

RESEARCH CENTRE

**Inria Paris Centre at Sorbonne  
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2024

ACTIVITY REPORT

Project-Team

CAGE

## **Control and Geometry**

IN COLLABORATION WITH: Laboratoire Jacques-Louis Lions (LJLL)

### **DOMAIN**

**Applied Mathematics, Computation and  
Simulation**

### **THEME**

**Optimization and control of dynamic  
systems**

*Inria*

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## Project-Team CAGE

*Creation of the Project-Team: 2018 August 01*

### Keywords

#### Computer sciences and digital sciences

- A6. – Modeling, simulation and control
- A6.1. – Methods in mathematical modeling
- A6.1.1. – Continuous Modeling (PDE, ODE)
- A6.4. – Automatic control
- A6.4.1. – Deterministic control
- A6.4.3. – Observability and Controlability
- A6.4.4. – Stability and Stabilization
- A6.4.5. – Control of distributed parameter systems
- A6.4.6. – Optimal control

#### Other research topics and application domains

- B2. – Health
- B2.6. – Biological and medical imaging
- B4.2.2. – Fusion
- B5.2.4. – Aerospace
- B5.11. – Quantum systems

# 1 Team members, visitors, external collaborators

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## Faculty Members

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- Jean-Michel Coron [UNIV PARIS, Professor]
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- Jeremy Martin [INRIA, Post-Doctoral Fellow, until Aug 2024]
- Jingrui Niu [INRIA, Post-Doctoral Fellow]
- Tommaso Rossi [SORBONNE UNIVERSITE, Post-Doctoral Fellow]
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- Bettina Kazandjian [PLYTECH SORBONNE, from Oct 2024, Encadrants: Ugo Boscain (CAGE), Eugenio Pozzoli (CNRS), Mario Sigalotti (CAGE)]
- Xiangyu Ma [SORBONNE UNIVERSITE, Encadrants: Ugo Boscain (CAGE), Dario Prandi (CNRS), Giuseppina Turco (CNRS)]
- Rayane Mouhli [UNIV PARIS - CITE, Encadrants: Barbara Gris (CAGE), Irène Kaltenmark (UPC)]
- Robin Roussel [SORBONNE UNIVERSITE]
- Liang Ruikang [POLYTECH SORBONNE]
- Lia Sela [Sorbonne Université, from Sep 2024, Encadrants: Emmanuel Trélat (CAGE), Jean Clairambault (SU), Jean-Philippe Foy (SU)]
- Lucia Tessarolo [SORBONNE UNIVERSITE]

## Interns and Apprentices

- Bettina Kazandjian [INRIA, Intern, from Mar 2024 until Jul 2024]
- Nicolas Llorens [INRIA, Intern, from May 2024 until Aug 2024]
- Guenole Cocou Odah [INRIA, Intern, from Jul 2024 until Sep 2024]

## Administrative Assistant

- Laurence Bourcier [INRIA]

## Visiting Scientist

- Andrey Agrachev [SISSA, from Mar 2024 until May 2024]

## 2 Overall objectives

CAGE's activities take place in the field of mathematical control theory, with applications in several directions: control of quantum mechanical systems, stability and stabilization, in particular in presence of uncertain dynamics, optimal control, and geometric models for vision. Although control theory is nowadays a mature discipline, it is still the subject of intensive research because of its crucial role in a vast array of applications. Our focus is on the analytical and geometrical aspects of control applications.

At the core of the scientific activity of the team is the **geometric control** approach, that is, a distinctive viewpoint issued in particular from (elementary) differential geometry, to tackle questions of controllability, motion planning, stability, and optimal control. The emphasis of such a geometric approach is in intrinsic properties, and it is particularly well adapted to study nonlinear and nonholonomic phenomena [90, 66]. The geometric control approach has historically been associated with the development of finite-dimensional control theory. However, its impact in the study of distributed parameter control systems and, in particular, systems of **controlled partial differential equations** has been growing in the last decades, complementing analytical and numerical approaches by providing dynamical, qualitative, and intrinsic insight [82]. CAGE has the ambition to be at the core of this development.

One of the features of the geometric control approach is its capability of exploiting **symmetries and intrinsic structures** of control systems. Symmetries and intrinsic structures (e.g., Lagrangian or Hamiltonian structures) can be used to characterize minimizing trajectories, prove regularity properties, and describe invariants. The geometric theory of **quantum control**, in particular, exploits the rich geometric structure encoded in the Schrödinger equation to design adapted control schemes and to characterize their qualitative properties.

## 3 Research program

### 3.1 Research domain

Our contributions are in the area of **mathematical control theory**, which is to say that we are interested in the analytical and geometrical aspects of control applications. In this approach, a control system is modeled by a system of equations (of many possible types: ordinary differential equations, partial differential equations, stochastic differential equations, difference equations, . . .), possibly not explicitly known in all its components, which are studied in order to establish qualitative and quantitative properties concerning the actuation of the system through the control.

**Motion planning** is, in this respect, a cornerstone property: it denotes the design and validation of algorithms for identifying a control law steering the system from a given initial state to (or close to) a target one. Initial and target positions can be replaced by sets of admissible initial and final states as, for instance, in the motion planning task towards a desired periodic solution. Many specifications can be added to the pure motion planning task, such as robustness to external or endogenous disturbances, obstacle avoidance or penalization criteria. A more abstract notion is that of **controllability**, which denotes the property of a system for which any two states can be connected by a trajectory corresponding to an admissible control law. In mathematical terms, this translates into the surjectivity of the so-called **end-point map**, which associates with a control and an initial state the final point of the corresponding trajectory. The analytical and topological properties of endpoint maps are therefore crucial in analyzing the properties of control systems.

One of the most important additional objective which can be associated with a motion planning task is **optimal control**, which corresponds to the minimization of a cost (or, equivalently, the maximization

of a gain) [118]. Optimal control theory is clearly deeply interconnected with calculus of variations, even if the non-interchangeable nature of the time-variable results in some important specific features, such as the occurrence of **abnormal extremals** [94]. Research in optimal control encompasses different aspects, from numerical methods to dynamic programming and non-smooth analysis, from regularity of minimizers to high order optimality conditions and curvature-like invariants.

Another domain of control theory with countless applications is **stabilization**. The goal in this case is to make the system converge towards an equilibrium or some more general safety region. The main difference with respect to motion planning is that here the control law is constructed in feedback form. One of the most important properties in this context is that of **robustness**, i.e., the performance of the stabilization protocol in presence of disturbances or modeling uncertainties. A powerful framework which has been developed to take into account uncertainties and exogenous non-autonomous disturbances is that of hybrid and switched systems [105, 95, 111]. The central tool in the stability analysis of control systems is that of **control Lyapunov function**. Other relevant techniques are based on algebraic criteria or dynamical systems. One of the most important stability property which is studied in the context of control system is **input-to-state stability** [109], which measures how sensitive the system is to an external excitation.

One of the areas where control applications have nowadays the most impressive developments is in the field of **biomedicine and neurosciences**. Improvements both in modeling and in the capability of finely actuating biological systems have concurred in increasing the popularity of these subjects. Notable advances concern, in particular, identification and control for biochemical networks [103] and models for neural activity [87]. Therapy analysis from the point of view of optimal control has also attracted a great attention [107].

Biological models are not the only one in which stochastic processes play an important role. Stock-markets and energy grids are two major examples where optimal control techniques are applied in the non-deterministic setting. Sophisticated mathematical tools have been developed since several decades to allow for such extensions. Many theoretical advances have also been required for dealing with complex systems whose description is based on **distributed parameters** representation and **partial differential equations**. Functional analysis, in particular, is a crucial tool to tackle the control of such systems [115].

Let us conclude this section by mentioning another challenging application domain for control theory: the decision by the European Union to fund a flagship devoted to the development of quantum technologies is a symptom of the role that quantum applications are going to play in tomorrow's society. **Quantum control** is one of the bricks of quantum engineering, and presents many peculiarities with respect to standard control theory, as a consequence of the specific properties of the systems described by the laws of quantum physics. Particularly important for technological applications is the capability of inducing and reproducing coherent state superpositions and entanglement in a fast, reliable, and efficient way [88].

### 3.2 Scientific foundations

At the core of the scientific activity of the team is the **geometric control** approach. One of the features of the geometric control approach is its capability of exploiting **symmetries and intrinsic structures** of control systems. Symmetries and intrinsic structures can be used to characterize minimizing trajectories, prove regularity properties and describe invariants. An egregious example is given by mechanical systems, which inherently exhibit Lagrangian/Hamiltonian structures which are naturally expressed using the language of symplectic geometry [78]. The geometric theory of quantum control, in particular, exploits the rich geometric structure encoded in the Schrödinger equation to engineer adapted control schemes and to characterize their qualitative properties. The Lie–Galerkin technique that we proposed starting in [79] builds on this premises in order to provide powerful tests for the controllability of quantum systems defined on infinite-dimensional Hilbert spaces.

Although the focus of geometric control theory is on qualitative properties, its impact can also be disruptive when it is used in combination with quantitative analytical tools, in which case it can dramatically improve the computational efficiency. This is the case in particular in optimal control. Classical optimal control techniques (in particular, Pontryagin Maximum Principle, conjugate point theory, associated numerical methods) can be significantly improved by combining them with powerful modern techniques of geometric optimal control, of the theory of numerical continuation, or of dynamical

system theory [114, 106]. Geometric optimal control allows the development of general techniques, applying to wide classes of nonlinear optimal control problems, that can be used to characterize the behavior of optimal trajectories and in particular to establish regularity properties for them and for the cost function. Hence, geometric optimal control can be used to obtain powerful optimal synthesis results and to provide deep geometric insights into many applied problems. Numerical optimal control methods with geometric insight are in particular important to handle subtle situations such as rigid optimal paths and, more generally, optimal syntheses exhibiting abnormal minimizers.

