RESEARCH CENTRE

Inria Saclay Centre at Institut Polytechnique de Paris

IN PARTNERSHIP WITH:

CNRS, Institut Polytechnique de Paris

2024 ACTIVITY REPORT

Project-Team ASCII

Analysis of Stochastic Cooperative Intelligent Interactions

IN COLLABORATION WITH: Centre de Mathématiques Appliquées (CMAP)

DOMAIN

Applied Mathematics, Computation and Simulation

THEME

Stochastic approaches

Contents

Project-Team ASCII

Creation of the Project-Team: 2019 November 01

Keywords

Computer sciences and digital sciences

A3.1. – Data

A3.2. – Knowledge

A3.3. – Data and knowledge analysis

A6. – Modeling, simulation and control

Other research topics and application domains

B1. – Life sciences

B1.1. – Biology

B1.2. – Neuroscience and cognitive science

B2.3. – Epidemiology

B4. – Energy

B6. – IT and telecom

1 Team members, visitors, external collaborators

Research Scientists

- Carl Graham [Team leader, CNRS, Researcher, HDR]
- Quentin Cormier [INRIA, Researcher]
- Denis Talay [INRIA, Emeritus, HDR]

Faculty Member

- Josselin Garnier [École polytechnique, Professor, HDR]

Post-Doctoral Fellow

- Charlie Sire [INRIA, Post-Doctoral Fellow, until Sep 2024]

PhD Students

- Samuel Chan-Ashing [ENS Paris, from Sep 2024]
- Guillaume Chennetier [EDF, until Sep 2024]
- Paul Lartaud [École polytechnique, until Oct 2024]
- Songbo Wang [École polytechnique, until Sep 2024]

Technical Staff

- Nicolas Baradel [INRIA, Engineer, from May 2024]
- Maxime Colomb [INRIA, Engineer]

Interns and Apprentices

- Martin Duffaud [ENSAE, Intern, from Jun 2024 until Jul 2024]

Administrative Assistant

- Amandine Sainsard [INRIA, from Mar 2024]

Visiting Scientist

- Assil Fadle [École polytechnique]

2 Overall objectives

The ASCII team investigates stochastic systems of interacting particles. A particular focus is brought on such systems that behave as a collection of agents striving to cooperate intelligently in order to achieve a common objective by solving complex optimisation problems.

Target applicative fields include energy production, neuroscience, economics, communication networks, and epidemiology. The goal is the modelisation of relevant phenomena, the mathematical analysis of the resulting models, and the rigorous development and calibration of effective simulation and numerical methods for the quantitative evaluation of pertinent quantities.

Our innovative approach raises many challenges. The models are complex and often are non-Markovian and exhibit singularities. Appropriate new mathematical and numerical tools, stochastic and deterministic, have to be developed, such as non-standard stochastic control and optimization methods coupled with specific calibration techniques. We need to combine techniques from varied mathematical fields such as stochastic analysis, partial differential equations, numerical probability, optimization, and stochastic control.

3 Research program

In the field of stochastic particle systems with singular interactions, we investigate convergence to mean-field limits and analyse the convergence rates to this limit as well as of relevant dizcretizations of the limit. One of our main challenges is the simulation of *complex, singular and large scale* McKean-Vlasov particle systems and stochastic partial differential equations, with a strong emphasis on the detection of numerical instabilities and potential big approximation errors. The determination of blow-up times is also a major issue for spectrum approximation and criticality problems in neutron transport theory, Keller-Segel models for chemotaxis, etc.

Reliability assessment for power generation systems or subsystems is another target application of our research. For such complex systems, standard Monte Carlo methods are inefficient due to the difficulty to appropriately simulate rare events. Hence we develop rare event simulation algorithms based on particle filters methods combined with suitable variance reduction methods.

Our research program on interacting agents concerns various types of networks. Devising optimal regulation procedures in such complex stochastic environments is an important and difficult challenge. In the situations we are interested in, the agents often do not compete but on the contrary cooperate in their regulation. Here are some examples: control of cancer therapies, bacteria interaction in Keller-Segel chemotaxis models, distributed control for the planning problem, and distributed algorithms for stochastic telecomunication networks.

The numerous mathematical tools necessary to analyse stochastic agent systems depend on the network type and the analysis objectives. For instance, they include propagation of chaos theory, queueing theory, large deviation theory, ergodic theory, population dynamics, and partial differential equation analysis, in order to respectively determine mean-field limits, congestion rates or spike train distributions, failure probabilities, equilibrium measures, evolution dynamics, and macroscopic regimes.

To develop an example, recently proposed neuron models consider diverse populations of neurons and set up stochastic time evolutions of the membrane potential of every neuron in a way which depends on its population. When the number of populations is fixed, interaction intensities between individuals in different populations have similar orders of magnitude, and the total number of neurons tends to infinity, mean-field limits have been identified and fluctuation theorems have been proven.

However, to the best of our knowledge, no theoretical analysis is available on interconnected networks of networks with different populations of interacting individuals which naturally arise in biology. We aim to study the effects of interconnections between sub-networks resulting from individual and local connections. Of course, the problem needs to be posed in terms of the geometry of the big network and of the scales between connectivity intensities and network sizes.

A related research concerns stochastic, continuous state and time opinion models where each agent's opinion locally interacts with other agents' opinions in the systemic. Due to some exogenous randomness, the interaction tends to create clusters of common opinion. By using linear stability analysis of the associated nonlinear Fokker-Planck equation that governs the empirical density of opinions in the limit of infinitely many agents, we can estimate the number of clusters, the time to cluster formation, the critical strength of randomness so as to have cluster formation, the cluster dynamics after their formation, the width and the effective diffusivity of the clusters.

Another type of network systems we are interested in derives from risk modeling. We consider evolving systems with a large number of inter-connected components, each of which can be in a normal state or in a failed state. These components also have mean field interactions and a cooperative behavior. We will also include diversity as well as other more complex interactions such as hierarchical ones. We aim to study the probability of overall failure of the system. We therefore have to model the intrinsic stability of each component, the strength of external random perturbations to the system, and the degree of inter-connectedness or cooperation between the components. Some important target applications are the following:

- Engineering systems with a large number of interacting parts: Components can fail but the system fails only when a large number of components fail simultaneously.
- Power distribution systems: Individual components of the system are calibrated to withstand fluctuations in demand by sharing loads, but sharing also increases the probability of an overall failure.

