RESEARCH CENTRE

Inria Centre at the University of Bordeaux

IN PARTNERSHIP WITH:

Institut Polytechnique de Bordeaux, Université de Pau et des Pays de l'Adour, CNRS, TotalEnergies

2024
ACTIVITY REPORT

Project-Team

MAKUTU

Experimental-based modeling and simulation of wave propagation to characterize geophysical and heliophysical media and to design complex objects

IN COLLABORATION WITH: Laboratoire de mathématiques et de leurs applications (LMAP)

DOMAIN

Digital Health, Biology and Earth

THEME

Earth, Environmental and Energy Sciences

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Project-Team MAKUTU

Creation of the Project-Team: 2021 February 01

Keywords

Computer sciences and digital sciences

A3.4.5. – Bayesian methods

A3.4.6. – Neural networks

A6.1. – Methods in mathematical modeling

A6.1.1. – Continuous Modeling (PDE, ODE)

A6.1.4. – Multiscale modeling

A6.1.5. – Multiphysics modeling

A6.2. – Scientific computing, Numerical Analysis & Optimization

A6.2.1. – Numerical analysis of PDE and ODE

A6.2.5. – Numerical Linear Algebra

A6.2.6. – Optimization

A6.2.7. – High performance computing

A6.3.1. – Inverse problems

A6.3.4. – Model reduction

A6.5. – Mathematical modeling for physical sciences

A6.5.1. – Solid mechanics

A6.5.4. – Waves

Other research topics and application domains

B3. – Environment and planet

B3.3. – Geosciences

B3.3.1. – Earth and subsoil

B4. – Energy

B9. – Society and Knowledge

B9.5. – Sciences

B9.5.2. – Mathematics

B9.5.3. – Physics

B9.5.5. – Mechanics

B9.5.6. – Data science

1 Team members, visitors, external collaborators

Research Scientists

- Helene Barucq [Team leader, INRIA, Senior Researcher]
- Henri Calandra [TotalEnergies, Industrial member]
- Julien Diaz [INRIA, Senior Researcher]
- Augustin Ernoult [INRIA, Researcher, until Jan 2024]
- Florian Faucher [INRIA, Researcher]
- Stefano Frambati [TotalEnergies, Industrial member]
- Gaetan Fuss [TotalEnergies, Industrial member]
- Ha Ngoc Pham Howard Faucher [INRIA, Researcher]
- Jie Meng [TotalEnergies, Industrial member]
- Rami Nammour [TotalEnergies, Industrial member]
- Chengyi Shen [INRIA, Starting Research Position, from Jul 2024]

Faculty Members

- Marc Durufle [BORDEAUX INP, Associate Professor]
- Sebastien Tordeux [UPPA, Associate Professor, HDR]

Post-Doctoral Fellows

- Thibaud Beltzung [INRIA, Post-Doctoral Fellow, until Mar 2024]
- Victor Martins Gomes [INRIA, Post-Doctoral Fellow, from Sep 2024]
- Chengyi Shen [INRIA, Post-Doctoral Fellow, until Jun 2024]

PhD Students

- Corto Bastien [INRIA, until Jan 2024]
- Julien Besset [INRIA]
- Lola Chabat [UPPA]
- Ibrahima Djiba [INRIA]
- Pierre Dubois [CEA, from Nov 2024]
- Arjeta Heta [UPPA]
- Andrea Lagardere [AIRBUS, from Apr 2024]
- Augustin Leclerc [INSA ROUEN NORMANDIE, until Nov 2024]
- Matthias Rivet [UPPA]
- Manon Sarrouilhe [INRIA]
- Nicolas Victorion [INRIA]

Technical Staff

- Aurélien Citrain [INRIA, Engineer]
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Interns and Apprentices

- Jean Dutheil [INRIA, Intern, from Apr 2024 until Sep 2024]
- Andrea Lagardere [INRIA, until Mar 2024]

Administrative Assistant

- Fabienne Cuyollaa [INRIA]

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- Patrick Amestoy [INP Toulouse – MUMPSTech, HDR]
- Damien Fournier [Max Planck Institute for Solar System Research]
- Laurent Gizon [Max Planck Institute for Solar System Research, Professor, HDR]
- Jean-Yves L'excellent [MUMPS Tech, HDR]
- Mamadou Ndiaye [UNIV VALENCIENNES]
- Nathan Rouxelin [INSA ROUEN NORMANDIE]
- Maarten de Hoop [Rice University, Rice University, professor, HDR]

2 Overall objectives

Imagine trying to describe a place with exactness from more or less numerous and precise memories, or guessing the content and internal structures of an object after having observed it only partially, without ever touching it because it is inaccessible or very fragile? These are the objectives of Makutu team, where recordings of reflected waves correspond to the memories. Waves can be seismic, electromagnetic or acoustic and Makutu focuses its research on the characterization of Earth's subsurface and the internal dynamic of the Sun. An important component of Makutu's work is the improvement of the resolution methods for direct problems, in order to simulate the propagation of waves in complex media. The characterization and reconstruction of objects using non-invasive approaches then need the resolution of an inverse problem, with efficient forward modeling at the center.

Makutu is an industrial Inria project-team joint with TotalEnergies, in partnership with University of Pau and Pays de l'Adour, Institut Polytechnique de Bordeaux and CNRS. The team is bi-located, one part is hosted by UPPA and the other is hosted by Inria on the University of Bordeaux campus. It is a follow-up of Magique-3D (Advanced Modeling in 3D Geophysics) and its research topics have expanded are summarized as "*Experimental-based modeling and simulation of wave propagation to characterize geophysical and heliophysical media and to design complex objects*" with the name Makutu (magicians in Maori).

The numerical simulation of waves propagating in complex media (the direct problems solved by Makutu) requires the development of advanced numerical methods but the research does not stop there. Indeed, to probe a medium or reconstruct an object from reflected waves measurements (the inverse problems addressed by Makutu), it is important to accurately solve systems of partial differential equations that model all the waves that can be measured. As the complexity of the physical models goes hand in hand with the complexity of the calculations, Makutu is particularly committed in the development and analysis of appropriate mathematical models as well as in the design and study of advanced numerical methods taking into account the characteristics of the physics considered.

Makutu's research has many facets, with final goals ranging from the development of open-source prototype codes written to assess new ideas, to software packages to be ported to an industrial environment. Makutu's research activities are inspired by a strong interdisciplinary industrial and academic partnership. The team's contributions are at the interface of applied analysis, numerical analysis, and scientific computing. The size of some of our problems projects us into the High Performance Computing (HPC) environment and dictates the choices we make for certain approximation spaces that are conducive to massive parallelism.

Makutu has important contributions in the field of high-order discretization methods along with high-order time schemes. Whatever the application is, numerical schemes are all designed with a view to reduce computational time or limit memory consumption, while maintaining a high level of accuracy. Sometimes, it is also necessary to work on the mathematical models themselves whose brute complexity can be a source of difficulty or even blockage for the numerical simulations. One of the originality of the team is to collaborate with experimenters to compare measurements and numerical data in order to calibrate the models. All these contributions are significant steps to reach the final team's objectives which are expressed as the resolution of complex inverse problems. Large-scale computing is then an important part of our activity, which we carry out taking into account the three pillars that make up HPC, i.e. computing time, storage and precision. Recently, the team has started to work on the use of machine learning to assist the numerical schemes they develop in order to control the numerical pollution (or dispersion) which becomes very strong in large-scale computations. This is particularly the case in geophysics where several hundred wavelengths can be propagated. It is worth noting that we take particular care in developing our numerical methods so that they can be used for a wide range of applications, whether the calculations are done in an HPC environment or on a simple laptop.

3 Research program

Makutu's research program decomposes itself into four axes that are: (1) Methodological contributions to the simulation of mechanical and electromagnetic waves in complex media; (2) Seismic imaging; (3) Helioseismology. Each axis shares the same objective to realize simulations of real phénomena. To achieve this, one needs real data and advanced mathematical models and high-order numerical schemes that are compatible with high-performance computing architectures.

To obtain real data, in addition to its current collaborations with scientists both from Academia and Industry, Makutu is developing a new branch of research activities by carrying out its own laboratory measurements. For instance, in order to take into account porosity, parameters such as viscosity, attenuation, thermodynamic effects, etc., must be integrated, and their impact must be properly analyzed before considering using them to characterize the propagation media. This constitutes a clear step ahead for Makutu, and opens up new prospects of contributing to the characterization of very complex media based on wave field measurements.

Regarding the development of numerical schemes, Makutu is developing and analyzing high-order finite elements like Discontinuous Galerkin (DG) methods and spectral element methods possibly coupled with high-order time schemes in the time-dependent regime. The coupling of DG methods with other techniques of discretization is also under consideration. Trefftz-DG and Hybridizable DG methods have been developed both for poro-elastic waves and electromagnetic waves. HDG and HDG+ formulations are also under study for helioseismology with with coupling to integral equation methods.

The research activities of Makutu members have in common the use of wave field measurements. These data are either real or synthetic, produced in the latter case by a different numerical method. The medium can be reconstructed by identifying the physical or geometric parameters that characterize it. In each case, the aim is to solve a non-linear, ill-posed inverse problem. To solve it, Makutu focuses on full waveform inversion (FWI), which is a high-definition imaging method widely used in geophysics.

4 Application domains

Makutu research program is organized around three principal domains of applications: geophysical exploration, solar imaging, and music. Each of them requires a relevant panel of significant contributions requiring achievements in laboratory measurements, modeling, mathematical analysis, advanced numerical schemes and massively parallel software development. Recently, the team has added experimental contributions to feed simulations with real data and also improve modeling through better calibration. Makutu's application domains can be regrouped into a long-standing activity dedicated to subsurface imaging, and two more recent activities dedicated to solar imaging and the development of numerical wind instruments. Each field of application is not compartmentalized in the methodological sense of the term: equations, numerical schemes and programming practices are shared and possibly adapted to the underlying application.

4.1 Geophysical exploration

Geophysical exploration is a historical field for the team (see e.g [41, 43, 44, 45].Geophysical exploration has been driven for a very long time by the goal of finding hydrocarbons. Today, it is evolving towards a very proactive direction in favor of renewable energies and Makutu commits part of its research activities in this direction, in the framework of industrial and international collaborations. Industrial partnership with TotalEnergies has evolved to the transformation of Makutu into an industrial project-team since January 2022. The dedicated research project targets monitoring of CO2 storage through the development of a new numerical branch in GEOSX for seismic propagation and inversion. As far as geothermical energy is concerned, Makutu is member of the international project SEE4GEO lead by C. Morency from Lawrence Livermore National Laboratory. The project combines experimental research in the field and in laboratory with numerical developements in the continuity of CHICkPEA project previously funded by UPPA (2018-2021).

Inversion is central for geophysical exploration and Makutu focuses on Full Waveform Inversion (FWI) as a high-fidelity solution methodology for reconstructing the physical parameters from observed data. FWI can be carried out in time-domain [42, 55, 62, 61] or in frequency domain [58, 59, 57]. Its main feature is to avoid the formation of the large Jacobian matrix by computing the gradient of the misfit functional using the adjoint-state method [46]. A detailed review of FWI for geophysical applications can be found in [56].

4.1.1 Deep geothermal energy

Obtaining accurate images of natural reservoirs is critical for their management and exploitation and seismic imaging is an efficient tool (see [54, 53] and their references therein. One example is with deep geothermal energy which requires precise imaging of deep fractured reservoirs filled with geothermal fluids. Standard seismic imaging is based upon inverting mechanical waves which have difficulties to detect them, whereas electromagnetic waves are more sensitive. We see here a clear interest of coupling seismic with electromagnetic methods and this is what Makutu began developing with CHICkPEA project ended in 2021. The team is now involved in project **SEE4GEO** funded by ADEME, in the framework of Geothermica call.

