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ACTIVITY REPORT

Project-Team MINGUS

MultI-scale Numerical Geometric Schemes

IN COLLABORATION WITH: Institut de recherche mathématique de Rennes (IRMAR)

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Applied Mathematics, Computation and Simulation

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Numerical schemes and simulations

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Project-Team MINGUS

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2 Overall objectives

2.1 Presentation

In applications involving complex physics, such as plasmas and nanotechnologies, numerical simulations serve as a prediction tool supplementing real experiments and are largely endorsed by engineers or researchers. Their performances rely not only on computational power, but also on the efficiency of the underlying numerical method and the complexity of the underlying models. The contribution of applied mathematics is then required, on the one hand for a better understanding of qualitative properties and a better identification of the different regimes present in the model, and on the other hand, for a more sounded construction of new models based on asymptotic analysis. This mathematical analysis is expected to greatly impact the design of *multiscale* numerical schemes.

The proposed research group MINGUS will be dedicated to the mathematical and numerical analysis of (possibly stochastic) partial differential equations (PDEs), originating from plasma physics and nanotechnologies, with emphasis on *multiscale* phenomena either of **highly-oscillatory**, of **dissipative** or **stochastic** types. These equations can be also encountered in applications to rarefied gas dynamics, radiative transfer, population dynamics or laser propagation, for which the multiscale character is modelled by a scale physical parameter ε.

Producing accurate solutions of multiscale equations is extremely challenging owing to severe restrictions to the numerical methods imposed by fast (or stiff) dynamics. *Ad-hoc* numerical methods should aim at capturing the slow dynamics solely, instead of resolving finely the stiff dynamics at a formidable computational cost. At the other end of the spectrum, the separation of scales -as required for numerical efficiency- is envisaged in asymptotic techniques, whose purpose is to describe the model in the limit where the small parameter ε tends to zero. MINGUS aspires to accommodate sophisticated tools of mathematical analysis and heuristic numerical methods in order to produce simultaneously rich asymptotic models and efficient numerical methods.
To be more specific, MINGUS aims at finding, implementing and analysing new multiscale numerical schemes for the following physically relevant multiscale problems:

- **Highly-oscillatory Schrödinger equation for nanoscale physics:** In quantum mechanics, the Schrödinger equation describes how the quantum state of some physical system changes with time. Its mathematical and numerical study is of paramount importance to fundamental and applied physics in general. We wish to specifically contribute to the mathematical modeling and the numerical simulation of confined quantum mechanical systems (in one or more space dimensions) possibly involving stochastic terms. Such systems are involved in quantum semi-conductors or atom-chips, as well as in cold atom physics (Bose-Einstein condensates) or laser propagation in optical fibers.

  The prototypical equation is written

$$\boxed{i\varepsilon\partial_t\psi^\varepsilon = \frac{\varepsilon^2}{\beta}\Delta\psi^\varepsilon + |\psi^\varepsilon|^2\psi^\varepsilon + \psi^\varepsilon\xi} \tag{1}$$

  where the function $\psi^\varepsilon = \psi^\varepsilon(t,x) \in \mathbb{C}$ depends on time $t \geq 0$ and position $x \in \mathbb{R}^3$, $\xi = \xi(x,t)$ is a white noise and where the small parameter ε is the Planck's constant describing the microscopic/macroscopic ratio. The limit $\varepsilon \to 0$ is referred to as the semi-classical limit. The regime $\varepsilon = 1$ and $\beta \to 0$ (this can be for instance the relative length of the optical fiber) is highly-oscillatory. The noise ξ acts as a potential, it may represent several external perturbations. For instance temperature effects in Bose-Einstein condensation or amplification in optical fibers. The highly oscillatory regime combined with noise introduces new challenges in the design of efficient schemes.

- **Highly-oscillatory or highly-dissipative kinetic equations:** Plasma is sometimes considered as the fourth state of matter, obtained for example by bringing a gas to a very high temperature. A globally

neutral gas of neutral and charged particles, called plasma, is then obtained and is described by a kinetic equation as soon as collective effects dominate as compared to binary collisions. A situation of major importance is magnetic fusion in which collisions are not predominant. In order to confine such a plasma in devices like tokamaks (ITER project) or stellarators, a large magnetic field is used to endow the charged particles with a cyclotronic motion around field lines. Note that kinetic models are also widely used for modeling plasmas in earth magnetosphere or in rarefied gas dynamics.

Denoting $f^\varepsilon = f^\varepsilon(t,x,v) \in \mathbb{R}^+$ the distribution function of charged particles at time $t \geq 0$, position $x \in \mathbb{R}^3$ and velocity $v \in \mathbb{R}^3$, a typical kinetic equation for f^ε reads

$$\boxed{\partial_t f^\varepsilon + v \cdot \nabla_x f^\varepsilon + \left(E + \frac{1}{\varepsilon}(v \times B)\right) \cdot \nabla_v f^\varepsilon = \frac{1}{\beta} Q(f^\varepsilon) + f^\varepsilon m^\varepsilon} \tag{2}$$

where (E,B) is the electro-magnetic field (which may itself depend on f through Maxwell's equations), m^ε is a random process (which may describe absorption or creation of particles) and Q is a collision operator. The dimensionless parameters ε, β are related to the cyclotronic frequency and the mean free path. Limits $\varepsilon \to 0$ and $\beta \to 0$ do not share the same character (the former is oscillatory and the latter is dissipative) and lead respectively to gyrokinetic and hydrodynamic models. The noise term m^ε is correlated in space and time. At the limit $\varepsilon \to 0$, it converges formally to a white noise and stochastic PDEs are obtained.

The objective of MINGUS is twofold: the construction and the analysis of numerical schemes for multiscale (S)PDEs originating from physics. In turn, this requires (i) a deep mathematical understanding of the (S)PDEs under consideration and (ii) a strong involvement into increasingly realistic problems, possibly resorting to parallel computing. For this aspect, we intend to benefit from the Inria Selalib software library which turns out to be the ideal complement of our activities.

During the last period, some results have been obtained by the members of the team [2, 3, 4, 5, 6].

3 Research program

The MINGUS project is devoted to the mathematical and numerical analysis of models arising in plasma physics and nanotechnology. The main goal is to construct and analyze numerical methods for the approximation of PDEs containing multiscale phenomena. Specific multiscale numerical schemes will be proposed and analyzed in different regimes (namely highly-oscillatory and dissipative). The ultimate goal is to dissociate the physical parameters (generically denoted by ε) from the numerical parameters (generically denoted by h) with a uniform accuracy. Such a task requires mathematical prerequisite of the PDEs.

Then, for a given stiff (highly-oscillatory or dissipative) PDE, the methodology of the MINGUS team will be the following

- Mathematical study of the asymptotic behavior of multiscale models.
  This part involves averaging and asymptotic analysis theory to derive asymptotic models, but also long-time behavior of the considered models.
- Construction and analysis of multiscale numerical schemes.
  This part is the core of the project and will be deeply inspired from the mathematical prerequisite. In particular, our ultimate goal is the design of *Uniformly Accurate* (UA) schemes, whose accuracy is independent from ε.
- Validation on physically relevant problems.
  The last goal of the MINGUS project is to validate the new numerical methods, not only on toy problems, but also on realistic models arising in physics of plasmas and nanotechnologies. We will benefit from the Selalib software library which will help us to scale-up our new numerical methods to complex physics.

3.1 Dissipative problems

In the dissipative context, the asymptotic analysis is quite well understood in the deterministic case and multiscale numerical methods have been developed in the last decades. Indeed, the so-called Asymptotic-Preserving schemes has retained a lot of attention, in particular in the context of collisional kinetic equations. But, there is still a lot to do if one is interested in deriving high order asymptotic models, which enable to capture the original solution for all time. Moreover, this analysis is still misunderstood when more complex systems are considered, involving non homogeneous relaxation rates or stochastic terms for instance. Following the methodology we aim at using, we first address the mathematical analysis before deriving multiscale efficient numerical methods.

A simple model of dissipative systems is governed by the following differential equation

$$\left\{\begin{array}{rcll} \dfrac{dx^\varepsilon(t)}{dt} & = & \mathscr{G}(x^\varepsilon(t),y^\varepsilon(t)), & x^\varepsilon(0)=x_0, \\ \dfrac{dy^\varepsilon(t)}{dt} & = & -\dfrac{y^\varepsilon(t)}{\varepsilon}+\mathscr{H}(x^\varepsilon(t),y^\varepsilon(t)), & y^\varepsilon(0)=y_0, \end{array}\right. \tag{3}$$

for given initial condition $(x_0,y_0)\in\mathbb{R}^2$ and given smooth functions $\mathscr{G},\mathscr{H}$ which possibly involve stochastic terms.

3.1.1 Asymptotic analysis of dissipative PDEs (*F. Castella, A. Debussche, E. Faou*)

Derivation of asymptotic problems

Our main goal is to analyze the asymptotic behavior of dissipative systems of the form (3) when ε goes to zero. The *center manifold theorem* [35] is of great interest but is largely unsatisfactory from the following points of view

- a constructive approach of h and x_0^ε is clearly important to identify the high-order asymptotic models: this would require expansions of the solution by means of B-series or word-series [37] allowing the derivation of error estimates between the original solution and the asymptotic one.
- a better approximation of the transient phase is strongly required to capture the solution for small time: extending the tools developed in averaging theory, the main goal is to construct a suitable change of variable which enables to approximate the original solution for all time.

Obviously, even at the ODE level, a deep mathematical analysis has to be performed to understand the asymptotic behavior of the solution of (3). But, the same questions arise at the PDE level. Indeed, one certainly expects that dissipative terms occurring in collisional kinetic equations (2) may be treated theoretically along this perspective. The key new point indeed is to see the center manifold theorem as a change of variable in the space of unknowns, while the standard point of view leads to considering the center manifold as an asymptotic object.

