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ACTIVITY REPORT

Project-Team

ACUMES

Analysis and Control of Unsteady Models for Engineering Sciences

IN COLLABORATION WITH: Laboratoire Jean-Alexandre Dieudonné (JAD)

DOMAIN

Applied Mathematics, Computation and Simulation

THEME

Numerical schemes and simulations

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Project-Team ACUMES

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2 Overall objectives

ACUMES aims at developing a rigorous framework for numerical simulations and optimal control for transportation and buildings, with focus on multi-scale, heterogeneous, unsteady phenomena subject to uncertainty. Starting from established macroscopic Partial Differential Equation (PDE) models, we pursue a set of innovative approaches to include small-scale phenomena, which impact the whole system. Targeting applications contributing to sustainability of urban environments, we couple the resulting models with robust control and optimization techniques.

Modern engineering sciences make an important use of mathematical models and numerical simulations at the conception stage. Effective models and efficient numerical tools allow for optimization before production and to avoid the construction of expensive prototypes or costly post-process adjustments. Most up-to-date modeling techniques aim at helping engineers to increase performances and safety and reduce costs and pollutant emissions of their products. For example, mathematical traffic flow models are used by civil engineers to test new management strategies in order to reduce congestion on the existing road networks and improve crowd evacuation from buildings or other confined spaces without constructing new infrastructures. Similar models are also used in mechanical engineering, in conjunction with concurrent optimization methods, to reduce energy consumption, noise and pollutant emissions of cars, or to increase thermal and structural efficiency of buildings while, in both cases, reducing ecological costs.

Nevertheless, current models and numerical methods exhibit some limitations:

- Most simulation-based design procedures used in engineering still rely on steady (time-averaged) state models. Significant improvements have already been obtained with such a modeling level, for instance by optimizing car shapes, but finer models taking into account unsteady phenomena are required in the design phase for further improvements.
- The classical purely macroscopic approach, while offering a framework with a sound analytical basis, performing numerical techniques and good modeling features to some extent, is not able to reproduce some particular phenomena related to specific interactions occurring at a lower (possibly micro) level. We refer for example to self-organizing phenomena observed in pedestrian flows, or to the dynamics of turbulent flows for which large scale / small scale vortical structures interfere. These flow characteristics need to be taken into account to obtain more precise models and improved optimal solutions.
- Uncertainty related to operational conditions (e.g. inflow velocity in aerodynamics), or models (e.g. individual behavior in crowds) is still rarely considered in engineering analysis and design, yielding solutions of poor robustness.

This project focuses on the analysis and optimal control of classical and non-classical evolutionary systems of Partial Differential Equations (PDEs) arising in the modeling and optimization of engineering problems related to safety and sustainability of urban environments, mostly involving fluid-dynamics and structural mechanics. The complexity of the involved dynamical systems is expressed by multi-scale, time-dependent phenomena, possibly subject to uncertainty, which can hardly be tackled using classical approaches, and require the development of unconventional techniques.

3 Research program

3.1 Research directions

The project develops along the following two axes:

- modeling complex systems through novel (unconventional) PDE systems, accounting for multi-scale phenomena and uncertainty;
- optimization and optimal control algorithms for systems governed by the above PDE systems.

These themes are motivated by the specific problems treated in the applications, and represent important and up-to-date issues in engineering sciences. For example, improving the design of transportation means and civil buildings, and the control of traffic flows, would result not only in better performances of the object of the optimization strategy (vehicles, buildings or road networks level of service), but also in enhanced safety and lower energy consumption, contributing to reduce costs and pollutant emissions.

3.2 PDE models accounting for multi-scale phenomena and uncertainties

Dynamical models consisting of evolutionary PDEs, mainly of hyperbolic type, appear classically in the applications studied by the previous Project-Team Opale (compressible flows, traffic, cell-dynamics, medicine, etc). Yet, the classical purely macroscopic approach is not able to account for some particular phenomena related to specific interactions occurring at smaller scales. These phenomena can be of greater importance when dealing with particular applications, where the "first order" approximation given by the purely macroscopic approach turns out to be inadequate. We refer for example to self-organizing phenomena observed in pedestrian flows [111], or to the dynamics of turbulent flows for which large scale / small scale vortical structures interfere [137].

Nevertheless, macroscopic models offer well known advantages, namely a sound analytical framework, fast numerical schemes, the presence of a low number of parameters to be calibrated, and efficient optimization procedures. Therefore, we are convinced of the interest of keeping this point of view as dominant, while completing the models with information on the dynamics at the small scale / microscopic level. This can be achieved through several techniques, like hybrid models, homogenization, mean field games. In this project, we will focus on the aspects detailed below.

The development of adapted and efficient numerical schemes is a mandatory completion, and sometimes ingredient, of all the approaches listed below. The numerical schemes developed by the team are based on finite volumes or finite elements techniques, and constitute an important tool in the study of the considered models, providing a necessary step towards the design and implementation of the corresponding optimization algorithms, see Section 3.3.

3.2.1 Micro-macro couplings

Modeling of complex problems with a dominant macroscopic point of view often requires couplings with small scale descriptions. Accounting for systems heterogeneity or different degrees of accuracy usually leads to coupled PDE-ODE systems.

In the case of heterogeneous problems the coupling is "intrinsic", i.e. the two models evolve together and mutually affect each-other. For example, accounting for the impact of a large and slow vehicle (like a bus or a truck) on traffic flow leads to a strongly coupled system consisting of a (system of) conservation law(s) coupled with an ODE describing the bus trajectory, which acts as a moving bottleneck. The coupling is realized through a local unilateral moving constraint on the flow at the bus location, see [81] for an existence result and [67, 82] for numerical schemes.

If the coupling is intended to offer higher degree of accuracy at some locations, a macroscopic and a microscopic model are connected through an artificial boundary, and exchange information across it through suitable boundary conditions. See [73, 99] for some applications in traffic flow modeling, and [92, 96, 98] for applications to cell dynamics.

The corresponding numerical schemes are usually based on classical finite volume or finite element methods for the PDE, and Euler or Runge-Kutta schemes for the ODE, coupled in order to take into account the interaction fronts. In particular, the dynamics of the coupling boundaries require an accurate handling capturing the possible presence of non-classical shocks and preventing diffusion, which could produce wrong solutions, see for example [67, 82].

We plan to pursue our activity in this framework, also extending the above mentioned approaches to problems in two or higher space dimensions, to cover applications to crowd dynamics or fluid-structure interaction.

3.2.2 Micro-macro limits

Rigorous derivation of macroscopic models from microscopic ones offers a sound basis for the proposed modeling approach, and can provide alternative numerical schemes, see for example [74, 87] for the derivation of Lighthill-Whitham-Richards [123, 136] traffic flow model from Follow-the-Leader and [93] for results on crowd motion models (see also [113]). To tackle this aspect, we will rely mainly on two (interconnected) concepts: measure-valued solutions and mean-field limits.

The notion of **measure-valued solutions** for conservation laws was first introduced by DiPerna [88], and extensively used since then to prove convergence of approximate solutions and deduce existence results, see for example [94] and references therein. Measure-valued functions have been recently advocated as the appropriate notion of solution to tackle problems for which analytical results (such as existence and uniqueness of weak solutions in distributional sense) and numerical convergence are missing [56, 95]. We refer, for example, to the notion of solution for non-hyperbolic systems [101], for which no general theoretical result is available at present, and to the convergence of finite volume schemes for systems of hyperbolic conservation laws in several space dimensions, see [95].

In this framework, we plan to investigate and make use of measure-based PDE models for vehicular and pedestrian traffic flows. Indeed, a modeling approach based on (multi-scale) time-evolving measures (expressing the agents probability distribution in space) has been recently introduced (see the monograph [78]), and proved to be successful for studying emerging self-organized flow patterns [77]. The theoretical measure framework proves to be also relevant in addressing micro-macro limiting procedures of mean field type [102], where one lets the number of agents going to infinity, while keeping the total mass constant. In this case, one must prove that the *empirical measure*, corresponding to the sum of Dirac measures concentrated at the agents positions, converges to a measure-valued solution of the corresponding macroscopic evolution equation. We recall that a key ingredient in this approach is the use of the *Wasserstein distances* [145, 144]. Indeed, as observed in [130, Section 6], the usual L^1 spaces are not natural in this context, since they do not guarantee uniqueness of solutions.

This procedure can potentially be extended to more complex configurations, like for example road networks or different classes of interacting agents, or to other application domains, like cell-dynamics.

Another powerful tool we shall consider to deal with micro-macro limits is the so-called **Mean Field Games (MFG)** technique (see the seminal paper [122]). This approach has been recently applied to some of the systems studied by the team, such as traffic flow and cell dynamics. In the context of crowd dynamics, including the case of several populations with different targets, the mean field game approach has been adopted in [64, 63, 89, 121], under the assumption that the individual behavior evolves according to a stochastic process, which gives rise to parabolic equations greatly simplifying the analysis of the system. Besides, a deterministic context is studied in [132], which considers a non-local velocity field. For cell dynamics, in order to take into account the fast processes that occur in the migration-related machinery, a framework such as the one developed in [80] to handle games "where agents evolve their strategies according to the best-reply scheme on a much faster time scale than their social configuration variables" may turn out to be suitable. An alternative framework to MFG is also considered. This framework is based on the formulation of -Nash- games constrained by the **Fokker-Planck** (FP, [54]) partial differential equations that govern the time evolution of the probability density functions -PDF- of stochastic systems and on objectives that may require to follow a given PDF trajectory or to minimize an expectation functional.

3.2.3 Non-local flows

Non-local interactions can be described through macroscopic models based on integro-differential equations. Systems of the type

$$\partial_t u + div_{\mathbf{x}} F(t, \mathbf{x}, u, W) = 0, \qquad t > 0, \mathbf{x} \in R^d, d \geq 1, \tag{1}$$

where $u = u(t, \mathbf{x}) \in R^N$, $N \geq 1$ is the vector of conserved quantities and the variable $W = W(t, \mathbf{x}, u)$ depends on an integral evaluation of u, arise in a variety of physical applications. Space-integral terms are considered for example in models for granular flows [51], sedimentation [58], supply chains [106], conveyor belts [104], biological applications like structured populations dynamics [129], or more general problems like gradient constrained equations [53]. Also, non-local in time terms arise in conservation laws with memory, starting from [79]. In particular, equations with non-local flux have been recently introduced in traffic flow modeling to account for the reaction of drivers or pedestrians to the surrounding density of other individuals, see [59, 66, 70, 103, 140]. While pedestrians are likely to react to the presence of people all around them, drivers will mainly adapt their velocity to the downstream traffic, assigning a greater importance to closer vehicles. In particular, and in contrast to classical (without integral terms) macroscopic equations, these models are able to display finite acceleration of vehicles through Lipschitz bounds on the mean velocity [59, 103] and lane formation in crossing pedestrian flows.

General analytical results on non-local conservation laws, proving existence and possibly uniqueness of solutions of the Cauchy problem for (1), can be found in [52] for scalar equations in one space dimension ($N = d = 1$), in [71] for scalar equations in several space dimensions ($N = 1$, $d \geq 1$) and in [49], [72, 76] for multi-dimensional systems of conservation laws. Besides, specific finite volume numerical methods have been developed recently in [49, 103] and [120].

Relying on these encouraging results, we aim to push a step further the analytical and numerical study of non-local models of type (1), in particular concerning well-posedness of initial - boundary value problems, regularity of solutions and high-order numerical schemes.

3.2.4 Uncertainty in parameters and initial-boundary data

Different sources of uncertainty can be identified in PDE models, related to the fact that the problem of interest is not perfectly known. At first, initial and boundary condition values can be uncertain. For instance, in traffic flows, the time-dependent value of inlet and outlet fluxes, as well as the initial distribution of vehicles density, are not perfectly determined [65]. In aerodynamics, inflow conditions like velocity modulus and direction, are subject to fluctuations [109, 128]. For some engineering problems, the geometry of the boundary can also be uncertain, due to structural deformation, mechanical wear or disregard of some details [91]. Another source of uncertainty is related to the value of some parameters in the PDE models. This is typically the case of parameters in turbulence models in fluid mechanics, which have been calibrated according to some reference flows but are not universal [138, 143], or in traffic flow models, which may depend on the type of road, weather conditions, or even the country of interest (due to differences in driving rules and conductors behavior). This leads to equations with flux functions depending on random parameters [139, 142], for which the mean and the variance of the solutions can be computed using different techniques. Indeed, uncertainty quantification for systems governed by PDEs has become a very active research topic in the last years. Most approaches are embedded in a probabilistic framework and aim at quantifying statistical moments of the PDE solutions, under the assumption that the characteristics of uncertain parameters are known. Note that classical Monte-Carlo approaches exhibit low convergence rate and consequently accurate simulations require huge computational times. In this respect, some enhanced algorithms have been proposed, for example in the balance law framework [127]. Different approaches propose to modify the PDE solvers to account for this probabilistic context, for instance by defining the non-deterministic part of the solution on an orthogonal basis (Polynomial Chaos decomposition) and using a Galerkin projection [109, 119, 124, 147] or an entropy closure method [86], or by discretizing the probability space and extending the numerical schemes to the stochastic components [48]. Alternatively, some other approaches maintain a fully deterministic PDE resolution, but approximate the solution in the vicinity of the reference parameter values by Taylor series expansions based on first- or second-order sensitivities [133, 143, 146].

Our objective regarding this topic is twofold. In a pure modeling perspective, we aim at including uncertainty quantification in models calibration and validation for predictive use. In this case, the choice of the techniques will depend on the specific problem considered [57]. Besides, we plan to extend previous works on sensitivity analysis [91, 125] to more complex and more demanding problems. In particular, high-order Taylor expansions of the solution (greater than two) will be considered in the framework of the Sensitivity Equation Method [60] (SEM) for unsteady aerodynamic applications, to improve the accuracy of mean and variance estimations. A second targeted topic in this context is the study of the uncertainty related to turbulence closure parameters, in the sequel of [143]. We aim at exploring the capability of the SEM approach to detect a change of flow topology, in case of detached flows. Our ambition is to contribute to the emergence of a new generation of simulation tools, which will provide solution densities rather than values, to tackle real-life uncertain problems. This task will also include a reflection about numerical schemes used to solve PDE systems, in the perspective of constructing a unified numerical framework able to account for exact geometries (isogeometric methods), uncertainty propagation and sensitivity analysis with respect to control parameters.

3.3 Optimization and control algorithms for systems governed by PDEs

The non-classical models described above are developed in the perspective of design improvement for real-life applications. Therefore, control and optimization algorithms are also developed in conjunction with these models. The focus here is on the methodological development and analysis of optimization algorithms for PDE systems in general, keeping in mind the application domains in the way the problems are mathematically formulated.

3.3.1 Sensitivity vs. adjoint equation

Adjoint methods (achieved at continuous or discrete level) are now commonly used in industry for steady PDE problems. Our recent developments [135] have shown that the (discrete) adjoint method can be efficiently applied to cost gradient computations for time-evolving traffic flow on networks, thanks to the special structure of the associated linear systems and the underlying one dimensionality of the problem. However, this strategy is questionable for more complex (e.g. 2D/3D) unsteady problems, because it requires sophisticated and time-consuming check-pointing and/or re-computing strategies [55, 105] for the backward time integration of the adjoint variables. The sensitivity equation method (SEM) offers a promising alternative [90, 114], if the number of design parameters is moderate. Moreover, this approach can be employed for other goals, like fast evaluation of neighboring solutions or uncertainty propagation [91].
Regarding this topic, we intend to apply the continuous sensitivity equation method to challenging problems. In particular, in aerodynamics, multi-scale turbulence models like Large-Eddy Simulation (LES) [137] , Detached-Eddy Simulation (DES) [141] or Organized-Eddy Simulation (OES) [61], are more and more employed to analyze the unsteady dynamics of the flows around bluff-bodies, because they have the ability to compute the interactions of vortices at different scales, contrary to classical Reynolds-Averaged Navier-Stokes models. However, their use in design optimization is tedious, due to the long time integration required. In collaboration with turbulence specialists (M. Braza, CNRS - IMFT), we aim at developing numerical methods for effective sensitivity analysis in this context, and apply them to realistic problems, like the optimization of active flow control devices. Note that the use of SEM allows computing cost functional gradients at any time, which permits to construct new gradient-based optimization strategies like instantaneous-feedback method [117] or multiobjective optimization algorithm (see section below).