4.1.2 CO2 injection monitoring

The reduction of greenhouse gases in the atmosphere is a societal topic of the utmost importance, with the Paris Agreement setting ambitious goals for many countries. One fundamental pillar of greenhouse emission management is Carbon Capture Utilisation and Storage (CCUS) [63]. With this strategy, carbon dioxide produced on- or off-site is sequestered and injected into depleted reservoirs, thus offsetting an important portion of current CO2 emissions. The successful and safe implementation of this strategy requires the prediction, monitoring and surveillance of stored CO2 over long periods, which presents significant challenges in terms of seismic acquisition, seismic inversion and numerical simulation. These tools, coupled with state-of-the-art flow simulations, are vital in order to support the injection operations with vital real-time and long-term information. Moreover, specific challenges related to the physics of injected CO2, such as viscosity, temperature and multi-phase fluid conditions push to the limits our current numerical models, and require ambitious new multi-physics simulations to support safe and cost-effective CO2 injection operations. For example, some recent publications like [60, 64] have shown that the combination of CO2-brine flow with wave propagation provides efficient simulations for the monitoring of sequestered CO2. Makutu is currently developing numerical methods for this new application, in collaboration with TotalEnergies, as a new computational branch of the open-source multiphysics simulator GEOSX.

4.2 Solar imaging

Helioseismology studies the interior and dynamics of the Sun based on the observation of wave oscillation in the solar photosphere. These movements can be observed at the surface by the Dopplergrams given by ground-based or satellite-borne observatories. In recent years, methods for understanding Earth subsurfaces have opened up new ways to study the interior of the Sun as in the case with helioseismology and the interior of stars with aesteroseismology from oscillation observed at their surface. Techniques in helioseisomolgy is generally divided into global and local helioseismology. The first approach studies frequencies of oscillations modes, cf. [47]. On the other hand, local helioseimology, which adapts techniques of geophysical seismic interferometry studies, measures local wave propagation and works with the full 3D observed wavefield, and is thus more adapted to study additional features such large-scale flows in active region, sun spots and plage, cf. [52, 51].

Makutu extends its activity on terrestrial seismology to studying the Sun, for the latter offers a vast wealth of problems to be explored both for direct modeling as well as inversion. The collaboration between Makutu and the solar group at the Max Planck institute for Solar research MPS at MPS brings together the expertise of MPS in solar physics and seismology and that of Makutu in numerical simulation of wave propagation and large-scale inversion in geophysics. This ongoing collaboration dating from 2016 with the creation of associated team ANTS which started in 2019 and ended in 2022. The main goal of the collaboration is the creation of a computational framework for accurate and efficient simulation of solar oscillation to be used in full wave-form inversion, e.g. for 3D solar flow.

The stochastic nature of solar oscillation is described by random right-hand source term, and in using statistical analysis, under appropriate assumptions (e.g. the convenient source assumption), power spectrums and time-distance diagrams can obtained from the deterministic Green kernel of modeling wave equation, cf. [50]. In this way, the Green kernel becomes a crucial object in local helioseismology, and its accurate and efficient computation is the main goal of forward modeling. In addition to appropriate numerical schemes, investigation of radiation boundary conditions is required in order to describe accurately waves above cut-off frequencies.

Up until 2021, the focus has been put on acoustic waves which are identified with p-ridges in observed solar power spectrum. Acoustic waves at low frequencies can be adequately described by a scalar equation. Recent and ongoing works extend the investigation to vector wave equation to include gravity and differential rotation. The latter is particularly of interest due to the recent discovery of inertial waves in the Sun. This is subject of the thesis of Lola Chabat which starts in October 2022, and the goal of which is to create an in-house software to compare accurately eigenvalues for the solar wave equation with differential rotation. The remaining challenge is to include full 3d flow to the vector equation.

The above works lay the necessary foundation for inversion of solar parameters such as flow and active region sound speed. Current state-of-the art tools in these references is linear inversion using Born approximation [49]. In additional they are carried out in 1D or 2D. It is thus interesting to apply nonlinear inversion such as Full Waveform Inversion in 3D cf. [48] to these problems.

5 Social and environmental responsibility

Makutu recognizes the importance of conducting research in a responsible and sustainable way. We are committed to ensuring that our work has a positive impact on society and the environment.

In terms of social responsibility, Makutu members ensure that their research is inclusive and accessible to all members of society. The team prides itself on bringing together researchers from diverse social and cultural backgrounds. It makes its results and publications available to the general public and is involved in scientific dissemination activities.

In terms of environmental responsibility, Makutu strives to minimize the environmental impact of its research. Wherever possible, the team works to reduce its carbon footprint by implementing environmentally friendly practices and maintaining remote collaborations to limit international travel. It is also engaged in a research program dedicated to sustainable energy. In particular, it is contributing to the development of advanced software for monitoring CO2 storage and is studying complex models that can assist in the development of geothermal drilling by avoiding the devastating creation of micro-earthquakes.

Overall, Makutu is committed to conducting research in a responsible and sustainable manner and is committed to having a positive impact on society and the environment.

6 Highlights of the year

6.1 Habilitation

- Juliette Chabassier has successfully defended her habilitation (HDR) on June 24th, titled “Modélisation physique et simulation numérique d’instruments de musique”.

6.2 Awards

- Florian Faucher has been awarded the **EAIP young scientist award** by the the Eurasian Association on Inverse Problem (EAIP) during the 11th International Conference “Inverse Problems: Modeling and Simulation”.

6.3 Start of the ANR-DFG project BUTTERFLY

Project BUTTERFLY has started in May 2024. It is a bilateral research project with Göttingen University supported by ANR and DFG (German Research Foundation). It is a follow up of the associate team ANTS which ended in 2022. Its objective is to combine expertise in applied mathematics (Makutu) and helioseismology (U. Göttingen) to develop the computational tools required to learn about the surface distribution of magnetic activity on the surfaces of distant Sun-like stars using asteroseismology.

6.4 Start of the ERC Starting Grant INCORWAVE

The ERC Starting Grant INCORWAVE has started in 2024. The project studies the nonlinear inversion of correlation waveforms with hierarchical reconstructions. It proposes to create new mathematical and computational framework for nonlinear inversion in terrestrial and extra-terrestrial seismology. Two specific problems are considered: first, for the reconstruction of visco-elasticity properties with applications to Earth’s subsurface monitoring; secondly, for the reconstruction of three-dimensional flows in the Sun to characterize the properties of deep solar convection. It is a five-years project with Florian Faucher as the PI.

7 New software, platforms, open data

7.1 New software

7.1.1 OpenWind

Name: Open Wind Instrument Design

Keywords: Wave propagation, Inverse problem, Experimental mechanics, Time Domain, Physical simulation

Scientific Description: Implementation of first order finite elements for wind musical instrument simulation. Implementation of the Full Waveform inversion method for wind musical instrument inversion. Implementation of energy consistent numerical schemes for time domain simulation of reed-type wind musical instrument.

Functional Description: Simulation and inversion of wind musical instruments using one-dimensional finite element method with tonholes or valves and fingering chart. The software has three functionnalities. First, the software takes the shape of a wind instrument and computes the acoustical response (answer to a given frequential excitation). Second, the software takes the instrument shape and the control parameters of a musician, and computes the produced sound and the time evolution of many acoustical quantities. Last, the software takes a measured acoustical response and computes the corresponding instrument geometry (inner bore and tone holes parameters).

Release Contributions: - Time domain and frequency domain simulations of flute-like instruments (sound synthesis and impedance computation) - Scripts used for generating figures for Alexis Thibault Thesis - Roughness parametric model - New scheme for Reed/lips model with dimensionless parameters - Effect of air composition (humidity and CO2 rates) on the acoustic propagation - Possibility to impose geometric constraints (linear and non-linear) during optimization process

URL: `https://openwind.inria.fr`

Publications: hal-02984478, hal-02996142, hal-03132474, hal-02917351, hal-02432750, hal-02019515, hal-03231946, hal-03328715, hal-03794474, hal-01963674, hal-04008847, hal-04217988

Contact: Augustin Ernoult

Participants: Juliette Chabassier, Augustin Ernoult, Alexis Thibault, Robin Tournemenne, Olivier Geber, Guillaume Castera, Tobias Van Baarsel

7.1.2 Hou10ni

Keywords: 2D, 3D, Elastodynamic equations, Acoustic equation, Elastoacoustic, Frequency Domain, Time Domain, Discontinuous Galerkin

Scientific Description: Hou10ni simulates acoustic and elastic wave propagation in time domain and in harmonic domain, in 2D and in 3D. It is also able to model elasto acoustic coupling. The time domain solver is based on the second order formulation of the wave equation and the space discretization is achieved using Interior Penalty Discontinuous Galerkin (IPDG) Method. Both IPDG and Hybridizable Discontinuous Galerkin (HDG) Methods are implemented in the frequency domain solver. Recently, the HDG version has been extender to poroelastic and conducting poroelastic (poroelastic+electromagnetic) media.

Functional Description: This software simulates the propagation of waves in heterogeneous 2D and 3D media in time-domain and in frequency domain. It is based on an Interior Penalty Discontinuous Galerkin Method (IPDGM) and allows for the use of meshes composed of cells of various order (p-adaptivity in space).

News of the Year: In 2023, we compared numerical results provided by Hou10ni to numerical simulations in the time-domain, based on a spectral-element method, in order to assess the accuracy of using a fictitious dielectric permittivity.

URL: `https://team.inria.fr/magique3d/software/hou10ni/`

Publications: hal-04394440, hal-03948879, hal-01513597, hal-01957131, hal-01388195, hal-01972134, hal-01957147, hal-02152117, hal-02486942, hal-02408315, hal-02911686, hal-03464413, tel-03442300, tel-03014772, hal-01656440, hal-01662677, hal-01623953, hal-01623952, hal-01513597, hal-01519168, hal-01254194, hal-01400663, hal-01400656, hal-01400643, hal-01313013, hal-01303391, hal-01408981, tel-01304349, hal-01184090, hal-01223344, hal-01207897, hal-01184111, hal-01184110, hal-01184107, hal-01207906, hal-01184104, hal-01207886, hal-01176854, hal-01408705, hal-01408700, tel-01292824, hal-01656440, hal-00931852, hal-01096390, hal-01096392, hal-01096385, hal-01096324, hal-01096318, tel-01133713, tel-00880628

Contact: Julien Diaz

Participants: Conrad Hillairet, Elodie Estecahandy, Julien Diaz, Lionel Boillot, Marie Bonnasse, Marc Fuentes, Rose-Cloe Meyer, Vinduja Vasanthan, Arjeta Heta

7.1.3 Hawen

Name: time-HArmonic waVe modEling and INversion using Hybridizable Discontinuous Galerkin Discretization

Keyword: Digital twin

Scientific Description: Many applications such as seismic and medical imaging, material sciences, helioseismology, and planetary science, aim to reconstruct properties of a non-directly accessible or non-visible interior. For this purpose, they rely on waves whose propagation through a medium interrelates with the physical properties (density, sound speed, etc.) of this medium. Hawen is a software designed to perform imaging with waves, following an algorithm that comprises of two main stages: In the data acquisition stage, the medium response to probing waves is recorded (e.g., seismic waves from Earthquakes recorded by ground network). In the second stage, we rely on a reconstruction procedure that iteratively updates an initial model of physical parameters, so that numerical simulations approach the measurements. This procedure is employed, for instance, for seismic (reconstruction of subsurface layers) and medical (disease diagnostic) imaging.

Functional Description: The software allows the reconstruction of the physical properties of a media using waves propagating therein. For instance, Hawen allows the recovery of the physical properties of the Earth and the Sun for the observations of surface oscillations. Such applications are of interest in geophysics and helioseismology.

Release Contributions: Compared with the 2/1/2023 version (v1.2.1) - Improved simulations in elastic media, - Improved inversion methods, - New features, such as the consideration of new type of sources for the wave propagation, - Compilation with CMAKE, - Incorporation of validation tests.

URL: `https://ffaucher.gitlab.io/hawen-website/`

Publications: hal-04336798, hal-04356602, hal-04087228, hal-03871831, hal-03877239, hal-03406861, hal-02982650, hal-03101659, hal-03101642, hal-02982619

Contact: Florian Faucher

Participants: Florian Faucher, Marc Fuentes, Ha Howard

7.1.4 MONTJOIE

Keywords: High order finite elements, Edge elements, Aeroacoustics, High order time schemes

Scientific Description: Montjoie is designed for the efficient solution of time-domain and time-harmonic linear partial differential equations using high-order finite element methods. This code is mainly written for quadrilateral/hexahedral finite elements, partial implementations of triangular/tetrahedral elements are provided. The equations solved by this code, come from the ”wave propagation” problems, particularly acoustic, electromagnetic, aeroacoustic, elastodynamic problems.