Stochastic PDEs

We aim at analyzing the asymptotic behavior of stochastic collisional kinetic problems, that is equation of the type (2). The noise can describe creation or absorption (as in (2)), but it may also be a forcing term or a random magnetic field. In the parabolic scaling, one expects to obtain parabolic SPDEs at the limit. More precisely, we want to understand the fluid limits of kinetic equations in the presence of noise. The noise is smooth and non delta correlated. It contains also a small parameter and after the rescaling, it converges formally to white noise. Thus, this adds another scale in the multiscale analysis. Following the pioneering work by Debussche and Vovelle [38], substantial progresses have been done in this topic.

More realistic problems may be addressed such as high field limit describing sprays, or even hydrodynamic limit. The full Boltzmann equation is a very long term project and we wish to address simpler problems such as convergence of BGK models to a stochastic Stokes equation.

The main difficulty is that when the noise acts as a forcing term, which is a physically relevant situation, the equilibria are affected by the noise and we face difficulties similar to that of high field limit problems. Also, a good theory of averaging lemma in the presence of noise is lacking. The methods we use are generalization of the perturbed test function method to the infinite dimensional setting. We work at the level of the generator of the infinite dimensional process and prove convergence in the sense of

the martingale problems. A further step is to analyse the speed of convergence. This is a prerequisite if one wants to design efficient schemes. This requires more refined tools and a good understanding of the Kolmogorov equation.

3.1.2 Numerical schemes for dissipative problems (*All members*)

The design of numerical schemes able to reproduce the transition from the microscopic to macroscopic scales largely matured with the emergence of Asymptotic Preserving schemes which have been developed initially for collisional kinetic equations (actually, for solving (2) when $\beta \to 0$). Several techniques have flourished in the last decades. As said before, AP schemes entail limitations which we aim at overcoming by deriving

- AP numerical schemes whose numerical cost diminishes as $\beta \to 0$,
- Uniformly accurate numerical schemes, whose accuracy is independent of β.

Time diminishing methods
The main goal consists in merging Monte-Carlo techniques [33] with AP methods for handling *automatically* multiscale phenomena. As a result, we expect that the cost of the so-obtained method decreases when the asymptotic regime is approached; indeed, in the collisional (i.e. dissipative) regime, the deviational part becomes negligible so that a very few number of particles will be generated to sample it. A work in this direction has been done by members of the team.

We propose to build up a method which enables to realize the transition from the microscopic to the macroscopic description without domain decomposition strategies which normally requires to fix and tune an interface in the physical space and some threshold parameters. Since it will permit to go over domain decomposition and AP techniques, this approach is a very promising research direction in the numerical approximation of multiscale kinetic problems arising in physics and engineering.

Uniformly accurate methods
To overcome the accuracy reduction observed in AP schemes for intermediate regimes, we intend to construct and analyse multiscale numerical schemes for (3) whose error is uniform with respect to ε. The construction of such a scheme requires a deep mathematical analysis as described above. Ideally one would like to develop schemes that preserve the center manifold (without computing the latter!) as well as schemes that resolve numerically the stiffness induced by the fast convergence to equilibrium (the so-called transient phase). First, our goal is to extend the strategy inspired by the central manifold theorem in the ODE case to the PDE context, in particular for collisional kinetic equations (2) when $\beta \to 0$. The design of Uniformly Accurate numerical schemes in this context would require to generalize two-scale techniques introduced by members of the team in the framework of highly-oscillatory problems [36].

Multiscale numerical methods for stochastic PDEs
AP schemes have been developed recently for kinetic equations with noise in the context of Uncertainty Quantification UQ [41]. These two aspects (multiscale and UQ) are two domains developed in separate communities. UQ has drawn a lot of attention recently to control the propagation of data pollution; undoubtedly UQ has a lot of applications and one of our goals will be to study how sources of uncertainty are amplified or not by the multiscale character of the model. We also wish to go much further in developing AP schemes when the noise is also rescaled and the limit is a white noise driven SPDE, as described in section (3.1.1). For simple nonlinear problem, this should not present much difficulties but new ideas will definitely be necessary for more complicated problems when noise deeply changes the asymptotic equation.

3.2 Highly-oscillatory problems

As a generic model for highly-oscillatory systems, we will consider the equation

$$\frac{d\,u^\varepsilon(t)}{d\,t} = \mathscr{F}(t/\varepsilon, u^\varepsilon(t)), \quad u^\varepsilon(0) = u_0, \tag{4}$$

for a given u_0 and a given periodic function $\mathscr{F}$ (of period P w.r.t. its first variable) which possibly involves stochastic terms. Solution u^ε exhibits high-oscillations in time superimposed to a slow dynamics.

Asymptotic techniques -resorting in the present context to *averaging* theory [45]- allow to decompose

$$u^\varepsilon(t) = \Phi_{t/\varepsilon} \circ \Psi_t \circ \Phi_0^{-1}(u_0), \tag{5}$$

into a fast solution component, the εP-periodic change of variable $\Phi_{t/\varepsilon}$, and a slow component, the flow Ψ_t of a non-stiff *averaged* differential equation. Although equation (5) can be satisfied only up to a small remainder, various methods have been recently introduced in situations where (4) is posed in $\mathbb{R}^n$ or for the Schrödinger equation (1).

In the asymptotic behavior $\varepsilon \to 0$, it can be advantageous to replace the original singularly perturbed model (for instance (1) or (2)) by an approximate model which does not contain stiffness any longer. Such reduced models can be derived using asymptotic analysis, namely averaging methods in the case of highly-oscillatory problems. In this project, we also plan to go beyond the mere derivation of limit models, by searching for better approximations of the original problem. This step is of mathematical interest *per se* but it also paves the way of the construction of multiscale numerical methods.

3.2.1 Asymptotic analysis of highly-oscillatory PDEs (*All members*)

Derivation of asymptotic problems

We intend to study the asymptotic behavior of highly-oscillatory evolution equations of the form (4) posed in an infinite dimensional Banach space.

Recently, the stroboscopic averaging has been extended to the PDE context, considering nonlinear Schrödinger equation (1) in the highly-oscillatory regime. A very exciting way would be to use this averaging strategy for highly-oscillatory kinetic problem (2) as those encountered in strongly magnetized plasmas. This turns out to be a very promising way to re-derive gyrokinetic models which are the basis of tokamak simulations in the physicists community. In contrast with models derived in the literature (see [34]) which only capture the average with respect to the oscillations, this strategy allows for the complete recovery of the exact solution from the asymptotic (non stiff) model. This can be done by solving companion transport equation that stems naturally from the decomposition (5).

Long-time behavior of Hamiltonian systems

The study of long-time behavior of nonlinear Hamiltonian systems have received a lot of interest during the last decades. It enables to put in light some characteristic phenomena in complex situations, which are beyond the reach of numerical simulations. This kind of analysis is of great interest since it can provide very precise properties of the solution. In particular, we will focus on the dynamics of nonlinear PDEs when the initial condition is close to a stationary solution. Then, the long-time behavior of the solution is studied through mainly three axes

- *linear stability*: considering the linearized PDE, do we have stability of a stationary solution ? Do we have linear Landau damping around stable non homogeneous stationary states?
- *nonlinear stability*: under a criteria, do we have stability of a stationary solution in energy norm like in [42], and does this stability persist under numerical discretization? For example one of our goals is to address the question of the existence and stability of discrete travelling wave in space and time.
- do we have existence of damped solutions for the full nonlinear problem ? Around homogeneous stationary states, solutions scatter towards a modified stationary state (see [43, 39]). The question of existence of Landau damping effects around non homogeneous states is still open and is one of our main goal in the near future.

Asymptotic behavior of stochastic PDEs

The study of SPDEs has known a growing interest recently, in particular with the fields medal of M. Hairer in 2014. In many applications such as radiative transfer, molecular dynamics or simulation of optical fibers, part of the physical interactions are naturally modeled by adding supplementary random terms (the noise) to the initial deterministic equations. From the mathematical point of view, such terms change drastically the behavior of the system.

- In the presence of noise, highly-oscillatory dispersive equations present new problems. In particular, to study stochastic averaging of the solution, the analysis of the long time behavior of stochastic

dispersive equations is required, which is known to be a difficult problem in the general case. In some cases (for instance highly-oscillatory Schrödinger equation (1) with a time white noise in the regime $\varepsilon \ll 1$), it is however possible to perform the analysis and to obtain averaged stochastic equations. We plan to go further by considering more difficult problems, such as the convergence of a stochastic Klein-Gordon-Zakharov system to a stochastic nonlinear Schrödinger equation.

- The long-time behavior of stochastic Schrödinger equations is of great interest to analyze mathematically the validity of the Zakharov theory for wave turbulence (see [44]). The problem of wave turbulence can be viewed either as a deterministic Hamiltonian PDE with random initial data or a randomly forced PDEs where the stochastic forcing is concentrated in some part of the spectrum (in this sense it is expected to be a hypoelliptic problem). One of our goals is to test the validity the Zakharov equation, or at least to make rigorous the spectrum distribution spreading observed in the numerical experiments.

3.2.2 Numerical schemes for highly-oscillatory problems (*All members*)

This section proposes to explore numerical issues raised by highly-oscillatory nonlinear PDEs for which (4) is a prototype. Simulating a highly-oscillatory phenomenon usually requires to adapt the numerical parameters in order to solve the period of size ε so as to accurately simulate the solution over each period, resulting in an unacceptable execution cost. Then, it is highly desirable to derive numerical schemes able to advance the solution by a time step independent of ε. To do so, our goal is to construct *Uniformly Accurate* (UA) numerical schemes, for which the numerical error can be estimated by Ch^p (h being any numerical parameters) with C independent of ε and p the order of the numerical scheme.

Recently, such numerical methods have been proposed by members of the team in the highly-oscillatory context [36]. They are mainly based on a separation of the fast and slow variables, as suggested by the decomposition (5). An additional ingredient to prove the uniformly accuracy of the method for (4) relies on the search for an appropriate initial data which enables to make the problem smooth with respect to ε.

