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2024
ACTIVITY REPORT

Project-Team

AROMATH

AlgebRa, geOmetry, Modeling and AlgoriTHms

DOMAIN

Algorithmics, Programming, Software and Architecture

THEME

Algorithmics, Computer Algebra and Cryptology

Contents

Project-Team AROMATH

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Keywords

Computer sciences and digital sciences

A5.5.1. – Geometrical modeling

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Other research topics and application domains

B9.5.1. – Computer science

B9.5.2. – Mathematics

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2 Overall objectives

Our daily life environment is increasingly interacting with digital information. An important amount of this information is of geometric nature. It concerns the representation of our environment, the analysis and understanding of “real” phenomena, the control of physical mechanisms or processes. The interaction between physical and digital worlds is two-way. Sensors are producing digital data related to measurements or observations of our environment. Digital models are also used to “act” on the physical world. Objects that we use at home, at work, to travel, such as furniture, cars, planes, ... are nowadays produced by industrial processes which are based on digital representation of shapes. CAD-CAM (Computer Aided Design – Computer Aided Manufacturing) software is used to represent the geometry of these objects and to control the manufacturing processes which create them. The construction capabilities themselves are also expanding, with the development of 3D printers and the possibility to create daily-life objects “at home” from digital models.

The impact of geometry is also important in the analysis and understanding of phenomena. The 3D conformation of a molecule explains its biological interaction with other molecules. The profile of a wing determines its aeronautic behavior, while the shape of a bulbous bow can decrease significantly the wave resistance of a ship. Understanding such a behavior or analyzing a physical phenomenon can nowadays be achieved for many problems by numerical simulation. The precise representation of the geometry and the link between the geometric models and the numerical computation tools are closely related to the quality of these simulations. This also plays an important role in optimization loops where the numerical simulation results are used to improve the “performance” of a model.

Geometry deals with structured and efficient representations of information and with methods to treat it. Its impact in animation, games and VAMR (Virtual, Augmented and Mixed Reality) is important. It also has a growing influence in e-trade where a consumer can evaluate, test and buy a product from its digital description. Geometric data produced for instance by 3D scanners and reconstructed models are nowadays used to memorize old works in cultural or industrial domains.

Geometry is involved in many domains (manufacturing, simulation, communication, virtual world...), raising many challenging questions related to the representations of shapes, to the analysis of their properties and to the computation with these models. The stakes are multiple: the accuracy in numerical engineering, in simulation, in optimization, the quality in design and manufacturing processes, the capacity of modeling and analysis of physical problems.

3 Research program

3.1 High order geometric modeling

The accurate description of shapes is a long standing problem in mathematics, with an important impact in many domains, inducing strong interactions between geometry and computation. Developing precise geometric modeling techniques is a critical issue in CAD-CAM. Constructing accurate models, that can be exploited in geometric applications, from digital data produced by cameras, laser scanners, observations or simulations is also a major issue in geometry processing. A main challenge is to construct models that can capture the geometry of complex shapes, using few parameters while being precise.

Our first objective is to develop methods, which are able to describe accurately and in an efficient way, objects or phenomena of geometric nature, using algebraic representations.

The approach followed in Computer Aided Geometric Design (CAGD) to describe complex geometry is based on parametric representations called NURBS (Non Uniform Rational B-Spline). The models are constructed by trimming and gluing together high order patches of algebraic surfaces. These models are built from the so-called B-Spline functions that encode a piecewise algebraic function with a prescribed regularity at knots. Although these models have many advantages and have become the standard for designing nowadays CAD models, they also have important drawbacks. Among them, the difficulty to locally refine a NURBS surface and also the topological rigidity of NURBS patches that imposes to use many such patches with trims for designing complex models, with the consequence of the appearing of cracks at the seams. To overcome these difficulties, an active area of research is to look for new blending functions for the representation of CAD models. Some examples are the so-called T-Splines, LR-Spline blending functions, or hierarchical splines, that have been recently devised in order to perform efficiently local refinement. An important problem is to analyze spline spaces associated to general subdivisions, which is of particular interest in higher order Finite Element Methods. Another challenge in geometric modeling is the efficient representation and/or reconstruction of complex objects, and the description of computational domains in numerical simulation. To construct models that can represent efficiently the geometry of complex shapes, we are interested in developing modeling methods, based on alternative constructions such as skeleton-based representations. The change of representation, in particular between parametric and implicit representations, is of particular interest in geometric computations and in its applications in CAGD.

We also plan to investigate adaptive hierarchical techniques, which can locally improve the approximation of a shape or a function. They shall be exploited to transform digital data produced by cameras, laser scanners, observations or simulations into accurate and structured algebraic models.

The precise and efficient representation of shapes also leads to the problem of extracting and exploiting characteristic properties of shapes such as symmetry, which is very frequent in geometry. Reflecting the symmetry of the intended shape in the representation appears as a natural requirement for visual quality, but also as a possible source of sparsity of the representation. Recognizing, encoding and exploiting symmetry requires new paradigms of representation and further algebraic developments. Algebraic foundations for the exploitation of symmetry in the context of non linear differential and polynomial equations are addressed. The intent is to bring this expertise with symmetry to the geometric models and computations developed by AROMATH.

3.2 Robust algebraic-geometric computation

In many problems, digital data are approximated and cannot just be used as if they were exact. In the context of geometric modeling, polynomial equations appear naturally as a way to describe constraints between the unknown variables of a problem. *An important challenge is to take into account the input error in order to develop robust methods for solving these algebraic constraints.* Robustness means that a small perturbation of the input should produce a controlled variation of the output, that is forward stability, when the input-output map is regular. In non-regular cases, robustness also means that the output is an exact solution, or the most coherent solution, of a problem with input data in a given neighborhood, that is backward stability.

Our second long term objective is to develop methods to robustly and efficiently solve algebraic problems that occur in geometric modeling.

Robustness is a major issue in geometric modeling and algebraic computation. Classical methods in computer algebra, based on the paradigm of exact computation, cannot be applied directly in this context. They are not designed for stability against input perturbations. New investigations are needed to develop methods which integrate this additional dimension of the problem. Several approaches are investigated to tackle these difficulties.

One relies on linearization of algebraic problems based on “elimination of variables” or projection into a space of smaller dimension. Resultant theory provides a strong foundation for these methods, connecting the geometric properties of the solutions with explicit linear algebra on polynomial vector spaces, for families of polynomial systems (e.g., homogeneous, multi-homogeneous, sparse). Important progress has been made in the last two decades to extend this theory to new families of problems with specific geometric properties. Additional advances have been achieved more recently to exploit the syzygies between the input equations. This approach provides matrix based representations, which are particularly powerful for approximate geometric computation on parametrized curves and surfaces. They are tuned to certain classes of problems and an important issue is to detect and analyze degeneracies and to adapt them to these cases.

A more adaptive approach involves linear algebra computation in a hierarchy of polynomial vector spaces. It produces a description of quotient algebra structures, from which the solutions of polynomial systems can be recovered. This family of methods includes Gröbner Basis, which provides general tools for solving polynomial equations. Border Basis is an alternative approach, offering numerically stable methods for solving polynomial equations with approximate coefficients. An important issue is to understand and control the numerical behavior of these methods as well as their complexity and to exploit the structure of the input system.