Functional Description: Montjoie is a code that provides a C++ framework for solving partial differential equations on unstructured meshes with finite element-like methods (continuous finite element, discontinuous Galerkin formulation, edge elements and facet elements). The handling of mixed elements (tetrahedra, prisms, pyramids and hexahedra) has been implemented for these different types of finite elements methods. Several applications are currently available : wave equation, elastodynamics, aeroacoustics, Maxwell’s equations.

URL: https://www.math.u-bordeaux.fr/~durufle/montjoie

Contact: Marc Durufle

Participants: Juliette Chabassier, Marc Durufle, Morgane Bergot

7.1.5 GEOSX

Keywords: Physical simulation, Multiphysics modelling

Functional Description: GEOSX is an open-source, multiphysics simulator developed cooperatively by Lawrence Livermore National Laboratory, Stanford University, and TotalEnergies. Its goal is to open up new horizons in modeling carbon storage and other subsurface energy systems. This includes: - taking advantage of the ongoing revolution in high-performance computing hardware, which is enabling orders-of-magnitude gains in performance, but also forcing a fundamental rethink of our software designs, - enriching the physics used in industrial simulations, allowing complex fluid flow, thermal, and geomechanical effects to be handled in a seamless manner, - developing highly-scalable algorithms for solving these coupled systems, - and improving workflows for modeling faults, fractures, and complex geologic formations. Inria contributes to the seismic wave propagators of GEOSX, and to its python interface. Inria also contributes advanced workflows for seismic inversion, and CO2 storage an monitoring.

URL: http://www.geosx.org/

Contact: Randolph Settgast

7.1.6 Gar6more2D

Keywords: Validation, Wave propagation

Functional Description: This code computes the analytical solution of problems of waves propagation in two layered 3D media such as- acoustic/acoustic- acoustic/elastodynamic- acoustic/porous- porous/porous,based on the Cagniard-de Hoop method.

URL: https://gitlab.inria.fr/jdiaz/gar6more2d

Publications: inria-00274136, inria-00404224, inria-00305395, gregor:hal-03471065

Contact: Julien Diaz

Participants: Abdelaaziz Ezziani, Julien Diaz

Partner: Université de Pau et des Pays de l'Adour

7.1.7 GoTem3

Keywords: Trefftz, Computational electromagnetics, HPC, Domain decomposition

Functional Description: GoTem3 is domain decomposition platform based on the ultra-weak formulation of Cessenat and Després for the solution of diffraction problems posed on regular grids. It uses matrix free strategies as well as local and global preconditioners to solve cases involving more than a billion degrees of freedom on a single computational core.

News of the Year: The code has been endowed with the ability to account for basic quasiTrefftz functions derived from an auxiliary code for solving electromagnetic wave equations using the flux reconstruction and spectral difference methods. This work was implemented as part of Matthias Rivet's thesis.

Publications: hal-03945383, tel-04172930, hal-03642116

Contact: Sebastien Tordeux

Participants: Sebastien Pernet, Matthias Rivet, Margot Sirdey, Sebastien Tordeux

7.2 Open data

7.2.1 WaveBench: Benchmarking Data-driven Solvers for Linear Wave Propagation PDEs

This is a joint work of Florian Faucher with Tianlin Liu, Ivan Dokmanic and AmirEhsan Khorashadizadeh (University of Basel), Antonio Lara-Benitez and Maarten V. de Hoop (Rice University).

Publication: [11].

Description: We present WaveBench, a comprehensive collection of benchmark datasets for wave propagation PDEs. WaveBench (1) contains 24 datasets that cover a wide range of forward and inverse problems for time-harmonic and time-varying wave phenomena in 2D; (2) in- cludes a user-friendly PyTorch environment for comparing learning-based methods; and (3) comprises reference performance and model checkpoints of popular PDE surrogates such as U-Nets and Fourier neural operators.

Data-set link: `https://zenodo.org/records/8015145`,

Data-set link: `https://github.com/wavebench/wavebench`.

7.2.2 Out-of-distributional risk bounds for neural operators with applications to the Helmholtz equation

This is a joint work of Florian Faucher with Antonio Lara-Benitez and Maarten V. de Hoop (Rice University), Takashi Furuya (Shimane University), Anastasis Kratsios (McMaster University and the Vector Institute), and Xavier Tricoche (Purdue University).

Publication: [10].

Description: We propose a data-set corresponding to wave propagation, which is used to study a subfamily of Neural Operators (NOs) enabling an enhanced empirical approximation of the nonlinear operator mapping wave speed to solution, or boundary values for the Helmholtz equation on a bounded domain. The latter operator is commonly referred to as the “forward” operator in the study of inverse problems, and we propose a hypernetwork version of the subfamily of NOs as a surrogate model

Data-set link: `https://github.com/JALB-epsilon/Fine-tuning-NOs`,

Data-set link: `https://rice.box.com/s/haczq8oad4b5cvi8pf8cp01sz4f0vfey`.

8 New results

8.1 Methodological contributions to the simulation of mechanical and electromagnetic waves in complex media

8.1.1 Hybridizable Discontinuous Galerkin discretization for linear anisotropic elastic wave equation: Voigt-notation and stabilization

Participants: Hélène Barucq, Florian Faucher, Ha Pham.

This work is concerned with implementing the hybridizable discontinuous Galerkin (HDG) method to solve the linear anisotropic elastic equation in the frequency domain, focusing in particular on providing a compact description of the discrete problem and an optimal choice of stabilization in defining the HDG numerical traces. Voigt notation is employed in the description of the discrete problem in order to facilitate matrix operation and to provide efficient book-keeping of physical parameters. Additionally, a first-order formulation working with the compliance elasticity tensor is employed to allow for parameter variation within a mesh cell, for better representation of complex media. We determine an optimal choice of stabilization by constructing a hybridized Godunov-upwind flux for anisotropic elasticity possessing three distinct wavespeeds. This stabilization removes the need to choose judiciously scaling factors and can be used as a versatile choice for all materials. Its optimality is established by comparing with identity matrix-based stabilization in a wide range of values for the scaling factor. Numerical investigations are carried out in two and three dimensions, for isotropic elasticity and material with varying degree of anisotropy.

Details are available in the article published in journal Computer Methods in Applied Mechanics and Engineering, [12]. This work has also been presented at Waves conference in Berlin [17], ECCOMAS in Lisbon, [16] and in a BIRS worshop, [26]. In parallel, we are developing optimization methods for the numerical fluxes of the Trefftz method. The first method is based on local optimization of information transfer for a criterion that ensures the convergence of the iterative algorithm. The second involves approximating the theoretical Dirichlet-to-Neumann operator through a non-trivial approximation obtained via supervised learning.

8.1.2 Coupled Quasi-Trefftz Method for Aeroacoustics

Participants: Andréa Lagardère, Sébastien Tordeux.

The acoustic propagation of noise generated by engines or by turbulence around, for instance, the landing gear determines the external noise level produced by an aircraft during various phases of flight (parking, taxiing, takeoff, cruising, landing) as well as the sound level inside the cabin. Numerical simulation of this noise must account for the presence of an airflow with variable temperature and pressure characteristics, which produces significant diffraction and refraction effects.

Currently, the method used at Airbus for this type of simulation combines two types of modeling: on the one hand, volumetric finite elements (FEM) in regions with high heterogeneity, and on the other, boundary element methods (BEM) in the rest of the domain (where the assumption of a uniform fluid and flow is made). These two methods are solved in a coupled manner and are collectively referred to as FEM-BEM.

The downside of the FEM volumetric approach used here is that it requires a very fine mesh to accurately capture variations in the fluid's characteristics. This makes computation very expensive in the jet of the engine and almost impossible in the boundary layer on the fuselage of the aircraft. We are working on an alternative formulation for regions with high heterogeneity, using a well-known Discontinuous Galerkin Method (DGM) but with an innovative aspect: it employs Quasi-Trefftz finite elements, which vary in each element and are tailored to the local variations in the fluid's characteristics. The GPW finite elements (Generalized Plane Wave) in question were co-developed by L.M. Imbert-Gérard. Since they are custom-designed in each mesh element to suit the equation being solved and the local variations of the PDE coefficients, we can increase the size of each element and significantly reduce the total number of unknowns in the calculation.

Furthermore, to handle large-scale cases, we aim to maintain the ability to couple the quasi-Trefftz method with integral equations. This is why we are working on coupling DGM-BEM in the frequency domain, as the time-domain case has already been addressed.

8.1.3 Enhanced finite element methods using neural networks

Participants: Hélène Barucq, Florian Faucher, Nicolas Victorion.

In this work, we present a preliminary study combining two approaches in the context of solving PDEs: the classical finite element method (FEM) and more recent techniques based on neural networks. Indeed, in recent years, physics-informed neural networks (PINNs) have become particularly interesting for rapidly solving such problems, especially in high dimensions. However, their lack of accuracy is a major draw- back in this context, hence the interest in combining them with FEM, for which error estimators are already known. The complete pipeline proposed here therefore consists in modifying classical FEM approximation spaces by taking information from a prior, chosen here as the prediction of a neural network. On the one hand, this combination improves and certifies the prediction of neural networks in order to obtain a fast and accurate solution. On the other hand, error estimates are proven, showing that such strategies outperform clas- sical ones by a factor that depends only on the quality of the prior. We validate our approach with numerical results, obtained for this preliminary work on parametric problems with one- and two-dimensional geometries. They demonstrate that, to achieve a fixed error target, a coarser mesh can be used with our enhanced FEM compared to the standard one, leading to reduced computation time, particularly for parametric problems.

This is a joint work with Frédérique Lecourtier, Michel Duprez, Emmanuel Franck, Vanessa Lleras and Victor Michel-Dansac at the University of Strasbourg. A draft is currently in its final stage before submission. The results have been presented in different workshops and conferences.

8.1.4 Modal computation for open waveguides

Participants: Hélène Barucq, Marc Duruflé, Augustin Leclerc.

We are interested in the modeling and calculation of electromagnetic (EM) modes in waveguides with complex geometries, particularly in twisted electric cables in open environments. The aim is to develop numerical methods for solving Maxwell's equations in order to better understand the propagation of electromagnetic fields in realistic configurations. This study is motivated by the challenges of reducing electromagnetic radiation and improving cable performance in terms of field confinement.

We deal with two main configurations: straight and twisted waveguides. For straight waveguides, semi-analytical methods are used, particularly for coaxial cables, to test numerical models by comparison with accurate solutions. We extend these methods to open waveguides, where absorbing boundary conditions (ALC) are introduced to model an infinite environment while limiting spurious reflections. In twisted cables, a helical geometry is exploited to reformulate the EM wave propagation equations in a suitable numerical framework, enabling low-frequency simulations. Work was then initiated on the construction of ABCs within this framework.

The results obtained provide a better understanding of low-frequency electromagnetic phenomena, and open up prospects for the design of more efficient devices and the study of electromagnetic systems in real heterogeneous environments. This work has been presented at the conference Waves 2024 in Berlin [25]. An article is also under review.

8.1.5 Trefftz methods for Helmholtz equation

Participants: Hélène Barucq, Julien Diaz, Sébastien Tordeux.

Several topics of the plane-wave ultra-weak variational formulations for numerically solving the Helmholtz equation are revisited: treatment of the case of piecewise constant anisotropic coefficients, derivation of the formulation, coercivity properties, etc. The construction of one of these formulations, compatible with the transmission and reflection of plane waves, at normal incidence on an interface shared by two elements of the mesh, leads to a general framework which covers all the previously used ultra-weak formulations. We thus show that any ultra-weak formulation can be characterized by its equivalence with a unique discontinuous Galerkin method for which the numerical fluxes are expressed by outgoing traces. It is also shown that the particular ultra-weak formulation, which provided the general framework, can be considered as an upwind scheme in the sense that these numerical fluxes can be obtained from a Riemann solver. Based on the theory of elliptic interface boundary-value problems, conditions on the coefficients and the geometry ensuring the coercivity properties of the formulation are brought out in the 2D case. The identification of two ways of describing plane waves in the anisotropic case and an appropriate change of variables reduce the estimates of the convergence error to those related to the usual Helmholtz equation. This also allows us to derive a theoretical basis for the choice of local plane-wave bases for an efficient coverage of the anisotropic case. Some numerical experiments illustrating the efficiency of the approach complete the study. A report has been published [37].