Such an approach is assuredly powerful since it provides a numerical method which enables to capture the high oscillations in time of the solution (and not only its average) even with a large time step. Moreover, in the asymptotic regime, the potential gain is of order $1/\varepsilon$ in comparison with standard methods, and finally averaged models are not able to capture the intermediate regime since they miss important information of the original problem. We are strongly convinced that this strategy should be further studied and extended to cope with some other problems. The ultimate goal being to construct a scheme for the original equation which degenerates automatically into a consistent approximation of the averaged model, without resolving it, the latter can be very difficult to solve.

- Space oscillations:
  When rapidly oscillating coefficients in **space** (*i.e.* terms of the form $a(x, x/\varepsilon)$) occur in elliptic or parabolic equations, homogenization theory and numerical homogenization are usually employed to handle the stiffness. However, these strategies are in general not accurate for all $\varepsilon \in]0,1]$. Then, the construction of numerical schemes which are able to handle both regimes in an uniform way is of great interest. Separating fast and slow *spatial* scales merits to be explored in this context. The delicate issue is then to extend the choice of suitable initial condition to an *appropriate choice of boundary conditions* of the augmented problem.

- Space-time oscillations:
  For more complex problems however, the recent proposed approaches fail since the main oscillations cannot be identified explicitly. This is the case for instance when the magnetic field B depends on t or x in (2) but also for many other physical problems. We then have to deal with the delicate issue of space-time oscillations, which is known to be a very difficult problem from a mathematical and a numerical points of view. To take into account the space-time mixing, a periodic motion has to be detected together with a phase S which possibly depends on the time and space variables. These techniques originate from **geometric optics** which is a very popular technique to handle high-frequency waves.

- Geometrical properties:
  The questions related to the geometric aspects of multiscale numerical schemes are of crucial importance, in particular when long-time simulations are addressed (see [40]). Indeed, one of the main questions of geometric integration is whether intrinsic properties of the solution may be passed onto its numerical approximation. For instance, if the model under study is Hamiltonian, then the exact flow is symplectic, which motivates the design of symplectic numerical approximation. For practical simulations of Hamiltonian systems, symplectic methods are known to possess very nice properties (see [40]). It is important to combine multiscale techniques to geometric numerical integration. All the problems and equations we intend to deal with will be addressed with a view to preserve intrinsic geometric properties of the exact solutions and/or to approach the asymptotic limit of the system in presence of a small parameter. An example of a numerical method developed by members of the team is the multi-revolution method.

- Quasi-periodic case:
  So far, numerical methods have been proposed for the periodic case with single frequency. However, the quasi-periodic case [1] is still misunderstood although many complex problems involve multi-frequencies. Even if the quasi-periodic averaging is doable from a theoretical point of view in the ODE case, (see [45]), it is unclear how it can be extended to PDEs. One of the main obstacle being the requirement, usual for ODEs like (4), for $\mathscr{F}$ to be analytic in the periodic variables, an assumption which is clearly impossible to meet in the PDE setting. An even more challenging problem is then the design of numerical methods for this problem.

- Extension to stochastic PDEs:
  All these questions will be revisited within the stochastic context. The mathematical study opens the way to the derivation of efficient multiscale numerical schemes for this kind of problems. We believe that the theory is now sufficiently well understood to address the derivation and numerical analysis of multiscale numerical schemes. Multi-revolution composition methods have been recently extended to highly-oscillatory stochastic differential equations The generalization of such multiscale numerical methods to SPDEs is of great interest. The analysis and simulation of numerical schemes for highly-oscillatory nonlinear stochastic Schrödinger equation under diffusion-approximation for instance will be one important objective for us. Finally, an important aspect concerns the quantification of uncertainties in highly-oscillatory kinetic or quantum models (due to an incomplete knowledge of coefficients or imprecise measurement of data). The construction of efficient multiscale numerical methods which can handle multiple scales as well as random inputs have important engineering applications.

4 Application domains

4.1 Application domains

The MINGuS project aims at applying the new numerical methods on realistic problems arising for instance in physics of nanotechnology and physics of plasmas. Therefore, in addition to efforts devoted to the design and the analysis of numerical methods, the inherent large size of the problems at hand requires advanced mathematical and computational methods which are hard to implement. Another application is concerned with population dynamics for which the main goal is to understand how the spatial propagation phenomena affects the demography of a population (plankton, parasite fungi, ...). Our activity is mostly at an early stage in the process of transfer to industry. However, all the models we use are physically relevant and all have applications in many areas (ITER, Bose-Einstein condensate, wave turbulence, optical tomography, transport phenomena, population dynamics, ...). As a consequence, our research aims at reaching theoretical physicists or computational scientists in various fields who have strong links with industrial applications. In order to tackle as realistic physical problems as possible, a fundamental aspect will consist in working on the realization of numerical methods and algorithms which are able to make an efficient use of a large number of processors. Then, it is essential for the numerical methods developed in the MINGuS project to be thought through this prism. We will benefit

[1] replacing t/ε by $t\omega/\varepsilon$ in (4), with $\omega \in \mathbb{R}^d$ a vector of non-resonant frequencies

from the strong expertise of P. Navaro in scientific computing and more precisely on the Selalib software library (see description below). Below, we detail our main applications: first, the modeling and numerical approximation of magnetized plasmas is our major application and will require important efforts in terms of software developments to scale-up our multiscale methods; second, the transport of charged particles in nanostructures has very interesting applications (like graphene material), for which our contributions will mainly focus on dedicated problems; lastly, applications on population dynamics will be dedicated to mathematical modeling and some numerical validations.

4.2 Plasmas problems

The SeLaLib (Semi-Lagrangian Library) software library is a modular library for kinetic and gyrokinetic simulations of plasmas in fusion energy devices. Selalib is a collection of fortran modules aimed at facilitating the development of kinetic simulations, particularly in the study of turbulence in fusion plasmas. Selalib offers basic capabilities and modules to help parallelization (both MPI and OpenMP), as well as pre-packaged simulations. Its main objective is to develop a documented library implementing several numerical methods for the numerical approximation of kinetic models. Another objective of the library is to provide physicists with easy-to-use gyrokinetic solvers. It has been originally developed by E. Sonnendrücker and his collaborators in the past CALVI Inria project, and has played an important role in the activities of the IPL FRATRES. P. Navaro is one of the main software engineer of this library and as such he played an important daily role in its development and its portability on supercomputers. Though Selalib has reached a certain maturity, additional work is needed to make it available to the community. There are currently discussions for a possible evolution of Selalib, namely the writing of a new release which will be available for free download. At the scientific level, Selalib is of great interest for us since it provides a powerful tool with which we can test, validate and compare our new methods and algorithms (user level). Besides numerical algorithms the library provides low-level utilities, input-output modules as well as parallelization strategies dedicated to kinetic problems. Moreover, a collection of simulations for typical test cases (of increasing difficulties) with various discretization schemes supplements the library. This library turns out to be the ideal complement of our activities and it will help us to scale-up our numerical methods to high-dimensional kinetic problems. During the last years, several experiments have been successfully performed in this direction (especially with PhD students) and it is important for us that this approach remains thorough. Then, we intend to integrate several of the numerical methods developed by the team within the Selalib library, with the strong help of P. Navaro (contributor level). This work has important advantages: (i) it will improve our research codes (in terms of efficiency software maintenance); (ii) it will help us to promote our research by making our methods available to the research community.

4.3 Quantum problems

Nowadays, a great challenge consists in the downscaling at the nanometer scale of electronic components in order to improve speed and efficiency of semiconductor materials. In this task, modeling and numerical simulations play an important role in the determination of the limit size of the nanotransistors. At the nanoscale, quantum effects have to be considered and the Schrödinger equation is a prominent equation in this context. In the so-called semiclassical regime or when the transport is strongly confined, the solution endows space-time high oscillations which are very difficult to capture numerically. An important application is the modeling of charged particles transport in graphene. Graphene is a sheet of carbone made of a single layer of molecule, organized in a bidimensional honeycomb crystal. The transport of charged particles in this structure is usually performed by Dirac equation (which is the relativistic counterpart of the Schrödinger equation). Due to the unusual properties of graphene -at room temperature, electrons moving in graphene behave as massless relativistic particles- physicists and compagnies are nowadays actively studying this material. In this context, predicting how the material properties are affected by the uncertainties in the hexagonal lattice structure or in external potentials, is a major issue.

4.4 Population dynamics

The main goal is to characterize how spatial propagation phenomena (diffusion, transport, advection, ...) affect the time evolution of the demography of a population. In collaboration with Y. Lagadeuc (ECOBIO, Rennes), this question has been studied for plankton. In this context, mathematical models have been proposed and it has been shown that the spatial dynamic (in this context, due to the marine current) which is fast compared to demographic scales, can strongly modify the demographic evolution of the plankton. In collaboration with Ecole d'Agronomie de Rennes, a mathematical study on the demography of a parasite fungi of plants has been performed. In this context, the demography is specific: the fungi can proliferate through sexual reproduction or through parthenogenesis. These two ways of reproduction give rise mathematically to quadratic and linear growth rates with respect to the population variable. The demography is then coupled with transport (transport of fungi spore by wind). Here, the goal is to characterize the propagation of the fungi population by finding travelling waves solutions which are well adapted to describe the evolution of invasive fronts. Moreover, this approach enables to recover with a good agreement realistic examples (infection of ash or banana tree) for which experimental data are available. In these contexts, mathematical models are a powerful tool for biologists since measurements are very complicated to obtain and laboratory experiments hardly reproduce reality. The models derived are multiscale due to the nature of the underlying phenomena and the next step is to provide efficient numerical schemes.