Optimal control is not the only area where the geometric approach has a great impact. Let us mention, for instance, motion planning, where different geometric approaches have been developed: those based on the **Lie algebra** associated with the control system [99, 96], those based on the differentiation of nonlinear flows such as the **return method** [83, 84], and those exploiting the **differential flatness** of the system [86].

Geometric control theory is not only a powerful framework to investigate control systems, but also a useful tool to model and study phenomena that are not *a priori* control-related. Two occurrences of this property play an important role in the activities of CAGE:

- geometric control theory as a tool to investigate properties of mathematical structures;
- geometric control theory as a modeling tool for neurophysical phenomena and for synthesizing biomimetic algorithms based on such models.

Examples of the first type, concern, for instance, hypoelliptic heat kernels [65] or shape optimization [69]. Examples of the second type are inactivation principles in human motricity [71] or neurogeometrical models for image representation of the primary visual cortex in mammals [76].

A particularly relevant class of control systems, both from the point of view of theory and applications, is characterized by the linearity of the controlled vector field with respect to the control parameters. When the controls are unconstrained in norm, this means that the admissible velocities form a distribution in the tangent bundle to the state manifold. If the distribution is equipped with a point-dependent quadratic form (encoding the cost of the control), the resulting geometrical structure is said to be **sub-Riemannian**. Sub-Riemannian geometry appears as the underlying geometry of nonlinear control systems: in a similar way as the linearization of a control system provides local informations which are readable using the Euclidean metric scale, sub-Riemannian geometry provides an adapted non-isotropic class of lenses which are often much more informative. As such, its study is fundamental for control design. The importance of sub-Riemannian geometry goes beyond control theory and it is an active field of research both in differential geometry [98], geometric measure theory [67] and hypoelliptic operator theory [72].

## 4 Application domains

### 4.1 First axis: Quantum control

Quantum control is one of the bricks of quantum engineering, since manipulation of quantum mechanical systems is ubiquitous in applications such as quantum computation, quantum cryptography, and quantum sensing (in particular, imaging by nuclear magnetic resonance).

Quantum control presents many peculiarities with respect to standard control theory, as a consequence of the specific properties of the systems described by the laws of quantum physics. Particularly important for technological applications is the capability of inducing and reproducing coherent state superpositions and entanglement in a fast, reliable, and efficient way. The efficiency of the control action has a dramatic impact on the quality of the coherence and the robustness of the required manipulation. Minimal time constraints and interaction of time scales are important factors for characterizing the efficiency of a quantum control strategy. CAGE works for the improvement of quantum control paradigms, especially for what concerns quantum systems evolving in infinite-dimensional Hilbert spaces. The controllability of quantum system is a well-established topic when the state space is finite-dimensional [85], thanks to general controllability methods for left-invariant control systems on compact Lie groups [77, 91]. When the state space is infinite-dimensional, it is known that in general the bilinear Schrödinger equation is not exactly controllable [116]. The Lie–Galerkin technique [79] combines finite-dimensional geometric control techniques and the distributed parameter framework in order to provide the most

powerful available tests for the approximate controllability of quantum systems defined on infinite-dimensional Hilbert spaces. Another important technique to the development of which we contribute is **adiabatic quantum control**. Adiabatic approximation theory and, in particular, adiabatic evolution [100, 112, 119] is well-known to improve the robustness of the control strategy and is strongly related to time scales analysis. The advantage of the adiabatic control is that it is constructive and produces control laws which are both smooth and robust to parameter uncertainty [120, 93, 75].

## 4.2 Second axis: Stability and stabilization

A control application with a long history and still very challenging open problems is **stabilization**. For infinite-dimensional systems, in particular nonlinear ones, the richness of the possible functional analytical frameworks makes feedback stabilization a challenging and active domain of research. Of particular interest are the different types of stabilization that may be obtained: exponential, polynomial, finite-time, ... Another important aspect of stabilization concerns control of systems with uncertain dynamics, i.e., with dynamics including possibly non-autonomous parameters whose value and dependence on time cannot be anticipated. **Robustification**, i.e., offsetting uncertainties by suitably designing the control strategy, is a widespread task in automatic control theory, showing up in many applicative domains such as electric circuits or aerospace motion planning. If dynamics are not only subject to static uncertainty, but may also change as time goes, the problem of controlling the system can be recast within the theory of switched and hybrid systems, both in a deterministic and in a probabilistic setting. **Switched and hybrid systems** constitute a broad framework for the description of the heterogeneous systems in which continuous dynamics (typically pertaining to physical quantities) interact with discrete/logical components. The development of the switched and hybrid paradigm has been motivated by a broad range of applications, including automotive and transportation industry [108], energy management [101] and congestion control [97]. Even if both controllability [110] and observability [92] of switched and hybrid systems raise several important research issues, the central role in their study is played by uniform stability and stabilizability [95, 111]. Uniformity is considered with respect to all signals in a given class, and it is well-known that stability of switched systems depends not only on the dynamics of each subsystem but also on the properties of the considered class of switching signals. In many situations it is interesting for modeling purposes to specify the features of the switched system by introducing **constrained switching rules**. A typical constraint is that each mode is activated for at least a fixed minimal amount of time, called the dwell-time. Our approach to constrained switching is based on the idea of relating the analytical properties of the classes of constrained switching laws (shift-invariance, compactness, closure under concatenation, ...) to the stability behavior of the corresponding switched systems. One can introduce **probabilistic uncertainties** by endowing the classes of admissible signals with suitable probability measures. The interest of this approach is that probabilistic stability analysis filters out highly 'exceptional' worst-case trajectories. Although less explicitly characterized from a dynamical viewpoint than its deterministic counterpart, the probabilistic notion of uniform exponential stability can be studied using several reformulations of Lyapunov exponents proposed in the literature [70, 81, 117].

## 4.3 Third axis: Motion planning and optimal control

Geometric optimal control allows the development of general techniques, applying to wide classes of nonlinear optimal control problems, that can be used to characterize the behavior of optimal trajectories and in particular to establish regularity properties for them and for the cost function. Hence, geometric optimal control can be used to obtain powerful optimal synthesis results and to provide deep geometric insights into many applied problems. Geometric optimal control methods are in particular important to handle subtle situations such as rigid optimal paths and, more generally, optimal syntheses exhibiting abnormal minimizers.

Although the focus of geometric control theory is on qualitative properties, its impact can also be disruptive when it is used in combination with quantitative analytical tools, in which case it can dramatically improve the computational efficiency. This is the case in particular in **optimal control**. Classical optimal control techniques (in particular, Pontryagin Maximum Principle, conjugate point theory, associated numerical methods) can be significantly improved by combining them with powerful modern techniques



of geometric optimal control, of the theory of numerical continuation, or of dynamical system theory [114, 106]. Applications of optimal control theory considered by CAGE concern, in particular, motion planning problems for aerospace (atmospheric re-entry, orbit transfer, low cost interplanetary space missions, ...) [73, 113].

#### 4.4 Fourth axis: Geometric models for vision and sub-Riemannian geometry

Geometric control theory is not only a powerful framework to investigate control systems, but also a useful tool to model and study phenomena that are not *a priori* control-related. In particular, we use control theory to investigate the properties of sub-Riemannian structures, both for the sake of mathematical understanding and as a modeling tool for image and sound perception and processing. We recall that **sub-Riemannian geometry** is a geometric framework which is used to measure distances in nonholonomic contexts and which has a natural and powerful optimal control interpretation in terms control-linear systems with quadratic cost. Sub-Riemannian geometry, and in particular the theory of their associated (hypoelliptic) diffusive processes, plays a crucial role in the neurogeometrical model of the primary visual cortex due to Petitot, Citti and Sarti, based on the functional architecture first described by Hubel and Wiesel [89, 102, 80, 104]. Such a model can be used as a powerful paradigm for bio-inspired image processing, as already illustrated in the literature [76, 74]. Our contributions to **geometry of vision** are based not only on this approach, but also on another geometric and sub-Riemannian framework for vision, based on pattern matching in the group of diffeomorphisms. In this case admissible diffeomorphisms correspond to deformations which are generated by vector fields satisfying a set of nonholonomic constraints. A sub-Riemannian metric on the infinite-dimensional group of diffeomorphisms is induced by a length on the tangent distribution of admissible velocities [68]. Nonholonomic constraints can be especially useful to describe distortions of sets of interconnected objects (e.g., motions of organs in medical imaging).

## 5 Highlights of the year

### 5.1 Awards

- Bettina Kazandjian won the Prix Junior Maryam Mirzakhani 2024, awarded by the Fondation Mathématique Jacques Hadamard for her M1 work entitled “Optimal control: Minimization of the  $L^1$  -norm cost for a linear dynamical system”
- Emmanuel Trélat was elected member of the *Academia Europaea*.