4 Application domains

Our short and mid-term potential industrial impact concerns notably energy market regulation, power distribution companies, and nuclear plant maintenance. It also concerns all industrial sectors in which massive stochastic simulations at nano scales are becoming unavoidable and certified results are necessary.

From a more scientific perspective, we aim to have impact in cell biology, neuroscience, and communication networks, notably by using applied mathematics tools at the crossroads of stochastic integration theory, optimization and control, partial differential equation analysis, and stochastic numerical methods.

A long-term ongoing program is the development of an agent-based simulation tool for the spread of epidemics on numerical twins of large territorial zones furnished with statistically conform synthetic populations of individuals moving between their hourly activities. The goal is to allow health agencies to evaluate quantitatively the spatial and temporal effects of varied sanitary actions during actual epidemics in order to enable them to take educated decisions based on solid figures on hospital ward occupation, mortality, economic costs, social costs, etc.

5 Social and environmental responsibility

5.1 Footprint of research activities

Classic footprint for researchers: massive computer runs and travel for international conferences and cooperations.

5.2 Impact of research results

The research is useful for the risk managment of global interlocked economic instances and actors, of energy production in complex power plants, and of epidemics and pandemics.

This year we are particpating in a program on the numerical twin of France and its territories. We have obtained advanced results on the twins of large urban zones furnished with statistically valid synthetic populations in movement between their hourly activiities. Our main aim was and remains to develop on these twins agent-based models for epidemics, with the goal to provide tools for health deciders to evaluate the impact of varied sanitary measures on the spread of the epidemic as well as on the economy and on social structures. These twins can be useful for many other pruposes such as urban planning.

6 Highlights of the year

6.1 Awards

The paper [6] is on the front cover of the journal SIAM Review (Vol. 66, Issue 3).

7 New software, platforms, open data

7.1 New platforms

7.1.1 The ICI epidemy propagation simulation platform

Participants: Maxime Colomb, Quentin Cormier, Josselin Garnier, Nicolas Gilet, Carl Graham, Denis Talay.

In 2020, D. Talay launched the ICI project, a collaboration between INRIA and IGN, of which he is the coordinator. This project aims to provide a platform to simulate an individual-based model of epidemic propagation on a finely represented large geoographic environment. Statistical studies of the simulation results should allow to better understand the epidemy propagation and to compare *in silico* the performances of varied public health strategies to control it.

Maxime Colomb (INRIA-IGN research engineer) and Nicolas Gilet (INRIA research engineer) have been the main developers of the code of a prototype, but Nicolas has left us in December 2022 for a permanent postion at CEA. Quentin Cormier has intervened directly on the code to help optimize it for further extension to a much larger scale. All permanent members of Ascii collaborate on the modeling and algorithmic issues. The following researchers also contribute to this project:

- CRESS (Inserm, Hôtel-Dieu hospital, Paris-Centre University): Profs. Isabelle Boutron, Raphaël Porcher, Philippe Ravaud, Viet-Thi Tran.
- IGN: Julien Perret.
- Inria: Aline Carneiro Viana (TRIBE), Razvan Stanica (INSA-Lyon and AGORA), Milica Tomasevic (CNRS, École Polytechnique, and MERGE).
- EPFL (Switzerland): Prof. Laura Grigori.

The prototype couples diverse models in order to construct a digital twin of a geographical area and launch agent-based simulations of epidemic spreads on it:

- A precise model of the area of interest is constructed using fine scale data bases.
- Synthetic populations are generated in the area by crossing socio-economic data bases. These ar represnetative of the actual populations.
- The spatial evolutions of the individuals are modeled in accordance with data bases furnished, *e.g*, by transport and mobile communication companies.
- Contamination events between individuals interacting during their evolutions at specific places (offices, apartments, restaurants, shops, etc.) are then modeled.

The geographic model is built from multiple geographic sources such as IGN, INSEE, OpenStreetMap, and Local authority open data portals. A three-layered synthetic population is generated in order to represent housing and to populate it by households composed of individuals. Multiple characteristics are added in order to allow to represent the living conditions and inner household interactions of the population. Shops and activities are generated by matching multi-sourced data which allow to enrich information about each amenity such as opening hours and surface.

We simulate the socio-professional structures and hourly trips of the population using probability laws related to the urban space (probability of going out, going to work, shopping, etc.) and to social characteristics (age, job, etc.). Multiple Markov chains are constructed and calibrated for various geographical and socio-demographic profiles using precise values from global surveys. Micro-spatialization of travel objectives are realized using mobile phone data.

In addition, person-to-person contamination is modeled between individuals located in the same space at the same time using transmission probability laws specific to each individual's characteristics, parameterized by the distance between a healthy and a contagious individual as well as by the contact duration.

Since the model is stochastic, obtaining accurate and robust statistics on the evolution of the epidemic requires to simulate firstly a large number of independent socio-professional structures within a given urban area and then, for each population, a large number of realizations of daily trips and contaminations.

Therefore, in order to carry out a very large number of simulations covering all parameters of the model, the model requires high performance computing. The code is written in the Julia language and is currently parallelized using the SLURM resource manager. The model has been launched on the internal cluster of INRIA Saclay called Margaret (200 CPU cores corresponding to 10 CPU nodes) which allows to check the code for a few different epidemiological parameters. We have also obtained the support of AMD to launch our model on a cluster, equipped with AMD EPYC™ processors and AMD Instinct™ Accelerators, into the national GRID5000/SILECS infrastructure. The ICI project has obtained 4 millions CPU hours from DARI/GENCI which can be used on the CEA cluster called Irene-Rome (up to 300 000 CPU cores) in order to launch simulations for a large panel of epidemiological parameters. In addition, we use the OpenMole platform to run and diagnose our numerical model.

Maxime Colomb and Nicolas Gilet have developed a website describing the ICI project and model. They have developed a user interface by including the back-end of the application on an INRIA web server and building an automatic pipeline between the interface and the server in order to display all the results of the simulations to the user. From this it is possible to study the effect of health policies on the epidemic propagation by displaying the main epidemic indicators computed by the model.

Several parts of this project have been presented in various scientific events such as Journées de la recherche de l'IGN 2024, SocSimFest 2023, GT Échelle, Health GIS: Spatial Thinking in Applied Research (STAR).

We are part of the Mobidec PEPR which aims to create toolboxes for various transportation simulations. The creation of spatialized schedules for well-described individuals should benefit from the knowledge of the various research programs involved in this PEPR. This part of the ICI project should then become available for multiple usages through the Mobidec toolbox.