5 Social and environmental responsibility

5.1 Footprint of research activities

A group called ECO-IRMAR has been created in the IRMAR laboratory to inform about the footprint of research activities at the level of the laboratory. The members of the team follow the advices proposed by this group.

6 Highlights of the year

Ludovic Martaud has been hired as a permanent researcher.

7 New software, platforms, open data

7.1 New software

7.1.1 Selalib

Name: SEmi-LAgrangian LIBrary

Keywords: Plasma physics, Semilagrangian method, Parallel computing, Plasma turbulence

Scientific Description: The objective of the Selalib project (SEmi-LAgrangian LIBrary) is to develop a well-designed, organized and documented library implementing several numerical methods for kinetic models of plasma physics. Its ultimate goal is to produce gyrokinetic simulations.

Another objective of the library is to provide to physicists easy-to-use gyrokinetic solvers, based on the semi-lagrangian techniques developed by Eric Sonnendrücker and his collaborators in the past CALVI project. The new models and schemes from TONUS are also intended to be incorporated into Selalib.

Functional Description: Selalib is a collection of modules conceived to aid in the development of plasma physics simulations, particularly in the study of turbulence in fusion plasmas. Selalib offers basic capabilities from general and mathematical utilities and modules to aid in parallelization, up to pre-packaged simulations.

URL: https://selalib.github.io

Contact: Philippe Helluy

Participants: Edwin Chacon Golcher, Pierre Navaro, Sever Hirstoaga, Eric Sonnendrucker, Michel Mehrenberger

Partners: Max Planck Insitute - Garching, Université de Strasbourg, CNRS, Université de Rennes 1

7.1.2 HOODESolver.jl

Name: Julia package for solving numerically highly-oscillatory ODE problems

Keywords: Ordinary differential equations, Numerical solver

Functional Description: Julia is a programming language for scientists with a syntax and functionality similar to MATLAB, R and Python. HOODESolver.jl is a julia package allowing to solve ordinary differential equations with sophisticated numerical techniques resulting from research within the MINGUS project team. To use it, just install Julia on your workstation.

Release Contributions: This is the first version of the package. It will evolve further because we want to have a better integration with the Julia organization on differential equations. This one already includes a lot of methods to numerically solve differential equations. This integration will allow us to have a larger audience and thus more feedback and possibly external collaborations.

Contact: Nicolas Crouseilles

Participants: Yves Mocquard, Pierre Navaro, Nicolas Crouseilles

Partners: Université de Rennes 1, CNRS

7.1.3 PhaseLifting

Keyword: Python

Functional Description: Performs the phase bearing of a complex function defined on a grid, taking into account two criteria based on a non uniform weight map.

Contact: Erwan Faou

8 New results

Our results of the year are presented in the following three sections:

- multiscale numerical schemes (for dissipative or highly-oscillatory problems),
- numerical schemes for Hamiltonian problems,
- analysis of PDEs and SPDEs.

8.1 Multiscale numerical schemes

Participants: G. Beck, N. Crouseilles, E. Faou, A. Laurent, Y. Le Hénaff, D. Prel.

Multiscale (ie highly oscillatory or dissipative) ordinary differential equations (ODEs) have a long history since they are ubiquitous to describe dynamical multiscale physical phenomena in physics or chemistry. They can be obtained by appropriate spatial discretization of the partial differential equation or can directly describe the behavior of dynamical quantities. In addition to the standard difficulties coming from their numerical resolution, multiscale ODEs involve a stiffness (characterized by a parameter $\varepsilon \in]0,1]$). Creating strong gradients in the solution. Hence, to capture these small scales, conventional

methods have to consider a time step smaller than ε leading to unacceptable computational cost. The team members proposed several strategies to overcome this stiffness.

In [7], the design and analysis of high order accurate IMEX finite volume schemes for the compressible Euler-Poisson (EP) equations in the quasineutral limit is presented. As the quasineutral limit is singular for the governing equations, the time discretisation is tantamount to achieving an accurate numerical method. To this end, the EP system is viewed as a differential algebraic equation system (DAEs) via the method of lines. As a consequence of this vantage point, high order linearly semi-implicit (SI) time discretisation are realised by employing a novel combination of the direct approach used for implicit discretisation of DAEs and, two different classes of IMEX-RK schemes: the additive and the multiplicative. For both the time discretisation strategies, in order to account for rapid plasma oscillations in quasineutral regimes, the nonlinear Euler fluxes are split into two different combinations of stiff and non-stiff components. The high order scheme resulting from the additive approach is designated as a classical scheme while the one generated by the multiplicative approach possesses the asymptotic preserving (AP) property. Time discretisations for the classical and the AP schemes are performed by standard IMEX-RK and SI-IMEX-RK methods, respectively so that the stiff terms are treated implicitly and the non-stiff ones explicitly. In order to discretise in space a Rusanov-type central flux is used for the non-stiff part, and simple central differencing for the stiff part. AP property is also established for the space-time fully-discrete scheme obtained using the multiplicative approach. Results of numerical experiments are presented, which confirm that the high order schemes based on the SI-IMEX-RK time discretisation achieve uniform second order convergence with respect to the Debye length and are AP in the quasineutral limit.

In [28], we focus on developing a new class of numerical methods able to both handle quasineutrality and charge separation in plasmas. At large temporal and spatial scales, plasmas tend to be quasineutral, meaning that the local net charge density is nearly zero. However, when the scale at which one observes the plasma dynamics is smaller than the characteristic distance over which the electric field and charges are typically screened, then quasineutrality breaks down. In such regimes, standard numerical methods face severe stability constraints, rendering them practically unusable. To address this issue, in this work, we introduce and analyze a new class of finite volume penalized-IMEX Runge-Kutta methods for the Euler-Poisson (EP) system, specifically designed to handle the quasineutral limit. We show that, these proposed schemes are uniformly stable with respect to the Debye length and degenerate into high order methods as the quasineutral limit is approached. Several numerical tests confirm that this new class of methods exhibits the desired properties.

This work [10] deals with wave propagation into a coaxial cable, which can be modelled by the 3D Maxwell equations or 1D simplified models. The usual model, called the telegrapher's model, is a 1D wave equation of the electrical voltage and current. We derived a more accurate model from the Maxwell equations that takes into account dispersive effects. These two models aim to be a good approximation of the 3D electromagnetic fields in the case where the thickness of the cable is small. We perform some numerical simulations of the 3D Maxwell equations and of the 1D simplified models in order to validate the usual model and the new one. Moreover, we show that, while the usual telegrapher model is of order one with respect to the thickness of the cable, the dispersive 1D model is of order two.

In [13], Lawson type numerical methods are studied to solve Vlasov type equations on a phase space grid. These time integrators are known to satisfy enhanced stability properties in this context since they do not suffer from the stability condition induced from the linear part. We introduce here a class of modified Lawson integrators in which the linear part is approximated in such a way that some geometric properties of the underlying model are preserved, which has important consequences for the analysis of the scheme. Several Vlasov-Maxwell examples are presented to illustrate the good behavior of the approach.

In [15], an exponential Discontinuous Galerkin (DG) method is proposed to solve numerically Vlasov type equations. The DG method is used for space discretization which is combined exponential Lawson Runge-Kutta method for time discretization to get high order accuracy in time and space. In addition to get high order accuracy in time, the use of Lawson methods enables to overcome the stringent condition on the time step induced by the linear part of the system. Moreover, it can be proved that a discrete Poisson equation is preserved. Numerical results on Vlasov-Poisson and Vlasov Maxwell equations are presented to illustrate the good behavior of the exponential DG method.

In [16], we propose an Eulerian-Lagrangian (EL) Runge-Kutta (RK) discontinuous Galerkin (DG)

method for a linear hyperbolic system. The method is designed based on the ELDG method for transport problems (J. Comput. Phys. 446:110,632, 2021), which tracks solutions along approximations to characteristics in the DG framework, allowing extra large time stepping sizes with stability with respect to the classical RKDG method. Considering each characteristic family, a straightforward application of ELDG for the hyperbolic system will be to transform to the characteristic variables, evolve them on associated characteristic-related space-time regions, and transform them back to the original variables. However, the conservation could not be guaranteed in a general setting. In this paper, we formulate a conservative semi-discrete ELDG method by decomposing each variable into two parts, each of them associated with a different characteristic family. As a result, four different quantities are evolved in EL fashion and recombined to update the solution. The fully discrete scheme is formulated by using method-of-lines RK methods, with intermediate RK solutions updated on the background mesh. Numerical results for 1D and 2D wave equations are presented to demonstrate the performance of the proposed ELDG method. These include the high order spatial and temporal accuracy, stability with extra large time stepping size, and conservative property.

This work [22] deals with the numerical approximation of plasmas which are confined by the effect of a fast oscillating magnetic field (see [12]) in the Vlasov model. The presence of this magnetic field induces oscillations (in time) to the solution of the characteristic equations. Due to its multiscale character, a standard time discretization would lead to an inefficient solver. In this work, time integrators are derived and analyzed for a class of highly oscillatory differential systems. We prove the uniform accuracy property of these time integrators, meaning that the accuracy does not depend on the small parameter ε. Moreover, we construct an extension of the scheme which degenerates towards an energy preserving numerical scheme for the averaged model, when $\varepsilon \to 0$. Several numerical results illustrate the capabilities of the method.

The main concern of the paper [29] is the mathematical modelling and numerical simulation of thermonuclear fusion plasmas, constituted of two different kinds of particles, namely the thermal electron/ion bulk and an energetic α-particle population, created by the fusion reaction at very high speeds (3.5 MeV). This α-particle population behaves differently than the thermal plasma bulk, and due to its high energy contain, can have a considerable impact on the stability and thus confinement of the plasma bulk. An adequate modelling of this α-population is therefore crucial, and the present paper intends to make use of kinetic Fokker-Planck models to describe the evolution of these energetic particles experiencing Coulomb collisions. The main mathematical and numerical difficulties encountered in this study are related to the multi-scale nature of the overall problem, involving different types of particles and their multiscale dynamics.