3.3.2 Integration of Computer-Aided Design and analysis for shape optimization

A major difficulty in shape optimization is related to the multiplicity of geometrical representations handled during the design process. From high-order Computer-Aided Design (CAD) objects to discrete mesh-based descriptions, several geometrical transformations have to be performed, that considerably impact the accuracy, the robustness and the complexity of the design loop. This is even more critical when multiphysics applications are targeted, including moving bodies.

To overcome this difficulty, we intend to investigate *isogeometric analysis* [115] methods, which propose to use the same CAD representations for the computational domain and the physical solutions yielding geometrically exact simulations. In particular, hyperbolic systems and compressible aerodynamics are targeted.

3.3.3 Multi-objective descent algorithms for multi-disciplinary, multi-point, unsteady optimization or robust-design

In differentiable optimization, multi-disciplinary, multi-point, unsteady optimization or robust-design can all be formulated as multi-objective optimization problems. In this area, we have proposed the *Multiple-Gradient Descent Algorithm (MGDA)* to handle all criteria concurrently [84] [83]. Originally, we have stated a principle according to which, given a family of local gradients, a descent direction common to all considered objective-functions simultaneously is identified, assuming the Pareto-stationarity condition is not satisfied. When the family is linearly-independent, we have access to a direct algorithm. Inversely, when the family is linearly-dependent, a quadratic-programming problem should be solved. Hence, the technical difficulty is mostly conditioned by the number m of objective functions relative to the search space dimension n. In this respect, the basic algorithm has recently been revised [85] to handle the case where $m > n$, and even $m \gg n$, and is currently being tested on a test-case of robust design subject to a periodic time-dependent Navier-Stokes flow.

The multi-point situation is very similar and, being of great importance for engineering applications, will be treated at large.

Moreover, we intend to develop and test a new methodology for robust design that will include uncertainty effects. More precisely, we propose to employ MGDA to achieve an effective improvement of all criteria simultaneously, which can be of statistical nature or discrete functional values evaluated in confidence intervals of parameters. Some recent results obtained at ONERA [131] by a stochastic variant of our methodology confirm the viability of the approach. A PhD thesis has also been launched at ONERA/DADS.

Lastly, we note that in situations where gradients are difficult to evaluate, the method can be assisted by a meta-model [149].

3.3.4 Bayesian Optimization algorithms for efficient computation of general equilibria

Bayesian Optimization (BO) relies on Gaussian processes, which are used as emulators (or surrogates) of the black-box model outputs based on a small set of model evaluations. Posterior distributions provided by the Gaussian process are used to design acquisition functions that guide sequential search strategies that balance between exploration and exploitation. Such approaches have been transposed to frameworks other than optimization, such as uncertainty quantification. Our aim is to investigate how the BO apparatus can be applied to the search of general game equilibria, and in particular the classical Nash equilibrium (NE). To this end, we propose two complementary acquisition functions, one based on a greedy search approach and one based on the Stepwise Uncertainty Reduction paradigm [97]. Our proposal is designed to tackle derivative-free, expensive models, hence requiring very few model evaluations to converge to the solution.

3.3.5 Decentralized strategies for inverse problems

Most if not all the mathematical formulations of inverse problems (a.k.a. reconstruction, identification, data recovery, non destructive engineering,...) are known to be ill posed in the Hadamard sense. Indeed, in general, inverse problems try to fulfill (minimize) two or more very antagonistic criteria. One classical example is the Tikhonov regularization, trying to find artificially smoothed solutions close to naturally non-smooth data.

We consider here the theoretical general framework of parameter identification coupled to (missing) data recovery. Our aim is to design, study and implement algorithms derived within a game theoretic framework, which are able to find, with computational efficiency, equilibria between the "identification related players" and the "data recovery players". These two parts are known to pose many challenges, from a theoretical point of view, like the identifiability issue, and from a numerical one, like convergence, stability and robustness problems. These questions are tricky [50] and still completely open for systems like coupled heat and thermoelastic joint data and material detection.

4 Application domains

4.1 Active flow control for vehicles

The reduction of CO_2 emissions represents a great challenge for the automotive and aeronautic industries, which committed respectively a decrease of 20% for 2020 and 75% for 2050. This goal will not be reachable, unless a significant improvement of the aerodynamic performance of cars and aircrafts is achieved (e.g. aerodynamic resistance represents 70% of energy losses for cars above 90 km/h). Since vehicle design cannot be significantly modified, due to marketing or structural reasons, active flow control technologies are one of the most promising approaches to improve aerodynamic performance. This consists in introducing micro-devices, like pulsating jets or vibrating membranes, that can modify vortices generated by vehicles. Thanks to flow non-linearities, a small energy expense for actuation can significantly reduce energy losses. The efficiency of this approach has been demonstrated, experimentally as well as numerically, for simple configurations [148].

However, the lack of efficient and flexible numerical tools, that allow to simulate and optimize a large number of such devices on realistic configurations, is still a bottleneck for the emergence of this technology in industry. The main issue is the necessity of using high-order schemes and complex models to simulate actuated flows, accounting for phenomena occurring at different scales. In this context, we intend to contribute to the following research axes:

- *Sensitivity analysis for actuated flows.* Adjoint-based (reverse) approaches, classically employed in design optimization procedure to compute functional gradients, are not well suited to this context. Therefore, we propose to explore the alternative (direct) formulation, which is not so much used, in the perspective of a better characterization of actuated flows and optimization of control devices.
- *Isogeometric simulation of control devices.* To simulate flows perturbed by small-scale actuators, we investigate the use of isogeometric analysis methods, which allow to account exactly for CAD-based geometries in a high-order hierarchical representation framework. In particular, we try to exploit the features of the method to simulate more accurately complex flows including moving devices and multiscale phenomena.

4.2 Vehicular and pedestrian traffic flows

Intelligent Transportation Systems (ITS) is nowadays a booming sector, where the contribution of mathematical modeling and optimization is widely recognized. In this perspective, traffic flow models are a commonly cited example of "complex systems", in which individual behavior and self-organization phenomena must be taken into account to obtain a realistic description of the observed macroscopic dynamics [110]. Further improvements require more advanced models, keeping into better account interactions at the microscopic scale, and adapted control techniques, see [62] and references therein.

In particular, we will focus on the following aspects:

- *Junction models.* We are interested in designing a general junction model both satisfying basic analytical properties guaranteeing well-posedness and being realistic for traffic applications. In particular, the model should be able to overcome severe drawbacks of existing models, such as restrictions on the number of involved roads and prescribed split ratios [75, 100], which limit their applicability to real world situations. Hamilton-Jacobi equations could be also an interesting direction of research, following the recent results obtained in [116].
- *Data assimilation.* In traffic flow modeling, the capability of correctly estimating and predicting the state of the system depends on the availability of rich and accurate data on the network. Up to now, the most classical sensors are fixed ones. They are composed of inductive loops (electrical wires) that are installed at different spatial positions of the network and that can measure the traffic flow, the occupancy rate (i.e. the proportion of time during which a vehicle is detected to be over the loop) and the speed (in case of a system of two distant loops). These data are useful / essential to calibrate the phenomenological relationship between flow and density which is known in the traffic literature as the Fundamental Diagram. Nowadays, thanks to the wide development of mobile internet and geolocalization techniques and its increasing adoption by the road users, smartphones have turned into perfect mobile sensors in many domains, including in traffic flow management. They can provide the research community with a large database of individual trajectory sets that are known as Floating Car Data (FCD), see [112] for a real field experiment. Classical macroscopic models, say (hyperbolic systems of) conservation laws, are not designed to take into account this new kind of microscopic data. Other formulations, like Hamilton-Jacobi partial differential equations, are most suited and have been intensively studied in the past five years (see [68, 69]), with a stress on the (fixed) Eulerian framework. Up to our knowledge, there exist a few studies in the time-Lagrangian as well as space-Lagrangian frameworks, where data coming from mobile sensors could be easily assimilated, due to the fact that the Lagrangian coordinate (say the label of a vehicle) is fixed.

- *Control of autonomous vehicles.* Traffic flow is usually controlled via traffic lights or variable speed limits, which have fixed space locations. The deployment of autonomous vehicles opens new perspectives in traffic management, as the use of a small fraction of cars to optimize the overall traffic. In this perspective, the possibility to track vehicles trajectories either by coupled micro-macro models [81, 99] or via the Hamilton-Jacobi approach [68, 69] could allow to optimize the flow by controlling some specific vehicles corresponding to internal conditions.

4.3 Combined hormone and brachy therapies for the treatment of prostate cancer

The latest statistics published by the International Agency for Research on Cancer show that in 2018, 18.1 million new cancer cases have been identified and 9.6 million deaths have been recorded worldwide making it the second leading cause of death globally. Prostate cancer ranks third in incidence with 1.28 million cases and represents the second most commonly diagnosed male cancer.

Prostate cells need the hormone androgen to survive and function properly. For this to happen, the androgens have to bind to a protein in the prostate cells called Androgen Receptor and activate it. Since androgens act as a growth factor for the cells, one way of treating prostate cancer is through the antihormone therapy that hinder its activity. The Androgen Deprivation Therapy (ADT) aims to either reduce androgen production or to stop the androgens from working through the use of drugs. However, over time, castration-resistant cells that are able to sustain growth in a low androgen environment emerge. The castration-resistant cells can either be androgen independent or androgen repressed meaning that they have a negative growth rate when the androgen is abundant in the prostate. In order to delay the development of castration resistance and reduce its occurrence, the Intermittent Androgen Deprivation Therapy is used.

On the other hand, brachytherapy is an effective radiation therapy used in the treatment of prostate cancer by placing a sealed radiation source inside the prostate gland. It can be delivered in high dose rates (HDR) or low dose rates (LDR) depending on the radioactive source used and the duration of treatment.

In the HDR brachytherapy, the source is placed temporarily in the prostate for a few minutes to deliver high dose radiation while for the LDR brachytherapy low radiations dose are delivered from radioactive sources permanently placed in the prostate. The radioactivity of the source decays over time, therefore its presence in the prostate does not cause any long-term concern as its radioactivity disappears eventually. In practice, brachytherapy is prescribed either as monotherapy, often for localized tumors, or combined with another therapy such as external beam radiation therapy for which the total dose prescribed is divided between internal and external radiation. Brachytherapy can also be prescribed in combination with hormone therapy.

However, in the existing literature there is currently no mathematical model that explores this combination of treatments. Our aim is to develop a computational model based on partial differential equations to assess the effectiveness of combining androgen deprivation therapy with brachytherapy in the treatment of prostate cancer. The resulting simulations can be used to explore potential unconventional therapeutic strategies.

4.4 Other application fields

Besides the above mentioned axes, which constitute the project's identity, the methodological tools described in Section 3 have a wider range of application. We currently carry on also the following research actions, in collaboration with external partners.

- **Game strategies for thermoelastography.** Thermoelastography is an innovative non-invasive control technology, which has numerous advantages over other techniques, notably in medical imaging [126]. Indeed, it is well known that most pathological changes are associated with changes in tissue stiffness, while remaining isoechoic, and hence difficult to detect by ultrasound techniques. Based on elastic waves and heat flux reconstruction, thermoelastography shows no destructive or aggressive medical sequel, unlike X-ray and comparables techniques, making it a potentially prominent choice for patients.

  Physical principles of thermoelastography originally rely on dynamical structural responses of tissues, but as a first approach, we only consider static responses of linear elastic structures.

The mathematical formulation of the thermoelasticity reconstruction is based on data completion and material identification, making it a harsh ill-posed inverse problem. In previous works [107, 118], we have demonstrated that Nash game approaches are efficient to tackle ill-posedness. We intend to extend the results obtained for Laplace equations in [107], and the algorithms developed in Section 3.3.5 to the following problems (of increasing difficulty):
- Simultaneous data and parameter recovery in linear elasticity, using the so-called Kohn and Vogelius functional (ongoing work, some promising results obtained).
- Data recovery in coupled heat-thermoelasticity systems.
- Data recovery in linear thermoelasticity under stochastic heat flux, where the imposed flux is stochastic.
- Data recovery in coupled heat-thermoelasticity systems under stochastic heat flux, formulated as an incomplete information Nash game.
- Application to robust identification of cracks.

- **Constraint elimination in Quasi-Newton methods.** In single-objective differentiable optimization, Newton's method requires the specification of both gradient and Hessian. As a result, the convergence is quadratic, and Newton's method is often considered as the target reference. However, in applications to distributed systems, the functions to be minimized are usually "functionals", which depend on the optimization variables by the solution of an often complex set of PDE's, through a chain of computational procedures. Hence, the exact calculation of the full Hessian becomes a complex and costly computational endeavor.

  This has fostered the development of *quasi-Newton's methods* that mimic Newton's method but use only the gradient, the Hessian being iteratively constructed by successive approximations inside the algorithm itself. Among such methods, the Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm is well-known and commonly employed. In this method, the Hessian is corrected at each new iteration by rank-one matrices defined from several evaluations of the gradient only. The BFGS method has "super-linear convergence".

  For constrained problems, certain authors have developed so-called *Riemannian BFGS*, e.g. [134], that have the desirable convergence property in constrained problems. However, in this approach, the constraints are assumed to be known formally, by explicit expressions.

  In collaboration with ONERA-Meudon, we are exploring the possibility of representing constraints, in successive iterations, through local approximations of the constraint surfaces, splitting the design space locally into tangent and normal subspaces, and eliminating the normal coordinates through a linearization, or more generally a finite expansion, and applying the BFGS method through dependencies on the coordinates in the tangent subspace only. Preliminary experiments on the difficult Rosenbrock test-case, although in low dimensions, demonstrate the feasibility of this approach. On-going research is on theorizing this method, and testing cases of higher dimensions.

- **Multi-objective optimization for nanotechnologies.** Our team takes part in a larger collaboration with CEA/LETI (Grenoble), initiated by the Inria Project-Team Nachos (now Atlantis), and related to the Maxwell equations. Our component in this activity relates to the optimization of nanophotonic devices, in particular with respect to the control of thermal loads. We have first identified a gradation of representative test-cases of increasing complexity:
  - infrared micro-source;
  - micro-photoacoustic cell;
  - nanophotonic device.

  These cases involve from a few geometric parameters to be optimized to a functional minimization subject to a finite-element solution involving a large number of degrees of freedom. CEA disposes of such codes, but considering the computational cost of the objective functions in the complex cases, the first part of our study is focused on the construction and validation of meta-models, typically of RBF-type (Radial Basis Functions). Multi-objective optimization will be carried out subsequently by MGDA, and possibly Nash games.

5 Social and environmental responsibility

5.1 Impact of research results

Acumes's research activity in traffic modeling and control is intended to improve road network efficiency, thus reducing energy consumption and pollutant emission.

The research activities related to isogeometric analysis and physics-informed neural networks (PINNs) aim at facilitating the use of numerical simulations and design optimization in engineering, yielding a gain of efficiency, for instance in transportation industry (cars, aircrafts) or energy industry (air conditioning, turbines).