In order to compute "only" the (real) solutions of a polynomial system in a given domain, duality techniques can also be employed. They consist in analyzing and adding constraints on the space of linear forms which vanish on the polynomial equations. Combined with semi-definite programming techniques, they provide efficient methods to compute the real solutions of algebraic equations or to solve polynomial optimization problems. The main issues are the completness of the approach, their scalability with the degree and dimension and the certification of bounds.

Singular solutions of polynomial systems can be analyzed by computing differentials, which vanish at these points. This leads to efficient deflation techniques, which transform a singular solution of a given problem into a regular solution of the transformed problem. These local methods need to be combined with more global root localisation methods.

Subdivision methods are another type of methods which are interesting for robust geometric computation. They are based on exclusion tests which certify that no solution exists in a domain and inclusion tests, which certify the uniqueness of a solution in a domain. They have shown their strength in addressing many algebraic problems, such as isolating real roots of polynomial equations or computing the topology of algebraic curves and surfaces. The main issues in these approaches is to deal with singularities and degenerate solutions.

4 Application domains

4.1 Geometric modeling for Design and Manufacturing.

The main domain of applications that we consider for the methods we develop is Computer Aided Design and Manufacturing.

Computer-Aided Design (CAD) involves creating digital models defined by mathematical constructions, from geometric, functional or aesthetic considerations. Computer-aided manufacturing (CAM) uses the geometrical design data to control the tools and processes, which lead to the production of real objects from their numerical descriptions.

CAD-CAM systems provide tools for visualizing, understanding, manipulating, and editing virtual shapes. They are extensively used in many applications, including automotive, shipbuilding, aerospace industries, industrial and architectural design, prosthetics, and many more. They are also widely used to produce computer animation for special effects in movies, advertising and technical manuals, or for digital content creation. Their economic importance is enormous. Their importance in education is also growing, as they are more and more used in schools and educational purposes.

CAD-CAM has been a major driving force for research developments in geometric modeling, which leads to very large software, produced and sold by big companies, capable of assisting engineers in all the steps from design to manufacturing.

Nevertheless, many challenges still need to be addressed. Many problems remain open, related to the use of efficient shape representations, of geometric models specific to some application domains, such as in architecture, naval engineering, mechanical constructions, manufacturing ... Important questions on the robustness and the certification of geometric computation are not yet answered. The complexity of the models which are used nowadays also appeals for the development of new approaches. The manufacturing environment is also increasingly complex, with new type of machine tools including: turning, 5-axes machining and wire EDM (Electrical Discharge Machining), 3D printer. It cannot be properly used without computer assistance, which raises methodological and algorithmic questions. There is an increasing need to combine design and simulation, for analyzing the physical behavior of a model and for optimal design.

The field has deeply changed over the last decades, with the emergence of new geometric modeling tools built on dedicated packages, which are mixing different scientific areas to address specific applications. It is providing new opportunities to apply new geometric modeling methods, output from research activities.

4.2 Geometric modeling for Numerical Simulation and Optimization

A major bottleneck in the CAD-CAM developments is the lack of interoperability of modeling systems and simulation systems. This is strongly influenced by their development history, as they have been following different paths.

The geometric tools have evolved from supporting a limited number of tasks at separate stages in product development and manufacturing, to being essential in all phases from initial design through manufacturing.

Current Finite Element Analysis (FEA) technology was already well established 40 years ago, when CAD-systems just started to appear, and its success stems from using approximations of both the geometry and the analysis model with low order finite elements (most often of degree ≤ 2).

There has been no requirement between CAD and numerical simulation, based on Finite Element Analysis, leading to incompatible mathematical representations in CAD and FEA. This incompatibility makes interoperability of CAD/CAM and FEA very challenging. In the general case today, this challenge is addressed by expensive and time-consuming human intervention and software developments.

Improving this interaction by using adequate geometric and functional descriptions should boost the interaction between numerical analysis and geometric modeling, with important implications in shape optimization. In particular, it could provide a better feedback of numerical simulations on the geometric model in a design optimization loop, which incorporates iterative analysis steps.

The situation is evolving. In the past decade, a new paradigm has emerged to replace the traditional Finite Elements by B-Spline basis element of any polynomial degree, thus in principle enabling exact representation of all shapes that can be modeled in CAD. It has been demonstrated that the so-called isogeometric analysis approach can be far more accurate than traditional FEA.

It opens new perspectives for the interoperability between geometric modeling and numerical simulation. The development of numerical methods of high order using a precise description of the shapes raises questions on piecewise polynomial elements, on the description of computational domains and of their interfaces, on the construction of good function spaces to approximate physical solutions. All these problems involve geometric considerations and are closely related to the theory of splines and to the geometric methods we are investigating. We plan to apply our work to the development of new interactions between geometric modeling and numerical solvers.

5 Highlights of the year

A workshop to celebrate Bernard Mourrains' 60 birthday and his contributions to computer algebra, effective algebraic geometry and geometric modeling was organized on August 3rd, 2024, at University of Leipzig, Germany, as a satellite event of the MEGA 2024 conference. It gathered over 50 participants and was very successfull.

6 New software, platforms, open data

6.1 New software

6.1.1 G+Smo

Name: Geometry plus Simulation Modules

Keyword: Isogeometric analysis

Functional Description: G+Smo (pronounced gismo or gizmo) is a C++ library for isogeometric analysis (IGA).

G+Smo (Geometry + Simulation Modules, pronounced "gismo") is an open-source C++ library that brings together mathematical tools for geometric design and numerical simulation. It implements the relatively new paradigm of isogeometric analysis, which suggests the use of a unified framework in the design and analysis pipeline. G+Smo is an object-oriented, cross-platform, template C++ library and follows the generic programming principle, with a focus on both efficiency and ease of use. The library aims at providing access to high quality, open-source software to the forming isogeometric numerical simulation community and beyond. Geometry plus simulation modules aims at the seamless integration of Computer-aided Design (CAD) and high order Finite Element Analysis (FEA).

The library and its documentation are available at https://gismo.github.io/

Release Contributions: Main changes in release v.25.01.0: Impove dramatically OpenMp parallelization. Introduced new point cloud fitting methods notably with joint optimization of control points and point parameters. Added support for the OptimLib library for optimization. Improved meshing of multipatch geometries. Added ParaView export using Bezier extraction tags, which are now supported by ParaView. Added new functionality to perform L2 projection of arbitrary functions with extra control. Added support for PreCICE library for coupled problems. Improved the XML file format by introducing labels, as well as the I/O of multipatch geometries. Expanded the Python bindings and introduced Julia bindings. The Julia package is available in the Julia registry. Many bug fixes, efficiency improvements and code cleanup.

URL: `https://github.com/gismo`

Contact: Angelos Mantzaflaris

6.1.2 MomentPolynomialOpt

Name: MomentPolynomialOpt

Keywords: Global optimization, Moment, Polynomial equations, Semi-algebraic set, Convex relaxation

Functional Description: The package provides efficient tools to build convex relaxations of moment sequences and their dual Sum-of-Squares relaxations, to optimize vectors of moment sequences that satisfy positivity constraints or mass constraints, to compute global minimizers of polynomial and moment optimization problems from moment sequences, polar ideals, approximate real radical. It also provides tools for computing minimum enclosing ellipsoids of basic semi-algebraic sets. It uses a connection with SDP solvers via the JuMP interface.