8.2 Geometric numerical schemes

Participants: I. Almuslimani, N. Crouseilles, X. Hong, A. Laurent.

The MINGuS team has a long history in the design and study of numerical schemes for Hamiltonian PDEs. The main examples are Schroedinger or Vlasov equations.

In [8], we construct a mean-field model that describes the nonlinear dynamics of a spinpolarized electron gas interacting with fixed, positively-charged ions possessing a magnetic moment that evolves in time. The mobile electrons are modeled by a four-component distribution function in the two-dimensional phase space (x, v), obeying a Vlasov-Poisson set of equations. The ions are modeled by a Landau-Lifshitz equation for their spin density, which contains ion-ion and electron-ion magnetic exchange terms. We perform a linear response study of the coupled Vlasov-Poisson-Landau-Lifshitz (VPLL) equations for the case of a Maxwell-Boltzmann equilibrium, focussing in particular on the spin dispersion relation. Condition of stability or instability for the spin modes are identied, which depend essentially on the electron spin polarization rate η and the electron-ion magnetic coupling constant K. We also develop an Eulerian grid-based computational code for the fully nonlinear VPLL equations, based on the geometric Hamiltonian method rst developed in a previous work. This technique allows us to achieve great accuracy for the conserved quantities, such as the modulus of the ion spin vector and the total energy. Numerical tests in the linear regime are in accordance with the estimations of the linear

response theory. For two-stream equilibria, we study the interplay of instabilities occurring in both the charge and the spin sectors. The set of parameters used in the simulations, with densities close to those of solids ($10^{29} m^{-3}$) and temperatures of the order of $10\, eV$ may be relevant to the warm dense matter regime appearing in some inertial fusion experiments.

[24] Our goal is to highlight some of the deep links between numerical splitting methods and control theory. We consider evolution equations of the form $\dot{x} = f_0(x) + f_1(x)$ where f_0 encodes a non-reversible dynamic, so that one is interested in schemes only involving forward flows of f_0. In this context, a splitting method can be interpreted as a trajectory of the control-affine system $\dot{x} = f_0(x(t)) + u(t) f_1(x(t))$, associated with a control u which is a finite sum of Dirac masses. The general goal is then to find a control such that the flow of $f_0 + u(t) f_1$ is as close as possible to the flow of $f_0 + f_1$. Using this interpretation and classical tools from control theory, we revisit well-known results concerning numerical splitting methods, and we prove a handful of new ones, with an emphasis on splittings with additional positivity conditions on the coefficients. First, we show that there exist numerical schemes of any arbitrary order involving only forward flows of f_0 if one allows complex coefficients for the flows of f_1. Equivalently, for complex-valued controls, we prove that the Lie algebra rank condition is equivalent to the small-time local controllability of a system. Second, for real-valued coefficients, we show that the well-known order restrictions are linked with so-called "bad" Lie brackets from control theory, which are known to yield obstructions to small-time local controllability. We use our recent basis of the free Lie algebra to precisely identify the conditions under which high-order methods exist.

While backward error analysis does not generalise straightforwardly to the strong and weak approximation of stochastic differential equations, it extends for the sampling of ergodic dynamics. The calculation of the modified equation relies on tedious calculations and there is no expression of the modified vector field, in opposition to the deterministic setting. We uncover in this paper [25] the Hopf algebra structures associated to the laws of composition and substitution of exotic aromatic S-series, relying on the new idea of clumping. We use these algebraic structures to provide the algebraic foundations of stochastic numerical analysis with S-series, as well as an explicit expression of the modified vector field as an exotic aromatic B-series.

In [14], we consider a particular discretization of the harmonic oscillator which admits an orthogonal basis of eigenfunctions called Kravchuk functions possessing appealing properties from the numerical point of view. We analytically prove the almost second-order convergence of these discrete functions towards Hermite functions, uniformly for large numbers of modes. We then describe an efficient way to simulate these eigenfunctions and the corresponding transformation. We finally show some numerical experiments corroborating our different results.

In [32], we generalize the spectral concentration problem as formulated by Slepian, Pollak and Landau in the 1960s. We show that a generalized version with arbitrary space and Fourier masks is well-posed, and we prove some new results concerning general quadratic domains and gaussian filters. We also propose a more general splitting representation of the spectral concentration operator allowing to construct quasi-modes in some situations. We then study its discretization and we illustrate the fact that standard eigen-algorithms are not robust because of a clustering of eigenvalues. We propose a new alternative algorithm that can be implemented in any dimension and for any domain shape, and that gives very efficient results in practice.

In [12], we focus on an adaptation of the method described in a previous in order to deal with source term in the 2D Euler equations. This method extends classical 1D solvers (such as VFFC, Roe, Rusanov) to the two-dimensional case on unstructured meshes. The resulting schemes are said to be composite as they can be written as a convex combination of a purely node-based scheme and a purely edge-based scheme. We combine this extension with the ideas developed by Alouges, Ghidaglia and Tajchman in an unpublished work – focused mainly on the 1D case – and we propose two attempts at discretizing the source term of the Euler equations in order to better preserve stationary solutions. We compare these discretizations with the “usual” centered discretization on several numerical examples.

The aromatic bicomplex is an algebraic tool based on aromatic Butcher trees and used in particular for the explicit description of volume-preserving affine-equivariant numerical integrators. The present work [17]defines new tools inspired from variational calculus such as the Lie derivative, different concepts of symmetries, and Noether’s theory in the context of aromatic forests. The approach allows to draw a correspondence between aromatic volumepreserving methods and symmetries on the Euler-Lagrange complex, to write Noether’s theorem in the aromatic context, and to describe the aromatic B-series of

volume-preserving methods explicitly with the Lie derivative.

8.3 Analysis of SPDEs

Participants: G. Beck, F. Castella, A. Debussche, E. Faou.

In view of the construction of efficient multiscale numerical schemes, the study and analysis of PDEs or SPDEs is of great importance. Below is a list of some contributions of the team on this aspect.

In [9], we prove that the stochastic Nonlinear Schrödinger (NLS) equation is the limit of NLS equation with random potential with vanishing correlation length. We generalize the perturbed test function method to the context of dispersive equations. Apart from the difficulty of working in infinite dimension, we treat the case of random perturbations which are not assumed uniformly bounded.

In [19], we study a model describing the interaction between waves at the surface of a sea and a freely floating partially immersed cylinder in the Boussinesq regime. This wave-interaction system can be reduced to an initial boundary value problem for the Boussinesq equations in an exterior domain. The boundary condition depends on the motion of the floating cylinder which is in turn determined by a nonlinear ODE with forcing terms coming from the exterior wave field.

In [21], we study how relaxing the classical hydrostatic balance hypothesis affects theoretical aspects of the LU primitive equations well-posedness. We focus on models that sit between incompressible 3D LU Navier-Stokes equations and standard LU primitive equations, aiming for numerical manageability while capturing non-hydrostatic phenomena. Our main result concerns the well-posedness of a specific stochastic interpretation of the LU primitive equations. This holds with rigid-lid type boundary conditions, and when the horizontal component of noise is independent of z. In fact these conditions can be related to the dynamical regime in which the primitive equations remain valid. Moreover, under these conditions, we show that the LU primitive equations solution tends toward the one of the deterministic primitive equations for a vanishing noise, thus providing a physical coherence to the LU stochastic mode.

Motivated by the modeling of the spatial structure of the velocity field of three-dimensional turbulent flows, and the phenomenology of cascade phenomena, a linear dynamics was recently proposed that can generate high velocity gradients from a smooth-in-space forcing term. It is based on a linear partial differential equation stirred by an additive random forcing term that is δ-correlated in time. The underlying proposed deterministic mechanism corresponds to a transport in Fourier space that aims to transfer energy injected at large scales towards small scales. The key role of the random forcing is to realize these transfers in a statistically homogeneous way. Whereas at finite times and positive viscosity the solutions are smooth, a loss of regularity is observed for the statistically stationary state in the inviscid limit. We present in this work [11] some simulations, based on finite volume methods in the Fourier domain and a splitting method in time, which are more accurate than the pseudospectral simulations. We show that our algorithm is able to reproduce accurately the expected local and statistical structure of the predicted solutions. We conduct numerical simulations in one, two, and three spatial dimensions, and we display the solutions both in physical and Fourier space. We additionally display key statistical quantities such as second-order structure functions and power spectral densities at various viscosities.

In [20], we consider a stochastic nonlinear formulation of classical coastal waves models under location uncertainty (LU). In the formal setting investigated here, stochastic versions of the Serre-Green-Nagdi, Boussinesq and classical shallow water wave models are obtained through an asymptotic expansion, which is similar to the one operated in the deterministic setting. However, modified advection terms emerge, together with advection noise terms. These terms are well-known features arising from the LU formalism, based on momentum conservation principle.

In [23], We study the convergence of a Zakharov system driven by a time white noise, colored in space, to a multiplicative stochastic nonlinear Schrödinger equation, as the ion-sound speed tends to infinity. In the absence of noise, the conservation of energy gives bounds on the solutions, but this evolution becomes singular in the presence of the noise. To overcome this difficulty, we show that the problem may be recasted in the diffusion-approximation framework, and make use of the perturbed test-function method. We also obtain convergence in probability. The result is limited to dimension one, to avoid too

much technicalities. As a prerequisite, we prove the existence and uniqueness of regular solutions of the stochastic Zakharov system.

In [30], we address a slow-fast system of coupled three dimensional Navier-Stokes equations where the fast component is perturbed by an additive Brownian noise. By means of the rough path theory, we establish the convergence in law of the slow component towards a Navier-Stokes system with an Itô-Stokes drift and a rough path driven transport noise. This gives an alternative, more general and direct proof to a former paper. Notably, the limiting rough path is identified as a geometric rough path, which does not necessarily coincide with the Stratonovich lift of the Brownian motion.