8.1.6 Trefftz methods for Maxwell equations

Participants: Matthias Rivet, Sébastien Tordeux.

The simulation of time-harmonic electromagnetic waves requires a matrix inversion whose cost, especially in three-dimensional cases, increases quickly with the size of the computational domain. This is a tangible issue regarding the memory consumption. Trefftz methods consist in using a discontinuous Galerkin method whose basis functions are specific to the considered physical problem and thus reduce numerical dispersion phenomena. We propose an iterative Trefftz solver based on a domain decomposition method which will reduce considerably the memory consumption. However, iterative Trefftz methods based on a plane wave approximation are ill-conditioned. To overcome this problem, we propose improvements of the Cessenat and Deprés preconditioner and a basis reduction. A matrix-free strategy allows to avoid the assembly of the matrix associated to the linear system and GMRES solver does not require the computation of the inverse anymore. This is a joint work with Sébastien Pernet (ONERA).

This year, we developed a local solver based on the flux reconstruction approach, which improved the conditioning of the Trefftz method by transforming it into a quasi-Trefftz method. A paper is under review and a paper has been published [14].

In parallel, we are developing optimization methods for the numerical fluxes of the Trefftz method. The first method is based on local optimization of information transfer for a criterion that ensures the convergence of the iterative algorithm. The second involves approximating the theoretical Dirichlet-to-Neumann operator through a non-trivial approximation obtained via supervised learning.

Moreover, a pedagogical research report has been published about the unique continuation Theorem for general symmetric hyperbolic problems [39]. This work was presented at the Eccomas conference in Lisbon, the Waves conference in Berlin, and the Numélec conference in Toulouse and in Oaxaca, [21], [34], [31], [32].

8.1.7 Iterative Trefftz methods for anisotropic acoustic problems equations

Participants: Hélène Barucq, Ibrahima Djiba, Sébastien Tordeux.

Trefftz methods are discontinuous finite element methods, in which the basis functions are local solutions of the problem under study. Applied to wave equations in the harmonic regime, they have the particularity of leading to a variational formulation based only on the mesh skeleton, with each element linked to its neighbor via a numerical trace, the definition of which determines the method's efficiency. Studies show that these methods, like all discontinuous Galerkine methods, are more resistant to numerical pollution than continuous finite element methods. In addition, the size of the linear Trefftz system is considerably reduced, which is of great interest for geophysical applications involving domains containing a very large number of wavelengths. However, these methods have one major drawback: they are highly dependent on numerical traces, which play a central role in the variational formulation of the problem. To better explain this dependence, we revisit Trefftz's method, written in the formalism of Friedrichs systems extended to the harmonic regime. To do this, we express the problem using new variables, derived from the spectral decomposition of the hyperbolic flow into an incoming and an outgoing flow. The communication between the elements respects the hyperbolic character of the problem, and the resulting Trefftz formulation is well-posed. The new method is studied in the case where the approximation spaces are plane wave spaces. As the stability of the formulation tends to weaken with the number of plane waves, we propose regularization techniques to enhance the stability of the method while maintaining good accuracy. This work has been presented at Eccomas conference in Lisboa [19] and we have written a chapter of book [35].

8.1.8 A HDG+ method for the diffusive-flux formulation of the convected Helmholtz equation

Participants: Hélène Barucq, Nathan Rouxelin, Sébastien Tordeux.

We construct HDG methods based on the diffusive flux formulation of the convected acoustic wave equation.We mostly describe the HDG+ method which involves different polynomial degrees for approximating the unknowns, hence leading to a more efficient method with a super-convergencelike behaviour. A detailed analysis of the methods including local and global well-posedness, as well as convergence estimates is carried out. The HDG+ method is also compared with a more conventional HDG method to demonstrate its effectiveness. A paper is currently under review.

8.1.9 Out-of-distributional risk bounds for neural operators with applications to the Helmholtz equation

Participants: Florian Faucher.

Despite their remarkable success in approximating a wide range of operators defined by PDEs, existing neural operators (NOs) do not necessarily perform well for all physics problems. We focus here on high-frequency waves to highlight possible shortcomings. To resolve these, we propose a subfamily of NOs enabling an enhanced empirical approximation of the nonlinear operator mapping wave speed to solution, or boundary values for the Helmholtz equation on a bounded domain. The latter operator is commonly referred to as the "forward" operator in the study of inverse problems. Our methodology draws inspiration from transformers and techniques such as stochastic depth. Our experiments reveal certain surprises in the generalization and the relevance of introducing stochastic depth. Our NOs show superior performance as compared with standard NOs, not only for testing within the training distribution but also for out-of-distribution scenarios. To delve into this observation, we offer an in-depth analysis of the Rademacher complexity associated with our modified models and prove an upper bound tied to their stochastic depth that existing NOs do not satisfy. Furthermore, we obtain a novel out-of-distribution risk bound tailored to Gaussian measures on Banach spaces, again relating stochastic depth with the bound. We conclude by proposing a hypernetwork version of the subfamily of NOs as a surrogate model for the mentioned forward operator.

This is a joint work with Antonio Lara-Benitez and Maarten V. de Hoop (Rice University), Takashi Furuya (Shimane University), Anastasis Kratsios (McMaster University) and Xavier Tricoche (Purdue University).

This work has been published in Journal of Computational Physics, [10].

8.1.10 Application of the Sparse Direct Solver MUMPS to Hybridizable Discontinuous Galerkin Discretization for Wave Modeling.

Participants: Florian Faucher.

We consider the Hybridizable Discontinuous Galerkin (HDG) discretization method for the numerical simulation of time-harmonic waves in acoustic and elastic media. The discretization results in large sparse systems of linear equations of the form AX = B to be solved for different frequencies. Although a significant amount of memory is required, these systems can be efficiently solved using sparse direct solvers, that exploit the block structure of A and that factorize A only once for multiple right-hand sides. In this study, we revisit the potential of the massively-parallel sparse multifrontal solver MUMPS and its recent features to efficiently solve our problem. We analyze in more details the influence of the HDG discretization which, being more appropriate for high-order polynomials, can lead to denser blocks. We also study the consideration of acoustic and elastic wave propagation on the data sparsity of the matrix, and the capacity to use low rank compression possibly with mixed precision [1,2] in the sparse direct solver for efficiency (time and memory footprint). We provide numerical experiments in a parallel setting for the simulation of wave propagation with the code hawen [3]. In particular, we consider a 3D earth domain with an adapted mesh (refined for topography and coarser below), exploiting p-adaptivity. We also illustrate how today challenges related to heterogeinity of computers, mixed-precision arithmetics, new models and applications, motivate research and developments in the field of sparse direct solvers.

This is a joint work with the MUMPS group, in particular Patrick Amestoy (ENS Lyon), Jean-Yves L'Excellent (ENS Lyon), Théo Mary (Sorbonne University) and Chiara Puglisi. This work is part has been presented at the ECCOMAS conference in Lisbon, and at SIAM Conference on Applied Linear Algebra, [23].

8.1.11 Multiphysics coupling algorithms for black box solvers in an HPC framework

Participants: Hélène Barucq, Pierre Dubois.

To numerically solve a coupled multiphysics problem, there are two main paradigms of numerical methods: - Monolithic methods, which involve solving a global system integrating all physics with a single solver. Although these approaches are costly in terms of computation and memory, they provide an accurate numerical solution when the solver converges. Their main limitation lies in their low modularity: any modification of the multiphysics problem implies a complete revision of the solver. The scalability of these methods to the Exascale depends mainly on the parallelization of the solver used. The latter is often a direct linear solver (due to the conditioning of the multiphysics system), which is intrinsically limited in terms of parallelization. - Partitioned methods, which rely on independent solvers for each physics and require efficient coupling algorithms to ensure convergence and robustness, via techniques such as suite convergence acceleration. These methods seem better suited to the transition to Exascale, as efficient parallel solving can be applied asynchronously to each solver, enabling optimal exploitation of massively parallel architectures. The aim of this work is to design a generic method for solving multiphysics problems based on black-box solvers (without modifying the solver). In this context, the partitioned approach appears to be the most suitable. This method will be adapted to high-performance computing and compared with monolithic approaches. This is a collaboration with Isabelle Ramière and Raphaël Prat from CEA Cadarache, within the project Exa-MA.

8.1.12 On the well-posedness of Pride system for modeling seismo-electric effects wave propagation in conductive porous media

Participants: Arjeta Heta, Hélène Barucq, Julien Diaz.

A conductive porous medium excited by a seismic wave is the scene of seismo-electric effects. Understanding them opens new prospects for imaging geological objects invisible to seismic imaging. This phenomenon has been modeled by Pride in the 90s, by a strong (two-way) coupling of Maxwell's (electromagnetics) with Biot's (porous elasticity) equations. A simplified system considering only Maxwell-to-Biot (electroseismic) or Biot-to-Maxwell (seismoelectric) coupling is often considered. In this context, it is enough to show existence and uniqueness of the solutions to the separate systems. We analyze the well-posedness of the boundary value problem resulting from strong Maxwell-Biot coupling. We consider the low-frequency regime which yields a constant coupling term and seek solutions in time domain. The main idea to prove the existence and uniqueness of the solution is to reformulate the considered Maxwell-Biot system such that the fixed-point theorem can be employed. Such a proof requires the derivation of various a priori estimates on both elastic and electromagnetic fields, that appear to be of general interest. Constraints on the physical parameters are respected for a quite general class of physical media.

This work is part of the thesis of Arjeta Heta, it is a collaboration with Rabia Djellouli (CSUN). It has been presented at Waves 2024 [30].

8.2 Seismic imaging

8.2.1 Cross-correlation of time-harmonic wavefields for passive imaging: numerical modeling and inversion

Participants: Jean Dutheil, Florian Faucher.

The objective of passive imaging is to use these ambient data that come from the superposition of stochastic events, to reconstruct the inner properties of the medium. This approach is also referred to as 'ambient noise imaging'. In order to be able to reconstruct the properties of the medium, one must first connect to some deterministic objects that we can analyze with the wave equations. This task can be achieved by using the cross-correlation of the measured signals, which gives us the relation with the deterministic solution of the wave equation which is the Green's function. Then we study the quantitative inversion algorithm based upon an iterative minimization for passive imaging. Contrary to active-source imaging, additional operations have to be added, in particular, to compute the gradient of the misfit function. Eventually, we carry out numerical experiments of inversion.

This work has been the topic of the Master Thesis of Jean Dutheil.

8.2.2 Experimental characterization and modeling of electromagnetic waves generated by seismo-electric conversion at porous media interfaces

Participants: Hélène Barucq, Victor Martins Gomes.

Seismo-electromagnetic phenomena, electrokinetic in nature, take place whenever a seismic wave propagates in fluid-bearing media, its energy depending mainly on the electrical properties of the fluid and the hydraulic properties of the porous medium. They result from a conversion of mechanical into electromagnetic (EM) energy due to the transient ionic interactions occurring at the pore scale. Two of these phenomena are usually studied: the electric field accompanying seismic waves, and the EM field that travels independently, generated at discontinuities of physicochemical properties in the porous medium. Although the first event is sensitive to physical parameters of the surrounding medium, the second catches information about interfaces in the subsurface, with the resolution of seismic methods, making it very attractive to near surface exploration. In this context, we propose a new experimental setup where both phenomena can be simultaneously studied. At first, we use a porous medium composed of homogeneous water-saturated sand and study the characteristics of the coseismic electric field. Afterwards, a thin layer of Vosges sandstone is inserted into the sand, which allows the study of the EM waves generated at the two closely spaced sand-sandstone interfaces. We record the seismic displacement field at the upper surface of the sand volume using a laser vibrometer, and use stainless steel electrodes buried in the sand to acquire individual electric potentials rather than electric fields, seeking to favour the measurement of the EM interface-generated signals. With the help of direct numerical simulations, we compare experimental measurements and theoretical predictions, based on a well established set of seismoelectric governing equations. In both types of experiments, this comparison shows very good agreements between experimental and numerical waveforms, thus confirming the relevant theory. The electric potential data also show that the EM signals generated at interfaces are clearly recorded at distances of about 10 seismic P wavelengths away from the interface. By contrast, the same events are barely noticeable near the inserted layer when measured using classical electric dipolar arrays.

