must be the projection of such a solution.

Cut and conjugate loci. The cut locus —made of the points where the extremals lose optimality— is obviously crucial in optimal control, but usually out of reach (even in low dimensions), and anyway does not have an analytic characterization because it is a non-local object. Fortunately, conjugate points —where the extremals lose *local* optimality— can be effectively computed with high accuracy for many control systems. Elaborating on the seminal work of the Russian and French schools (see [81, 30, 32] and [50] among others), efficient algorithms were designed to treat the smooth case. This was the starting point of a series of papers of members of the team culminating in the outcome of the *cotcot* software [44], followed by the *HamPath* [56] code. Over the years, these codes have allowed for the computation of conjugate loci in a wealth of situations including applications to space mechanics, quantum control, and more recently swimming at low Reynolds number. With in mind the two-dimensional analytic Riemannian framework, a heuristic approach to the global issue of determining cut points is to search for singularities of the conjugate loci; this line is however very delicate to follow on problems stemming from applications in three or more dimensions (see *e.g.* [57] and [41]). In all these situations, the fundamental object underlying the analysis is the curvature tensor. In Hamiltonian terms, one considers the dynamics of subspaces (spanned by Jacobi fields) in the Lagrangian Grassmannian [29]. This point of view withstands generalizations far beyond the smooth case: In L^1-minimization, for instance, discontinuous curves in the Grassmannian have to be considered (instantaneous rotations of Lagrangian subspaces still obeying symplectic rules [62]). The cut locus is a central object in Riemannian geometry, control and optimal transport. This is the motivation for a series of conferences on "The cut locus: A bridge over differential geometry, optimal control, and transport", co-organized by team members and Japanese colleagues.

Riemann and Finsler geometry. Studying the distance and minimizing geodesics in Riemannian Geometry or Finsler Geometry is a particular case of optimal control, simpler because there are no differential constraints; it is studied in the team for the following two reasons. On the one hand, after some tranformations, like averaging or reduction, some more difficult optimal control problems lead to a Riemann or Finsler geometry problem. On the other hand, optimal control, mostly the Hamiltonian setting, brings a fresh viewpoint on problems in Riemann and Finsler geometry. On Riemannian ellipsoids of revolution, the optimal control approach allowed to decide on the convexity of the injectivity domain, which, associated with non-negativity of the Ma-Trudinger-Wang curvature tensor, ensures continuity of the optimal transport on the ambient Riemannian manifold [66, 67]. The analysis in the oblate geometry

[42] was completed in [60] in the prolate one, including a preliminary analysis of non-focal domains associated with conjugate loci. Averaging in systems coming from space mechanics control with L^2-minimization yields a Riemannian metric, thoroughly computed in [40] together with its geodesic flow; in reduced dimension, its conjugate and cut loci were computed in [43] with Japanese Riemannian geometers. Averaging the same systems for minimum time yields a Finsler Metric, as noted in [38]. In [48], the geodesic convexity properties of these two types of metrics were compared. When perturbations (other than the control) are considered, they introduce a "drift", *i.e.* the Finsler metric is no longer symmetric.

Sub-Riemannian Geometry. Optimal control problems that pertain to sub-Riemannian Geometry bear all the difficulties of optimal control, like the role of singular/abnormal trajectories, while having some useful structure. They lead to many open problems, see the monograph [75] for an introduction. The sub-Riemannian problem can be encoded by a non-linear control system with no drift, subjected to a quadratic energy minimization objective. This allows the sub-Riemannian problem to serve as rich model spaces for optimal control. The interest of sub-Riemannian geometry can go beyond these aspects however. It was proved by Hormander in 1967 [73] that local controllability of the system (given in terms of Lie-brackets of vector fields) is equivalent to sub-ellipticity of a second order differential operator associated with the vector fields. In this way, sub-Riemannian geometry acts as a bridge between elements of analysis of PDEs and geometric control theory. For instance, many recent works focus on framing properties of sub-elliptic operators in terms of minimizers of the optimal control problem (such as the influence of cut and conjugate points on diffusion asymptotics [36]). This link even allowed to successfully introduce concepts of sub-elliptic diffusions in computer vision algorithms thanks to sub-Riemannian geometric structures identified in mammal visual mechanisms [52].

Small controls and conservative systems, averaging. Using averaging techniques to study small perturbations of integrable Hamiltonian systems is as old an idea as celestial mechanics. It is very subtle in the case of multiple periods but more elementary in the single period case, here it boils down to taking the average of the perturbation along each periodic orbit [33, 80]. This line of research stemmed out of applications to space engineering (see Section 4.1): the control of the super-integrable Keplerian motion of a spacecraft orbiting around the Earth is an example of a slow-fast controlled system. Since weak propulsion is used, the control itself acts as a perturbation, among other perturbations of similar magnitudes: higher order terms of the Earth potential (including J_2 effect, first), potential of more distant celestial bodies (such as the Sun and the Moon), atmospheric drag, or even radiation pressure. Properly qualifying the convergence properties (when the small parameter goes to zero) is important and is made difficult by the presence of control. In [38], convergence is seen as convergence to a differential inclusion; this applies to minimum time; a contribution of this work is to put forward the metric character of the averaged system by yielding a Finsler metric (see Section 3.2). Proving convergence of the extremals (solutions of the Pontryagin Maximum Principle) is more intricate. In [59], standard averaging ([33, 80]) is performed on the minimum time extremal flow after carefully identifying slow variables of the system thanks to a symplectic reduction. This alternative approach allows to retrieve the previous metric approximation, and to partly address the question of convergence. Under suitable assumptions on a given geodesic of the averaged system (disconjugacy conditions, namely), one proves existence of a family of quasi-extremals for the original system that converge towards the geodesic when the small perturbation parameter goes to zero. This needs to be improved, but convergence of all extremals to extremals of an "averaged Pontryagin Maximum Principle" certainly fails. In particular, one cannot hope for C^1-regularity on the value function when the small parameter goes to zero as swallowtail-like singularities due to the structure of local minima in the problem are expected. (A preliminary analysis has been made in [58].)

Optimality of periodic solutions/periodic controls. When seeking to minimize a cost with the constraint that the controls and/or part of the states are periodic (and with other initial and final conditions), the notion of conjugate points is more difficult than with straightforward fixed initial point. In [46], for the problem of optimizing the efficiency of the displacement of some micro-swimmers with periodic deformations, we used the sufficient optimality conditions established by R. Vinter's group [84, 70] for systems with non unique minimizers due to the existence of a group of symmetry (always present with a periodic minimizer-candidate control). This takes place in a long term collaboration with P. Bettiol

(Univ. Bretagne Ouest) on second order sufficient optimality conditions for periodic solutions, or in the presence of higher dimensional symmetry groups, following [84, 70]. Another question relevant to locomotion is the following. Observing animals (or humans), or numerically solving the optimal control problem associated with driftless micro-swimmers for various initial and final conditions, we remark that the optimal strategies of deformation seem to be periodic, at least asymptotically for large distances. This observation is the starting point for characterizing dynamics for which some optimal solutions are periodic, and asymptotically attract other solutions as the final time grows large; this is reminiscent of the "turnpike theorem" (classical, recently applied to nonlinear situations in [83]).

3.3 Software

Optimal control applications (but also the development of theory where numerical experiments can be very enlightening) require many algorithmic and numerical developments that are an important side of the team activity. We develop on-demand algorithms and pieces of software, for instance we have to interact with a production software developed by Thales Alenia Space. A strong asset of the team is the interplay of its expertise in geometric control theory with applications and algorithms, and the team has a long-lasting commitment to the development of numerical codes for the efficient resolution of optimal control problems. Methods for solving optimal control problems with ordinary differential equations more or less fall into three main categories. Dynamic Programming (or Hamilton Jacobi Bellman method) computes the global optimum but suffers from high computational costs, the so-called *curse of dimensionality*. Indirect methods based on Pontryagin Maximum Principle are extremely fast and accurate but often require more work to be applied, in terms of mathematical analysis and a priori knowledge of the solution; this kind of fine geometrical analysis is one of the strong know-how of McTAO. Direct transcription methods offer a good tradeoff between robustness and accuracy and are widely used for industrial applications. For challenging problems, an effective strategy is to start with a direct method to find a first rough solution, then refine it through an indirect method. We develop this further in a book chapter [17] published this year. Such a combined approach has been for instance used between McTAO, the former COMMANDS team (Inria Saclay), and CNRS team APO (Université Toulouse, CNRS, ENSEEIHT) for the optimization of contrast in medical imaging (MRI), and fuel-effective trajectories for airplanes. This combination of direct and indirect methods has a lot of interest to solve optimal control problems that contain state or control constraints. In the collaborations mentioned above, the interfacing between the two solvers BOCOP and HamPath were done manually by *ad hoc* `python` or `matlab` layers. In collaboration with COMMANDS and colleagues from ENSEEIHT, McTAO leads the ct: control toolbox project whose goal is to interoperate these solvers using a high level common interface. The project is an Inria Sophia ADT[1] (2019-) in AMDT[1] mode supported by Inria Sophia SED. The last sprint session, closing the project, is planned for February 2023.