The package is available at https://github.com/AlgebraicGeometricModeling/MomentTools.jl and its documentation at https://algebraicgeometricmodeling.github.io/MomentTools/

Release Contributions: New data structure for moment sequences. New functionalities for the construction of SDP relaxations of Generalized Moment Problems (GMP).

URL: `https://github.com/AlgebraicGeometricModeling/MomentPolynomialOpt.jl`

Contact: Bernard Mourrain

Participants: Lorenzo Baldi, Bernard Mourrain

6.1.3 TensorDec

Keywords: Tensor decomposition, Multivariate series, Low rank models, Hankel

Functional Description: TensorDec is a Julia package for the decomposition of tensors and polynomial-exponential series. It provides tools to compute rank decomposition or Waring decomposition of symmetric tensors or multivariate homogeneous, of multilinear tensors.

It also allows computing low rank tensor approximations of given tensors, using Riemannian optimization techniques, with well-chosen initial start. It also provides tools to compute catalecticant or Hankel operators associated to tensors and their apolar ideal.

The package is accessible at https://github.com/AlgebraicGeometricModeling/TensorDec.jl and its documentation at https://algebraicgeometricmodeling.github.io/TensorDec.jl/.

Release Contributions: New functions for the decomposition of symmetric and multilinear tensors. Improved documentation. Tutorials.

URL: `https://github.com/AlgebraicGeometricModeling/TensorDec.jl`

Contact: Bernard Mourrain

Participants: Rima Khouja, Bernard Mourrain

7 New results

7.1 Exact Moment Representation in Polynomial Optimization

Participants: Lorenzo Baldi, Bernard Mourrain.

In [21], we investigate the problem of representing moment sequences by measures in the context of Polynomial Optimization Problems, that consist in finding the infimum of a real polynomial on a real semialgebraic set defined by polynomial inequalities. We analyze the exactness of Moment Matrix (MoM) hierarchies, dual to the Sum of Squares (SoS) hierarchies, which are sequences of convex cones introduced by Lasserre to approximate measures and positive polynomials. We investigate in particular flat truncation properties, which allow testing effectively when MoM exactness holds and recovering the minimizers. We show that the dual of the MoM hierarchy coincides with the SoS hierarchy extended with the real radical of the support of the defining quadratic module Q. We deduce that flat truncation happens if and only if the support of the quadratic module associated with the minimizers is of dimension zero. We also bound the order of the hierarchy at which flat truncation holds. As corollaries, we show that flat truncation and MoM exactness hold when regularity conditions, known as Boundary Hessian Conditions, hold (and thus that MoM exactness holds generically); and when the support of the quadratic module Q is zero-dimensional. Effective numerical computations illustrate these flat truncation properties.

7.2 On Łojasiewicz Inequalities and the Effective Putinar's Positivstellensatz

Participants: Lorenzo Baldi, Bernard Mourrain, Adam Parusinski.

The representation of positive polynomials on a semi-algebraic set in terms of sums of squares is a central question in real algebraic geometry, which the Positivstellensatz answers. In [22], we study the effective Putinar's Positivestellensatz on a compact basic semialgebraic set S and provide a new proof and new improved bounds on the degree of the representation of positive polynomials. These new bounds involve a parameter ε measuring the non-vanishing of the positive function, the constant c and exponent L of a Łojasiewicz inequality for the semi-algebraic distance function associated to the inequalities $g = (g_1, ..., g_r)$ defining S. They are polynomial in c and ε^{-1} with an exponent depending only on L. We analyze in details the Łojasiewicz inequality when the defining inequalities g satisfy the Constraint Qualification Condition. We show that, in this case, the Łojasiewicz exponent L is 1 and we relate the Łojasiewicz constant c with the distance of g to the set of singular systems.

7.3 Orbit spaces of Weyl groups acting on compact tori: a unified and explicit polynomial description

Participants: Evelyne Hubert, Tobias Metzlaff, Cordian Riener.

The Weyl group of a crystallographic root system has a nonlinear action on the compact torus. The orbit space of this action is a compact basic semi-algebraic set. In [29], we present a polynomial description of this set for the Weyl groups of type A, B, C, D and G. Our description is given through a polynomial matrix inequality. The novelty lies in an approach via Hermite quadratic forms and a closed formula for the matrix entries. The orbit space of the nonlinear Weyl group action is the orthogonality region of generalized Chebyshev polynomials. In this polynomial basis, we show that the matrices obtained for the five types follow the same, surprisingly simple pattern. This is applied to the optimization of trigonometric polynomials with crystallographic symmetries.

7.4 Optimization of trigonometric polynomials with crystallographic symmetry and spectral bounds for set avoiding graphs

Participants: Evelyne Hubert, Tobias Metzlaff, Philippe Moustrou, Cordian Riener.

In [28], we provide a new approach to the optimization of trigonometric polynomials under the hypothesis of a crystallographic symmetry. This approach widens the bridge between trigonometric and polynomial optimization. The trigonometric polynomials considered are supported on weight lattices associated to crystallographic root systems and are assumed invariant under the associated reflection group. On one hand, the invariance allows us to rewrite the objective function in terms of generalized Chebyshev polynomials of the generalized cosines; On the other hand, the generalized cosines parametrize a compact basic semi algebraic set, this latter being given by an explicit polynomial matrix inequality. The initial problem thus boils down to a polynomial optimization problem that is straightforwardly written in terms of generalized Chebyshev polynomials. The minimum is to be computed by a converging sequence of lower bounds as given by a hierarchy of relaxations based on the Hol-Scherer Positivstellensatz and indexed by the weighted degree associated to the root system. This new method for trigonometric optimization was motivated by its application to estimate the spectral bound on the chromatic number of set avoiding graphs. We examine cases of the literature where the avoided set affords crystallographic symmetry. In some cases, we obtain new analytic proofs for sharp bounds on the chromatic number while, in others, we compute new lower bounds numerically.

7.5 Isolated singularities, inverse systems and the punctual Hilbert scheme

Participant: Bernard Mourrain.

In [34], we study the punctual Hilbert scheme from an algorithmic point of view. We first present algorithms, which allow to compute the inverse system of an isolated point. We define the punctual Hilbert scheme as a subvariety of a Grassmannian variety and provide explicit equations defining it. Then we localized our study to the algebraic variety Hilb_B of inverse systems, which admits a given dual (monomial) basis B. Building on the algorithm to compute the inverse system and exploiting combinatorial properties of the monomial basis B, we derive a minimal set of equations defining Hilb_B. Using this effective presentation, we prove new properties of transversality of Hilb_B, give new dimension formula for its irreducible components and prove that they are birational.

7.6 Toric Sylvester Forms

Participants: Laurent Busé, Carles Checa.

