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ACTIVITY REPORT

Project-Team PARADYSE

PARticles And DYnamical SystEms

IN COLLABORATION WITH: Laboratoire Paul Painlevé (LPP)

DOMAIN

Applied Mathematics, Computation and Simulation

THEME

Numerical schemes and simulations

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Project-Team PARADYSE

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Keywords

Computer sciences and digital sciences

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1 Team members, visitors, external collaborators

Research Scientists

- Guillaume Dujardin [Team leader, INRIA, Senior Researcher]
- Quentin Chauleur [INRIA, ISFP, from Oct 2024]
- Guillaume Ferriere [INRIA, Researcher]

Faculty Members

- Vianney Combet [UNIV LILLE, Associate Professor Delegation, from Sep 2024]
- Stephan De Bièvre [UNIV LILLE, Professor Delegation]
- André De Laire Peirano [UNIV LILLE, Associate Professor, from Sep 2024, HDR]
- André De Laire Peirano [UNIV LILLE, Associate Professor Delegation, until Aug 2024, HDR]
- Olivier Goubet [UNIV LILLE, Professor]

Post-Doctoral Fellows

- Matthieu Arnhem [UNIV LILLE, Post-Doctoral Fellow]
- Quentin Chauleur [UNIV LILLE, Post-Doctoral Fellow, until Aug 2024]
- Gabriel Silva Nahum [ANR, Post-Doctoral Fellow, from Mar 2024]

PhD Students

- Abbas El Hajj [IFPEN, CIFRE, from Nov 2024]
- Christopher Langrenez [UNIV LILLE]
- Erwan Le Quiniou [UNIV LILLE]
- Gabriel Silva Nahum [Instituto superior técnico (Lisbonne), until Feb 2024]
- Mateo Spriet [CNRS, from Oct 2024]
- Sebastian Tapia Mandiola [INRIA, from Oct 2024]
- Celine Wang [UNIV LILLE]

Technical Staff

- Alexandre Roget [INRIA, Engineer, until Feb 2024]

Interns and Apprentices

- Alonso Salvador Carrasco Urbina [INRIA, Intern, until Mar 2024]
- Sebastian Tapia Mandiola [UNIV LILLE, Intern, until Mar 2024]

Administrative Assistant

- Lucille Leclercq [INRIA]

Visiting Scientist

- Imen Manoubi [UNIV GABES, from May 2024 until Jun 2024]

2 Overall objectives

The PARADYSE team gathers mathematicians from different communities with the same motivation: to provide a better understanding of dynamical phenomena involving particles. These phenomena are described by fundamental models arising from several fields of physics. We shall focus on model derivation, study of stationary states and asymptotic behaviors, as well as links between different levels of description (from microscopic to macroscopic) and numerical methods to simulate such models. Applications include non-linear optics, thermodynamics and ferromagnetism. Research in this direction has a long history, that we shall only partially describe in the sequel. We are confident that the fact that we come from different mathematical communities (PDE theory, mathematical physics, probability theory and numerical analysis), as well as the fact that we have strong and effective collaborations with physicists, will bring new and efficient scientific approaches to the problems we plan to tackle and will make our team strong and unique in the scientific landscape. Our goal is to obtain original and important results on a restricted yet ambitious set of problems that we develop in this document.

3 Research program

3.1 Time asymptotics: Stationary states, solitons, and stability issues

The team investigates the existence of *solitons* and their link with the global dynamical behavior for non-local problems such as the Gross–Pitaevskii (GP) equation which arises in models of dipolar gases. These models, in general, also introduce non-zero boundary conditions which constitute an additional theoretical and numerical challenge. Numerous results are proved for local problems, and numerical simulations allow to verify and illustrate them, as well as making a link with physics. However, most fundamental questions are still open at the moment for non-local problems.

The non-linear Schrödinger (NLS) equation finds applications in numerous fields of physics. We concentrate, in a continued collaboration with our colleagues from the physics department (PhLAM) at Université de Lille (U-Lille) in the framework of the Laboratoire d'Excellence CEMPI, on its applications in non-linear optics and cold atom physics. Issues of orbital stability and modulational instability are central here (see Section 4.1 below).

Another typical example of problem that the team wishes to address concerns the Landau–Lifshitz (LL) equation, which describes the dynamics of the spin in ferromagnetic materials. This equation is a fundamental model in the magnetic recording industry [53] and solitons in magnetic media are of particular interest as a mechanism for data storage or information transfer [54]. It is a quasilinear PDE involving a function that takes values on the unit sphere $\mathbb{S}^2$ of $\mathbb{R}^3$. Using the stereographic projection, it can be seen as a quasilinear Schrödinger equation and the questions about the solitons, their dynamics and potential blow-up of solutions evoked above are also relevant in this context. This equation is less understood than the NLS equation: even the Cauchy theory is not completely understood [44, 52]. In particular, the geometry of the target sphere imposes that the solution has a norm equal to one everywhere, so in particular the boundary conditions cannot be zero, and, even in dimension one, there are kink-type solitons having different limits at $\pm\infty$.

3.2 Derivation of macroscopic laws from microscopic dynamics

The team investigates, from a microscopic viewpoint, the dynamical mechanism at play in the phenomenon of relaxation towards thermal equilibrium for large systems of interacting particles. For instance, a first step consists in giving a rigorous proof of the fact that a particle repeatedly scattered by random obstacles through a Hamiltonian scattering process will eventually reach thermal equilibrium, thereby completing previous works in this direction by the team. As a second step, models similar to the ones considered classically will be defined and analyzed in the quantum mechanical setting, and more particularly in the setting of quantum optics.

Another challenging problem is to understand the interaction of large systems with the boundaries, which is responsible for most energy exchanges (forcing and dissipation), even though it is concentrated in very thin layers. The presence of boundary conditions to evolution equations sometimes lacks understanding from a physical and mathematical point of view. In order to legitimate the choice done at the macroscopic level of the mathematical definition of the boundary conditions, we investigate systems of particles with different local interactions. We apply various techniques to understand how diffusive and driven systems interact with the boundaries.