This work has been done in collaboration with Daniel Brito (LFCR, CNRS, UPPA), Stéphane Garambois (Isterre, Grenoble) and Clarisse Bordes (LFCR, CNRS, UPPA) and Michel Dietrich (Isterre, Grenoble). This work has been presented at EGU conference.

8.2.3 Quantitative inverse wave problems in visco-elastic anisotropic media

Participant: Florian Faucher.

We consider the quantitative inverse problem for the reconstruction of physical properties of a medium are from wave measurements. We consider linear viscoelasticity to model the propagation of waves in solid materials, taking into account attenuation and anisotropy of the media. In this work, we employ the time-harmonic wave equations to facilitate working with different attenuation models, which are frequency-dependent. For numerical discretization, we use the hybridizable discontinuous Galerkin (HDG) method which employs static condensation to reduce the computational cost. We perform quantitative inversion by following a minimization algorithm where the elastic properties are iteratively updated. We carry out reconstructions with attenuation model uncertainty, and highlight the importance of considering anisotropy in the model.

This work has been presented at the Workshop at University College London, [29], and conferences WAVES [20], and IPMS [15].

8.2.4 On the detectability of CO2 plumes with seismic records: a numerical simulation study

Participants: Hélène Barucq, Henri Calandra, Florian Faucher, Stefano Frambati, Chengyi Shen.

One of the prevailing questions on the monitoring of CO2 plumes in a CCUS project is the detectability of changes in petrophysical and geophysical properties induced by the substitution of initial fluids by CO2 . Numerous works on the field data have shown encouraging results on the ability of seismic attributes to detect and monitor changes in petrophysical properties, which can be linked with the geophysical properties through Petro-Elastic Models. Seismic attributes are also believed to have the potential for tracking straightforwardly changes in geophysical properties such as the elastic wave velocities, on which we present the simulation results on the 3D Otway case study conducted within project-team Makutu Inria and TotalEnergies. The numerical case study features a baseline and a monitor as geophysical models, with geophones and fibre optics (DAS) as receivers in a Vertical Seismic Profile (VSP) configuration. The wave propagation simulations are run on the GEOS platform featuring GPU accelerations. We aim at constructing a reliable and efficient numerical tool for precise simulations. Numerically we will show the impact of adaptative meshes on both the performance and precision; in terms of geophysical settings, we will illustrate the need of considering attenuation (Trinh et al., 2019) to match reference or real data. Finally we will compare the synthetic VSP records from DAS and geophones for the purpose of discussing the CO2 plume detectability.

This work is presented at Mathias conference [33]

8.2.5 Frequency analysis of the seismoelectric coupling operator

Participants: Hélène Barucq, Julien Diaz, Arjeta Heta.

This work focuses on the mathematical modeling of the seismo-electric effects that occur in porous media consisting of a charged fluid within a solid matrix of opposite charge. The medium is neutral on a macroscopic scale, but relative displacements between solid and fluid induce electric currents (charge displacement). Seismic waves propagating in these media induce local fluid flows, and thus charge displacements, leading to the creation of electromagnetic waves. Mathematically, this is modeled by Biot's poro-elasticity equations coupled to Maxwell's equations. The coupling theory derived by S. Pride in 1994 is realized in the frequency domain, and the coupling parameters (coupling coefficient and dynamic permeability) are frequency-dependent. Interpreting these parameters as symbols of global pseudo-differential operators in time, we understand that writing the model in the time domain involves operators whose discretization will generate very heavy computational costs. This explains why simulations performed in the time domain always involve a very low-frequency model in which the frequency dependence of the parameters is masked. Two main developments are underway. The first is concerned with the approximation of frequency-dependent coupling operators, in order to obtain temporal seismo-electric equations of higher fidelity, i.e. valid for a frequency regime broader than that of the very low frequency. This opens up the prospect of using valid time models on a laboratory scale. At present, the temporal seismo-electric equations are solved at very low frequencies, in which case the coupling terms are independent of frequency. Several types of approximation, both polynomial and rational, are introduced and studied numerically. At low frequencies, local approximants such as Padé or Taylor (very low frequency) can be used. Global approximations — of the Legendre and Chebyshev type — are used with high frequencies, or a wide frequency band. These approximations are tested numerically by comparing first with semi-analytical solutions and then with solutions computed with the Hou10ni solver based on a hybrid discontinuous Galerkine (HDG) method in a parallel computing context. The second development concerns the mathematical analysis of the system of Pride equations in the time domain, with strong coupling at very low frequencies. We show that by adopting a simplified version of the feedback term, we can apply Hille-Yosida theory to establish that the system is well-posed. It is also shown that a fixed-point theorem can be applied, opening up the prospect of considering higher-order couplings.

8.2.6 Optimized Full Waveform Inversion for seismic in GEOS

Participants: Hélène Barucq, Julien Besset, Henri Calandra, Stefano Frambati.

The reduction of energy carbon footprint justifies the recent major programs launched on CO2 storage in existing reservoirs known to geologists. Numerical simulation plays a key role in their implementation by providing a low-cost means of monitoring. This is a global concern that explains the use of open-source software platforms facilitating knowledge sharing and collaborations. Regarding monitoring aspects, Full Waveform Inversion (FWI) has demonstrated its ability in probing the subsurface accurately. FWI is an iterative process in which we need to solve forward problems in large-scale propagation domains whose discretization involves more than 108 cells each. Implementing an FWI algorithm needs thus an optimized architecture in terms of memory management and GPU-CPU computation. Among the existing open-source platforms, GEOS targets such architecture. Moreover, GEOS offers a multi-physics approach ready for reservoir simulation and offering the perspective of coupling with seismic.

We have addressed the idea of constructing reduced bases to provide faster computational software for large scale inversion. It turns out that standard construction techniques deliver reduced bases which depend on the physical parameters. Hence, with the purpose of reconstructing the velocity model, it is necessary to reconstruct the basis functions at each update of the model in the inversion workflow. We have thus proposed a new method of construction which involves Fréchet derivatives. This work is a collaboration with Rabia Djellouli (CSUN). The algorithm has been presented at Waves 2024 and numerical aspects of the problem have been presented at ECCOMAS 2024 [18]. An article is currently being written.

8.2.7 Machine Learning approaches for CO2 geological storage monitoring: repeatability, inversion and generative models

Participants: Hélène Barucq, Henri Calandra, Stefano Frambati, Manon Sarrouilhe.

The goal of this work just begun is to investigate how current and future Machine Learning (ML) techniques can be used to improve CO2 injection monitoring techniques and help current approaches, based mainly on numerical analysis, raise to the standards required for large-scale CO2 injection operations. In particular, classical noise-reduction neural networks used in image processing are employed more and more for physical systems, and represent good candidate solutions to acquisition repeatability issues. Using current simulation codes, one can generate synthetic data that includes sources of systematic error such as source and receiver position, reservoir uplifting and specific reservoir response function change, which can be used to train networks and models. One of the goals of this thesis is therefore to investigate how these techniques can be adapted to include the knowledge of spatial distribution of the acquisition geometry, and used to distinguish the specific signature of systematic error in repeated seismic acquisition campaigns. Recent works on supervised learning for geophysics applications have shown that models based on convolutional neural networks (CNN) are able to achieve grater precision in the reconstruction of reservoir CO2 saturation distribution from surface gravimetry data, compared to standard inversion techniques. Adapting these approaches to seismic data, which is able to carry more information due to the hyperbolic character of its governing equations, is a very promising research avenue and could lead to much improved reservoir imaging with much sparser acquisition data. In particular, a significant reduction in the number of source shots can be achieved, with positive effect on both the computational and the experimental cost of an acquisition campaign. One goal of this thesis is to investigate the role of similar supervised learning models for direct reservoir inversion from seismic data. Furthermore, achieving this results paves the way for the integration of multiple physics (including, for example, gravity and electromagnetic waves, as well as reservoir production and injection data) into the constraining data of the reservoir model, leading to a much more complete and robust inversion scheme. This work has been presented at the poster session of Mathias days 2024 [40].

8.2.8 Bringing seismic monitoring for CO2 geological storage into GEOS and PyGeos: wave propagation kernels with anisotropy and attenuation, DAS acquisition, integrated workflows and more

Participants: Hélène Barucq, Henri Calandra, Aurélien Citrain, Florian Faucher, Stefano Frambati.

CO2 geological storage has become a major issue in recent years, and with it the need for digital tools for seismic monitoring of CO2 injection. As part of the development of such a tool, the Makutu joint team (Inria-TotalEnergies) was set up in January 2022 with the ambitiousd goal to integrate the tools needed to simulate wave propagation and seismic imaging into GEOS, an open-source platform designed to run on exascale machines and developed by the Lawrence Livermore National Lab, Stanford University, TotalEnergies and Chevron. In this presentation we will talk about the development made by the Makutu team during the last years, in particular the solvers bases on high order Spectral Element Methods and the different physics they can handle (acoustic, elastic and elasto-acoustic). We also present the last acheivements which include the implementation of VTI anisotropy for acoustic and elastic physics, the introduction of attenuation inside the elastic solver but also the optimization made to improve time computation and reduce the numerical costs. We will also briefly touch the subject of Distributed Acoustic Sensing (DAS) acquisition and the python interface PyGEOS is used for out workflows. Finally, we will discuss future developments, such as the integration of the Discontinuous Galerkin method, which will make it possible to more closely integrate our kernels with reservoir and geomechanics simulations in GEOS, but also the challenges that this will entail, such as meshing and degree of freedoms management in GEOS. To conclude we will also discuss how we can include Machine Learning in our workflows for tasks such as data management and pre-processing for the inverse problem.

This work has been presented at Mathias days 2024 [28]

8.3 Helioseismology

8.3.1 Effects of Cowling approximation on solar oscillations in radial symmetry

Participants: Hélène Barucq, Lola Chabat, Florian Faucher, Ha Pham.

Helioseismology investigates the interior structures and the dynamics of the Sun from oscillations observed on its visible surface. Ignoring flow and rotation, time-harmonic adiabatic waves in a selfgravitating Sun in Eulerian-Lagrangian description are described by the Lagrangian displacement and the gravitational potential perturbation which satisfy Galbrun's equation coupled with a Poisson equation. In most works, perturbation to gravitational potential is neglected under Cowling's approximation. However, this approximation is known to shift the eigenvalues of the forward operator for low-order harmonic modes. Here, we study the effects of this approximation on numerical solutions and discuss its implication for the inverse problem. Removing Cowling's approximation allows us to accurately simulate waves for low-degree modes, and help us better characterize the deep interior of the Sun. The investigation is carried out for a Sun with minimum activity, called quiet Sun, whose background coefficients are given by the radially symmetric standard solar model Model S in the interior, with a choice of extension beyond the surface to include the presence of atmosphere. Radial symmetry is exploited to decouple the problem on each spherical harmonic mode to give a system of ordinary differential equations in radial variable.

This is the Ph.D. thesis of Lola Chabat, and it has been presented at the conference WAVES [27], the Journées Jeunes Chercheurs [24].

8.3.2 Computation of Green's tensor and absorbing boundary conditions for the Galbrun's equation in radial symmetry and with Cowling's approximation

Participants: Hélène Barucq, Florian Faucher, Ha Pham.