4 Application domains

4.1 Aerospace Engineering

Participants: Jean-Baptiste Caillau, Thierry Dargent, Lamberto Dell'Elce, Frank de Veld, Alesia Herasimenka, Jean-Baptiste Pomet.

Space engineering is very demanding in terms of safe and high-performance control laws. It is therefore prone to fruitful industrial collaborations. McTAO now has an established expertise in space and celestial mechanics. Our collaborations with industry are mostly on orbit transfer problems with low-thrust propulsion. It can be orbit transfer to put a commercial satellite on station, in which case the dynamics are a Newtonian force field plus perturbations and the small control. There is also, currently, a renewed interest in low-thrust missions such as Lisa Pathfinder (ESA mission towards a Lagrange point of the Sun-Earth system) or BepiColombo (joint ESA-JAXA mission towards Mercury). Such missions look

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[1] ADT is the name of software development actions supported by the service "SED", by devoting some engineers to these projects. AMDT means that a group of engineers works on the project part time, rather than sending one engineer in the team.

more like a controlled multibody system. In all cases the problem involves long orbit transfers, typically with many revolutions around the primary celestial body. When minimizing time, averaging techniques provide a good approximation. Another important criterion in practice is fuel consumption minimization (crucial because only a finite amount of fuel is onboard a satellite for all its “life”), which amounts to L^1-minimization. Both topics are studied by the team. We have a steady relationship with CNES and Thales Alenia Space (Cannes), that have financed or co-financed 4 PhDs and 2 post-docs in the decade and are a source of inspiration even at the methodological level. Team members also have connections with Airbus-Safran (Les Mureaux) on launchers.

Some of the authoritative papers in the field were written by team members, with an emphasis on the geometric analysis and on algorithms (coupling of shooting and continuation methods). There are also connections with peers more on the applied side, like D. Scheeres (Colorado Center for Astrodynamics Research at Boulder), the group of F. Bernelli (Politecnico Milano), and colleagues from University of Barcelona (A. Farrès, A. Jorba).

Two new directions have been taken recently. The first one is about the control of solar sails (see Section 7.6), the second one about collision avoidance for spacecrafts (see Section 7.8). Collision avoidance is becoming very important in nowadays space missions due to the growing number of various bodies (garbage, micro-satellites…) orbiting around the earth. A PhD (Frank de Veld) started in December, supported by Thales Alenia Space. Solar sailing has been actively studied for two decades and recent missions have demonstrated its interest for "zero-fuel" missions; it poses delicate control questions due to drastic constraints on the control direction. Alesia Herasimenka, whose PhD had been selected by ESA for a three-year research co-sponsorship, defended her thesis in September and is now a postdoc at University of Luxembourg.

4.2 Optimal control of microbial cells, and other biological applications

Participants: Bernard Bonnard, Jean-Baptiste Caillau, Martin Fleurial, Sandrine Gayrard‘ , Jean-Baptiste Pomet, Ludovic Sacchelli, Toufik Bakir *(Université de Bourgogne Franche Comté, Dijon)*, Walid Djema *(BIOCORE project-team)*, Jean-Luc Gouzé *(BIOCORE project-team)*, Sofya Maslovskaya *(Paderborn University, Germany)*, Jérémy Rouot *(Université de Bretagne Occidentale, Brest)*, Agustín Yabo *(INRAE, Montpellier)*.

The growth of microorganisms is fundamentally an optimization problem which consists in dynamically allocating resources to cellular functions so as to maximize growth rate or another fitness criterion. Simple ordinary differential equation models, called self-replicators, have been used to formulate this problem in the framework of optimal and feedback control theory, allowing observations in microbial physiology to be explained. The resulting control problems are very challenging due to the nonlinearity of the models, parameter uncertainty, the coexistence of different time-scales, a dynamically changing environment, and various other physical and chemical constraints. In the framework of the ANR Maximic (PI Hidde de Jong, Inria Grenoble Rhône-Alpes), we aim at developing novel theoretical approaches for addressing these challenges in order to (i) study natural resource allocation strategies in microorganisms and (ii) propose new synthetic control strategies for biotechnological applications. In order to address (i), we develop extended self-replicator models accounting for the cost of regulation and energy metabolism in bacterial cells. We study these models by a combination of analytical and numerical approaches to derive optimal control solutions and a control synthesis, dealing with the bang-bang-singular structure of the solutions. Moreover, we define quasi-optimal feedback control strategies inspired by known regulatory mechanisms in the cell. To test whether bacteria follow the predicted optimal strategies, we quantify dynamic resource allocation in the bacterium Escherichia coli by monitoring, by means of time-lapse fluorescent microscopy, the expression of selected genes in single cells growing in a microfluidics device. In order to address (ii), we build self-replicator models that include a pathway for the production of a metabolite of interest. We also add a mechanism to turn off microbial growth by means of an external input signal, at the profit of the production of the metabolite. We formulate the maximization of the amount of metabolite produced as an optimal control problem, and derive optimal solutions and a control synthesis, as well as quasi-optimal feedback strategies satisfying chemical and physical design constraints. The proposed synthetic control strategies are being tested experimentally by growing E. coli strains capable of producing glycerol from glucose in a mini-bioreactor system. We aim at quantifying the amount of glucose consumed and glycerol produced, in the case of a predefined input signal (open-loop control) and the adaptive regulation of the input signal based on on-line measurements of the growth rate and the expression of fluorescent reporters of selected genes (closed-loop control). New results are presented in Section 7.9.

The team is also involved in other problems related to biological or medical applications, namely muscular functional electro-stimulation (new results presented in Section 7.10), Lotka-Volterra models (new results presented in Section 7.11), and alcoholic fermentation (new results presented in Section 7.12).

4.3 Neural dynamics

Participants: Adel Annabi, Dario Prandi *(CNRS, CentraleSupélec)*, Jean-Baptiste Pomet, Ludovic Sacchelli.

Neural fields serve as integro-differential dynamical models for the transmission of activity within cortical areas [54]. Originating in the 1970s, these models prove particularly advantageous when exploring the mesoscopic scale. At this level, the neuronal clusters under examination are sufficiently large to be understood as a continuum, yet compact enough to enable a targeted investigation of specific cortical functions. A significant appeal of these models lies in their efficacy in describing phenomena within the perceptual mechanisms of vision and audition. Notably, they have paved the way for sub-Riemannian-inspired geometric models addressing the anisotropic diffusion of information [51, 53].

Given their successes in characterizing cortical areas, their interplay and their scale, these models also offer valuable insights into experiments involving the measurement and stimulation of neural activity via electrodes. Consequently, substantial interest has been directed toward these models from the point of view of control, where the input-output formalism provides strategic avenues for deep-brain stimulation techniques. This interest has manifested in recent applications, including the treatment of Parkinson's disease [61]. The exploration of this perspective is the topic of A. Annabi's PhD research, which delves into the visual cortex, specifically concentrating on observability and observer design for low-dimensional models within the V1 cortical area.

5 Highlights of the year

Awards

Alesia Herasimenka, who defended her PhD in the team in September, was one of the 35 winners of the "Prix Jeunes Talents France 2023 L'Oréal-UNESCO Pour les Femmes et la Science" for her doctoral work, see [18], Section 7.6, and the press release from *Fondation L'Oréal*.

Other

Ivan Beschastnyi was hired by Inria this year and joined the team in December, 2023.

6 New software, platforms, open data

6.1 New software

6.1.1 ct

Name: control toolbox

Keywords: Optimal control, Ordinary differential equations, Mathematical Optimization, Differential homotopy, Automatic differentiation

Scientific Description: Numerical resolution of optimal control problems

Functional Description: The project gathers and allows to interoperate tools designed to solve numerically optimal control problems on ordinary differential equations. The available approaches include direct methods (based on a transcription of optimal control problems into mathematical programs) as well as indirect ones (based on Pontrjagin maximum principle, like the shooting method). The latter can be coupled to differential continuation. Automatic differentiation (aka Differentiable Programming) plays a crucial a role in all these algorithms. The project strongly leverages on SED Sophia support.

Release Contributions: - bocop refactoring - nutopy library - project gallery

URL: `https://control-toolbox.org`

Contact: Jean-Baptiste Caillau

Participants: Jean-Baptiste Caillau, Pierre Martinon, Olivier Cots, Thibaud Kloczko, Tristan Cabel, Jean-Luc Szpyrka, Erwan Demairy, Julien Wintz, Carlos Zubiaga Pena, Nicolas Niclausse, Joseph Gergaud

Partners: Université de Toulouse, CNRS, IRIT, ENSEEIHT

7 New results

7.1 Output feedback stabilization of non-uniformly observable systems

Participants: Ludovic Sacchelli, Lucas Brivadis *(CentraleSupélec, Gif-sur-Yvette)*, Jean-Paul Gauthier *(Université de Toulon)*.