At the moment, the ICI simulations have been applied to the whole of Paris for epidemics which propagate by means of aerosols such as Covid.

In Fall 2024 ICI was selected as one of the priority projects of the 'Digital Twin of France' national initiative launched by Inria, IGN and Cerema. The objective now is to build an epidemic numerical simulation platform which concerns various French areas (Ile de France, rural communities, middle sized cities) and diverse epidemics.

The main originality of the project consists in aiming to provide accurate statistical informations which are differentially computed for various geographic areas with variable size, various groups of individuals categorised by age, socio-professional status, place of residence, etc. The computation of all these statistical indicators takes into account the characteristics of the epidemics under consideration: infectiousness, transmission, mode, incubation period, etc.

The statistical informations deduced from the ICI simulations will allow the health authorities to objectively compare future effects and risks of various potential strategies to control future epidemics, to determine optimal strategies in terms of territories and groups of individuals, to anticipate the supply of care at local or national level, to help determine the relevant sets of data to be collected in order to develop precise epidemic simulations, etc.

During an epidemic crisis, the ICI platform should also provide spatially localised informations which might usefully complete the global predictions deduced in real time from macroscopic models such as the SEIR compartmental model and its variants.

The ICI application to the "Digital Twin of France" program was supported by the French Agency for Health Innovation (AIS), the General Directorate for Healthcare Provision (DGOS) and the the General Directorate of Health (DGS) at the Ministry of Health, the Ile de France regional health agence (ARS) and by the ANRS Emerging infectious diseases. These institutions are helping the ICI team to define the priority objectives and deliverables of the project.

Web sites

`http://ici.gitlabpages.inria.fr/website`
`https://gitlab.inria.fr/ici/website`
`https://ici.saclay.inria.fr`

7.1.2 Our contribution to the PyCATSHOO toolbox

Participants: Josselin Garnier.

Our second topical activity concerns the PYCATSHOO toolbox developed by EDF which allows the modeling of dynamical hybrid systems such as nuclear power plants or dams. Hybrid systems mix two kinds of behaviour. First, the discrete and stochastic behaviour which is in general due to failures and repairs of the system's constituents. Second, the continuous and deterministic physical phenomena which evolve inside the system.

PyCATSHOO is based on the theoretical framework of Piecewise Deterministic Markov Processes (PDMPs). It implements this framework thanks to distributed hybrid stochastic automata and object-oriented modeling. It is written in C++. Both Python and C++ APIs are available. These APIs can be used either to model specific systems or for generic modelling i.e. for the creation of libraries of component models. Within PyCATSHOO special methods can be developed.

J. Garnier is contributing, and will continue to contribute, to this toolbox within joint Cifre programs with EdF. The PhD theses are aimed to add new functionalities to the platform. For instance, an importance sampling with cross entropy method

8 New results

8.1 Stochastic cooperative numerical methods for complex industrial systems

Participants: Josselin Garnier, Guillaume Chennetier, Paul Lartaud, Charlie Sire.

Bayesian network approach to multi-fidelity surrogate modeling

In [15], Baptiste Kerleguer, Claire Cannamela and Josselin Garnier address the surrogate modeling of a computer code output in a hierarchical multi-fidelity context, i.e., when the output can be evaluated at different levels of accuracy and computational cost. Using observations of the output at low- and high-fidelity levels, they propose a method that combines Gaussian process (GP) regression and the Bayesian neural network (BNN), called the GPBNN method. The low-fidelity output is treated as a single-fidelity code using classical GP regression. The high-fidelity output is approximated by a BNN that incorporates, in addition to the high-fidelity observations, well-chosen realizations of the low-fidelity output emulator. The predictive uncertainty of the final surrogate model is then quantified by a complete characterization of the uncertainties of the different models and their interaction.

Adaptive importance sampling based on fault trees for PDMP

In [7], Guillaume Chennetier, Hassane Chraibi, Anne Dutfoy, and Josselin Garnier introduce an original adaptive importance sampling stratgey based on fault tree analysis for Piecewise Deterministic Markov Processes (PDMPs). PDMPs can be used to model complex dynamical industrial systems. The counterpart of this modeling capability is their simulation cost, which makes reliability assessment untractable with standard Monte Carlo methods. A significant variance reduction can be obtained with an adaptive importance sampling method based on a cross-entropy procedure. The success of this method relies on the selection of a good family of approximations of the committor function of the PDMP. In this paper original families are proposed. Their forms are based on reliability concepts related to fault tree analysis: minimal path sets and minimal cut sets. They are well adapted to high-dimensional industrial systems. The proposed method is discussed in detail and applied to academic systems and to a realistic system from the nuclear industry.

Reduced order modeling for full waveform inversion

In [6], [1], Liliana Borcea, Josselin Garnier, Alexander Mamonov, and Jorn Zimmerling introduce and study a new formulation of a classical inverse problem: Full Waveform Inversion. Full Waveform inversion is concerned with estimating a heterogeneous medium, modeled by variable coefficients of wave equations, using sources that emit probing signals and receivers that record the generated waves. It is an old and intensively studied inverse problem with a wide range of applications, but the existing inversion methodologies are still far from satisfactory. The typical mathematical formulation is a nonlinear least squares data fit optimization and the difficulty stems from the nonconvexity of the objective function that displays numerous local minima at which local optimization approaches stagnate. This pathological behavior has at least three unavoidable causes: (1) The mapping from the unknown coefficients to the wave field is nonlinear and complicated. (2) The sources and receivers typically lie on a single side of the medium, so only backscattered waves are measured. (3) The probing signals are band limited and with high frequency content. There is a lot of activity in the computational science and engineering communities that seeks to mitigate the difficulty of estimating the medium by data fitting. In this paper a different point of view is presented which is based on reduced order models (ROMs) of two operators that control the wave propagation. The ROMs are called data driven because they are computed directly from the measurements, without any knowledge of the wave field inside the inaccessible medium. This computation is noniterative and uses standard numerical linear algebra methods. The resulting ROMs capture features of the physics of wave propagation in a complementary way and have surprisingly good approximation properties that facilitate waveform inversion.

