RESEARCH CENTRE

Inria Saclay Centre at Université Paris-Saclay

IN PARTNERSHIP WITH:

CentraleSupélec, CNRS

2024

ACTIVITY REPORT

Project-Team

DISCO

Dynamical Interconnected Systems: Control and Optimization

IN COLLABORATION WITH: Laboratoire des signaux et systèmes (L2S)

DOMAIN

Applied Mathematics, Computation and Simulation

THEME

Optimization and control of dynamic systems

Inria

Contents

Project-Team DISCO

Creation of the Project-Team: 2024 June 01

Keywords

Computer sciences and digital sciences

A6.1.1. – Continuous Modeling (PDE, ODE)

A6.1.2. – Stochastic Modeling

A6.1.3. – Discrete Modeling (multi-agent, people centered)

A6.2.1. – Numerical analysis of PDE and ODE

A6.2.6. – Optimization

A6.3.4. – Model reduction

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A6.4.4. – Stability and Stabilization

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A8.11. – Game Theory

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Other research topics and application domains

B1.1.8. – Mathematical biology

B1.1.10. – Systems and synthetic biology

B2.3. – Epidemiology

B2.5. – Handicap and personal assistances

B3.6. – Ecology

B4. – Energy

B5.2.3. – Aviation

B5.10. – Biotechnology

B7.2.1. – Smart vehicles

1 Team members, visitors, external collaborators

Research Scientists

- Catherine Bonnet [Team leader, INRIA, Senior Researcher, HDR]
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- Guilherme Mazanti [INRIA, ISFP]
- Frederic Mazenc [INRIA, Senior Researcher, HDR]
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- Laurent Pfeiffer [INRIA, Researcher]

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2 Overall objectives

2.1 Objectives

Control of interacting complex natural or artificial systems guaranteeing performance, safety and low computational burden is a challenge for the years to come.

Things have drastically changed from a material point of view in the last 15 years in engineering, biological and medical fields. For instance, in the field of robotics, the time when large robots, evolving in secure spaces where no human was supposed to be able to enter, could be managed with very basic control laws is over. Developing conntrol algorithms for robots of smaller size and having to adapt online their behaviour to the dynamics of the environment is much more difficult.

These topics are clearly the same in fields such as autonomous vehicles, unmanned aerial vehicles, traffic and energy networks etc. Moreover, in the case where several robots/vehicles/agents are cooperating, the question of communication is crucial and delays as well as disconnection are major issues in this context. The size of the network is certainly another key question.

Adopting a model-based approach, the aim of the team is to develop (optimal) control methods for (possibly large size) interconnected systems with the ultimate goal to produce implementable and low computational burden solutions (in the modest sense of favoring low complexity controllers from an implementation point of view).

We contribute to the modeling of chosen applications in Energy, bio-systems, Engineering and medicine. Partial Differential Equations (PDEs), Ordinary Differential Equations (ODEs) (possibly with delays), models will be considered in the linear as well as the nonlinear setting. Discretization of PDEs as well as the modeling of discrete measurements/actuation arising in continuous model systems will give rise to infinite or finite-dimensional discrete-time models. We also consider general classes of systems which encompass the framework of our applications such as Fractional Differential or Difference Equations.

Our goal is to analyze these models as much as possible without further simplifications in order to capture most of the phenomena (and, in particular, time-heterogeneity) and to develop control algorithms for them. This includes stability analysis, observation, robustness analysis and (optimal) control.

Note that we also perform some research at the confluence of Control Methods and Machine Learning (ML), not developing ourselves ML techniques to synthesize controllers from data but rather studying how *System Theory and Optimization Theory* can help analyzing closed-loop systems containing Neural Networks-based controllers.

3 Research program

3.1 Stability Analysis and Control of Linear Interconnected Systems

This first axis is devoted to the study of stability properties of (a small number of) interconnected linear systems, an area where many fundamental questions are still open. Algebraic tools, time-domain (including semi-group Theory) and frequency domain (where Hypergeometric functions should play an important role) techniques are involved to characterize stabilizability and controllability properties as well as asymptotic, exponential, H_∞ and L^p (in particular L^∞ *i.e.* BIBO)-stability properties.

For standard or fractional systems with delays, we are particularly interested in questions such as :

- The location in the complex plane of the roots of quasi-polynomials of the type $p_0(s) + \sum_{i=1}^{n} p_i(s) e^{-h_i s}$ where $p_i \;\; i = 0, \cdots, n$ are real polynomials in the complex variable s satisfying $\deg p_0 \geq \deg p_i$ and $h_i > 0$.
- The stability properties of linear delay systems with time-varying delays and/or time varyning coefficients via both time-domain and input-output methods. Robustness issues will be investigated.
- Structural properties of time-delay systems, such as controllability or stabilizability.
- Partial pole placement for delay systems : we aim here at establishing a general approach to control the decay rate of the solutions via the assignment of the spectral abscissa.

For Systems Modeled by Partial Differential Equations, we want to perform the analysis of :

- Stability properties of propagation phenomena modeled by partial differential equations.
- Partial differential equations with different time scales
- Dynamical systems with Integral delay equations
- Delayed control of systems modeled by PDEs

3.2 Observation and Control of Nonlinear Interconnected Systems

Observation and Control of Nonlinear Interconnected Systems

The second axis explores the control and observation (in particular finite-time observation) of various classes of (a small number of interconnected) nonlinear delay systems. Lyapunov analysis will be a central tool.

Concerning the Control of nonlinear systems, we will focus on:

- The local analysis of nonlinear systems with delays in particular via the trajectory-based approach.
- The Stability analysis and performance assessment of Piecewise Affine systems described by the equations

$$\begin{aligned} x[k+1] &= F_1 x[k] + F_2 \phi(y(x[k])) \\ y(x[k]) &= F_3 x[k] + F_4 \phi(y(x[k])) + f_5 \end{aligned}$$

  where the vector-valued function $\phi : \mathbb{R}^m \to \mathbb{R}^m$ is a vector of ramp functions on each argument.
- Dynamical systems with Neural Networks (NN) in the loop

Concerning Observers and estimation of parameters, we wish to develop:

- Finite-time observers and estimation of unknown parameters for classes of systems that encompass discrete-time systems and systems with unknown parameters in the measurements.
- Advanced estimation and control techniques for traffic networks (new mesoscopic models will be developed).

3.3 Optimal control of mean-field type dynamical systems

Our research activity in optimal control is motivated by large-scale problems and aims at addressing both theoretical and numerical aspects. We have a focus on situations involving a large number of interacting entities: for example, a fleet of electrical vehicles, a cell population in biology, or a large training set in supervized learning. In such situations, the mean-field approximation consists in considering the probability distribution of these entities rather than an enumeration of their states. This point of view allows for a simpler mathematical treatment and numerical resolution.