Stabilization of the state of a system by means of a feedback control is a fundamental problem in control theory. When only part of the system is known, a usual strategy is to rely on a dynamic algorithm, known as an observer, in order to provide an estimate of the state that can be fed to the controller. This is known as output feedback control. Designing a stable closed-loop based on an observer requires that some necessary information on the state can be accessed through this partial measurement. Critically, for nonlinear systems, whether or not it is possible to reliably estimate the state can depend on the control. This fact is known as non-uniform observability and is a root issue for observer design.

Regarding output feedback control, if singular controls exist for observability, there is no clear definitive answer as to how to achieve stabilization. The now published [6] reviews some strategies that showed to be efficient in tackling the difficulties posed by non-uniform observability, and explores the genericity side of the matter, including a proof that this critical situation can be generic in some key classes of systems. In 2023's IFAC (International Federation of Automatic control) world conference, [11] explores the issue through the point of view of hybrid systems, a new framing for the group. In the paper, we propose a method to monitor observability of the system online. We use this method as the backbone of a dynamically switching Kalman-type observer and feedback to achieve stabilization of bilinear systems. Allowing Kalman filters in that context was a long term goal but had remained an open problem. This hybrid approach used to maintain observability of non-uniformly observable systems has been explored more thoroughly, and will be the topic of future works.

7.2 Geometry and optimal control for navigation problem

Participants: Bernard Bonnard, Joseph Gergaud, Boris Wembe *(ENSEEIHT, Toulouse)*, Olivier Cots, Jérémy Rouot *(Université de Bretagne Occidentale, Brest)*.

This is a long term research contribution that revisits and generalizes the Navigation Problem set by Carathéodory and Zermelo of a ship navigating on a river with a linear current and aiming to reach the opposite shore in minimum time. This work is motivated by the displacement of particles in a two dimensional fluid, in presence of a vortex (initially, a singularity in the Helhmoltz-Kirchhoff equations) inducing a strong current that hampers local controllability. To define a minimum time Zermelo navigation problem, we consider the particle as the ship of the navigation problem and the control is defined as the heading angle of the ship axis. It turns out that the historical problem and our recent vortex study are two examples of the general case of Zermelo navigation problems on surfaces of revolution on which some contributions appeared this year. Our main contributions in this setting are multiple. In [5] and [47], we analyze the role of abnormal geodesics in the problem, in particular in relation with cusp singularities of the geodesics and non continuity properties of the value function. In [2], we relate the existence, in the geodesics dynamic, of separatrices interpreted as Reeb components, with the Morse–Reeb classification of the geodesics. Furthermore, we provide, still in [2], explicit computations of the conjugate and cut loci in case studies such as the averaged Kepler case in space mechanics .

7.3 Small time local controllability for some degenerate two-input systems

Participants: Laetitia Giraldi *(CALISTO project-team)*, Pierre Lissy *(Université Paris Dauphine, Paris)*, Clément Moreau *(Kyoto University, Japan)*, Jean-Baptiste Pomet.

Here, we investigate small time local controllability (STLC) for affine control systems with two controls around an equilibrium such that the two control vector fields are co-linear at this point. Such a problem was motivated by the control of planar articulated magnetically actuated swimmers at low Reynolds number around the straight configuration with all magnetic moments aligned, see C. Moreau's PhD for details [76]. We pursued a more general study of local controllability with two controls with the above mentionned properties; in the paper [8], accepted for publication this year, we introduce novel necessary conditions for STLC of these systems, based on Chen-Fliess expansions of solutions, in the spirit of [74] or the more recent [37]. On top of "generalizing" the case of micro-swimmers, this work is the first attempt to give obstruction to local controllability in the spirit of these references for multi-input systems.

7.4 Stability of linear time-varying time-delay systems

Participants: Laurent Baratchart *(FACTAS project-team)*, Sébastien Fueyo *(Tel-Aviv University)*, Jean-Baptiste Pomet.

A linear time-periodic difference-delay systems (periodic LDDS for short) is a dynamical system of the form $z(t) = A_1(t)z(t-\tau_1)+\cdots+A_N(t)z(t-\tau_N)$, where z is finite dimensional and the matrices A_j depend periodically on time. The state of this dynamical system is infinite dimensional. S. Fueyo's doctoral work was about testing the stability of nonlinear amplifiers for high frequency signals by frequency domain methods; linearizing along an internal periodic solution yields a model based on networks of 1-D hyperbolic PDEs, and a periodic LDDS appears as its 'high frequency limit", whose stability conditions stability of the PDE dynamical system (see [69], or the long introduction of [35]). Coming back to hyperbolic stability of time-varying LDDS, a well known necessary and sufficient condition for stability holds in the time-invariant case, due to Hale and Henry [72, 71]; it gives a final answer to the question, but it is not so easy to check explicitly this criteria, and there is still a vast literature on more specific sufficient conditions. We obtained a generalization to the periodic time-varying case, presented in the manuscript [20], submitted for publication, with the proof of a technical part given in [21], separately submitted for publication.

7.5 Second-order averaging of fast-oscillating control systems

Participants: Jean-Baptiste Caillau, Lamberto Dell'Elce, Jean-Baptiste Pomet.

Research on fast-oscillating optimal control systems is a long-standing topic in McTAO. We investigated in 2021 how trajectories of fast-oscillating control system with a single fast variable converge to their averaged counterpart [64]. During 2022, further insight was gained on the averaging of systems with two fast variables [63]. Outcomes of these studies were mostly deduced from numerical experiments, but no proof of convergence of trajecories of the original system to their averaged counterpart was offered, since such proof requires the inclusion of second-order terms, which were neglected in these works. In 2023, we carried out a formal computation of second-order terms, which was exploited in the methodology described in Section 7.7. These developments pave the way to a proof of convergence and to a more general study for systems with several fast angles (which entails to address resonance issues).

7.6 Control of solar sails: controllability and optimal control

Participants: Jean-Baptiste Caillau, Lamberto Dell'Elce, Alesia Herasimenka, Jean-Baptiste Pomet.

The PhD thesis of A. Herasimenka [18], defended in September, 2023, is devoted to the control of solar sails, which offer a propellantless solution to perform interplanetary transfers, planet escapes, and de-orbiting maneuvers by leveraging on solar radiation pressure (SRP).

The thesis offers two main contributions, both of which were finalized in 2023. The first one is dedicated to the controllability study of solar sails. The primary challenge in assessing their controllability arises from the specific constraints imposed on the control set. Due to the nature of solar radiation pressure, a solar sail can only generate force whose directions belong to a convex cone with axis toward the direction of the Sun. Traditional methods for evaluating controllability are inadequate due to these specific constraints. To address this challenge, we proposed a novel necessary condition and a novel sufficient condition for controllability. The former involves identifying forbidden dual directions in the co-tangent bundle associated with the system state manifold that eventually constrain the state in some "half space" and the latter asserts global or local controllability under conditions that involve both the system and the shape of the control constraints. These theoretical results are applicable to any periodic system with a conical constraint on its control set. We also developed an algorithm aimed at efficiently verifying this necessary condition, based on convex optimization tools and fine properties of trigonometric polynomials as well as Nesterov's technique of sum of squares relaxation. This requirement is aimed at assessing whether a non-ideal solar sail with given optical parameters is capable of decreasing or increasing all possible functions of the Keplerian integrals of motion over an orbital period. In other words, we verify whether a solar sail can perturb its orbit in any arbitrary way given its optical properties. These developments were, for one part, published in the *Journal of Guidance, Control, and Dynamics* [9] (a paper that insists on constructive method to check the necessary conditions) and are, for another part, under revision for publication in *SIAM J. on Control & Optim.* [24] (a more mathematical paper that establishes these controllability results in a fairly general context). An extension of the numerical algorithm to the assessment of the local controllability of periodic orbits in the circular-restricted, three-body problem was presented at the Spaceflight Mechanics Meeting [13].

The second contribution of the thesis is an algorithm that computes the optimal control inputs for steering a sail towards a desired direction of the phase space. The algorithm employs convex optimization to obtain an admissible yet suboptimal control as an initial input. Subsequently, an optimal control problem is solved to maximize the displacement in the desired direction. By analyzing the Hamiltonian dynamics of the system, the relevant switching function that governs the structure of the solution is identified. Additionally, an upper bound on the number of zeros of this function is established, enabling the efficient implementation of a multiple shooting code using differential continuation. The publication of this work is currently under review [26].

7.7 Efficient solution of the low-thrust Lambert’s problem

Participants: Lamberto Dell’Elce, Alesia Herasimenka, Nicola Baresi *(University of Surrey)*, Aaron J. Rosengren *(University of California San Diego)*.