Solar oscillations can be modeled by Galbrun's equation which describes Lagrangian wave displacement in a self-gravitating stratified medium. For spherically symmetric backgrounds, we construct an algorithm to compute efficiently and accurately the coefficients of the Green's tensor of the time-harmonic equation in vector spherical harmonic basis. With only two resolutions, our algorithm provides values of the kernels for all heights of source and receiver, and prescribes analytically the singularities of the kernels. We also derive absorbing boundary conditions (ABC) to model wave propagation in the atmosphere above the cut-off frequency. The construction of ABC, which contains varying gravity terms, is rendered difficult by the complex behavior of the solar potential in low atmosphere and for frequencies below the Lamb frequency. We carry out extensive numerical investigations to compare and evaluate the efficiency of the ABCs in capturing outgoing solutions. Finally, as an application towards helioseismology, we compute synthetic solar power spectra that contain pressure modes as well as internal-gravity (g-) and surface-gravity (f-) ridges which are missing in simpler approximations of the wave equation. For purpose of validation, the location of the ridges in the synthetic power spectra are compared with observed solar modes.

This work is joint with Laurent Gizon and Damien Fournier (Max Planck Institute in Göttingen, it has been published in Journal of Computational Physics, [8], an extended report with more details is also available, [13].

8.3.3 Absorbing boundary conditions and an algorithm for computing Green's tensor for Galbrun's equation in radial symmetry

Participants: Hélène Barucq, Florian Faucher, Ha Pham.

For the time-harmonic Galbrun's equation in spherical symmetry, we present an algorithm to compute the Green's tensor in vector spherical harmonic basis, and construct absorbing boundary conditions (ABC). The algorithm employs two resolutions to provide values at all heights of source and receiver with singularity prescribed analytically. The constructed ABCs contain gravity and are robust above atmospheric Lamb frequencies. Solar power spectra computed with these results display pressure and gravity ridges, the latter absent in scalar approximation.

This work is joint with Laurent Gizon and Damien Fournier (Max Planck Institute in Göttingen, it has been presented at WAVES conference in Berlin, [22].

9 Bilateral contracts and grants with industry

- Makutu research agreement

  Participants: Hélène Barucq, Henri Calandra, Julien Diaz, Marc Duruflé, Florian Faucher, Stefano Frambati.

  Period: January 2022 – December 2025; Management: INRIA Bordeaux Sud-Ouest, Amount: 350000 euro per year.

- Numerical schemes assisted with Machine Learning for solving time-dependent seismic wave problems

  Participants: Hélène Barucq, Henri Calandra, Florian Faucher, Stefano Frambati, Nicolas Victorion.

  Period: 2021 November - 2024 October, Management: INRIA Bordeaux Sud-Ouest, Amount: 90000 euros.

- Predoctoral contract with Airbus

  Participants: Andrea Lagardère, Sébastien Tordeux.

  Period: October 2023 - April 2024, Management: INRIA Bordeaux Sud-Ouest Amount: 19200 euros

- CIFRE thesis support contract with Airbus

  Participants: Andrea Lagardère, Sébastien Tordeux.

  Period: May 2024 - April 2027,
  PhD scholarship, management Airbus,
  Research Support contract Management: INRIA Bordeaux Sud-Ouest Amount: 15000 euros per year

10 Partnerships and cooperations

10.1 European initiatives

10.1.1 Horizon Europe

INCORWAVE INCORWAVE project on cordis.europa.eu

Title: Nonlinear inversion of correlation waveforms with hierarchical reconstructions

Duration: From January 1, 2024 to December 31, 2028

Partners:

- INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET AUTOMATIQUE (INRIA), France

Inria contact: Florian Faucher

Coordinator:

Summary: Waves propagating through a complex medium provide a non-invasive way to probe its interior structures. In ambient noise imaging, the input data are the cross-correlation of the stochastic wavefields. To reconstruct the properties of the medium, the waveform inversion is formulated as an optimization problem involving a misfit function whose convexity plays a critical role in the achievable spatial resolution of the inversion results, especially in the absence of a priori information about the medium. Current inversions are often limited by computational cost, cross-talk between the physical quantities, and the use of single-scattering approximations. Project INCORWAVE proposes to create a new mathematical and computational framework for nonlinear inversion of full waveform cross-correlation. Two specific problems are considered: first, for the reconstruction of geophysical visco-elasticity tensors with applications to Earth's subsurface monitoring; secondly, for the reconstruction of three-dimensional flows in the Sun to characterize the poorly understood properties of deep solar convection. To improve the convexity of misfit functions, the inversion procedure of project INCORWAVE will follow a hierarchical progression which is established by selecting subsets of input data, unknown parameters, and frequencies. The choice of each of these subsets, as well as the associated misfit function, is controlled by criteria in form of convergence estimates. Indispensable to meaningful inversion is accurate modeling operators that describe the physics under consideration and that are adapted to the treatment of real data. For the reconstruction of the elasticity tensor, the project will develop a solver in terms of P- and S-potentials for heterogeneous media. A 3D global Sun vector-wave solver is created for the inversion of the convection component of the solar flow that does not bear symmetry.

10.1.2 Other european programs/initiatives

Butterfly

Participants: Hélène Barucq, Florian Faucher, Sébastien Tordeux, Ha Pham.

Title: Stellar butterfly diagrams

Duration: From May 1, 2024 to April 30, 2027

Partners:

- INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET AUTOMATIQUE (INRIA), France
- University of Göttingen.

Inria contact: Hélène Barucq

Coordinator: Hélène Barucq and Laurent Gizon

Summary: This project has emerged out of an ongoing collaboration started in 2016 between the INRIA-Makutu team in Pau led by Hélène Barucq and the Göttingen helioseismology group led by Laurent Gizon. This collaboration focuses on the development of computational and theoretical techniques to study the forward and inverse problems of helioseismology using advanced discretization methods and high- performance computing. In project Butterfly we propose to further develop the computational models and tools that are required to recover information about stellar activity from observations of stellar oscillations (asteroseismology).

SEE4GEO

Participants: Hélène Barucq, Julien Diaz, Arjeta Heta
.

Title: SeismoElectric Effects for GEOthermal resource assessment and monitoring

Duration: From November 1, 2021 to October 30, 2024

Partners:

- University of Hawaii at Mänoa (USA);
- University of Pau and the Pays de l'Adour, UPPA (France);
- TLS Geothermics, TLS (France),
- NORCE (Norway)

Inria contact: Hélène Barucq

Coordinator: Christina Morency (morency1@llnl.gov) LLNL (Lawrence Livermore National Laboratory, US department of Energy)

See also : SEE4GEO on Geothermica website.

Summary: Geothermal systems involve the injection of large amounts of fluid into the subsurface. Identifying fracture networks is of great importance to assess geothermal resources. Traditional seismic imaging techniques fail to resolve fluid-phase properties, while purely electromagnetic (EM) approaches provide limited, low-resolution constraints on the rock structure. Seismoelectric effects (SEE) arise from the seismic-to-electromagnetic conversion in naturally charged porous media with a certain degree of fluid saturation. With SEE, we leverage seismic and EM technique sensitivities. In this project, we offer an integrated SEE assessment for geothermal systems relying on numerical modelling, laboratory experiments and field surveys. Makutu is involved in the project as a joint team with UPPA (University of Pau and Pays de l'Adour).

10.2 National initiatives

PEPR Numpex - Focused project Exa-MA (Methods and Algorithms for Exascale)

Participants: Hélène Barucq, Henri Calandra, Julien Diaz, Marc Duruflé, Florian Faucher, Stefano Frambati, Sébastien Tordeux.

The french exascale program aims at designing and developing software bricks that will equip the future exascale computers. Makutu contributes to the topic of advanced discretization and to the design of demonstrators with a focus on large scale inverse problems as demonstrators. Members of Makutu participate in the targeted project Exa-MA working on Mathematical Methods and Algorithms for Exascale. A PhD thesis started in November 2024 in collaboration with CEA Cadarache.

11 Dissemination

11.1 Promoting scientific activities

11.1.1 Scientific events: organisation

Member of the organizing committees

- Hélène Barucq has co-organized a BIRS workshop with Lise-Marie Imbert (Arizona University at Tucson), Ilaria Perugia (Vienne University) and Virginia Selgas (Oviedo University). Entitled Contemporary Challenges in Trefftz Methods, from Theory to Applications, the program is available at link.
- Hélène Barucq has been a member of the organization committee of the Waves 2024 conference that was held in Berlin by the end of June. The program is available at link.

11.1.2 Journal

Reviewer - reviewing activities
Members of Makutu are regular reviewers for Journal of Computational Physics, M2AN, Geophysics, Inverse Problems, IEEE Transactions in Medical Imaging, Surveys in Geophysics, SIAM Journal of Applied Mathematics.

11.1.3 Invited talks

- Hélène Barucq has been invited to ORAP 2024, BNF, Paris, 24/10/24.
- Florian has been invited to give a plenary talk at the International Conference "Inverse Problems: Modeling and Simulation" (IPMS 2024), June 2024.
- Florian Faucher has beeen invited to the Workshop on Computational methods for Inverse and Ill-posed problems at University College London, november 2024.

11.1.4 Scientific expertise

- Hélène Barucq is member of the scientific committee of BRGM.
- Hélène Barucq is member of the scientific committee of CERFACS.
- Hélène Barucq has been member of the HCERES committee for the evaluation of CMAP.

11.1.5 Research administration

- Hélène Barucq is co-heading with Christophe Prud'homme (UNISTRA) the targeted project Exa-MA which is part of the PEPR Numpex.
- Julien Diaz is elected member of the "Comité Social d'Administration" and of the "Formation Spécialisée en Santé et Sécurité au Travail". He was elected member of the Inria Administrative Board until october 2024. He is appointed member of the Bureau du Comité des Projets (BCP) of Inria Bordeaux Sud-Ouest. Since 2018, he has been the head of the Mescal team of LMAP.

11.2 Teaching - Supervision - Juries

11.2.1 Teaching

- Master : Julien Diaz, Parallel computing, 32 eq. TD, Master, UPPA, France
- Master : Sébastien Tordeux, Analyse EDP 2 : Fourier problèmes aux limites S2, 20 eq. TD, Master 1, UPPA, France
- Master : Sébastien Tordeux, éléments finis, 21 eq. TD, Master 2, UPPA, France
- Licence : Sébastien Tordeux, Espaces Lp et séries de Fourier, Licence 3 Mathématiques, UPPA, France
- Licence : Sébastien Tordeux, Algèbre numérique et Python, Licence 3 Mathématiques, UPPA, Pau, France, 60 eq. TD.

- Licence : Sébastien Tordeux, Mathématiques générales, Licence 1 Informatique, UPPA, Pau, France, 20 eq. TD.
- Licence : Sébastien Tordeux, Probabilités et statistiques, Licence 1 Informatique, UPPA, Pau, France, 20 eq. TD.
- Licence : Matthias Rivet, Analyse numérique, Insa-Toulouse, 40 eq. TD.
- Master : Matthias Rivet, Eléments finis, volumes finis, différences finies, Galerkine discontinu, Isae-Supaéro-Toulouse, 24 eq. TD.

11.2.2 Supervision

- PhD of Lola Chabat. Supervisors: Hélène Barucq, Ha Pham. Additional supervision: Florian Faucher.
- Phd of Nicolas Victorion. Supervisors: Hélène Barucq and Florian Faucher.
- PhD of Augustin Leclerc. Supervisors: Hélène Barucq and Christian Gout. Additional supervisor: Antoine Tonnoir
- PhD of Arjeta Heta. Supervisors: Hélène Barucq and Julien Diaz.
- PhD of Manon Sarrouilhe. Supervisors: Hélène Barucq and Stefano Frambati
- PhD of Julien Besset. Supervisor: Hélène Barucq and Stefano Frambati
- PhD of Mathias Rivet. Supervisors: Sébastien Tordeux and Sébastien Pernet
- PhD of Ibrahima Djiba. Supervisors: Hélène Barucq and Sébastien Tordeux
- PhD of Andrea Lagardère. Supervisors: Sébastien Tordeux and Guillaume Sylvand
- PhD of Pierre Dubois. Supervisors: Hélène Barucq and Isabelle Ramière. Additional supervisor: Raphaël Prat.
- Master thesis of Jean Dutheil. Supervisor: Florian Faucher.

