RESEARCH CENTRE
Inria Centre at Université de Lorraine

Project-Team SPHINX

# Heterogeneous Systems: Inverse Problems, Control and Stabilization, Simulation 

IN COLLABORATION WITH: Institut Elie Cartan de Lorraine (IECL)

DOMAIN
Applied Mathematics, Computation and Simulation

THEME
Optimization and control of dynamic systems

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## Project-Team SPHINX

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## Keywords

Computer sciences and digital sciences
A6.1. - Methods in mathematical modeling
A6.1.1. - Continuous Modeling (PDE, ODE)
A6.2.1. - Numerical analysis of PDE and ODE
A6.2.6. - Optimization
A6.3.1. - Inverse problems
A6.3.2. - Data assimilation
A6.4. - Automatic control
A6.4.1. - Deterministic control
A6.4.3. - Observability and Controlability
A6.4.4. - Stability and Stabilization
A6.5. - Mathematical modeling for physical sciences
A6.5.1. - Solid mechanics
A6.5.2. - Fluid mechanics
A6.5.4. - Waves
A6.5.5. - Chemistry
Other research topics and application domains
B2. - Health
B2.6. - Biological and medical imaging
B5.3. - Nanotechnology
B5.11. - Quantum systems
B9. - Society and Knowledge
B9.5. - Sciences
B9.5.2. - Mathematics
B9.5.3. - Physics
B9.5.4. - Chemistry

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## 2 Overall objectives

In this project, we investigate theoretical and numerical mathematical issues concerning heterogeneous physical systems. The heterogeneities we consider result from the fact that the studied systems involve subsystems of different physical nature. In this wide class of problems, we study two types of systems: fluid-structure interaction systems (FSIS) and complex wave systems (CWS). In both situations, one has to develop specific methods to take the coupling between the subsystems into account.
(FSIS) Fluid-structure interaction systems appear in many applications: medicine (motion of the blood in veins and arteries), biology (animal locomotion in a fluid, such as swimming fishes or flapping birds but also locomotion of microorganisms, such as amoebas), civil engineering (design of bridges or any structure exposed to the wind or the flow of a river), naval architecture (design of boats and submarines, researching into new propulsion systems for underwater vehicles by imitating the locomotion of aquatic animals). FSIS can be studied by modeling their motions through Partial Differential Equations (PDE) and/or Ordinary Differential Equations (ODE), as is classical in fluid mechanics or in solid mechanics. This leads to the study of difficult nonlinear free boundary problems which have constituted a rich and active domain of research over the last decades.
(CWS) Complex wave systems are involved in a large number of applications in several areas of science and engineering: medicine (breast cancer detection, kidney stone destruction, osteoporosis diagnosis, etc.), telecommunications (in urban or submarine environments, optical fibers, etc.), aeronautics (target detection, aircraft noise reduction, etc.) and, in the longer term, quantum supercomputers. Direct problems consist of finding a solution with respect to the parameters of the problem, for instance, the propagation of waves with respect to the knowledge of the speed of propagation of the medium, most theoretical issues are now widely understood. However, substantial efforts remain to be undertaken concerning the simulation of wave propagation in complex media. Such situations include heterogeneous media with strong local variations of the physical properties (high frequency scattering, multiple scattering media) or quantum fluids (Bose-Einstein condensates). In the first case for instance, the numerical simulation of such direct problems is a hard task, as it generally requires solving ill-conditioned possibly indefinite large size problems, following from space or space-time discretizations of linear or nonlinear evolution PDE set on unbounded domains. Inverse problems are the converse problem of the direct problems, as they aim to find properties of the direct problem, for instance, the speed of propagation in a medium, with respect to the solution or a partial observation of the solution. These problems are often ill-posed and many questions are open at both the theoretical (identifiability, stability and robustness, etc.) and practical (reconstruction methods, approximation and convergence analysis, numerical algorithms, etc.) levels.

## 3 Research program

### 3.1 Analysis, control, stabilization and optimization of heterogeneous systems

Fluid-Structure Interaction Systems are present in many physical problems and applications. Their study involves solving several challenging mathematical problems:

- Nonlinearity: One has to deal with a system of nonlinear PDE such as the Navier-Stokes or the Euler systems;
- Coupling: The corresponding equations couple two systems of different types and the methods associated with each system need to be suitably combined to successfully solve the full problem;
- Coordinates: The equations for the structure are classically written with Lagrangian coordinates whereas the equations for the fluid are written with Eulerian coordinates;
- Free boundary: The fluid domain is moving and its motion depends on the motion of the structure. The fluid domain is thus an unknown of the problem and one has to solve a free boundary problem.

In order to control such FSIS, one has first to analyze the corresponding system of PDE. The oldest works on FSIS go back to the pioneering contributions of Thomson, Tait and Kirchhoff in the 19th century
and Lamb in the 20th century, who considered simplified models (potential fluid or Stokes system). The first mathematical studies in the case of a viscous incompressible fluid modeled by the Navier-Stokes system and a rigid body whose dynamics is modeled by Newton's laws appeared much later [105, 100, 79], and almost all mathematical results on such FSIS have been obtained in the last twenty years.

The most studied FSIS is the problem modeling a rigid body moving in a viscous incompressible fluid [61, 58, 99, 68, 73, 102, 104, 89, 71]. Many other FSIS have been studied as well. Let us mention [91, $76,72,62,50,67,49,69$ for different fluids. The case of deformable structures has also been considered, either for a fluid inside a moving structure (e.g., blood motion in arteries) or for a moving deformable structure immersed in a fluid (e.g., fish locomotion). The obtained coupled FSIS is a complex system and its study raises several difficulties. The main one comes from the fact that we gather two systems of different nature. Some studies have been performed for approximations of this system: [54,50, 82, 63, 52]). Without approximations, the only known results [59,60] were obtained with very strong assumptions on the regularity of the initial data. Such assumptions are not satisfactory but seem inherent to this coupling between two systems of different natures. In order to study self-propelled motions of structures in a fluid, like fish locomotion, one can assume that the deformation of the structure is prescribed and known, whereas its displacement remains unknown [96]. This permits to start the mathematical study of a challenging problem: understanding the locomotion mechanism of aquatic animals. This is related to control or stabilization problems for FSIS. Some first results in this direction were obtained in [77, 51, 93].

### 3.2 Inverse problems for heterogeneous systems

The area of inverse problems covers a large class of theoretical and practical issues which are important in many applications (see for instance the books of Isakov [78] or Kaltenbacher, Neubauer, and Scherzer [80]). Roughly speaking, an inverse problem is a problem where one attempts to recover an unknown property of a given system from its response to an external probing signal. For systems described by evolution PDE, one can be interested in the reconstruction from partial measurements of the state (initial, final or current), the inputs (a source term, for instance) or the parameters of the model (a physical coefficient for example). For stationary or periodic problems (i.e., problems where the time dependency is given), one can be interested in determining from boundary data a local heterogeneity (shape of an obstacle, value of a physical coefficient describing the medium, etc.). Such inverse problems are known to be generally ill posed and their study raises the following questions:

- Uniqueness. The question here is to know whether the measurements uniquely determine the unknown quantity to be recovered. This theoretical issue is a preliminary step in the study of any inverse problem and can be a hard task.
- Stability. When uniqueness is ensured, the question of stability, which is closely related to sensitivity, deserves special attention. Stability estimates provide an upper bound for the parameter error given some uncertainty on the data. This issue is closely related to the so-called observability inequality in systems theory.
- Reconstruction. Inverse problems are usually ill-posed, one needs to develop specific reconstruction algorithms which are robust to noise, disturbances and discretization. A wide class of methods is based on optimization techniques.

We can split our research in inverse problems into two classes which both appear in FSIS and CWS:

1. Identification for evolution PDE.

Driven by applications, the identification problem for systems of infinite dimension described by evolution PDE has seen in the last three decades a fast and significant growth. The unknown to be recovered can be the (initial/final) state (e.g., state estimation problems [44, 70, 74, 101] for the design of feedback controllers), an input (for instance source inverse problems [41, 53, 64]) or a parameter of the system. These problems are generally ill-posed and many regularization approaches have been developed. Among the different methods used for identification, let us mention optimization techniques [57], specific one-dimensional techniques (like in [45]) or observer-based methods as in [85].

In the last few years, we have developed observers to solve initial data inverse problems for a class of linear systems of infinite dimension. Let us recall that observers, or Luenberger observers [84], have been introduced in automatic control theory to estimate the state of a dynamical system of finite dimension from the knowledge of an output (for more references, see for instance [90] or [103]). Using observers, we have proposed in [92, 75] an iterative algorithm to reconstruct initial data from partial measurements for some evolution equations. We are deepening our activities in this direction by considering more general operators or more general sources and the reconstruction of coefficients for the wave equation. In connection with this problem, we study the stability in the determination of these coefficients. To achieve this, we use geometrical optics, which is a classical albeit powerful tool to obtain quantitative stability estimates on some inverse problems with a geometrical background, see for instance [47, 46].