Lambert’s problem, in its classical form, consists in finding a Keplerian orbit joining two position vectors in a given transfer time. Solutions of this problem are extensively used for preliminary mission design since they offer the identification of launch opportunities and a rough evaluation of their fuel cost by assuming impulsive maneuvers at the two boundary points. Specifically, the concept of "launch windows" and the use of graphical representations to find feasible trajectories for planetary missions were introduced in the early space age. These graphical representations, which include what are now called pork-chop plots, help mission planners to visualize and analyze the trade-offs between departure and arrival dates, taking into account the positions and velocities of the planets involved. Pork-chop plots are often generated to illustrate total ΔV cost (the “ΔV” of a maneuver is a normalized estimate of the total impulse that the thrusters need to deliver, clearly related to the mass of propellant to be consumed) as a function of the departure and arrival dates by recursively solving Lambert’s problems. When considering the low-thrust counterpart of this transfer problem in fixed time, the entire state vector is imposed at the two boundaries, *i.e.*, both position and velocity of the satellite have to match the ones of departure and arrival bodies, because impulsive maneuvers are not allowed. In contrast to the original problem, no exact closed-form solution exists.

Two problems need to be addressed when tackling low-thrust transfers for a fixed maneuvering time. First, the minimum thrust magnitude necessary to carry out the maneuver has to be identified. Solutions to this problem are characterized by the absence of coasting arcs and the exploitation of the maximum control force throughout the entire trajectory. Second, once a sufficiently-large thrust is chosen, minimum-energy maneuvers can be found. Our research during the 2023 focused on the first problem, which is of interest because it offers a lower bound on the thrust that cannot be violated when optimizing other cost functions and it may serve as initial guess for the minimum fuel problem. A numerical methodology based on the averaging of the extremal flow of the optimal-control system (see Section 7.5) was proposed: First, a reduced-order solution of the averaged two-point boundary value problem (TPBVP) parametrized by the adjoint variable of the fast variable is solved. This step requires the solution of a single shooting problem followed by a numerical continuation procedure. This problem is independent of the thrust magnitude. Second, sensitivities of the shooting function are computed. These sensitivities are then used to evaluate perturbations of the averaged TPBVP associated to short-periodic variations and second-order terms, which are hereby retained to obtain a first-order approximation of the fast variable from the averaged solution. This information is finally used to find the minimum thrust required for the transfer.

The methodology was presented at the AIAA/AAS Spaceflight Mechanics Meeting [16], and published in the *Journal of Guidance, Control, and Dynamics* [7].

7.8 Low-thrust satellite collision avoidance

Participants: Lamberto Dell’Elce, Frank de Veld, Jean-Baptiste Pomet.

This topic is at the core of Frank de Veld’s research, whose PhD started in December, 2022. The presence of space debris in Earth orbits became an important factor to consider for day-to-day spacecraft operations. Future trends regarding the number of satellites, number of space debris objects, and tracking capabilities of these objects suggest that satellites in low-Earth orbits will continue to require collision avoidance maneuvres (CAM’s) on a regular basis within their lifetime to resume operational activities safely. At the same time, low thrust satellites are becoming more popular in space flight replacing conventional chemical propulsion systems with more efficient electrical ones. While propellant consumption is reduced using low-thrust propulsion systems, CAM design becomes more challenging. Smaller thrust levels go hand in hand with less maneuvrability and a longer time required to perform a certain maneuvre, such as a collision avoidance maneuvre.

In this context, our research focused on the relationship between the thrust arc duration, time of initiating thrust and thrust magnitude, when performing a collision avoidance maneuvre. We investigated the relationship between the time instant of initiating a CAM compared to the time of closest approach (TCA) and the separation distance at the time of closest approach. The resulting outcome of our study has consequences for CAM design for low-thrust satellites, which often differs significantly from high-thrust CAM design, as well as space traffic management in general.

7.9 Optimal allocation of resources in bacteria

Participants: Agustín Yabo *(INRAE, Montpellier)*, Jean-Baptiste Caillau, Jean-Luc Gouzé *(BIOCORE project-team)*, Mohab Safey El Din *(Sorbonne Université)*.

In the framework of the ANR Maximic, these last years, we carried on the study of self-replicator models. These models describe the allocation of resources inside the bacteria: the substrate is used to produce precursors that, in turn, can be employed either to produce genetic machinery (and increase the biomass) or metabolic machinery (that will further catalyze the transformation of substrate into precursors). To this internal control, the model adds and external action that aims, after some genetic engineering on the bacteria (to create a strain that reacts to light stimuli), at producing a new metabolite of interest. Then, while the behavior of the untouched bacteria tends to be very well mimicked by biomass maximization strategies, maximizing the production of the metabolite of interest induces new biological strategies. This kind of model (and refinements) were studied in [86, 87]. Key properties of the system are: (i) the Fuller phenomenon as connection between bang and singular arcs requires an infinite number of switchings in finite time; (ii) the turnpike phenomenon. Indeed, for large fixed final times, trajectories of the system are essentially singular and close to the optimal (w.r.t. a constant static control) equilibrium which is a hyperbolic fixed point of the singular flow. See ct gallery for an example, and the recently defended PhD thesis of A. Yago's PhD thesis [85] for a discussion of these results. In collaboration with M. Safey El Din, stability properties of the system were established thanks to a consistency check of a system of polynomial inequalities [10].

Colleagues from McTAO and Biocore teams at Sophia, together with former students (Agustin Yabo, now researcher at INRAE), are involved in this task. New results concern the definition on extended (higher dimensional) models for the bacteria dynamics, check of second order optimality conditions on the resulting optimal control problem, and study of the turnpike phenomenon for these optimization problems. One can also mention results on stability of the equilibria of these systems; the analysis in [10] resorts to real algebraic geometry and associated algorithmic tools in collaboration with Mohab Safey el Dinh (Sorbonne Université / CNRS / LIP6).

7.10 Biomechanics: control of muscular force response using FES with application to the conception of a smart electrostimulator

Participants: Bernard Bonnard, Sandrine Gayrard, Jérémy Rouot *(Université de Bretagne Occidentale, Brest)*, Toufik Bakir *(Université de Bourgogne Franche Comté, Dijon)*.

This topic started in McTAO in 2017 with a collaboration between B. Bonnard and T. Bakir (ImVia-UBFC), and J. Rouot (LMBA, Brest), in a collaboration with *Segula Technologies*. The problem of control of muscular force is posed in terms of optimization of the train pulses of a Functional Electro-Stimulation (FES) signal to produce the muscular contraction. Based on preliminary experimental studies, the dynamical model that was chosen for muscular control is known as *Ding et al. force-fatigue model* in the literature. It is a refinement of the historical Hill model (Medicine Nobel Prize in 1922) that takes into account the variations of the fatigue variable. From the control methodology point of view, this required some developments on optimal control for sample control systems. This is by itself already a rich topic.

In 2020, this project took the industrial transfer direction with a Cifre PhD funding in partnership with Segula Technologies (see Section 8) whose goal is the design of a smart electrostimulator for force reinforcement or rehabilitation in the framework of S. Gayrard's PhD. This PhD was defended in September, 2023, see Section 10.2.2; the manscript is condifential, a summary will soon be available online. The contribution is twofold. From the theoretical point of view, we have derived a finite dimensional approximation of the forced dynamics based on the *Ding et al.* model to provide fast optimizing schemes aiming to track a reference force or maximize the force, see [34]. On the other hand, S. Gayrard finalized a prototype of the smart electrostimulator, which was a major objective of the collaboration with Segula Technologies; extensive tests have been performed; the principle of these tests and parameter estimation was presented at 2023 European Control Conference [12].

7.11 Feedback invariants and optimal control with biological applications and numerical calculations

Participants: Bernard Bonnard, Jérémy Rouot *(Université de Bretagne Occidentale, Brest)*, Cristiana J. Silva *(Instituto Universitário de Lisboa, Lisbon)*.

The starting point of the study was the problem of controlled Lotka Volterra dynamics motivated by curing microbiote infection by a pathogenic agent. This leads to complicated optimal control problems in the frame of permanent or sampled-data controls, in relation with medical constraints. The problem can be set as a time-minimal control problem with terminal manifold of codimension one. Our contributions are presented in the series of works [22, 3, 23, 4]. In relation with previous work regarding the optimal control of chemical networks, the time minimal synthesis has been described near the terminal manifold up to codimension two situations in the jets spaces. We have compared both permanent controls and sampled-data control in relation with numerical issues. Finally, we have obtained feedback invariants to classify the geodesic dynamics associated to rays of abnormal geodesics which are related to shifted equilibria of the free Lotka-dynamics and which can be calculated using only linear computations.

7.12 State estimation in alcoholic fermentation models

Participants: Agustín Yabo *(INRAE, Montpellier)*, Ludovic Sacchelli, Martin Fleurial.

This inquiry folds into the ANR research project STARWINE on real time control of aroma production in wine fermentation processes, of which A. Yabo is a member. Focused on alcoholic fermentation in wine-making conditions, the study addresses the challenge of online state estimation during wine fermentation. This is relevant in industrial scenarios where control laws rely on estimating the full state from partial measurements of the system, mainly biogas production. This topic has been the subject of M. Fleurial's M2 internship and led to the submission of [25] in an international conference on control.

The primary emphasis of [25] lies in investigating the observability properties of an alcoholic fermentation model. Second, a full information estimator algorithm was developed. This type of algorithms are based on prediction error minimization on expanding time windows. While this method may be costly for non linear systems (algorithmically speaking), it is well adapted to the context of fermentation processes that are typically slow (a hundred hours in timeframe and new measurements added at intervals of 30 minutes). To validate the algorithm, comprehensive testing was conducted using both simulated and experimental data provided by the MISTEA research unit. This work provides the basis for further application of the estimation algorithm to more modern fermentation models.