Mean field game model for renewable investment

In [9], Celia Escribe,Josselin Garnier, and Emmanuel Gobet study a stylized model for investment into renewable power plants under long-term uncertainty. They model risk-averse agents facing heterogeneous weather conditions and a common noise including uncertainty on demand trends, future fuel prices and the average national weather conditions. The objective of each agent is to maximize multistage profit by controlling investment in discrete time steps. They analyze this model in a noncooperative game setting with N players, where the interaction among agents occurs through the spot price mechanism. Their model extends to a mean field game with common noise when the number of agents is infinite. They prove that the N -player game admits a Nash equilibrium. Moreover, they prove that under appropriate assumptions, any sequence of Nash equilibria to the N-player game converges to the unique solution of the MFG game. Numerical experiments highlight the impact of the risk aversion parameter and the importance of correctly specifying the distribution of the heterogeneity among agents. Moreover, they demonstrate that the results obtained by their model cannot be replicated by a model based on a representative agent with a unique parameter that would represent homogenized weather conditions. This emphasizes the importance of including explicit modeling of heterogeneity in prospective models when a heterogeneous parameter is expected to have a significant influence on the outcomes.

Sensitivity analysis of interacting particles driven by colored noise

In [11], Josselin Garnier, Harry Ip, and Laurent Mertz propose an efficient sensitivity analysis method for a wide class of colored-noise-driven interacting particle systems (IPSs). Their method is based on unperturbed simulations and significantly extends the Malliavin weight sampling method proposed by Szamel in 2017 for evaluating sensitivities such as linear response functions of IPSs driven by simple Ornstein-Uhlenbeck processes. They show that the sensitivity index depends not only on two effective parameters that characterize the variance and correlation time of the noise, but also on the noise spectrum. In the case of a single particle in a harmonic potential, they obtain exact analytical formulas for two types of linear response functions. By applying their method to a system of many particles interacting via a repulsive screened Coulomb potential, they compute the mobility and effective temperature of the system. Their results show that the system dynamics depend, in a nontrivial way, on the noise spectrum.

Model error quantification for Bayesian calibration

In [29], Gilles Defaux, Cédric Durantin, Josselin Garnier, Baptiste Kerleguer, Guillaume Perrin and Charlie Sire deal with Bayesian calibration, which is used to ensure that numerical simulations accurately reflect the behavior of physical systems. An important issue is the quantification of the model error, i.e., the discrepancy between the model outputs and the observed data. Conventional methods cannot be implemented in transposition situations, such as when a model has multiple outputs but only one is experimentally observed. To account for the model error in this context, the authors augment the calibration process by introducing additional input numerical parameters through a hierarchical Bayesian model, which includes hyperparameters for the prior distribution of the calibration variables. Importance sampling estimators are used to avoid increasing computational costs. Performance metrics are introduced to assess the proposed probabilistic model and the accuracy of its predictions. The method is applied on a computer code with three outputs that models the Taylor cylinder impact test. The outputs are considered as the observed variables one at a time, to work with three different transposition situations. The proposed method is compared with other approaches that embed model errors to demonstrate the significance of the hierarchical formulation.

8.2 Modeling, analysis, and simulation of cooperative stochastic systems

Participants: Quentin Cormier, Carl Graham, Denis Talay.

Communication networks and their algorithms.

Carl Graham studies communication networks and the algorithms (or protocols) used to manage efficiently their resources in real-time in a distributed and cooperative fashion. For instance, load balancing algorithms (LBA) strive to avoid server idleness and queue build-up, and are the topic of a lively example-based litterature.

In [30], Carl Graham has rigorously defined a wide class of LBA for which he has devised perfect simulation methods. The state space is infinite, and the methods use dominated coupling from the past. The dominating process is a network with uniform routing in a coupling preserving a preorder related to the increasing convex order. The use of a preorder is novel in this context. This allows performance evaluation in equilibrium of any LBA in the class using Monte Carlo estimation.

A stochastic numerical method for the parabolic-parabolic Keller-Segel system.

The parabolic-parabolic Keller-Segel model is a set of equations that model the process of cell movement. It takes into account the evolution of different chemical components that can aid, hinder or change the direction of movement, a process called chemotaxis.

In collaboration with Radu Maftei (who is a past Inria post-doc student) Milica Tomasevic (CMAP, Ecole Polytechnique) and Denis Talay have continued to analyse the numerical performances of a stochastic particle numerical method for the parabolic-parabolic Keller-Segel model. They also propose and test various algorithmic improvements to the method in order to substantially decrease its execution time without altering its global accuracy.

An hypothesis test for complex stochastic simulations.

In a joint work with Héctor Olivero [18], D. Talay has proposed and analyzed an asymptotic hypothesis test for independent copies of a given random variable which is supposed to belong to an unknown domain of attraction of a stable law. The null hypothesis $\mathbf{H_0}$ is: '$X = \sqrt{V}$ is in the domain of attraction of the Normal law' and the alternative hypothesis is $\mathbf{H_1}$: 'X is in the domain of attraction of a stable law with index smaller than 2'.

Surprisingly, the proposed hypothesis test is based on a statistic which is inspired by methodologies to determine whether a semimartingale has jumps from the observation of one single path at discrete times. The authors have justified their test by proving asymptotic properties of discrete time functionals of Brownian bridges. They also have discussed many numerical experiments which allowed them to illustrate satisfying properties of the proposed test.

Long time behavior of particle systems and their mean-field limit, application to spiking neural networks

Quentin Cormier has studied the long time behavior of a family of McKean-Vlasov stochastic differential equations. He has given conditions ensuring the local stability of an invariant probability measure. The criterion involves the location of the roots of an explicit holomorphic function associated to the dynamics. When all the roots lie on the left-half plane, local stability holds and convergence is proven in Wasserstein norms. The optimal rate of convergence is provided. This method is then applied to study a large class of models of interacting particles on the torus, see [27]. This work will appear in Annales de l'Institut Henri Poincaré. In addition, he has adapted the method to study the long time behavior of a model of interacting Integrate-and-Fire neurons [8], [3]. This last result provides new insights concerning the bistability behavior of such networks. In particular, a conjecture formulated by P. Robert and J. Touboul regarding the number of stationary solutions is proved. This last work has been published in the journal Mathematical Neuroscience and Applications.

Renewal theorems in a periodic environment

Quentin Cormier has studied a renewal problem within a periodic environment, departing from the classical renewal theory by relaxing the assumption of independent and identically distributed inter-arrival times, see [28]. In this work, the conditional distribution of the next arrival time, given the current one, is governed by a periodic kernel, denoted as H. The periodicity property of H is expressed as $\mathbb{P}(T_{k+1} > t \mid T_k) = H(t, T_k)$, where $H(t+T, s+T) = H(t, s)$. For a fixed time t, we define N_t as the count of events occurring up to time t. The focus is on two temporal aspects: Y_t, the time elapsed since the last event, and X_t, the time until the next event occurs, given by $Y_t = t - T_{N_t}$ and $X_t = T_{N_t+1} - t$. The study explores the long-term behavior of the distributions of X_t and Y_t. As an application, we describe the long time behavior of a spiking neuron subject to a time-periodic input current.