## 6 New results

The volume [39] is a tribute to Ivan Kupka who passed away at Easter 2023. It contains a collection of articles written by colleagues working on dynamical systems or control theory. Those colleagues interacted with Ivan Kupka directly or indirectly through his articles. In his earliest work (1963) Ivan proved independently the Kupka–Smale theorem: “in a compact manifold the set of vector fields with the following properties are generic: all closed orbits are hyperbolic and heteroclinic orbits are transversal”. At the end of the 70s he made significant contributions to controllability properties of invariant vector fields on semi-simple Lie groups, which are being applied to space mechanics and quantum control decades later. In the mid 80s Ivan proved the ubiquity of Fuller phenomenon which allows to describe the complex behaviors of the extremal trajectories in optimal control and provides an obstruction to the construction of optimal syntheses in the sub-analytic category. In 2001, he co-authored a well-cited book “Deterministic Observation Theory and Applications” which deals with geometric observation, state and parameters estimation. Since the end of the 20th century and for the next two decades he made significant contributions to stochastic systems with life and chemical science applications. This book is a legacy to Ivan Kupka and his research interests and provides a modern perspective for researchers and graduate students to geometric control and its applications connecting applied mathematics and control engineering.

## 6.1 Quantum control: new results

Let us list here our new results in quantum control theory.

- In [58] we discuss how a general bilinear finite-dimensional closed quantum system with dispersed parameters can be steered between eigenstates. We show that, under suitable conditions on the separation of spectral gaps and the boundedness of parameter dispersion, rotating wave and adiabatic approximations can be employed in cascade to achieve population inversion between arbitrary eigenstates. We propose an explicit control law and test numerically the sharpness of the conditions on several examples.
- In [50] we investigate when linearly independent eigenfunctions of the Schrödinger operator may have the same modulus. General properties are established and the one-dimensional case is treated in full generality. The study is motivated by its application to the bilinear control of the Schrödinger equation. By assuming that the potentials of interaction satisfy a saturation property and by adapting a strategy recently proposed by Duca and Nersesyan, we discuss when the system can be steered arbitrarily fast between energy levels. Extensions of the previous results to quantum graphs are finally presented.
- Achiral molecules can be made temporarily chiral by excitation with electric fields, in the sense that an average over molecular orientations displays a net chiral signal. In [30], we go beyond the assumption of molecular orientations to remain fixed during the excitation process. Treating both rotations and vibrations quantum mechanically, we identify conditions for the creation of chiral vibrational wavepackets – with net chiral signals – in ensembles of achiral molecules which are initially randomly oriented. Based on the analysis of symmetry and controllability, we derive excitation schemes for the creation of chiral wavepackets using a combination of (a) microwave and IR pulses and (b) a static field and a sequence of IR pulses. These protocols leverage quantum rotational dynamics for pump-probe spectroscopy of chiral vibrational dynamics, extending the latter to regions of the electromagnetic spectrum other than the UV.
- In the conference proceeding [38], we discuss how a three-level closed quantum system with dispersed parameters can be steered between eigenstates via a scalar control. The technique exploits a dynamical decoupling of the control based on the rotating wave approximation, which works under suitable conditions on the spectral gaps of the system and on the bounds on the parameter dispersion. We test numerically the sharpness of the conditions on several examples.

## 6.2 Stability and stabilization: new results

Let us list here our new results about stability and stabilization of control and hybrid systems.

- The paper [48] deals with the stability of linear periodic time-varying difference delay systems, i.e., dynamical systems where a finite dimensional signal at a certain time is given as a linear time-varying function of its values at a finite number of delayed times. We give a necessary and sufficient condition for exponential stability, that is a generalization of the one by Henry and Hale in the 1970s. It has a control theoretic interpretation in terms of the harmonic transfer function of a corresponding linear control system.
- In [36], we establish exponential contraction results for the diameter and variance of general first-order multiagent systems. Our approach is based on compactification techniques, and works under rather mild assumptions. Namely, we posit that either the scrambling coefficient, or the algebraic connectivity of the averaged interaction graphs of the system over all time windows of a given length are uniformly positive.
- In [37] we consider linear singularly perturbed dynamics in which the set of fast variables is a switching parameter. We introduce auxiliary switching systems (in a single time-scale) whose instability implies the instability of the original dynamics. This translates into necessary conditions for the stability of the considered dynamics.

- We study in [20] the numerical approximation of the stabilization of the semidiscrete linearized Boussinesq system around an unstable stationary state. The stabilization is achieved by internal feedback controls applied on the velocity and the temperature equations, localized in an arbitrary open subset.
- Considering deterministic finite particle systems, we elaborate on various ways to pass to the limit as the number of agents tends to infinity, either by mean field limit, deriving the Vlasov equation, or by hydrodynamic or graph limit, obtaining the Euler equation. In [60], we provide convergence estimates. We also show how to pass from Liouville to Vlasov or to Euler by taking adequate moments. Our results encompass and generalize a number of known results of the literature.
- The work [44] concerns feedback global stabilization of the sterile insect technique dynamics. The Sterile Insect Technique (SIT) is presently one of the most ecological methods for controlling insect pests responsible for crop destruction and disease transmission worldwide. This technique consists in releasing sterile males among the insect pest population, the aim being to reduce fertility and, consequently, reduce significantly the wild insect population after a few generations. In this work, we study the global stabilization of a pest population at extinction equilibrium by the SIT method and construct explicit feedback laws that stabilize the model. Numerical simulations show the efficiency of our feedback laws.
- There exist many ways to stabilize an infinite-dimensional linear autonomous control systems when it is possible. Anyway, finding an exponentially stabilizing feedback control that is as simple as possible may be a challenge. The Riccati theory provides a nice feedback control but may be computationally demanding when considering a discretization scheme. Proper Orthogonal Decomposition (POD) offers a popular way to reduce large-dimensional systems. In [35] we establish that, under appropriate spectral assumptions, an exponentially stabilizing feedback Riccati control designed from a POD finite-dimensional approximation of the system stabilizes as well the infinite-dimensional control system.
- In [54], we consider the control problem for an infinite chain of coupled harmonic oscillators with a Langevin thermostat at the origin. We study the effect of two types of open-loop boundary controls, impulsive control and linear memory-feedback control, in the high frequency limit. We investigate their action on the reflection-transmission coefficients for the wave energy for the scattering of the thermostat. Our study shows that impulsive boundary controls have no impact on the rates and are thus not appropriate to act on the system, despite their physical meaning and relevance. In contrast, the second kind of control that we propose, which is less standard and uses the past of the state solution of the system, is adequate and relevant. We prove that any triple of rates satisfying appropriate assumptions is asymptotically reachable thanks to linear memory-feedback controls that we design explicitly.

### 6.3 Motion planning and optimal control: new results

Let us list here our new results on controllability and motion planning algorithms, including optimal control, optimization beyond the quantum control framework.

- The monograph [41] is the result of various master and summer school courses taught by Emmanuel Trélat. The objective is to introduce the readers to mathematical control theory, both in finite and infinite dimension. In the finite-dimensional context, we consider controlled ordinary differential equations (ODEs); in this context, existence and uniqueness issues are easily resolved thanks to the Picard-Lindelöf (Cauchy-Lipschitz) theorem. In infinite dimension, in view of dealing with controlled partial differential equations (PDEs), the concept of well-posed system is much more difficult and requires to develop a bunch of functional analysis tools, in particular semigroup theory – and this, just for the setting in which the control system is written and makes sense. The book is organized into two parts, the first being devoted to finite-dimensional control systems, and the second to infinite-dimensional ones.

- There exist many examples of systems which have some symmetries, and which one may monitor with symmetry preserving controls. Since symmetries are preserved along the evolution, full controllability is not possible, and controllability has to be considered inside sets of states with same symmetries. In [46] we prove that generic systems with symmetries are controllable in this sense. This result has several applications, for instance: (i) generic controllability of particle systems when the kernel of interaction between particles plays the role of a mean-field control; (ii) generic controllability for families of vector fields on manifolds with boundary; (iii) universal interpolation for neural networks architectures with "generic" self attention-type layers - a type of layers ubiquitous in recent neural networks architectures, e.g., in the Transformers architecture. The tools we develop could help address various other questions of control of equivariant systems.
- The monograph [40] is aimed at students and researchers who want to learn how to efficiently solve constrained optimization problems involving partial differential equations (PDE) using the FreeFEM software.
- In [45], we study  $L^1$ -optimal stabilization of linear systems with finite and infinite horizons. Main results concern the existence, uniqueness and structure of optimal solutions, and the robustness of optimal cost.
- In [29], we consider a nonlinear system of two parabolic equations, with a distributed control in the first equation and an odd coupling term in the second one. We prove that the nonlinear system is smalltime locally null-controllable. The main difficulty is that the linearized system is not null-controllable. To overcome this obstacle, we extend in a nonlinear setting the strategy introduced by one of the authors that consists in constructing odd controls for the linear heat equation. The proof relies on three main steps. First, we obtain from the classical  $L^2$  parabolic Carleman estimate, conjugated with maximal regularity results, a weighted  $L^p$  observability inequality for the nonhomogeneous heat equation. Secondly, we perform a duality argument, close to the well-known Hilbert Uniqueness Method in a reflexive Banach setting, to prove that the heat equation perturbed by a source term is null-controllable thanks to odd controls. Finally, the nonlinearity is handled with a Schauder fixed-point argument.
- Consider, on the one part, a general nonlinear finite-dimensional optimal control problem and assume that it has a unique solution whose state is denoted by  $x^*$ . On the other part, consider the sampled-data control version of it. Under appropriate assumptions, we prove in [22] that the optimal state of the sampled-data problem converges uniformly to  $x^*$  as the norm of the corresponding partition tends to zero. Moreover, applying the Pontryagin maximum principle to both problems, we prove that, if  $x^*$  has a unique weak extremal lift with a costate  $p$  that is normal, then the costate of the sampled-data problem converges uniformly to  $p$ . In other words, under a nondegeneracy assumption, control sampling commutes, at the limit of small partitions, with the application of the Pontryagin maximum principle.
- In [62], we establish an exponential periodic turnpike property for linear quadratic optimal control problems governed by periodic systems in infinite dimension. We show that the optimal trajectory converges exponentially to a periodic orbit when the time horizon tends to infinity. Similar results are obtained for the optimal control and adjoint state. Our proof is based on the large time behavior of solutions of operator differential Riccati equations with periodic coefficients.
- The paper [24] is devoted to the local null-controllability of the nonlinear KdV equation equipped the Dirichlet boundary conditions using the Neumann boundary control on the right. Rosier proved that this KdV system is small-time locally controllable for all non-critical lengths and that the uncontrollable space of the linearized system is of finite dimension when the length is critical. Concerning critical lengths, Coron and Crépeau showed that the same result holds when the uncontrollable space of the linearized system is of dimension 1, and later Cerpa, and then Cerpa and Crépeau established that the local controllability holds at a finite time for all other critical lengths. In this paper, we prove that, for a class of critical lengths, the nonlinear KdV system is *not* small-time locally controllable.