## 2. Geometric inverse problems.

We investigate some geometric inverse problems that appear naturally in many applications, like medical imaging and non-destructive testing. A typical problem we have in mind is the following: given a domain $\Omega$ containing an (unknown) local heterogeneity $\omega$, we consider the boundary value problem of the form

$$
\begin{cases}L u=0, & (\Omega \backslash \omega) \\ u=f, & (\partial \Omega) \\ B u=0, & (\partial \omega)\end{cases}
$$

where $L$ is a given partial differential operator describing the physical phenomenon under consideration (typically a second order differential operator), $B$ the (possibly unknown) operator describing the boundary condition on the boundary of the heterogeneity and $f$ the exterior source used to probe the medium. The question is then to recover the shape of $\omega$ and/or the boundary operator $B$ from some measurement on the outer boundary $\partial \Omega$. This setting includes in particular inverse scattering problems in acoustics and electromagnetics (in this case $\Omega$ is the whole space and the data are far field measurements) and the inverse problem of detecting solids moving in a fluid. It also includes, with slight modifications, more general situations of incomplete data (i.e., measurements on part of the outer boundary) or penetrable inhomogeneities. Our approach to tackle this type of problems is based on the derivation of a series expansion of the input-to-output map of the problem (typically the Dirichlet-to-Neumann map of the problem for the Calderón problem) in terms of the size of the obstacle.

### 3.3 Numerical analysis and simulation of heterogeneous systems

Within the team, we have developed in the last few years numerical codes for the simulation of FSIS and CWS. We plan to continue our efforts in this direction.

- In the case of FSIS, our main objective is to provide computational tools for the scientific community, essentially to solve academic problems.
- In the case of CWS, our main objective is to build tools general enough to handle industrial problems. Our strong collaboration with Christophe Geuzaine's team in Liège (Belgium) makes this objective credible, through the combination of DDM (Domain Decomposition Methods) and parallel computing.

Below, we explain in detail the corresponding scientific program.

- Simulation of FSIS: In order to simulate fluid-structure systems, one has to deal with the fact that the fluid domain is moving and that the two systems for the fluid and for the structure are strongly coupled. To overcome this free boundary problem, three main families of methods are usually applied to numerically compute in an efficient way the solutions of the fluid-structure interaction systems. The first method consists in suitably displacing the mesh of the fluid domain in order to follow the displacement and the deformation of the structure. A classical method based on this idea is the A.L.E. (Arbitrary Lagrangian Eulerian) method: with such a procedure, it is possible to keep a good precision at the interface between the fluid and the structure. However, such methods are
difficult to apply for large displacements (typically the motion of rigid bodies). The second family of methods consists in using a fixed mesh for both the fluid and the structure and to simultaneously compute the velocity field of the fluid with the displacement velocity of the structure. The presence of the structure is taken into account through the numerical scheme. Finally, the third class of methods consists in transforming the set of PDEs governing the flow into a system of integral equations set on the boundary of the immersed structure. The members of SPHINX have already worked on these three families of numerical methods for FSIS systems with rigid bodies (see e.g., [97], [81], [98], [94], [95], [86]).
- Simulation of CWS: Solving acoustic or electromagnetic scattering problems can become a tremendously hard task in some specific situations. In the high frequency regime (i.e., for small wavelength), acoustic (Helmholtz's equation) or electromagnetic (Maxwell's equations) scattering problems are known to be difficult to solve while being crucial for industrial applications (e.g., in aeronautics and aerospace engineering). Our particularity is to develop new numerical methods based on the hybridization of standard numerical techniques (like algebraic preconditioners, etc.) with approaches borrowed from asymptotic microlocal analysis. Most particularly, we contribute to building hybrid algebraic/analytical preconditioners and quasi-optimal Domain Decomposition Methods (DDM) [48, 65], [66] for highly indefinite linear systems. Corresponding three-dimensional solvers (like for example GetDDM) will be developed and tested on realistic configurations (e.g., submarines, complete or parts of an aircraft, etc.) provided by industrial partners (Thales, Airbus). Another situation where scattering problems can be hard to solve is the one of dense multiple (acoustic, electromagnetic or elastic) scattering media. Computing waves in such media requires us to take into account not only the interactions between the incident wave and the scatterers, but also the effects of the interactions between the scatterers themselves. When the number of scatterers is very large (and possibly at high frequency [42, 43]), specific deterministic or stochastic numerical methods and algorithms are needed. We introduce new optimized numerical methods for solving such complex configurations. Many applications are related to this problem, such as osteoporosis diagnosis where quantitative ultrasound is a recent and promising technique to detect a risk of fracture. Therefore, numerical simulation of wave propagation in multiple scattering elastic media in the high frequency regime is a very useful tool for this purpose.


## 4 Application domains

### 4.1 Robotic swimmers

Some companies aim at building biomimetic robots that can swim in an aquarium, as toys but also for medical purposes. An objective of SPHINX is to model and to analyze several models of these robotic swimmers. For the moment, we focus on the motion of a nanorobot. In that case, the size of the swimmers leads us to neglect the inertia forces and to only consider the viscosity effects. Such nanorobots could be used for medical purposes to deliver some medicine or perform small surgical operations. In order to get a better understanding of such robotic swimmers, we have obtained control results via shape changes and we have developed simulation tools (see [55, 56, 86, 83]). Among all the important issues, we aim to consider the following ones:

1. Solve the control problem by limiting the set of admissible deformations.
2. Find the "best" location of the actuators, in the sense of being the closest to the exact optimal control.

The main tools for this investigation are the 3D codes that we have developed for simulating the fish in a viscous incompressible fluid (SUSHI3D) or in an inviscid incompressible fluid (SOLEIL).

### 4.2 Aeronautics

We will develop robust and efficient solvers for problems arising in aeronautics (or aerospace) like electromagnetic compatibility and acoustic problems related to noise reduction in an aircraft. Our interest for
these issues is motivated by our close contact with companies like Airbus or "Thales Systèmes Aéroportés". We will propose new applications needed by these partners and assist them in integrating these new scientific developments in their home-made solvers. In particular, in collaboration with C. Geuzaine (Université de Liège), we are building a freely available parallel solver based on Domain Decomposition Methods that can handle complex engineering simulations, in terms of geometry, discretization methods as well as physics problems, see here.

## 5 Highlights of the year

### 5.1 Awards

- Yannick Privat was awarded the Blaise Pascal Prize of the French Academy of Sciences in 2023.


## 6 New results

### 6.1 Analysis, control, stabilization and optimization of heterogeneous systems

Participants: Rémi Buffe, Imene Djebour, Alessandro Duca, Ludovick Gagnon, Julien Lequeurre, Karim Ramdani, Jean-François Scheid, Takéo Takahashi, Julie Valein, Christophe Zhang.

## Analysis

In [38], we extend the theory of single and double layer potentials (well documented for functions with $H_{l o c}^{1}$ regularity) to locally square integrable functions. Having in mind numerical simulations for which functions are usually defined on a polygonal mesh, we wish this theory to cover the cases of non-smooth domains (i.e., with Lipschitz continuous or polygonal boundaries).

In [11], we consider a wave equation with a structural damping coupled with an undamped wave equation located at its boundary. We prove that, due to the coupling, the full system is parabolic. In order to show that the underlying operator generates an analytical semigroup, we study in particular the effect of the damping of the "interior" wave equation on the "boundary" wave equation and show that it generates a structural damping.

In [25], we study the weak uniqueness and the regularity of the weak solutions of a fluid-structure interaction system. More precisely, we consider the motion of a rigid ball in a viscous incompressible fluid and we assume that the fluid-rigid body system fills the entire space $\mathbb{R}^{3}$. We prove that the corresponding weak solutions that additionally satisfy a classical Prodi-Serrin condition, including a critical one, are unique. We also show that the weak solutions are regular under the Prodi-Serrin conditions, with a smallness condition in the critical case.

In [34], we consider a fluid-structure interaction system coupling a viscous fluid governed by the compressible Navier-Stokes equations and a rigid body immersed in the fluid and modeled by Newton's law. In this work, we consider the Navier slip boundary conditions. Our aim is to show the local existence and uniqueness of the strong solution to the corresponding problem. The main step of this work is that we use a Lagrangian change of variables in order to handle the transport equation and to reduce the problem in the initial domain. However, the specificity here is that the Lagrangian coordinates do not coincide with the Eulerian coordinates at the boundaries since we consider slip boundary conditions. Therefore, it brings some extra nonlinear terms in the boundary conditions. The strategy is based on the study of the linearized system with nonhomogeneous boundary conditions and on the Banach fixed-point theorem.

## Control

Controlling coupled systems is a complex issue depending on the coupling conditions and the equations themselves. Our team has a strong expertise to tackle this kind of problems in the context of fluid-structure interaction systems. More precisely, we obtained the following results.

In [12], we consider the controllability of an abstract parabolic system by using switching controls. More precisely, we show that under general hypotheses, if a parabolic system is null controllable for any positive time with $N$ controls, then it is also null controllable with the property that at each time, only one of these controls is active. The main difference with previous results in the literature is that we can handle the case where the main operator of the system is not self-adjoint. We give several examples to illustrate our result: coupled heat equations with terms of orders 0 and 1 , the Oseen system or the Boussinesq system.