Optimal control under unknown intensity with Bayesian learning

Nicolas Baradel and Quentin Cormier have studied an optimal control problem inspired by neuroscience, where the dynamics is driven by a Poisson process with a controlled stochastic intensity and an uncertain parameter, see [25]. Given a prior distribution for the unknown parameter, we describe its evolution according to Bayes' rule. We reformulate the optimization problem using Girsanov's theorem and establish a dynamic programming principle. Finally, we characterize the value function as the unique viscosity solution to a finite-dimensional Hamilton-Jacobi-Bellman equation, which can be solved numerically.

9 Bilateral contracts and grants with industry

Participants: Josselin Garnier.

9.1 CIROQUO Research & Industry Consortium

Several INRIA teams, including ASCII, are involved in the CIROQUO Research & Industry Consortium – Consortium Industrie Recherche pour l'Optimisation et la QUantification d'incertitude pour les données Onéreuses – (Industry Research Consortium for the Optimization and QUantification of Uncertainty for Onerous Data). Josselin Garnier is the INRIA Saclay representative on the steering committee.

The principle of the CIROQUO Research & Industry Consortium is to bring together academic and technological research partners to solve problems related to the exploitation of numerical simulators, such as code transposition (how to go from small to large scale when only small-scale simulations are possible), taking into account the uncertainties that affect the result of the simulation, validation and calibration (how to validate and calibrate a computer code from collected experimental data).

This project is the result of a simple observation: industries using computer codes are often confronted with similar problems during the exploitation of these codes, even if their fields of application are very diverse. Indeed, the increase in the availability of computing cores is counterbalanced by the growing complexity of the simulations, whose computational times are usually of the order of an hour or a day. In practice, this limits the number of simulations. This is why the development of mathematical methods to make the best use of simulators and the data they produce is a source of progress. The experience acquired over the last thirteen years in the DICE and ReDICE projects and the OQUAIDO Chair shows that the formalization of real industrial problems often gives rise to first-rate theoretical problems that can feed scientific and technical advances.

The creation of the CIROQUO Research & Industry Consortium, led by the Ecole Centrale de Lyon and co-animated with the IFPEN, follows these observations and responds to a desire for collaboration between technological research partners and academics in order to meet the challenges of exploiting large computing codes.

Scientific approach. The limitation of the number of calls to simulators implies that some information – even the most basic information such as the mean value, the influence of a variable or the minimum value of a criterion – cannot be obtained directly by the usual methods. The international scientific community, structured around computer experiments and uncertainty quantification, took up this problem more than twenty years ago, but a large number of problems remain open. On the academic level, this is a dynamic field which is notably the subject of the French CNRS Research Group MascotNum since 2006 and renewed in 2020.

Composition. The CIROQUO Research & Industry Consortium aims to bring together a limited number of participants in order to make joint progress on test cases from the industrial world and on the upstream research that their treatment requires. The overall approach that it will focus on is metamodeling and related areas such as experiment planning, optimization, inversion and calibration. IRSN, STORENGY, CEA, IFPEN, and BRGM are the Technological Research Partners and Mines Saint-Etienne, Centrale Lyon, CNRS, UCA, UPS, UT3 and Inria the Academic Partners of the consortium.

Scientific objectives. On the practical level, the expected impacts of the project are a concretization of the progress of numerical simulation by a better use of computational time, which allows the determination of better solutions and associated uncertainties. On the theoretical level, this project will allow to create an emulation around the major scientific locks of the discipline such as code transposition/calibration/validation, modeling for complex environments, or stochastic codes. In each of these scientific axes, a particular attention will be paid to the large dimension. Real problems sometimes involve several tens or hundreds of inputs. Methodological advances will be proposed to take into account this additional difficulty. The work expected from the consortium differs from the dominant research in machine learning by specificities linked to the exploration of expensive numerical simulations. However, it seems important to build bridges between the many recent developments in machine learning and the field of numerical simulation.

Philosophy. The CIROQUO Research & Industry Consortium is a scientific collaboration project aiming to mobilize means to achieve methodological advances. The project promotes cross-fertilization between partners coming from different backgrounds but confronted with problems related to a common methodology. Its main goals are:

- The development of exchanges between technological research partners and academic partners on issues, practices and solutions through periodic scientific meetings and collaborative work, particularly through the co-supervision of students,

- The contribution of common scientific skills thanks to regular training in mathematics and computer science,

- The recognition of the Consortium at the highest level thanks to publications in international journals and the diffusion of free reference software.

Example of a case study. Calibrating a numerical code means determining the best values for the input parameters by comparing its predictions with the experimental data available. In some cases, physical experiments cannot be carried out at full scale or on all the observables we are interested in. Only small scale or partial experiments can be carried out, and these are known as simplified experiments. The aim of Charlie Sire's postdoc was to develop a methodology, known as transposition, enabling a code to be calibrated in the context of a simplified experiment and then transposed to the real case. The postdoc was short (10 months) due to Charlie Sire finding a position at Ecole des Mines. In spite of this, [29] has been written in collaboration with CEA (who brought a real test case, a Taylor cylinder impact test) and submitted in October 2024.

Extension. CIROQUO was supposed to end in December 2024, but has been extended for four years under the same terms. New partners will be included, in particular, EDF (Électricité de France).

9.2 Collaboration with EdF on industrial risks

This collaboration has been going on for several years, with Josselin Garnier as the leader for ASCII. It concerns the assessment of the reliability of hydraulic and nuclear power plants built and operated by EDF (Électricité de France). The failure of a power plant may have major consequences, such as floods, dam failures, or core meltdowns. For regulatory and safety reasons EDF must ensure that the probability of failure of a power plant is suitably small.

The failure of such systems occurs when the values of certain physical variables (temperature, pressure, water level) exceed some critical thresholds. Typically, these variables enter this critical region when several components of the system are deteriorated. Therefore, in order to estimate the probability of system failure, it is necessary to model jointly the behavior of the components and of the physical variables.