The research conducted with the startup Mycophyto aims at reducing the use of chemical fertilizers and phytopharmaceutical products by developing natural biostimulants (mycorrhyzal fungi). It started with the arrival of Khadija Musayeva in October 2020.

The research conducted in ANR NEMATIC aims at exploring the ability to control the growth of filamentous fungi, which have the potential for the production of biofuels and biosourced chemicals. Investigations started with the arrival of Nicolas Fricker in January 2023.

The research conducted with company Altair on code OpenRadioss aims at improving the resolution of multimaterial flows in presence of large density ratios, with a view to security applications such as shaped-charge detonation. It started with the arrival of Alexandre Vieira in October 2024.

The research conducted with Konstantin Brenner during Amal Chakib's M2 internship on shallow water equations with moving obstacles has important applications for flooding events and impacts on infrastructures.

With the increasing demands of modern applications such as electric and hybrid vehicles and renewable energy storage, the limitations of current commercial batteries using liquid or gel electrolytes have become critical. We initiated a research activity on solid state batteries. Their better understanding and hence optimizing is expected to strikingly improve industrial properties, such as higher energy densities, longer lifespans, cost-effectiveness, low maintenance, and enhanced safety.

6 Highlights of the year

Our work on traffic flow control by connected and automated vehicles (see Setion 8.1) has been extensively cited during the Bode Lecture *"Semper in motu: Transforming mobility through learning and control"* delivered by Karl H. Johansson (KTH Royal Institute of Technology, Sweden) at the 2024 Conference on Decision and Control held in Milan, Italy, in December 2024.

7 New software, platforms, open data

7.1 New software

7.1.1 Igloo

Name: Iso-Geometric anaLysis using discOntinuOus galerkin methods

Keywords: Numerical simulations, Isogeometric analysis

Scientific Description: Igloo contains numerical methods to solve partial differential equations of hyperbolic type, or convection-dominant type, using an isogeometric formulation (NURBS bases) with a discontinuous Galerkin method.

Functional Description: Simulation software for NURBS meshes

URL: `https://gitlab.inria.fr/igloo/igloo/-/wikis/home`

Contact: Régis Duvigneau

7.1.2 pinnacle

Name: Physics-Informed Neural Networks Computational Library and Environment

Keywords: Neural networks, Partial differential equation, Physical simulation, Data assimilation, Inverse problem, Multiphysics modelling

Scientific Description: Set of methods for rapid implementation of physics-informed neural networks to solve direct and inverse problems: space-time sampling with refinement algorithms, dense multilayer neural networks, library of physical models (mechanics, fluid, heat transfer, electromagnetics), optimisation algorithms, import/export tools for meshes and solutions.

Functional Description: Software library for implementation of physics informed neural networks.

Contact: Régis Duvigneau

Participants: Régis Duvigneau, Stéphane Lanteri, Alexis Gobe, Maxime Le

7.1.3 MovingBottleneck

Keywords: Finite volume methods, Numerical optimization

Functional Description: Matlab code for solving numerically a system coupling a first order traffic model and ODEs describing moving bottleneck trajectories in one space dimension, based on original ideas developed in https://hal.inria.fr/hal-01070262 In particular, we use Godunov scheme to solve the PDE, with a specific flux correction at the moving bottleneck positions consisting in a conservative reconstruction of the jump discontinuity. The code also allows for Model Predictive Control implementation. It has been used to produce results published in https://hal.inria.fr/hal-01644823 , https://www.aimspress.com/article/doi/10.3934/nhm.2023040 , https://inria.hal.science/hal-03648482 , https://hal.science/hal-04366870

URL: `https://gitlab.inria.fr/pgoatin/movingbottleneck`

Publications: hal-01644823, hal-03648482, hal-04366870, hal-01070262

Contact: Paola Goatin

Participants: Chiara Daini, Maria Laura Delle Monache, Paola Goatin, Giulia Piacentini

7.1.4 PyLate

Name: Python Library for Aggregate Traffic Estimation

Keywords: Macroscopic traffic flow models, Numerical simulations

Scientific Description: PyLate: Python Library for Aggregate Traffic Estimation

PyLate is a Python library designed for macroscopic traffic simulation. It enables the creation of road networks using the NetworkX library, representing the network as a directed graph object (networkx.DiGraph). The library implements the Godunov numerical scheme and supports multiclass traffic flows, with each class having its own fundamental diagram and routing strategy.

Currently supported fundamental diagrams: • Triangular Fundamental Diagram • Greenshields Fundamental Diagram

Each class is characterised by: • An origin node (where its demand flow is generated). • A set of parameters for the fundamental diagram. • A routing strategy.

The following routing strategies are currently available: • Fixed-ratios: At each node, flows are routed according to a fixed, predefined distribution matrix. • Follow1Path: All flows are routed along a single path connecting the origin node to a destination node. • LogitDynamic: At each node, flows are distributed across up to n paths ending at the destination node in proportions defined by a Logit distribution. • LogitPredefined: Users are distributed according to a Logit model across a predefined set of paths between their origin and destination nodes. • LogitOD: Users are distributed according to a Logit model across up to n paths connecting their origin and destination nodes (different from LogitDynamic because the set of paths is fixed between origin and destination).

The library supports extending routing strategies and fundamental diagrams by creating subclasses of the respective abstract classes, without altering the core source code.

Integration with OpenStreetMap: Thanks to the osmnx library, PyLate allows importing road network data directly from OpenStreetMap, simplifying the setup of realistic traffic networks. However, when importing networks from OpenStreetMap, the generated graph is typically a MultiDiGraph, which allows multiple edges between nodes. To use PyLate, it is necessary to convert the MultiDiGraph into a DiGraph, which simplifies the handling of the network for the macroscopic traffic simulation.

Functional Description: PyLate is a Python library for macroscopic traffic simulation, built on the NetworkX library. It enables the creation of road networks as directed graphs (DiGraph) and implements the Godunov numerical scheme. PyLate supports multi-class traffic flows, each with its own fundamental diagram and routing strategy.

Currently supported fundamental diagrams include Triangular and Greenshields. Routing strategies include Fixed-ratios, Follow1Path, LogitDynamic, LogitPredefined, and LogitOD, each offering different methods for flow distribution across paths.

PyLate allows extension of routing strategies and diagrams via subclassing. It integrates with OpenStreetMap using osmnx to import road network data.

URL: https://gitlab.inria.fr/pgoatin/pylate

Publications: hal-04206281, hal-04206328

Contact: Enrico Siri

Participants: Enrico Siri, Paola Goatin

7.1.5 CELIA2D

Keywords: 2D, Finite volume methods, Computational Fluid Dynamics, Free surface flows

Functional Description: The CELIA2D implements the Finite Volume method with cut cells for hyperbolic systems of conservation laws in 2D : compressible Euler and Saint-Venant. Mobile obstacles interact with fluid: the obstacles can be deformable, crack or come into contact.

Contact: Laurent Monasse

7.1.6 Precis

Keywords: Finite volume methods, 3D, Computational Fluid Dynamics

Functional Description: The Precis code implements Finite Volumes for compressible Euler equations on cut cells in three space dimensions.

Contact: Laurent Monasse

7.1.7 ShockFitting

Keywords: 2D, 3D, Discontinuous Galerkin, Finite volume methods, Compressible flows

Functional Description: The ShockFitting code implements in Julia the simulation of discontinuity interface tracking (contact discontinuities and shocks) for compressible fluids. The methods used are space-time cut cells in dimensions 1, 2 and 3 for a space discretization of Finite Volume of Discontinuous Galerkin type.

Contact: Laurent Monasse

8 New results

8.1 Macroscopic traffic flow models on networks

Participants: Eric Andoni, Paola Goatin, Chiara Daini *(KOPERNIC Project-Team, Inria Paris)*, Maria Laura Delle Monache *(UC Berkeley, USA)*, Antonella Ferrara *(Univ. Pavia, Italy)*, Agatha Joumaa, Carmen Nieto, Benedetto Piccoli *(Rutgers U, USA)*, Alessandra Rizzo *(Univ. Messina, Italy)*, Enrico Siri.

Traffic control by Connected and Automated Vehicles

We rely on a multi-scale approach to model mixed traffic composed of a small fleet of CAVs in the bulk flow. In particular, CAVs are allowed to overtake (if on distinct lanes) or queuing (if on the same lane). Controlling CAVs desired speeds allows to act on the system to minimize the selected cost function. For the proposed control strategies, we apply both global optimization and a Model Predictive Control approach. In particular, we perform numerical tests to investigate how the CAVs number and positions impacts the result, showing that few, optimally chosen vehicles are sufficient to significantly improve the selected performance indexes, even using a decentralized control policy. Simulation results support the attractive perspective of exploiting a very small number of vehicles as endogenous control actuators to regulate traffic flow on road networks, providing a flexible alternative to traditional control methods. Moreover, we compare the impact of the proposed control strategies (decentralized, quasi-decentralized, centralized). See [29].

In the same light, in [44] we propose a new model for multi-lane traffic with moving bottlenecks, e.g., autonomous vehicles (AV). It consists of a system of balance laws for traffic in each lane, coupled in the source terms for lane changing, and fully coupled to ODEs for the AVs' trajectories. More precisely, each AV solves a controlled equation depending on the traffic density, while the PDE on the corresponding lane has a flux constraint at the AV's location. We prove existence of entropy weak solutions, and we characterize the limiting behavior for the source term converging to zero (without AVs), corresponding to a scalar conservation law for the total density. The convergence in the presence of AVs is more delicate and we show that the limit does not satisfy an entropic equation for the total density as in the original coupled ODE-PDE model. Finally, we illustrate our results via numerical simulations.

In the aim of modeling the formation of stop-and-go waves (to be controlled employing CAVs), in [32] we prove the existence of weak solutions for a class of second order traffic models with relaxation, without requiring the sub-characteristic stability condition to hold. Therefore, large oscillations may arise from small perturbations of equilibria, capturing the formation of stop-and-go waves observed in reality. An analysis of the corresponding travelling waves completes the study.

In [30], we study the boundary stabilization of Generic Second Order Macroscopic traffic models in Lagrangian coordinates. These consist in 2×2 nonlinear hyperbolic systems of balance equations with a relaxation-type source term. We provide the existence of weak solutions of the Initial Boundary Value problem for generic relaxation terms. In particular, we do not require the sub-characteristic stability condition to hold, so that equilibria are unstable and perturbations may lead to the formation of large oscillations, modeling the appearance and persistence of stop-and-go waves. Moreover, since the largest eigenvalue of the system is null, the boundaries are characteristic, and the available results on boundary controllability do not apply. Therefore, we perform a detailed analysis of the Wave Front Tracking approximate solutions to show that weak solutions can be steered to the corresponding equilibrium state by prescribing the equilibrium speed at the right boundary. This corresponds to controlling the speed of one vehicle to stabilize the upstream traffic flow. The result is illustrated through a numerical example.

Routing strategies in traffic flows on networks

The research focused on the development of a general multi-class macroscopic traffic model that can be applied to large-scale real-world networks. The study investigated the impact of a Stackelberg control strategy on a "compliant" fraction of the traffic flow, defining their tendency to comply as a function of the difference in travel costs between the routes independently chosen by users and those proposed by the controller. The optimization process also accounts for the independent reaction of the non-controlled fraction of users, framing the problem as a Stackelberg game. Furthermore, the study explored the effects of the control strategy on the system at different penetration rates, defined as the percentage of users targeted by the controller. It also considered varying levels of user willingness to comply with the proposed routing strategy. The analysis aimed to understand how these two factors — penetration rate and user compliance — interact and influence the overall system efficiency, traffic flow, and congestion patterns.

To implement this model, we developed the Python library `PyLate` (7.1.4). This library enables the rapid setup of macroscopic traffic simulations on synthetic networks and, by leveraging the `osmnx` library, allows for the seamless import of real-world road networks from `OpenStreetMap`.

Hyperbolic-parabolic models for the management of traffic generated pollution

In [41], vehicular traffic flows through a merge regulated by traffic lights and produces pollutants that diffuse in the surrounding region. This situation motivates a general hyperbolic-parabolic system, whose well-posedness and stability are here proved in L1. Roads are allowed to be also 2–dimensional. The effects of stop & go waves are comprised, leading to measure source terms in the parabolic equation. The traffic lights, as well as inflows and outflows, can be regulated to minimize the presence of pollutant in given regions.

8.2 Nonlocal pedestrian flow models

Participants: Paola Goatin, Ilaria Ciaramaglia, Harols Deivi Contreras *(Universidad San Sebastian, Chile)*, Daniel Inzunza *(Universidad San Sebastian, Chile)*, Gabriella Puppo *(Univ. Roma La Sapienza, Italy)*, Elena Rossi *(Univ. Modena - Reggio Emilia, Italy)*, Luis-Miguel Villada *(Universidad del Bio Bio, Chile)*.

In the framework of Ilaria Ciaramaglia's PhD thesis, [27] provides the well-posedness of weak entropy solutions of a scalar non-local traffic flow model with time delay. Existence is obtained by convergence of finite volume approximate solutions constructed by Lax-Friedrich and Hilliges-Weidlich schemes, while the L1 stability with respect to the initial data and the delay parameter relies on a Kruzkov-type doubling of variable technique. Numerical tests are provided to illustrate the efficiency of the proposed schemes, as well as the solution dependence on the delay and look-ahead parameters.

In [28], we propose and study a nonlocal system of balance laws, which models the traffic dynamics on a two-lane and two-way road where drivers have a preferred lane (the lane on their right) and the other one is used only for overtaking. In this model, the convective part is intended to describe the intralane dynamics of vehicles: the flux function includes local and nonlocal terms, namely, the velocity function in each lane depends locally on the density of the class of vehicles traveling on their preferred lane and in a nonlocal form on the density of the class of vehicles overtaking in the opposite direction. The source terms are intended to describe the coupling between the two lanes: the overtaking and return criteria depend on weighted means of the downstream traffic density of the class of vehicles traveling in their preferred lane and of the class of vehicles traveling in the opposite direction on the same lane. We construct approximate solutions using a finite volume scheme and we prove existence of weak solutions by means of compactness estimates. We also show some numerical simulations to describe the behavior of the numerical solutions in different situations and to illustrate some features of model.

In [45], we consider a class of multi-population pedestrian models consisting in a system of nonlocal conservation laws coupled in the nonlocal components and describing several groups of pedestrians moving towards their respective targets while trying to avoid each other and the obstacles limiting the walking domain. Specifically, the nonlocal operators account for interactions occurring at the microscopic level as a reaction to the presence of other individuals or obstacles along the preferred path. In particular, the presence of obstacles is implemented in the nonlocal terms of the equations and not as classical boundary conditions. This allows to rewrite domain shape optimization problems as PDE-constrained problems. In this paper, we investigate the well-posedness of such optimization problems by proving the stability of solutions with respect to the positions and shapes of the obstacles. A differentiability result in the linear case is also provided. These properties are illustrated with a numerical example. See also [36].

8.3 Mean Field Games

Participants: Abderrahmane Habbal, Amal Machtalay *(U Mohamed VI Polytech, Morocco)*(UM6P) , Imad Kissami *(UM6P)*, Ahmed Ratnani *(UM6P)*, Meryeme Jahid *(UM6P)*, Lahcen Maniar *(Univ. Cadi Ayyad, Marrakech, Morocco)*, S.E. Chorfi *(Univ. Cadi Ayyad, Marrakech, Morocco)*.

- **Two-class Traffic Flows** We have explored a multi-class traffic model and examined the computational feasibility of mean-field games (MFG) in obtaining approximate Nash equilibria for traffic flow games involving a large number of players. We introduced a two-class traffic mean-field game framework, building upon classical multi-class formulations. To facilitate our analysis, we employed various numerical techniques, including high-performance computing and regularization of LGMRES solvers. By utilizing these tools, we conducted simulations at significantly larger spatial and temporal scales.