In the Mean Field Game domain, we want to:

- Address the case of Agents with constraints and free-final time
- Study a new class of interactions for MFGs, which we called pairwise interactions the cost of each agent is the sum, with respect to all other agents, of some interaction function of the considered agent and any other agent.
- Develop variants of the Frank-Wolfe algorithm (whose interest go beyond MFGs) which are capable to address constrained or non-smooth MFGs.
- Export of our expertise on classical MFG models to more applied problems, in interactions with other mathematical fields : Management of fisheries, Energy, biology

Concerning the Theoretical analysis of some modern machine-learning methods, we aim to to continue to exploit their natural connection to Mean Field Models by analyzing:

- QSome machine learning methods that explicitly incorporate terms that can be interpreted as mean-field interactions.
- Deep neural networks that may be viewed as the discretization of ODE's
- Neural networks that may be viewed as the discretization of PDEs (such as convolutional residual networks)

4 Application domains

4.1 Analysis and Control of life sciences systems

The team is involved in life sciences applications. The actual two main lines are the analysis of bioreactors models (microorganisms; bacteria, microalgae, yeast, etc..) and the management of fisheries.

4.2 Energy Systems

4.2.1 Energy Management

The team is interested in Energy management and considers control problems in energy networks.

In the framework of a joint project with EDF, we aim at investigating large-scale and non-convex optimization problems related to the management of large population of "flexibilities", which are small-size storage devices (batteries, thermostatic loads), which can be used to facilitate the energy balance when sources of renewable energy are introduced into the network.

4.2.2 Electrical Machines

Advanced techniques of nonlinear control are currently studied to account for the input constraints and embedded in electrical drives of AC motors. These platforms are benchmarks for the nonlinear control design and analysis techniques developed in by the team members.

4.3 Mechanical systems

4.3.1 Biomechanics

In collaboration with colleagues from BME Budapest and the Robert Merle d'Aubigné Institute, we are interested in understanding the human balance from mechanical/neural point of views. We mainly seek to better understand the principle functioning of the flow process generated by our nervous system and its effect on motor abilities through the experiment of human stance on a rolling board.

4.3.2 Mechanical engineering

The team is interested in vibration control (in link with the so-called multiplicity-induced-dominancy, MID and partial pole placement) and in developing advanced delay algorithms for compensating and tracking periodic signals (related to the repetitive control).

4.4 Transportation Systems

The team is interested in control applications in transportation systems. In particular, the problem of collision avoidance of autonomous vehicels has been investigated under the framework of Time Varying systems. The goal is to obtain closed-loop control laws that guarantee the execution of a trajectory under uncertainties such as road and vehicle conditions.

5 Highlights of the year

5.1 Awards

S.I Niculescu received the IFAC French NMO Award (annual prize which recognizes a personality who has made fundamental contributions to IFAC and its functioning).

6 New results

6.1 Stability analysis for linear systems with a switched rapidly varying delay

Participants: Frédéric Mazenc, Silviu Niculescu, Diego Torres-García .

In [16], we have studied linear continuous-time systems with switched pointwise delays. We have developed a technique enabling to establish the stability of these systems with a rapidly varying periodic delay. It relies on two ingredients: a representation of the systems as time-varying systems with constant delays and an averaging approach. Illustrative examples showed the effectiveness of the proposed methodology.

6.2 Event-triggered control

Participants: Frédéric Mazenc, Michael Malisoff *(LSU)*, Corina Barbalata *(LSU)*, Safeyya Alyahia *(LSU)*
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In the contribution [1], we provided new dynamic event-triggered controls for continuous-time linear systems that contain additive uncertainties. We established input-to-state stability properties that imply uniform global exponential stability when the additive uncertainties are zero. Significant novel features include (a) new dynamic extensions and new trigger rules that provide a new positive systems analog of significant prior dynamic event-triggered work of A. Girard and (b) the application to a BlueROV2 underwater vehicle model, where we provided significantly larger lower bounds on the inter-execution times, and usefully fewer trigger times, compared with standard dynamic event-triggered approaches that used the usual Euclidean norm, and as compared with static event-triggered controls that instead used positive systems approaches.

6.3 New developments on the multiplicity-induced-dominancy property

Participants: Islam Boussaada *(DISCO)*, Guilherme Mazanti *(DISCO)*, Wim Michiels *(K.U. Leuven)*, Silviu-Iulian Niculescu *(DISCO)*.

The multiplicity-induced-dominancy (MID) property consists on the fact that, if a given time-delay system admits a spectral value of “large enough” multiplicity, then this spectral value is dominant (i.e., the rightmost one in the spectrum) and thus determines the asymptotic behavior of the system. The validity of this property has an important impact in the stabilization of time-delay systems, since, when it holds, one may stabilize a system by selecting its free parameters in order to ensure that it admits a spectral value of large multiplicity which is dominant and has negative real part, ensuring exponential stability. Since the seminal works [70, 71], the MID property has been established for some classes of time-delay systems and has been applied to many contexts [89, 69], and is an active research topic of the DISCO team. During the past year, several new developments on the MID property were established in the team.

The focus of [5] is the MID property for delay-differential equations (DDEs) with a single delay and spectral values with overorder multiplicity, i.e., a multiplicity larger than the order of the DDE. More precisely, that article considered the particular case of retarded DDEs in which the non-delayed part is of order n, the delayed part is of order $n-1$, and one is interested in roots of multiplicity at least $n+1$. We have proved that a root of overorder multiplicity is necessarily a root of a particular polynomial, called the *elimination-produced polynomial*, and addressed the MID property using a suitable factorization of the corresponding characteristic function involving special functions of Kummer type. Additional results and discussion were provided in the case of the nth order integrator, in particular on the local optimality of a multiple root. The derived results showed how the delay can be further exploited as a control parameter and were applied to some problems of stabilization of standard benchmarks with prescribed exponential decay.

A generalization of the results of [5] to the case of single-delay DDEs was done in [4]. For a single-delay DDE with non-delayed part of order n and delayed part of order $m \leq n$, that reference characterized the existence of spectral values of multiplicities between $n+1$ and $n+m+1$ in terms of a suitable integral factorization, which can be rewritten as a linear combination of Kummer functions, the number of Kummer functions appearing in the combination being the difference between the degree of the corresponding quasipolynomial and the multiplicity of the spectral value. We have also considered stabilizability issues with an arbitrary configuration of known and unknown parameters of the system, proving that a root of overorder multiplicity is necessarily a root of a particular function, called the *elimination-produced function*, which extends the previous notion of elimination-produced polynomial. Theoretical results of the paper were concluded by a sufficient condition for validity of the MID property, which relies on the Green–Hille transformation from [85]. The article was concluded by illustrative examples.

As observed in [89, 69], Kummer confluent hypergeometric functions appear naturally when considering the MID property for time-delay systems. In [53], we have established new properties of Kummer functions with integer coefficients, showing a new representation formula for these functions in terms of iterated integrals of an exponential kernel.

It has been known since at least [91] that, in many examples, spectral values of large multiplicity of a time-delay system appear naturally when minimizing the spectral abscissa of the system, but a general analysis of this phenomenon is still missing from the literature. The article [20] has considered, in its first part, the case of second-order retarded delay-differential equations with first-order delayed term coming from a full state feedback, proving that, in this configuration, triple real roots or pairs of complex-conjugate roots of multiplicity 2 each are minimizers of the spectral abscissa. The proofs rely on perturbation theory of nonlinear eigenvalue problems and exploit the quasi-convexity of the spectral abscissa function. In the second part, a computational characterization of minima of the spectral abscissa is made for output feedback, yielding a more complex picture, which includes configurations with both multiple and simple rightmost roots. In the analysis, the pivotal role of the invariant zeros is highlighted, which translate into restrictions on the tunable parameters in the closed-loop quasi-polynomial.