Finally, we aim at obtaining results on the macroscopic behavior of large scale interacting particle systems subject to kinetic constraints. In particular, we study the behavior in one and two dimensions of the Facilitated Exclusion Process (FEP), on which several results have already been obtained. The latter is a very interesting prototype for kinetically constrained models because of its unique mathematical features (explicit stationary states, absence of mobile cluster to locally shuffle the configuration). There are very few mathematical results on the FEP, which was put forward by the physics community as a toy model for phase separation.

Our goal is to develop collaboration at the interface between probability and PDE theory, and use the rich PDE background of the team to provide tools to be used on statistical physics problems put forward by the probability side of the team.

3.3 Numerical methods: analysis and simulations

The team addresses both questions of precision and numerical cost of the schemes for the numerical integration of non-linear evolution PDEs, such as the NLS equation. In particular, we aim at developing, studying and implementing numerical schemes with high order that are more efficient for these problems. We also want to contribute to the design and analysis of schemes with appropriate qualitative properties. These properties may as well be “asymptotic-preserving” properties, energy-preserving properties, or convergence to an equilibrium properties. Other numerical goals of the team include the numerical simulation of standing waves of non-linear non-local GP equations. We also keep on developing numerical methods to efficiently simulate and illustrate theoretical results on instability, in particular in the context of the modulational instability in optical fibers, where we study the influence of randomness in the physical parameters of the fibers.

The team also designs simulation methods to estimate the accuracy of the physical description via microscopic systems, by computing precisely the rate of convergence as the system size goes to infinity. One method under investigation is related to cloning algorithms, which were introduced very recently and turn out to be essential in molecular simulation.

4 Application domains

4.1 Optical fibers

In the propagation of light in optical fibers, the combined effect of non-linearity and group velocity dispersion (GVD) may lead to the destabilization of the stationary states (plane or continuous waves). This phenomenon, known under the name of modulational instability (MI), consists in the exponential growth of small harmonic perturbations of a continuous wave. MI has been pioneered in the 60s in the context of fluid mechanics, electromagnetic waves as well as in plasmas, and it has been observed in non-linear fiber optics in the 80s. In uniform fibers, MI arises for anomalous (negative) GVD, but it may also appear for normal GVD if polarization, higher order modes or higher order dispersion are considered. A different kind of MI related to a parametric resonance mechanism emerges when the dispersion or the non-linearity of the fiber are periodically modulated.

As a follow-up of our work on MI in periodically modulated optical fibers, we investigate the effect of random modulations in the diameter of the fiber on its dynamics. It is expected on theoretical grounds that such random fluctuations can lead to MI and this has already been illustrated for some models of the randomness. We investigate precisely the conditions under which this phenomenon can be strong enough to be experimentally verified. For this purpose, we investigate different kinds of random processes describing the modulations, taking into account the manner in which such modulations can be created experimentally by our partners of the fiber facility of the PhLAM. This necessitates a careful modeling of the fiber and a precise numerical simulation of its behavior as well as a theoretical analysis of the statistics of the fiber dynamics.

This application domain involves in particular S. De Bièvre and G. Dujardin.

4.2 Ferromagnetism

The Landau–Lifshitz (LL) equation describes the dynamics of the spin in ferromagnetic materials. Depending on the properties of the material, the LL equation can include a dissipation term (the so-called Gilbert damping) and different types of anisotropic terms. The LL equation belongs to a larger class of non-linear PDEs which are often referred to as geometric PDEs, and some related models are the Schrödinger map equation and the harmonic heat flow. We focus on the following aspects of the LL equation.

Solitons In the absence of Gilbert damping, the LL equation is Hamiltonian. Moreover, it is integrable in the one-dimensional case and explicit formulas for solitons can be given. In the easy-plane case, the orbital and asymptotic stability of these solitons have been established. However, the stability in other cases, such as in biaxial ferromagnets, remains an open problem. In higher dimensional cases, the existence of solitons is more involved. In a previous work, a branch of semitopological solitons with different speeds has been obtained numerically in planar ferromagnets. A rigorous proof of the existence of such solitons is established using perturbation arguments, provided that the speed is small enough. However, the proof does not give information about their stability. We would like to propose a variational approach to study the existence of this branch of solitons, that would lead to the existence and stability of the whole branch of ground-state solitons as predicted. We also investigate numerically the existence of other types of localized solutions for the LL equation, such as excited states or vortices in rotation.

On the other hand, with the inclusion of the Gilbert damping, the Landau-Lifshitz-Gilbert (LLG) equation becomes (partially) dissipative. Interestingly, in the one-dimensional case, the same solitons, referred to as *domain walls*, emerge as significant structures. Not only do they demonstrate asymptotic stability, even in the presence of a small magnetic field ([49]), but they also serve as crucial building blocks for various stable configurations, such as 2-domain wall structures ([48]). Numerical simulations further suggest that any general solution should decompose over time into a superposition of domain walls, though this still presents an open problem at the theoretical level. Exploring the scenario of a notched nanowire ([47]) reveals yet another context where generalized domain walls manifest. They exhibit an even better asymptotic stability compared to their non-notched counterparts, which may lead to applications in information storage.

Approximate models An important physical conjecture is that the LL model is to a certain extent universal, so that the non-linear Schrödinger and Sine-Gordon equations can be obtained as its various limit cases. In a previous work, A. de Laire has proved a result in this direction and established an error estimate in Sobolev norms, in any dimension. A next step is to produce numerical simulations that will enlighten the situation and drive further developments in this direction.