In [24], we investigate the structure of the saturation of ideals generated by sparse homogeneous polynomials over a projective toric variety X with respect to the irrelevant ideal of X. As our main results, we establish a duality property and make it explicit by introducing toric Sylvester forms, under a certain positivity assumption on X. In particular, we prove that toric Sylvester forms yield bases of some graded components of Isat/I, where I denotes an ideal generated by n+1 generic forms, n is the dimension of I and Isat the saturation of I with respect to the irrelevant ideal of the Cox ring of X. Then, to illustrate the relevance of toric Sylvester forms, we provide three consequences in elimination theory: (1) we introduce a new family of elimination matrices that can be used to solve sparse polynomial systems by means of linear algebra methods, including overdetermined polynomial systems; (2) by incorporating toric Sylvester forms to the classical Koszul complex associated to a polynomial system, we obtain new expressions of the sparse resultant as a determinant of a complex; (3) we give a new formula for computing toric residues of the product of two forms.

7.7 Mixed Subdivisions Suitable for the Greedy Canny–Emiris Formula

Participants: Carles Checa, Ioannis Emiris.

The Canny–Emiris formula (Canny and Emiris in International symposium on applied algebra, algebraic algorithms, and error-correcting codes, 1993) gives the sparse resultant as the ratio of the determinant of a Sylvester-type matrix over a minor of it, both obtained via a mixed subdivision algorithm. In Checa and Emiris (International symposium on symbolic and algebraic computation, 2022), the authors gave an explicit class of mixed subdivisions for the greedy approach so that the formula holds, and the dimension of the constructed matrices is smaller than that of the subdivision algorithm, following the approach of Canny and Pedersen (Algorithm for the Newton resultant, 1993). Our method improves upon the dimensions of the matrices when the Newton polytopes are zonotopes and the systems are multihomogeneous. In [25], we provide more such cases, and we conjecture which might be the liftings providing minimal size of the resultant matrices. We also describe two applications of this formula, namely in computer vision and in the implicitization of surfaces, while offering the corresponding JULIA code. We finally introduce a novel tropical approach that leads to an alternative proof of a result in Checa and Emiris (2022).

7.8 From CAD to Representations Suitable for Isogeometric Analysis: a Complete Pipeline

Participants: Michelangelo Marsala, Angelos Mantzaflaris, Bernard Mourrain.

In [31], we present a complete pipeline to convert CAD models into smooth G1 spline representations, which are suitable for isogeometric analysis. Starting from a CAD boundary representation of a mechanical object, we perform an automatic control cage extraction by means of quadrangular faces, such that its limit Catmull-Clark subdivision surface approximates accurately the input model. Then we compute a basis of the G1 spline space over the quad mesh in order to carry out least squares fitting over a point cloud, acquired by sampling the original CAD geometry. The resulting surface is a collection of Bézier patches with G1 regularity, except at the sharp edges. Finally, we use the basis functions to perform isogeometric analysis simulations of realistic PDEs on the reconstructed G1 model. The quality of the construction is demonstrated via several numerical examples performed on a collection of CAD objects presenting various challenging realistic shapes.

This is a joint work with Sam Whyman and Mark Gammon (ITI CADFIX, Cambridge, UK).

7.9 A spline-based regularized method for the reconstruction of complex geological models

Participants: Ayoub Belhachmi, Bernard Mourrain.

The study and exploration of the subsurface requires the construction of geological models. This task can be difficult, especially in complex geological settings, with various unconformities. These models are constructed from seismic or well data, which can be sparse and noisy. In [23], we propose a new method to compute a stratigraphic function that represents geological layers in arbitrary settings. This function interpolates the data using piecewise quadratic C 1 Powell-Sabin splines, and is regularized via a selfadaptive diffusion scheme. For the discretization, we use Powell-Sabin splines on triangular meshes. Compared to classical interpolation methods, the use of piecewise quadratic splines has two major advantages. First, their ability to produce surfaces of higher smoothness and regularity. Second, it is straightforward to discretize high order smoothness energies like the squared Hessian energy (Stein et al. 2018). The regularization is considered as the most challenging part of any implicit modeling approach. Often, existing regularization methods produce inconsistent geological models, in particular for data with high thickness variations. To handle this kind of data, we propose a new scheme in which a diffusion term is introduced and iteratively adapted to the shapes and variations in the data, while minimizing the interpolation error.

This is a joint work with Azeddine Benabbou (SLB, Montpellier).

7.10 Goal-Adaptive Meshing of Isogeometric Kirchhoff-Love Shells

Participants: Angelos Mantzaflaris.

Mesh adaptivity is a technique to provide detail in numerical solutions without the need to refine the mesh over the whole domain. Mesh adaptivity in isogeometric analysis can be driven by Truncated Hierarchical B-splines (THB-splines) which add degrees of freedom locally based on finer B-spline bases. Labeling of elements for refinement is typically done using residual-based error estimators. In [32], an adaptive meshing workflow for isogeometric Kirchhoff-Love shell analysis is developed. This framework includes THB-splines, mesh admissibility for combined refinement and coarsening and the Dual-Weighted Residual (DWR) method for computing element-wise error contributions. The DWR can be used in several structural analysis problems, allowing the user to specify a goal quantity of interest which is used to mark elements and refine the mesh. This goal functional can involve, for example, displacements, stresses, eigenfrequencies etc. The proposed framework is evaluated through a set of different benchmark problems, including modal analysis, buckling analysis and non-linear snap-through and bifurcation problems, showing high accuracy of the DWR estimator and efficient allocation of degrees of freedom for advanced shell computations.

This is a joint work with Hugo M. Verhelst, Matthias M. Möller, J. den Besten (Univ. Delft).

7.11 Adaptive isogeometric gear contact analysis: geometry generation, truncated hierarchical B-Spline refinement and validation

Participants: Christos Karampatzakis, Angelos Mantzaflaris.

Gears are one of the most widely used transmission components. Their operation relies on the contact between mating gear teeth flanks for the transmission of power. Accurate prediction of the contact stresses at these regions, is crucial for the design and dimensioning of these systems. Gear design is centered around highly smooth involute curves that greatly influence their contact behavior. In [30], a fully adaptive isogeometric contact modeling scheme, based on hierarchical splines, is presented and applied to the simulation of gear contact problems. In particular, isogeometric simulation is performed for the modeling of mating pair of gear teeth, regarded as linearly elastic bodies. A boundary fitted B-Spline representation of the teeth is automatically generated from engineering design parameters and is used to define the initial discretisation basis. The numerical integration over the contact region is addressed using the so called, Gauss-Point to Surface formulation and a closest point projection procedure. Truncated hierarchical B-Splines are used to capture the highly localized nature of contact, while effectively reducing the number of degrees of freedom. The adaptivity is driven by the strain energy density gradient, which allows to automatically localize the mesh without a priori knowledge of the contact region between the teeth flanks. In our experiments, we justify the choices made in different steps of our algorithm and we assess the performance of our adaptive solver with respect to classical tensor product B-Splines.

7.12 Parameterization learning with convolutional neural networks for gridded data fitting

Participants: Angelos Mantzaflaris.