6.4 New developments on the coexistence of real roots inducing dominancy

Participants: Souad Amrane *(Université Mouloud Mammerie Tizi-Ouzou, Algérie)*, Fazia Bedouhene *(Université Mouloud Mammerie Tizi-Ouzou, Algérie)*, Islam Boussaada *(DISCO)*, Silviu-Iulian Niculescu *(DISCO)*, Timothée Schmoderer *(DISCO)*, Cyprien Tamekue *(Laboratoire des Signaux et Systèmes (L2S))*.

The spectral property coexistent-real-roots-inducing-dominancy (CRRID) first introduced in [60, 66] has the originality of assigning a maximal number of simple real zeros of the characteristic function rather than a multiple zero as is proposed in the MID approach. Thus, compared to the MID, the CRRID allows more parametric freedom and in general more robust design. During the past year, several new developments on the CRRID property were established in the team.

We contributed in [24] by showing that the coexistence of the maximal number of real spectral values of generic single-delay retarded second-order differential equations guarantees the realness of the rightmost spectral value. From a control theory standpoint, this entails that a delayed proportional-derivative (PD) controller can stabilize a delayed second-order differential equation. By assigning the maximum number of negative roots to the corresponding characteristic function (a quasipolynomial), we establish the conditions for asymptotic stability. If the assigned real spectral values are uniformly distributed, we specify a necessary and sufficient condition for the rightmost root to be negative, thus guaranteeing the exponential decay rate of the systems solutions. We illustrate the proposed design methodology in the delayed PD control of the damped harmonic oscillator.

As emphasized in [28], it appears that the CRRID is an appropriate attack angle to investigate the MID property in low multiplicities. Indeed, despite the fact that this assumption is not met, an analytical proof for the MID is proposed. The coexistence of real spectral values is the main ingredient. The obtained result is illustrated through the stabilization of an unstable second-order plant by a delayed PD controller.

Finally, in [57], a control-oriented delay-based modeling of a one-layer neural network of Hopfield-type subject to an external input designed as delayed feedback is investigated. The specificity of such a model is that it makes the considered neuron less susceptible to seizure caused by its inherent dynamic instability. This modeling exploits a recently set partial pole placement for linear functional differential equations, which relies on the coexistence of real spectral values, allowing the explicit prescription of the closed-loop solution's exponential decay. The proposed framework improves some pioneering and scarce results from the literature on the characterization of the exact solution's exponential decay when a simple real spectral value exists. Indeed, it improves neural stability when the inherent dynamic is stable and provides insights into the design of a one-layer neural network that can be stabilized exponentially with delayed feedback and with a prescribed decay rate regardless of whether the inherent neuron dynamic is stable or unstable.

6.5 Implementation of Stabilizing Controllers for Retarded Delay Systems

Participants: Catherine Bonnet *(DISCO)*, Suat Gumussoy *(Siemens Technology Princeton)*, Hitay Özbay *(Bilkent University)*, Mustafa Oguz Yegin *(Bilkent University)*.

We have proposed in [29] a way to efficiently implement Hinfty-stabilizing controllers for delay systems of retarded type. We indeed chose a particular controller (central controller) in the set of all stabilizing controllers and decomposed its transfer function in terms of stable delay systems and Finite Impulse Response (FIR) Systems. The central controller is then written in such a way that the corresponding Matlab/Simulink implementation scheme is obvious. We illustrated that all stabilizing controllers can be implemented using stable components only around the central controller. We also discussed stability robustness in the presence of implementation errors. The method was illustrated in an example.

6.6 Stability analysis of systems with delay-dependent coefficients and commensurate delays

Participants: Islam Boussaada *(DISCO)*, Chi Jin *(TuSimple Inc. Shanghai,China)*, Keqin Gu *(Illinois University Edwardsville, USA)*, Qin Ma *(Nanjing University of Science and Technology, China)*, Silviu-Iulian Niculescu *(DISCO)*.

Systems with coefficients depending on time-delay may arise from various scientific disciplines, such as population models with age structure, the stellar dynamos, and hematopoiesis dynamics. In the SISO case, this class of system has been the subject of a thesis of Jin Chi (DISCO 2015-2017) where an analysis method has been proposed, see for instance [86]. A generalizion to the MIMO case is obtained in [10] where a method of stability analysis of linear time-delay systems with commensurate delays and delay-dependent coefficients is proposed. The method is based on a D-decomposition formulation that consists of identifying the critical pairs of delay and frequency, and determining the corresponding crossing directions. The process of identifying the critical pairs consists of a magnitude condition and a phase condition. The magnitude condition utilizes the OrlandoÕs formula, and generates frequency curves within the delay interval of interest. Such frequency curves correspond to the delay-frequency pairs such that the decomposition equation has at least one solution on the unit circle. The delay interval of interest is divided into continuous frequency curve intervals (CFCIs). Under some non-degeneracy assumptions, the number of frequency curves remains constant within each CFCI, and the associated decomposition equation has one and only one solution on the unit circle at any point on a frequency curve. By traversing through the frequency curves, all the crossing points can be identified. The crossing direction is related to the sign of the lowest-order nonzero derivative of the phase angle with respect to the delay, which is a generalization of the existing literature even for the case with single delay. This conclusion allows one to determine the crossing direction by examining the phase angle vs delay diagram. An example is presented to illustrate how a stability analysis can be conducted if some non-degeneracy assumptions are violated.

6.7 Spectral methods for the stability of linear hyperbolic systems

Participants: Kais Ammari *(Université de Monastir, Tunisie)*, Jean Auriol *(Laboratoire des Signaux et Systèmes (L2S))*, Ismaïla Balogoun *(DISCO)*, Islam Boussaada *(DISCO)*, Guilherme Mazanti *(DISCO)*, Silviu Niculescu *(DISCO)*, Timothée Schmoderer *(DISCO)*, Sami Tliba *(Laboratoire des Signaux et Systèmes (L2S))*.

Systems of first-order hyperbolic partial differential equations (PDEs) have been extensively studied over the years due to their application in modeling various physical phenomena, such as drilling devices, water management systems, aeronomy, cable vibration dynamics, pipelines, and traffic networks [65, 84, 62, 78]. When there are no in-domain coupling terms, the method of characteristics can be used to relate the behavior of these systems to time-delay systems, a link extensively explored in the literature [8, 74, 75, 76, 96]. The situation is more delicate when in-domain coupling terms are present, but, as recently shown in [61, 68], a transformation into a time-delay system is still possible, using first a backstepping transformation to convert the in-domain couplings into integral boundary terms, and then applying the method of characteristics to obtain a time-delay system, which presents a distributed delay.