Self-similar behavior Self-similar solutions have attracted a lot of attention in the study of non-linear PDEs because they can provide some important information about the dynamics of the equation. While self-similar expanders are related to non-uniqueness and long time description of solutions, self-similar shrinkers are related to a possible singularity formation. However, there is not much known about the self-similar solutions for the LL equation. A. de Laire and S. Gutierrez (University of Birmingham) have studied expander solutions and proved their existence and stability in the presence of Gilbert damping. We will investigate further results about these solutions, as well as the existence and properties of self-similar shrinkers.

This application domain involves in particular A. de Laire, G. Dujardin and G. Ferriere.

4.3 Bose-Einstein condensates and nonlinear optics

In quantum physics and nonlinear optics, the Gross-Pitaevskii equation with non-zero boundary conditions is employed to describe the behavior of quantum fluids and Bose-Einstein condensates. The primary challenges are to comprehend new realistic physical effects, such as nonlocal interactions, quasilinear effects and variations in the width of the domain.

In order to establish a rigorous understanding of the dynamics of these models, the study of particular solutions such as dark solitons, which play a key role in the large-time behavior, is a crucial first step. For instance, proving the stability of dark solitons, based on various physical considerations, implies that these structures are good candidates to be controlled experimentally and to be considered in new applications.

Although the properties of dark solitons are well-known in classical models described by the Gross-Pitaevskii equation, the situation becomes more intricate when adding terms to model new realistic physical effects. Each characteristic introduces a range of new theoretical and numerical difficulties. This complexity emphasizes the need for a careful and detailed examination to enhance our understanding of these intricate systems.

This application domain involves in particular A. de Laire, G. Dujardin, G. Ferriere and Q. Chauleur.

4.4 Cold atoms

The cold atoms team of the PhLAM Laboratory is reputed for having realized experimentally the so-called Quantum Kicked Rotor, which provides a model for the phenomenon of Anderson localization. The latter was predicted by Anderson in 1958, who received in 1977 a Nobel Prize for this work. Anderson localization is the absence of diffusion of quantum mechanical wave functions (and of waves in general) due to the presence of randomness in the medium in which they propagate. Its transposition to the Quantum Kicked Rotor goes as follows: a freely moving quantum particle periodically subjected to a “kick” will see its energy saturate at long times. In this sense, it “localizes” in momentum space since its momenta do not grow indefinitely, as one would expect on classical grounds. In its original form, Anderson localization applies to non-interacting quantum particles and the same is true for the saturation effect observed in the Quantum Kicked Rotor.

The challenge is now to understand the effects of interactions between the atoms on the localization phenomenon. Transposing this problem to the Quantum Kicked Rotor, this means describing the interactions between the particles with a Gross–Pitaevskii equation, which is a NLS equation with a local (typically cubic) non-linearity. So the particle’s wave function evolves between kicks following the Gross–Pitaevskii equation and not the linear Schrödinger equation, as is the case in the Quantum Kicked Rotor. Preliminary studies for the Anderson model have concluded that in that case the localization phenomenon gives way to a slow subdiffusive growth of the particle’s kinetic energy. A similar phenomenon is expected in the non-linear Quantum Kicked Rotor, but a precise understanding of the dynamical mechanisms at work, of the time scale at which the subdiffusive growth will occur and of the subdiffusive growth exponent is lacking. It is crucial to design and calibrate the experimental setup intended to observe the phenomenon. The analysis of these questions poses considerable theoretical and numerical challenges due to the difficulties involved in understanding and simulating the long term dynamics of the non-linear system. A collaboration of the team members with the PhLAM cold atoms group is currently under way.

This application domain involves in particular S. De Bièvre, G. Dujardin and Q. Chauleur.

4.5 Modelling shallow water dynamics

The understanding of the propagation of waves in shallow water is of importance for the modelling of tsunamis and other rogue waves. This requires a better understanding of dispersive shallow water systems as ABCD systems, that are related to the classical Boussines systems, and classifying particular travelling waves solutions for these systems. To deal with systems is at forefront of research. Analogous questions for single equations as KdV equations are well-documented.

A. de Laire and O. Goubet are involved in these topics, together with researchers in Chile : C. Muñoz (Universidad de Chile), M. E. Martinez (University of Chile) and F. Poblete (Austral University of Chile).

The applications for tsunamis is of interest for people in Chile.

4.6 Qualitative and quantitative properties of numerical methods

Numerical simulation of multimode fibers The use of multimode fibers is a possible way to overcome the bandwidth crisis to come in our worldwide communication network consisting in singlemode fibers. Moreover, multimode fibers have applications in several other domains, such as high power fiber lasers and femtosecond-pulse fiber lasers which are useful for clinical applications of non-linear optical microscopy and precision materials processing. From the modeling point of view, the envelope equations are a system of non-linear non-local coupled Schrödinger equations. For a better understanding of several physical phenomena in multimode fibers (e.g. continuum generation, condensation) as well as for the design of physical experiments, numerical simulations are a suitable tool. However, the huge number of equations, the coupled non-linearities and the non-local effects are very difficult to handle numerically. Some attempts have been made to develop and make available efficient numerical codes for such simulations. However, there is room for improvement: one may want to go beyond MATLAB prototypes, and to develop an alternative parallelization to the existing ones, which could use the linearly implicit methods that we plan to develop and analyze. In link with the application domain 4.1, we develop in particular a code for the numerical simulation of the propagation of light in multimode fibers, using high-order efficient methods, that is to be used by the physics community.

This application domain involves in particular G. Dujardin and A. Roget.

Qualitative and long-time behavior of numerical methods We contribute to the design and analysis of schemes with good qualitative properties. These properties may as well be “asymptotic-preserving” properties, energy-preserving properties, decay properties, or convergence to an equilibrium properties. In particular, we contribute to the design and analysis of numerically hypocoercive methods for Fokker–Planck equations [51], as well as energy-preserving methods for Hamiltonian problems [45].

This application domain involves in particular G. Dujardin.