  We led extensive numerical experiments considering three different scenarios involving cars and trucks, as well as three different cost functionals. Our results primarily focused on the dynamics of autonomous vehicles (AVs) in traffic, yielding results which support the effectiveness of the approach.

  Moreover, we conducted original comparisons between macroscopic Nash mean-field speeds and their microscopic counterparts. These comparisons allowed us to computationally validate the $\epsilon-$Nash approximation, demonstrating a slightly improved convergence rate compared to theoretical expectations.

  Future directions encompass second order traffic models, the multi-lane case, particularly prone to non-cooperative game considerations, and addressing some theoretical issues, see [108].

- **Degenerate mean-Field Game Systems** We investigate inverse backward-in-time problems for a class of second-order degenerate Mean-Field Game (MFG) systems.

  More precisely, given the final datum at time $t = T$ of a solution to the one-dimensional mean-field game system with a degenerate diffusion coefficient, we aim to determine the intermediate states, at some $t = t_0$ for any $0 \leq t_0 < T$, i.e., the value function and the mean distribution at intermediate times, respectively.

  We prove conditional stability estimates under suitable assumptions on the diffusion coefficient and the initial state $t = 0$. The proofs are based on Carleman's estimates with a simple weight function. We first prove a Carleman estimate for the Hamilton-Jacobi-Bellman (HJB) equation. A second Carleman estimate will be derived for the Fokker-Planck (FP) equation. Then, by combining the two estimates, we obtain a Carleman estimate for the mean-field game system, leading to the stability of the backward problems [26].

8.4 Fluid-structure interaction using isogeometric analysis

Participants: Régis Duvigneau.

The isogeometric analysis framework is used to develop an accurate numerical scheme for fluid-structure interaction problems. The approach relies on a Discontinuous Galerkin method for the flow (compressible Navier-Stokes equations) whereas a Continuous Galerkin method is employed for the structure (non-linear membrane or linear elasticity), both of them based on NURBS bases allowing an exact matching interface with a different resolution for each discipline. The approach is applied to the study of membrane wings.

8.5 Simulation of windblown sand

Participants: Régis Duvigneau, Adrien Bousseau *(GraphDeco Project-Team)*, Guillaume Cordonnier *(GraphDeco Project-Team)*, Nicolas Rosset *(GraphDeco Project-Team)*.

In collaboration with GraphDeco team, numerical methods for fast simulations of sand erosion and deposition patterns has been developed for computer graphics applications in the context of N. Rosset's PhD thesis. The proposed approach based on geosciences principles has been validated against experimental results as well as real world observations in [34].

8.6 Learning strategies for PDEs

Participants: Guillaume Coulaud, Régis Duvigneau, Nathan Ricard, Maxime Le *(SED Centre Inria d'Université Côte d'Azur)*.

We investigate the use of novel machine learning paradigms in the context of complex PDE systems, including the following research axes:

- **Multiphysic coupling using physics-informed neural networks**
  Physics-Informed Neural Networks (PINNs) have emerged as a promising paradigm to handle diverse scenarios to simulate multiphysic systems, by embedding the different PDE models and coupling conditions in a single learning task. This is investigated in [35] in the context of conjugate heat transfer, during G. Coulaud's Master thesis.

- **Turbulence inference using physics-informed neural networks**
  Turbulence modeling is still a major issue in complex flow simulations, due to the limitations of turbulence models in terms of application range. PINNs offer a promising framework to overcome this difficulty, by allowing to build simulation tools based on both PDE models and experimental data. Thus, we investigate this approach in [35] to simulate turbulent flows by infering turbulence characteristics from mean flow data in replacement to classical turbulence closures.

- **Non-classical training approaches for physics-informed neural networks**
  Above-mentioned studies put in light some specific issues for the training of PINNs, related to the presence of unbalanced terms in the loss functions, yielding the failure of first-order gradient algorithms classically used in machine learning. Thus, we investigate non-classical training strategies based on Nash games or hierarchical methods. This work is part of N. Ricard's PhD thesis.

The three latter activities have benefited from SED support for the development of pinnacle software (7.1.2) devoted to PINNs.

8.7 Advanced Bayesian optimization

Participants: Ayoub Bellouch *(Atlantis team)*, Luca Berti *(IRMA, Institut de Recherche Mathématique Avancée)*, Mickaël Binois, Nicholson Collier *(Argonne, USA)*, Régis Duvigneau, Arindam FadikarArgonne, USA , Laëtitia Giraldi *(Calisto team)*, Stéphane Lanteri *(Atlantis team)*, Jonathan Ozik *(Argonne, USA)*, Lucas Palazzolo *(Calisto team)*, Victor Picheny *(SecondMind, GB)*, Abby Stevens *(Argonne, USA)*.

- **Handling of noisy simulators**

  In [38], we present specific aspects of Gaussian process modeling in the presence of complex noise. Starting from the standard homoscedastic model, various generalizations from the literature are presented: input varying noise variance, non-Gaussian noise, or quantile modeling. These approaches are compared in terms of goal, data availability and inference procedure. A distinction is made between methods depending on their handling of repeated observations at the same location, also called replication. The chapter concludes with the corresponding adaptations of the sequential design procedures. These are illustrated in an example from epidemiology.

  A more realistic example is described by [46], showing a significant reduction of the number of simulations required to calibrate the parameters of a Covid-19 simulator.

- **High-dimensional Gaussian process modeling**

  In an effort to improve the scalability of Bayesian optimization based on Gaussian processes, we propose a new approach in [39]. Gaussian processes are a widely embraced technique for regression and classification due to their good prediction accuracy, analytical tractability and built-in capabilities for uncertainty quantification. However, they suffer from the curse of dimensionality whenever the number of variables increases. This challenge is generally addressed by assuming additional structure in the problem, the preferred options being either additivity or low intrinsic dimensionality. Our contribution for high-dimensional Gaussian process modeling is to combine them with a multi-fidelity strategy, showcasing the advantages through experiments on synthetic functions and datasets.

- **Bayesian optimization of micro-swimmers**

  In [47], we are interested in understanding and optimizing the design of helical micro-swimmers. This is crucial for advancing their application in various fields. This study presents an innovative approach combining Free-Form Deformation with Bayesian Optimization to enhance the shape of these swimmers. Our method facilitates the computation of generic swimmer shapes that achieve optimal average speed and efficiency. Applied to both monoflagellated and biflagellated swimmers, our optimization framework has led to the identification of new optimal shapes. These shapes are compared with biological counterparts, highlighting a diverse range of swimmers, including both pushers and pullers.

  Massively parallel Bayesian optimization

  Motivated by a large scale multi-objective optimization problem for which thousands of evaluations can be conducted in parallel, we develop an efficient approach to tackle this issue in [25].

  One way to reduce the time of conducting optimization studies is to evaluate designs in parallel rather than just one-at-a-time. For expensive-to-evaluate black-boxes, batch versions of Bayesian optimization have been proposed. They work by building a surrogate model of the black-box that can be used to select the designs to evaluate efficiently via an infill criterion. Still, with higher levels of parallelization becoming available, the strategies that work for a few tens of parallel evaluations become limiting, in particular due to the complexity of selecting more evaluations. It is even more crucial when the black-box is noisy, necessitating more evaluations as well as repeating experiments. Here we propose a scalable strategy that can keep up with massive batching natively, focused on the exploration/exploitation trade-off and a portfolio allocation. We compare the approach with related methods on deterministic and noisy functions, for mono- and multi-objective optimization tasks. These experiments show similar or better performance than existing methods, while being orders of magnitude faster.

- **Multi-fidelity modeling and optimization**

  To reduce the computational cost related to the use of high-fidelity simulations when evaluating the cost function, we investigate the construction of multi-fidelity Gaussian Process models, that can rely on different physical models (e.g. inviscid or viscous flows) or numerical accuracy (e.g. coarse or fine meshes). The objective is to construct a model that is accurate regarding the high-fidelity evaluations, but mostly based on low-fidelity simulations. In the context of design optimization, we especially investigate the use of a multi-task entropy search approach, with applications to aerodynamics and nano-photonics (in collaboration with the Atlantis team).

8.8 Pareto optimality and Nash games

Participants: Jean-Antoine Désidéri, Mickaël Binois, Nathalie Bartoli *(ONERA/DTIS, Université de Toulouse)*, Christophe David *(ONERA/DTIS, Université de Toulouse)*, Sébastien Defoort *(ONERA/DTIS, Université de Toulouse)*, Julien Wintz *(SED, Inria Sophia Antipolis)*.

In the multi-objective optimization of a complex system, establishing the Pareto front associated with the whole set of cost functions is usually a computationally demanding task, whose results are not always easy to analyze, while the final decision still remains to be made among Pareto-optimal solutions. These observations led us to propose a prioritized approach in which the Pareto front is calculated only for a subset of primary cost functions, those of preponderant importance, followed by an economical and decisive step in which a continuum of Nash equilibria accounting for secondary functions is calculated [8].

The method has been applied to the multi-objective optimization of the flight performance of an Airbus-A320-type aircraft in terms of take-off fuel mass and operational empty weight (primary cost functions) concurrently with ascent-to-cruise altitude duration (secondary) [12]. These results have been presented at a Conference on “New Greener and Digital Modern Transport” (JyU., Finland, May 2023), and recently completed by Bayesian optimization in [43] and are currently in press for proceedings.

That work reflects our cooperation with the Information Processing and Systems Department (DTIS) of Onera Toulouse. It will be continued to account for additional criteria related to environmental impact and operational performance.

In the present prioritized approach for multiobjective optimization, after a first phase of optimization has produced a Primary Pareto Front relative to the sole primary cost functions (under functional constraints), considered to be preponderant, a second phase of optimization is initiated to yield a continuum of Nash equilibria of quasi-Pareto-optimal solutions with respect to the whole set of objective functions (primary and secondary). This second phase relies on an orthogonal decomposition of the working space, a subset of $\mathbf{R}^N$, referred to as “territory splitting”.

We have generalized the original method [8] by relaxing the convergence condition on the construction of the territory splitting to isolate the affine subspace locally tangent to the constraints, proposed several alternatives, and tested their efficacy on a testcase of optimal sizing of an aluminum sandwich panel [42].

8.9 Modeling, Simulation and Optimization of Solid State Electrolyte batteries

Participants: Abderrahmane Habbal, Mustapha Bouchaara *(U Mohamed VI Polytech, Morocco, UM6P)*, Ahmed Ratnani *(UM6P)*.

The demand for advanced energy storage drives an urgency to accelerate material discovery in solid-state electrolytes. In pursuit of this aim, this study presents an innovative methodology that integrates materials science insights with machine learning techniques to improve the ionic conductivity prediction in garnet-based solid electrolytes. Utilizing an expanded dataset comprising 362 data points, and exploiting easily obtainable pre-synthesis inputs, our approach incorporates rigorous data preprocessing inspired by materials science and machine learning methodologies. Through systematic feature selection and hyperparameter tuning, the model achieved an improved R-squared value of 0.85. This study highlights the efficacy of the proposed approach and underscores the potential of machine learning in streamlining materials discovery and design for next-generation solid-state batteries [33].

8.10 Optimal transport and isogeometric analysis

Participants: Abderrahmane Habbal, Mustapha Bahari *(U Mohamed VI Polytech, Morocco,UM6P)*, Ahmed Ratnani *(UM6P)*, Eric Sonnendrücker *(Max Planck Institute)*.

- **Adaptive mesh generation using fast diagonalization** In this work, we devise fast solvers and adaptive mesh generation procedures based on the Monge–Ampère Equation using B-Splines Finite Elements, within the Isogeometric Analysis framework. Our approach ensures that the constructed mapping is a bijection, which is a major challenge in Isogeometric Analysis. First, we use standard B-Splines Finite Elements to solve the Monge–Ampère Equation. An analysis of this approach shows serious limitations when dealing with high variations near the boundary. In order to solve this problem, a new formulation is derived using compatible B-Splines discretization based on a discrete DeRham sequence. A new fast solver is devised in this case using the Fast Diagonalization method. Different tests are provided and show the performance of our new approach, see [24].

- **Mesh adaptation for time dependent problems** We introduce a new algorithm designed to create a dynamic r-adaptive mesh within the framework of isogeometric analysis. The approach is based on the simultaneous computation of adaptive meshes using a nonlinear parabolic Monge–Ampere equation with a resolution of partial differential equations in multidimensional spaces. The technique ensures the absence of geometric boundary errors and is simple to implement, requiring the solution of only one Laplace scalar equation at each time step. It utilizes a fast diagonalization method that can be adapted to any dimension. Various numerical experiments were conducted to validate an original parabolic Monge–Ampere solver. The solver was respectively applied to Burgers, Allen–Cahn, and Cahn–Hilliard problems to demonstrate the efficiency of the new approach [23].

8.11 Cut-cells for Saint-Venant equations with moving obstacles

Participants: Laurent Monasse, Amal Chakib, Konstantin Brenner *(Université Côte d'Azur)*.

During Amal Chakib's M2 internship, we investigated the extension of the cut-cell methodology developed in code CELIA2D for Euler equations to Saint-Venant equations for free-surface flows with moving obstacles. The ultimate goal is to simulate flooding phenomena where vehicles float and block the fluid flow, increasing the damage to structures and casualties. The investigations allowed to extend the numerical scheme for well-balanced shallow water on real topography data with bottom friction. Validation and benchmarking against state-of-the-art methods are in development.

8.12 Shock fitting with cut cell methods

Participants: Laurent Monasse, Alexandre Vieira, Régis Duvigneau.

Compressible fluids develop shocks in finite time and transport initial material discontinuities. Accurately tracking these discontinuities in the fluid state is numerically challenging and can lead to numerical smearing of discontinuities and loss of accuracy. This is especially difficult for discontinuities in material density and behavior laws. Instead of discontinuity capturing methods, we have proposed to use discontinuity tracking methods by following material discontinuities in a Lagrangian way. In order to enable wave interaction and topology changes, we combine discontinuity tracking with cut cell methods. We have extended the classical Finite Volume framework to Discontinuous Galerkin methods in space-time.

8.13 Fungal growth modeling and simulation

Participants: Laurent Monasse, Yves D'Angelo *(Université Côte d'Azur)*, Rémi Catellier *(Université Côte d'Azur)*, Claire Guerrier *(Université Côte d'Azur)*, Nicolas Fricker *(Université Côte d'Azur)*.

Fungi develop growing networks in the form of mycellium which explore space by branching at the tips (apex branching) and on the existing network (lateral branching). We have studied fungal growth at two scales: a large scale (of the order of the Petri dish) where the behavior of the fungus is homogenized, and a small scale (of the order of the fungus cell) in order to understand the undelying biochemical phenomena at hand in growth and branching. On the large scale, we have proposed a new partial differential equation (PDE) model for the dynamics of growth. We have computed front propagation velocities and proposed numerical schemes to approximate the solution in space and time of the PDE. The work is in its final process to submission. On the other hand, on the small scale, we have proposed a biologically informed model of filament growth. We have developed a fast and accurate numerical scheme accounting for the absorption of nutrients, their conversion to vesicles and the transport of vesicles in the cell, inducing the growth of the filament. The model is in the process of fitting unknown parameters to experimental biological data.

9 Bilateral contracts and grants with industry

9.1 Bilateral contracts with industry

- **Mycophyto** (2020-2024): this research contract involving Université Côte d'Azur is financing the post-doctoral contract of Khadija Musayeva. The goal is to develop prediction algorithms based on environmental data.

  Participants: Mickaël Binois, Khadija Musayeva.

- **IFPEN** (2022-2025): this research contract is financing the PhD thesis of Agatha Joumaa on "A multi-mode macroscopic traffic model for the improvement of mobility and air quality in our cities via optimal modal share and routing"'.