The approximation of point clouds in terms of parametric representations is a fundamental task for geometric modeling and processing applications to properly analyze the (re-)constructed model in its full complexity. A necessary and key step in this process requires the identification of a suitable data parameterization. If the data connectivity is determined by a rectilinear grid, standard closed-form methods are available for general multivariate configurations. Existing data-driven parameterization methods instead are limited to the univariate case and require pre/post-processing steps of the input data to handle point sequences without fixed length. In [35], we propose a dimension independent method based on convolutional neural networks to assign parameter values to gridded point clouds of arbitrary size, without the need for additional data processing steps. We train the proposed networks by considering polynomial least squares approximations and demonstrate, both in the univariate and bivariate settings, that the accuracy of the final model properly scales when uniform and adaptive spline refinement is considered. A selection of numerical experiments on point clouds of different sizes highlights the performance of our parameterization scheme. Noisy data sets which simulate measurement errors are also considered.

This is joint work with Michele de Vita, Carlotta Giannelli, Sofia Imperatore (Univ. Firenze).

7.13 BIDGCN: Boundary informed dynamic graph convolutional network for adaptive spline fitting of scattered data

Participants: Angelos Mantzaflaris.

In [26], we propose a Boundary Informed Dynamic Graph Convolutional Network (BIDGCN) characterized by a novel boundary informed input layer, with special focus on applications related to adaptive spline approximation of scattered data. The newly introduced layer propagates given boundary information to the interior of the point cloud, in order to let the input data be suitably processed by successive graph convolutional network layers. We apply our BIDGCN model to the problem of parameterizing three-dimensional unstructured data sets over a planar domain. The parameterization problem is a key step in the solution of different geometric modeling tasks and in particular for the design of surface reconstruction schemes with smooth spline surfaces. A selection of numerical examples shows the effectiveness of the proposed approach for adaptive spline fitting with (truncated) hierarchical B-spline constructions.

This is a joint work with Carlotta Giannelli, Sofia Imperatore (Univ. Firenze), Felix Scholz (MTU, Munich).

7.14 Improved markerless gait kinematics measurement using a biomechanically-aware algorithm with subject-specific geometric modeling

Participants: Laurent Busé, Mehran Hatamzadeh.

Despite the advancements in developing markerless gait analysis systems, they still demonstrate lower accuracy compared to gold-standard systems. Hence, in [27], a novel approach is presented to improve the lower limb kinematics accuracy in markerless gait analysis. This approach refines the 3D lower-limb skeletons obtained by AI-based pose estimation algorithms in a subject-specific geometric manner, preserves skeleton links' length, benefits from gait phases information that adds biomechanical awareness to the algorithm, and utilizes an embedded trajectory smoothing. Validation of the proposed method shows that it reduces 12.6%-43.5% of root mean square error (RMSE) and significantly improves kinematic curves' similarity to the gold-standard ones. Results also prove the feasibility of more accurate lower limb kinematics calculation using a single (2.02°-7.57° RMSE) or dual RGB-D camera (1.66°-7.25° RMSE). Development of such algorithms could result in requirement of fewer cameras that deliver comparable or even superior measurement accuracy compared to multi-camera approaches.

7.15 HydraProt: A New Deep Learning Tool for Fast and Accurate Prediction of Water Molecule Positions for Protein Structures

Participants: Ioannis Emiris.

Water molecules are integral to the structural stability of proteins and vital for facilitating molecular interactions. However, accurately predicting their precise position around protein structures remains a significant challenge, making it a vibrant research area. In [33], we introduce HydraProt (deep Hydration of Proteins), a novel methodology for predicting precise positions of water molecule oxygen atoms around protein structures, leveraging two interconnected deep learning architectures: a 3D U-net and a Multi-Layer Perceptron (MLP). Our approach starts by introducing a coarse voxel-based representation of the protein, which allows for rapid sampling of candidate water positions via the 3D U-net. These water positions are then assessed by embedding the water–protein relationship in the Euclidean space by means of an MLP. Finally, a postprocessing step is applied to further refine the MLP predictions. HydraProt surpasses existing state-of-the-art approaches in terms of precision & recall and has been validated on large data sets of protein structures. Notably, our method offers rapid inference runtime and should constitute the method of choice for protein structure studies and drug discovery applications. Our pretrained models, data, and the source code required to reproduce these results are accessible at github.com/azamanos/HydraProt.

This is a joint work with Andreas Zamanos and George Ioannakis from Athena Research and Innovation Centre (Greece).

8 Bilateral contracts and grants with industry

8.1 Bilateral grants with industry

- **Geometric computing.**

Participants: Ioannis Emiris, Panagiotis Repouskos, Ioannis Psarros.

Ioannis Emiris coordinates a research contract with the industrial partner ANSYS Inc. (Greece), in collaboration with Athena Research Center. MSc students P. Repouskos, M. Dioletis, and T. Pappas, postdoc fellow I. Psarros, and Athena researcher George Ioannakis are partially funded.

Electronic design automation (EDA) and simulating Integrated Circuits requires robust geometric operations on thousands of electronic elements (capacitors, resistors, coils etc) represented by polyhedral objects in 2.5 dimensions, not necessarily convex. A special case may concern axis-aligned objects but the real challenge is the general case. The project, extended into 2024, focuses on 3 axes: (1) efficient data structures and prototype implementations for storing the aforementioned polyhedral objects so that nearest neighbor queries are fast in the L-max metric, which is the primary focus of the contract, (2) random sampling of the free space among objects, (3) data-driven algorithmic design for problems concerning data-structures and their construction and initialization. The implementation of prototypes has led into the development of a software library including implementations of parallel algorithms (Cuda).

It has continued during the entire year 2024 along with a tripartite grant from the Greek ministry of Development (Athena RC, ANSYS, and University of Patras, Greece).

- **Interactive construction of 3D models - Application to the modeling of complex geological structures.**

Participants: Ayoub Belhachmi, Bernard Mourrain.

CIFRE collaboration between Schlumberger Montpellier (A. Azzedine) and Inria Sophia Antipolis (B. Mourrain). The PhD candidate is A. Belhachmi. The objective of the work is the development of a new spline based high quality geomodeler for reconstructing the stratigraphy of geological layers from the adaptive and efficient processing of large terrain information.

9 Partnerships and cooperations

9.1 European initiatives

9.1.1 Horizon Europe

Participants: Enrica Barrilli, Evelyne Hubert, Angelos Mantzaflaris, Mattia Matucci, Bernard Mourrain.