High-order methods We contribute to the design of efficient numerical methods for the simulation of non-linear evolution problems. In particular, we focus on a class of linearly implicit high-order methods, that have been introduced for ODEs and generalized to PDEs [23]. We wish both to extend their analysis to PDE contexts, and to analyze their qualitative properties in such contexts.

This application domain involves in particular G. Dujardin.

4.7 Modeling of the liquid-solid transition and interface propagation

Analogously to the so-called Kinetically Constrained Models (KCM) that have served as toy models for glassy transitions, stochastic particle systems on a lattice can be used as toy models for a variety of physical phenomena. Among them, the kinetically constrained lattice gases (KCLG) are models in which particles jump randomly on a lattice, but are only allowed to jump if a local constraint is satisfied by the system.

Because of the hard constraint, the typical local behavior of KCLGs will differ significantly depending on the value of local conserved fields (e.g. particle density), because the constraint will either be typically satisfied, in which case the system is locally diffusive (liquid phase), or not, in which case the system quickly freezes out (solid phase).

Such a toy model for liquid-solid transition is investigated by the former member of the team C. Erignoux and his co-authors in [4] and [46]. The focus of these articles is the so-called facilitated exclusion process, which is a terminology coined by physicists for a specific KCLG, in which particles can only jump on an empty neighbor if another neighboring site is occupied. They derive the macroscopic behavior of the model, and show that in dimension 1 the hydrodynamic limit displays a phase separated behavior where the liquid phase progressively invades the solid phase.

Both from a physical and mathematical point of view, much remains to be done regarding these challenging models: in particular, they present significant mathematical difficulties because of the way the local physical constraints put on the system distort the equilibrium and steady-states of the model. For this reason, A. Roget is currently working with C. Erignoux (DRACULA Project-team, Institut Camille Jordan, Université Lyon 1), M. Simon (Institut Camille Jordan, Université Lyon 1) and A. Shapira (MAP5, Paris) to generate numerical results on generalizations of the facilitated exclusion process, in order to shine some light on the microscopic and macroscopic behavior of these difficult models.

This application domain involves in particular A. Roget.

4.8 Mathematical modeling for ecology

This application domain is at the interface of mathematical modeling and numerics. Its object of study is a set of concrete problems in ecology. The landscape of the south of the Hauts-de-France region is made of agricultural land, encompassing forest patches and ecological corridors such as hedges. The issues are

- the study of the invasive dynamics and the control of a population of beetles which damages the oaks and beeches of our forests;
- the study of native protected species (the purple wireworm and the pike-plum) which find refuge in certain forest species.

Running numerics on models co-constructed with ecologists is also at the heart of the project. In our model, the timescales of animals and plants compare. The life cycle of a tree is one year. For animals we consider mainly insects whose life cycle is also of one year, even for the propagation of insects. Beetle larvae spend a few years in the earth before moving. As a by-product, the mathematical model may tackle other major issues such as the interplay between heterogeneity, diversity and invasibility.

The models use Markov chains at a mesoscopic scale and evolution advection-diffusion equations at a macroscopic scale.

This application domain involves O. Goubet. Interactions with PARADYSE members concerned with particle models and hydrodynamic limits are planned.

5 New software, platforms, open data

5.1 New software

5.1.1 MM_Propagation

Name: MultiMode Propagation

Keywords: Optics, Numerical simulations, Computational electromagnetics

Functional Description: This C++ software, which is interfaced with MatLab, simulates the propagation of light in multimode optical fibers. It takes into account several physical effects such as dispersion, Kerr effect, Raman effect, coupling between the modes. It uses high order numerical methods that allow for precision at reasonable computational cost.

URL: https://github.com/alexandreroget/MM_Propagation

Contact: Alexandre Roget

6 New results

Participants: Quentin Chauleur, Stephan De Bièvre, André De Laire Peirano, Guillaume Dujardin, Olivier Goubet, Guillaume Ferrière, Christopher Langrenez, Erwan Le Quiniou, Gabriel Nahum.

Some of the results presented below overlap several of the main research themes presented in section 3. However, results presented in paragraphs 6.1-6.11 are mainly concerned with research axis 3.1, whereas the paragraph 6.12 concern axis 3.2. Paragraphs 6.13-6.14 are related to quantum information and computing, and Paragraphs 6.15-6.22 concern numerics-oriented results, so that they are all encompassed in axis 3.3.

6.1 Asymptotic stability of 2-domain walls for the Landau-Lifshitz-Gilbert equation in a nanowire with Dzyaloshinskii-Moriya interaction

The article [20] extends the study of magnetization dynamics in an infinite ferromagnetic nanowire, where the evolution is governed by the Landau-Lifshitz-Gilbert (LLG) equation. The energy functional considered includes an easy-axis anisotropy along the direction e_1 and incorporates the Dzyaloshinskii-Moriya interaction. In a previous work [49], R. Côte (University of Strasbourg) and R. Ignat (Toulouse Mathematics Institute) analyzed a specific structure, called *domain wall*, which connects $-e_1$ at $-\infty$ to e_1 at $+\infty$. They proved its uniqueness and asymptotic stability, up to translations and rotations around e_1, under a small external magnetic field. Their approach relied on energy properties near the domain wall and its evolution via the LLG flow.

In this paper [20], R. Côte (University of Strasbourg) and G. Ferriere investigate solutions which look like 2-domain walls, *i.e.* configurations with two well-separated transitions, each resembling a domain wall. They establish that, under specific conditions on the external magnetic field such that it drives the transitions further apart, these structures are also asymptotically stable, up to translations and rotations around e_1 for each transition. The proof employs energy methods analogous to those used in multi-soliton results for dispersive equations. By localizing the energy around each transition, they recover similar stability properties, with additional negligible terms.