In [31], we continue some investigations on the periodic NLSE started by Lebowitz, Rose and Speer and by Bourgain with the addition of a distributional multiplicative potential. We prove that the equation is globally wellposed for a set of data of full normalized Gibbs measure, after suitable truncation in the focusing case. The set and the measure are invariant under the flow. The main ingredients used are Strichartz estimates on periodic NLS with distributional potential to obtain local well-posedness for low regularity initial data.

In [26], we consider the moments and the distribution of hitting times on the lollipop graph which is the graph exhibiting the maximum expected hitting time among all the graphs having the same number of nodes. We obtain recurrence relations for the moments of all order and we use these relations to analyze the asymptotic behavior of the hitting time distribution when the number of nodes tends to infinity.

In [27], we consider the moments and the distribution of the hitting and cover times of a random walk in the complete graph. We study both the time needed to reach any subset of states and the time needed to visit all the states of a subset at least once. We obtain recurrence relations for the moments of all orders and we use these relations to analyze the asymptotic behavior of the hitting and cover times distributions when the number of states tends to infinity.

9 Bilateral contracts and grants with industry

Participants: E. Faou.

9.1 Bilateral contracts with industry

Participants: E. Faou.

- Contract with the Cailabs compagny.
  A long standing collaboration has emerged between MINGuS and the company CAILABS whose main aim is the conception and construction of optical fibers. Most of the main objectives of this collaboration are strictly confidential. However they have strong common point with the scientific goals of the MINGuS project, for instance the development of efficient numerical methods for quantum simulation and many aspects of mathematical and physical analysis of quantum systems. The impact of this collaboration are very important both from the transfer of technological points of view and from the interaction with a very active startup providing very practical problems that are often very close to hot academic topics. We believe that this interaction will last long and continue to feed the scientific activity of the whole project with problem directly coming from the industrial and economical worlds.
- Agence Lebesgue.

  Participants: E. Faou.

Since 2019 and up to August 2023, E. Faou was head of the *Agence Lebesgue pour les mathématiques* whose role is precisely to increase the role of mathematics in the socio-economic world by facilitating contact between mathematicians and companies or institute working in distant sector of activity.

An important activity of the Agence Lebesgue is formation where mathematicians go to industries, companies of the private sector or other institutes to organize some crash course in some hot topics in mathematics, or on demand depending on the requirement of the partners.

10 Partnerships and cooperations

10.1 International initiatives

10.1.1 Inria associate team not involved in an IIL or an international program

Bubbles

Title: Bubbles and modulations for solving Schroedinger equations

Duration: 2022 → 2026

Coordinator: Pierre Raphaël (pr463@cam.ac.uk)

Partners:

- University of Cambridge Cambridge (Royaume-Uni)

Inria contact: Erwan Faou

Summary: Schroedinger equations: long time analysis, weak turbulence and numerical integrators using modulations.

10.1.2 Participation in other International Programs

SIMONS project E. Faou is PI of the Simons collaboration on wave turbulence gathering mathematicians and physicists from New-York University, ENS Paris, ENS Lyon and Torino.

CNRS International Research Project PICASSO L. Martaud is a member of the CNRS IRP PICASSO between french and spanish researchers. The kickoff meeting is scheduled for the end of march. See .

10.2 International research visitors

The MINGuS members welcame and visited several international laboratories.

10.2.1 Visits of international scientists

The MINGuS received some international colleagues

- Megala Anandan (PhD, IIS Bangalore), february to may
- Tomas Morales (associate professor, Universidad de Málaga), 1-4 July
- Saurav Samantaray (postdoc, IIT Chennai), 10-22 july
- K. Arun (associate professor, IISER Thiruvanatapuram) 15-30 november
- Liu Liu (associate professor, CUHK, Hong Kong), 7-12 december.
- Yassine Tahraoui (associate professor, SNS, Pisa), 25-29 november.

10.2.2 Visits to international teams

Research stays abroad

N. Crouseilles

Visited institution: UW Madison

Country: US

Dates: february 2024

Context of the visit: collaboration with Q. Li

Mobility program/type of mobility: (sabbatical, internship, research stay, lecture...) research stay

A. Laurent

Visited institution: Bergen

Country: Norway

Dates: December 2024

Context of the visit: collaboration with H. Munthe-Kaas

Mobility program/type of mobility: (sabbatical, internship, research stay, lecture...) research stay

A. Laurent

Visited institution: Oxford

Country: UK

Dates: October 2024

Context of the visit: collaboration with G. Maierhofer

Mobility program/type of mobility: (sabbatical, internship, research stay, lecture...) research stay

A. Laurent

Visited institution: Taipei

Country: Taiwan

Dates: July 2024

Context of the visit: collaboration with M. Sutti

Mobility program/type of mobility: (sabbatical, internship, research stay, lecture...) research stay

A. Laurent

Visited institution: Geneva

Country: Switzerland

Dates: April 2024

Context of the visit: collaboration with G. Vilmart

Mobility program/type of mobility: (sabbatical, internship, research stay, lecture...) research stay

10.3 National initiatives

- 2024: project funded by Fédération de Recherche Fusion par Confinement Magnétique, headed by N. Crouseilles. 5000 euros.

  This project is focused on the design of numerical schemes for tokamak plasmas and involve members of the team but also colleagues from university of Nantes.

  Participants: Nicolas Crouseilles.

- 2019-2024 GdR TRAG on rough path theory.

  Participants: Arnaud Debussche.

  The goal of the TRAG GDR is to gather french mathematicians who work on the rough path theory. GDR TRAG.

- 2023-2027: E. Faou is the PI of the ANR project KEN (Kinetic, PDE and Numerics). The partners are R. Krikorian (Ecole Polytechnique) and B. Grébert (University Nantes). Budget total 391000 euros

  Participants: Nicolas Crouseilles, Erwan Faou.

  The project involved a group in Nantes, Ecole Polytechnique and some MINGuS members. It gathers people from the Dynamical system community, specialists of the analysis of Partial Differential Equations, as well as people coming from the numerical analysis and scientific computing worlds.

- 2023-2027: F. Castella is a member of the ANR project BEEP (Behavioural epidemiology and evolution of plant pathogens) headed by F. Hamelin. The partners are Inrae Rennes and Sophia-Antipolis, CIRAD Montpellier, Univ. Cambridge and Univ. Osnabrueck.

  Participants: Francois Castella.

  The project involved a group in Nantes, Ecole Polytechnique and some MINGuS members. It gathers people from the Dynamical system community, specialists of the analysis of Partial Differential Equations, as well as people coming from the numerical analysis and scientific computing worlds.

- 2023-2027: G. Beck is a member of the ANR project BOURGEONS, headed by A.-L. Dalibard (Sorbonne university).

  Participants: Geoffrey Beck.

- 2023-2027: A. Debussche is a member of the CNRS-MITI project SpatialBioNet.

  Participants: Arnaud Debussche.

  This project focuses on some theoretical challenges and environmental applications on boundary, congestion and vorticity in fluids.

10.4 Regional initiatives

G. Beck obtained an AIS grant from Britanny region council.

11 Dissemination

G. Beck, F. Castella, N. Crouseilles, A. Debussche, E. Faou, A. Laurent, L. Martaud, P. Navaro

11.1 Promoting scientific activities

11.1.1 Scientific events: organisation

International conferences

- G. Beck, N. Crouseilles and A. Laurent organized a workshop (Rennes, France, May 2024) BMS
- G. Beck co-organizes the advanced summer school 'mathematical fluid dynamics' (Cargèse, France, april 2025)

Julia/R events

- P. Navaro organized Julia events webinars
- P. Navaro organized Julia programming school
- P. Navaro organized a R programming school

Seminars

- G. Beck organizes the weekly seminar of the team 'Numerical analysis and modeling' at IRMAR
- E. Faou organizes the IRMAR colloquium at IRMAR.
- A. Laurent co-organizes the weekly seminar 'Mathematics and applications' at ENS Rennes

General chair, scientific chair

- E. Faou was member of the scientific council of the Scicade-2024 conference, June 2024, Singapour

11.1.2 Journal

Member of the editorial boards

- A. Debussche: Editor in chief of "Stochastics and Partial Differential Equations: Analysis and Computations" (2013-)
- A. Debussche: Member of the editorial committee of Annales de l'IHP Probabilités et Statisques (2020-).
- A. Debussche: Member of the editorial committee of Journal of Evolution equation (2014-).
- A. Debussche: Member of the editorial committee of Annales Henri Lebesgue (2018-).
- A. Debussche: Member of the editorial committee of the collection of books: Mathématiques and Applications, Springer.

Reviewer - reviewing activities The MINGuS members do several reviews for the journal in which they publish.