TENORS (see on cordis.europa.eu)

Title: Tensor modEliNg, geOmetRy and optimiSation

Duration: From January 1, 2024 to December 31, 2027

Partners:

- INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET AUTOMATIQUE (INRIA), France
- UNIVERSITETET I TROMSOE - NORGES ARKTISKE UNIVERSITET (UiT), Norway
- Bluetensor Srl (Bluetensor Srl), Italy
- HSBC BANK PLC, United Kingdom
- UNIVERSITE COTE D'AZUR, France
- Arva AS (Arva), Norway
- FUNDACIO INSTITUT DE CIENCIES FOTONIQUES (ICFO-CERCA), Spain
- Cambridge Quantum Computing Limited (Cambridge Quantum Computing Limited), United Kingdom
- UNIVERSITA DEGLI STUDI DI FIRENZE (UNIFI), Italy
- UNIVERSITAET LEIPZIG (ULEI), Germany
- UNIVERSITAT POLITECNICA DE CATALUNYA (UPC), Spain
- CESKE VYSOKE UCENI TECHNICKE V PRAZE (CVUT), Czechia
- QUANDELA, France
- UNIVERSITA DEGLI STUDI DI TRENTO (UNITN), Italy
- UNIVERSITAT KONSTANZ (UKON), Germany
- TILBURG UNIVERSITY- UNIVERSITEIT VAN TILBURG (TILBURG UNIVERSITY), Netherlands
- MAX-PLANCK-GESELLSCHAFT ZUR FORDERUNG DER WISSENSCHAFTEN EV (MPG), Germany
- UNIVERSITE PAUL SABATIER TOULOUSE III (UNIVERSITE TOULOUSE III - Paul Sabatier), France
- ARTELYS, France

- CENTRE NATIONAL DE LA RECHERCHE SCIENTIFIQUE CNRS (CNRS), France
- STICHTING NEDERLANDSE WETENSCHAPPELIJK ONDERZOEK INSTITUTEN (NWO-I), Netherlands

Inria contact: Bernard MOURRAIN

Coordinator:

Summary: TENORS aims to form the next generation of researchers and engineers in scientific computing and data analysis, disrupting the current paradigms of tensor calculus by exploiting cutting-edge research in geometry and optimization. Tensors are nowadays ubiquitous in many domains of applied mathematics, computer science, signal processing, data processing, machine learning and in the emerging area of quantum computing. The demand for highly trained scientists with a deep understanding of tensor methods and with advanced knowledge in the geometry of tensor spaces, with skills on the design of efficient algorithms and software handling tensor computation and in the applications of high performance tensor computation is raising, in many fields including machine learning and quantum computation, which are nowadays expanding very quickly.

TENORS contributes to satisfy this demand by fostering scientific and technological advances in the area of tensor sciences, stimulating interdisciplinary and intersectoriality knowledge exchange between algebraists, geometers, computer scientists, numerical analysts, data analysts, physicists, quantum scientists, and industrial actors facing real-life tensor-based problems, in a network of PhD students at its core.

A unique strength of the network is to gather top-researchers of these different domains.

TENORS will train young scientists in academy or industry in how to exploit the best of these techniques efficiently and disseminate this knowledge to industry. As a truly multidisciplinary network, TENORS will at the same time seek to apply these new techniques to real-life applications thanks to the industrial actors involved in the network.

9.1.2 H2020 projects

Participants: Laurent Busé, Ioannis Emiris, Angelos Mantzaflaris, Bernard Mourrain.

GRAPES (see on cordis.europa.eu)

Title: learninG, pRocessing, And oPtimising shapES

Duration: From December 1, 2019 to May 31, 2024

Partners:

- INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET AUTOMATIQUE (INRIA), France
- UNIVERSITA DEGLI STUDI DI ROMA TOR VERGATA (UNITOV), Italy
- RHEINISCH-WESTFAELISCHE TECHNISCHE HOCHSCHULE AACHEN (RWTH AACHEN), Germany
- ATHINA-EREVNITIKO KENTRO KAINOTOMIAS STIS TECHNOLOGIES TIS PLIROFORIAS, TON EPIKOINONION KAI TIS GNOSIS (ATHENA - RESEARCH AND INNOVATION CENTER), Greece
- UNIVERSITAT LINZ (JOHANNES KEPLER UNIVERSITAT LINZ UNIVERSITY OF LINZ JOHANNES KEPLER UNIVERSITY OF LINZ JKU), Austria
- SINTEF AS (SINTEF), Norway
- VILNIAUS UNIVERSITETAS (Vilniaus universitetas), Lithuania

- UNIVERSITA DELLA SVIZZERA ITALIANA (USI), Switzerland
- UNIVERSITAT DE BARCELONA (UB), Spain
- GEOMETRY FACTORY SARL, France
- UNIVERSITY OF STRATHCLYDE, United Kingdom

Inria contact: Laurent Busé

Coordinator: Ioannis Emiris

Summary: GRAPES aims at considerably advancing the state of the art in Mathematics, Computer-Aided Design, and Machine Learning in order to promote game changing approaches for generating, optimizing, and learning 3D shapes, along with a multisectoral training for young researchers. Recent advances in the above domains have solved numerous tasks concerning multimedia and 2D data. However, automation of 3D geometry processing and analysis lags severely behind, despite their importance in science, technology and everyday life, and the well-understood underlying mathematical principles. The CAD industry, although well established for more than 20 years, urgently requires advanced methods and tools for addressing new challenges.

The scientific goal of GRAPES is to bridge this gap based on a multidisciplinary consortium composed of leaders in their respective fields. Top-notch research is also instrumental in forming the new generation of European scientists and engineers. Their disciplines span the spectrum from Computational Mathematics, Numerical Analysis, and Algorithm Design, up to Geometric Modelling, Shape Optimization, and Deep Learning. This allows the 15 PhD candidates to follow either a theoretical or an applied track and to gain knowledge from both research and innovation through a nexus of intersectoral secondments and Network-wide workshops.

Horizontally, our results lead to open-source, prototype implementations, software integrated into commercial libraries as well as open benchmark datasets. These are indispensable for dissemination and training but also to promote innovation and technology transfer. Innovation relies on the active participation of SMEs, either as a beneficiary hosting an ESR or as associate partners hosting secondments. Concrete applications include simulation and fabrication, hydrodynamics and marine design, manufacturing and 3D printing, retrieval and mining, reconstruction and visualisation, urban planning and autonomous driving.

9.2 National initiatives

Participants: Angelos Mantzaflaris, Bernard Mourrain, Régis Duvigneau *(ACUMES)*.

Title: ANR PRCI: *"RFF-Splines: High-order Isogeometric simulation with geometrically continuous functions"*

Duration: 2025–2028, administrative start October 2025

Coordinator: Angelos Mantzaflaris

Partners: University of Linz, Austria

Summary: Our project aims at the development of a novel framework for high-order discretization of partial differential equations on general domains. Smoothness requirements and superior approximation power are paramount for efficient simulations. We focus on the paradigm of *isogeometric analysis* that uses spline functions for design and analysis on real-world (both man-made and occurring in nature) geometries. General domains pose challenges due to their topology, in particular in the vicinity of so-called extraordinary vertices, which are essentially artifacts created by the discretization (i.e., meshing) procedures. We propose a framework of geometrically continuous splines, called Refinable FreeForm (RFF-) Splines, to enable numerical schemes for *topologically unrestricted design and analysis.*

The novelty of the construction is based on three main points: First, we will establish *theoretical guarantees for approximation power*. These will be derived based on the property of local polynomial reproduction, using an adapted construction for spline projectors on domain manifolds. Second, we will focus on the *efficient construction* of the basis functions, notably concerning the evaluation and the matrix assembly for simulation via numerical integration. Third, we will emphasize *adaptivity and approximate evaluation through local refinement*. We will employ the truncation mechanism to reduce the support of the basis functions (thereby increasing sparsity) and to preserve the partition of unity property, again with theoretical guarantees regarding the approximation power.