- The paper [23] deals with the controllability of linear one-dimensional hyperbolic systems. Reformulating the problem in terms of linear difference equations and making use of infinite-dimensional realization theory, we obtain both necessary and sufficient conditions for approximate and exact controllability, expressed in the frequency domain. The results are applied to flows in networks.

In [42] motivated by physical applications, the Zermelo navigation problem on the two-dimensional sphere with a revolution metric is analyzed within the framework of minimal time optimal control. The Pontryagin maximum principle is used to compute extremal curves and a neat geometric frame is introduced using the Carathéodory-Zermelo-Goh transformation. Assuming that the current is of revolution, the geodesics are sorted according to a Morse-Reeb classification. We then illustrate the relevance of this classification using various examples from physics: the Lindblad equation in quantum control, the averaged Kepler case in space mechanics and the Landau-Lifshitz equation in ferromagnetism.

- In [17] we consider a smooth system of the form  $\dot{q} = f_0(q) + \sum_{i=1}^k u_i f_i(q)$ ,  $q \in M$ ,  $u_i \in \mathbb{R}$ , and study controllability issues on the group of diffeomorphisms of  $M$ . It is well-known that the system can arbitrarily well approximate the movement in the direction of any Lie bracket polynomial of  $f_1, \dots, f_k$ . Any Lie bracket polynomial of  $f_1, \dots, f_k$  is good in this sense. Moreover, some combinations of Lie brackets which involve the drift term  $f_0$  are also good but surely not all of them. In this paper we try to characterize good ones and, in particular, all universal good combinations, which are good for any nilpotent truncation of any system.
- In [56], we study linear backward parabolic SPDEs and present new a priori estimates for their weak solutions. Inspired by the seminal work of Y. Hu, J. Ma and J. Yong from 2002 on strong solutions, we establish  $L^p$ -estimates requiring minimal assumptions on the regularity of the coefficients, the terminal data, and the external force. To this end, we derive a new Itô's formula for the  $L^p$ -norm of the solution, extending the classical result in the  $L^2$ -setting. This formula is then used to improve further the regularity of the first component of the solution up to  $L^\infty$ . Additionally, we present applications such as the controllability of backward SPDEs with  $L^p$ -controls and a local existence result for a semilinear equation without imposing any growth condition on the nonlinear term.
- In [55], we study the internal control for stochastic lattice dynamics, with the goal of controlling the transition kernel of the kinetic equation in the limit. A major novelty of the work is the introduction of a new geometric combinatorial argument, used to establish paths for the controls.
- In [63] we consider the heat equation set on a bounded  $C^1$  domain of  $\mathbb{R}^n$  with Dirichlet boundary conditions. The first purpose of this paper is to prove that the heat equation is observable from any measurable set  $\omega$  with positive  $(n - 1 + \delta)$ -Hausdorff content, for  $\delta > 0$  arbitrary small. The proof relies on a new spectral estimate for linear combinations of Laplace eigenfunctions, obtained via a Remez type inequality, and the use of the so-called Lebeau-Robbiano's method. Even if this observability result is sharp with respect to the scale of Hausdorff dimension, our second goal is to construct families of sets  $\omega$  which have codimension greater than or equal to 1 for which the heat equation remains observable.
- In [47], we investigate quantitative propagation of smallness properties for the Schrödinger operator on a bounded domain in  $\mathbb{R}^d$ . We extend Logunov, Malinnikova's results concerning propagation of smallness for  $A$ -harmonic functions to solutions of divergence elliptic equations perturbed by a bounded zero order term. We also prove similar results for gradient of solutions to some particular equations. This latter result enables us to follow the recent strategy of Burq, Moyano for the obtaining of spectral estimates on rough sets for the Schrödinger operator. Applications to observability estimates and to the null-controllability of associated parabolic equations posed on compact manifolds or the whole euclidean space are then considered.
- Motivated by applications requiring sparse or nonnegative controls, we investigate in [61] the reachability properties of linear infinite-dimensional control problems under conic constraints. Relaxing the problem to convex constraints if the initial cone is not already convex, we provide

a constructive approach based on minimising a properly defined dual functional, which covers both the approximate and exact reachability problems. Our main results heavily rely on convex analysis, Fenchel duality and the Fenchel-Rockafellar theorem. As a byproduct, we uncover new sufficient conditions for approximate and exact reachability under convex conic constraints. We also prove that these conditions are in fact necessary. When the constraints are nonconvex, our method leads to sufficient conditions ensuring that the constructed controls fulfill the original constraints, which is in the flavour of bang-bang type properties. We show that our approach encompasses and generalises several works, and we obtain new results for different types of conic constraints and control systems.

- In [19], we consider a linear quadratic (LQ) optimal control problem in both finite and infinite dimensions. We derive an asymptotic expansion of the value function as the fixed time horizon  $T$  tends to infinity. The leading term in this expansion, proportional to  $T$ , corresponds to the optimal value attained through the classical turnpike theory in the associated static problem. The remaining terms are associated with optimal stabilization problems towards the turnpike.
- In [18], considering a general nonlinear dissipative finite dimensional optimal control problem in fixed time horizon  $T$ , we establish a two-term asymptotic expansion of the value function as  $T \rightarrow +\infty$ . The dominating term is  $T$  times the optimal value obtained from the optimal static problem within the classical turnpike theory. The second term, of order unity, is interpreted as the sum of two values associated with optimal stabilization problems related to the turnpike.
- In [25], we consider the small-time local controllability property of a water tank modeled by 1D Saint-Venant equations, where the control is the acceleration of the tank. It is known from the work of Dubois et al. that the linearized system is not controllable. Moreover, concerning the linearized system, they showed that a traveling time  $*$  is necessary to bring the tank from one position to another for which the water is still at the beginning and at the end. Concerning the nonlinear system, Coron showed that local controllability around equilibrium states holds for a time large enough. In this paper, we show that for the local controllability of the nonlinear system around the equilibrium states, the necessary time is at least  $2*$  even for the tank being still at the beginning and at the end. The key point of the proof is a coercivity property for the quadratic approximation of the water-tank system.
- In [52] we study the global controllability and stabilization problems of the harmonic map heat flow from a circle to a sphere. Combining ideas from control theory, heat flow, differential geometry, and asymptotic analysis, we obtain several important properties, such as small-time local controllability, local quantitative rapid stabilization, obstruction to semi-global asymptotic stabilization, and global controllability to geodesics. Surprisingly, due to the geometric feature of the equation we also discover the small-time global controllability between harmonic maps within the same homotopy class for general compact Riemannian manifold targets, which is to be compared with the analogous but longstanding problem for the nonlinear heat equations.
- In [33], we introduce a new shape functional defined for toroidal domains that we call harmonic helicity, and study its shape optimization. Given a toroidal domain, we consider its associated harmonic field. The latter is the magnetic field obtained uniquely up to normalization when imposing zero normal trace and zero electrical current inside the domain. We then study the helicity of this field, which is a quantity of interest in magneto-hydrodynamics corresponding to the  $L^2$  product of the field with its image by the Biot-Savart operator. To do so, we begin by discussing the appropriate functional framework and an equivalent PDE characterization. We then focus on shape optimization, and we identify the shape gradient of the harmonic helicity. Finally, we study and implement an efficient numerical scheme to compute harmonic helicity and its shape gradient using finite elements exterior calculus. The article is part of the PhD thesis of Robin Roussel [43].
- In [34], we study Poincaré maps of harmonic fields in toroidal domains using a shape variational approach. Given a bounded domain of  $\mathbb{R}^3$ , we define its harmonic fields as the set of magnetic fields which are curl free and tangent to the boundary. For toroidal domains, this space is one dimensional, and one may thus single out a harmonic field by specifying a degree of freedom, such

as the circulation along a toroidal loop. We are then interested in the Poincaré maps of such fields restricted to the boundary, which produce diffeomorphisms of the circle. We begin by proving a general shape differentiability result of such Poincaré maps in the smooth category, and obtain a general formula for the shape derivative. We then investigate two specific examples of interest; axisymmetric domains, and domains for which the harmonic field has a diophantine rotation number on the boundary. We prove that, in the first case, the shape derivative of the Poincaré map is always identically zero, whereas in the second case, assuming an additional condition on the geometry of the domain, the shape derivative of the Poincaré map may be any smooth function of the circle by choosing an appropriate perturbation of the domain. The article is part of the PhD thesis of Robin Roussel [43].