  Participants: Paola Goatin, Agatha Joumaa.

- **Altair** (2024-2025): this research contract involving AMIES support finances the post-doctoral contract of Alexandre Vieira. The goal is to develop compressible multimaterial flow simulation using cut-cell methods in the open-source code OpenRadioss.

  Participants: Laurent Monasse, Alexandre Vieira.

10 Partnerships and cooperations

10.1 European initiatives

10.1.1 Horizon Europe

DATAHYKING DATAHYKING project on cordis.europa.eu

Title: Data-driven simulation, uncertainty quantification and optimization for hyperbolic and kinetic models

Duration: From March 1, 2023 to February 28, 2027

Partners:

- TRANSPORT & MOBILITY LEUVEN (TML), Belgium
- Autovie Venete S.p.A., Italy
- INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET AUTOMATIQUE (INRIA), France
- UNIVERSITE COTE D'AZUR, France
- UNIVERSITE COTE D'AZUR, France
- RHEINISCH-WESTFAELISCHE TECHNISCHE HOCHSCHULE AACHEN (RWTH AACHEN), Germany
- ESI GROUP (ESI GROUP), France
- Cassa di Compensazione e Garanzia s.p.a. (CC&G), Italy
- NEOVYA Mobility by Technology (NEOVYA Mobility by Technology), France
- INCICO (INICIO SPA), Italy
- SIEMENS INDUSTRY SOFTWARE NETHERLANDS BV (Siemens Industry Software Netherlands B.V.), Netherlands
- RHEINLAND-PFALZISCHE TECHNISCHE UNIVERSITAT, Germany
- CENTRE DE RECHERCHE EN AERONAUTIQUE ASBL - CENAERO (CENAERO), Belgium
- UNIVERSITE DE LILLE (UNIVERSITE DE LILLE), France
- ZENSOR (ZENSOR), Belgium
- KATHOLIEKE UNIVERSITEIT LEUVEN (KU Leuven), Belgium
- UNIVERSITA DEGLI STUDI DI ROMA LA SAPIENZA (UNIROMA1), Italy
- UNIVERSITA DEGLI STUDI DI FERRARA (Unife), Italy

Inria contact: Paola Goatin

Coordinator: Giovanni Samaey (KU Leuven)

Summary: Europe faces major challenges in science, society and industry, induced by the complexity of our dynamically evolving world. To tackle these challenges, mathematical models and computer simulations are indispensable, for instance to design and optimize systems using virtual prototypes. Moreover, while the big data revolution provides additional possibilities, it is currently unclear how to optimally combine simulation results with observation data into a digital. Many systems of interest consist of large numbers of particles with highly non-trivial interaction (e.g., fine dust in pollution, vehicles in mobility).

However, to date, computer simulation of such systems is usually done with highly approximate (macroscopic) models to reduce computational complexity. Facing these challenges without sacrificing the complexity of the underlying particle interactions requires a fundamentally new type of scientist that uses an interdisciplinary approach and a solid mathematical underpinning. Hence, we aim at training a new generation of modeling and simulation experts to develop virtual experimentation tools and workflows that can reliably and efficiently exploit the potential of mathematical modeling and simulation of interacting particle systems.

To this end, we create a data-driven simulation framework for kinetic models of interacting particle systems, and define a common methodology for these future modeling and simulation experts. The network focuses on (i) reliable and efficient simulation; (ii) robust consensus-based optimization, also for machine learning; (iii) multifidelity methods for uncertainty quantification and Bayesian inference; and (iv) applications in fluid flow, traffic flow, and finance, also in collaboration with industry. Moreover, the proposed EJD program will create a closely connected new generation of highly demanded European scientists, and initiate long-term partnerships to exploit synergy between academic and industrial partners.

10.2 National initiatives

10.2.1 ANR

- **Institute 3IA Côte d'Azur**: The 3IA Côte d'Azur is one of the four "Interdisciplinary Institutes of Artificial Intelligence" that were created in France in 2019. Its ambition is to create an innovative ecosystem that is influential at the local, national and international levels, and a focal point of excellence for research, education and the world of AI.
  ACUMES is involved with the project "Data driven traffic management" in the axis *AI for smart and secure territories* (2020-2024), for which P. Goatin is chair holder. This project aims at contributing to the transition to intelligent mobility management practices through an efficient use of available resources and information, fostering data collection and provision. We focus on improving traffic flow on road networks by using advanced mathematical models and statistical techniques leveraging the information recovered by real data.

  Participants: Paola Goatin, Daniel Inzunza, Alexandra Würth.

- **COSS - COntrol on Stratified Structures** (ANR-22-CE40-0010, PI Nicolas Forcadel, INSA Rouen): The central theme of this project lies in the area of control theory and partial differential equations (in particular Hamilton-Jacobi equations), posed on stratified structures and networks. These equations appear very naturally in several applications. Indeed, many practical optimal control problems, such as traffic flow modeling or energy management in smart-grids networks or sea-land trajectories with different dynamics, involve a state space in a stratified form (a collection of manifolds with different dimensions and associated to different dynamics). These control problems can be studied within the framework of Hamilton Jacobi equations theory; in particular, they involve admissible trajectories that have to stay in the stratified domain.

  Participants: Paola Goatin.

- **NEMATIC - Analysis Modelling Simulation Multiscale (ANR-21-CE45-0010, PI Eric Herbert, LIED, Université de Paris)**: The objective of the project is to experimentally characterize, analyse, model and simulate the multiscale dynamics of complex and growing branching random networks. Both analytical and numerical means as well as experimental realizations are used and developed. In a biological context, the growth of the filamentous fungus Podospora anserina will be used as a model, by systematically comparing modeling and experiments. The project brings together biologists, who are specialists in this field, as well as physicists and mathematicians in charge of acquiring and analyzing experimental data and designing the models as well as simulations. On the one hand, we plan to develop the numerical reconstruction of the network, by transforming the raw experimental data into a spatio-temporal graph, the dynamics of which will be included in an efficient labelling of the temporal evolution of the nodes, capable of interpreting anastomosis and branching, and thus of following through time and space a node of the network. By varying the type of constraints applied during model validation, we expect a fine-grained understanding of emergent processes (such as branching) and resilience. NEMATIC aims to provide the scientific community with the experimental, theoretical and numerical data and tools necessary for such analyses.

Participants: Laurent Monasse, Nicolas Fricker.

- **NEMO - ControlliNg a magnEtic Micro-swimmer in a cOnfined area (ANR-21-CE45-0013, PI Laetitia Giraldi, EPI CALISTO, Inria)**: NEMO aims to develop numerical methods to control a micro-robot swimmer in the arteries of the human body. These robots could deliver drugs specifically to cancer cells before they form new tumors, thus avoiding metastasis and the traditional chemotherapy side effects. NEMO will focus on micro-robots, called Magnetozoons, composed of a magnetic head and an elastic tail immersed into a laminar fluid possibly non-Newtonian. These robots imitate the propulsion of spermatozoa by propagating a wave along their tail. Their movement is controlled by an external magnetic field that produces a torque on the head of the robot, producing a deformation of the tail. The tail then pushes the surrounding fluid and the robot moves forward. The advantage of such a deformable swimmer is its aptness to carry out a large set of swimming strategies, which could be selected according to the geometry or the rheology of the biological media where the swimmer evolves (blood, eye retina, or other body tissues). Although the control of a such micro-robots has mostly focused on simple unconfined environnement, the main challenge is today to design external magnetic fields that allow them to navigate efficiently in complex realistic environments. NEMO aims to elaborate efficient controls, which will be designed by tuning the external magnetic field, through a combination of Bayesian optimization and accurate simulations of the swimmer's dynamics with Newtonian or non-Newtonian fluids. Then, the resulting magnetic fields will be validated experimentally in a range of confined environments. In such an intricate situation, where the surrounding fluids is bounded laminar and possibly non-Newtonian, optimization of a strongly nonlinear, and possibly chaotic, high-dimensional dynamical system will lead to new paradigms. The results of NEMO will be the subject of several publications in mathematical modeling, numerical analysis, optimization, control, physics and multidisciplinary journals. The numerical developments will be provided as open-source softwares. The experiments will contribute as a proof of concept validating the NEMO control approach.

  Participants: Mickaël Binois, Laurent Monasse.

11 Dissemination

11.1 Promoting scientific activities

11.1.1 Scientific events: organisation

General chair, scientific chair

- Mickael Binois co-chaired the 2024 annual conference of the Groupement de Recherche MascotNum, now RT-UQ.

Member of the organizing committees

- Régis Duvigneau was a member of the organizing committee of the 2024 annual conference of the Groupement de Recherche MascotNum.
- Abderrahmane Habbal was a member of the organizing committee of the UM6P Winter-School on Model Reduction Methods for Control and Machine Learning, February 26-27-28-29, 2024, Benguerir, Morocco.
- Abderrahmane Habbal was a member of the organizing committee of the 2nd Edition of the International Conference on Mathematics and Decision, December 17–20, 2024, Rabat.

11.1.2 Scientific events: selection

Member of the conference program committees

- Mickael Binois participated in the following conference program committees:
  - 2024 annual conference of the Groupement de Recherche MascotNum,
  - Conference on Uncertainty in Artificial Intelligence 2024,
  - AAAI Conference on Artificial Intelligence 2025.

Reviewer

- Mickael Binois reviewed for the following conferences: AISTATS 2025, EMO 2025, GECCO 2024, ICLR25, ICML 2024, ITSC24 and NeurIPS 2024.
- Abderrahmane Habbal reviewed for the conference CARI 2024

11.1.3 Journal

Member of the editorial boards

- Mickael Binois is Associate Editor of ACM Transactions on Evolutionary Learning and Optimization
- Paola Goatin is Managing Editor of *Networks and Heterogeneous Media* and Associate Editor of *SIAM Journal on AppliedMathematics* and *ESAIM: Mathematical Modelling and Numerical Analysis.*

Reviewer - reviewing activities

- Mickael Binois is a reviewer for the following international journals: Comput. Optim. Appl., Comput. Stat. Data Anal, J. Comput. Sci., J. Mach. Learn. Res, J Multi-Criteria Dec, Philos. Trans. R. Soc. A, SIAM-ASA J. Uncertain. Quantif., Technometrics, Transactions on Machine Learning Research.
- Régis Duvigneau is reviewer for the following international journals: Comp. & Fluids, Comp. Meth. Appl. Mech. Eng., J. Fluids & Struct., int. J. Num. Meth. in Fluids.
- Paola Goatin reviewed for: Applied Mathematics and Computation, Analysis & PDE, Journal of Hyperbolic Differential Equations, Mathematical Modelling of Natural Phenomena, Networks and Heterogeneous Media, Nonlinear Differential Equations and Applications, Zeitschrift fuür angewandte Mathematik und Physik.
- Laurent Monasse reviewed for: Journal of Fluid Mechanics, La Matematica, SIAM Journal on Scientific Computing.
- Abderrahmane Habbal reviewed for J. of Optim Theory and App., Springer Proc on Math and Stats, J. of Comp and App. Math., Journal of Computational Science, Mathematical Modelling of Natural Phenomena,

11.1.4 Invited talks

- Mickael Binois: SIAM Conference on Uncertainty Quantification, February 2024, Trieste, Italy. Conference Talk: Trajectory-Oriented Optimization of Stochastic Epidemiological Models.
- Mickael Binois: Design and Analysis of Experiments conference, May 2024, Blacksburg, USA. Conference Talk: Scalable Bayesian optimization for noisy problems.
- Mickael Binois: Warwick Business School, October 2024, Paris. Invited Seminar: Leveraging replication in active learning.

- Mickael Binois: INRAE MaIAGE, November 2024, Paris. Invited Seminar: Leveraging replication in active learning.
- Mickael Binois: Workshop SAMOURAI, December 2024, Paris. Invited talk: *Options for high-dimensional Bayesian optimization*
- Régis Duvigneau: Nat. Renewable Energy Lab., Colorado (USA), August 2024. Invited talk: *A fully integrated geometry-simulation-optimization framework for aerodynamic design using a NURBS-based Discontinuous Galerkin method*
- Régis Duvigneau: Workshop SciML 2024, Strasbourg, July 2024. Invited talk: *Physics-Informed Neural Networks for fluid dynamics : opportunities, current issues and possible paths forward*
- Paola Goatin: WONAPDE 2024 - Seventh Chilean Workshop on Numerical Analysis of Partial Differential Equations, Concepción (Chile), January 2024. Plenary talk: *Nonlocal conservation laws: theory, numerics and applications.* Minisymposium: "Nonlinear hyperbolic PDEs: numerical techniques and related models". Talk: *Multi-class and multi-population traffic flow models on networks.*
- Paola Goatin: EQUADIFF 2024, Karlstad (Sweden), June 2024. Minisymposium: "Hyperbolic PDEs: models and theorems". Talk: *A multi-scale multi-lane model for traffic regulation via autonomous vehicles.*
- Paola Goatin: Workshop SPP-2410: "Nonlocal Modelling in Fluidmechanical Applications", Mannheim (Germany), September 2024. Invited talk: *Non-local traffic flow models with time delay: analysis and numerical experiments.*
- Paola Goatin: PGMOdays 2024, Palaiseau (France), November 2024. Invited talk: *Traffic flow models for current and future mobility challenges.*
- Paola Goatin: Socialysm: Social Sciences and Mathematics, Rome (Italy), November 2024. Invited talk: *Multi-class and multi-population traffic flow models on networks.*
- Paola Goatin: CDC2024 - 63rd IEEE Conference on Decision and Control, Milano (Italy), December 2024. Invited session: "Macroscopic Traffic Modelling and Control". Talk: *Dissipation of Stop-and-Go waves in traffic flows using controlled vehicles.*
- Abderrahmane Habbal "Modelling pedestrian avoidance with Fokker-Planck Nash games" Invited Seminar, May 2024, at The Mathematics Institute of Univ. Sevilla, Spain.
- Abderrahmane Habbal "Nash strategies for boundary data completion coupled to shape and distributed parameter identification problems" Invited talk at the e-Series EDP E MATEMATICA APLICADA.
- Abderrahmane Habbal : "Exploring new applications of PDE-constrained games" Invited Plenary Lecture and short course on game theory at the 13th Euro-Maghrebian Workshop on Evolution Equations, September 2024, Castro Urdiales, Spain.

11.1.5 Scientific expertise

Mickael Binois was sollicitated for the evaluation of projects by the ANR and the Czech Science Foundation.

11.1.6 Research administration

- Régis Duvigneau is head of the Scientific Committee of Platforms (cluster and immersive space) for Inria Centre at Université Côte d'Azur.
- Régis Duvigneau is member of the Scientific Committee of OPAL computing Platform at Université Côte d'Azur

- Régis Duvigneau is member of the Steering Committee of "Maison de la Simulation et Interactions" at Université Côte d'Azur.
- Paola Goatin was adjunct director of the Doctoral School of Fundamental and Applied Sciences (ED SFA) of Université Côte D'Azur.
- Paola Goatin was member of the Full Professor (PR) hiring committee of Université Côte d'Azur in Applied Mathematics.
- Laurent Monasse was an elected member of the Scientific Committee of EUR Spectrum of Université Côte d'Azur until November 2024.
- Laurent Monasse is head of the Committee of Technology Development (CDT) for Inria Centre at Université Côte d'Azur since December 1st, 2024.