In [52], we explore the latter transformation in order to establish a necessary and sufficient stability condition for a class of two coupled first-order linear hyperbolic partial differential equations with in-domain coupling. The stability of the associated time-delay system, which is of neutral type, is studied through spectral methods, using Cauchy's argument principle in order to count the number of unstable roots. This is done by adapting to the present setting the Stépán–Hassard argument variation approach for retarded time-delay systems, first introduced in [97] and then refined in [83]. Our theoretical findings are also validated in [52] through simulations.

In [24] the boundary control of the transport equation is addressed. Namely, based on the CRRID property, a control method is proposed, which is merely a delayed output feedback relying on a partial pole placement idea, that consists in assigning an appropriate exponential decay rate to the closed-loop system's solution. The proposed control structure appearing in the transport boundary, which has proven its effectiveness in controlling finite dimensional systems, consists of an autoregressive relation linking the transport equation's input and output. The obtained result provides an analytical lower bound for the solution's exponential decay.

As a mechanical engineering application, the transverse vibration midpoint control of a string is considered in [39]. More precisely, we explore the use of a control block that includes delays in both input and output signals under the assumption that the memory-window length is also a parameter of the control law. Relatively simple to construct, such a control law was successfully used in the boundary control of the transport and wave equations. The effectiveness of the proposed method is illustrated through numerical simulations and the corresponding finite-difference scheme is detailed.

In [26], we design a controller for an interconnected system consisting of a linear Stochastic Differential Equation (SDE) actuated through a linear hyperbolic Partial Differential Equation (PDE). Our approach aims to minimize the variance of the state of the SDE component. We leverage a backstepping technique to transform the original PDE into an uncoupled stochastic PDE. As such, we reformulate our initial problem as the control of a delayed SDE with a non-deterministic drift. Under standard controllability assumptions, we design a controller steering the mean of the states to zero while keeping its covariance bounded. As final step, we address the optimal control of the delayed SDE employing Artsteins transformation and Linear Quadratic stochastic control techniques.

6.8 Averaging for time-varying systems

Participants: Rami Katz *(Tel Aviv Univ)*, Frédéric Mazenc, Emilia Fridman *(Tel Aviv Univ)*.

We have have developed a new avaraging technique for fast-varying continuous-time systems.

In [11], we treated the input-to-state stability-like estimates for perturbed linear continuous-time systems with multiple time-scales, under the assumption that the averaged unperturbed system is exponentially stable. Such systems contain rapidly-varying, piecewise continuous and almost periodic coefficients with small parameters. Our method relies on a novel delay-free system transformation in conjunction with a new system presentation, where the rapidly-varying coefficients are scalars that have zero average. We employ time-varying Lyapunov functions for ISS-like analysis. The analysis yields LMI conditions, leading to explicit bounds on the small parameters, decay rate and ISS gains. The novel system presentation plays a crucial role in the ISS-like analysis by allowing to derive essentially less conservative upper bounds on terms containing the small parameters. We further extended our approach to rapidly-varying systems subject to either discrete (constant or fast-varying) or distributed delays, where our approach decouples the effects of the delay and small parameters on the stability of the system and leads to LMI conditions for stability of systems with non-small delays. Numerical examples showed that, compared to the existing results, the approach essentially enlarges the small parameter and delay bounds for which the ISS-like/stability property of the original system is preserved.

6.9 Local Halanay's Inequality

Participants: Frédéric Mazenc, Michael Malisoff *(LSU)*.

In contributions [13] and [33], we provided a local version of an approach to proving asymptotic stability that is based on the celebrate Halanay's inequality. Our results are applicable to a family of nonlinear systems that contain state and input delays. We determine input-to-state stability inequalities when the systems contain additive uncertainty. We combined the results with an observer and a Gramian approach, to solve an output feedback stabilization problem. Our numerical examples illustrated how our theorems lead to new basin of attraction estimates.

6.10 Extremum seeking

Participants: Frédéric Mazenc, Michael Malisoff *(LSU)*, Emilia Fridman *(Tel Aviv Univ)*.

We solved in [34] a gradient based bounded extremum seeking problem for single-variable static maps in the presence of time-varying piecewise continuous measurement uncertainty. Instead of using previously reported averaging-based methods, we introduced a new state transformation, allowing us to use new comparison function and generalized Lyapunov function approaches to obtain our ultimate bounds on the parameter estimation error. We illustrated significant advantages of our new method, including less restrictive conditions on the extremum seeking parameters, as compared with previous methods.

6.11 Stability Analysis of Piecewise affine systems

Participants: Giorgio Valmorbida, Joao Manoel Gomes da Silva Jr *(Universidade Federal do Rio Grande do Sul)*, Leonardo Cabral *(Universidade Federal do Rio Grande do Sul)*, Diego Munoz Carpintero *(Universidad O'Higgins, Chile)*.

Piece-wise affine systems appear when linear dynamics are defined in different partitions of the state space. This type of systems naturally appears whenever actuators have different stages or saturate or whenever non-linear control laws are obtained as the solution to a parameterised optimization problem as, for instance for systems with feedback laws based on the so-called explicit Model Predictive Control. Even though the dynamics is simple to describe, the stability analysis, performance assessment and robustness analysis are difficult to perform since, due to the often used explicit representation, the Lyapunov stability and dissipation tests are often described in terms of a number of inequalities that increases exponentially on the number of sets in the partition since they are based on the enumeration of the partition transitions. Moreover regional stability and uncertainties corresponding to modification on the partition are difficult to study in this scenario.

To overcome these difficulties we have proposed an implicit representation for this class of systems in terms of ramp functions. The main advantage of such a representation lies on the fact that the ramp function can be exactly characterized in terms of linear inequalities and a quadratic equation, namely a linear complementarity condition. Thanks to the characterization of the ramp function and the implicit description of the PWA system, the verification of Lyapunov inequalities related to piecewise quadratic functions can be formulated as a semidefine programming whenever some co-positivity constraints are relaxed.

We have applied the results to the local analysis and synthesis of PWA control laws [7]. Such a local formulation is based on local conditions for co-positivity of matrices. The proposed results encompass regional stability analysis formulations in the literature.

We have also studied continuous-time systems with piecewise quadratic Lyapunov functions [6]. The proposed stabilty conditions allowed to compute tight convergence rates with refined partition and more complex tests for copositivity.

For linear systems with state constraints, we proposed a parametrization of piecewise-quadratic Lyapunov functions to compute compute nonlinear control laws allowing to enlarge invariant sets [21]. The main advantage of the proposed method is the computation of asymmetric invariant sets with low memory footprint.

6.12 Implicit Neural Networks and Piecewise affine functions from Model Predictive Control

Participants: Giorgio Valmorbida, Ross Drummond *(University of Sheffield)*.

Model predictive control (MPC) for linear systems with quadratic costs and linear constraints were shown to admit an exact representation as an implicit neural network. A method to “unravel” the implicit neural network of MPC into an explicit one is also proposed. As well as building links between model-based and data-driven control, these results emphasize the capability of implicit neural networks for representing solutions of optimisation problems, as such problems are themselves implicitly defined functions.

6.13 Nonsmooth optimization algorithms through duality

Participants: Guilherme Mazanti *(DISCO)*, Thibault Moquet *(DISCO)*, Laurent Pfeiffer *(DISCO)*.