11.1.3 Invited talks

- G. Beck gave a talk at the workshop "5th Stochastic Transport in Upper Ocean Dynamics" (Rennes, France), September.
- G. Beck gave a talk at the conference WAVES 2024 (16th International Conference on Mathematical and Numerical Aspects of Wave Propagation) (Brlin, Germany), July.
- G. Beck gave a talk at the conference CEA-SMAI/GAMNI (Paris, France), January.
- N. Crouseilles gave a talk at the Vlasovia conference (Florence, Italy), february.
- N. Crouseilles gave a talk at the IMSI conference (Chicago, US), february.
- N. Crouseilles gave a talk at the NumKin workshop (Munich, Germany), november.
- A. Debussche gave a talk at the workshop on Stochastic Partial Differential Equations (Vienne, Austria), february.
- A. Debussche gave a talk at the 3rd school and workshop "Hamiltonian and dispersive PDE" (Milan, Italy), september.
- A. Debussche gave a talk at the 5th workshop "Stochastic Transport in Upper Ocean Dynamics" (Rennes, France), september.
- E. Faou gave a talk at the conference 'Nonlinear wave and Hamiltonian PDEs' (Courmayer, Italy), february.
- E. Faou gave a talk at the workshop 'Molecular dynamics' (Angers, France), february.
- E. Faou gave a talk at the webinar event 'Kinetic and fluid equations for collective behavior', organized by the French-Korean IRL, may.
- E. Faou gave a talk at the workshop 'Singularity formation for nonlinear PDE' (Cambridge, UK), september.
- E. Faou gave a talk at the Simons Foundations workshop on wave turbulence (New-York, US), december.
- A. Laurent gave a talk at the Lie-Stormer colloquium (Tromso, Norway), december.
- A. Laurent gave a talk at the "rencontres de Besse, Besse-et-Saint-Anastaise" (Besse, France), october.
- A. Laurent gave a talk at the NUMDIFF-17 conference (Halle, Germany), september.
- A. Laurent gave a talk at the 'journées MAS' (Poitiers, France), august.
- A. Laurent gave a talk at the Scicade conference (Singapore), july.
- A. Laurent gave a talk at the Workshop "Constrained Dynamics, Stochastic Numerical Methods and the Modeling of Complex Systems" (Oberwolfach, Germany), may. See [18]

11.1.4 Leadership within the scientific community

- N. Crouseilles is head of the MINGuS Inria team.
- A. Debussche is co-head of the Labex Lebesgue (since July 2023).

11.1.5 Scientific expertise

- N. Crouseilles was member of the hiring committee of the 'MCF' position MCF1599 at university of Toulouse.
- N. Crouseilles was member of the hiring committee of the CPJ position at university of Rennes 2.
- A. Debussche was member of the HCERES committee for the evaluation of the Laboratory Jacques-Louis Lions (Paris).
- E. Faou was president of the HCERES committee for the evaluation of the Laboratory CERMICS (Marne-la-Vallée).
- P. Navaro was member of the hiring committee of CNRS research engineer position.

11.1.6 Research administration

- N. Crouseilles is member of the IRMAR council.
- N. Crouseilles is member of the "comité des projets" of Inria Rennes.
- N. Crouseilles is responsible for the university of Rennes of the Fédération de recherche pour la fusion confinement magnétique.
- N. Crouseilles is member of the "comité' de centre Inria de Rennes" and of the bureau.
- N. Crouseilles participated to the working group "chantier simplifications" between IRMAR laboratory and Inria center of Rennes.
- A. Debussche is member of the administration council of ENS Rennes.
- A. Debussche is member of the External Advisory Board du projet ERC STUOD.
- A. Debussche is member of the steering comittee of the EUR "Digisport", Rennes.
- A. Debussche is member of the steering comittee of the cofund "MathPhDInFrance" (FSMP).
- P. Navaro animates the Julia french community at CNRS (see newsletters).

11.2 Teaching - Supervision - Juries

11.2.1 Teaching

The members of team are involved in teaching activities. We list below the Master courses only.

- G. Beck, Dynamical system with noise, 40 hours, Master 1, University of Rennes.
- G. Beck, Wave propagation, 27 hours, Master 2, University of Rennes.
- F. Castella, transport equations, 40 hours, Master 1, University of Rennes.
- F. Castella, Boltzmann equation as a limit of a N-particles system, 36 hours, Master 2, University of Rennes.
- N. Crouseilles, Numerical methods for PDEs, 24 hours, Master 1, ENS Rennes.
- A. Debussche, Calcul stochastique, 36 hours, Master 2, University Rennes.
- A. Laurent, Geometric numerical integration, 18 hours, Master 2, University of Rennes.

11.2.2 Supervision

- G. Beck and L. Martaud coadvised E. Contentin (master 2 intern)
- G. Beck and E. Faou coadvise E. Contentin (PhD)
- F. Castella coadvised H. Martin (postdoc, with F. Hamelin). Left in september 2024, he joined the EHESP (Rennes) as an engineer.
- F. Castella coadvises Y. Fotso-Fotso (PEPR postdoc, with F. Hamelin).
- F. Castella advised M. Bouchereau (PhD). Defended in 2024, now postdoc at CEA-Saclay.
- N. Crouseilles coadvised D. Prel (PhD). Defended in 2024, now teacher at University of Nantes.
- N. Crouseilles and E. Faou coadvised Y. Le Hénaff (PhD). Defended in 2024, now postdoc in Tubingen (Germany).
- N. Crouseilles and A. Laurent coadvised M. de Penguern (master intern).
- A. Debussche coadvised B. Hug (PhD, with E. Mémin). Defended in december 2024, now teacher in high school.
- A. Debussche coadvises A. Moneyron (PhD, with E. Mémin).
- A. Debussche coadvises S. Moskowitz (PhD, with E. Mémin).
- A. Debussche advised R. Nader (postdoc). Left in september 2024, she joined an engineer school in Cergy.
- E. Faou advises T. Seetohul (master 2 intern and PhD).
- A. Laurent advised D. Coudière (master intern).

11.2.3 Juries

- N. Crouseilles and E. Faou were members of the PhD thesis committee of Y. Le Hénaff, Rennes (june 2024).
- N. Crouseilles was president of the PhD thesis committee of F. Tucciarone, Rennes (june 2024).
- N. Crouseilles was reviewer of the PhD thesis of A. Chrisment, Bordeaux (July 2024).
- N. Crouseilles was reviewer of the HdR of J. Chabassier, Pau (July 2024).
- N. Crouseilles was member of the PhD thesis of E. Lehmann, Toulouse (July 2024).
- N. Crouseilles was member of the PhD thesis of M. Anandan, Bangalore, India (July 2024).
- N. Crouseilles was president of the PhD committee of T. Laidin, Lille (September 2024).
- N. Crouseilles was member of the PhD committee of D. Prel, Nantes (November 2024).
- N. Crouseilles was reviewer of the PhD committee of L. Reboul, Palaiseau (December 2024).
- A. Debussche was member of the committee of H. Eulry, Rennes (June 2024).
- A. Debussche was member of the committee of B. Hug, Rennes (December 2024).
- E. Faou was reviewer of the HdR of R. Duboscq, Toulouse (February 2024).
- E. Faou was member of the PhD thesis of C. Abou Khalil, Nantes (September 2024).
- E. Faou was president of the PhD committee of R. Bug, Rennes (December 2024).

11.3 Popularization

- G. Beck, N. Crouseilles, A. Laurent, P. Navaro collaborated with Thomas Menuet (musician, who spent two years at the IRMAR laboratory) to create an enigma combining music and mathematics. More informations can be found Résidence Thomas Menuet but also in OuestFrance and in canalb
- A. Laurent gave a talk at the 5' Lebesgue
- A. Laurent participated at 'la fête de la science' in Rennes.
- A. Laurent participated at the 'club Rennes et Maths'
- A. Laurent participated at the livestorm on the scientific outreach dedicated to PhD students
- A. Laurent participated to the event 'les Marmottes'

12 Scientific production

12.1 Major publications

[1] J. Bernier, E. Faou and B. Grebert. 'Long time behavior of the solutions of NLW on the d-dimensional torus'. In: *Forum of Mathematics, Sigma* 8 (2020), E12. DOI: 10.1017/fms.2020.8. URL: https://hal.archives-ouvertes.fr/hal-02151338.

[2] F. Casas, N. Crouseilles, E. Faou and M. Mehrenberger. 'High-order Hamiltonian splitting for Vlasov-Poisson equations'. In: *Numerische Mathematik* 135.3 (2017), pp. 769–801. DOI: 10.1007/s00211-016-0816-z. URL: https://hal.inria.fr/hal-01206164 (cit. on p. 4).

[3] P. Chartier, N. Crouseilles, M. Lemou, F. Méhats and X. Zhao. 'Uniformly accurate methods for three dimensional Vlasov equations under strong magnetic field with varying direction'. In: *SIAM Journal on Scientific Computing* 42.2 (2020), B520–B547. DOI: 10.1137/19M127402X. URL: https://hal.archives-ouvertes.fr/hal-02179534 (cit. on p. 4).

[4] N. Crouseilles, P.-A. Hervieux, Y. Li, G. Manfredi and Y. Sun. 'Geometric Particle-in-Cell methods for the Vlasov-Maxwell equations with spin effects'. In: *Journal of Plasma Physics* 87.3 (28th May 2021), article n° 825870301. DOI: 10.1017/S0022377821000532. URL: https://hal.inria.fr/hal-03148534 (cit. on p. 4).

[5] A. Debussche and J. Vovelle. 'Diffusion-approximation in stochastically forced kinetic equations'. In: *Tunisian Journal of Mathematics* 3.1 (2021), pp. 1–53. DOI: 10.2140/tunis.2021.3.1. URL: https://hal.archives-ouvertes.fr/hal-01567138 (cit. on p. 4).

[6] E. Faou. 'Linearized wave turbulence convergence results for three-wave systems'. In: *Communications in Mathematical Physics* 378.2 (Sept. 2020), pp. 807–849. DOI: 10.1007/s00220-020-03799-w. URL: https://hal.inria.fr/hal-01801810 (cit. on p. 4).

12.2 Publications of the year

International journals

[7] K. Arun, N. Crouseilles and S. Samantaray. 'High order asymptotic preserving and classical semi-implicit RK schemes for the euler-poisson system in the quasineutral limit'. In: *Journal of Scientific Computing* 100 (6th June 2024), p. 24. DOI: 10.1007/s10915-024-02577-3. URL: https://hal.science/hal-03783065 (cit. on p. 13).

[8] B. Bakry, N. Crouseilles, P.-A. Hervieux, X. Hong and G. Manfredi. 'Ultrafast dynamics of a spin-polarized electron plasma with magnetic ions'. In: *Journal of Plasma Physics* (2024), pp. 1–38. URL: https://hal.science/hal-04844507. In press (cit. on p. 14).