- In the work [27], we present a general framework which guarantees the existence of optimal domains for isoperimetric problems within the class of  $C^{1,1}$ -regular domains satisfying a uniform ball condition as long as the desired objective function satisfies certain properties. We then verify that the helicity isoperimetric problem studied by Cantarella, DeTurck, Gluck and Teytel in 2002 satisfies the conditions of our framework and hence establish the existence of optimal domains within the given class of domains. We additionally use the same framework to prove the existence of optimal domains among uniform  $C^{1,1}$ -domains for a first curl eigenvalue problem which has been studied recently for other classes of domains.
- In [28], we investigate properties of the image and kernel of the Biot-Savart operator in the context of stellarator designs for plasma fusion. We first show that for any given coil winding surface (CWS) the image of the Biot-Savart operator is  $L^2$ -dense in the space of square-integrable harmonic fields defined on a plasma domain surrounded by the CWS. Then we show that harmonic fields which are harmonic in a proper neighbourhood of the underlying plasma domain can in fact be approximated in any  $C^k$ -norm by elements of the image of the Biot-Savart operator. In the second part of this work we establish an explicit isomorphism between the space of harmonic Neumann fields and the kernel of the Biot-Savart operator which in particular implies that the dimension of the kernel of the Biot-Savart operator coincides with the genus of the coil winding surface and hence turns out to be a homotopy invariant among regular domains in 3-space. Lastly, we provide an iterative scheme which we show converges weakly in  $W^{-\frac{1}{2},2}$ -topology to elements of the kernel of the Biot-Savart operator.
- In [53] we investigate properties of surface helicity and in particular answer two open questions posed by Cantarella and Parsley: (i) We give a precise mathematically rigorous physical interpretation of surface helicity in terms of linking of distinct field lines. (ii) We prove that surface helicity is non-trivial if and only if the underlying surface has non-trivial topology (i.e. at least one hole). We then focus on toroidal surfaces which are of relevance in plasma physics and express surface helicity in terms of average poloidal and toroidal windings of the individual field lines of the underlying vector field which enables us to provide a connection between surface helicity and rotational transform. Further, we show how some of our results may be utilised in the context of coil designs for plasma fusion confinement devices in order to obtain coil configurations of particular "simple" shape. Lastly, we consider the problem of optimising surface helicity among toroidal surfaces of fixed area and show that toroidal surfaces admitting a symmetry constitute global minimisers.
- The article [32] deals with the existence of hypersurfaces minimizing general shape functionals under certain geometric constraints. We consider as admissible shapes orientable hypersurfaces satisfying a so-called reach condition, also known as the uniform ball property, which ensures  $C^{1,1}$  regularity of the hypersurface. In this paper, we revisit and generalise the results of Guo et al and, J. Dalphin. We provide a simpler framework and more concise proofs of some of the results contained in these references and extend them to a new class of problems involving PDEs. Indeed, by using the signed distance introduced by Delfour and Zolesio, we avoid the intensive and technical use of local maps, as was the case in the above references.
- Given a well-posed linear evolution system settled on a domain  $\Omega$  of  $\mathbb{R}^d$ , an observation subset  $\omega \subset \Omega$  and a time horizon  $T$ , the observability constant is defined as the largest possible nonnegative constant such that the observability inequality holds for the pair  $(\omega, T)$ . In [59] we investigate the

large-time behavior of the observation domain that maximizes the observability constant over all possible measurable subsets of a given Lebesgue measure. We prove that it converges exponentially, as the time horizon goes to infinity, to a limit set that we characterize. The mathematical technique is new and relies on a quantitative version of the bathtub principle.

- In [31], we consider the internal control of linear parabolic equations through on-off shape controls, i.e., controls of the form  $M(t)1_{\omega(t)}$  with  $M(t) \geq 0$  and  $\omega(t)$  with a prescribed maximal measure. We establish small-time approximate controllability towards all possible final states allowed by the comparison principle with nonnegative controls. We manage to build controls with constant amplitude  $M(t) = M$ . In contrast, if the moving control set  $\omega(t)$  is confined to evolve in some region of the whole domain, we prove that approximate controllability fails to hold for small times. The method of proof is constructive. Using Fenchel-Rockafellar duality and the bathtub principle, the on-off shape control is obtained as the bang-bang solution of an optimal control problem, which we design by relaxing the constraints. Our optimal control approach is outlined in a rather general form for linear constrained control problems, paving the way for generalisations and applications to other PDEs and constraints.

## 6.4 Geometric models for vision and sub-Riemannian geometry: new results

Let us list here our new results in the geometry of vision axis and, more generally, on hypoelliptic diffusion and sub-Riemannian geometry.

- In [21] we prove Goh conditions of order  $n$  for strictly singular length minimizing curves of corank 1, under the assumption that the lower order intrinsic differentials of the end-point map vanish. This result relies upon the proof of an open mapping theorem for maps with non-singular  $n$ th differential.
- In [26] we prove (sub)mean value formulas at the point  $0 \in \Sigma$  for (sub)harmonic functions  $a$  on a hypersurface  $\Sigma \subset \mathbb{R}^{n+1}$  where the differentiable structure and the surface measure depend on the ambient Grushin structure.
- In [49] we investigate the validity of synthetic curvature-dimension bounds in the sub-Finsler Heisenberg group, equipped with a positive smooth measure.
- We begin [57] by characterizing metabelian distributions in terms of principal bundle structures. Then, we prove that in sub-Riemannian manifolds with metabelian distributions of rank  $r$ , the projection of strictly singular trajectories to some  $r$ -dimensional manifold must remain within an analytic variety. As a consequence, for rank-2 metabelian distributions, geodesics are of class  $C^1$ .
- The paper [51] is motivated by recent works on inverse problems for acoustic wave propagation in the interior of gas giant planets. In such planets, the speed of sound is isotropic and tends to zero at the surface. Geometrically, this corresponds to a Riemannian manifold with boundary whose metric blows up near the boundary. Here, the spectral analysis of the corresponding Laplace-Beltrami operator is presented and the Weyl law is derived. The involved exponents depend on the Hausdorff dimension which, in the supercritical case, is larger than the topological dimension.

## 7 Partnerships and cooperations

### 7.1 International research visitors

#### 7.1.1 Visits of international scientists

##### Inria International Chair

- Andrei Agrachev (SISSA, Trieste, Italy) made two visits to CAGE (5/3–4/5 and 15/10–24/11) in the framework of his Inria International Chair 2020-2024.



### Other international visits to the team

- Shirshendu Chowdhury (IISER Kolkata) visited the team CAGE at LJLL for one month.
- Felipe Wallison Chaves Silva (Federal University of Paraíba) and Maurício Cardoso Santos (Federal University of Paraíba) visited together the team CAGE at LJLL for one month.

## 7.2 National initiatives

### 7.2.1 ANR

- ANR TRECOS, for *New Trends in Control and Stabilization: Constraints and non-local terms*, coordinated by Sylvain Ervedoza, University of Bordeaux. The ANR started in 2021 and runs up to 2025. TRECOS' focus is on control theory for partial differential equations, and in particular models from ecology and biology. Kévin Le Balc'h and Emmanuel Trélat are member of TRECOS.
- ANR/DFG CoRoMo for *Efficient quantum control of molecular rotations – time and controllability*, 2023–2025. The grant is co-coordinated by Ugo Boscain (CAGE) and Christiane Koch (Berlin). In this project, we seek to elucidate the role of time in quantum control, using the important benchmark of molecular rotations as testbed. We will leverage controllability analysis to tackle the role of time in quantum control, combining physical intuition from the control of molecular rotations with recent advances of mathematical methods. Ugo Boscain, Tommaso Rossi, and Mario Sigalotti are member of CoRoMo.
- ANR EINSTEIN-PPF for *Contraintes d'Einstein : passé, présent et futur*, coordinated by Philippe Lefloch. Relying on a close collaboration between analysts and geometers, the ANR project is aimed at advancing our knowledge of the analytic and geometric properties of Einstein spacetimes, especially when the metrics under consideration have low regularity. Emmanuel Trélat is member of EINSTEIN-PPF.

### 7.2.2 Other national initiatives

- The Inria Exploratory Action “StellaCage” is supporting since Spring 2020 a collaboration between CAGE, Yannick Privat (Inria team TONUS), and the startup Renaissance Fusion, based in Grenoble. StellaCage approaches the problem of designing better stellarators (yielding better confinement, with simpler coils, capable of higher fields) by combining geometrical properties of magnetic field lines from the control perspective with shape optimization techniques.
- The 80 prime project BioSpeech (2023–2024), coordinated by Ugo Boscain, studies a bio-inspired geometric model for speech sound reconstruction. It is a collaboration between mathematicians, automatic control scientists, and linguists.

## 7.3 Regional initiatives

- The Bourse Emergence(s) de la Ville de Paris “Morphométrie sous contrainte pour l'analyse de données biologiques : un nouvel outil pour la communauté scientifique”, whose principal investigator is Barbara Gris, runs from 2022 to 2025.
- Project CURED: appel à projets Tremplins nouveaux entrants de Sorbonne Université for Kévin Le Balc'h, 8000 euro. The project ended in December 2024.
- Project “Quantum control of rotational dynamics”, funded by the DIM QUANTIP, 3000 euro, supports the organization of a workshop in 2025.