In [14], we prove the null-controllability of a fluid-structure system coupling the Navier-Stokes equation for the fluid and a plate equation at the boundary. The control acts on arbitrarily small subsets of the fluid domain and in a small subset of the vibrating boundary. By proving a proper observability inequality, we obtain the local controllability for the nonlinear system. The proof relies on microlocal argument to handle the pressure terms.

In [32], we consider the controllability of a fluid-structure interaction system, where the fluid is modeled by the Navier-Stokes system and where the structure is a damped beam located on a part of its boundary. The motion of the fluid is bi-dimensional whereas the deformation of the structure is one-dimensional and we use periodic boundary conditions in the horizontal direction. Our result is the local null-controllability of this free-boundary system by using only one scalar control acting on an arbitrary small part of the fluid domain. This improves a previous result obtained by the authors where three scalar controls were needed to achieve the local null-controllability. In order to show the result, we prove the final-state observability of a linear Stokes-beam interaction system in a cylindrical domain. This is done by using a Fourier decomposition, proving Carleman inequalities for the corresponding system for the low-frequency solutions and in the case where the observation domain is a horizontal strip. Then we conclude this observability result by using a Lebeau-Robbiano strategy for the heat equation and a uniform exponential decay for the high-frequency solutions. Then, the result on the nonlinear system can be obtained by a change of variables and a fixed-point argument.

In [17], we study the local null-controllability of a modified Navier-Stokes system which includes nonlocal spatial terms. We generalize a previous work where the nonlocal spatial term is given by the linearization of a Ladyzhenskaya model for a viscous incompressible fluid. Here the nonlocal spatial term is more complicated and we consider a control with one vanishing component. The proof of the result is based on a Carleman estimate where the main difficulty consists of handling the nonlocal spatial terms. One of the key points is a particular decomposition of the solution of the adjoint system that allows us to overcome regularity issues. With a similar approach, we also show the existence of insensitizing controls for the same system.

In [18], we show the boundary controllability to stationary states of the Stefan problem with two phases and in one dimension in the space variable. For an initial condition that is a stationary state and for a time of control large enough, we also obtain the controllability to stationary states together with the sign constraints associated with the problem. Our method is based on the flatness approach that consists of writing the solution and the controls through two outputs and their derivatives. We construct these outputs as Gevrey functions of order $\sigma$ so that their solution and controls are also in a Gevrey class.

In [19], we study a one-dimensional cross diffusion system with a free boundary modeling physical vapor deposition. Using the flatness approach, we show several results of boundary controllability for this system in spaces of Gevrey class functions. One of the main difficulties consists in the physical constraints on the state and the control. More precisely, the state corresponds to volume fractions of the $n+1$ chemical species and to the thickness of the film produced in the process, whereas the controls are the fluxes of the chemical species. We obtain the local controllability in the case where we apply $n+1$ nonnegative controls and a controllability result for large time in the case where we apply $n$ controls without any sign constraints. We also show in this last case that the controllability may fail for small times. We illustrate these results with some numerical simulations.

In [33], we study the boundary controllability of $2 \times 2$ system of heat equations by using a flatness approach. According to the relation between the diffusion coefficients of the heat equation, it is known that the system can be not null controllable or null controllable for any $T>T_{0}$ where $T_{0} \in[0, \infty]$. Here we recover this result in the case that $T_{0} \in[0, \infty)$ by using the flatness method, and we obtain an explicit formula for the control and for the corresponding solutions. In particular, the state and the control have Gevrey regularity in time and in space.

In [27], we deal with the controllability properties of a system of $m$ coupled Stokes systems or $m$
coupled Navier-Stokes systems. We show the null-controllability of such systems in the case where the coupling is in a cascade form and when the control acts only on one of the systems. Moreover, we impose that this control has a vanishing component so that we control an $m \times N$ state (corresponding to the velocities of the fluids) by $N-1$ distributed scalar controls. The proof of the controllability of the coupled Stokes system is based on a Carleman estimate for the adjoint system. The local null-controllability of the coupled Navier-Stokes systems is then obtained by means of the source term method and a Banach fixed point.

In [28], we consider the controllability of a class of systems of $n$ Stokes equations, coupled through terms of order zero and controlled by $m$ distributed controls. Our main result states that such a system is null controllable if and only if a Kalman type condition is satisfied. This generalizes the case of finitedimensional systems and the case of systems of coupled linear heat equations. The proof of the main result relies on the use of the Kalman operator introduced and on a Carleman estimate for a cascade type system of Stokes equations. Using a fixed-point argument, we also obtain that if the Kalman condition is verified, then the corresponding system of Navier-Stokes equations is locally null controllable.

In [40], we consider a Stackelberg control strategy applied to the Boussinesq system. More precisely, we act on this system with a hierarchy of two controls. The aim of the "leader" control is the nullcontrollability property whereas the objective of "follower" control is to keep the state close to a given trajectory. By solving first the optimal control problem associated with the follower control, we are led to show the null-controllability property of a system coupling a forward with a backward Boussinesq type systems. Our main result states that for an adequate weighted functional for the optimal control problem, this coupled system is locally null controllable. To show this result, we first study the adjoint system of the linearized system and obtain a weighted observability estimate by combining several Carleman estimates and an adequate decomposition for the heat and the Stokes system.

In [31], we ensure an observability inequality, also known as spectral inequality, within spaces spanned by the first eigenfunctions for a family of one-dimensional degenerate operators $\partial_{x} x^{\alpha} \partial_{x}$. We give a bound for the blow-up of the constant when the frequency cut goes to infinity, which is known to be optimal for the Laplace operator. A new application to observability for degenerate parabolic equations is given. Finally, the associated degenerate parabolic equation is known to be not null controllable when $\alpha \geq 2$. we prove a bound for the blow-up of the constants of the spectral inequality when $\alpha \rightarrow 2^{-}$. The proof relies on a combination of the moment method and Carleman estimates.

Finally, in [39], we consider the internal control of linear parabolic equations through on-off shape controls with a prescribed maximal measure. They establish small-time approximate controllability towards all possible final states allowed by the comparison principle with non-negative controls and manage to build controls with constant amplitude.

## Stabilization

Stabilization of infinite dimensional systems governed by PDEs is a challenging problem. In our team, we have investigated this issue for different kinds of systems (fluid systems and wave systems) using different techniques.

In [35], we consider the stabilization of a class of linear evolution systems $z^{\prime}=A z+B v$ under the observation $y=C z$ by means of a finite dimensional control $v$. The control is based on the design of a Luenberger observer which can be infinite or finite dimensional (of dimension large enough). In the infinite dimensional case, the operator $A$ is supposed to generate an analytical semigroup with compact resolvent and the operators $B$ and $C$ are unbounded operators whereas, in the finite dimensional case, $A$ is assumed to be a self-adjoint operator with compact resolvent, $B$ and $C$ are supposed to be bounded operators. In both cases, we show that if $(A, B)$ and $(A, C)$ verify the Fattorini-Hautus Criterion, then we can construct an observer-based control $v$ of finite dimension (greater or equal to the largest geometric multiplicity of the unstable eigenvalues of $A$ ) such that the evolution problem is exponentially stable. As an application, we study the stabilization of the $N$-dimensional convection-diffusion system with Dirichlet boundary control and an internal observation.

In [26], we consider the Korteweg-de Vries equation with time-dependent delay on the boundary or internal feedbacks. Under some assumptions on the time-dependent delay, on the weights of the feedbacks, and on the length of the spatial domain, we prove the exponential stability results, using appropriate Lyapunov functionals. We finish with some numerical simulations that illustrate the stability
results and the influence of the delay on the decay rate.
In [23], we show the stabilization by a finite number of controllers of a fluid-structure interaction system where the fluid is modeled by the Navier-Stokes system into a periodical canal and where the structure is an elastic wall localized on top of the fluid domain. The elastic deformation of the structure follows a damped beam equation. We also assume that the fluid can slip on its boundaries and we model this by using the Navier slip boundary conditions. Our result states the local exponential stabilization around a stationary state of strong solutions by using dynamical controllers in order to handle the compatibility conditions at initial time. The proof is based on a change of variables to write the fluid-structure interaction system in a fixed domain and on the stabilization of the linearization of the corresponding system around the stationary state. One of the main difficulties consists in handling the nonlinear terms coming from the change of variables in the boundary conditions.

In [36], we prove the rapid stabilization of the linearized water waves equation with the Fredholm backstepping method. This result is achieved by overcoming an important theoretical threshold imposed by the classical methodology, namely, the quadratically close criterion. Indeed, the spatial operator of the linearized water waves exhibits an insufficient growth of the eigenvalues and the quadratically close criterion is not true in this case. We introduce the duality compactness method for general skew-adjoint operators to circumvent this difficulty. In turn, we prove the existence of a Fredholm backstepping transformation for a wide range of equations, opening the path to an abstract framework for this widely used method.