11.2 Teaching - Supervision - Juries

11.2.1 Teaching

- Master: M. Binois, Optimisation bayésienne, 9 hrs, M2, Polytech Nice Sophia - Université Côte d'Azur.
- Master: M. Binois, Optimization, 24 hrs, M1, Polytech Nice Sophia - Université Côte d'Azur.
- Master: J.-A. Désidéri, Multidisciplinary Optimization, ISAE Supaéro (Toulouse), 5 hrs.
- Master: Régis Duvigneau, Advanced Optimization, 28 hrs, M2, Polytech Nice Sophia - Université Côte d'Azur.
- Advanced course: Régis Duvigneau, 3 hrs, Deep Learning School 2024 3IA Côte d'Azur, Sophia-Antipolis (France), July 2024: *"Deep learning for physics"*.
- Master: A. Habbal, Optimization, 18 hrs, M1, Polytech Nice Sophia - Université Côte d'Azur.
- Master: A. Habbal, Numerical methods for PDEs, 24 hrs, M1, Polytech Nice Sophia - Université Côte d'Azur.
- Master: A. Habbal, Stochastic Processes, 24 hrs, M1, Polytech Nice Sophia - Université Côte d'Azur.
- Master: A. Habbal, Introduction to optimization, 15 hrs, M1, Mohammed VI Polytechnic University, Morocco.
- Master: A. Habbal, Fall projects M1, 20 hrs, Polytech Nice Sophia - Université Côte d'Azur.
- Licence (L3): A. Habbal, Mathematical model of addiction, 48 hrs, L3 Semester Project, Polytech Nice Sophia - Université Côte d'Azur.
- Licence (L1): A. Habbal, Mathematics reinforcement, 36 hrs, Polytech Nice Sophia - Université Côte d'Azur.
- Master: L. Monasse, Numerical methods for PDEs, 12 hrs, M1, Polytech Nice Sophia - Université Côte d'Azur.
- License (L3): L. Monasse, Mathematics for Engineers 2, 36 hrs, Polytech Nice Sophia - Université Côte d'Azur.

11.2.2 Supervision

- PhD defense: Nicolas Rosset, *Fast simulation of wind-obstacle interactions Applications to desertscape modeling and car design*, Université Côte d'Azur, December 2024. Supervisors: Adrien Bousseau *(GraphDeco)*, Guillaume Cordonnier *(GraphDeco)*, Régis Duvigneau.
- PhD defense: Mustapha Bahari, *Optimal Transportation for Adaptive mesh generation and r-refinement*, July 2024. Université Côte d'Azur and UM6P, Morocco. Supervisors : Abderrahmane Habbal, A. Ratnani (UM6P)
- PhD in progress: Nathan Ricard, *Physics informed neural networks for multidisciplinary design*, Université Côte d'Azur. Supervisors: Régis Duvigneau, Mickael Binois.
- PhD in progress: Lucas Palazzolo *(Calisto)*, *Numerical methods for optimising the locomotion of flagellate microswimmers*, Université Côte d'Azur. Supervisors: Laetitia Giraldi *(Calisto)*, Mickael Binois.
- PhD in progress: Agatha Joumaa, *Pseudo-real-time optimization of the environmental performance of urban mobility using macroscopic and multimodal modeling approaches*, Université Côte d'Azur/IFPEN. Supervisors: Paola Goatin, Giovanni De Nunzio.
- PhD in progress: Ilaria Ciaramaglia, *Interactions between microscopic and macroscopic models for autonomous vehicles in human-driven environments*, Université Côte d'Azur and Univerità di Roma La Sapienza. Supervisors: Paola Goatin, Gabriella Puppo.
- PhD in progress: Carmen Mezquita Nieto, *Modeling and optimization of multi-modal transportation networks based on kinetic and hyperbolic equations*, Université Côte d'Azur and RPTU Kaiserslautern. Supervisors: Paola Goatin, Axel Klar.
- PhD in progress: Martin Fleurial, *Microscopic and macroscopic models for multilane and multispecies traffic flow*, Université Côte d'Azur and Univerità di Roma La Sapienza. Supervisors: Paola Goatin, Gabriella Puppo.
- PhD in progress: Eric Andoni, *Bayesian model calibration with uncertainty for traffic flow models*, Université Côte d'Azur and KU Leuven. Supervisors: Paola Goatin, Giovanni Samaey.
- PhD in progress: Nicolas Fricker, *Multi-scale modeling of voltage and ionic propagation in neurons: finding the rules of experience-driven neuronal encoding,* Université Côte d'Azur. Supervisors: Yves D'Angelo, Claire Guerrier and Laurent Monasse
- PhD in progress: Amal Machtalay, *« from mean-field games to agent based models (and back) through markov chain agregation »*, Université Côte d'Azur and UM6P, Morocco. Supervisors : Abderrahmane Habbal, A. Ratnani (UM6P)

11.2.3 Juries

- Régis Duvigneau was reviewer of Philippe Farjon's PhD thesis, *Développement et mise en oeuvre de méthodes d'optimisation sur des chambres de combustion aéronautiques fonctionnant à l'hydrogène*, ISAE Sup'Aero Toulouse, November 2024.
- Paola Goatin was member of the committee of T. Toso's PhD thesis *"Dynamics and games for information-aware routing in traffic networks"*, Université de Grenoble, October 4th, 2024.
- Paola Goatin was member of the committee of K. Mitsuzawa's PhD thesis *"Comparing high-dimension data: Interpretable two-sample testing by variable selection"*, Sorbonne Université, October 2nd, 2024.
- Laurent Monasse was reviewer of Antonin Leprevost's PhD thesis, *Couplage partitionné fluide-structure à l'échelle locale avec grille mobile pour des transitoires non-linéaires*, Université Paris-Saclay, April 23rd, 2024.

- Laurent Monasse was reviewer of Paul Larousse's PhD thesis, *Modélisation d'interface endommageable en dynamique explicite dédiée au démoulage de pneumatiques*, Insa Lyon, November 29th, 2024.

11.3 Popularization

11.3.1 Specific official responsibilities in science outreach structures

- Régis Duvigneau is member of the editorial board of Interstices web journal that targets digital sciences popularization.
- Laurent Monasse is local correspondant of Fondation Blaise Pascal at Université Côte d'Azur.

12 Scientific production

12.1 Major publications

[1] A. Aggarwal, R. M. Colombo and P. Goatin. 'Nonlocal systems of conservation laws in several space dimensions'. In: *SIAM Journal on Numerical Analysis* 52.2 (2015), pp. 963–983. URL: https://hal.inria.fr/hal-01016784.

[2] B. Andreianov, P. Goatin and N. Seguin. 'Finite volume schemes for locally constrained conservation laws'. In: *Numer. Math.* 115.4 (2010). With supplementary material available online, pp. 609–645.

[3] S. Blandin and P. Goatin. 'Well-posedness of a conservation law with non-local flux arising in traffic flow modeling'. In: *Numerische Mathematik* (2015). DOI: 10.1007/s00211-015-0717-6. URL: https://hal.inria.fr/hal-00954527.

[4] R. M. Colombo and P. Goatin. 'A well posed conservation law with a variable unilateral constraint'. In: *J. Differential Equations* 234.2 (2007), pp. 654–675.

[5] M. L. Delle Monache and P. Goatin. 'Scalar conservation laws with moving constraints arising in traffic flow modeling: an existence result'. In: *J. Differential Equations* 257.11 (2014), pp. 4015–4029.

[6] M. L. Delle Monache, J. Reilly, S. Samaranayake, W. Krichene, P. Goatin and A. Bayen. 'A PDE-ODE model for a junction with ramp buffer'. In: *SIAM J. Appl. Math.* 74.1 (2014), pp. 22–39.

[7] J.-A. Desideri and R. Duvigneau. 'Parametric optimization of pulsating jets in unsteady flow by Multiple-Gradient Descent Algorithm (MGDA)'. In: *Numerical Methods for Differential Equations, Optimization, and Technological Problems, Modeling, Simulation and Optimization for Science and Technology*. 1st Jan. 2017. URL: https://hal.inria.fr/hal-01414741.

[8] J.-A. Désidéri. 'Adaptation by Nash games in gradient-based multi-objective/multi-disciplinary optimization'. In: JANO13 - Mathematical Control and Numerical Applications. Vol. 372. Springer Proceedings in Mathematics & Statistics Series. Khouribga, Morocco, 22nd Feb. 2021. URL: https://hal.inria.fr/hal-03430972 (cit. on p. 21).

[9] J.-A. Désidéri. 'COOPERATION AND COMPETITION IN MULTIDISCIPLINARY OPTIMIZATION Application to the aero-structural aircraft wing shape optimization'. In: *Computational Optimization and Applications* 52.1 (2012), pp. 29–68. DOI: 10.1007/s10589-011-9395-1. URL: https://hal.inria.fr/hal-00645787.

[10] J.-A. Désidéri. 'Multiple-gradient descent algorithm (MGDA) for multiobjective optimization / Algorithme de descente à gradients multiples pour l'optimisation multiobjectif'. In: *Comptes Rendus. Mathématique* Tome 350.Fascicule 5-6 (20th Mar. 2012), pp. 313–318. DOI: 10.1016/j.crma.2012.03.014. URL: https://hal.inria.fr/hal-00768935.

[11] J.-A. Désidéri and R. Duvigneau. 'Prioritized optimization by Nash games : towards an adaptive multi-objective strategy'. In: *ESAIM: Proceedings and Surveys* 71 (Aug. 2021), pp. 54–63. DOI: 10.1051/proc/202171106. URL: https://hal.inria.fr/hal-03430912.

[12] J.-A. Désidéri, J. Wintz, N. Bartoli, C. David and S. Defoort. *Combining Pareto Optimality with Nash Games in Multi-Objective Prioritized Optimization of an Aircraft Flight Performance.* RR-9490. Inria - Sophia Antipolis; Acumes, 17th Oct. 2022, p. 29. URL: https://inria.hal.science/hal-03817789 (cit. on p. 21).

[13] R. Duvigneau and P. Chandrashekar. 'Kriging-based optimization applied to flow control'. In: *Int. J. for Numerical Methods in Fluids* 69.11 (2012), pp. 1701–1714.

[14] A. Habbal and M. Kallel. 'Neumann-Dirichlet Nash strategies for the solution of elliptic Cauchy problems'. In: *SIAM J. Control Optim.* 51.5 (2013), pp. 4066–4083.

[15] M. Kallel, R. Aboulaich, A. Habbal and M. Moakher. 'A Nash-game approach to joint image restoration and segmentation'. In: *Appl. Math. Model.* 38.11-12 (2014), pp. 3038–3053. DOI: 10.1016/j.apm.2013.11.034. URL: http://dx.doi.org/10.1016/j.apm.2013.11.034.

[16] M. Martinelli and R. Duvigneau. 'On the use of second-order derivative and metamodel-based Monte-Carlo for uncertainty estimation in aerodynamics'. In: *Computers and Fluids* 37.6 (2010).

[17] Q. Mercier, F. Poirion and J.-A. Desideri. 'A stochastic multiple gradient descent algorithm'. In: *European Journal of Operational Research* (31st May 2018), p. 10. DOI: 10.1016/j.ejor.2018.05.064. URL: https://hal.archives-ouvertes.fr/hal-01833165.

[18] S. Roy, A. Borzì and A. Habbal. 'Pedestrian motion modelled by Fokker–Planck Nash games'. In: *Royal Society open science* 4.9 (2017), p. 170648.

[19] G. Todarello, F. Vonck, S. Bourasseau, J. Peter and J.-A. Desideri. 'Finite-volume goal-oriented mesh adaptation for aerodynamics using functional derivative with respect to nodal coordinates'. In: *Journal of Computational Physics* 313 (15th May 2016), p. 21. DOI: 10.1016/j.jcp.2016.02.063. URL: https://hal.inria.fr/hal-01410153.

[20] M. Twarogowska, P. Goatin and R. Duvigneau. 'Macroscopic modeling and simulations of room evacuation'. In: *Appl. Math. Model.* 38.24 (2014), pp. 5781–5795.

[21] G. Xu, B. Mourrain, A. Galligo and R. Duvigneau. 'Constructing analysis-suitable parameterization of computational domain from CAD boundary by variational harmonic method'. In: *J. Comput. Physics* 252 (Nov. 2013).

[22] B. Yahyaoui, M. Ayadi and A. Habbal. 'Fisher-KPP with time dependent diffusion is able to model cell-sheet activated and inhibited wound closure'. In: *Mathematical biosciences* 292 (2017), pp. 36–45.

12.2 Publications of the year

International journals

[23] M. Bahari, A. Habbal and A. Ratnani. 'Isogeometric analysis based mesh adaptation for time dependent problems'. In: *Engineering with Computers* 40.6 (1st July 2024), pp. 3851–3874. DOI: 10.1007/s00366-024-02009-8. URL: https://inria.hal.science/hal-04892403 (cit. on p. 22).

[24] M. Bahari, A. Habbal, A. Ratnani and E. Sonnendrücker. 'Adaptive isogeometric analysis using optimal transport and their fast solvers'. In: *Computer Methods in Applied Mechanics and Engineering* 418 (Jan. 2024), p. 116570. DOI: 10.1016/j.cma.2023.116570. URL: https://inria.hal.science/hal-03936627 (cit. on p. 22).

[25] M. Binois, N. Collier and J. Ozik. 'A portfolio approach to massively parallel Bayesian optimization'. In: *Journal of Artificial Intelligence Research* 82 (14th Jan. 2025), pp. 137–167. DOI: 10.1613/jair.1.16868. URL: https://inria.hal.science/hal-03383097 (cit. on p. 20).

[26] S. Chorfi, A. Habbal, M. Jahid, L. Maniar and A. Ratnani. 'Stability of backward inverse problems for degenerate mean-field game systems'. In: *arXiv* (2024). DOI: 10.48550/arXiv.2410.21541. URL: https://inria.hal.science/hal-04892449 (cit. on p. 18).

[27] I. Ciaramaglia, P. Goatin and G. Puppo. 'Non-local traffic flow models with time delay: well-posedness and numerical approximation'. In: *Discrete and Continuous Dynamical Systems - Series B* 30.3 (2025), pp. 874–907. DOI: 10.3934/dcdsb.2024113. URL: https://hal.science/hal-04547467 (cit. on p. 17).

[28] H. D. Contreras, P. Goatin and L.-M. Villada. 'A two-lane bidirectional nonlocal traffic model'. In: *Journal of Mathematical Analysis and Applications* 543.2 (Mar. 2025), p. 129027. DOI: 10.1016/j.jmaa.2024.129027. URL: https://hal.science/hal-04528991 (cit. on p. 17).

[29] C. Daini, M. L. D. Monache, P. Goatin and A. Ferrara. 'Traffic Control via Fleets of Connected and Automated Vehicles'. In: *IEEE Transactions on Intelligent Transportation Systems* (2024), p. 1. DOI: 10.1109/TITS.2024.3506703. URL: https://hal.science/hal-04366870 (cit. on p. 16).

[30] P. Goatin. 'Dissipation of Stop-and-Go Waves in Traffic Flows Using Controlled Vehicles: A Macroscopic Approach'. In: *IEEE Control Systems Letters* 8 (2024), pp. 628–633. DOI: 10.1109/LCSYS.2024.3401092. URL: https://hal.science/hal-04590482 (cit. on p. 16).

[31] P. Goatin, D. Inzunza and L. M. Villada. 'Nonlocal macroscopic models of multi-population pedestrian flows for walking facilities optimization'. In: *Applied Mathematical Modelling* (2025), p. 115927. DOI: 10.1016/j.apm.2025.115927. URL: https://hal.science/hal-04191630.

[32] P. Goatin and A. Rizzo. 'Instabilities in generic second order traffic models with relaxation'. In: *Zeitschrift für Angewandte Mathematik und Physik = Journal of Applied mathematics and physics = Journal de mathématiques et de physique appliquées* 75.5 (2024), p. 167. DOI: 10.1007/s00033-024-02307-7. URL: https://hal.science/hal-04321589 (cit. on p. 16).