7.13 Observer design for Blumenfeld's model of V1

Participants: Adel Annabi, Dario Prandi *(CNRS, CentraleSupélec)*, Jean-Baptiste Pomet, Ludovic Sacchelli.

The goal of this research is to provide insights into observability and observer synthesis of neural fields equations, the general topic of A. Annabi PhD. In the case of the visual cortex, neural fields models can be used to describe the activity dynamics in the specific case of orientation sensitivity of neurons. This focus allows to map neural fields in the visual cortex onto neural fields in the orientation domain. This reframing allows to move to Fourier series, which can be truncated to give a significant enough model in only 3 dimensions, a model of V1 due to Blumenfeld. The work of A. Annabi describes the observability of this model and highlights the symmetries of the system, and introduces hybrid elements to mitigate their effects. Crucially, this research also gives insights into the persistence conditions necessary for accurate estimation of the system's state, laying groundwork for further exploration of neural field models.

7.14 ct: control toolbox

Participants: Jean-Baptiste Caillau, Olivier Cots, Joseph Gergaud, Pierre Martinon *(CAGE project-team, on leave)*.

The ADT `ct: control toolbox` had its final sprint in 2023. The focus was on initiating new developments in Julia to take advantage of the powerful features of the language. Julia is indeed a perfect match for our needs in scientific computing for numerical optimal control; the language has a high level of abstraction well suited for mathematical descriptions, but still makes no compromise when it comes to performance thanks to efficient just-in-time compilation. Moreover, it currently has several efficient backends for AD / DP (automatic differentiation / differentiable programming), including ForwarDiff, Zygote of Enzyme: this is a crucial step for our project, both for direct and indirect methods. (Some examples of the project gallery require up to five levels of nested automatic differentiation.) The toolbox is now a full ecosystem available at control-toolbox.org. These achievements and the use of Julia have been presented in conferences [15, 14].

8 Bilateral contracts and grants with industry

8.1 Intelligent muscle electrostimulator, Segula Technologies

Participants: Toufik Bakir *(Université de Bourgogne Franche Comté, Dijon)*, Bernard Bonnard, Sandrine Gayrard.

In the framework of a **CIFRE grant**, a contract (title: “Réalisation d'un prototype d'électrostimulateur intelligent”) between Segula Technologies and Université de Bourgogne is partially funding (together with ANRT) Sandrine Gayrard's PhD.

This is completed by an **additionnal collaboration contract** between Segula and SAYENS (representing Université de Bourgogne), aiming at constructing the prototype of the smart electrostimulator.

- PI: Bernard Bonnard and Toufik Bakir (ImViA, Université de Bourgogne Franche Comté, Dijon).
- Period: 2020–2023.

8.2 Optimal Control of Solar Sails, ESA (European Space Agency)

Participants: Jean-Baptiste Caillau, Lamberto Dell'Elce, Alesia Herasimenka, Jean-Baptiste Pomet.

Three year contract starting in 2021 between the team and the European Space agency. Its purpose is to support the environment of Alesia Herasimenka's PhD on this topic.

- Partners: McTAO and ESA.
- Total amount: 24k€.
- Period: 2021–2024.
- Inria reference: 16016.

8.3 Méthodes de contrôle pour l'évitement de collisions entre satellites, Thales Alenia Space

Participants: Thierry Dargent, Lamberto Dell'Elce, Frank de Veld, Jean-Baptiste Pomet.

Thales Alenia Space is co-funding the thesis of Frank de Veld named "Méthodes de Contrôle pour l'évitement de collisions entre satellites".

- Partners: McTAO and Thales Alenia Space.
- Period: 2022–2025
- Total amount: 75k€
- Inria reference: 022-0674

9 Partnerships and cooperations

Participants: Adel Malik Annabi, Antonin Bavoil, Ivan Beschastnyi, Bernard Bonnard, Jean-Baptiste Caillau, Sandrine Gayrard, Alesia Herasimenka, Jean-Baptiste Pomet, Ludovic Sacchelli.

9.1 National initiatives

9.1.1 ANR

MAXIMIC: optimal control of microbial cells by natural and synthetic strategies. Started in 2017, ended in March, 2023. J.-B. Caillau and J.-B. Pomet were participants. More information and news on the site of this project.

PDE-AI: partial differential equations for AI. This project on "Numerical analysis, optimal control and optimal transport for AI", funded by PEPR IA from 2023 to 2027, is led by A. Chambolle (CNRS / Dauphine) and involves 10 French nodes, including a node in Nice / Sophia supervised by J.-B. Caillau. Total amount for the node 390 K€.

9.1.2 Other

- Labex CIMI grant on "Singular control and numerical optimisation in Julia". Participants are J.-B. Caillau, J. Gergaud (PI) and O. Cots (both Université de Toulouse). Total amount 8k€.
- "Recherche en réseaux" Université de Bourgogne grant on "Design of an asymmetric copepod-microswimmer for 2d-motion". Participants are T. Bakir (P.I., Université de Bourgogne), B. Bonnard. Total amount 5k€.
- McTAO project-team participates in the GdR MOA, a CNRS network on Mathematics of Optimization and Applications.

9.2 Regional initiatives

"Emplois Jeunes Doctorants" Grant from Région SUD – Provence Alpes Côte d'Azur "Emplois jeunes Chercheurs", 2022-2025, that co-funds Frank de Veld's PhD. See also the contract with Thales Alenia Space in Section 8. Total amount: 54k€.

10 Dissemination

Participants: Adel Malik Annabi, Antonin Bavoil, Ivan Beschastnyi, Bernard Bonnard, Jean-Baptiste Caillau, Sandrine Gayrard, Alesia Herasimenka, Jean-Baptiste Pomet, Ludovic Sacchelli.

10.1 Promoting scientific activities

10.1.1 Scientific events: organisation

The McTAO project team maintains a recurring seminar on topics of control theory, optimization and applications (organizer: L. Sacchelli). The seminar has a monthly periodicity and has hosted 9 sessions in 2023:

16/01/23 Ivan Beschastnyi (Université d'Aveiro): optimal control.

24/02/23 Daniel J. Scheeres (University of Colorado Boulder): Quasi-periodic orbits.

08/03/23 Giovanni Federico Gronchi (Università di Pisa): Averaging techniques in space mechanics.

07/04/23 Bruno Cessac (Inria, Biovision): chaos in retinal neuron dynamics.

08/06/23 Agustin Yabo (Inrae, Montpellier): Wine fermentation process and control.

28/06/23 Aaron Rosengren (UC San Diego): xGEO space mechanics.

08/09/23 Bernd Dachwald (Aachen University): neural networks for solar sails trajectory planning.

19/10/23 Jingrui Niu (Inria CAGE, LJLL): controllability of nonlinear Schrödinger equations.

10/11/23 Frédéric Jean (ENSTA Paris): constrained optimal control for reusable lauchers.

10.1.2 Scientific events: selection

Member of conference program committees

J.-B. Caillau is a member of the PGMO scientific council of *Programme Gaspard Monge* (PGMO), and was a member of the scientific board of PGMO days' 2023.

Reviewing activities

All team members take part in a continued effort to offer reviews in various conferences of importance to the community.

10.1.3 Journal

Member of the editorial boards

- B. Bonnard is member of the editorial board of Pacific Journal of Mathematics for Industry.
- J.-B. Caillau is member of the editorial board of ESAIM: Mathematical Modelling and Numerical Analysis (ESAIM: M2AN).

Reviewing activities

All team members take part in a continued effort to offer reviews in various journals of importance to the community.

10.1.4 Invited talks

L. Sacchelli gave invited talks at: Journées Contrôle Optimal, FRUMAM, Marseille, 09/06/23; Lehigh mathematics department colloquium, Lehigh University, USA, 11/10/2023.

10.1.5 Research administration

- J.-B. Caillau is member of 3IA Côte d'Azur scientific council
- J.-B. Caillau is member of Inria Sophia CDT
- J.-B. Caillau co-organizes LJAD colloquium
- J.-B. Pomet is a member of the steering comitee of Academy of excellence on complex systems, Université Côte d'Azur (IDEX) and a member of the executive bureau of "EUR SPECTRUM" (graduate school, Université Côte d'Azur).
- J.-B. Pomet was an elected member of Inria permanent Evaluation Committee until August 31 (first elected in 2014).

10.2 Teaching - Supervision - Juries

10.2.1 Teaching

- Engineering school and University: J.-B. Caillau has a full teaching duty of Professor at L (BSc) and M (Master) level at Polytech Nice Sophia and Université Côte d'Azur.
- Engineering school: L. Dell'Elce, A. Herasimenka and L. Sacchelli each took part as teaching assistants at L3 and M1 levels at Polytech Nice Sophia, Université Côte d'Azur.

10.2.2 Supervision

PhD students.