## Optimization

In [16], an optimal shape problem for a general functional depending on the solution of a bidimensional Fluid-Structure Interaction problem (FSI) is studied. The system is composed of a coupling stationary Stokes-Elasticity sub-system for modeling the deformation of an elastic structure immersed in a viscous fluid. The differentiability with respect to the reference domain of the elastic structure is proved under shape perturbations with diffeomorphisms. The shape-derivative is then calculated with the use of the velocity method. This derivative involves the material derivatives of the solution of this fluid-structure interaction problem. The adjoint method is then used to obtain a simplified expression for the shape derivative. The main difficulty for studying the shape sensitivity of this FSI problem lies in the coupling between the Stokes problem written in a Eulerian frame and the linear elasticity problem written in a Lagrangian form.

### 6.2 Direct and inverse problems for heterogeneous systems

Participants: Alessandro Duca, Anthony Gerber-Roth, Alexandre Munnier, Karim Ramdani, Jean-François Scheid.

## Direct problems

Negative materials are artificially structured composite materials (also known as metamaterials), whose dielectric permittivity and magnetic permeability are simultaneously negative in some frequency ranges. K. Ramdani continued his collaboration with R. Bunoiu on the homogenization of composite materials involving both positive and negative materials. Due to the sign-changing coefficients in the equations, classical homogenization theory fails, since it is based on uniform energy estimates which are known only for positive (more precisely constant sign) coefficients.

In [15], in collaboration with C. Timofte, K. Ramdani and R. Bunoiu investigate the homogenization of a diffusion-type problem, for sign-changing conductivities in the case of imperfect interface conditions is considered, by allowing flux jumps across their oscillating interface. The main difficulties of this study are due to the sign-changing coefficients and the appearance of an unsigned surface integral term in the variational formulation. A proof by contradiction (nonstandard in this context) and $T$-coercivity techniques are used in order to cope with these difficulties.

## Inverse problems

Supervised by A. Munnier and K. Ramdani, the PhD of Anthony Gerber-Roth is devoted to the investigation of some geometric inverse problems, and can be seen as a continuation of the work initiated by the two supervisors in [88] and [87]. In these papers, the authors addressed a particular case of Calderòn's inverse problem in dimension two, namely the case of a homogeneous background containing a finite number of cavities (i.e., heterogeneities of infinitely high conductivities). The first contribution of Anthony Gerber-Roth was to apply the method proposed in [87] to tackle a two-dimensional inverse gravimetric problem. The strong connection with the important notion of quadrature domains in this context has been highlighted. An efficient reconstruction algorithm has been proposed (and rigorously justified in some cases) for this geometric inverse problem. These results are detailed in [5].

In [29], we are interested in an inverse problem set on a tree shaped network where each edge behaves according to the wave equation with potential, external nodes have Dirichlet boundary conditions and internal nodes follow the Kirchoff law. The main goal is the reconstruction of the potential everywhere on the network, from the Neumann boundary measurements at all but one external vertices. Leveraging from the Lipschitz stability of this inverse problem, we aim to provide an efficient reconstruction algorithm based on the use of a specific global Carleman estimate. The proof of the main tool and of the convergence of the algorithm are provided; along with a detailed description of the numerical illustrations given at the end of the article.

In [37], we address the classical inverse problem of recovering the position and shape of obstacles immersed in a planar Stokes flow using boundary measurements. We prove that this problem can be transformed into a shape-from-moments problem to which ad hoc reconstruction methods can be applied. The effectiveness of this approach is confirmed by numerical tests that show significant improvements over those available in the literature to date.

### 6.3 Numerical analysis and simulation of heterogeneous systems

Participants: Yannick Privat, Jean-François Scheid.

The work in [13] is devoted to the modeling and numerical simulations of a one-dimensional model for localized corrosion phenomena. Localized corrosion involves the dissolution of metal in an aqueous solution of a number of chemical species together with their mass transport by diffusion and migration, and their reactions in solution. From a mathematical point of view, this problem can be identified as a Stefan problem involving a convection-reaction-diffusion system of PDEs with a moving boundary between the aqueous solution and the metal. The unknowns of this system are the concentrations of the chemical species, the electric potential and the position of the free boundary. The dissolution law steering the evolution of the free boundary is given by the nonlinear Butler-Volmer formula. In this work, the mathematical procedure for solving this strongly coupled differential equations system and the numerical development for simulations are presented. A finite-difference ALE (Abritrary Lagrangian Eulerian) scheme is used for the numerical computation of the solutions of this free boundary problem, leading to a nonlinear discrete system which is then solved using a Newton procedure. The numerical simulations obtained are in good agreement with experimental results.

## 7 Partnerships and cooperations

### 7.1 International initiatives

### 7.1.1 Inria associate team not involved in an IIL or an international program

 MOUSTIQParticipants: Rémi Buffe, Ludovick Gagnon, Takéo Takahashi, Julie Valein.

Title: Modelization and control of infectious diseases, wave propagation in heterogeneous media and nonlinear dispersive equations

Duration: 2020-2024
Coordinator: Ludovick Gagnon
Partners: Universidade Federale da Paraiba (Brésil)
Summary: This project is divided into three research axes, all in the field of control theory and within the field of expertise of the Sphinx project team.

The first axis consists in improving a network transport model of virus spread by mosquitoes such as Zika, Dengue or Chikungunya. The objective is to introduce time-delay terms into the model to take into account delays such as incubation time or reaction time of health authorities. The study of the controllability of the model will then be carried out in order to optimize the reaction time as well as the coverage of the population in the event of an outbreak.
The second axis concerns the controllability of waves in a heterogeneous environment. These media are characterized by discontinuous propagation speed at the interface between two media, leading to refraction phenomena according to Snell's law. Only a few controllability results are known in restricted geometric settings, the last result being due to the Inria principal investigator. Examples of applications of the controllability of these models range from seismic exploration to the clearance of anti-personnel mines.
Finally, the last axis aims to study the controllability of nonlinear dispersive equations. These equations are distinguished by a decrease of the solutions due to the different propagation speed of each frequency. There only exists few tools available to obtain controllability results of these equations in arbitrarily small time and many important questions remain open. These equations can be used to model, for example, the propagation of waves in shallow waters as well as the propagation of signals in an optical fiber.

### 7.1.2 STIC/MATH/CLIMAT AmSud projects

## SCIPInPDES

Participants: Rémi Buffe, Ludovick Gagnon, Takéo Takahashi, Julie Valein.

Title: Stabilization, Control and Inverse Problems in PDEs
Program: MATH-AmSud
Duration: January 1, 2023 - December 31, 2024
Local supervisor: Ludovick Gagnon
Partners:

- Universidad Técnica Federico Santa María (Chili)
- Universidade Federal da Paraíba (Brazil)


## Inria contact: Ludovick Gagnon

Summary: This project is devoted to the analysis of several physical processes and phenomena modeled by Partial Differential Equations. In particular, the problems to be addressed are of great interest in biology, engineering, fluid mechanics, aeronautics, chemistry and medicine. From a mathematical point of view, we will study several qualitative properties for these problems, such as controllability, observability, unique continuation, asymptotic behavior, uniqueness, stability, etc.

### 7.2 National initiatives

## ANR TRECOS

Participants: Ludovick Gagnon, Yannick Privat, Takéo Takahashi, Julie Valein.

Title: New Trends in Control and Stabilization: Constraints and non-local terms
Duration: 2021-2024
Coordinator: Sylvain Ervedoza
Partners:

- University of Bordeaux
- Sorbonne University of Paris
- University of Toulouse

Summary: The goal of this project is to address new directions of research in control theory for partial differential equations, triggered by models from ecology and biology. In particular, our projet will deal with the development of new methods which will be applicable in many applications, from the treatment of cancer cells to the analysis of the thermic efficiency of buildings, and from control issues for the biological control of pests to cardiovascular fluid flows. To achieve these objectives, we will have to solve several theoretical issues in order to design efficient control methods.

## ANR ODISSE

Participants: Ludovick Gagnon, Takéo Takahashi, Karim Ramdani, Julie Valein, JeanClaude Vivalda.

Title: Observer Design for Infinite-dimensional Systems
Duration: 2019-2024
Coordinator: Vincent Andrieu

## Partners:

- University Claude Bernard of Lyon
- University of Toulouse
- Inria Saclay - Ile de France

Summary: Methodologically, this project is at the crossroads of inverse problems and observers theory. These two disciplines have a long and rich history of interactions and their overlap is becoming more and more obvious. The project proposes fundamental/theoretical contributions in observer design to reconstruct missing parameters in infinite dimensional systems. The methodological contributions expected of this fundamental project will be illustrated on different test-beds.

## 8 Dissemination

Participants: Mabrouk Ben Jaba, Rémi Buffe, Blaise Colle, Baparou Danhane, Imene Aicha Djebour, David Dos Santos Ferreira, Alessandro Duca, Benjamin Florentin, Ludovick Gagnon, Anthony Gerber-Roth, Julien Lequeurre, Alexandre Munnier, Yannick Privat, Karim Ramdani, JeanFrançois Scheid, Takéo Takahashi, Julie Valein, Jean-Claude Vivalda, Christophe Zhang.