[33] Z. Kharbouch, M. Bouchaara, F. Elkouihen, A. Habbal, A. Ratnani and A. Faik. 'Data refinement for enhanced ionic conductivity prediction in garnet-type solid-state electrolytes'. In: *Solid State Ionics* 417 (Dec. 2024), p. 116713. DOI: 10.1016/j.ssi.2024.116713. URL: https://inria.hal.science/hal-04892418 (cit. on p. 21).

[34] N. Rosset, R. Duvigneau, A. Bousseau and G. Cordonnier. 'Windblown sand around obstacles -simulation and validation of deposition patterns'. In: *Proceedings of the ACM on Computer Graphics and Interactive Techniques* 7.1 (2024), p. 13. DOI: 10.1145/3651284. URL: https://inria.hal.science/hal-04566241. In press (cit. on p. 19).

International peer-reviewed conferences

[35] G. Coulaud, M. Le and R. Duvigneau. 'Investigations on Physics-Informed Neural Networks for Aerodynamics'. In: 58th 3AF International Conference on Applied Aerodynamics. Orleans, France, France, Mar. 2024. URL: https://hal.science/hal-04519693 (cit. on p. 19).

[36] P. Goatin, D. Inzunza and L. M. Villada. 'Numerical comparison of nonlocal macroscopic models of multi-population pedestrian flows with anisotropic kernel'. In: *Hyperbolic Problems: Theory, Numerics, Applications. Volume II, SEMA SIMAI Springer Series.* XVI International Conference on Hyperbolic Problems: Theory, Numerics, Applications. Vol. 35. SEMA SIMAI Springer Series. Malaga, Spain: Springer Nature Switzerland, 6th June 2024, pp. 371–381. DOI: 10.1007/978-3-031-55264-9_32. URL: https://hal.science/hal-03964740 (cit. on p. 17).

[37] A. Würth, P. Goatin and L.-M. Villada. 'A cheap and easy-to-implement upwind scheme for second order traffic flow models'. In: *Hyperbolic Problems: Theory, Numerics, Applications. Volume II, SEMA SIMAI Springer Series.* XVI International Conference on Hyperbolic Problems: Theory, Numerics, Applications. Vol. 35. SEMA SIMAI Springer Series. Malaga, Spain: Springer Nature Switzerland, 6th June 2024, pp. 209–219. DOI: 10.1007/978-3-031-55264-9_18. URL: https://hal.science/hal-03957121.

Reports & preprints

[38] M. Binois, A. Fadikar and A. Stevens. *Gearing Gaussian process modeling and sequential design towards stochastic simulators.* 6th Dec. 2024. URL: https://inria.hal.science/hal-04824305 (cit. on p. 20).

[39] M. Binois and V. Picheny. *Combining additivity and active subspaces for high-dimensional Gaussian process modeling*. 2nd Feb. 2024. URL: https://hal.science/hal-04434927 (cit. on p. 20).

[40] I. Ciaramaglia, P. Goatin and G. Puppo. *A multi-class non-local macroscopic model with time delay for mixed autonomous / human-driven traffic*. 10th Jan. 2025. URL: https://hal.science/hal-04880080.

[41] R. M. Colombo, P. Goatin and E. Rossi. *A Hyperbolic -Parabolic Framework to Manage Traffic Generated Pollution*. 12th Feb. 2024. URL: https://hal.science/hal-04504572 (cit. on p. 17).

[42] J.-A. Désidéri. *Territory splitting revisited in prioritized multiobjective optimization*. RR-9568. Inria & Université Nice Sophia Antipolis, CNRS, I3S, Sophia Antipolis, France, 8th Jan. 2025, p. 25. URL: https://inria.hal.science/hal-04876427 (cit. on p. 21).

[43] J.-A. Désidéri, J. Wintz, M. Binois, N. Bartoli, C. David and S. Defoort. *Prioritized multi-objective optimization of an aircraft flight performance based on Nash games from preponderant Pareto-optimal points*. 9th Dec. 2024. URL: https://inria.hal.science/hal-04826712 (cit. on p. 21).

[44] P. Goatin and B. Piccoli. *A multi-scale multi-lane model for traffic regulation via autonomous vehicles*. 2024. URL: https://hal.science/hal-04559168 (cit. on p. 16).

[45] P. Goatin and E. Rossi. *Well-posedness of nonlocal macroscopic models of multi-population pedestrian flows for domain shape optimization*. 3rd Oct. 2024. URL: https://hal.science/hal-04719383 (cit. on p. 17).

[46] D. O'Gara, C. C. Kerr, D. J. Klein, M. Binois, R. Garnett and R. A. Hammond. *Improving Policy-Oriented Agent-Based Modeling with History Matching: A Case Study*. 31st Dec. 2024. URL: https://hal.science/hal-04868131 (cit. on p. 20).

[47] L. Palazzolo, M. Binois, L. Berti and L. Giraldi. *Parametric Shape Optimization of Flagellated Micro-Swimmers Using Bayesian Techniques*. 2024. URL: https://hal.science/hal-04699705 (cit. on p. 20).

12.3 Cited publications

[48] R. Abgrall and P. M. Congedo. 'A semi-intrusive deterministic approach to uncertainty quantification in non-linear fluid flow problems'. In: *J. Comput. Physics* (2012) (cit. on p. 7).

[49] A. Aggarwal, R. M. Colombo and P. Goatin. 'Nonlocal systems of conservation laws in several space dimensions'. In: *SIAM Journal on Numerical Analysis* 52.2 (2015), pp. 963–983. URL: https://hal.inria.fr/hal-01016784 (cit. on p. 7).

[50] G. Alessandrini. 'Examples of instability in inverse boundary-value problems'. In: *Inverse Problems* 13.4 (1997), pp. 887–897. DOI: 10.1088/0266-5611/13/4/001. URL: http://dx.doi.org/10.1088/0266-5611/13/4/001 (cit. on p. 9).

[51] D. Amadori and W. Shen. 'An integro-differential conservation law arising in a model of granular flow'. In: *J. Hyperbolic Differ. Equ.* 9.1 (2012), pp. 105–131 (cit. on p. 6).

[52] P. Amorim, R. M. Colombo and A. Teixeira. 'On the Numerical Integration of Scalar Nonlocal Conservation Laws'. In: *ESAIM M2AN* 49.1 (2015), pp. 19–37 (cit. on p. 7).

[53] P. Amorim. 'On a nonlocal hyperbolic conservation law arising from a gradient constraint problem'. In: *Bull. Braz. Math. Soc. (N.S.)* 43.4 (2012), pp. 599–614 (cit. on p. 6).

[54] M. Annunziato and A. Borzì. 'A Fokker-Planck control framework for multidimensional stochastic processes'. In: *Journal of Computational and Applied Mathematics* 237 (2013), pp. 487–507 (cit. on p. 6).

[55] A. Belme, F. Alauzet and A. Dervieux. 'Time accurate anisotropic goal-oriented mesh adaptation for unsteady flows'. In: *J. Comput. Physics* 231.19 (2012), pp. 6323–6348 (cit. on p. 8).

[56] S. Benzoni-Gavage, R. M. Colombo and P. Gwiazda. 'Measure valued solutions to conservation laws motivated by traffic modelling'. In: *Proc. R. Soc. Lond. Ser. A Math. Phys. Eng. Sci.* 462.2070 (2006), pp. 1791–1803 (cit. on p. 6).

[57] E. Bertino, R. Duvigneau and P. Goatin. 'Uncertainties in traffic flow and model validation on GPS data'. 2015 (cit. on p. 7).

[58] F. Betancourt, R. Bürger, K. H. Karlsen and E. M. Tory. 'On nonlocal conservation laws modelling sedimentation'. In: *Nonlinearity* 24.3 (2011), pp. 855–885 (cit. on p. 6).

[59] S. Blandin and P. Goatin. 'Well-posedness of a conservation law with non-local flux arising in traffic flow modeling'. In: *Numer. Math.* 132.2 (2016), pp. 217–241. URL: https://doi.org/10.1007/s00211-015-0717-6 (cit. on p. 7).

[60] J. Borggaard and J. Burns. 'A {PDE} Sensitivity Equation Method for Optimal Aerodynamic Design'. In: *Journal of Computational Physics* 136.2 (1997), pp. 366–384. DOI: http://dx.doi.org/10.1006/jcph.1997.5743. URL: http://www.sciencedirect.com/science/article/pii/S0021999197957430 (cit. on p. 7).

[61] R. Bourguet, M. Brazza, G. Harran and R. El Akoury. 'Anisotropic Organised Eddy Simulation for the prediction of non-equilibrium turbulent flows around bodies'. In: *J. of Fluids and Structures* 24.8 (2008), pp. 1240–1251 (cit. on p. 8).

[62] A. Bressan, S. Čanić, M. Garavello, M. Herty and B. Piccoli. 'Flows on networks: recent results and perspectives'. In: *EMS Surv. Math. Sci.* 1.1 (2014), pp. 47–111 (cit. on p. 10).

[63] M. Burger, J. Haskovec and M.-T. Wolfram. 'Individual based and mean-field modelling of direct aggregation'. In: *Physica D* 260 (2013), pp. 145–158 (cit. on p. 6).

[64] M. Burger, M. Di Francesco, P. A. Markowich and M.-T. Wolfram. 'Mean field games with nonlinear mobilities in pedestrian dynamics'. In: *Discrete Contin. Dyn. Syst. Ser. B* 19.5 (2014), pp. 1311–1333 (cit. on p. 6).

[65] A. Cabassi and P. Goatin. *Validation of traffic flow models on processed GPS data*. Tech. rep. Research Report RR-8382. 2013. URL: https://hal.inria.fr/hal-00876311 (cit. on p. 7).

[66] J. A. Carrillo, S. Martin and M.-T. Wolfram. 'A local version of the Hughes model for pedestrian flow'. Preprint. 2015 (cit. on p. 7).

[67] C. Chalons, M. L. Delle Monache and P. Goatin. 'A conservative scheme for non-classical solutions to a strongly coupled PDE-ODE problem'. Preprint. 2015 (cit. on p. 5).

[68] C. Claudel and A. M. Bayen. 'Lax-Hopf Based Incorporation of Internal Boundary Conditions Into Hamilton-Jacobi Equation. Part II: Computational Methods'. In: *Automatic Control, IEEE Transactions on* 55.5 (May 2010), pp. 1158–1174 (cit. on pp. 10, 11).

[69] C. G. Claudel and A. M. Bayen. 'Convex formulations of data assimilation problems for a class of Hamilton-Jacobi equations'. In: *SIAM J. Control Optim.* 49.2 (2011), pp. 383–402 (cit. on pp. 10, 11).

[70] R. M. Colombo, M. Garavello and M. Lécureux-Mercier. 'A Class Of Nonloval Models For Pedestrian Traffic'. In: *Mathematical Models and Methods in Applied Sciences* 22.04 (2012), p. 1150023 (cit. on p. 7).

[71] R. M. Colombo, M. Herty and M. Mercier. 'Control of the continuity equation with a non local flow'. In: *ESAIM Control Optim. Calc. Var.* 17.2 (2011), pp. 353–379 (cit. on p. 7).

[72] R. M. Colombo and M. Lécureux-Mercier. 'Nonlocal crowd dynamics models for several populations'. In: *Acta Math. Sci. Ser. B Engl. Ed.* 32.1 (2012), pp. 177–196 (cit. on p. 7).

[73] R. M. Colombo and F. Marcellini. 'A mixed ODE-PDE model for vehicular traffic'. In: *Mathematical Methods in the Applied Sciences* 38.7 (2015), pp. 1292–1302 (cit. on p. 5).

[74] R. M. Colombo and E. Rossi. 'On the micro-macro limit in traffic flow'. In: *Rend. Semin. Mat. Univ. Padova* 131 (2014), pp. 217–235 (cit. on p. 6).

[75] G. Costeseque and J.-P. Lebacque. 'Discussion about traffic junction modelling: conservation laws vs Hamilton-Jacobi equations'. In: *Discrete Contin. Dyn. Syst. Ser. S* 7.3 (2014), pp. 411–433 (cit. on p. 10).

[76] G. Crippa and M. Lécureux-Mercier. 'Existence and uniqueness of measure solutions for a system of continuity equations with non-local flow'. In: *Nonlinear Differential Equations and Applications NoDEA* (2012), pp. 1–15 (cit. on p. 7).

[77] E. Cristiani, B. Piccoli and A. Tosin. 'How can macroscopic models reveal self-organization in traffic flow?' In: *Decision and Control (CDC), 2012 IEEE 51st Annual Conference on.* Dec. 2012, pp. 6989–6994 (cit. on p. 6).

[78] E. Cristiani, B. Piccoli and A. Tosin. *Multiscale modeling of pedestrian dynamics.* Vol. 12. MS&A. Modeling, Simulation and Applications. Springer, Cham, 2014 (cit. on p. 6).

[79] C. M. Dafermos. 'Solutions in L^∞ for a conservation law with memory'. In: *Analyse mathématique et applications.* Montrouge: Gauthier-Villars, 1988, pp. 117–128 (cit. on p. 7).

[80] P. Degond, J.-G. Liu and C. Ringhofer. 'Large-scale dynamics of mean-field games driven by local Nash equilibria'. In: *J. Nonlinear Sci.* 24.1 (2014), pp. 93–115. DOI: 10.1007/s00332-013-9185-2. URL: http://dx.doi.org/10.1007/s00332-013-9185-2 (cit. on p. 6).

[81] M. L. Delle Monache and P. Goatin. 'Scalar conservation laws with moving constraints arising in traffic flow modeling: an existence result'. In: *J. Differential Equations* 257.11 (2014), pp. 4015–4029 (cit. on pp. 5, 11).

[82] M. L. Delle Monache and P. Goatin. 'A front tracking method for a strongly coupled PDE-ODE system with moving density constraints in traffic flow'. In: *Discrete Contin. Dyn. Syst. Ser. S* 7.3 (2014), pp. 435–447 (cit. on p. 5).

[83] J.-A. Désidéri. 'Multiple-Gradient Descent Algorithm (*MGDA*) for Pareto-Front Identification'. In: *Numerical Methods for Differential Equations, Optimization, and Technological Problems.* Vol. 34. Modeling, Simulation and Optimization for Science and Technology, Fitzgibbon, W.; Kuznetsov, Y.A.; Neittaanmäki, P.; Pironneau, O. Eds. J. Périaux and R. Glowinski Jubilees. Springer-Verlag, 2014. Chap. 1 (cit. on p. 8).

[84] J.-A. Désidéri. 'Multiple-gradient descent algorithm (MGDA) for multiobjective optimization'. In: *Comptes Rendus de l'Académie des Sciences Paris.* I 350 (2012), pp. 313–318. URL: http://dx.doi.org/10.1016/j.crma.2012.03.014 (cit. on p. 8).

[85] J.-A. Désidéri. *Révision de l'algorithme de descente à gradients multiples (MGDA) par orthogonalisation hiérarchique.* Research Report 8710. INRIA, Apr. 2015 (cit. on p. 9).

[86] B. Després, G. Poëtte and D. Lucor. 'Robust uncertainty propagation in systems of conservation laws with the entropy closure method'. In: *Uncertainty quantification in computational fluid dynamics.* Vol. 92. Lect. Notes Comput. Sci. Eng. Springer, Heidelberg, 2013, pp. 105–149 (cit. on p. 7).

[87] M. Di Francesco and M. D. Rosini. 'Rigorous Derivation of Nonlinear Scalar Conservation Laws from Follow-the-Leader Type Models via Many Particle Limit'. In: *Archive for Rational Mechanics and Analysis* (2015) (cit. on p. 6).