## 8 Dissemination

### 8.1 Promoting scientific activities

#### 8.1.1 Scientific events: organisation

##### Member of the organizing committees

- Ugo Boscain was member of the organizing committee of the “Colloque Energie du CNRS”, Paris.
- Ugo Boscain was member of the scientific and of the organizing committee of the “Colloque PFAS : enjeux et alternatives”, Siège du CNRS, Paris.
- Ugo Boscain was member of the scientific and of the organizing committee of the "Colloque Jumeaux numériques : nouvelles frontières", Siège du CNRS, Paris.
- Ugo Boscain was member of the scientific committee of the conference “Frontiers in Sub-Riemannian Geometry”, CIRM, Marseille.
- Mario Sigalotti was co-organizer (with D. Barilari and V. Francheschi) of the 2nd edition of the Padua Paris Sub-Riemannian seminar, Padua, Italy.
- Emmanuel Trélat was member of the scientific committee of SMAI Mode, Lyon.

#### 8.1.2 Journal

##### Member of the editorial boards

- Ugo Boscain is Associate editor of SIAM Journal on Control and Optimization and he is Corresponding editor of the special section “Control of Quantum Mechanical Systems”.
- Ugo Boscain is Managing editor of Journal of Dynamical and Control Systems
- Jean-Michel Coron is Editor-in-chief of Comptes Rendus Mathématique
- Jean-Michel Coron is Associate editor of Journal of Evolution Equations
- Jean-Michel Coron is Associate editor of Asymptotic Analysis
- Jean-Michel Coron is Associate editor of ESAIM: Control, Optimisation and Calculus of Variations
- Jean-Michel Coron is Associate editor of Applied Mathematics Research Express
- Jean-Michel Coron is Associate editor of Advances in Differential Equations
- Jean-Michel Coron is Associate editor of Mathematics of Control, Signals, and Systems
- Jean-Michel Coron is Associate editor of Annales de l’IHP, Analyse non linéaire
- Mario Sigalotti is Associate editor of SIAM Journal on Control and Optimization
- Mario Sigalotti is Associate editor of ESAIM: Control, Optimisation and Calculus of Variations
- Mario Sigalotti is Associate editor of Journal on Dynamical and Control Systems
- Emmanuel Trélat is Editor-in-chief of ESAIM: Control, Optimisation and Calculus of Variations
- Emmanuel Trélat is Associate editor of SIAM Review
- Emmanuel Trélat is Associate editor of Systems & Control Letters
- Emmanuel Trélat is Associate editor of Journal on Dynamical and Control Systems
- Emmanuel Trélat is Associate editor of Bollettino dell’Unione Matematica Italiana

- Emmanuel Trélat is Associate editor of ESAIM: Mathematical Modelling and Numerical Analysis
- Emmanuel Trélat is Editor of BCAM Springer Briefs
- Emmanuel Trélat is Associate editor of IEEE Transactions on Automatic Control
- Emmanuel Trélat is Associate editor of Journal of Optimization Theory and Applications
- Emmanuel Trélat is Associate editor of Mathematical Control & Related Fields
- Emmanuel Trélat is Associate editor of Mathematics of Control, Signals, and Systems
- Emmanuel Trélat is Associate editor of Optimal Control Applications and Methods
- Emmanuel Trélat is Associate editor of Advances in Continuous and Discrete Models: Theory and Modern Applications
- Emmanuel Trélat is Associate editor of Comptes Rendus Mathématique

### 8.1.3 Invited talks

- Ugo Boscain was invited speaker at the conference ConQuEr24, Erlangen, Germany.
- Ugo Boscain was invited speaker at the research seminar on Applied Mathematics: Analysis, Numerics and Computing, Research Center for Systems and Technologies (SYSTEC), Centre for Mathematics of the University of Porto (CMUP), Portugal.
- Ugo Boscain was invited speaker at the conference “Differential evolutive models in spaces with singularities”, Rome, Italy.
- Ugo Boscain was invited speaker at the conference “Future Perspectives on Perturbative Linear and Nonlinear Modeling of Contact-Type Perturbations”, Rome, Italy.
- Ugo Boscain was invited speaker at the conference “On the Road to New Horizons: A Lighthearted Conference on Control Theory”, Rouen.
- Ugo Boscain was invited speaker at the workshop “Quantum Optimal Control From Mathematical Foundations to Quantum Technologies”, Berlin Mathematics Research Center, Zuse Institute, Berlin, Germany.
- Kévin Le Balc’h was invited speaker at the conference “Recent advances on control theory of PDE systems”, Bangalore, India.
- Kévin Le Balc’h was invited speaker at the Rencontres ANR Trecos, Nancy.
- Kévin Le Balc’h was invited speaker at the conference “Théorie du contrôle et problèmes inverses”, Monastir, Tunisia.
- Kévin Le Balc’h was invited speaker at the Journées EDP, Aussois.
- Kévin Le Balc’h was invited speaker at two thematic sessions at the conference “X Partial differential equations, optimal design and numerics”, Benasque, Spain.
- Kévin Le Balc’h was invited speaker at the séminaire EDP, IECL, Nancy.
- Kévin Le Balc’h was invited speaker at the PDE/Analysis seminar, MIT, Boston, USA.
- Kévin Le Balc’h was invited speaker at the GT Contrôle, LJLL, Paris.
- Wadim Gerner was invited speaker at the GT CalVa (groupe de travail de Calcul des Variations), Université Paris-Dauphine.
- Mario Sigalotti was invited speaker at the workshop “New trends in quantum control”, Nice.

- Mario Sigalotti was plenary speaker at the 4th IFAC Conference of Modelling, Identification and Control of Nonlinear Systems, Lyon.
- Emmaneul Trélat was plenary speaker at the conference MOAD'24, Algeria.
- Emmaneul Trélat was invited speaker at the conference "Frontiers in Sub-Riemannian Geometry", CIRM, Marseille.
- Emmaneul Trélat was invited speaker at the Workshop on Non-Linear Analysis and Control Theory, Chile (online).
- Emmaneul Trélat was invited speaker at the conference "Analysis and PDEs", Hannover, Germany.
- Emmaneul Trélat was invited speaker at the Padova-Paris Sub-Riemannian Workshop, Padova, Italy.
- Emmaneul Trélat was invited speaker at the ANR Workshop Einstein, Avignon.
- Emmaneul Trélat was invited speaker at the Conference on control of PDEs, Sichuan Univ., Chengdu, China.
- Emmaneul Trélat was invited speaker at the Workshop "Num. meth. for opt. transport problems, MFG and multi-agent dyn.", Valparaíso, Chile.
- Emmaneul Trélat was invited speaker at the seminar of the Chinese Academy of Sciences.
- Emmaneul Trélat was invited speaker at the seminar of the Peking University, China.
- Emmaneul Trélat was invited speaker at the seminar of the Univ. Lille.

#### 8.1.4 Leadership within the scientific community

- Ugo Boscain is Délégué Scientifique at INSMI in charge of interdisciplinarity and member of the Comité de pilotage of the Mission pour les initiatives transverses et interdisciplinaires (MITI).
- Emmanuel Trélat is Head of the Laboratoire Jacques-Louis Lions (LJLL).

#### 8.1.5 Scientific expertise

- Ugo Boscain was responsable scientifique (with Anne-Marie Gué) of the appel à projets 2025 "Le large spectre du son : du cognitif au quantique"
- Ugo Boscain was responsable scientifique (with Philippe Grandcolas) of the appel à projets 2024 "Suivis à long terme".
- Ugo Boscain was responsable scientifique (with Annick Lesne) of the appel à projets 2025 "Interactions complexes et comportements collectifs".
- Ugo Boscain was member of the committee for the AAPs 2024/2025:
  - AAP Données massives pour la découverte scientifique : production, sélection, curation et analyse
  - OSEZ L'INTERDISCIPLINARITE ! 2025
  - Mobilité interdisciplinaire immersive 2025
  - PFAS : enjeux et alternatives
  - Pépinière interdisciplinaire des Antilles françaises
  - Jumeaux numériques : nouvelles frontières et futurs développements
  - OSEZ L'INTERDISCIPLINARITE ! 2024
  - Mobilité interdisciplinaire immersive 2024
  - Ressources et sobriété

- 80prime 2024.
- Conditions Extrêmes
- Pollution et dépollution : solutions et trajectoires
- Emmanuel Trélat is member of the conseil scientifique de la Fédération de Mathématiques de CentraleSupélec.
- Emmanuel Trélat is member of the Advisory Board of the Department of Data Science, FAU (Erlangen), Germany.

#### 8.1.6 Research administration

- Kévin Le Balc'h is SMAI correspondent for the Laboratoire Jacques-Louis Lions.
- Emmanuel Trélat is member of the Bureau de comité des équipes-projets, Inria Paris center.