The Frank–Wolfe Algorithm (FWA), also known as Conditional Gradient Algorithm, is an iterative minimization algorithm for minimizing a convex function with Lipschitz continuous gradient over a closed convex bounded subset K of a Banach space. Introduced in [79], this method assumes that one can easily minimize linear functions orver K, through a so-called Linear Minimization Oracle (LMO), and proceeds by using the result of this oracle to update the candidate to optimality through convex combinations. Many extensions of the FWA were considered in the literature, such as those from [80, 95, 98].

In [56], we have proposed an extension of the FWA, named Dualized Level-Set (DLS) algorithm, which allows to address a class of nonsmooth costs, involving in particular support functions. The key idea behind the construction of the DLS method is a general interpretation of the FWA as a dual of a cutting-plane algorithm, a fact previously observed and exploited in [63, 64, 99]. The DLS algorithm essentially results from a dualization of a specific cutting-plane algorithm from [88], based on projections on some level sets, and it generates a sequence of primal-dual candidates. We prove that the corresponding sequence of primal-dual gaps converges to 0 with a rate of $O(1/\sqrt{t})$. With respect to other extensions of the FWA to nonsmooth costs, the main novelty here is that we do not rely on regularizations of the nonsmooth term, dealing with it through duality instead.

6.14 Existence and asymptotic behavior of equilibria for general optimal-exit mean field games

Participants: Guilherme Mazanti *(DISCO)*.

In recent years, many works, such as [73, 67, 81, 82, 72], have considered mean field games (MFGs) in which the time at which an agent exits the game is part of their optimization criterion. In particular, [90, 77, 92, 93, 94] have considered several variants of a similar MFG model motivated by crowd motion in which agents evolve in a given set and wish to reach a given target set, typically in minimal time and possibly with an additional cost in the exit position. In these MFGs, agents interact through their dynamics, the maximal speed of an agent being assumed to be a function of their position and the distribution of other agents, an interaction that may model, in particular, congestion phenomena.

The main aim of [14] was to provide a unifying theory for the existence of equilibria of this class of MFGs, and to detail results on the asymptotic behavior of equilibria and on their dependence with respect to the initial distribution of agents. The main result of [14] on existence of equilibria generalizes all previous existence results proved in [90, 77, 92, 93, 94] and can also be applied to other classes of games, such as MFGs on networks. In addition, [14] also provides results on the convergence of the distribution of agents to a limit distribution, giving precise convergence rates. Finally, it is shown in [14] that, even though equilibria and limit distributions of such MFGs may not be continuous functions of the initial distribution of agents m_0, they are necessarily upper semicontinuous set-valued functions of m_0, which is shown to be the best result one may expect in full generality.

6.15 Mechanical engineering application

Participants: Islam Boussaada *(DISCO)*, Jaroslav Busek *(Czech Technical University in Prague, Czech Republic)*, Matej Kure *(Czech Technical University in Prague, Czech Republic)*, Silviu Niculescu *(DISCO)*, Tomas Vyhlidal *(Czech Technical University in Prague, Czech Republic)*, Wim Michiels *(KU Leuven, Belgium)*.

In [12], a combination of analytical and numerical time-delay-system spectrum-shaping tools are applied to the design of the robust delayed resonator with an acceleration feedback. First, the delayed resonator model is turned into a dimension-less form with the objective to generalize the derived results. The main theoretical result is then provided as a complete parameterization of the proposed resonator feedback with two delay terms to assign a pair of roots with multiplicity two on the imaginary axis. In the frequency-domain, the double roots are projected to widening the stop-band in the active absorber frequency response, which increases its robustness in vibration suppression. On the other hand, they have a destabilizing effect on the overall system dynamics. The stabilization is subsequently performed by an additional controller via spectral optimization. The design is thoroughly validated by both simulations and experiments where the results are compared with the classical delayed resonator with lumped delay acceleration feedback.

6.16 Chemostat with delay

Participants: Frédéric Mazenc, Gonzalo Robeldo *(Univ. of Chile)*, Daniel Sepulveda *(UTEM)*.

In [17], we have revisited a recently introduced chemostat model of one–species with a periodic input of a single nutrient which is described by a system of delay differential equations. Previous results provided sufficient conditions ensuring the existence and uniqueness of a periodic solution for arbitrarily small delays. This paper partially extends these results by proving—with the construction of Lyapunov–like functions—that the evoked periodic solution is globally asymptotically stable when considering Monod uptake functions and a particular family of nutrient inputs.

6.17 Management of fisheries using a mean-field-game approach

Participants: Ziad Kobeissi, Idriss Mazari-Fouquer *(University Paris Dauphine-PSL)*, Domenec Ruiz-Balet *(University Paris Dauphine-PSL)*.

We propose a novel model for managing fisheries, described by a system of three coupled partial differential equations. The first is a reaction-diffusion equation representing the dynamics of the fish population, which follows standard approaches in the mathematical literature on spatial ecology. The other two equations are derived using a mean-field-game (MFG) framework to model a large population of fishermen, where the number of fishermen is assumed to be large enough to be treated as infinite. Each fisherman aims to maximize their individual profit, calculated as the revenue from selling fish minus the cost of moving their boat. Under two different structural assumptions about the nonlinearities in the fish dynamics, we prove theoretical results illustrating the tragedy of the commons. Specifically, we show that a lack of coordination among fishermen can significantly harm, or even lead to the extinction of the fish population. Our findings are supported by several numerical simulations.

The paper [87] has been published in Nonlinearity in 2024. In this paper, we assume that, when no fisherman is present, the fishes' population is invading (mathematically, there is an invading travelling front). We prove that fishermen, when acting selfishly, each in their own best interest, might lead to a reversal of the travelling wave and, consequently, to an extinction of the global population. We then show that, at least in some cases, if the fishermen coordinated instead of acting selfishly, each of them could make more benefit, while still guaranteeing the survival of the population.

In [54], the main focus is on the derivation of analytical results (e.g existence, uniqueness) and of long time behaviour (here, convergence to the ergodic system) on the mean-field-game system of fishing introduced in [87]. We develop novel and versatile tools to address this new class of MFG systems, where agent interactions are indirect and mediated through the solution of a third coupled partial differential equation. The challenging issue of uniqueness, a well-known difficulty in the mean-field game literature, is partially tackled using an innovative approach based on spectral methods inspired by shape optimization theory. We establish the convergence of the time-dependent system to its ergodic counterpart as the time horizon tends to infinity, both in the first- and second-order cases.

7 Bilateral contracts and grants with industry

7.1 Bilateral contracts with industry

Participants: Laurent Pfeiffer, François Caffier.

The team signed in 2024 a contract with IFPEN in the framework of the Inria-IFPEN contract 20 June 2020. A PhD is funded for 3 years.

8 Partnerships and cooperations

8.1 International initiatives

8.1.1 CNRS-Africa Joint Research Program

SPECTRE-EDP

Title: *SPECTRE-EDP - Stabilisation PrEsCriTe : effet du REtard sur les Equations aux Dérivées Partielles*

Program: CNRS-Africa JRP

Duration: July 1, 2024 – December 31, 2028

Holder: Islam Boussaada

Partners : Kaïs Ammari, Université de Monastir, TUNISIA

Participants: Jean Auriol(L2S) , Guilherme Mazanti, Silviu Niculescu, Sami Tliba(L2S) , Karim Trabelsi(IPSA) .