6.2 Logarithmic Gross-Pitaevskii equation

The logarithmic nonlinearity in the context of Schrödinger equations has recently regained interest in various domains of physics. For instance, this model may generalize the Gross-Pitaevskii equation, used in the case of two-body interaction, to the case of three-body interaction. R. Carles (University of Rennes) and G. Ferriere study this equation, named the logarithmic Gross-Pitaevskii equation (or logGP), on the whole space $\mathbb{R}^d$ in [16]. As the first mathematical study of this equation in this framework, they focus on its global wellposedness in the energy space, which turns out to correspond to the energy space for the standard Gross-Pitaevskii equation with a cubic nonlinearity in small dimensions, and on the characterization of solitary and traveling waves in the one-dimensional case. This works opens the door to further studies on this equation, especially on its asymptotic and long-time dynamics : multidimensional solitary and traveling waves and their orbital stability, scattering, multi-solitons, convergence towards other models...

6.3 On the stationary solution of the Landau-Lifshitz-Gilbert equation on a nanowire with constant external magnetic field

In the preprint [36], G. Ferriere examines the Landau-Lifshitz-Gilbert (LLG) equation governing the magnetization dynamics in an infinite ferromagnetic nanowire with easy-axis anisotropy along the e_1 direction and subjected to a constant external magnetic field $h_0 e_1$. Under specific conditions on h_0, the study establishes the existence of stationary solutions with identical asymptotic behavior at infinity, their uniqueness up to the symmetries of the LLG equation, and the instability of their orbits under the LLG flow. These findings provide new insights into the behavior of solutions to the LLG equation, complemented by numerical simulations that explore the stability of 2-domain wall structures and the interactions between domain walls.

6.4 Existence and Uniqueness of Domain Walls for Notched Ferromagnetic Nanowires

In the preprint [34], R. Côte (University of Strasbourg), C. Courtès (University of Strasbourg), G. Ferriere, L. Godard-Cadillac (University of Bordeaux), and Y. Privat (University of Lorraine, Nancy) explore the existence and properties of domain walls in a model of notched ferromagnetic nanowires. They employ variational methods and critical point theory to investigate the energy functional describing the system.

The authors first establish the equivalence of the critical points of this functional and the critical points of another, more suitable functional through lifting. The existence of a minium is then achieved under the assumption that the residual cross-section area function s is strictly below 1 in a bounded interval and is equal to 1 outside this interval. They then demonstrate the uniqueness of the critical point under the proper constraints on the limits at $\pm\infty$ by leveraging a Mountain-Pass argument. The uniqueness requires stronger monotonicity assumptions, mainly that s is unimodal, all the more as it is expected that non-uniqueness should hold in the case of many notches.

The identified critical point corresponds to a domain wall structure, *i.e.* a transition from $-e_1$ to e_1. The authors also prove that the transition is mainly performed inside the notch. Furthermore, the study analyzes the asymptotic behavior of the solution, showing that the magnetization decays to a uniform state at infinity. In the special case of a symmetric notch, additional insights are obtained using rearrangement techniques.

6.5 Numerical computation of dark solitons of a nonlocal nonlinear Schrödinger equation

The existence and decay properties of dark solitons for a large class of nonlinear nonlocal Gross-Pitaevskii equations with nonzero boundary conditions in dimension one was established recently by A. de Laire and S. López-Martínez (Autonomous University of Madrid) in [14]. Mathematically, these solitons correspond to minimizers of the energy at fixed momentum and are orbitally stable. In the paper [25], A. de Laire, G. Dujardin and S. López-Martínez (Autonomous University of Madrid) provide a numerical method to compute approximations of such solitons for these types of equations, and give actual numerical experiments for several types of physically relevant nonlocal potentials. These simulations allow them to obtain a variety of dark solitons, and to comment on their shapes in terms of the parameters of the nonlocal potential. In particular, they suggest that, given the dispersion relation, the speed of sound and the Landau speed are important values to understand the properties of these dark solitons. They also allow them to test the necessity of some sufficient conditions in the theoretical result proving existence of the dark solitons.

6.6 Gray and black solitons of nonlocal Gross-Pitaevskii equations

In the preprint [39], A. de Laire and S. López-Martínez (Autonomous University of Madrid) continue the investigation started in [14, 25], concerning the qualitative aspects of dark solitons of one-dimensional Gross-Pitaevskii equations with general nonlocal interactions. Under general conditions on the potential interaction term, they provide uniform bounds, demonstrate the existence of symmetric solitons, and identify conditions under which monotonicity is lost. Additionally, they present new properties of black solitons. Moreover, they establish the nonlocal-to-local convergence, i.e. the convergence of the soliton of the nonlocal model toward the explicit dark solitons of the local Gross-Pitaevskii equation.

6.7 Exotic traveling waves for a quasilinear Schrödinger equation with nonzero background

A. de Laire and E. Le Quiniou have studied a quasilinear Schrödinger equation with nonzero conditions at infinity in dimension one. This quasilinear model corresponds to a weakly nonlocal approximation of the nonlocal Gross–Pitaevskii equation, and can also be derived by considering the effects of surface tension in superfluids. When the quasilinear term is neglected, the resulting equation is the classical Gross–Pitaevskii equation, which possesses a well-known stable branch of subsonic traveling waves solution, given by dark solitons.

In the preprint [38], they investigate how the quasilinear term affects the traveling-waves solutions. They provide a complete classification of finite energy traveling waves of the equation, in terms of the two parameters: the speed and the strength of the quasilinear term. This classification leads to the existence of dark and antidark solitons, as well as more exotic localized solutions like dark cuspons, compactons, and composite waves, even for supersonic speeds. Depending on the parameters, these types of solutions can coexist, showing that finite energy solutions are not unique. Furthermore, they prove that some of these dark solitons can be obtained as minimizers of the energy, at fixed momentum, and that they are orbitally stable.