For this purpose, a model based on Piecewise Deterministic Markovian Piecewise Processes (PDMP) is devised and used. The platform called PYCATSHOO has been developed by EDF in order to simulate this type of process. This platform allows to estimate the probability of failure of the system by Monte Carlo simulation as long as the probability of failure is not too low. When the probability becomes too low, the classical Monte Carlo estimation method requires a very large number of simulations in order to estimate the probabilities of such rare events, and is much too slow to be used in our context. It is then necessary to use methods using fewer simulations to estimate the probability of system failure, such as variance reduction methods. Among the variance reduction methods are “importance sampling” and “splitting” methods, but these methods present difficulties when used with PDMPs.

This had first lead to the defense of a CIFRE thesis by Thomas Galtier in 2019. It was followed by a CIFRE thesis by Guillaume Chennetier, defended on September 24, 2024, after which he moved to a postdoc position at Ecole des Ponts. In his thesis Guillaume has proposed new methods for estimating rare event probabilities for PDMPs, which offer the flexibility needed to represent complex dynamic industrial systems. The industrial challenge was to enable the tool PyCATSHOO, used by EDF for its probabilistic safety assessment studies, to efficiently estimate the failure probability of such systems with guaranteed accuracy. Guillaume has proposed a theoretical framework for implementing importance sampling of PDMPs, and has highlighted the connection between the optimal biased distribution and the so-called “committor function" of the process. Using tools from reliability analysis and the theory of random walks on graphs, new families of approximations of the committor function were introduced in the thesis. The proposed methodology is adaptive: an approximation of the committor function is constructed a priori and then refined during the simulations of a cross-entropy procedure. The simulations are then recycled to produce an importance sampling estimator of the target probability. Convergence results have been obtained, making it possible to overcome the dependence between simulations and to construct asymptotic confidence intervals. The method produced excellent results on the tested industrial systems. Theoretical articles have been written and submitted to journals.

10 Partnerships and cooperations

Participants: Quentin Cormier, Philip Protter, René Carmona, Mete Soner.

10.1 International initiatives

10.1.1 Inria associate team not involved in an IIL or an international program

CIRCUS

Title: Columbia Inria Research on Collaborative Ultracritical Systems

Duration: 2020 ->

Coordinator: Philip Protter (pep2117@columbia.edu)

Partners:

- Columbia University (États-Unis)

Inria contact: Quentin Cormier

Summary: CIRCUS will focus on collaborative stochastic agent and particle systems. In standard models, the agents and particles have ‘blind’ interactions generated by an external interaction kernel or interaction mechanism which their empirical distribution does not affect. A contrario, agent and particle systems which will be developed, analysed, simulated by CIRCUS will have the key property that the agents and particles dynamically optimize their interactions.

Two main directions of research will be investigated: optimal regulation in stochastic environments, and optimized simulations of particle systems with singular interactions. In both cases, the interactions (between the agents or the numerical particles) are optimized, non Markovian, and the singularities reflect ultracritical phenomena such as aggregations or finite-time blow-ups.

10.2 International research visitors

10.2.1 Visits to international teams

As part as the CIRCUS associate team, Quentin Cormier has visited René Carmona and Mete Soner for ten days in November 2024 at Princeton. During this stay, they continued to collaborate on the study of synchronization phenomena in mean-field games.

So far, they have studied a mean-field game version of the Kuramoto model in the case where all agents (oscillators) are identical. A phase transition is observed: when the noise is sufficiently low, the agents, despite being selfish, synchronize.

They now aim to study the case where the oscillators are heterogeneous. In this scenario, the oscillators do not naturally oscillate at the same frequency; one only assume that the distribution of the oscillators' natural frequencies is known. The goal is to analyze the Nash equilibria of the system and their long-term stability as a function of this distribution. Preliminary results suggest that when the frequency distribution is Gaussian, the Nash equilibria of the system are qualitatively similar to the homogeneous case. For other types of distributions, new phenomena arise, particularly the existence of traveling waves.

10.3 National initiatives

Quentin Cormier visited Julien Tugaut at Université Jean Monnet (UJM, Saint-Étienne). They work together on the long time behavior of some McKean-Vlasov equations.

11 Dissemination

Participants: Quentin Cormier, Josselin Garnier, Carl Graham, Denis Talay, Maxime Colomb.

11.1 Promoting scientific activities

11.1.1 Scientific events: organisation

Carl Graham was member of the Organizing Committee for the "Conférence pour les 50 ans du CMAP" in Palaiseau, France, 11–13 September 2024.

11.1.2 Scientific events: selection

J. Garnier was a member of the scientific committee of the Joint Chinese-French Conference on Partial Differential Equations and Its Applications (Hong Kong, 24 -28 November 2024).

11.1.3 Journal

J. Garnier is a member of the editorial boards of the journals *Asymptotic Analysis*, *Discrete and Continuous Dynamical Systems – Series S, ESAIM P&S, SIAM Journal on Applied Mathematics*, and *SIAM/ASA Journal on Uncertainty Quantification (JUQ)*.

D. Talay is serving as an Area Editor of *Stochastic Processes and their Applications* and as an Associate Editor of *Journal of the European Mathematical Society, SMAI Journal of Computational Mathematics*, and *Monte Carlo Methods and Applications.* He is also serving as Co-editor in chief of *MathematicS in Action.*

C. Graham is member of the editorial board of *Markov Processes and Related Fields.*

11.1.4 Invited talks

J. Garnier was a speaker at:

- GDR Complexe Annual Meeting (December 2024),

- 14th AIMS Conference at Abu Dhabi (December 2024),
- Stochastic Analysis and Mathematical Finance Seminar at Oxford (May 2024),
- International Conference on Applied Mathematics at Hong Kong (May 2024),
- Mathematics Colloquium at New York University Abu Dhabi (April 2024),
- Colloquium CEREMADE at Paris Dauphine University (February 2024).

Denis Talay presented the ICI platform at the Workshop on Spatial Epidemic Models (Including Graphs and Graphons) May 29-31, 2024 at Rice Global Paris Center. He also gave an invited lecture at the Third Biostochastic Workshop on March 25-28th, 2024 in Valparaíso (Chile). During this workshop a ceremony, launched by the university of Valparaiso and Inria Chile, celebrated his long-time cooperation with Prof. R. Rebolledo (university of Valparaiso) and their several successful collaborative programs which allowed numerous exchanges between French and Chilean students and researchers in Probability.