Inria contact: Islam Boussaada

Summary: The main objective of this project is to develop unified methods for obtaining control algorithms with reduced complexity for the control of PDE systems. The methods developed in this context have the original feature of controlling the exponential decay of the controlled systems' solutions.

8.1.2 STIC/MATH/CLIMAT AmSud projects

NetConHybSDP

Title: *NetConHybSDP - Networked control of hybrid systems by semidefinite programming with applications in industry 4.0*

Program: STIC-AmSud

Duration: January 1, 2023 – December 31, 2024

Local supervisor: Giorgio Valmorbida

Partners:

Participants: Pedro Luis Dias Peres *(UNICAMP, Brazil)*, Daniel Sbarbaro Hofer *(Universidad de Concepción, Chile)*, Joao Manoel Gomes da Silva Jr *(UFRGS, Brazil)*, Valter Leite Jr. *(CEFET-MG, Brazil)*.

Inria contact: Giorgio Valmorbida

Summary: This project focuses on stability analysis, control and estimator design for hybrid systems operating in Industry 4.0 networks. The aim is to develop methods and algorithms that are able to cope with challenges such as fault detection, monitoring, effects of sampling, quantization, limitations of bandwidth and signal magnitude, presence of uncertainties, time-delays, nonlinear behaviors and packet losses. In the search for nonconservative results, semidefinite programming tools, that can be accurately and efficiently solved, are used

8.2 International research visitors

8.2.1 Visits of international scientists

Other international visits to the team

Kaïs Ammari

Status Full Professor

Institution of origin: Université de Monastir

Country: TUNISIA

Dates: 1 Month in 2024

Context of the visit: CNRS-Africa JRP; SPECTRE-EDP

Mobility program/type of mobility: Research Stay

Daniel Amaral

Status PhD student

Institution of origin: Universidade Federal do Ceará

Country: Brazil

Dates: 1 January - 30 June

Context of the visit: Research visit to collaboration on desing for Robust Smith Predictor schemes

Mobility program/type of mobility: Research Stay

Andressa Moura

Status PhD student

Institution of origin: UNICAMP

Country: Brazil

Dates: 1 January - 30 April

Context of the visit: Research visit to collaboration on the design of Observers for Hybrid Systems

Mobility program/type of mobility: Research Stay

Gabryelle de Souza

Status Undergraduate Intern

Institution of origin: UNICAMP

Country: Brazil

Dates: 2 February - 1 June

Context of the visit: Research visit to collaboration on the tests for polynomial positivity in the context of the Stability Criterion of Liénard-Chipart.

Mobility program/type of mobility: internship

Gonzalo Arias

Status PhD student

Institution of origin: Pontificia Universidad Católica de Chile

Country: Chile

Dates: 15–19 April

Context of the visit: Research collaboration on singular perturbation methods for the stability and control of systems involving hyperbolic partial differential equations.

Mobility program/type of mobility: Research stay

8.2.2 Visits to international teams

Research stays abroad

Valmorbida

Visited institution: Universidad O'Higgins

Country: Chile

Dates: 19-27 July 2024

Context of the visit: Collaboration within the NetConHybSDP SticAmSud project

Mobility program/type of mobility: research stay

8.3 National initiatives

- L. Pfeiffer is involved in the EDF-Inria DEFI on the "management of tomorrow's electrical systems".

9 Dissemination

9.1 Promoting scientific activities

9.1.1 Scientific events: organisation

Since 2023, I. Boussaada organizes jointly with K. Ammari (Université de Monastir) the annual conference CTIP (Control Theory and Inverse Problems). Since its creation, each edition of CTIP Conference attracted more than 60 scientific participants.

I. Boussaada organized jointly with K. Ammari (Université de Monastir) the kickoff meeting of SPECTRE-EDP project funded by CNRS-Africa Joint Research Program. This event attracted more than 60 scientific participants from eight different countries.

9.1.2 Scientific events: selection

Chair of conference program committees I. Boussaada was the IPC co-chair of the 2024 *IFAC Time-Delay Systems.*

Member of the conference program committees C.Bonnet is SIAM Associate Editor of the *American Control Conference* since 2019. She was a member of the IPC (and Associate Editor) of the Conferences *Mathematical Theory of Networks and Systems* 2024 and *IFAC Time-Delay Systems* 2024.

I. Boussaada was Associate Editor for the 2024 *IFAC Time-Delay Systems* as well the 2024 *European Control Conference.*

F. Mazenc was Associate Editor of the *European Control Conference, Stockholm, Sweden (2024).*

Reviewer Members of the team have reviewed papers for several conferences covering the topics of the team, including the Conference on Decision and Control, American Control Conference, European Control Conference.

9.1.3 Journal

Member of the editorial boards F. Mazenc was Associate Editor of Automatica

F. Mazenc was Associate Editor of IEEE Control Systems Letters.

F. Mazenc was Editor of the Asian Journal of Control.

G. Mazanti is Associate Editor of Matemática Contemporânea

Reviewer - reviewing activities Members of the team have reviewed papers for several journals covering the topics of the team, including Acta Applicandae Mathematicae, Applied Mathematics and Optimization, Automatica, IEEE Transactions on Automatic Control, IMA Journal of Mathematical Control and Information, Journal of Optimization Theory and Applications, and Systems & Control Letters

9.1.4 Invited talks

I. Boussaada gave a plenary talk at MOAD conference held in November 2024, Tizi-Ouzou, Algeria.

I. Boussaada gave a plenary talk at ICAAM conference held in December 2024, Monastir, Tunisia.

In January, G. Mazanti gave an online talk at the Seminar of the Association Sciences for Africa (ASA). Title of the talk: Stabilité et contrôle d'équations aux différences.

In March, F. Mazenc gave a talk at the Laboratory Jacques-Louis Lion, Paris. Title of the talk: A stability analysis technique called trajectory-based approach.

In March, G. Mazanti gave a talk at the "Mini-workshop on analysis and geometry", Shandong University, Jinan, China. Title of the talk: Stability of continuous-time difference equations.

In March, G. Mazanti gave a talk at the workshop "Optimization, Control, and Aerospace Applications", Zhejiang University, Hangzhou, China. Title of the talk: A minimal-time mean field game model inspired by crowd motion.

In March, G. Mazanti gave a talk at the "Seminar of the China–France Mathematics Talent Class", University of Science and Technology of China, Hefei, China. Title of the talk: Game theory, crowd motion, and mean field games.

In March, G. Mazanti gave a talk at the seminar of the School of Electronic Information and Communications (EIC), Huazhong University of Science and Technology, Wuhan, China. Title of the talk: A nonsmooth Frank–Wolfe algorithm through a dual cutting-plane approach.

In March, G. Mazanti gave a talk at the Mathematics Seminar Series of the Great Bay University, Dongguan, China. Title of the talk: Game theory, crowd motion, and mean field games.