The starting point of our work is provided by a now classic work of Prautzsch in 1997, which focused on the fundamental property of polynomial reproduction. Obviously, this property is essential for the derivation of theoretical guarantees for the approximation performance. The project goes all the way from the design of the construction and the theory development to the algorithmic aspects and the efficient implementation in C++, as well as experimental evaluation in demanding applications that involve high order partial differential equations.

10 Dissemination

Participants: Laurent Busé, Ioannis Emiris, Evelyne Hubert, Martin Jalard, Angelos Mantzaflaris, Bernard Mourrain.

10.1 Promoting scientific activities

10.1.1 Scientific events: organisation

Member of the organizing committees

Angelos Mantzaflaris was a member of the organizing committee of the worksop G+Smo Developer Days 2024, March 2024, Thessaloniki, Greece, where he also delivered tutorials on the use of G+Smo in C++.

10.1.2 Scientific events: selection

Member of the conference program committees

- Laurent Busé and Bernard Mourrain were members of the program committees of the *Symposium on Solid and Physical Modeling* SPM2024 and of the *International Conference on Geometric Modeling and Processing* GMP2024.
- Evelyne Hubert refereed submissions for the conferences MEGA and GMP 2024.
- Angelos Mantzaflaris was a member of the program committees of GMP 2024 and Isogeometric Analysis (IGA) 2024.

Reviewer

- Laurent Busé was a reviewer for the international conference SIGGRAPH Asia.
- Bernard Mourrain was a reviewer for the conference ISSAC'24 (International Symposium on Symbolic and Algebraic Computation) and for the conference MEGA (Effective Methods in Algebraic Geometry), in Leipzig, Germany, July 29 - August 2, 2024.

10.1.3 Journal

Member of the editorial boards

- Laurent Busé is an editor of the *Journal of Pure and Applied Algebra*.
- Ioannis Emiris is an associate editor of the *Journal of Applicable Algebra in Engineering, Communication and Computing*.
- Evelyne Hubert is member of the editorial board of the journal *Foundations of Computational Mathematics* and *Journal of Symbolic Computation*. She was a guest editor for the special issue *Computational Algebra and Geometry: A special issue in memory and honor of Agnes Szanto* of the *Journal of Symbolic Computation*.
- Bernard Mourrain is a member of the editorial board of the *Journal of Symbolic Computation*.

Reviewer - reviewing activities

- Laurent Busé was a reviewer for the *Computer-Aided Design* journal, the *Journal of Algebra*, the *Computer-Aided and Geometric Design* journal, the *Michigan Mathematical Journal*, the *Communications in Mathematics and Statistics* journal and the journal *Transactions on Graphics*.
- Evelyne Hubert refereed submissions at the SIAM journal *Applied Algebra and Geometry, Journal of Geometry and Physics, Applied and Computational Harmonic Analysis*.
- Angelos Mantzaflaris reviewed for the journals *Computer Aided Geometric Design*, *Computer Methods in Applied Mechanics and Engineering, Computer-Aided Design*, *Applied Numerical Mathematics, Mathematics and Computers in Simulation*, *Journal of Scientific Computing, Journal of Computational and Applied Mathematics*.
- Bernard Mourrain reviewed submissions for the following journals: *Journal of Algebra and Applications*, *Journal of Symbolic Computation, Mathematics of Computation*, *Journal of Computational and Applied Mathematics*, *Journal de Mathématiques Pures et Appliquées, Optimization Letter*, *Journal of Numerical Algebra, Control and Optimization, SIAM Journal on Applied Algebra and Geometry*, *SIAM Journal of Optimization*, *Journal of Computer Aided Geometric Design*.

10.1.4 Invited talks

- Laurent Busé was invited to give an invited course at the University of Hue, Vietnam, from January 16th to February 1st, to give an invited talk at the algraic geoemtry seminar of the University of Toulouse, March 27th and to give an invited technical talk at the ANSYS seminar, May 23rd (online).
- Evelyne Hubert was invited to give tutorial lectures at the ACM conference *International Symposium on Symbolic and Algebraic Computation* that was held in Raleigh, USA. She was also invited to present her results on the computation of fundamental equivariants at the conference *Equivariant Methods in Algebraic and Differential Geometry* held at the Isaac Newton Institute in Cambridge (UK) and at the conference *Algebraic Combinatorics and Finite Groups*. She was invited to give a talk on the computation of symmetric cubatures at the conference *Singular and oscillatory integration - advances and applications* held at University College London, UK. She was also invited to present her results
- Angelos Mantzaflaris was invited to the BIRS (Banff International Research Station for Mathematical Innovation and Discovery) workshop *Homological Perspective on Splines and Finite Elements*, May 2024 in Kelowna, Canada.

- Bernard Mourrain was invited to give a talk at the conference High order statistics and tensors, ICERM, Providence, USA, January 8-13, at the mini-symposium "Tensor decompositions and their applications" of the SIAM conference on Linear Algebra, Paris, France, May 12-15, at the Journées d'Approximation, Lille, France, May 15-17, at the Workshop "Positive Solutions of Polynomial Systems Arising from Real-life Applications", BIRS ((Banff International Research Station for Mathematical Innovation and Discovery), Grenada, Spain, May 19-24, at workshop LoRAINNe'24 on LOw-Rank Approximations and their Interactions with Neural NEtworks, Nancy, France, November 26-27, 2024.

10.1.5 Leadership within the scientific community

- Laurent Busé is co-chair of the Computer Algebra French Research Community (GT CF of the GDR IFM).
- Evelyne Hubert is chair of the society *Foundations of Computational Mathematics*.

10.1.6 Scientific expertise

Bernard Mourrain was reviewer for the ANR (Fondements du numérique : informatique, automatique, traitement du signal et des images) and for the Royal Society Wolfson Fellowship, UK.

10.1.7 Research administration

- Laurent Busé is chair of the Commité de Suivi Doctoral (CSD) of the Inria centre at UniCA, member of the Comité NICE of the Inria centre at UniCA, representative member of Inria centre at UniCA at the Département Disciplinaires de Mathématiques of UniCA, member of the Comité de Pilotage of the EUR SPECTRUM of UniCA.
- Ioannis Emiris is President of BoD and Director General of Athena Research Center, Greece.
- Angelos Mantzaflaris is a member of the Bureau of AMIES (Agence pour les Mathématiques en Interaction avec l'Entreprise et la Société).
- Bernard Mourrain is member of the Bureau du Commité des Equipes Projets at Inria of UniCA.

10.2 Teaching - Supervision - Juries

10.2.1 Teaching

- License : Martin Jalard, Analyse, 64h (L1), Polytech Nice Sophia - Université Côte d'Azur
- Licence : Ioannis Emiris, Algorithms and complexity, 52 h (L2), NKU Athens
- Licence : Ioannis Emiris, Software development, 26 h (L3), NKU Athens
- Licence : Angelos Mantzaflaris, Mathematics 1, 64 h (L1), ESDHEM, SKEMA Business School in Sophia-Antipolis
- Licence : Issam Tauil: L1 course on Linear Algebra (38h), personnal interrogationolles en algèbre multilinéaire en L3
- Master : Ioannis Emiris, Structural bioinformatics, 39 h (M2), NKU Athens
- Master : Laurent Busé, Geometric Modeling, 18h (M2), Polytech Nice Sophia - Université Côte d'Azur.
- Master : Angelos Mantzaflaris, Numerical Interpolation 20 h (M2), Polytech Nice Sophia - Univ Côte d'Azur.