- Antonin Bavoil, "Génération optimale d'énergie par un cerf-volant" (Optimal energy generation from a kite), Université Côte d'Azur, co-supervised by J.-B. Caillau and Alain Nême (ENSTA Bretagne). Funded by CNRS. Started October, 2023.
- Adel Malik Annabi, "Observability and observer synthesis for neural fields equations", Université Côte d'Azur, co-supervised by J.-B. Pomet, L. Sacchelli and D. Prandi (CentraleSupélec). Started October, 2022.
- Frank de Veld, "Méthodes de Contrôle pour l'évitement de collisions entre satellites", Inria, co-supervised by J.-B. Pomet, L. Dell'Elce and T. Dargent. Started December 2022.
- Sandrine Gayrard, "Réalisation d'un prototype d'électrostimulateur intelligent", Université de Bourgogne Franche Comté, co-supervised by B. Bonnard and T. Bakir (Université de Bourgogne Franche Comté), defended her PhD on September 21st, 2023. [condifential manuscript, summary online soon.]
- Alesia Herasimenka, "Optimal control of solar sails", Université Côte d'Azur, co-supervised by L. Dell'Elce and J.-B. Caillau, defended her PhD on September 7th, 2023. See [18].

Interns.

- Antonin Bavoil, 5th year Polytech Nice. Supervised by J.-B. Caillau. March to September, 2023, funded by CNRS (Prix défi Défense 2022 des Assises des maths du CNRS).
- Martin Fleurial, INSA Rouen. Co-supervised by L. Sacchelli and A. Yabo (INRAE, Montpellier) on the topic of "State and parameter estimation of wine fermentation models for real-time aroma control". March to August, 2023.

10.2.3 Juries

- J.-B. Caillau was reviewer and sat at the jury of *Clara Leparoux* PhD thesis (ENSTA Paris).
- J.-B. Pomet was reviewer and sat at the jury of *Bruno Hérissé* HDR thesis (Université Paris saclay).

11 Scientific production

11.1 Publications of the year

International journals

[1] N. Baresi and L. Dell'Elce. 'Periodic and Quasi-Periodic Orbits near Close Planetary Moons'. In: *Journal of Guidance, Control, and Dynamics* (2023), pp. 1–15. DOI: `10.2514/1.G007221`. URL: `https://inria.hal.science/hal-03937917`.

[2] B. Bonnard, O. Cots and B. Wembe. 'Zermelo Navigation Problems on Surfaces of Revolution and Geometric Optimal Control'. In: *ESAIM: Control, Optimisation and Calculus of Variations* 29.60 (31st July 2023), p. 34. DOI: `10.1051/cocv/2023052`. URL: `https://hal.science/hal-03209491`.

[3] B. Bonnard and J. Rouot. 'Optimal Control of the Controlled Lotka-Volterra Equations with Applications - The Permanent Case'. In: *SIAM Journal on Applied Dynamical Systems* 22.4 (2023), pp. 2761–2791. DOI: `10.1137/22M151978X`. URL: `https://inria.hal.science/hal-03757060`.

[4] B. Bonnard, J. Rouot and C. Silva. 'Geometric Optimal Control of the Generalized Lotka-Volterra Model of the Intestinal Microbiome'. In: *Optimal Control Applications and Methods* (2024). DOI: `10.1002/oca.3089`. URL: `https://hal.science/hal-03861565`.

[5] B. Bonnard, J. Rouot and B. Wembe. 'Accessibility Properties of Abnormal Geodesics in Optimal Control Illustrated by Two Case Studies.' In: *Mathematical Control and Related Fields* 13.4 (Dec. 2023), pp. 1618–1638. DOI: `10.3934/mcrf.2022052`. URL: `https://inria.hal.science/hal-03310348`.

[6] L. Brivadis, J.-P. Gauthier and L. Sacchelli. 'Output feedback stabilization of non-uniformly observable systems'. In: *Proceedings of the Steklov Institute of Mathematics* 321.1 (June 2023), pp. 69–83. DOI: `10.1134/S0081543823020050`. URL: `https://hal.science/hal-03589760`.

[7] L. Dell'Elce, A. Herasimenka, A. J. Rosengren and N. Baresi. 'Second-order Averaging of Low-Thrust Transfers in Fixed Time *'. In: *Journal of Guidance, Control, and Dynamics* (2024). URL: `https://inria.hal.science/hal-04387944`.

[8] L. Giraldi, P. Lissy, C. Moreau and J.-B. Pomet. 'Necessary conditions for local controllability of a particular class of systems with two scalar controls'. In: *ESAIM: Control, Optimisation and Calculus of Variations* 30.4 (2024). DOI: `10.1051/cocv/2023073`. URL: `https://hal.science/hal-02178973`.

[9] A. Herasimenka, L. Dell'Elce, J.-B. Caillau and J.-B. Pomet. 'Controllability Properties of Solar Sails'. In: *Journal of Guidance, Control, and Dynamics* (2023). DOI: `10.2514/1.G007250`. URL: `https://hal.science/hal-03331300`.

[10] A. G. Yabo, M. Safey El Din, J.-B. Caillau and J.-L. Gouzé. 'Stability analysis of a bacterial growth model through computer algebra'. In: *MathematicS In Action* 12.1 (26th Sept. 2023), pp. 175–189. DOI: `10.5802/msia.37`. URL: `https://hal.science/hal-03671432`.

International peer-reviewed conferences

[11] L. Brivadis and L. Sacchelli. 'Output feedback stabilization of non-uniformly observable systems by means of a switched Kalman-like observer'. In: *IFAC-PapersOnLine*. 22nd World Congress of the International Federation of Automatic Control (IFAC 2023). Yokohama, Japan, 9th July 2023. DOI: 10.1016/j.ifacol.2023.10.250. URL: https://hal.science/hal-03845706.

[12] S. Gayrard. 'Fast Optimization Scheme for the Muscular Response to FES Stimulation to Design a Smart Electrostimulator'. In: 2023 European Control Conference (ECC). Bucharest, France: IEEE, 13th June 2023, pp. 1–6. DOI: 10.23919/ECC57647.2023.10178228. URL: https://inria.hal.science/hal-03891925.

[13] A. Herasimenka, A. Farrés and L. Dell'Elce. 'Station-keeping under conical constraint on the control force'. In: 9th International Conference on Astrodynamics Tools and Techniques (ICATT). Sopot, Poland, 12th June 2023. URL: https://inria.hal.science/hal-04156856.

Conferences without proceedings

[14] J.-B. Caillau, O. Cots, J. Gergaud and P. Martinon. 'Solving optimal control problems with Julia'. In: Julia and Optimization Days 2023. Paris, France, 4th Oct. 2023. URL: https://hal.science/hal-04277441.

[15] J.-B. Caillau, O. Cots, J. Gergaud and P. Martinon. 'Solving optimal control problems with Julia (talk)'. In: JuliaCon 2023. Cambridge, Boston, United States, 26th July 2023. URL: https://hal.science/hal-04277383.

[16] L. Dell'Elce, A. Herasimenka, A. J. Rosengren and N. Baresi. 'Efficient Numerical Solution of the Low-Thrust Lambert Problem'. In: SFMM 2023 / 33rd AAS/AIAA Space Flight Mechanics Meeting. Austin, United States, 15th Jan. 2023. URL: https://inria.hal.science/hal-04050277.

Scientific book chapters

[17] J.-B. Caillau, R. Ferretti, E. Trélat and H. Zidani. 'An algorithmic guide for finite-dimensional optimal control problems'. In: *Handbook of numerical analysis: Numerical control, Part B*. Vol. 24. Handbook of Numerical Analysis. North-Holland; Elsevier, 2023, pp. 559–626. DOI: 10.1016/bs.hna.2022.11.006. URL: https://hal.science/hal-03883822.

Doctoral dissertations and habilitation theses

[18] A. Herasimenka. 'Optimal control of solar sails'. Université Côte d'Azur, 7th Sept. 2023. URL: https://theses.hal.science/tel-04304035.

Reports & preprints

[19] V. Andrieu, L. Brivadis, J.-P. Gauthier, L. Sacchelli and U. Serres. *Exponential stabilizability and observability at the target imply semiglobal exponential stabilizability by templated output feedback*. 11th Jan. 2024. URL: https://hal.science/hal-04387614.

[20] L. Baratchart, S. Fueyo and J.-B. Pomet. *Exponential stability of linear periodic difference-delay equations*. 2023. URL: https://inria.hal.science/hal-03500720.

[21] L. Baratchart, S. Fueyo and J.-B. Pomet. *Integral representation formula for linear non-autonomous difference-delay equations*. 2023. URL: https://hal.science/hal-04204500.

[22] B. Bonnard and J. Rouot. *Feedback Classification and Optimal Control with Applications to the Controlled Lotka-Volterra Model*. 1st Jan. 2023. URL: https://inria.hal.science/hal-03917363.

[23] B. Bonnard and J. Rouot. *Optimal Control of the Lotka-Volterra Equations with Applications*. 2024. URL: https://hal.science/hal-03829465.