### 8.1 Promoting scientific activities

### 8.1.1 Scientific events: organisation

## Member of the organizing committees

- Rémi Buffe (since September 2023), Julien Lequeurre and Alexandre Munnier are co-organizers of the annual workshop Journées EDP de l'IECL.
- David Dos Santos Ferreira is a member of the scientific committee of the GDR Analyse des EDP, which organizes the "Journées des jeunes EDPistes".
- Rémi Buffe is the organizer of the IECL's working group on PDEs.
- Ludovick Gagnon is a co-organizer of the IECL's weekly seminar on PDEs in Nancy.
- Julien Lequeurre is a co-organizer of the IECL's weekly seminar on PDEs in Metz


### 8.1.2 Scientific events: selection

Reviewer Alessandro Duca was a reviewer for the 10th International Congress on Industrial and Applied Mathematics, ICIAM 2023 Tokyo.

### 8.1.3 Journal

Member of the editorial boards Yannick Privat is a member of the editorial boards of the following journals and book series: AIMS Applied Math. books, Computational and Applied Mathematics, Evolution Equations and Control Theory, Journal of Optimization Theory and Applications, Mathematical Control and Related Fields (MCRF) and Numerical Algebra, Control and Optimization.

Reviewer - reviewing activities SPHINX members were reviewers for several scientific journals on control theory and PDEs. We mention for instance: Annales de l'Institut Henri Poincaré, Analyse non linéaire; Automatica; Communications in Mathematical Sciences; Computational and Applied Mathematics; ESAIM: Control, Optimisation and Calculus of Variations; Evolution Equations and Control Theory; Inverse Problems; Inverse Problems and Imaging; Journal de Mathématiques Pures et Appliquées; Journal of Dynamical and Control Systems; Journal of Mathematical Physics; Journal of Optimization Theory and Applications; Mathematical Control and Related Fields (MCRF); Mathematical Methods in the Applied Sciences; SIAM Journal on Applied Mathematics; SIAM Journal on Control and Optimization; etc.

### 8.1.4 Invited talks

- Alessandro Duca was invited to give a talk at: 10th International Congress on Industrial and Applied Mathematics, ICIAM 2023, in Tokyo. He presented his work at the workshops: "Journées QUACO" (Laboratory Jacques-Louis Lions, Paris) and "Journée scientifique Matière, systèmes et dispositifs quantique" (Université de Lorraine, Nancy). He was also invited to give a seminar at the New York University of Shanghai.
- Julie Valein was invited to give a talk at the workshops: NANT and WG3 meeting "Graph structure generalizations in dynamical system modelling" (Będlewo, Pologne); "Contrôle : approches mathématique et automatique " (Fédération Charles Hermite, Nancy); workshop "Commande et observation des systèmes" (LAAS, Toulouse); "Control of Partial Differential Equations" (Polytechnical University Hauts-de-France). She was also invited to give a seminar at the "Laboratoire de Mathématiques de Besançon".
- David Dos Santos Ferreira was invited to give a talk at the Ghent Methusalem colloquium (Belgium) and Spectral and Resonance Problems for Imaging, Seismology and Materials Science (Reims).
- Christophe Zhang gave a seminar at the Laboratoire des Signaux et Systèmes (CNRS, Gif-sur-Yvette).


### 8.1.5 Research administration

- Karim Ramdani is the head of the PDE team of IECL laboratory (the Mathematics laboratory of Université de Lorraine) since June 2021.
- Julie Valein is an elected member of the scientific pole AM2I of Université de Lorraine since 2022. She was member of the CNU 26 in 2023 (elected member after the promotion of another member from MCF to Professor during the mandate 2019-2023). She was also elected as a substitute member for the CNU 26 for the mandate 2023-2027.


### 8.2 Teaching - Supervision - Juries

### 8.2.1 Teaching

Except for the researchers of the team (A. Duca, L. Gagnon, K. Ramdani, T. Takahashi and J.-C. Vivalda), SPHINX members have teaching obligations at "Université de Lorraine" and are teaching at least 192 hours each year. They teach mathematics at different levels (Licence, Master, Engineering school). Many of them also have pedagogical responsibilities. In 2023, several members of the team taught in the Mathematics Master 2 (second year) of Université de Lorraine: Rémi Buffe and Ludovick Gagnon a course on Control theory, David Dos Santos Ferreira a course on spectral theory, Alexandre Munnier a course on integral equations and potential theory, Yannick Privat and Christophe Zhang a course on Optimitazion, and Benjamin Florentin a course on Complex Analysis.

### 8.2.2 Supervision

- PhD in progress: Benjamin Florentin; "Déformations isospectrales pour l'opérateur de Steklov"; Oct 2022; David Dos Santos Ferreira.
- PhD in progress: Anthony Gerber Roth; "Autour de quelques problèmes inverses géométriques"; Set 2020; Alexandre Munnier and Karim Ramdani.
- PhD in progress: Mabrouk Ben Jaba; "Modélisation du transfert de dioxygène dans le poumon humain, une approche 'contrôle optimal'"; Oct 2023; Yannick Privat and Jean-François Scheid.
- PhD in progress: Killian Lutz; "Méthodes hybrides couplant apprentissage par renforcement et méthodes de contrôle optimal des EDP pour l'informatique quantique"; Sep 2023; Emmanuel Franck and Yannick Privat.
- PhD in progress: Ivan Hasenohr; "Contrôlabilité sous contraintes géométriques et topologiques"; Oct 2022; Sébastien Martin, Camille Pouchol, Yannick Privat and Christophe Zhang.
- PhD in progress: Blaise Colle; "Stabilisation et contrôlabilité du problème de Stefan"; Oct. 2021; Jérôme Lohéac and Takéo Takahashi.
- PhD in progress: Yingying Wu-Zhang; "Algunos problemas de control en mecánica de fluidos y algunas ecuaciones diferenciales parciales"; Sept. 2020; Luz de Teresa and Takéo Takahashi.


### 8.2.3 Juries

- Yannick Privat was the president of the jury of the Ph.D. thesis of M. Lajili (Université Haute Alsace, September 2023). He was a reviewer of the Ph.D. theses of P. Ascensio (INRIA Sophia Antipolis, September 2023) and E. Martinet (Université Savoie Mont Blanc, October 2023). He was also a member of the juries of the Ph.D. theses of T. Delaunay (École Polytechnique, December 2023) and V. Mons (Université Nice Côte d'Azur, December 2023).
- Karim Ramdani was president of the Ph.D. committee of Luis Alejandro Rosas Martinez (École Polytechnique, November 2023).
- Takéo Takahashi was a reviewer of the Ph.D. thesis of Tiphaine Delaunay (École Polytechnique, December 2023).
- David Dos Santos Ferreira was a reviewer of the Ph.D. thesis of Spyridon Fillipas (Laboratoire de Mathématiques d’Orsay, December 2023).
- Julie Valein was a reviewer of the Ph.D. thesis of F. Koudohode (Université Toulouse III - Paul Sabatier, October 2023). She was also member of the juries of the Ph.D. theses of Z. Mohamad Ali (Université de Lorraine - Metz, December 2023) and H. Parada (Laboratory Jean Kunztmann of Grenoble, July 2023).


### 8.3 Popularization

### 8.3.1 Internal or external Inria responsibilities

Karim Ramdani is a member (since October 2018) of the Working Group "Publications" of the "Committee for Open Science" of the French Ministry of Higher Education, Research and Innovation.

### 8.3.2 Interventions

Karim Ramdani gave several talks to review the most recent changes in scientific publishing, especially concerning the emergence of the dangerous author-pays model of open science.

## 9 Scientific production

### 9.1 Major publications

[1] L. Bălilescu, J. San Martín and T. Takahashi. 'Fluid-structure interaction system with Coulomb's law'. In: SIAM Journal on Mathematical Analysis (2017). URL: https : //hal . archives-ouverte s.fr/hal-01386574.
[2] R. Bunoiu, L. Chesnel, K. Ramdani and M. Rihani. 'Homogenization of Maxwell's equations and related scalar problems with sign-changing coefficients'. In: Annales de la Faculté des Sciences de Toulouse. Mathématiques. (2020). URL: https://hal.inria.fr/hal-02421312.
[3] N. Burq, D. Dos Santos Ferreira and K. Krupchyk. 'From semiclassical Strichartz estimates to uniform $L^{p}$ resolvent estimates on compact manifolds'. In: Int. Math. Res. Not. IMRN 16 (2018), pp. 5178-5218. DOI: $10.1093 / i m r n / r n x 042$. URL: https://doi.org/10.1093/imrn/rnx042.
[4] L. Gagnon. 'Lagrangian controllability of the 1-dimensional Korteweg-de Vries equation'. In: SIAM J. Control Optim. 54.6 (2016), pp. 3152-3173. DOI: 10.1137/140964783. URL: https://doi. org /10.1137/140964783.
[5] A. Gerber-Roth, A. Munnier and K. Ramdani. 'A reconstruction method for the inverse gravimetric problem'. In: SMAI Journal of Computational Mathematics 9 (2023), pp. 197-225. URL: https : //i nria.hal.science/hal-04157036.
[6] O. Glass, A. Munnier and F. Sueur. 'Point vortex dynamics as zero-radius limit of the motion of a rigid body in an irrotational fluid’. In: Inventiones Mathematicae 214.1 (2018), pp. 171-287. DOI: 10.1007/s00222-018-0802-4. URL: https://hal. archives-ouvertes.fr/hal-00950544.
[7] C. Grandmont, M. Hillairet and J. Lequeurre. 'Existence of local strong solutions to fluid-beam and fluid-rod interaction systems'. In: Annales de l'Institut Henri Poincaré (C) Non Linear Analysis 36.4 (July 2019), pp. 1105-1149. DOI: $10.1016 /$ j. anihpc.2018.10.006. URL: https://hal.in ria.fr/hal-01567661.
[8] A. Munnier and K. Ramdani. 'Calderón cavities inverse problem as a shape-from-moments problem'. In: Quarterly of Applied Mathematics 76 (2018), pp. 407-435. URL: https://hal. inri a.fr/hal-01503425.
[9] K. Ramdani, J. Valein and J.-C. Vivalda. 'Adaptive observer for age-structured population with spatial diffusion'. In: North-Western European Journal of Mathematics 4 (2018), pp. 39-58. URL: https://hal.inria.fr/hal-01469488.
[10] J.-F. Scheid and J. Sokolowski. 'Shape optimization for a fluid-elasticity system'. In: Pure Appl. Funct. Anal. 3.1 (2018), pp. 193-217.