10.2.2 Supervision

- Laurent Busé was the co-advisor of the PhD thesis of Mehran Hatamzadeh, on the topic "An innovative gait analysis technology", co-supervised with Raphaël Zory (LAMHESS, UniCA), who defended his PhD on November 4th.
- Laurent Busé is the co-advisor of the PhD thesis of Issam Tauil, on the topic "Enumerative geometry on toric surfaces and reduced elimination theory", co-supervised with Thomas Dedieu (Toulouse).
- Evelyne Hubert advises Martin Jalard since October 2022 on the *effective stratification by orbit type*, (mostly) funded by Ecole Normale Supérieure de Rennes.
- Angelos Mantzaflaris co-supervised the PhD thesis of Sofia Imperatore (together with Carlotta giannelli; thesis defended in May 2023 at the University of Florence, Italy).
- Angelos Mantzaflaris is the co-advisor (with Carlotta Gainnelli - U. Florence) of Mattia Matucci, on the topic *Low-rank approximation for tensor modeling*, funded by the TENORS project, started in November 2024, Inria.
- Bernard Mourrain was co-advisor (with Azzedine Benabou - Schlumberger Montpellier) of Ayoub Belhachmi, *Interactive construction of 3D models - Application to the modeling of complex geological structures* [36], CIFRE, PhD defended on January 2024.
- Bernard Mourrain is the co-advisor (with Alessandra Bernardi - U. Trento) of the PhD thesis of Enrica Barrilli, on the study of *Extensor varieties*, funded by the TENORS project, started in October 2024, Inria.
- Bernard Mourrain is the co-advisor (with Alessandra Bernardi - U. Trento) of Oriol Reig, on the topic of *Algorithms for tensor decomposition*, funded by the TENORS project, started in October 2024, Inria.
- Bernard Mourrain is the co-advisor (with Michael Gabay - Artelys) of the PhD thesis of Yassine Koubaa, on the topic of *Low rank approximation and optimization varieties*, funded by the TENORS project, started in November 2024, Inria.
- Bernard Mourrain is the co-advisor (with Salma Khumann - U. Konstanz) of the PhD thesis of Matteo Becchere, on the topic of *Tensor decompositions for sums of even powers of real polynomials*, funded by the TENORS project, started in November 2024, Inria.

10.2.3 Juries

Laurent Busé was the president of the PhD committee of Abid Ali, titled *Video Analysis using Deep Neural Networks: An Application for Autism*; thesis of University Côte d'Azur, defended on December 19, 2024.

Evelyne Hubert was part of the committee for selecting the junior researchers (CRCN and ISFP) at Centre Inria Université Côte d'Azur.

Angelos Mantzaflaris served in the PhD defence committee of Hugo Verhelst in TU Delft, the Netherlands, entitled *Isogeometric analysis of wrinkling*.

Bernard Mourrain was reviewer for the PhD thesis of Sofia Imperator, *Adaptive spline approximation: data-driven parameterization and CAD model (re-)construction*, U. Firenze, March 2024.

11 Scientific production

11.1 Major publications

[1] L. Baldi and B. Mourrain. 'On the Effective Putinar's Positivstellensatz and Moment Approximation'. In: *Mathematical Programming, Series A* (6th Sept. 2022). DOI: 10.1007/s10107-022-01877-6. URL: https://hal.science/hal-03437328.

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[3] E. Bartzos, I. Z. Emiris and R. Vidunas. 'New upper bounds for the number of embeddings of minimally rigid graphs'. In: *Discrete and Computational Geometry* 68.3 (2022), p. 796. DOI: 10.1007/s00454-022-00370-3. URL: https://hal.archives-ouvertes.fr/hal-03895585.

[4] L. Busé, Y. Cid-Ruiz and C. D'Andrea. 'Degree and birationality of multi-graded rational maps'. In: *Proceedings of the London Mathematical Society* 121.4 (2020), pp. 743–787. DOI: 10.1112/plms.12336. URL: https://hal.inria.fr/hal-01793578.

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[6] L. Calès, A. Chalkis, I. Z. Emiris and V. Fisikopoulos. 'Practical volume approximation of high-dimensional convex bodies, applied to modeling portfolio dependencies and financial crises'. In: *Computational Geometry* 109 (Feb. 2023), p. 101916. DOI: 10.1016/j.comgeo.2022.101916. URL: https://hal.science/hal-04294307.

[7] A. Chalkis, I. Z. Emiris and V. Fisikopoulos. 'A Practical Algorithm for Volume Estimation based on Billiard Trajectories and Simulated Annealing'. In: *ACM Journal of Experimental Algorithmics* 28 (11th May 2023), pp. 1–34. DOI: 10.1145/3584182. URL: https://hal.science/hal-04294298.

[8] A. Chalkis, I. Z. Emiris, V. Fisikopoulos, E. Tsigaridas and H. Zafeiropoulos. 'Geometric algorithms for sampling the flux space of metabolic networks'. In: *Journal of Computational Geometry* 14.1 (2023). DOI: 10.20382/jocg.v14i1a8. URL: https://inria.hal.science/hal-04310109.

[9] E. Christoforou, H. Leontiadou, F. Noé, J. Samios, I. Z. Emiris and Z. Cournia. 'Investigating the Bioactive Conformation of Angiotensin II Using Markov State Modeling Revisited with Web-Scale Clustering'. In: *Journal of Chemical Theory and Computation* 18.9 (13th Sept. 2022), pp. 5636–5648. DOI: 10.1021/acs.jctc.1c00881. URL: https://hal.science/hal-03895590.

[10] I. Z. Emiris and I. Psarros. 'Products of Euclidean Metrics, Applied to Proximity Problems among Curves'. In: *ACM Transactions on Spatial Algorithms and Systems* 6.4 (Aug. 2020), pp. 1–20. DOI: 10.1145/3397518. URL: https://hal.inria.fr/hal-03045134.

[11] A. J. Fuentes Suárez and E. Hubert. 'Scaffolding skeletons using spherical Voronoi diagrams: feasibility, regularity and symmetry'. In: *Computer-Aided Design* 102 (May 2018), pp. 83–93. DOI: 10.1016/j.cad.2018.04.016. URL: https://hal.inria.fr/hal-01774909.

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[20] K. Tertikas, D. Paschalidou, B. Pan, J. J. Park, M. A. Uy, I. Z. Emiris, Y. Avrithis and L. Guibas. 'PartNeRF: Generating Part-Aware Editable 3D Shapes without 3D Supervision'. In: CVPR 2023 - IEEE/CVF Conference on Computer Vision and Pattern Recognition. Vol. Proceedings of the 2023 IEEE/CVF Conference on Computer Vision and Pattern Recognition. Vancouver, Canada: IEEE, 21st Mar. 2023, pp. 4466–4478. DOI: 10.1109/CVPR52729.2023.00434. URL: https://hal.science/hal-04294277.

11.2 Publications of the year

International journals

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