Maxime Colomb gave a talk “Le projet ICI - Modélisations épidémiologiques pour évaluer diverses interventions sanitaires” at the “33ème Journée de la recherche sur le thème du Jumeau numérique de la France - Faire commun avec les jumeaux thématiques et territoriaux” of Université Gustave Eiffel, l’IGN, and l’ENSG-Géomatique, in Champs-sur-Marne on 28 March 2024.

Quentin Cormier gave an invited talk at the Séminaire de Mathématiques Appliquées of College de France. He gave a talk at the King’s College probability seminar in London. He participated to the groupe de travail "Rencontre McKean-Vlasov" in Jussieu, the 7 and 8 October 2024. He also gave a talk at ORFE during his visit at Princeton and a talk as part of the master MSV of Ecole Polytechnique.

11.1.5 Scientific expertise

J. Garnier is a member of the Comité de Prospective en Energie (Energy Foresight Committee) of the Académie des Sciences. This committee conducts in-depth discussions on energy, offering its scientific expertise to decision-makers and providing each citizen with the best possible insight into this constantly topical issue.

11.1.6 Research administration

D. Talay reported on proposals submitted to the Research Grants Council (RGC) of Hong Kong.
D. Talay is a member of the Comité National Français de Mathématiciens.
D. Talay is a member of the scientific committee of the national agency AMIES aimed to promote the collaborations between the mathematicans and the industry. D. Talay is also Vice-president of the Scientific and Administration Councils of the Natixis Foundation which supports academic research in financial mathematics, notably on risk analyses, and on machine learning applied to mathematical problems in economy, finance, and insurance.

11.2 Teaching - Supervision - Juries

11.2.1 Teaching

Josselin Garnier is a professor at the Ecole Polytechnique with a full teaching service. He also teaches the class "Inverse problems and imaging" at the Master Mathématiques, Vision, Apprentissage (M2 MVA)

Quentin Cormier, teaches the course “Probability: Stochastic Processes” at the Bachelor program of Ecole Polytechnique. Together with Romain Veltz (Inria Sophia-Antipolis), he also teaches the M2 course “Some Mathematical Methods for Neurosciences” at the Master Mathématiques, Vision, Apprentissage (M2 MVA).

Maxime Colomb is in charge of the module “Données massives spatialisées” for the MEDAS Master of the CNAM.

11.2.2 Supervision

D. Talay served as a member of the External Supervisory Committee of the Mathematics department of the University of Porto (Portugal).

Current PhD students of J. Garnier: Ali Abboud, Nils Baillie, Raphael Carpintero Perez (with S. Da Veiga and B. Staber), Paul Casteras (with J. Bect), Fatima-Zahrae El-Boukkouri (with O. Roustant), David Iagaru, Antoine Van Biesbroeck, Thomas Wasik (with A. Aubry).

Quentin Cormier supervises since September 2024 the PhD thesis of Samuel Chan-Ashing, jointly with Djalil Chafaï (Paris Dauphine and ENS Paris).

11.2.3 Juries

The team members were part of many PhD thesis juries.

12 Scientific production

12.1 Major publications

[1] L. Borcea, J. Garnier, A. Mamonov and J. Zimmerling. 'When Data Driven Reduced Order Modeling Meets Full Waveform Inversion'. In: *SIAM Review* 66.3 (8th Aug. 2024), pp. 501–532. DOI: 10.1137/23M1552826. URL: https://hal.science/hal-04686183 (cit. on p. 7).

[2] M. Colomb, Q. Cormier, J. Garnier, C. Graham, J. Perret and D. Talay. 'ICI Project: Individual-based epidemic propagation simulator based on a Digital Twin'. In: Health GIS: Spatial Thinking in Applied research 2024. Calgari, Canada, 20th Nov. 2024. URL: https://hal.science/hal-04794960.

[3] Q. Cormier. 'A mean-field model of Integrate-and-Fire neurons: non-linear stability of the stationary solutions'. In: *Mathematical Neuroscience and Applications* (27th Sept. 2024). DOI: 10.46298/mna.12583. URL: https://inria.hal.science/hal-02488915 (cit. on p. 9).

12.2 Publications of the year

International journals

[4] N. Baradel. 'Modeling frequency distribution above a priority in presence of IBNR'. In: *Scandinavian Actuarial Journal* (12th Dec. 2024), pp. 1–16. DOI: 10.1080/03461238.2024.2439815. URL: https://hal.science/hal-04903637.

[5] L. Borcea and J. Garnier. 'Enhanced wave transmission in random media with mirror symmetry'. In: *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences* 480.2292 (26th June 2024). DOI: 10.1098/rspa.2024.0073. URL: https://hal.science/hal-04686182.

[6] L. Borcea, J. Garnier, A. Mamonov and J. Zimmerling. 'When Data Driven Reduced Order Modeling Meets Full Waveform Inversion'. In: *SIAM Review* 66.3 (8th Aug. 2024), pp. 501–532. DOI: 10.1137/23M1552826. URL: https://hal.science/hal-04686183 (cit. on pp. 4, 7).

[7] G. Chennetier, H. Chraibi, A. Dutfoy and J. Garnier. 'Adaptive Importance Sampling Based on Fault Tree Analysis for Piecewise Deterministic Markov Process'. In: *SIAM/ASA Journal on Uncertainty Quantification* 12.1 (7th Mar. 2024), pp. 128–156. DOI: 10.1137/22M1522838. URL: https://hal.science/hal-04686162 (cit. on p. 7).

[8] Q. Cormier. 'A mean-field model of Integrate-and-Fire neurons: non-linear stability of the stationary solutions'. In: *Mathematical Neuroscience and Applications* (27th Sept. 2024). DOI: 10.46298/mna.12583. URL: https://inria.hal.science/hal-02488915 (cit. on p. 9).

[9] C. Escribe, J. Garnier and E. Gobet. 'A Mean Field Game Model for Renewable Investment under Long-Term Uncertainty and Risk Aversion'. In: *Dynamic Games and Applications* (2nd Mar. 2024). DOI: 10.1007/s13235-024-00554-x. URL: https://hal.science/hal-04055421 (cit. on p. 8).

[10] J. Garnier, H. Haddar and H. Montanelli. 'The linear sampling method for data generated by small random scatterers'. In: *SIAM Journal on Imaging Sciences* 17.4 (2024), pp. 2142–2173. DOI: 10.1137/24M1650417. URL: https://hal.science/hal-04526042.