### 9.2 Publications of the year

## International journals

[11] M. Badra and T. Takahashi. 'Analyticity of the semigroup corresponding to a strongly damped wave equation with a Ventcel boundary condition'. In: Dynamics of Partial Differential Equations (2023). URL: https://hal. science/hal-03784384.
[12] M. Badra and T. Takahashi. 'Switching controls for parabolic systems'. In: Palestine Journal of Mathematics (2023). URL: https://hal. science/hal-03417266.
[13] M. Bouguezzi, J.-F. Scheid, D. Hilhorst, H. Matano, C. Bataillon, F. Lequien and F. Rouillard. 'Anodic dissolution model with diffusion-migration transport for simulating localized corrosion'. In: Electrochimica Acta (2024). URL: https: //inria.hal. science/hal-04160934.
[14] R. Buffe and T. Takahashi. ‘Controllability of a fluid-structure interaction system coupling the Navier-Stokes system and a damped beam equation'. In: Comptes Rendus. Mathématique (2023). DOI: 10.5802/crmath.509. URL: https://hal. science/hal-03878011.
[15] R. Bunoiu, K. Ramdani and C. Timofte. 'Homogenization of a transmission problem with signchanging coefficients and interfacial flux jump'. In: Communications in Mathematical Sciences 21.7 (2023), pp. 2029-2049. DOI: 10.4310/CMS . 2023.v21.n7.a13. URL: https://inria.hal .science/hal-03712444.
[16] V. Calisti, I. Lucardesi and J.-F. Scheid. 'Shape sensitivity analysis of a 2D Fluid-Structure Interaction problem'. In: Journal of Optimization Theory and Applications 199 (2023), pp. 36-79. DOI: 10.1007/s10957-023-02213-4. URL: https://hal.science/hal-03810334.
[17] N. Carreño and T. Takahashi. 'Control problems for the Navier-Stokes system with nonlocal spatial terms'. In: Journal of Optimization Theory and Applications (2023). URL: https : //hal . science /hal-03819799.
[18] B. Colle, J. Lohéac and T. Takahashi. 'Controllability of the Stefan problem by the flatness approach'. In: Systems and Control Letters 174 (Apr. 2023), p. 105480. DOI: 10.1016/j. sysconle. 2 023.105480. URL: https://hal.science/hal-03721544.
[19] B. Colle, J. Lohéac and T. Takahashi. 'Controllability results for a cross diffusion system with a free boundary by a flatness approach'. In: Acta Applicandae Mathematicae 187 (30th Jan. 2023), p. 14. DOI: 10.1007/s10440-023-00607-0. URL: https://hal . science/hal-03969875.
[20] S. Cotin, G. Mestdagh and Y. Privat. 'Organ registration from partial surface data in augmented surgery from an optimal control perspective'. In: Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences 480 (22nd Mar. 2023), p. 20230197. DOI: 10.1098/rspa. 2023 .0197. URL: https://hal. science/hal-04043695.
[21] B. Danhane and J. Lohéac. 'Ensemble controllability of parabolic type equations'. In: Systems and Control Letters 183 (Jan. 2024), p. 105683. DOI: 10.1016/j. sysconle.2023.105683. URL: https://hal.science/hal-04174446.
[22] B. Danhane, J. Lohéac and M. Jungers. 'Conditions for uniform ensemble output controllability, and obstruction to uniform ensemble controllability'. In: Mathematical Control and Related Fields (2024). DOI: $10.3934 / \mathrm{mcrf}$. 2023036. URL: https://hal.science/hal-03824645.
[23] I. A. Djebour and T. Takahashi. 'Feedback boundary stabilization for a viscous incompressible fluid with Navier slip boundary conditions in interaction with a damped beam'. In: Nonlinear Analysis: Real World Applications (2024). DOI: 10.1016/j.nonrwa.2023.104022. URL: https: //hal.science/hal-04012901.
[24] A. Gerber-Roth, A. Munnier and K. Ramdani. 'A reconstruction method for the inverse gravimetric problem'. In: SMAI Journal of Computational Mathematics 9 (2023), pp. 197-225. URL: https://i nria.hal.science/hal-04157036.
[25] D. Maity and T. Takahashi. 'Uniqueness and regularity of weak solutions of a fluid-rigid body interaction system under the Prodi-Serrin condition'. In: Discrete and Continuous Dynamical Systems - Series A 44.3 (2024), pp. 718-742. DOI: $10.3934 / \mathrm{dcds} .2023123$. URL: https://hal . sc ience/hal-04075090.
[26] H. Parada, C. Timimoun and J. Valein. 'Stability results for the KdV equation with time-varying delay'. In: Systems and Control Letters 177 (July 2023), p. 105547. DOI: 10.1016/j. sysconle. 202 3.105547. URL: https://hal.science/hal-03819356.
[27] T. Takahashi, L. de Teresa and Y. Wu-Zhang. 'Controllability results for cascade systems of $m$ coupled $N$-dimensional Stokes and Navier-Stokes systems by $N=1$ scalar controls'. In: ESAIM: Control, Optimisation and Calculus of Variations (2023). Doi: 10.48550/arXiv . 2108 . 09856. URL: https://hal.science/hal-03325287.
[28] T. Takahashi, L. de Teresa and Y. Wu-Zhang. 'A Kalman condition for the controllability of a coupled system of Stokes equations'. In: Journal of Evolution Equations (2024). URL: https: //ha l. science/hal-03936869.

## Reports \& preprints

[29] L. Baudouin, M. de Buhan, E. Crépeau and J. Valein. Carleman-Based Reconstruction Algorithm on a wave Network. 1st Dec. 2023. URL: https: //hal. science/hal-04361363.
[30] M. Bestard, E. Franck, L. Navoret and Y. Privat. Optimal scenario for road evacuation in an urban environment. 20th Oct. 2023. URL: https://hal. science/hal-04253010.
[31] R. Buffe, K. D. Phung and A. Slimani. An optimal spectral inequality for degenerate operators. 26th Sept. 2023. URL: https://hal. science/hal-04219477.
[32] R. Buffe and T. Takahashi. Controllability with one scalar control of a system of interaction between the Navier-Stokes system and a damped beam equation. 24th June 2023. URL: https://hal . scie nce/hal-04140088.
[33] B. Colle, J. Lohéac and T. Takahashi. Flatness approach for the boundary controllability of a system of heat equations. 6th June 2023. URL: https://hal . science/hal-04119834.
[34] I. A. Djebour. Existence of strong solutions for a compressible fluid-solid interaction system with Navier slip boundary conditions. 13th Apr. 2023. URL: https://hal. science/hal-04068293.
[35] I. A. Djebour, K. Ramdani and J. Valein. Observer-based feedback-control for the stabilization of a class of parabolic systems. 7th July 2023. URL: https://hal. science/hal-04155214.
[36] L. Gagnon, A. Hayat, S. Xiang and C. Zhang. Fredholm backstepping for critical operators and application to rapid stabilization for the linearized water waves. 12th Jan. 2024. URL: https : //ha l.science/hal-03892656.
[37] A. Munnier. Reconstruction of obstacles in a Stokes flow as a shape-from-moments problem. 26th Sept. 2023. URL: https://hal. science/hal-04218495.
[38] A. Munnier. Square integrable surface potentials on non-smooth domains and application to the Laplace equation in $L^{2}$. 25th May 2023. URL: https : //hal . science/hal-03942972.
[39] C. Pouchol, E. Trélat and C. Zhang. Approximate control of parabolic equations with on-off shape controls by Fenchel duality. 11th Dec. 2023. URL: https: //hal . science/hal- 03889865.
[40] T. Takahashi, L. de Teresa and Y. Wu-Zhang. Stackelberg exact controllability for the Boussinesq system. 3rd Oct. 2023. URL: https://hal. science/hal-04228391.