[88] R. J. DiPerna. 'Measure-valued solutions to conservation laws'. In: *Arch. Rational Mech. Anal.* 8,8.3 (1985), pp. 223–270 (cit. on p. 6).

[89] C. Dogbé. 'Modeling crowd dynamics by the mean-field limit approach'. In: *Math. Comput. Modelling* 52.9-10 (2010), pp. 1506–1520 (cit. on p. 6).

[90] R. Duvigneau. *A Sensitivity Equation Method for Unsteady Compressible Flows: Implementation and Verification.* Tech. rep. INRIA Research Report No 8739, June 2015 (cit. on p. 8).

[91] R. Duvigneau and D. Pelletier. 'A sensitivity equation method for fast evaluation of nearby flows and uncertainty analysis for shape parameters'. In: *Int. J. of Computational Fluid Dynamics* 20.7 (Aug. 2006), pp. 497–512 (cit. on pp. 7, 8).

[92] R. Erban, M. B. Flegg and G. A. Papoian. 'Multiscale stochastic reaction-diffusion modeling: application to actin dynamics in filopodia'. In: *Bull. Math. Biol.* 76.4 (2014), pp. 799–818. DOI: 10.1007/s11538-013-9844-3. URL: http://dx.doi.org/10.1007/s11538-013-9844-3 (cit. on p. 5).

[93] R. Etikyala, S. Göttlich, A. Klar and S. Tiwari. 'Particle methods for pedestrian flow models: from microscopic to nonlocal continuum models'. In: *Math. Models Methods Appl. Sci.* 24.12 (2014), pp. 2503–2523 (cit. on p. 6).

[94] R. Eymard, T. Gallouët and R. Herbin. 'Finite volume methods'. In: *Handbook of numerical analysis, Vol. VII.* Handb. Numer. Anal., VII. North-Holland, Amsterdam, 2000, pp. 713–1020 (cit. on p. 6).

[95] U. Fjordholm, R. Kappeli, S. Mishra and E. Tadmor. *Construction of approximate entropy measure valued solutions for systems of conservation laws.* Tech. rep. 2014-33. Seminar for Applied Mathematics, ETH Zürich, 2014 (cit. on p. 6).

[96] M. B. Flegg, S. Hellander and R. Erban. 'Convergence of methods for coupling of microscopic and mesoscopic reaction-diffusion simulations'. In: *J. Comput. Phys.* 289 (2015), pp. 1–17. DOI: 10.1016/j.jcp.2015.01.030. URL: http://dx.doi.org/10.1016/j.jcp.2015.01.030 (cit. on p. 5).

[97] F. Fleuret and D. Geman. 'Graded learning for object detection'. In: *Proceedings of the workshop on Statistical and Computational Theories of Vision of the IEEE international conference on Computer Vision and Pattern Recognition (CVPR/SCTV).* Vol. 2. 1999 (cit. on p. 9).

[98] B. Franz, M. B. Flegg, S. J. Chapman and R. Erban. 'Multiscale reaction-diffusion algorithms: PDE-assisted Brownian dynamics'. In: *SIAM J. Appl. Math.* 73.3 (2013), pp. 1224–1247 (cit. on p. 5).

[99] M. Garavello and B. Piccoli. 'Coupling of microscopic and phase transition models at boundary'. In: *Netw. Heterog. Media* 8.3 (2013), pp. 649–661 (cit. on pp. 5, 11).

[100] M. Garavello and B. Piccoli. *Traffic flow on networks.* Vol. 1. AIMS Series on Applied Mathematics. Conservation laws models. American Institute of Mathematical Sciences (AIMS), Springfield, MO, 2006 (cit. on p. 10).

[101] P. Goatin and M. Mimault. 'A mixed system modeling two-directional pedestrian flows'. In: *Math. Biosci. Eng.* 12.2 (2015), pp. 375–392 (cit. on p. 6).

[102] P. Goatin and F. Rossi. 'A traffic flow model with non-smooth metric interaction: well-posedness and micro-macro limit'. Preprint. 2015. URL: http://arxiv.org/abs/1510.04461 (cit. on p. 6).

[103] P. Goatin and S. Scialanga. 'Well-posedness and finite volume approximations of the LWR traffic flow model with non-local velocity'. In: *Netw. Heterog. Media* 11.1 (2016), pp. 107–121 (cit. on p. 7).

[104] S. Göttlich, S. Hoher, P. Schindler, V. Schleper and A. Verl. 'Modeling, simulation and validation of material flow on conveyor belts'. In: *Applied Mathematical Modelling* 38.13 (2014), pp. 3295–3313 (cit. on p. 6).

[105] A. Griewank. 'Achieving logarithmic growth of temporal and spatial complexity in reverse automatic differentiation'. In: *Optimization Methods and Software* 1 (1992), pp. 35–54 (cit. on p. 8).

[106] M. Gröschel, A. Keimer, G. Leugering and Z. Wang. 'Regularity theory and adjoint-based optimality conditions for a nonlinear transport equation with nonlocal velocity'. In: *SIAM J. Control Optim.* 52.4 (2014), pp. 2141–2163 (cit. on p. 6).

[107] A. Habbal and M. Kallel. 'Neumann-Dirichlet Nash strategies for the solution of elliptic Cauchy problems'. In: *SIAM J. Control Optim.* 51.5 (2013), pp. 4066–4083 (cit. on p. 12).

[108] A. Habbal, I. Kissami, A. Machtalay and A. Ratnani. 'Computational investigations of a two-class traffic flow model : mean-field and microscopic dynamics'. working paper or preprint. Dec. 2021. URL: https://inria.hal.science/hal-03495749 (cit. on p. 18).

[109] X. Han, P. Sagaut and D. Lucor. 'On sensitivity of RANS simulations to uncertain turbulent inflow conditions'. In: *Computers & Fluids* 61.2-5 (2012) (cit. on p. 7).

[110] D. Helbing. 'Traffic and related self-driven many-particle systems'. In: *Rev. Mod. Phys.* 73 (4 2001), pp. 1067–1141 (cit. on p. 10).

[111] D. Helbing, P. Molnar, I. J. Farkas and K. Bolay. 'Self-organizing pedestrian movement'. In: *Environment and planning B* 28.3 (2001), pp. 361–384 (cit. on p. 5).

[112] J. C. Herrera, D. B. Work, R. Herring, X. J. Ban, Q. Jacobson and A. M. Bayen. 'Evaluation of traffic data obtained via GPS-enabled mobile phones: The Mobile Century field experiment'. In: *Transportation Research Part C: Emerging Technologies* 18.4 (2010), pp. 568–583 (cit. on p. 10).

[113] S. P. Hoogendoorn, F. L. van Wageningen-Kessels, W. Daamen and D. C. Duives. 'Continuum modelling of pedestrian flows: From microscopic principles to self-organised macroscopic phenomena'. In: *Physica A: Statistical Mechanics and its Applications* 416.0 (2014), pp. 684–694 (cit. on p. 6).

[114] H. Hristova, S. Etienne, D. Pelletier and J. Borggaard. 'A continuous sensitivity equation method for time-dependent incompressible laminar flows'. In: *Int. J. for Numerical Methods in Fluids* 50 (2004), pp. 817–844 (cit. on p. 8).

[115] T. Hughes, J. Cottrell and Y. Bazilevs. 'Isogeometric analysis: CAD, finite elements, NURBS, exact geometry, and mesh refinement'. In: *Computer Methods in Applied Mechanics and Engineering* 194 (2005), pp. 4135–4195 (cit. on p. 8).

[116] C. Imbert and R. Monneau. 'Flux-limited solutions for quasi-convex Hamilton–Jacobi equations on networks'. In: *arXiv preprint arXiv:1306.2428* (Oct. 2014) (cit. on p. 10).

[117] S. Jeon and H. Choi. 'Suboptimal feedback control of flow over a sphere'. In: *Int. J. of Heat and Fluid Flow* 31 (2010) (cit. on p. 8).

[118] M. Kallel, R. Aboulaich, A. Habbal and M. Moakher. 'A Nash-game approach to joint image restoration and segmentation'. In: *Appl. Math. Model.* 38.11-12 (2014), pp. 3038–3053. DOI: 10.1016/j.apm.2013.11.034. URL: http://dx.doi.org/10.1016/j.apm.2013.11.034 (cit. on p. 12).

[119] O. Knio and O. Le Maitre. 'Uncertainty propagation in CFD using polynomial chaos decomposition'. In: *Fluid Dynamics Research* 38.9 (Sept. 2006), pp. 616–640 (cit. on p. 7).

[120] A. Kurganov and A. Polizzi. 'Non-Oscillatory Central Schemes for a Traffic Flow Model with Arrehenius Look-Ahead Dynamics'. In: *Netw. Heterog. Media* 4.3 (2009), pp. 431–451 (cit. on p. 7).

[121] A. Lachapelle and M.-T. Wolfram. 'On a mean field game approach modeling congestion and aversion in pedestrian crowds'. In: *Transportation Research Part B: Methodological* 45.10 (2011), pp. 1572–1589 (cit. on p. 6).

[122] J.-M. Lasry and P.-L. Lions. 'Mean field games'. In: *Jpn. J. Math.* 2.1 (2007), pp. 229–260 (cit. on p. 6).

[123] M. J. Lighthill and G. B. Whitham. 'On kinematic waves. II. A theory of traffic flow on long crowded roads'. In: *Proc. Roy. Soc. London. Ser. A.* 229 (1955), pp. 317–345 (cit. on p. 6).

[124] G. Lin, C.-H. Su and G. Karniadakis. 'Predicting shock dynamics in the presence of uncertainties'. In: *Journal of Computational Physics* 217 (2006), pp. 260–276 (cit. on p. 7).

[125] M. Martinelli and R. Duvigneau. 'On the use of second-order derivative and metamodel-based Monte-Carlo for uncertainty estimation in aerodynamics'. In: *Computers and Fluids* 37.6 (2010) (cit. on p. 7).

[126] C. Merritt, F. Forsberg, J. Liu and F. Kallel. 'In-vivo elastography in animal models: Feasibility studies, (abstract)'. In: *J. Ultrasound Med.* 21.98 (2002) (cit. on p. 11).

[127] S. Mishra, C. Schwab and J. Sukys. 'Multi-level Monte Carlo finite volume methods for uncertainty quantification in nonlinear systems of balance laws'. In: *Lecture Notes in Computational Science and Engineering* 92 (2013), pp. 225–294 (cit. on p. 7).

[128] W. Oberkampf and F. Blottner. 'Issues in Computational Fluid Dynamics code verification and validation'. In: *AIAA Journal* 36 (1998), pp. 687–695 (cit. on p. 7).

[129] B. Perthame. *Transport equations in biology.* Frontiers in Mathematics. Birkhäuser Verlag, Basel, 2007 (cit. on p. 6).

[130] B. Piccoli and F. Rossi. 'Transport equation with nonlocal velocity in Wasserstein spaces: convergence of numerical schemes'. In: *Acta Appl. Math.* 124 (2013), pp. 73–105 (cit. on p. 6).

[131] F. Poirion. *Stochastic Multi Gradient Descent Algorithm*. RT 1/22295 DADS. ONERA, July 2014 (cit. on p. 9).

[132] F. S. Priuli. 'First order mean field games in crowd dynamics'. In: *ArXiv e-prints* (Feb. 2014). arXiv: 1402.7296 [math.AP] (cit. on p. 6).

[133] M. Putko, P. Newman, A. Taylor and L. Green. 'Approach for uncertainty propagation and robust design in CFD using sensitivity derivatives'. In: *15th AIAA Computational Fluid Dynamics Conference*. AIAA Paper 2001-2528. Anaheim, CA, June 2001 (cit. on p. 7).

[134] C. Qi, K. Gallivan and P.-A. Absil. 'Riemannian BFGS Algorithm with Applications'. English. In: *Recent Advances in Optimization and its Applications in Engineering*. Ed. by M. Diehl, F. Glineur, E. Jarlebring and W. Michiels. Springer Berlin Heidelberg, 2010, pp. 183–192. DOI: 10.1007/978-3-642-12598-0_16. URL: http://dx.doi.org/10.1007/978-3-642-12598-0%5C_16 (cit. on p. 12).

[135] J. Reilly, W. Krichene, M. L. Delle Monache, S. Samaranayake, P. Goatin and A. M. Bayen. 'Adjoint-based optimization on a network of discretized scalar conservation law PDEs with applications to coordinated ramp metering'. In: *J. Optim. Theory Appl.* 167.2 (2015), pp. 733–760 (cit. on p. 8).

[136] P. I. Richards. 'Shock waves on the highway'. In: *Operations Res.* 4 (1956), pp. 42–51 (cit. on p. 6).

[137] P. Sagaut. *Large Eddy Simulation for Incompressible Flows An Introduction*. Springer Berlin Heidelberg, 2006 (cit. on pp. 5, 8).

[138] J. Schaefer, T. West, S. Hosder, C. Rumsey, J.-R. Carlson and W. Kleb. 'Uncertainty Quantification of Turbulence Model Closure Coefficients for Transonic Wall-Bounded Flows'. In: *22nd AIAA Computational Fluid Dynamics Conference, 22-26 June 2015, Dallas, USA*. 2015 (cit. on p. 7).

[139] V. Schleper. 'A hybrid model for traffic flow and crowd dynamics with random individual properties'. In: *Math. Biosci. Eng.* 12.2 (2015), pp. 393–413 (cit. on p. 7).

[140] A. Sopasakis and M. A. Katsoulakis. 'Stochastic modeling and simulation of traffic flow: asymmetric single exclusion process with Arrhenius look-ahead dynamics'. In: *SIAM J. Appl. Math.* 66.3 (2006), pp. 921–944 (cit. on p. 7).

[141] P. R. Spalart. 'Detached-Eddy Simulation'. In: *Annual Review of Fluid Mechanics* 41 (2009), pp. 181–202 (cit. on p. 8).

[142] S. Tokareva, S. Mishra and C. Schwab. 'High Order Stochastic Finite Volume Method for the Uncertainty Quantification in Hyperbolic Conservtion Laws with Random Initial Data and Flux Coefficients'. In: *Proc. ECCOMAS*. Proc. ECCOMAS. 2012 (cit. on p. 7).

[143] É. Turgeon, D. Pelletier and J. Borggaard. 'Sensitivity and Uncertainty Analysis for Variable Property Flows'. In: *39th AIAA Aerospace Sciences Meeting and Exhibit*. AIAA Paper 2001-0139. Reno, NV, Jan. 2001 (cit. on p. 7).

[144] C. Villani. *Optimal transport*. Vol. 338. Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences]. Old and new. Springer-Verlag, Berlin, 2009 (cit. on p. 6).

[145] C. Villani. *Topics in optimal transportation*. Vol. 58. Graduate Studies in Mathematics. American Mathematical Society, Providence, RI, 2003 (cit. on p. 6).

[146] R. Walter and L. Huyse. *Uncertainty analysis for fluid mechanics with applications*. Tech. rep. 2002–1. ICASE, Feb. 2002 (cit. on p. 7).

[147] D. Xiu and G. Karniadakis. 'Modeling uncertainty in flow simulations via generalized Polynomial Chaos'. In: *Journal of Computational Physics* 187 (2003), pp. 137–167 (cit. on p. 7).

[148] D. You and P. Moin. 'Active control of flow separation over an airfoil using synthetic jets'. In: *J. of Fluids and Structures* 24 (2008), pp. 1349–1357 (cit. on p. 10).

[149] A. Zerbinati, A. Minelli, I. Ghazlane and J.-A. Désidéri. 'Meta-Model-Assisted MGDA for Multi-Objective Functional Optimization'. In: *Computers and Fluids* 102 (2014). http://www.sciencedirect.com/science/article/pii/S0045793014002576#, pp. 116–130 (cit. on p. 9).