6.8 Traveling waves for a quasilinear Schrödinger equation

In the paper [42], E. Le Quiniou studies a quasilinear Schrödinger equation with nonzero conditions at infinity. In the previous work [38] with A. de Laire, he obtained a continuous branch of traveling waves, given by dark solitons indexed by their speed. Neglecting the quasilinear term, one recovers the Gross–Pitaevskii equation, for which the branch of dark solitons is stable. It is known that the Vakhitov–Kolokolov (VK) stability criterion or momentum of stability criterion holds for general semilinear equations with nonvanishing conditions at infinity. In the quasilinear case, E. Le Quiniou proves that the VK stability criterion still applies and he deduces that the branch of dark solitons is stable for weak quasilinear interactions. For stronger quasilinear interactions, a cusp appears in the energy-momentum diagram, implying the stability of fast waves and the instability of slow waves.

6.9 Travelling waves for the Gross–Pitaevskii equation on the strip

In one space dimension, the Gross-Pitaevskii equation possesses a family of finite energy travelling waves, called dark solitons. These solitons extend trivially to the strip given by the product space $\mathbb{R} \times \mathbb{T}_L$, where $L > 0$ and $\mathbb{T}_L$ is the torus $\mathbb{T}_L = \mathbb{R}/L\mathbb{Z}$. In this two-dimensional context, the dark solitons are called planar (or line) dark solitons. However, it is well-known in the physics literature that these planar solitons can be unstable due to the tendency to develop distortions in their transverse profile. In addition, experimental observations have shown that the dynamics of planar dark solitons are stable when they are sufficiently confined in the transverse direction L, but unstable otherwise. In the latter case, the creation of vortices can occur.

In the articles [27] and [26], A. de Laire, P. Gravejat (CY Cergy Paris University) and D. Smets (Sorbonne University) provide a rigorous framework for studying this kind of phenomenon. Precisely, they prove the existence of nonconstant finite energy travelling wave solutions to the Gross-Pitaevskii equation on the strip $\mathbb{R} \times \mathbb{T}_L$, obtained as minimizers of the energy at fixed momentum. Moreover, by studying the associated variational problem, they deduce that these minimizers are exactly the planar dark solitons when L is less than a critical value, and that they are genuinely two-dimensional solutions otherwise. In particular, planar solitons do not minimize the energy in the presence of a large transverse direction. The proof of the existence of minimizers is based on the compactness of minimizing sequences, relying on a new symmetrization argument that is well-suited to the periodic setting.

6.10 Standing wave for two-dimensional Schrödinger equations with discontinuous dispersion

In collaboration with B. Alouini (University of Monastir) and I. Manoubi (Université of Gabès) [30], O. Goubet has studied the existence and stability of standing wave for an evolution nonlinear Schrödinger equation with discontinuous dispersion, the discontinuity being supported by a straight line. Both pure power nonlinearities and logarithmic nonlinearities are considered. The discontinuity destroys the invariance by space translation for the equation. The main result is that when restricted to a suitable subspace that contains the standing waves, these waves are orbitally stable in the H^1 subcritical regime in the pure power case or in the logarithmic case, and strongly unstable in the critical or supercritical case.

6.11 The logarithmic Schrödinger equation with spatial white noise on the full space

The logarithmic Schrödinger appears as a fundamental model in quantum gravity and nuclear physics, and adding a white noise potential can model strong media disorder. In [19], Q. Chauleur and A. Mouzard (University of Nanterre) prove the existence and uniqueness of solutions to the stochastic logarithmic Schrödinger. The proof relies on a particular exponential transform which have proved being useful in several contexts, in particular in models arising from quantum field theory.

6.12 Non-Linear Problems in Interacting Particle Systems

In his thesis [29], G. Nahum presents three studies on interacting particle systems where a form of nonlinearity emerges.

In the context of describing non-equilibrium steady states (NESS), he formulates an approach based on matrix products to characterize the steady state of a process defined on a lattice composed of sites numbered from 1 to N. This process evolves as a symmetric simple exclusion process (SSEP) on the intermediate sites, while being coupled with reaction-diffusion processes acting on pairs of boundary sites $\{1,2\}$ and $\{N-1,N\}$. Each pair of sites can adopt one of four possible states, resulting in 12 possible transitions. He derives a set of constraints ensuring the consistency of the underlying quadratic algebra, which are linked to reservoir correlations. He also presents a representation of the objects involved in this formulation and provides examples of transition rates that satisfy these constraints.

In the second study, he focuses on generalizing the porous media model (PMM), which is associated with a hydrodynamic equation where the diffusion rate depends on the density raised to a certain power. He extends this model by introducing a universal exclusion family parameterized by an exponent, which can take real values. This generalization allows the representation of the transition from the slow diffusion regime to the fast diffusion regime. He successfully addresses the case where the exponent lies in a specific interval, deriving the porous media equation for certain values of the exponent and the fast diffusion equation for others.

He further generalizes the PMM by constructing a diffusion coefficient that depends on a density multiplied by a function of the complementary density, parameterized by two exponents. The generalized model inherits certain theoretical properties of the original PMM, such as the presence of mobile clusters and blocked configurations. The construction of this model is delicate to preserve the gradient property of the PMM.

Finally, he extends this generalization to a long-range dynamics while maintaining the gradient property. This long-range dynamics is simple and can be applied to any exclusion process.

6.13 Kirkwood-Dirac distributions

The Kirkwood-Dirac (KD) quasiprobability distribution can describe any quantum state with respect to the eigenbases of two observables A and B. KD distributions behave similarly to classical joint probability distributions but can assume negative and nonreal values. In [50], S. De Bièvre provided an in-depth study of the notion of completely incompatible observables that he recently introduced and of its links to the support uncertainty and to the Kirkwood-Dirac nonpositivity of pure quantum states. The latter notion has recently been proven central to a number of issues in quantum information theory and quantum metrology. In this last context, it was shown that a quantum advantage requires the use of Kirkwood-Dirac nonclassical states.

Several papers have been published by members of PARADYSE on this subject in the last year. They are mentioned in the following sections.