11.2.3 Juries

- Hélène Barucq has been reviewer for the PhD dissertation presented by Maelys Ruello, entitled Méthodes de propagation de type One-Way pour les équations de Navier-Stokes : vers le calcul des perturbations optimales, defended at Onera Toulouse, 24/12/11.
- Hélène Barucq has been reviewer for the PhD dissertation presented by Valentin Ritzenthaler, entitled Stratégies de couplage des méthodes Compatible Discrete Operators appliquées aux équations de Maxwell dans le domaine temporel, defended at Onera Toulouse, 24/12/10.
- Hélène Barucq has been reviewer for the PhD dissertation presented by Tom Sprunck, entitled Can one hear the shape of a room?, defended at UNISTRA, Strasbourg, 24/12/11.
- Hélène Barucq has chaired the PhD jury of Kokou Michaelis DOTSE. The dissertation entitled Création de maillages pour optimiser les performances de solveurs haute-précision pour la résolution d'équations aux dérivées partielles, has been defended at ISAE, Toulouse, 24/06/12
- Hélène Barucq and Sébastien Tordeux have been members of the PhD jury of Lisa-Marie Mazzolo. The dissertation entitled Etude et développement d'un outil de simulation efficace pour l'évaluation de SER: application à la détection d'objets enfouis à partir de plate-formes aéroportées, has been defended at École de l'air et de l'espace, Salon de Provence, 24/11/25
- Hélène Barucq chaired the HDR committee of Juliette Chabassier, defended at Pau University, 24/06/26.

- Hélène Barucq chaired the HDR committee of Axel Modave, defender at Inria Saclay, 24/06/24.
- Julien Diaz chaired the PhD jury of Joyce GHANTOUS. The dissertation entitled "Prise en compte de conditions aux bords d'ordre élevé et analyse numérique de problèmes de diffusion sur maillages courbes à l'aide d'éléments finis d'ordre élevé.", has been defended at Pau University, 24/09/23.
- Julien Diaz chaired the PhD jury of Anthony BOSCO. The dissertation entitled "Développements de méthodes numériques rapides, précises et robustes pour les écoulements turbulents en maillages non structurés.", has been defended at Pau University, 24/12/05.
- Julien Diaz has been reviewer of the PhD dissertation presented by Dorian LEREVEREND, entitled "Étude analytique et numérique de problèmes inverses en diffraction acoustique pour la conception de microphones spatiaux.", been defended at Institut Polytechnique de Paris, 24/10/25.

11.3 Popularization

Participants: Hélène Barucq, Manon Sarrouilhe.

Promoting scientific computing to high school students
Saint-Vincent de Tyrosse, 24/10/08

12 Scientific production

12.1 Major publications

[1] H. Barucq, N. Rouxelin and S. Tordeux. 'Construction and analysis of a HDG solution for the total-flux-formulation of the convected Helmholtz equation'. In: *Mathematics of Computation* 92.343 (2023), pp. 2097–2131. DOI: 10.1090/mcom/3850. URL: https://hal.science/hal-04006555.

[2] S. Frambati, H. Barucq, H. Calandra and J. Diaz. 'Practical unstructured splines: Algorithms, multi-patch spline spaces, and some applications to numerical analysis'. In: *Journal of Computational Physics* 471 (15th Dec. 2022), p. 111625. DOI: 10.1016/j.jcp.2022.111625. URL: https://hal.science/hal-03788980.

[3] D. Gregor, P. Moczo, J. Kristek, A. Mesgouez, G. Lefeuve-Mesgouez, C. Morency, J. Diaz and M. Kristekova. 'Seismic waves in medium with poroelastic/elastic interfaces: a two-dimensional P-SV finite-difference modelling'. In: *Geophysical Journal International* 228.1 (24th Jan. 2022), pp. 551–588. DOI: 10.1093/gji/ggab357. URL: https://hal.inrae.fr/hal-03471065.

[4] P. Lalanne, T. Wu, D. Arrivault, M. Duruflé, A. Gras, F. Binkowski, S. Burger and W. Yan. 'Efficient hybrid method for the modal analysis of optical microcavities and nanoresonators'. In: *Journal of the Optical Society of America. A Optics, Image Science, and Vision* 38.8 (26th July 2021), p. 1224. DOI: 10.1364/JOSAA.428224. URL: https://hal.archives-ouvertes.fr/hal-03358012.

[5] J. A. Lara Benitez, T. Furuya, F. Faucher, A. Kratsios, X. Tricoche and M. de Hoop. 'Out-of-distributional risk bounds for neural operators with applications to the Helmholtz equation'. In: *Journal of Computational Physics* (6th June 2024), p. 113168. DOI: 10.1016/j.jcp.2024.113168. URL: https://hal.science/hal-03963324.

[6] V. Martins Gomes, D. Brito, S. Garambois, M. Dietrich, C. Bordes and H. Barucq. 'Seismoelectric wave conversions at an interface: a quantitative comparison between laboratory data and full-waveform modelling'. In: *Geophysical Journal International* 235.3 (18th Dec. 2023), pp. 2992–3011. DOI: 10.1093/gji/ggad409. URL: https://inria.hal.science/hal-04396286.

[7] S. Pernet, M. Sirdey and S. Tordeux. 'Ultra-weak variational formulation for heterogeneous maxwell problem in the context of high performance computing'. In: *ESAIM: Proceedings and Surveys* 75 (2023), pp. 96–121. DOI: 10.1051/proc/202375096. URL: https://hal.science/hal-03642116.

[8] H. Pham, F. Faucher, D. Fournier, H. Barucq and L. Gizon. ‘Assembling algorithm for Green's tensors and absorbing boundary conditions for Galbrun's equation in radial symmetry’. In: *Journal of Computational Physics* 519 (15th Dec. 2024), p. 113444. DOI: 10.1016/j.jcp.2024.113444. URL: https://hal.science/hal-04503374 (cit. on p. 21).

[9] M. Rivet, S. Pernet and S. Tordeux. ‘Optimised Correction Polynomial Functions for the Flux Reconstruction Method in Time-Harmonic Electromagnetism’. In: *Applied Mathematics Letters* 157 (Nov. 2024), p. 109187. DOI: 10.1016/j.aml.2024.109187. URL: https://inria.hal.science/hal-04517554.

12.2 Publications of the year

International journals

[10] J. A. Lara Benitez, T. Furuya, F. Faucher, A. Kratsios, X. Tricoche and M. de Hoop. ‘Out-of-distributional risk bounds for neural operators with applications to the Helmholtz equation’. In: *Journal of Computational Physics* (6th June 2024), p. 113168. DOI: 10.1016/j.jcp.2024.113168. URL: https://hal.science/hal-03963324 (cit. on pp. 11, 15).

[11] T. Liu, J. A. Lara Benitez, F. Faucher, A. Khorashadizadeh, M. V. de Hoop and I. Dokmanić. ‘WaveBench: Benchmarking Data-driven Solvers for Linear Wave Propagation PDEs’. In: *Transactions on Machine Learning Research Journal* (28th Feb. 2024). URL: https://hal.science/hal-04503454 (cit. on p. 11).

[12] H. Pham, F. Faucher and H. Barucq. ‘Numerical investigation of stabilization in the Hybridizable Discontinuous Galerkin method for linear anisotropic elastic equation’. In: *Computer Methods in Applied Mechanics and Engineering* (3rd June 2024), p. 117080. DOI: 10.1016/j.cma.2024.117080. URL: https://hal.science/hal-04503407 (cit. on p. 12).

[13] H. Pham, F. Faucher, D. Fournier, H. Barucq and L. Gizon. ‘Assembling algorithm for Green's tensors and absorbing boundary conditions for Galbrun's equation in radial symmetry’. In: *Journal of Computational Physics* 519 (15th Dec. 2024), p. 113444. DOI: 10.1016/j.jcp.2024.113444. URL: https://hal.science/hal-04503374 (cit. on p. 21).

[14] M. Rivet, S. Pernet and S. Tordeux. ‘Optimised Correction Polynomial Functions for the Flux Reconstruction Method in Time-Harmonic Electromagnetism’. In: *Applied Mathematics Letters* 157 (Nov. 2024), p. 109187. DOI: 10.1016/j.aml.2024.109187. URL: https://inria.hal.science/hal-04517554 (cit. on p. 14).

Invited conferences

[15] F. Faucher. ‘Numerical inverse wave problems for quantitative viscoelastic reconstructions’. In: IPMS 2024 - 11th International Conference "Inverse Problems: Modeling and Simulation". Il-Mellieħa, Malta, 26th May 2024. URL: https://inria.hal.science/hal-04854272 (cit. on p. 18).

International peer-reviewed conferences

[16] H. Barucq, F. Faucher and H. Pham. ‘New Stabilization of Hybridizable Discontinuous Galerkin discretization for anisotropic wave equation in Voigt notation’. In: ECCOMAS 2024 - 9th European Congress on Computational Methods in Applied Sciences and Engineering. Lisbon, Portugal, 3rd June 2024. URL: https://inria.hal.science/hal-04899069 (cit. on p. 12).

[17] H. Barucq, H. Pham and F. Faucher. ‘Hybridizable Discontinuous Galerkin method for solving anisotropic wave propagation’. In: WAVES 2024 - 16th International Conference on Mathematical and Numerical Aspects of Wave Propagation. Berlin, Germany, 30th June 2024. URL: https://inria.hal.science/hal-04898997 (cit. on p. 12).

[18] J. Besset, R. Djellouli, H. Barucq and S. Frambati. 'Fréchet-Taylor-Based approach for efficient ROM construction and Line Search acceleration.' In: Eccomas 2024 - 9th European Congress on Computational Methods in Applied Sciences and Engineering. Lisboa, Portugal, 3rd June 2024. URL: https://hal.science/hal-04898695 (cit. on p. 19).

[19] I. Djiba, H. Barucq, R. Djellouli and S. Tordeux. 'A Novel Trefftz-Based Computational Strategy for Solving Wave Propagation Problems'. In: ECCOMAS 2024 - 9th European Congress on Computational Methods in Applied Sciences and Engineering. Lisbon, Portugal, 3rd June 2024. URL: https://hal.science/hal-04877135 (cit. on p. 15).

[20] F. Faucher, H. Pham, O. Scherzer and H. Barucq. 'Time-harmonic inverse wave problem in linear viscoelasticity'. In: WAVES 2024 - 16th International Conference on Mathematical and Numerical Aspects of Wave Propagation. Berlin, Germany, 30th June 2024. URL: https://inria.hal.science/hal-04854275 (cit. on p. 18).

[21] S. Pernet, M. Rivet and S. Tordeux. 'Flux Reconstruction Type Auxiliary Solvers for a Trefftz Domain Decomposition Method Dealing with Time-Harmonic Electromagnetism'. In: ECCOMAS 2024 - 9th European Congress on Computational Methods in Applied Sciences and Engineering. Lisbonne, Portugal, 3rd June 2024. URL: https://inria.hal.science/hal-04877044 (cit. on p. 14).

[22] H. Pham, F. Faucher, D. Fournier, H. Barucq and L. Gizon. 'Absorbing boundary conditions and an algorithm for computing Green's tensor for Galbrun's equation in radial symmetry'. In: *Waves 2024*. Waves 2024 - The 16th International Conference on Mathematical and Numerical Aspects of Wave Propagation. Berlin, Germany, 30th June 2024. URL: https://inria.hal.science/hal-04880183 (cit. on p. 22).

Conferences without proceedings

[23] P. Amestoy, O. Boiteau, A. Buttari, L. Combe, F. Faucher, M. Gerest, F. Jézéquel, J.-Y. L'Excellent, T. Mary, S. Operto and C. Puglisi. 'Recent advances on sparse direct solver using Block Low-Rank and mixed precision for large scale applications'. In: SIAM Conference on Applied Linear Algebra (LA24). Paris, France, 13th May 2024. URL: https://hal.science/hal-04642783 (cit. on p. 16).

[24] H. Barucq, L. Chabat, F. Faucher, D. Fournier and H. Pham. 'The effects of Cowling's approximation on adiabatic wave propagation for radially symmetric backgrounds in helioseismology'. In: Rencontre JCJC Ondes 2024. PARIS, France, 18th Nov. 2024. URL: https://hal.science/hal-04884144 (cit. on p. 21).

[25] H. Barucq, M. Duruflé, C. Gout, A. Leclerc and A. Tonnoir. 'Computation of electromagnetic modes at low frequencies'. In: WAVES 2024 -The 16th International Conference on Mathematical and Numerical Aspects of Wave Propagation. Berlin, Germany, 30th June 2024. URL: https://inria.hal.science/hal-04907340 (cit. on p. 13).