### 9.3 Cited publications

[41] C. Alves, A. L. Silvestre, T. Takahashi and M. Tucsnak. 'Solving inverse source problems using observability. Applications to the Euler-Bernoulli plate equation'. In: SIAM J. Control Optim. 48.3 (2009), pp. 1632-1659.
[42] X. Antoine, C. Geuzaine and K. Ramdani. ‘Computational Methods for Multiple Scattering at High Frequency with Applications to Periodic Structures Calculations'. In: Wave Propagation in Periodic Media. Progress in Computational Physics, Vol. 1. Bentham, 2010, pp. 73-107.
[43] X. Antoine, K. Ramdani and B. Thierry. 'Wide Frequency Band Numerical Approaches for Multiple Scattering Problems by Disks'. In: Journal of Algorithms \& Computational Technologies 6.2 (2012), pp. 241-259.
[44] D. Auroux and J. Blum. 'A nudging-based data assimilation method : the Back and Forth Nudging (BFN) algorithm'. In: Nonlin. Proc. Geophys. 15.305-319 (2008).
[45] M. I. Belishev and S. A. Ivanov. 'Reconstruction of the parameters of a system of connected beams from dynamic boundary measurements'. In: Zap. Nauchn. Sem. S.-Peterburg. Otdel. Mat. Inst. Steklov. (POMI) 324.Mat. Vopr. Teor. Rasprostr. Voln. 34 (2005), pp. 20-42, 262.
[46] M. Bellassoued and D. Dos Santos Ferreira. 'Stability estimates for the anisotropic wave equation from the Dirichlet-to-Neumann map'. In: Inverse Probl. Imaging 5.4 (2011), pp. 745-773. DOI: 10.3934/ipi.2011.5.745. URL: http://dx.doi.org/10.3934/ipi.2011.5.745.
[47] M. Bellassoued and D. D. S. Ferreira. 'Stable determination of coefficients in the dynamical anisotropic Schrödinger equation from the Dirichlet-to-Neumann map'. In: Inverse Problems 26.12 (2010), pp. 125010, 30. DOI: 10.1088/0266-5611/26/12/125010. URL: http://dx.doi .org/10.1088/0266-5611/26/12/125010.
[48] Y. Boubendir, X. Antoine and C. Geuzaine. 'A Quasi-Optimal Non-Overlapping Domain Decomposition Algorithm for the Helmholtz Equation'. In: Journal of Computational Physics 2.231 (2012), pp. 262-280.
[49] M. Boulakia and S. Guerrero. ‘Regular solutions of a problem coupling a compressible fluid and an elastic structure'. In: J. Math. Pures Appl. (9) 94.4 (2010), pp. 341-365. DOI: 10.1016/j .matpur .2010.04.002. URL: http://dx.doi.org/10.1016/j.matpur.2010.04.002.
[50] M. Boulakia. 'Existence of weak solutions for an interaction problem between an elastic structure and a compressible viscous fluid'. In: J. Math. Pures Appl. (9) 84.11 (2005), pp. 1515-1554. Doi: 10.1016/j.matpur.2005.08.004. URL: http://dx.doi.org/10.1016/j.matpur. 2005. 08 . 004.
[51] M. Boulakia and A. Osses. 'Local null controllability of a two-dimensional fluid-structure interaction problem'. In: ESAIM Control Optim. Calc. Var. 14.1 (2008), pp. 1-42. DOI: 10.1051/cocv: 200 7031. URL: http://dx.doi.org/10.1051/cocv:2007031.
[52] M. Boulakia, E. Schwindt and T. Takahashi. 'Existence of strong solutions for the motion of an elastic structure in an incompressible viscous fluid'. In: Interfaces Free Bound. 14.3 (2012), pp. 273306. DOI: 10.4171/IFB/282. URL: http://dx.doi.org/10.4171/IFB/282.
[53] G. Bruckner and M. Yamamoto. 'Determination of point wave sources by pointwise observations: stability and reconstruction'. In: Inverse Problems 16.3 (2000), pp. 723-748.
[54] A. Chambolle, B. Desjardins, M. J. Esteban and C. Grandmont. 'Existence of weak solutions for the unsteady interaction of a viscous fluid with an elastic plate'. In: J. Math. Fluid Mech. 7.3 (2005), pp. 368-404. DOI: 10.1007/s00021-004-0121-y. URL: http://dx.doi.org/10.1007/s0002 1-004-0121-y.
[55] T. Chambrion and A. Munnier. 'Generic controllability of 3D swimmers in a perfect fluid'. In: SIAM J. Control Optim. 50.5 (2012), pp. 2814-2835. DOI: 10.1137/110828654. URL: http : //dx .doi. org/10.1137/110828654.
[56] T. Chambrion and A. Munnier. 'Locomotion and control of a self-propelled shape-changing body in a fluid'. In: J. Nonlinear Sci. 21.3 (2011), pp. 325-385. DOI: 10.1007/s00332-010-9084-8. URL: http://dx.doi.org/10.1007/s00332-010-9084-8.
[57] C. Choi, G. Nakamura and K. Shirota. 'Variational approach for identifying a coefficient of the wave equation'. In: Cubo 9.2 (2007), pp. 81-101.
[58] C. Conca, J. San Martín and M. Tucsnak. 'Existence of solutions for the equations modelling the motion of a rigid body in a viscous fluid'. In: Comm. Partial Differential Equations 25.5-6 (2000), pp. 1019-1042. DOI: 10.1080/03605300008821540. URL: http://dx.doi.org/10.1080/0360 5300008821540.
[59] D. Coutand and S. Shkoller. 'Motion of an elastic solid inside an incompressible viscous fluid'. In: Arch. Ration. Mech. Anal. 176.1 (2005), pp. 25-102. DOI: 10.1007/s00205-004-0340-7. URL: http://dx.doi.org/10.1007/s00205-004-0340-7.
[60] D. Coutand and S. Shkoller. 'The interaction between quasilinear elastodynamics and the NavierStokes equations'. In: Arch. Ration. Mech. Anal. 179.3 (2006), pp. 303-352. Doi: 10.1007/s00205-005-0385-2. URL: http://dx.doi.org/10.1007/s00205-005-0385-2.
[61] B. Desjardins and M. J. Esteban. 'Existence of weak solutions for the motion of rigid bodies in a viscous fluid'. In: Arch. Ration. Mech. Anal. 146.1 (1999), pp. 59-71. DOI: 10.1007/s00205005013 6. URL: http://dx.doi.org/10.1007/s002050050136.
[62] B. Desjardins and M. J. Esteban. 'On weak solutions for fluid-rigid structure interaction: compressible and incompressible models'. In: Comm. Partial Differential Equations 25.7-8 (2000), pp. 1399-1413. DOI: 10.1080/03605300008821553. URL: http://dx.doi.org/10.1080/0360 5300008821553.
[63] B. Desjardins, M. J. Esteban, C. Grandmont and P. Le Tallec. 'Weak solutions for a fluid-elastic structure interaction model'. In: Rev. Mat. Complut. 14.2 (2001), pp. 523-538.
[64] A. El Badia and T. Ha-Duong. 'Determination of point wave sources by boundary measurements'. In: Inverse Problems 17.4 (2001), pp. 1127-1139.
[65] M. El Bouajaji, X. Antoine and C. Geuzaine. ‘Approximate Local Magnetic-to-Electric Surface Operators for Time-Harmonic Maxwell's Equations'. In: Journal of Computational Physics 15.279 (2015), pp. 241-260.
[66] M. El Bouajaji, B. Thierry, X. Antoine and C. Geuzaine. 'A quasi-optimal domain decomposition algorithm for the time-harmonic Maxwell's equations'. In: Journal of Computational Physics 294.1 (2015), pp. 38-57. DOI: 10.1016/j.jcp.2015.03.041. URL: https://hal.archives-ouverte s.fr/hal-01095566.
[67] E. Feireisl. 'On the motion of rigid bodies in a viscous compressible fluid'. In: Arch. Ration. Mech. Anal. 167.4 (2003), pp. 281-308. DOI: $10.1007 /$ s00205-002-0242-5. URL: http://dx.doi.org /10.1007/s00205-002-0242-5.
[68] E. Feireisl. 'On the motion of rigid bodies in a viscous incompressible fluid’. In: J. Evol. Equ. 3.3 (2003). Dedicated to Philippe Bénilan, pp. 419-441. DOI: 10.1007/s00028-003-0110-1. URL: http://dx.doi.org/10.1007/s00028-003-0110-1.
[69] E. Feireisl, M. Hillairet and Š. Nečasová. 'On the motion of several rigid bodies in an incompressible non-Newtonian fluid'. In: Nonlinearity 21.6 (2008), pp. 1349-1366. DoI: 10. 1088/0951-7715/21 /6/012. URL: http://dx.doi.org/10.1088/0951-7715/21/6/012.
[70] E. Fridman. 'Observers and initial state recovering for a class of hyperbolic systems via Lyapunov method'. In: Automatica 49.7 (2013), pp. 2250-2260.
[71] G. P. Galdi and A. L. Silvestre. 'On the motion of a rigid body in a Navier-Stokes liquid under the action of a time-periodic force'. In: Indiana Univ. Math. J. 58.6 (2009), pp. 2805-2842. DOI: 10.1512/iumj.2009.58.3758. URL: http://dx.doi.org/10.1512/iumj.2009.58.3758.