[11] J. Garnier, H. Ip and L. Mertz. 'Sensitivity analysis of colored-noise-driven interacting particle systems'. In: *Physical Review E* 110.4 (17th Oct. 2024), p. 044119. DOI: 10.1103/PhysRevE.110.044119. URL: https://hal.science/hal-04854215 (cit. on p. 8).

[12] J. Garnier and K. Sølna. 'Shower curtain effect and source imaging'. In: *Inverse Problems and Imaging* 18.4 (2024), pp. 993–1023. DOI: 10.3934/ipi.2024004. URL: https://hal.science/hal-04686175.

[13] C. Gauchy, C. Feau and J. Garnier. 'UNCERTAINTY QUANTIFICATION AND GLOBAL SENSITIVITY ANALYSIS OF SEISMIC FRAGILITY CURVES USING KRIGING'. In: *International Journal for Uncertainty Quantification* 14.4 (2024), pp. 39–63. DOI: 10.1615/Int.J.UncertaintyQuantification.2023046480. URL: https://hal.science/hal-04902693.

[14] M. de Hoop, J. Garnier and K. Sølna. 'Three-Dimensional Random Wave Coupling Along a Boundary and an Associated Inverse Problem'. In: *Multiscale Modeling and Simulation: A SIAM Interdisciplinary Journal* 22.1 (9th Jan. 2024), pp. 39–65. DOI: 10.1137/23M1544842. URL: https://hal.science/hal-04686154.

[15] B. Kerleguer, C. Cannamela and J. Garnier. 'A Bayesian neural network approach to Multi-fidelity surrogate modelling'. In: *International Journal for Uncertainty Quantification* 14.1 (2024), pp. 43–60. DOI: 10.1615/Int.J.UncertaintyQuantification.2023044584. URL: https://hal.science/hal-03608580 (cit. on p. 7).

[16] P. Lartaud, P. Humbert and J. Garnier. 'Uncertainty quantification in Bayesian inverse problems with neutron and gamma time correlation measurements'. In: *Annals of Nuclear Energy* 213 (Apr. 2025), p. 111123. DOI: 10.1016/j.anucene.2024.111123. URL: https://hal.science/hal-04854205.

[17] A. Niclas and J. Garnier. 'Automated approach for recovering modal components in shallow waters'. In: *Journal of the Acoustical Society of America* 155.4 (1st Apr. 2024), pp. 2347–2358. DOI: 10.1121/10.0025471. URL: https://hal.science/hal-04686158.

[18] H. Olivero and D. Talay. 'A hypothesis test for the domain of attraction of a random variable'. In: *ESAIM: Probability and Statistics* 28 (10th Sept. 2024), pp. 292–328. DOI: 10.1051/ps/2024010. URL: https://inria.hal.science/hal-03817893 (cit. on p. 9).

[19] A. Richard and D. Talay. 'Lipschitz continuity in the Hurst parameter of functionals of stochastic differential equations driven by a fractional Brownian motion'. In: *Electronic Journal of Probability* 29.none (1st Jan. 2024). DOI: 10.1214/24-EJP1191. URL: https://inria.hal.science/hal-01323288.

International peer-reviewed conferences

[20] M. Colomb, Q. Cormier, J. Garnier, C. Graham, J. Perret and D. Talay. 'ICI Project: Individual-based epidemic propagation simulator based on a Digital Twin'. In: Health GIS: Spatial Thinking in Applied research 2024. Calgari, Canada, 20th Nov. 2024. URL: https://hal.science/hal-04794960.

Conferences without proceedings

[21] M. Colomb, Q. Cormier, C. Graham, J. Perret and D. Talay. 'ICI Project - use of a digital twin for epidemiological simulations'. In: Journée de la recherche de l'IGN 2024. Champs sur Marme, France, 28th Mar. 2024. URL: https://hal.science/hal-04654383.

[22] A. Niclas and J. Garnier. 'Automated approach for source location in shallow water'. In: WAVES 2024 - The 16th International Conference on Mathematical and Numerical Aspects of Wave Propagation. Berlin, France, 30th June 2024. URL: https://hal.science/hal-04712223.

[23] C. Sire, R. Le Riche, D. Rullière, J. Rohmer, L. Pheulpin and Y. Richet. 'Augmented Quantization: a General Approach to Mixture Models'. In: UQ 2024 - SIAM Conference on Uncertainty Quantification. Trieste, Italy, 27th Feb. 2024. URL: https://hal.science/hal-04527349.

Reports & preprints

[24] N. Baradel. *Continuous-time modeling and bootstrap for chain ladder reserving*. 14th Nov. 2024. URL: https://hal.science/hal-04903677.

[25] N. Baradel and Q. Cormier. *Optimal control under unknown intensity with Bayesian learning*. 7th Nov. 2024. URL: https://inria.hal.science/hal-04826347 (cit. on p. 10).

[26] R. Carpintero Perez, S. da Veiga, J. Garnier and B. Staber. *Gaussian process regression with Sliced Wasserstein Weisfeiler-Lehman graph kernels*. 7th Mar. 2024. URL: https://hal.science/hal-04440186.

[27] Q. Cormier. *On the stability of the invariant probability measures of McKean-Vlasov equations*. 30th Apr. 2024. URL: https://inria.hal.science/hal-03896874 (cit. on p. 9).

[28] Q. Cormier. *Renewal theorems in a periodic environment*. 12th Mar. 2024. URL: https://inria.hal.science/hal-04826340 (cit. on p. 9).

[29] G. Defaux, C. Durantin, J. Garnier, B. Kerleguer, G. Perrin and C. Sire. *Bayesian Calibration in a multi-output transposition context*. 2024. DOI: 10.48550/arXiv.2410.00116. URL: https://hal.science/hal-04717715 (cit. on pp. 8, 11).

[30] C. Graham. *Perfect simulation of Markovian load balancing queueing networks in equilibrium*. 3rd July 2024. URL: https://polytechnique.hal.science/hal-04633838 (cit. on p. 9).

[31] H. Olivero and D. Talay. *Supplementary Material to "A hypothesis test for the domain of attraction of a random variable"*. 28th Aug. 2024. URL: https://hal.science/hal-04266438.

Other scientific publications

[32] M. Colomb, Q. Cormier, J. Garnier, C. Graham, J. Perret and D. Talay. 'ICI: a data-driven individual-based epidemic propagation simulator'. In: Workshop on Spatial Epidemic Models (Including Graphs and Graphons). Paris, France, 29th May 2024. URL: https://hal.science/hal-04654431.