[26] H. Barucq, F. Faucher and H. Pham. 'HDG approximation of time-harmonic waves in elastic heterogeneous media with application to Full-Waveform Inversion'. In: Contemporary Challenges in Trefftz Methods, from Theory to Applications. Oaxaca, Mexico, 12th May 2024. URL: https://inria.hal.science/hal-04709800 (cit. on p. 12).

[27] L. Chabat, F. Faucher, H. Pham, H. Barucq and D. Fournier. 'The effects of Cowling's approximation on adiabatic wave propagation for radially symmetric backgrounds in helioseismology'. In: WAVES 2024 - he 16th International Conference on Mathematical and Numerical Aspects of Wave Propagation. BERLIN, Germany, July 2024. URL: https://hal.science/hal-04884116 (cit. on p. 21).

[28] A. Citrain, H. Barucq, S. Frambati, H. Calandra, F. Faucher and J. Meng. 'Bringing seismic monitoring for CO2 geological storage into GEOS and PyGeos: wave propagation kernels with anisotropy and attenuation, DAS acquisition, integrated workflows and more'. In: Mathias DAYS 2024. Magny le Hongre, France, 23rd Sept. 2024. URL: https://hal.science/hal-04908960 (cit. on p. 20).

[29] F. Faucher, H. Pham, O. Scherzer and H. Barucq. 'Numerical inverse wave problem in anisotropic elastic media'. In: Workshop on Computational methods for Inverse and Ill-posed problems. London, United Kingdom, 7th Nov. 2024. URL: https://inria.hal.science/hal-04854278 (cit. on p. 18).

[30] A. Heta, H. Barucq, R. Djellouli and J. Diaz. 'On the well-posedness of Pride system for modeling seismo-electric effects wave propagation in conductive porous media'. In: WAVES 2024 - 16th International Conference on Mathematical and Numerical Aspects of Wave Propagation. Berlin, Germany, 30th June 2024. URL: https://hal.science/hal-04902504 (cit. on p. 17).

[31] S. Pernet, M. Rivet and S. Tordeux. 'A Quasi-Trefftz Method Based on a Flux Reconstruction Local Solver for the Time-Harmonic Maxwell System'. In: WAVES 2024 - The 16th International Conference on Mathematical and Numerical Aspects of Wave Propagation. Berlin, Germany, 2024. URL: https://hal.science/hal-04843904 (cit. on p. 14).

[32] S. Pernet, M. Rivet and S. Tordeux. 'A quasi-Trefftz method for time-harmonic electromagnetism based on a Flux Reconstruction local solver'. In: NUMELEC 2024. Toulouse, France, 2024. URL: https://hal.science/hal-04843838 (cit. on p. 14).

[33] C. Shen, H. Barucq, H. Calandra, F. Faucher and S. Frambati. 'On the detectability of CO2 plumes with seismic records: a numerical simulation study on the Otway case'. In: MATHIAS DAYS 2024. Magny-le-Hongre, France, 23rd Sept. 2024. URL: https://inria.hal.science/hal-04887030 (cit. on p. 18).

[34] S. Tordeux. 'Trefftz variational iterative methods for solving linear hyperbolic systems'. In: Contemporary Challenges in Trefftz Methods, from Theory to Applications. Oaxaca, Mexico, 12th May 2024. URL: https://hal.science/hal-04637774 (cit. on p. 14).

Scientific book chapters

[35] H. Barucq, A. Bendali, I. Djiba and S. Tordeux. 'Trefftz methods for solving large-scale time-harmonic wave problems'. In: *Mathematical and Computational Modeling Across the Scales*. 2025. URL: https://hal.science/hal-04877519. In press (cit. on p. 15).

Doctoral dissertations and habilitation theses

[36] A. Leclerc. 'Computation of electromagnetic guided modes in twisted and open waveguides'. Normandie Université, 27th Nov. 2024. URL: https://theses.hal.science/tel-04837193.

Reports & preprints

[37] H. Barucq, A. Bendali, J. Diaz and S. Tordeux. *Revisiting the Plane-Wave Ultra-Weak Variational Formulation*. 28th May 2024. URL: https://hal.science/hal-04591459 (cit. on p. 14).

[38] H. Barucq, M. Duprez, F. Faucher, E. Franck, F. Lecourtier, V. Lleras, V. Michel-Dansac and N. Victorion. *Enriching continuous Lagrange finite element approximation spaces using neural networks*. 7th Feb. 2025. URL: https://hal.science/hal-04935072.

[39] S. Pernet, M. Rivet and S. Tordeux. *Unique continuation theorem for symmetric harmonic hyperbolic systems with constant coefficients*. RR-9544. Inria Bordeaux - Sud-Ouest, 14th Mar. 2024. URL: https://inria.hal.science/hal-04504052 (cit. on p. 14).

Other scientific publications

[40] M. Sarrouilhe, H. Barucq, H. Calandra and S. Frambati. 'Machine Learning approaches for CO2 geological storage monitoring: repeatability for passive seismic via autoencoders'. In: Mathias Days 2024. Magny-le-Hongre, France, 23rd Sept. 2024. URL: https://hal.science/hal-04894647 (cit. on p. 20).

12.3 Cited publications

[41] G. Alessandrini, M. V. de Hoop, F. Faucher, R. Gaburro and E. Sincich. 'Inverse problem for the Helmholtz equation with Cauchy data: reconstruction with conditional well-posedness driven iterative regularization'. In: *ESAIM: Mathematical Modelling and Numerical Analysis* 53.3 (May 2019), pp. 1005–1030. DOI: 10.1051/m2an/2019009. URL: https://hal.archives-ouvertes.fr/hal-02168474 (cit. on p. 5).

[42] A. Bamberger, G. Chavent and P. Lailly. 'About the stability of the inverse problem in 1-D wave equations—Application to the interpretation of seismic profiles'. In: *Applied Mathematics and Optimization* 5.1 (1979), pp. 1–47 (cit. on p. 5).

[43] H. Barucq, H. Calandra, J. Diaz and E. Shishenina. 'Space-time Trefftz-DG approximation for elasto-acoustics'. In: *Applicable Analysis* 00 (Aug. 2018), pp. 1–16. URL: https://hal.inria.fr/hal-01940623 (cit. on p. 5).

[44] H. Barucq, T. Chaumont-Frelet and C. Gout. 'Stability analysis of heterogeneous Helmholtz problems and finite element solution based on propagation media approximation'. In: *Mathematics of Computation* 86.307 (2017), pp. 2129–2157. DOI: 10.1090/mcom/3165. URL: https://hal.inria.fr/hal-01408934 (cit. on p. 5).

[45] M. Bonnasse-Gahot, H. Calandra, J. Diaz and S. Lanteri. 'Hybridizable discontinuous Galerkin method for the two-dimensional frequency-domain elastic wave equations'. In: *Geophysical Journal International* 213.1 (Apr. 2018), pp. 637–659. DOI: 10.1093/gji/ggx533. URL: https://hal.inria.fr/hal-01656440 (cit. on p. 5).

[46] G. Chavent. 'Identification of functional parameters in partial differential equations'. In: *Joint Automatic Control Conference*. 12. 1974, pp. 155–156 (cit. on p. 5).

[47] J. Christensen-Dalsgaard. 'Helioseismology'. In: *Reviews of Modern Physics* 74.4 (2002), p. 1073 (cit. on p. 6).

[48] F. Faucher. 'Contributions to Seismic Full Waveform Inversion for Time Harmonic Wave Equations: Stability Estimates, Convergence Analysis, Numerical Experiments involving Large Scale Optimization Algorithms'. Theses. Université de Pau et des Pays de l'Adour, Nov. 2017. URL: https://hal.archives-ouvertes.fr/tel-01807861 (cit. on p. 6).

[49] D. Fournier, C. S. Hanson, L. Gizon and H. Barucq. 'Sensitivity kernels for time-distance helioseismology-Efficient computation for spherically symmetric solar models'. In: *Astronomy & Astrophysics* 616 (2018), A156 (cit. on p. 6).

[50] L. Gizon, H. Barucq, M. Duruflé, C. Hanson, M. Leguèbe, A. Birch, J. Chabassier, D. Fournier, T. Hohage and E. Papini. 'Computational helioseismology in the frequency domain: acoustic waves in axisymmetric solar models with flows'. In: *Astronomy and Astrophysics - A&A* 600 (Apr. 2017), A35. DOI: 10.1051/0004-6361/201629470. URL: https://hal.archives-ouvertes.fr/hal-01403332 (cit. on p. 6).

[51] L. Gizon and A. C. Birch. 'Local helioseismology'. In: *Living Reviews in Solar Physics* 2.1 (2005), p. 6 (cit. on p. 6).

[52] L. Gizon, A. C. Birch and H. C. Spruit. 'Local helioseismology: three-dimensional imaging of the solar interior'. In: *Annual Review of Astronomy and Astrophysics* 48 (2010), pp. 289–338 (cit. on p. 6).

[53] J. Kasahara, Y. Hasada and T. Yamaguchi. 'Seismic imaging of supercritical geothermal reservoir using full-waveform inversion method'. In: *Proceedings*. 2019 (cit. on p. 5).

[54] C. M. Krawczyk, M. Stiller, K. Bauer, B. Norden, J. Henninges, A. Ivanova and E. Huenges. '3-D seismic exploration across the deep geothermal research platform GroßSchönebeck north of Berlin/Germany'. In: *Geothermal Energy* 7.1 (2019), pp. 1–18 (cit. on p. 5).

[55] P. Lailly and J. Bednar. 'The seismic inverse problem as a sequence of before stack migrations'. In: *Conference on inverse scattering: theory and application*. Siam Philadelphia, PA. 1983, pp. 206–220 (cit. on p. 5).

[56] R.-E. Plessix. 'A review of the adjoint-state method for computing the gradient of a functional with geophysical applications'. In: *Geophysical Journal International* 167.2 (2006), pp. 495–503 (cit. on p. 5).

[57] R. G. Pratt, C. Shin and G. Hick. 'Gauss–Newton and full Newton methods in frequency–space seismic waveform inversion'. In: *Geophysical Journal International* 133.2 (1998), pp. 341–362 (cit. on p. 5).

[58] R. G. Pratt and M. H. Worthington. 'INVERSE THEORY APPLIED TO MULTI-SOURCE CROSS-HOLE TOMOGRAPHY. PART 1: ACOUSTIC WAVE-EQUATION METHOD 1'. In: *Geophysical prospecting* 38.3 (1990), pp. 287–310 (cit. on p. 5).

[59] R. Pratt, Z.-M. Song, P. Williamson and M. Warner. ‘Two-dimensional velocity models from wide-angle seismic data by wavefield inversion’. In: *Geophysical Journal International* 124.2 (1996), pp. 323–340 (cit. on p. 5).

[60] G. Sosio, A. Gendrin, U. Miersemann, L. Pekot, R. Andrés, P. González, A. Giménez and J. C. Ballesteros. ‘Property modelling of a potential CO2 storage site using seismic inversion’. In: *EGUGA* (2013), EGU2013–10470 (cit. on p. 5).

[61] A. Tarantola. *Inverse problem theory: methods for data fitting and model parameter estimation*. Amsterdam, Netherlands: Elsevier Science Publishers, 1987 (cit. on p. 5).

[62] A. Tarantola. *Inverse problem theory and methods for model parameter estimation*. Vol. 89. siam, 2005 (cit. on p. 5).

[63] T. A. Torp and J. Gale. ‘Demonstrating storage of CO2 in geological reservoirs: The Sleipner and SACS projects’. In: *Energy* 29.9-10 (2004), pp. 1361–1369 (cit. on p. 5).

[64] H. Yamabe, T. Tsuji, Y. Liang and T. Matsuoka. ‘Influence of fluid displacement patterns on seismic velocity during supercritical CO2 injection: Simulation study for evaluation of the relationship between seismic velocity and CO2 saturation’. In: *International Journal of Greenhouse Gas Control* 46 (2016), pp. 197–204 (cit. on p. 5).