In July, G. Valmorbida gave a talk at the Universidad O'Higgins, Rancagua, Chile. Title of the talk: Stability Analysis of Piecewise-Affine Systems in Implicit Form and Evaluation of Piecewise-Linear Functions.

In August, G. Valmorbida gave a talk at theWorkshop "Methods and Algorithms for the Control of Complex Systems" . Title of the talk: Local Stability Analysis of Piecewise Affine Systems, 27-29 August 2024, Banyuls, France.

9.1.5 Scientific expertise

C. Bonnet is a member of the Scientific Council of CentraleSupélec since December 2021 and of the board of directors of the Gaspard Monge Program for Optimization since October 2023.

C. Bonnet was a member of the (IFAC Young Author Award finalist of the conference TDS 2024).

Since 2020, I. Boussaada is a member of the Scientific council of TéSa collaborative Laboratory.

Jan.–Sept. 2024 F. Mazenc was member of the Commission de Développement Technologique.

Oct.–Dec. 2024 G. Mazanti was member of the Commission de Développement Technologique.

9.1.6 Research administration

C. Bonnet was a member of the BCEP Inria Saclay Center until february 2024.

C. Bonnet is a member of the Coordination committee of the Mentoring Program of Inria Saclay Center and of the Council of L2S.

C. Bonnet is the co-President of the Parity Committee at Inria since January 2022. She is the Parity Referent at L2S for CNRS since its creation in November 2020.

Since 2019, I. Boussaada is a member of the administration Council of the association SAGIP.

Since 2020, I. Boussaada is a member of the skills development bord of Sup'Biotech engineering School.

9.2 Teaching - Supervision - Juries

9.2.1 Teaching

- Licence : Catherine Bonnet, Internship supervision of Léo Poisson, 3rd year, Université Paris-Saclay, 8-30 June 2024.
- Licence: Islam Boussaada, *Complex and harmonic analysis*, 127h, 3rd year, IPSA, France.
- Licence: Ziad Kobeissi, *Convergence, Integration and Probability*, 16.5h, 1st year, CentraleSupelec, Université Paris-Saclay.
- Licence: Ziad Kobeissi, *Partial Differential Equation*, 15h, 1st year, CentraleSupelec, Université Paris-Saclay.
- Licence: Guilherme Mazanti, *Partial differential equations*, 27h, 1st year, CentraleSupélec, Université Paris-Saclay.
- Licence: Silviu Niculescu, *Mathematics*, 15h, 1st year, ENSMP Paris, France.
- Licence: Silviu Niculescu, *Introduction to optimization*, 30h, 1st year, ESIEE Paris, France.
- Licence: Giorgio Valmorbida, *Signal Processing*, 1st year, 43h CentraleSupélec Université Paris-Saclay.
- Master: Silviu Niculescu, *Signals and Systems*, 12h, ESIEE Paris, France.
- Master: Giorgio Valmorbida, *Stability of Dynamical Systems*, Master ATSI (M2), Université Paris-Saclay.
- Master: Guilherme Mazanti, *Optimization*, 46h, 2nd year, CentraleSupélec, Université Paris-Saclay.
- Master: Laurent Pfeiffer, *Optimal control of ordinary differential equations*, 15h, Optimization Master, Université Paris-Saclay and Ensta-Paris.
- Master: Laurent Pfeiffer, *Continuous optimization*, 18h, Energy Master, Institut Polytechnique de Paris and Ensta-Paris.
- Master: Laurent Pfeiffer, *Optimization project*, 18h, Energy Master, Institut Polytechnique de Paris and Ensta-Paris.

- Master: Giorgio Valmorbida, *Optimization*, 2nd year, 43h CentraleSupélec Université Paris-Saclay.
- Master: Giorgio Valmorbida, *Hybrid Systems*, 3rd year, 18h CentraleSupélec Université Paris-Saclay.
- Master: Giorgio Valmorbida, *Projects and Internship supervision*, 2nd and 3rd years, 81h, CentraleSupélec Université Paris-Saclay.
- Master: Giorgio Valmorbida, *Introduction aux systèmes asservis*, 3h, CentraleSupélec Executive Education, Université Paris-Saclay.
- Doctorat: Silviu Niculescu, *Controlling Delayed Dynamics: Advances in Theory, Methods and Applications*, 7h, CISM Udine, Italy.

9.2.2 Supervision

I. Boussaada is the co-supervisor of the PhD students of Crédo Fanou as well as Gabriel Velho, both members of L2S.

I. Boussaada and G. Mazanti was the co-mentors of the postdoc of Ismaila Balogoun.

9.2.3 Juries

C. Bonnet was a member of the 2024 CRCN-ISFP recruiting committees of Inria Grenoble and of the Lecturer recruiting committee of Université de Dijon.

C. Bonnet was in the PhD committee of Hichem Bessafa, Université de Lorraine, France, 19 septembre 2024. Title: Algorithmes avancés d'estimation pour les applications des véhicules connectés et autonomes.

C. Bonnet was in the PhD committee of Hasni Arezki, Université de Lorraine, France, 15 novembre 2024. Title: Algorithmes d'estimation avancés pour les systèmes non linéaires : conception et applications.

C. Bonnet was in the committee of “Suivi individutel” of the PhD of Pauline Mazel, UCA and Inria, 6 juin 2024. Title: la modélisation et le contrôle optimal de la compétition entre cellules saines et cancéreuses. Thesis supervisors: Frédéric Grognard and Walid Djéma.

C. Bonnet was a reviewer of the 'Habilitation à diriger des recherches' of Rosa Abbou, Université de Nantes, France, 5 mars 2024. Title: Contributions à l'étude de la dynamique des systèmes en vue d'une commande supervisée : apports théoriques et applications.

G. Mazanti was in the committee of “Suivi individutel” of the PhD of Benoît Hureaux, Thesis title: Synthèse de lois de commande bio-inspirées à impédance variable pour interaction avec l'environnement dans des tâches rythmiques. Thesis supervisors: Pedro Rodriguez-Ayerbe, Isabelle Siegler, Maria Makarov. L2S, Gif-sur-Yvette.

F. Mazenc was in the Phd committee of Antoine Hugo; University of Rouen. Title of the Phd: Synthèse d'*o*bservateurs par intervalle p*o*ur le diagn*o*stic de fautes et le c*o*ntrôle r*o*buste, avec applicati*o*n aux dr*o*nes quadric*o*ptères. Supervisors: Etienne Craye, Helene Piet-Lahanier, Sofiane Ahmedali, Rihab El Houda Thabet, Luc Meyer.

F. Mazenc was in the committee of "Suivi individutel" of the Phd of Olyayo Reynaud, Thesis title: Safety of uncertain high other systems with delays. Thesis supervisors: Ahmad Hably and Mohamed Maghenem. Gipsa-Lab, Grenoble.

G. Valmorbida was in the committee of "Suivi individutel" of the Phd of Sara Callegari, Thesis title: Analyse et commande robuste de systèmes en dimension infinie connectés aux bords à des systèmes de dimension finies. Thesis supervisors: Dimitry Peaucelle. École doctorale EDSYS, Toulouse.