6.13.1 Properties and Applications of the Kirkwood-Dirac Distribution

In [15], S. De Bièvre, C. Langrenez and their collaborators provide an extensive review of the properties of KD-distributions and of their applications.

6.13.2 Characterizing the geometry of the Kirkwood-Dirac positive states

In [28], S. De Bièvre, C. Langrenez and D. R. M. Arvidsson-Shukur (Hitachi Cambridge Laboratory) analyse the geometry of the KD-positive and -nonpositive pure and mixed states. They analyze the dependence of the full convex set of states with positive KD distributions on the eigenbases of A and B and provide an algebraic necessary and sufficient condition for this set to be minimal, meaning that it contains only the basis projectors of A and B.

6.13.3 Convex roofs witnessing Kirkwood-Dirac nonpositivity

In [40], S. De Bièvre, C. Langrenez and D. R. M. Arvidsson-Shukur (Hitachi Cambridge Laboratory) introduce and study two witnesses for KD nonpositivity, through a convex roof construction and the notion of *support uncertainty*.

6.13.4 The set of Kirkwood-Dirac positive states is almost always minimal

In [41], S. De Bièvre, C. Langrenez and their collaborators show that, with probability one with respect to the choices of A and of B, the set of KD-positive states is indeed minimal, in the above sense. They also provide examples where this set is not minimal and contains "exotic" KD-positive states, which are mixed but cannot be written as mixtures of pure KD-positive states.

6.13.5 Contextuality Can be Verified with Noncontextual Experiments

In [43], S. De Bièvre and his collaborators show how the exotic states introduced in [41] can be used to evidence contextuality with a noncontextual experiment.

6.14 Rigorous results on approach to thermal equilibrium, entanglement, and non-classicality of an optical quantum field mode scattering from the elements of a non-equilibrium quantum reservoir

Rigorous derivations of the approach of individual elements of large isolated systems to a state of thermal equilibrium, starting from arbitrary initial states, are exceedingly rare. This is particularly true for quantum mechanical systems. In [21], S. De Bièvre and his collaborators demonstrate how, through a mechanism of repeated scattering, an approach to equilibrium of this type actually occurs in a specific quantum optics system.

6.15 Growth of Sobolev norms and strong convergence for the discrete nonlinear Schrödinger equation

As it is known, the nonlinear Schrödinger stands as a prime model in order to describe the propagation of waves in nonlinear optics or the dynamics of a superfluid in Bose-Einstein condensates. Its discretization in space stands as a first step in order to perform reliable and efficient numerical simulations. Q. Chauleur studies the convergence of the discrete nonlinear Schrödinger equation on a lattice $h\mathbb{Z}^d$ towards the continuous model as the step size of the lattice h tends to zero in [17]. The proof of the convergence relies on uniform dispersive estimates in order to control the growth of the discrete Sobolev norms of the solution, as well as bilinear estimates of the Shannon interpolation.

6.16 Strong convergence for the discrete nonlinear Klein-Gordon equation

In [31], Q. Chauleur extends the analysis of nonlinear dispersive equations, such as the nonlinear Klein-Gordon equation, on an infinite lattice $h\mathbb{Z}^d$ as the lattice spacing $h \to 0$ approaches the continuum limit. This work builds upon the framework established in [17], employing bilinear estimates of the Shannon interpolation combined with controls on the growth of discrete Sobolev norms of the solutions. Additionally, Q. Chauleur also provides some perspectives on uniform dispersive estimates for nonlinear waves on lattices.

6.17 Linearly implicit high-order numerical methods for evolution problems

G. Dujardin and I. Lacroix-Violet (University of Lorraine, Nancy) introduced a new class of numerical methods for the time integration of evolution equations set as Cauchy problems of ODEs or PDEs, in the research direction detailed in Section 3.3. The systematic design of these methods mixes the Runge–Kutta collocation formalism with collocation techniques, in such a way that the methods are linearly implicit and have high order. A specific analysis of Runge–Kutta collocation methods for this purpose was carried out by G. Dujardin and I. Lacroix-Violet (University of Lorraine, Nancy) [22]. The fact that these methods are implicit allows to avoid CFL conditions when the large systems to integrate come from the space discretization of evolution PDEs. Moreover, these methods proved to be efficient since they only require to solve one linear system of equations at each time step, and efficient techniques from the literature can be used to do so [23].

6.18 Vortex nucleation in 2D rotating Bose–Einstein condensates

In [24], G. Dujardin, I. Lacroix-Violet (University of Lorraine, Nancy) and A. Nahas (University of Lille) introduce a new numerical method for the minimization under constraints of a discrete energy modeling multicomponents rotating Bose-Einstein condensates in the regime of strong confinement and with rotation. Moreover, they consider both segregation and coexistence regimes between the components. It is well known that, depending on the regime, the minimizers may display different structures, sometimes with vorticity (from singly quantized vortices, to vortex sheets and giant holes). In order to study numerically the structures of the minimizers, the authors of [24] introduce a numerical algorithm for the computation of the indices of the vortices, as well as an algorithm for the computation of the indices of vortex sheets. Several computations are carried out, to illustrate the efficiency of the method, to cover different physical cases, to validate recent theoretical results as well as to support conjectures. Moreover, the new methods is compared with an alternative method from the literature. This work was part of A. Nahas' PhD thesis, co-advised by I. Lacroix-Violet (University of Lorraine, Nancy) and G. Dujardin.

6.19 Finite volumes for the Gross-Pitaevskii equation

In [32], Q. Chauleur studies the approximation of the Gross-Pitaevskii equation with a time-dependent potential using a Voronoi finite-volume scheme. The time integration is handled via an explicit splitting scheme, while the spatial integration employs a two-point flux approximation finite-volume method. This work serves as the theoretical counterpart to the companion paper [33], providing a numerical analysis of the new scheme designed to explore the dynamics of Bose-Einstein condensates in various geometries.