[24] J.-B. Caillau, L. Dell'Elce, A. Herasimenka and J.-B. Pomet. *On the controllability of nonlinear systems with a periodic drift*. Mar. 2024. URL: https://inria.hal.science/hal-03779482.

[25] M. Fleurial, L. Sacchelli and A. G. Yabo. *State estimation in alcoholic fermentation models: a case-study in wine-making conditions.* 6th Nov. 2023. URL: https://hal.science/hal-04269299.

[26] A. Herasimenka, L. Dell'Elce, J.-B. Caillau and J.-B. Pomet. *Optimal control of a solar sail.* 2023. URL: https://inria.hal.science/hal-04158202.

[27] A. G. Yabo, J.-B. Caillau and J.-L. Gouzé. *Optimal bacterial resource allocation strategies in batch processing.* 25th Sept. 2023. URL: https://inria.hal.science/hal-03710681.

11.2 Other

Scientific popularization

[28] A. Herasimenka, A. Farrés and L. Dell'Elce. 'Controllability of satellites on periodic orbits with cone-constraints on the thrust direction'. In: Complex Days 2023. Nice, France, 7th Feb. 2023. URL: https://hal.science/hal-04043023.

11.3 Cited publications

[29] A. A. Agrachev and R. V. Gamkrelidze. 'Symplectic methods for optimization and control'. In: *Geometry of feedback and optimal control.* Vol. 207. Textbooks Pure Appl. Math. Marcel Dekker, 1998, pp. 19–77.

[30] A. A. Agrachev and A. V. Sarychev. 'Strong minimality of abnormal geodesics for 2-distributions'. In: *J. Dynam. Control Systems* 1.2 (1995), pp. 139–176. DOI: 10.1137/S036301290138866X.

[31] A. Agrachev and Y. L. Sachkov. *Control theory from the geometric viewpoint.* Vol. 87. Encyclopaedia of Mathematical Sciences. Control Theory and Optimization, II. Berlin: Springer-Verlag, 2004. DOI: 10.1007/978-3-662-06404-7.

[32] A. A. Agrachev and A. V. Sarychev. 'Abnormal sub-Riemannian geodesics: Morse index and rigidity'. In: *Ann. Inst. H. Poincaré, Anal. non-linéaire* 13.6 (1996), pp. 635–690. URL: http://www.numdam.org/item/AIHPC_1996__13_6_635_0/.

[33] V. I. Arnold. *Mathematical methods of classical mechanics.* Vol. 60. Graduate Texts in Mathematics. New York: Springer-Verlag, 1989.

[34] T. Bakir, B. Bonnard, S. Gayrard and J. Rouot. 'Finite Dimensional Approximation to Muscular Response in Force-Fatigue Dynamics using Functional Electrical Stimulation'. In: *Automatica* (Oct. 2022). DOI: 10.1016/j.automatica.2022.110464. URL: https://inria.hal.science/hal-03154450.

[35] L. Baratchart, S. Fueyo, G. Lebeau and J.-B. Pomet. 'Sufficient Stability Conditions for Time-varying Networks of Telegrapher's Equations or Difference-Delay Equations'. In: *SIAM J. Math. Anal.* 53.2 (2021), pp. 1831–1856. DOI: 10.1137/19M1301795. URL: http://hal.inria.fr/hal-02385548/.

[36] D. Barilari, U. Boscain and R. W. Neel. 'Small-time heat kernel asymptotics at the sub-Riemannian cut locus'. In: *Journal of Differential Geometry* 92.3 (Nov. 2012). DOI: 10.4310/jdg/1354110195. URL: https://doi.org/10.4310/jdg/1354110195.

[37] K. Beauchard and F. Marbach. 'A unified approach of obstructions to small-time local controllability for scalar-input systems'. working paper or preprint. May 2022. URL: https://hal.science/hal-03683867.

[38] A. Bombrun and J.-B. Pomet. 'The averaged control system of fast oscillating control systems'. In: *SIAM J. Control Optim.* 51.3 (2013), pp. 2280–2305. DOI: 10.1137/11085791X. URL: http://hal.inria.fr/hal-00648330/.

[39] B. Bonnard. 'Feedback equivalence for nonlinear systems and the time optimal control problem'. In: *SIAM J. Control Optim.* 29.6 (1991), pp. 1300–1321. DOI: 10.1137/0329067. URL: https://doi.org/10.1137/0329067.

[40] B. Bonnard and J.-B. Caillau. 'Geodesic flow of the averaged controlled Kepler equation'. In: *Forum Mathematicum* 21.5 (Sept. 2009), pp. 797–814. DOI: 10.1515/FORUM.2009.038.

[41] B. Bonnard, J.-B. Caillau and O. Cots. 'Energy minimization in two-level dissipative quantum control: The integrable case'. In: *Proceedings of the 8th AIMS Conference on Dynamical Systems, Differential Equations and Applications*. Vol. suppl. Discrete Contin. Dyn. Syst. AIMS, 2011, pp. 198–208. DOI: `10.3934/proc.2011.2011.198`.

[42] B. Bonnard, J.-B. Caillau and L. Rifford. 'Convexity of injectivity domains on the ellipsoid of revolution: the oblate case'. In: *C. R. Math. Acad. Sci. Paris* 348.23-24 (2010), pp. 1315–1318. DOI: `10.1016/j.crma.2010.10.036`. URL: `https://hal.archives-ouvertes.fr/hal-00545768`.

[43] B. Bonnard, J.-B. Caillau, R. Sinclair and M. Tanaka. 'Conjugate and cut loci of a two-sphere of revolution with application to optimal control'. In: *Ann. Inst. H. Poincaré Anal. Non Linéaire* 26.4 (2009), pp. 1081–1098. DOI: `10.1016/j.anihpc.2008.03.010`.

[44] B. Bonnard, J.-B. Caillau and E. Trélat. 'Second order optimality conditions in the smooth case and applications in optimal control'. In: *ESAIM Control Optim. Calc. Var.* 13.2 (2007), pp. 207–236. DOI: `10.1051/cocv:2007012`.

[45] B. Bonnard and M. Chyba. *Singular trajectories and their role in control theory*. Vol. 40. Mathématiques & Applications. Berlin: Springer-Verlag, 2003.

[46] B. Bonnard, M. Chyba, J. Rouot, D. Takagi and R. Zou. 'Optimal Strokes : a Geometric and Numerical Study of the Copepod Swimmer'. working paper or preprint. Jan. 2016. URL: `https://hal.inria.fr/hal-01162407`.

[47] B. Bonnard, O. Cots, J. Gergaud and B. Wembe. 'Abnormal Geodesics in 2D-Zermelo Navigation Problems in the Case of Revolution and the Fan Shape of the Small Time Balls'. In: *Systems and Control Letters* 161 (Mar. 2022), p. 105140. DOI: `10.1016/j.sysconle.2022.105140`. URL: `https://hal.science/hal-02437507`.

[48] B. Bonnard, H. Henninger, J. Nemcova and J.-B. Pomet. 'Time Versus Energy in the Averaged Optimal Coplanar Kepler Transfer towards Circular Orbits'. In: *Acta Applicandae Math.* 135.2 (2015), pp. 47–80. DOI: `10.1007/s10440-014-9948-2`. URL: `https://hal.inria.fr/hal-00918633`.

[49] B. Bonnard, A. Jacquemard, M. Chyba and J. Marriott. 'Algebraic geometric classification of the singular flow in the contrast imaging problem in nuclear magnetic resonance'. In: *Math. Control Relat. Fields (MCRF)* 3.4 (2013), pp. 397–432. DOI: `10.3934/mcrf.2013.3.397`. URL: `https://hal.inria.fr/hal-00939495`.

[50] B. Bonnard and I. Kupka. 'Théorie des singularités de l'application entrée-sortie et optimalité des trajectoires singulières dans le problème du temps minimal'. In: *Forum Math.* 5 (1993), pp. 111–159.

[51] U. Boscain, R. A. Chertovskih, J. P. Gauthier and A. O. Remizov. 'Hypoelliptic Diffusion and Human Vision: A Semidiscrete New Twist'. In: *SIAM Journal on Imaging Sciences* 7.2 (2014), pp. 669–695. DOI: `10.1137/130924731`. eprint: `https://doi.org/10.1137/130924731`. URL: `https://doi.org/10.1137/130924731`.

[52] U. Boscain, J. Duplaix, J.-P. Gauthier and F. Rossi. 'Anthropomorphic Image Reconstruction via Hypoelliptic Diffusion'. In: *SIAM J. Control Optim.* 50.3 (Jan. 2012), pp. 1309–1336. DOI: `10.1137/11082405x`.

[53] U. Boscain, D. Prandi, L. Sacchelli and G. Turco. 'A bio-inspired geometric model for sound reconstruction'. In: *The Journal of Mathematical Neuroscience* 11.2 (2021). DOI: `10.1186/s13408-020-00099-4`. URL: `https://doi.org/10.1186/s13408-020-00099-4`.

[54] P. C. Bressloff. 'Spatiotemporal dynamics of continuum neural fields'. In: *Journal of Physics A: Mathematical and Theoretical* 45.3 (2011), p. 033001.