## 8.2 Teaching - Supervision - Juries

### 8.2.1 Teaching

- Uho Boscain taught “Design of stellarators for nuclear fusion, from shape optimization to KAM theory” at the Summer School “Methods and Models of Kinetic Theory”, Pesaro, Italy.
- Ugo Boscain and Mario Sigalotti thought “Geometric control theory” at the M2 Mathématiques de la Modélisation, Sorbonne Université.
- Barbara Gris was in charge of the supervision of projects for I3 students, Sorbonne Université.
- Kévin Le Balc'h thought “Agrégation externe (analyse, probabilités)” to M2 students at Sorbonne Université.
- Mario Sigalotti thought “Introduction to Geometric Control Theory” to PhD students at SISSA, Trieste, Italy.
- Emmanuel Trélat thought “Contrôle en dimension finie et infinie” to M2 students at Sorbonne Université
- Emmanuel Trélat thought “Optimisation et sciences des données” to M1 students at Sorbonne Université

### 8.2.2 Supervision

- PhD: Liangying Chen, “Sensitivity relations and verification theorem for infinite dimensional stochastic control systems”, December 2024. Supervisors: Emmanuel Trélat and Xu Zhang (Chengdu, China).
- PhD: Robin Roussel, “Confinement magnétique dans les stellarators : champs harmoniques et différentiation de forme”, December 2024. Supervisors: Ugo Boscain and Mario Sigalotti.
- PhD in progress: Kala Agbo Bidi, “Robust pest control strategies”. Supervisors: Luis Almeida and Jean-Michel Coron.
- PhD in progress: Bettina Kazandjian “Small-time controllability of bilinear partial differential equations via Lie bracket methods”, started in 2024. Supervisors: Ugo Boscain, Eugenio Pozzoli, and Mario Sigalotti.
- PhD in progress: Ruikang Liang, “The quantum speed limit in Quantum Control”, started in 2022. Supervisors: Ugo Boscain and Mario Sigalotti.
- PhD in progress: Xiangyu Ma, “A bio-inspired geometric model for speech sound reconstruction”, started in 2023. Supervisors: Ugo Boscain, Dario Prandi, and Giuseppina Turco.

- PhD in progress: Rayane Mouhli, “L’ontogénèse par grandes déformations”, started in 2023. Supervisors: Barbara Gris and Irène Kaltenmark.
- PhD in progress: Lia Sela, “Modélisation de la divergence phénotypique cellulaire dans la carcinogénèse orale pour améliorer la prévention et le traitement des cancers de la cavité buccale”, started in 2024. Supervisors: Jean Clairambault, Jean-Philippe Foy, and Emmanuel Trélat.
- PhD in progress: Lucia Tessarolo, “Sub- Riemannian geometry and pinwheels”, started in 2023. Supervisor: Ugo Boscain.

### 8.2.3 Juries

- Ugo Boscain was referee of the PhD thesis of Emanuel Malvetti, TUM Munich, Germany.
- Ugo Boscain was member of the jury for CR positions at INSMI
- Ugo Boscain was member of the jury for a professor position at Sorbonne Université, CNU 27, Intelligence artificielle pour la créativité et l’interaction musicales, IRCAM.
- Ugo Boscain and Mario Sigalotti were member of the PhD jury of Robin Roussel, Sorbonne Université.
- Mario Sigalotti was referee of the PhD thesis of Mohamed Bentaibi, University of Padua, Italy
- Mario Sigalotti was member of the PhD jury of Maël Bompais, Paris-Saclay University
- Mario Sigalotti and Emmanuel Trélat were members of the PhD jury of Liangying Chen, Sorbonne Université.
- Emmanuel Trélat was member of the HDR jury of E. Courtial. Univ. Orléans.
- Emmanuel Trélat was president of the Phd jury of A. Tendani-Soler, Univ. Bordeaux.
- Emmanuel Trélat was referee and member of the Phd jury of Y. Dubois de Mont-Marin, Univ. PSL.
- Emmanuel Trélat was president of the Phd jury of P. Zoghby. Univ. Nantes.
- Emmanuel Trélat was president of the Phd jury of M. Lambert, Univ. Paris Sciences et Lettres.
- Emmanuel Trélat was referee and member of the Phd jury of W. Jallet, Univ. Toulouse.
- Emmanuel Trélat wa, member of the Phd jury of E. Dionis. Univ. Bourgogne.
- Emmanuel Trélat was referee and member of the Phd jury of R. Elarayeh, Univ. Orléans.
- Emmanuel Trélat was referee and member of the Phd jury of B. Colle, Univ. Nancy.
- Emmanuel Trélat was president of the Phd jury of A. Fossà, ISAE, Toulouse.

## 8.3 Popularization

### 8.3.1 Productions (articles, videos, podcasts, serious games, ...)

- The paper [64] focuses on the feedback global stabilization and observer construction for a sterile insect technique model. The Sterile Insect Technique (SIT) is one of the most ecological methods for controlling insect pests responsible for worldwide crop destruction and disease transmission. In this work, we construct a feedback law that globally asymptotically stabilizes a SIT model at extinction equilibrium. Since the application of this type of control requires the measurement of different states of the target insect population, and in practice, some states are more difficult and very expensive to measure than others, it is important to know how to construct a state estimator which from a few measured states, estimates the other ones as the one we build in the second part of our work. In the last part of our work, we show that we can apply the feedback control with estimated states to stabilize the full system.

### 8.3.2 Participation in Live events

- Ugo Boscain gave a presentation on “Un modèle géométrique bio-inspiré pour la reconstruction du son” at the Maths club de Institut de Recherche sur l’Enseignement des Mathématiques.
- Ugo Boscain gave a presentation on “Conception de stellarators pour la fusion nucléaire : de systèmes dynamiques” at the event “Horizons interdisciplinaires” organized by MITI, CNRS, Paris.
- Tommaso Rossi participated to the PiDay at Lycée Louis le Grand, Paris.
- Emmanuel Trélat gave a presentation at the Lycée Français International de Pékin, China.

## 9 Scientific production

### 9.1 Major publications

- [1] D. Barilari, U. Boscain, D. Cannarsa and K. Habermann. ‘Stochastic processes on surfaces in three-dimensional contact sub-Riemannian manifolds’. In: *Annales de l’Institut Henri Poincaré (B) Probabilités et Statistiques* (2021). 25 pages, 2 figures. DOI: [10.1214/20-AIHP1124](https://doi.org/10.1214/20-AIHP1124). URL: <https://hal.archives-ouvertes.fr/hal-02557862>.
- [2] D. Barilari, Y. Chitour, F. Jean, D. Prandi and M. Sigalotti. ‘On the regularity of abnormal minimizers for rank 2 sub-Riemannian structures’. In: *Journal de Mathématiques Pures et Appliquées* 133 (2020), pp. 118–138. DOI: [10.1016/j.matpur.2019.04.008](https://doi.org/10.1016/j.matpur.2019.04.008). URL: <https://hal.archives-ouvertes.fr/hal-01757343>.
- [3] M. Bertalmio, L. Calatroni, V. Franceschi, B. Franceschiello and D. Prandi. ‘Cortical-inspired Wilson-Cowan-type equations for orientation-dependent contrast perception modelling’. In: *Journal of Mathematical Imaging and Vision* (June 2020). DOI: [10.1007/s10851-020-00960-x](https://doi.org/10.1007/s10851-020-00960-x). URL: <https://hal.archives-ouvertes.fr/hal-02316989>.
- [4] R. Bonalli, B. Hérisse and E. Trélat. ‘Optimal Control of Endo-Atmospheric Launch Vehicle Systems: Geometric and Computational Issues’. In: *IEEE Transactions on Automatic Control* 65.6 (2020), pp. 2418–2433. DOI: [10.1109/tac.2019.2929099](https://doi.org/10.1109/tac.2019.2929099). URL: <https://hal.archives-ouvertes.fr/hal-01626869>.
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- [6] Y. Colin de Verdière, L. Hillairet and E. Trélat. *Spectral asymptotics for sub-Riemannian Laplacians*. 5th Dec. 2022. URL: <https://hal.archives-ouvertes.fr/hal-03885610>.
- [7] J.-M. Coron, A. Hayat, S. Xiang and C. Zhang. ‘Stabilization of the linearized water tank system’. In: *Archive for Rational Mechanics and Analysis* 244.3 (2022), pp. 1019–1097. URL: <https://hal.archives-ouvertes.fr/hal-03161523>.
- [8] J.-M. Coron, F. Marbach and F. Sueur. ‘Small-time global exact controllability of the Navier-Stokes equation with Navier slip-with-friction boundary conditions’. In: *Journal of the European Mathematical Society* 22.5 (May 2020), pp. 1625–1673. DOI: [10.4171/JEMS/952](https://doi.org/10.4171/JEMS/952). URL: <https://hal.archives-ouvertes.fr/hal-01422161>.
- [9] J.-M. Coron and H.-M. Nguyen. ‘Finite-time stabilization in optimal time of homogeneous quasi-linear hyperbolic systems in one dimensional space’. In: *ESAIM: Control, Optimisation and Calculus of Variations* 26 (2020), p. 119. DOI: [10.1051/cocv/2020061](https://doi.org/10.1051/cocv/2020061). URL: <https://hal.archives-ouvertes.fr/hal-03080852>.
- [10] J.-M. Coron and H.-M. Nguyen. ‘Optimal time for the controllability of linear hyperbolic systems in one dimensional space’. In: *SIAM Journal on Control and Optimization* 57.2 (5th Apr. 2019), pp. 1127–1156. DOI: [10.1137/18M1185600](https://doi.org/10.1137/18M1185600). URL: <https://hal.archives-ouvertes.fr/hal-01952134>.