10 Scientific production

10.1 Publications of the year

International journals

[1] S. Alyahia, C. Barbalata, M. Malisoff and F. Mazenc. 'Dynamic Event-Triggered Control of Linear Continuous-Time Systems using a Positive Systems Approach'. In: *Nonlinear Analysis: Hybrid Systems* 54 (Nov. 2024), p. 101508. DOI: 10.1016/j.nahs.2024.101508. URL: https://inria.hal.science/hal-04704539 (cit. on p. 6).

[2] K. Ammari, I. Boussaada, S.-i. Niculescu and S. Tliba. 'Prescribing transport equation solution's decay via multiplicity manifold and autoregressive boundary control'. In: *International Journal of Robust and Nonlinear Control* (28th Jan. 2024). DOI: 10.1002/rnc.7214. URL: https://centralesupelec.hal.science/hal-04200203.

[3] A. Bertolin, G. Valmorbida, R. C. L. F. Oliveira and P. L. D. Peres. 'Output-feedback controllers with guaranteed L 2 -gain for continuous-time Lur'e systems using noncausal Zames–Falb multipliers'. In: *International Journal of Control* (9th Aug. 2024), pp. 1–14. DOI: 10.1080/00207179.2024.2386632. URL: https://hal.science/hal-04924899.

[4] I. Boussaada, G. Mazanti and S.-I. Niculescu. 'Over-order multiplicities and their application in controlling delay dynamics. On zeros' distribution of linear combinations of Kummer hypergeometric functions'. In: *ESAIM: Control, Optimisation and Calculus of Variations* (2024). DOI: 10.1051/cocv/2024055. URL: https://hal.science/hal-04266392. In press (cit. on p. 7).

[5] I. Boussaada, G. Mazanti, S.-I. Niculescu and W. Michiels. 'Decay Rate Assignment through Multiple Spectral Values in Delay Systems'. In: *IEEE Transactions on Automatic Control* (2024). DOI: 10.1109/TAC.2024.3447117. URL: https://hal.science/hal-04266228. In press (cit. on p. 7).

[6] L. Cabral, G. Valmorbida and J. M. Gomes da Silva Jr. 'Exponential Stability of Continuous-Time Piecewise Affine Systems'. In: *IEEE Control Systems Letters* 8 (2024), pp. 1649–1654. DOI: 10.1109/LCSYS.2024.3411509. URL: https://hal.science/hal-04924889 (cit. on p. 11).

[7] L. Cabral, G. Valmorbida and J. M. Gomes da Silva Jr. 'Regional Stability Analysis of Discrete-Time Piecewise Affine Systems'. In: *IEEE Transactions on Automatic Control* (2024), pp. 1–13. DOI: 10.1109/TAC.2024.3486656. URL: https://hal.science/hal-04924909 (cit. on p. 11).

[8] Y. Chitour, S. Fueyo, G. Mazanti and M. Sigalotti. 'Approximate and exact controllability criteria for linear one-dimensional hyperbolic systems'. In: *Systems and Control Letters* 190.105834 (2024). URL: https://inria.hal.science/hal-04228797 (cit. on p. 9).

[9] C. Dórea, S. Olaru, S.-I. Niculescu and V. Răsvan. 'Delay-dependent set-invariance for linear difference equations with multiple delays: a polyhedral approach'. In: *Journal of Difference Equations and Applications* (14th Feb. 2024), pp. 1–18. DOI: 10.1080/10236198.2024.2311674. URL: https://centralesupelec.hal.science/hal-04459220.

[10] C. Jin, K. Gu, Q. Ma, S.-I. Niculescu and I. Boussaada. 'Stability analysis of systems with delay-dependent coefficients and commensurate delays'. In: *Mathematics of Control, Signals, and Systems* (31st Aug. 2024). DOI: 10.1007/s00498-024-00399-0. URL: https://centralesupelec.hal.science/hal-04710700 (cit. on p. 9).

[11] R. Katz, E. Fridman and F. Mazenc. 'Constructive method for averaging-based stability via a delay free transformation'. In: *Automatica* (2024). URL: https://inria.hal.science/hal-04621064 (cit. on p. 10).

[12] M. Kuře, J. Bušek, I. Boussaada, W. Michiels, S.-I. Niculescu and T. Vyhlídal. 'Robust delayed resonator with acceleration feedback – design by double root assignment and experimental validation'. In: *Journal of Sound and Vibration* 576 (Jan. 2024), p. 118261. DOI: 10.1016/j.jsv.2024.118261. URL: https://hal.science/hal-04409473 (cit. on p. 13).

[13] M. Malisoff and F. Mazenc. 'Local Halanay's Inequality with Application to Feedback Stabilization *'. In: *Mathematical Control and Related Fields* (2024). URL: https://inria.hal.science/hal-04621069 (cit. on p. 10).

[14] G. Mazanti. 'A note on existence and asymptotic behavior of Lagrangian equilibria for first-order optimal-exit mean field games'. In: *Mathematical Control and Related Fields* 14.4 (Dec. 2024), pp. 1776–1813. DOI: 10.3934/mcrf.2024064. URL: https://hal.science/hal-04726422 (cit. on p. 12).

[15] F. Mazenc and M. Malisoff. 'Finite-Time Input-to-State Stabilization of Discrete-Time Systems'. In: *IEEE Control Systems Letters* 8 (2024), p. 6. URL: https://inria.hal.science/hal-04833269.

[16] F. Mazenc, S.-I. Niculescu and D. Torres-García. 'Stability analysis for linear systems with a switched rapidly varying delay'. In: *Automatica* (2024). URL: https://inria.hal.science/hal-04621061 (cit. on p. 6).

[17] F. Mazenc, G. Robledo and D. Sep Úlveda. 'A stability analysis of a time-varying chemostat with pointwise delay'. In: *Mathematical Biosciences and Engineering* 21.2 (Jan. 2024). DOI: 10.3934/mbe.2024xxx. URL: https://inria.hal.science/hal-04621063 (cit. on p. 13).

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[19] C.-F. Méndez-Barrios, J.-d. Torres-García and S.-i. Niculescu. 'Delay-Difference Approximations of PD-Controllers. Improperly-posed Systems in Multiple Delays Case'. In: *International Journal of Robust and Nonlinear Control* (31st Jan. 2024). DOI: 10.1002/rnc.7217. URL: https://hal.science/hal-04172009.

[20] W. Michiels, S.-I. Niculescu, I. Boussaada and G. Mazanti. 'On the relations between stability optimization of linear time-delay systems and multiple rightmost characteristic roots'. In: *Mathematics of Control, Signals, and Systems* (23rd Aug. 2024). DOI: 10.1007/s00498-024-00398-1. URL: https://centralesupelec.hal.science/hal-04710724 (cit. on p. 7).

[21] D. Muñoz-Carpintero and G. Valmorbida. 'Controller Design for Constrained Discrete-Time Uncertain Linear Systems Using Implicit Functions'. In: *IEEE Control Systems Letters* 8 (2024), pp. 1985–1990. DOI: 10.1109/LCSYS.2024.3416475. URL: https://hal.science/hal-04924895 (cit. on p. 11).

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