[55] L. Brivadis, J.-P. Gauthier, L. Sacchelli and U. Serres. 'Avoiding observability singularities in output feedback bilinear systems'. In: *SIAM J. Control Optim.* 59.3 (May 2021), pp. 1759–1780. DOI: `10.1137/19M1272925`. URL: `https://hal.archives-ouvertes.fr/hal-02172420`.

[56] J.-B. Caillau, O. Cots and J. Gergaud. 'Differential pathfollowing for regular optimal control problems'. In: *Optim. Methods Softw.* 27.2 (2012), pp. 177–196.

[57] J.-B. Caillau and B. Daoud. 'Minimum time control of the restricted three-body problem'. In: *SIAM J. Control Optim.* 50.6 (2012), pp. 3178–3202.

[58] J.-B. Caillau and A. Farrés. ‘On local optima in minimum time control of the restricted three-body problem’. In: *Recent Advances in Celestial and Space Mechanics*. Vol. Mathematics for Industry. 23. Springer, Apr. 2016, pp. 209–302. DOI: `10.1007/978-3-319-27464-5`. URL: `https://hal.archives-ouvertes.fr/hal-01260120`.

[59] J.-b. Caillau, J.-B. Pomet and J. Rouot. ‘Metric approximation of minimum time control systems’. working paper or preprint. Nov. 2017. URL: `https://hal.inria.fr/hal-01672001`.

[60] J.-B. Caillau and C. Royer. ‘On the injectivity and nonfocal domains of the ellipsoid of revolution’. In: *Geometric Control Theory and Sub-Riemannian Geometry*. Ed. by G. Stefani et al. Vol. 5. INdAM series. Springer, 2014, pp. 73–85. DOI: `10.1007/978-3-319-02132-4`. URL: `https://hal.archives-ouvertes.fr/hal-01315530`.

[61] R. Carron, A. Chaillet, A. Filipchuk, W. Pasillas-Lépine and C. Hammond. ‘Closing the loop of deep brain stimulation’. In: *Frontiers in systems neuroscience* 7 (2013), p. 112.

[62] Z. Chen, J.-B. Caillau and Y. Chitour. ‘L^1-minimization for mechanical systems’. In: *SIAM J. Control Optim.* 54.3 (May 2016), pp. 1245–1265. DOI: `10.1137/15M1013274`. URL: `https://hal.archives-ouvertes.fr/hal-01136676`.

[63] L. Dell’Elce, J.-B. Caillau and J.-B. Pomet. ‘Considerations on Two-Phase Averaging of Time-Optimal Control Systems’. In: *IFAC-PapersOnLine* 55.16 (2022). 18th IFAC Workshop on Control Applications of Optimization (CAO 2022), pp. 7–12. DOI: `10.1016/j.ifacol.2022.08.073`. URL: `https://inria.hal.science/hal-01793704`.

[64] L. Dell’Elce, J.-B. Caillau and J.-B. Pomet. ‘On the convergence of time-optimal maneuvers of fast-oscillating control systems’. In: *2021 European Control Conference (ECC)*. 2021, pp. 2008–2013. DOI: `10.23919/ECC54610.2021.9655101`. URL: `https://hal.inria.fr/hal-03391620`.

[65] L. Faubourg and J.-B. Pomet. ‘Control Lyapunov functions for homogeneous “Jurdjevic-Quinn” systems’. In: *ESAIM Control Optim. Calc. Var.* 5 (2000), pp. 293–311. DOI: `10.1051/cocv:2000112`.

[66] A. Figalli, L. Rifford and C. Villani. ‘Necessary and sufficient conditions for continuity of optimal transport maps on Riemannian manifolds’. In: *Tohoku Math. J.* 63.4 (2011), pp. 855–876. URL: `http://hal.inria.fr/hal-00923320v1`.

[67] A. Figalli, T. Gallouët and L. Rifford. ‘On the convexity of injectivity domains on nonfocal manifolds’. In: *SIAM J. Math. Anal.* 47.2 (2015), pp. 969–1000. DOI: `10.1137/140961821`. URL: `https://hal.inria.fr/hal-00968354`.

[68] M. Fliess, J. Lévine, P. Martin and P. Rouchon. ‘Flatness and Defect of Nonlinear Systems: Introductory Theory and Examples’. In: *Internat. J. Control* 61 (1995), pp. 1327–1361.

[69] S. Fueyo. ‘Systèmes à retard instationnaires et EDP hyperboliques 1-D instationnaires, fonctions de transfert harmoniques et circuits électriques non-linéaires’. Université Cote d’Azur, 30th Oct. 2020. URL: `https://hal.archives-ouvertes.fr/tel-03105344`.

[70] C. Gavriel and R. Vinter. ‘Second order sufficient conditions for optimal control problems with non-unique minimizers: an abstract framework’. In: *Appl. Math. Optim.* 70.3 (2014), pp. 411–442. DOI: `10.1007/s00245-014-9245-5`.

[71] J. K. Hale and S. M. Verduyn Lunel. *Introduction to functional-differential equations*. Vol. 99. Applied Mathematical Sciences. Springer-Verlag, New York, 1993. DOI: `10.1007/978-1-4612-4342-7`.

[72] D. Henry. ‘Linear autonomous neutral functional differential equations’. In: *J. Differential Equations* 15 (1974), pp. 106–128. DOI: `10.1016/0022-0396(74)90089-8`.

[73] L. Hörmander. ‘Hypoelliptic second order differential equations’. In: *Acta Mathematica* 119 (1967), pp. 147–171. DOI: `10.1007/bf02392081`. URL: `https://doi.org/10.1007/bf02392081`.

[74] M. Kawski. ‘High-order small-time local controllability’. In: *Nonlinear controllability and optimal control*. Vol. 133. Monogr. Textbooks Pure Appl. Math. Dekker, New York, 1990, pp. 431–467.

[75] R. Montgomery. *A tour of subriemannian geometries, their geodesics and applications*. Vol. 91. Mathematical Surveys and Monographs. American Mathematical Society, Providence, RI, 2002.

[76] C. Moreau. 'Contrôlabilité en dimension finie et infinie et applications à des systèmes non linéaires issus du vivant'. Université Côte d'Azur, 17th June 2020. URL: `https://hal.archives-ouvertes.fr/tel-03106682`.

[77] J.-B. Pomet. 'A necessary condition for dynamic equivalence'. In: *SIAM J. Control Optim.* 48 (2009), pp. 925–940. DOI: `10.1137/080723351`. URL: `http://hal.inria.fr/inria-00277531`.

[78] L. S. Pontryagin, V. G. Boltjanskiĭ, R. V. Gamkrelidze and E. Mitchenko. *Théorie mathématique des processus optimaux*. Moscou: Editions MIR, 1974.

[79] L. Rifford. 'Stratified semiconcave control-Lyapunov functions and the stabilization problem'. In: *Ann. Inst. H. Poincaré Anal. Non Linéaire* 22.3 (2005), pp. 343–384. DOI: `10.1016/j.anihpc.2004.07.008`.

[80] J. A. Sanders and F. Verhulst. *Averaging Methods in Nonlinear Dynamical Systems*. Vol. 56. Applied Mathematical Sciences. Springer-Verlag, 1985.

[81] A. V. Sarychev. 'The index of second variation of a control system'. In: *Mat. Sb.* 41 (1982), pp. 338–401.

[82] H. Shim and A. Teel. 'Asymptotic controllability and observability imply semiglobal practical asymptotic stabilizability by sampled-data output feedback'. In: *Automatica* 39.3 (2003), pp. 441–454. DOI: `10.1016/S0005-1098(02)00278-9`.

[83] E. Trélat and E. Zuazua. 'The turnpike property in finite-dimensional nonlinear optimal control'. In: *J. Differential Equations* 258.1 (2015), pp. 81–114. DOI: `10.1016/j.jde.2014.09.005`.

[84] R. Vinter. *Optimal control*. Modern Birkhäuser Classics. Birkhäuser Boston, Inc., 2000. DOI: `10.1007/978-0-8176-8086-2`.

[85] A. G. Yabo. 'Optimal resource allocation in bacterial growth : theoretical study and applications to metabolite production'. Theses. Université Côte d'Azur, Dec. 2021. URL: `https://theses.hal.science/tel-03636842`.

[86] A. G. Yabo, J. Caillau and J.-L. Gouzé. 'Optimal bacterial resource allocation: metabolite production in continuous bioreactors'. In: *Mathematical Biosciences and Engineering* 17.6 (2020), pp. 7074–7100. DOI: `10.3934/mbe.2020364`. URL: `https://hal.inria.fr/hal-02974185`.

[87] A. G. Yabo, J.-B. Caillau, J.-L. Gouzé, H. de Jong and F. Mairet. 'Dynamical analysis and optimization of a generalized resource allocation model of microbial growth'. In: *SIAM J. Appl. Dyn. Syst.* 21.1 (2022), pp. 137–165. DOI: `10.1137/21M141097X`. URL: `https://inria.hal.science/hal-03251044`.