- [11] S. Ervedoza, K. Le Balc'H and M. Tucsnak. 'Reachability results for perturbed heat equations'. In: *Journal of Functional Analysis* 283.10 (15th Nov. 2022). URL: <https://hal.archives-ouvertes.fr/hal-03380745>.
- [12] M. Leibscher, E. Pozzoli, C. Pérez, M. Schnell, M. Sigalotti, U. Boscain and C. P. Koch. 'Full quantum control of enantiomer-selective state transfer in chiral molecules despite degeneracy'. In: *Communications Physics* (6th May 2022). DOI: 10.1038/s42005-022-00883-6. URL: <https://hal.inria.fr/hal-02972059>.
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- [14] O. Öktem, B. Gris and C. Chen. 'Image reconstruction through metamorphosis'. In: *Inverse Problems* 36 (2020). DOI: 10.1088/1361-6420/ab5832. URL: <https://hal.archives-ouvertes.fr/hal-01773633>.
- [15] Y. Privat, R. Robin and M. Sigalotti. 'Optimal shape of stellarators for magnetic confinement fusion'. In: *Journal de Mathématiques Pures et Appliquées* (2022). DOI: 10.1016/j.matpur.2022.05.005. URL: <https://hal.inria.fr/hal-03472623>.
- [16] R. Robin, N. Augier, U. Boscain and M. Sigalotti. 'Ensemble qubit controllability with a single control via adiabatic and rotating wave approximations'. In: *Journal of Differential Equations* 318 (5th May 2022). DOI: 10.1016/j.jde.2022.02.042. URL: <https://hal.archives-ouvertes.fr/hal-02504532>.

## 9.2 Publications of the year

### International journals

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- [18] V. Askovic, E. Trélat and H. Zidani. 'Two-term large-time asymptotic expansion of the value function for dissipative nonlinear optimal control problems'. In: *Mathematical Control and Related Fields* 14.4 (2024), pp. 1501–1516. URL: <https://hal.science/hal-04345255> (cit. on p. 12).
- [19] V. Ašković, E. Trélat and H. Zidani. 'Linear quadratic optimal control turnpike in finite and infinite dimension: two-term expansion of the value function'. In: *Systems and Control Letters* 188 (2024), p. 105803. DOI: 10.1016/j.sysconle.2024.105803. URL: <https://hal.science/hal-04361557> (cit. on p. 12).
- [20] M. Azaïez and K. L. Balc'h. 'Semidiscrete approximation of the penalty approach to the stabilization of the Boussinesq system by localized feedback control'. In: *Annals of Applied Mathematics* (2024), pp. 191–218. URL: <https://hal.science/hal-03442394> (cit. on p. 9).
- [21] F. Boarotto, R. Monti and A. Socionovo. 'Higher order Goh conditions for singular extremals of corank 1'. In: *Archive for Rational Mechanics and Analysis* (2024). URL: <https://inria.hal.science/hal-04594888> (cit. on p. 14).
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- [31] C. Pouchol, E. Trélat and C. Zhang. ‘Approximate control of parabolic equations with on-off shape controls by Fenchel duality’. In: *Annales de l’Institut Henri Poincaré C, Analyse non linéaire* 41.6 (2024), pp. 1465–1507. DOI: [10.4171/AIHPC/114](https://doi.org/10.4171/AIHPC/114). URL: <https://hal.science/hal-03889865> (cit. on p. 14).
- [32] Y. Privat, R. Robin and M. Sigalotti. ‘Existence of surfaces optimizing geometric and PDE shape functionals under reach constraint’. In: *Interfaces and Free Boundaries : Mathematical Analysis, Computation and Applications* (24th June 2024). DOI: [10.4171/IFB/523](https://doi.org/10.4171/IFB/523). URL: <https://inria.hal.science/hal-03690069>. In press (cit. on p. 13).
- [33] R. Robin and R. Roussel. ‘Shape optimization of harmonic helicity in toroidal domains’. In: *Journal of Optimization Theory and Applications* (26th Jan. 2024). URL: <https://hal.science/hal-04419547>. In press (cit. on p. 12).
- [34] R. Roussel. ‘Shape differentiation for Poincaré maps of harmonic fields in toroidal domains’. In: *The Journal of Geometric Analysis* 35.1 (12th June 2024), p. 19. DOI: [10.1007/s12220-024-01849-6](https://doi.org/10.1007/s12220-024-01849-6). URL: <https://hal.science/hal-04611626> (cit. on p. 12).
- [35] E. Trélat, G. Wang and Y. Xu. ‘Stabilization of infinite-dimensional linear control systems by POD reduced-order Riccati feedback’. In: *Mathematical Control and Related Fields* 14.4 (2024), pp. 1705–1728. URL: <https://hal.science/hal-02163854> (cit. on p. 9).

#### International peer-reviewed conferences

- [36] B. Bonnet-Weill and M. Sigalotti. ‘Exponential Consensus Formation in Time-Varying Multiagent Systems via Compactification Methods’. In: *Proceedings of the 63rd IEEE Conference on Decision and Control*. 63rd IEEE Conference on Decision and Control. Milan, Italy, 2024. URL: <https://hal.science/hal-04600171> (cit. on p. 8).
- [37] Y. Chitour, J. Daafouz, I. Haidar, P. Mason and M. Sigalotti. ‘Necessary conditions for the stability of singularly perturbed linear systems with switching slow-fast behaviors’. In: 63rd IEEE Conference on Decision and Control, CDC 2024. Milan, Italy, 16th Dec. 2024. URL: <https://inria.hal.science/hal-04806068> (cit. on p. 8).
- [38] R. Liang, U. Boscain and M. Sigalotti. ‘Ensemble quantum control with a scalar input’. In: CDC 2024 - 63rd IEEE Conference on Decision and Control. Milan, Italy, 16th Dec. 2024. URL: <https://hal.science/hal-04813682> (cit. on p. 8).

### Scientific books

- [39] B. Bonnard, M. Chyba, D. Holcman and E. Trélat. *Ivan Kupka Legacy: A Tour Through Controlled Dynamics*. Vol. 12. AIMS, 2024. URL: <https://hal.science/hal-04869632> (cit. on p. 7).
- [40] F. Hecht, G. Lance and E. Trélat. *PDE-constrained optimization within FreeFEM*. 2024. URL: <https://hal.science/hal-04724788> (cit. on p. 10).
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### Scientific book chapters

- [42] B. Bonnard, O. Cots, Y. Privat and E. Trélat. ‘Zermelo navigation on the sphere with revolution metrics’. In: *IVAN KUPKA LEGACY: A Tour Through Controlled Dynamics*. Vol. 12. AIMS Applied Math Books - Special issue in honor of I. Kupka. 2024, pp. 35–66. URL: <https://hal.science/hal-04433828> (cit. on p. 11).

### Doctoral dissertations and habilitation theses

- [43] R. Roussel. ‘Magnetic confinement in stellarators: harmonic fields and shape differentiation’. Sorbonne Université, 9th Dec. 2024. URL: <https://theses.hal.science/tel-04865970> (cit. on pp. 12, 13).

### Reports & preprints

- [44] K. Agbo Bidi, L. Almeida and J.-M. Coron. *Global stabilization of a Sterile Insect Technique model by feedback laws*. 12th Nov. 2024. URL: <https://hal.science/hal-04777957> (cit. on p. 9).
- [45] A. Agrachev and B. Kazandjian. *Optimal Control for Linear Systems with  $L^1$ -norm Cost*. 11th Apr. 2024. URL: <https://inria.hal.science/hal-04546593> (cit. on p. 10).
- [46] A. Agrachev and C. Letrouit. *Generic controllability of equivariant systems and applications to particle systems and neural networks*. 5th Jan. 2025. URL: <https://hal.science/hal-04542990> (cit. on p. 10).
- [47] K. L. Balc’H and J. Martin. *Quantitative propagation of smallness and spectral estimates for the Schrödinger operator*. 22nd Mar. 2024. URL: <https://hal.science/hal-04625215> (cit. on p. 11).
- [48] L. Baratchart, S. Fueyo and J.-B. Pomet. *Exponential stability of linear periodic difference-delay equations*. 2024. URL: <https://inria.hal.science/hal-03500720> (cit. on p. 8).
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- [52] J.-M. Coron and S. Xiang. *Global controllability to harmonic maps of the heat flow from a circle to a sphere*. 1st Feb. 2024. URL: <https://hal.science/hal-04432880> (cit. on p. 12).
- [53] W. Gerner. *Asymptotic windings, surface helicity and their applications in plasma physics*. 1st Aug. 2024. URL: <https://hal.science/hal-04666332> (cit. on p. 13).
- [54] A. Hannani, M. N. Phung, M.-B. Tran and E. Trélat. *Controlling the rates of a chain of harmonic oscillators with a point Langevin thermostat*. 16th Mar. 2024. URL: <https://hal.science/hal-04507495> (cit. on p. 9).

- [55] A. Hannani, M. N. Phung, M.-B. Tran and E. Trélat. *Internal control of the transition kernel for stochastic lattice dynamics*. 30th Sept. 2024. URL: <https://hal.science/hal-04717306> (cit. on p. 11).
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- [59] I. Mazari, Y. Privat and E. Trélat. *Large-time optimal observation domain for linear parabolic systems*. 5th Feb. 2024. URL: <https://hal.science/hal-04440343> (cit. on p. 13).
- [60] T. Paul and E. Trélat. *From microscopic to macroscopic scale dynamics: mean field, hydrodynamic and graph limits*. 9th May 2024. URL: <https://hal.science/hal-04577170> (cit. on p. 9).
- [61] C. Pouchol, E. Trélat and C. Zhang. *Constructive reachability for linear control problems under conic constraints*. 10th May 2024. URL: <https://hal.science/hal-04572161> (cit. on p. 11).
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### Scientific popularization

- [64] K. Agbo Bidi. 'Feedback stabilization and observer design for sterile insect technique model'. In: *Mathematical Biosciences and Engineering* 21 (13th June 2024), pp. 6263–6288. DOI: [10.3934/mbe.2024274](https://doi.org/10.3934/mbe.2024274). URL: <https://hal.science/hal-04433395> (cit. on p. 20).

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