[9] G. Barrué, A. Debussche and M. Tusseau. 'Approximation diffusion for the nonlinear Schrödinger equation with a random potential'. In: *Asymptotic Analysis* (2024), pp. 1–42. DOI: 10.3233/asy-241894. URL: https://hal.science/hal-04383170. In press (cit. on p. 16).

[10] G. Beck and A. Beni Hamad. 'Electromagnetic waves propagation in thin heterogenous coaxial cables. Comparison between 3D and 1D models'. In: *AIMS Mathematics* 9.4 (2024), pp. 8981–9019. DOI: 10.3934/math.2024438. URL: https://hal.science/hal-04534336 (cit. on p. 13).

[11] G. Beck, C.-E. Bréhier, L. Chevillard, R. Grande and W. Ruffenach. 'Numerical simulations of a stochastic dynamics leading to cascades and loss of regularity: Applications to fluid turbulence and generation of fractional Gaussian fields'. In: *Physical Review Research* 6.3 (2024), p. 033048. DOI: 10.1103/PhysRevResearch.6.033048. URL: https://hal.science/hal-04651190 (cit. on p. 16).

[12] M. Boujoudar, E. Franck, P. Hoch, C. Lasuen, Y. Le Henaff and P. Paragot. 'A composite finite volume scheme for the Euler equations with source term on unstructured meshes'. In: *ESAIM: Proceedings and Surveys* (2024), pp. 1–22. URL: https://hal.science/hal-04543142. In press (cit. on p. 15).

[13] B. Boutin, A. Crestetto, N. Crouseilles and J. Massot. 'Modified Lawson methods for Vlasov equations'. In: *SIAM Journal on Scientific Computing* 46.3 (8th May 2024), A1574–A1598. DOI: 10.1137/22M154301X. URL: https://hal.science/hal-03911409 (cit. on p. 13).

[14] Q. Chauleur and E. Faou. 'Discrete quantum harmonic oscillator and kravchuk transform'. In: *ESAIM: Mathematical Modelling and Numerical Analysis* 58.6 (4th Dec. 2024), pp. 2155–2186. DOI: 10.1051/m2an/2024001. URL: https://hal.science/hal-03885282 (cit. on p. 15).

[15] N. Crouseilles and X. Hong. 'Exponential DG methods for Vlasov equations'. In: *Journal of Computational Physics* 498 (Feb. 2024), pp. 1–38. DOI: 10.1016/j.jcp.2023.112682. URL: https://hal.science/hal-04389308 (cit. on p. 13).

[16] X. Hong and J.-M. Qiu. 'A Conservative Eulerian–Lagrangian Runge–Kutta Discontinuous Galerkin Method for Linear Hyperbolic System with Large Time Stepping'. In: *Journal of Scientific Computing* 98.3 (17th Feb. 2024), p. 70. DOI: 10.1007/s10915-024-02452-1. URL: https://hal.science/hal-04500405 (cit. on p. 13).

[17] A. Laurent. 'The Lie derivative and Noether's theorem on the aromatic bicomplex for the study of volume-preserving numerical integrators'. In: *Journal of Computational Dynamics* 11.1 (2024), pp. 10–22. DOI: 10.3934/jcd.2023011. URL: https://hal.science/hal-04347291 (cit. on p. 15).

International peer-reviewed conferences

[18] A. Laurent. 'High order integration of stochastic dynamics in Rd, on manifolds, and in the neighbourhood of manifolds'. In: 2024 - Workshop on Constrained Dynamics, Stochastic Numerical Methods and the Modeling of Complex Systems. Oberwolfach, Germany: Mathematisches Forschungsinstitut Oberwolfach, 2024. DOI: 10.14760/OWR-2024-26. URL: https://hal.science/hal-04710249 (cit. on p. 22).

Conferences without proceedings

[19] E. Contentin, G. Beck and L. Martaud. 'Freely floating cylinder on a 3D fluid governed by the Boussinesq equations'. In: WAVES 2024 - 16th International Conference on Mathematical and Numerical Aspects of Wave Propagation. Berlin, Germany, 2024, pp. 1–2. URL: https://hal.science/hal-04847860 (cit. on p. 16).

Scientific book chapters

[20] A. Debussche, É. Mémin and A. Moneyron. 'Derivation of stochastic models for coastal waves'. In: *Stochastic Transport in Upper Ocean Dynamics III*. Springer nature, 2nd July 2024, pp. 183–222. URL: https://inria.hal.science/hal-04632538 (cit. on p. 16).

[21] A. Debussche, É. Mémin and A. Moneyron. 'Some properties of a non-hydrostatic stochastic oceanic primitive equations model'. In: *Stochastic Transport in Upper Ocean Dynamics III*. Springer Nature, 2nd July 2024, pp. 161–182. URL: https://inria.hal.science/hal-04632530 (cit. on p. 16).

Reports & preprints

[22] M. Anandan, B. Boutin and N. Crouseilles. *Uniformly higher order accurate schemes for dynamics of charged particles under fast oscillating magnetic fields*. 2024. URL: https://inria.hal.science/hal-04773938 (cit. on p. 14).

[23] G. Barrué, A. de Bouard and A. Debussche. *From Stochastic Zakharov System to Multiplicative Stochastic Nonlinear Schrödinger Equation*. 20th Sept. 2024. URL: https://hal.science/hal-04704413 (cit. on p. 16).

[24] K. Beauchard, A. Laurent and F. Marbach. *Control theory and splitting methods*. 2nd July 2024. URL: https://hal.science/hal-04710226 (cit. on p. 15).

[25] E. Bronasco and A. Laurent. *Hopf algebra structures for the backward error analysis of ergodic stochastic differential equations*. 10th July 2024. URL: https://hal.science/hal-04710238 (cit. on p. 15).

[26] F. Castella and B. Sericola. *Hitting times on the lollipop graph*. Centre Inria de l'université de Rennes, 13th Dec. 2024, pp. 1–30. URL: https://inria.hal.science/hal-04143403 (cit. on p. 17).

[27] F. Castella and B. Sericola. *Random walk in the complete graph : hitting and cover times*. Centre Inria, 17th Dec. 2024, p. 27. URL: https://inria.hal.science/hal-04365696 (cit. on p. 17).

[28] N. Crouseilles, G. Dimarco and S. Samantaray. *High order Asymptotic-Preserving penalized numerical schemes for the Euler-Poisson system in the quasineutral limit*. 2024. URL: https://hal.science/hal-04844514 (cit. on p. 13).

[29] N. Crouseilles and C. Negulescu. *Hybrid modelling of energetic α-particles interacting with the thermal bulk plasma*. 14th Dec. 2024. URL: https://hal.science/hal-04844525 (cit. on p. 14).

[30] A. Debussche and M. Hofmanová. *Rough analysis of two scale systems*. 18th Dec. 2024. DOI: 10.48550/arXiv.2306.15781. URL: https://hal.science/hal-04846233 (cit. on p. 17).

[31] A. Debussche and A. Mouzard. *Periodic nonlinear Schrödinger equation with distributional potential and invariant measures*. 1st May 2024. URL: https://hal.science/hal-04565547 (cit. on p. 17).

[32] E. Faou and Y. Le Henaff. *A generalized spectral concentration problem and the varying masks algorithm*. 2024. URL: https://hal.science/hal-04718024 (cit. on p. 15).

12.3 Cited publications

[33] C. Birdsall and A. Langdon. *Plasmas physics via computer simulations*. New York: Taylor and Francis, 2005 (cit. on p. 6).

[34] A. Brizard and T. Hahm. 'Foundations of nonlinear gyrokinetic theory'. In: *Reviews of Modern Physics* 79 (2007) (cit. on p. 7).

[35] J. Carr. 'Applications of Centre Manifold Theory'. In: *Applied Mathematical Sciences Series* 35 (1981) (cit. on p. 5).

[36] P. Chartier, N. Crouseilles, M. Lemou and F. Méhats. 'Uniformly accurate numerical schemes for highly-oscillatory Klein-Gordon and nonlinear Schrödinger equations'. In: *Numer. Math.* 129 (2015), pp. 513–536 (cit. on pp. 6, 8).

[37] P. Chartier, A. Murua and J. Sanz-Serna. 'Higher-order averaging, formal series and numerical integration III: error bounds'. In: *Foundation of Comput. Math.* 15 (2015), pp. 591–612 (cit. on p. 5).

[38] A. Debussche and J. Vovelle. 'Diffusion limit for a stochastic kinetic problem'. In: *Commun. Pure Appl. Anal.* 11 (2012), pp. 2305–2326 (cit. on p. 5).

[39] E. Faou and F. Rousset. 'Landau damping in Sobolev spaces for the Vlasov-HMF model'. In: *Arch. Ration. Mech. Anal.* 219 (2016), pp. 887–902 (cit. on p. 7).

[40] E. Hairer, C. Lubich and G. Wanner. *Geometric Numerical Integration. Structure-Preserving Algorithms for Ordinary Differential Equations, Second edition*. Springer Series in Computational Mathematics 31. Berlin: Springer, 2006 (cit. on p. 9).

[41] S. Jin and H. Lu. 'An Asymptotic-Preserving stochastic Galerkin method for the radiative heat transfer equations with random inputs and diffusive scalings'. In: *J. Comp. Phys.* 334 (2017), pp. 182–206 (cit. on p. 6).

[42] M. Lemou, F. Méhats and P. Raphaël. 'Orbital stability of spherical galactic models'. In: *Invent. Math.* 187 (2012), pp. 145–194 (cit. on p. 7).

[43] C. Mouhot and C. Villani. 'On Landau damping'. In: *Acta Math.* 207 (2011), pp. 29–201 (cit. on p. 7).

[44] S. Nazarenko. *Wave turbulence*. Springer-Verlag, 2011 (cit. on p. 8).

[45] L. Perko. 'Higher order averaging and related methods for perturbed periodic and quasi-periodic systems'. In: *SIAM J. Appl. Math.* 17 (1969), pp. 698–724 (cit. on pp. 7, 9).