[72] O. Glass and F. Sueur. 'The movement of a solid in an incompressible perfect fluid as a geodesic flow’. In: Proc. Amer. Math. Soc. 140.6 (2012), pp. 2155-2168. Doi: 10.1090/S0002-9939-2011-1 1219-X. URL: http://dx.doi.org/10.1090/S0002-9939-2011-11219-X.
[73] C. Grandmont and Y. Maday. 'Existence for an unsteady fluid-structure interaction problem’. In: M2AN Math. Model. Numer. Anal. 34.3 (2000), pp. 609-636. DOI: 10.1051/m2an : 2000159. URL: http://dx.doi.org/10.1051/m2an:2000159.
[74] G. Haine. 'Recovering the observable part of the initial data of an infinite-dimensional linear system with skew-adjoint generator'. In: Mathematics of Control, Signals, and Systems 26.3 (2014), pp. 435-462.
[75] G. Haine and K. Ramdani. 'Reconstructing initial data using observers: error analysis of the semi-discrete and fully discrete approximations'. In: Numer. Math. 120.2 (2012), pp. 307-343.
[76] J. Houot and A. Munnier. 'On the motion and collisions of rigid bodies in an ideal fluid'. In: Asymptot. Anal. 56.3-4 (2008), pp. 125-158.
[77] O. Y. Imanuvilov and T. Takahashi. 'Exact controllability of a fluid-rigid body system'. In: J. Math. Pures Appl. (9) 87.4 (2007), pp. 408-437. DOI: 10.1016/j.matpur.2007.01.005. URL: http://d x.doi.org/10.1016/j.matpur.2007.01.005.
[78] V. Isakov. Inverse problems for partial differential equations. Second. Vol. 127. Applied Mathematical Sciences. New York: Springer, 2006.
[79] N. V. Judakov. 'The solvability of the problem of the motion of a rigid body in a viscous incompressible fluid'. In: Dinamika Splošn. Sredy Vyp. 18 Dinamika Zidkost. so Svobod. Granicami (1974), pp. 249-253, 255.
[80] B. Kaltenbacher, A. Neubauer and O. Scherzer. Iterative regularization methods for nonlinear ill-posed problems. Vol. 6. Radon Series on Computational and Applied Mathematics. Walter de Gruyter GmbH \& Co. KG, Berlin, 2008.
[81] G. Legendre and T. Takahashi. 'Convergence of a Lagrange-Galerkin method for a fluid-rigid body system in ALE formulation'. In: M2AN Math. Model. Numer. Anal. 42.4 (2008), pp. 609-644. DoI: 10.1051/m2an:2008020. URL: http://dx.doi.org/10.1051/m2an:2008020.
[82] J. Lequeurre. 'Existence of strong solutions to a fluid-structure system'. In: SIAM J. Math. Anal. 43.1 (2011), pp. 389-410. DOI: 10.1137/10078983X. URL: http://dx.doi.org/10.1137/1007 8983X.
[83] J. Lohéac and A. Munnier. 'Controllability of 3D Low Reynolds Swimmers'. In: ESAIM:COCV (2013).
[84] D. Luenberger. 'Observing the state of a linear system’. In: IEEE Trans. Mil. Electron. MIL-8 (1964), pp. 74-80.
[85] P. Moireau, D. Chapelle and P. Le Tallec. 'Joint state and parameter estimation for distributed mechanical systems'. In: Computer Methods in Applied Mechanics and Engineering 197 (2008), pp. 659-677.
[86] A. Munnier and B. Pinçon. 'Locomotion of articulated bodies in an ideal fluid: 2D model with buoyancy, circulation and collisions'. In: Math. Models Methods Appl. Sci. 20.10 (2010), pp. 18991940. DOI: 10.1142/S0218202510004829. URL: http://dx.doi.org/10.1142/S0218202510 004829.
[87] A. Munnier and K. Ramdani. 'Calderón cavities inverse problem as a shape-from-moments problem'. In: Quarterly of Applied Mathematics 76 (2018), pp. 407-435. DOI: 10.1090/qam/1505. URL: https://hal.inria.fr/hal-01503425.
[88] A. Munnier and K. Ramdani. 'Conformal mapping for cavity inverse problem: an explicit reconstruction formula'. In: Applicable Analysis (2016). DOI: 10.1080/00036811.2016.1208816. URL: https://hal.inria.fr/hal-01196111.
[89] A. Munnier and E. Zuazua. 'Large time behavior for a simplified $N$-dimensional model of fluidsolid interaction'. In: Comm. Partial Differential Equations 30.1-3 (2005), pp. 377-417. DoI: 10.10 81/PDE-200050080. URL: http://dx.doi.org/10.1081/PDE-200050080.
[90] J. O'Reilly. Observers for linear systems. Vol. 170. Mathematics in Science and Engineering. Orlando, FL: Academic Press Inc., 1983.
[91] J. Ortega, L. Rosier and T. Takahashi. 'On the motion of a rigid body immersed in a bidimensional incompressible perfect fluid'. In: Ann. Inst. H. Poincaré Anal. Non Linéaire 24.1 (2007), pp. 139-165. DOI: 10.1016/j.anihpc.2005.12.004. URL: http://dx.doi.org/10.1016/j. anihpc. 2005 .12.004.
[92] K. Ramdani, M. Tucsnak and G. Weiss. 'Recovering the initial state of an infinite-dimensional system using observers'. In: Automatica 46.10 (2010), pp. 1616-1625.
[93] J.-P. Raymond. 'Feedback stabilization of a fluid-structure model’. In: SIAM J. Control Optim. 48.8 (2010), pp. 5398-5443. DOI: 10.1137/080744761. URL: http://dx.doi.org/10.1137/080744 761.
[94] J. San Martín, J.-F. Scheid and L. Smaranda. 'A modified Lagrange-Galerkin method for a fluid-rigid system with discontinuous density'. In: Numer. Math. 122.2 (2012), pp. 341-382. DoI: 10.1007/s 00211-012-0460-1. URL: http://dx.doi.org/10.1007/s00211-012-0460-1.
[95] J. San Martín, J.-F. Scheid and L. Smaranda. 'The Lagrange-Galerkin method for fluid-structure interaction problems'. In: Boundary Value Problems. (2013), pp. 213-246.
[96] J. San Martín, J.-F. Scheid, T. Takahashi and M. Tucsnak. 'An initial and boundary value problem modeling of fish-like swimming'. In: Arch. Ration. Mech. Anal. 188.3 (2008), pp. 429-455. Doi: 10.1007/s00205-007-0092-2. URL: http://dx.doi.org/10.1007/s00205-007-0092-2.
[97] J. San Martín, J.-F. Scheid, T. Takahashi and M. Tucsnak. ‘Convergence of the Lagrange-Galerkin method for the equations modelling the motion of a fluid-rigid system'. In: SIAM J. Numer. Anal. 43.4 (2005), 1536-1571 (electronic). DOI: 10.1137/S0036142903438161. URL: http://dx.doi .org/10.1137/S0036142903438161.
[98] J. San Martín, L. Smaranda and T. Takahashi. ‘Convergence of a finite element/ALE method for the Stokes equations in a domain depending on time'. In: J. Comput. Appl. Math. 230.2 (2009), pp. 521-545. DOI: 10.1016/j.cam.2008.12.021. URL: http://dx.doi.org/10.1016/j.cam .2008.12.021.
[99] J. San Martín, V. Starovoitov and M. Tucsnak. 'Global weak solutions for the two-dimensional motion of several rigid bodies in an incompressible viscous fluid'. In: Arch. Ration. Mech. Anal. 161.2 (2002), pp. 113-147. DOI: 10.1007/s002050100172. URL: http://dx.doi.org/10. 1007 /s002050100172.
[100] D. Serre. ‘Chute libre d'un solide dans un fluide visqueux incompressible. Existence’. In: Japan J. Appl. Math. 4.1 (1987), pp. 99-110. DOI: 10.1007/BF03167757. URL: http://dx.doi.org/10.1 007/BF03167757.
[101] P. Stefanov and G. Uhlmann. 'Thermoacoustic tomography with variable sound speed'. In: Inverse Problems 25.7 (2009). 075011, p. 16.
[102] T. Takahashi. 'Analysis of strong solutions for the equations modeling the motion of a rigid-fluid system in a bounded domain'. In: Adv. Differential Equations 8.12 (2003), pp. 1499-1532.
[103] H. Trinh and T. Fernando. Functional observers for dynamical systems. Vol. 420. Lecture Notes in Control and Information Sciences. Berlin: Springer, 2012.
[104] J. L. Vázquez and E. Zuazua. 'Large time behavior for a simplified 1D model of fluid-solid interaction'. In: Comm. Partial Differential Equations 28.9-10 (2003), pp. 1705-1738. DOI: 10.1081/PDE120024530. URL: http://dx.doi.org/10.1081/PDE-120024530.
[105] H. F. Weinberger. 'On the steady fall of a body in a Navier-Stokes fluid'. In: Partial differential equations (Proc. Sympos. Pure Math., Vol. XXIII, Univ. California, Berkeley, Calif., 1971). Providence, R. I.: Amer. Math. Soc., 1973, pp. 421-439.