6.20 Numerical study of the Gross-Pitaevskii equation on a two-dimensional ring and vortex nucleation

In [33], Q. Chauleur and G. Dujardin, in collaboration with physicists R. Chicireanu, J.-C. Garreau, and A. Rançon from the PhLAM laboratory at University of Lille, numerically investigate the dynamics of a Bose-Einstein condensate of cold potassium atoms confined by a ring potential with a Gaussian profile. By introducing a rotating sinusoidal perturbation, they demonstrate the nucleation of quantum vortices under specific dynamical regimes. The numerical simulations are carried out using Strang splitting for time integration and a two-point flux approximation finite-volume scheme applied to a carefully constructed admissible triangulation. Additionally, they develop numerical algorithms for vortex tracking tailored to the finite-volume framework.

6.21 Discrete quantum harmonic oscillator and Kravchuk transform

In [18], Q. Chauleur and E. Faou (University of Rennes) consider a particular discretization of the harmonic oscillator which admits an orthogonal basis of eigenfunctions called Kravchuk functions possessing appealing properties from the numerical point of view. We analytically prove the almost second-order convergence of these discrete functions towards Hermite functions, uniformly for large numbers of modes. We then describe an efficient way to simulate these eigenfunctions and the corresponding transformation. We finally show some numerical experiments corroborating our different results.

6.22 Numerical analysis of a semi-implicit Euler scheme for the Keller-Segel model

In a collaboration with X. Huang (School of Mathematical Science, Fujian, China) and J. Shen (School of Mathematical Science, Eastern Institute of Technology, Zhejiang, China) [37], O. Goubet has performed the numerical analysis of a discrete time scheme for a Keller-Segel model in dimension 2. This semi-implicit scheme preserves important features of the original equation as positivity of and diffusion.

7 Partnerships and cooperations

Participants: André De Laire Peirano, Olivier Goubet.

7.1 International initiatives

7.1.1 Associate Teams in the framework of an Inria International Lab or in the framework of an Inria International Program

PANDA

Title: Partial Differential Equations, Dispersive Models and Nonlinear Analysis

Duration: 3 years (2024 -> 2027)

Coordinator: Claudio Muñoz (cmunoz@dim.uchile.cl)

Partners: Universidad de Chile (Chili)

Inria contact: André de Laire

Summary: PANDA is a collaborative project between Chilean and French teams, in the field of applied mathematics. The main subject is the study of systems of dispersive partial differential equations, based on nonlinear analysis and numerical simulation techniques. One of the main applications of this project concerns the modeling of the propagation of waves on the ocean surface. Webpage.

7.2 International research visitors

7.2.1 Visits of international scientists

Inria International Chair

With the support of INRIA, I. Manoubi has visited PARADYSE for one month in May 2024. She has worked with O. Goubet and developped a research program around the existence and stability of standing waves for nonlinear Schrödinger equations with discontinuous and non standard dispersions.

Other international visits to the team

Salvador López Martínez

Status Assistant lecturer

Institution of origin: Universidad Autónoma de Madrid

Country: Spain

Dates: from July 8 to July 12, 2024

Context of the visit: Research collaboration with A. de Laire funded by LabEx CEMPI.

Claudio Muñoz

Status: Professor

Institution of origin: Universidad de Chile

Country: Chile

Dates: from November 4 to November 8, 2024

Context of the visit: Research collaboration with A. de Laire and O. Goubet, partially funded by LabEx CEMPI.

Felipe Poblete

Status: Associate Professor

Institution of origin: Universidad Austral de Chile

Country: Chile

Dates: from November 4 to November 8, 2024

Context of the visit: Research collaboration with A. de Laire and O. Goubet, partially funded by LabEx CEMPI.

Susana Gutiérrez

Status: Senior lecturer

Institution of origin: University of Birmingham

Country: United Kingdom

Dates: from December 16 to December 20 , 2024

Context of the visit: Research collaboration with A. de Laire, funded by LabEx CEMPI.

7.2.2 Visits to international teams

Visit to Chile

In the framework of the Associte team PANDA (between INRIA Lille and Universidad de Chile) quoted above, A. de Laire and O. Goubet have visited the DIM (Departement Mathematical Engeniering) of the Universidad de Chile. On this occasion, a workshop was organized in Santiago. In addition, this visit allowed the investigators to advance with the research program developed around the existence and stability of progressive waves for *abcd* systems, which generalize Boussinesq systems as asymptotic propagation models for long waves in shallow water.

7.3 National initiatives

7.3.1 LabEx CEMPI

Through their affiliation to the Laboratoire Paul Painlevé of Université de Lille, PARADYSE team members benefit from the support of the LabEx CEMPI. In addition, the LabEx CEMPI is funding the post-doc of Quentin Chauleur in the team, in an interdisciplinary initiative between PhLAM and LPP.

Title: Centre Européen pour les Mathématiques, la Physique et leurs Interactions

Partners: Laboratoire Paul Painlevé (LPP) and Laser Physics department (PhLAM), Université de Lille

ANR reference: 11-LABX-0007

Duration: February 2012 - December 2024 (the project has been renewed in 2019)

Budget: 6 960 395 euros

Coordinator: Emmanuel Fricain (LPP, Université de Lille)

The "Laboratoire d'Excellence" CEMPI (Centre Européen pour les Mathématiques, la Physique et leurs Interactions), a project of the Laboratoire de mathématiques Paul Painlevé (LPP) and the laboratoire de Physique des Lasers, Atomes et Molécules (PhLAM), was created in the context of the "Programme d'Investissements d'Avenir" in February 2012. The association Painlevé-PhLAM creates in Lille a research unit for fundamental and applied research and for training and technological development that covers a wide spectrum of knowledge stretching from pure and applied mathematics to experimental and applied physics. The CEMPI research is at the interface between mathematics and physics. It is concerned with key problems coming from the study of complex behaviors in cold atoms physics and nonlinear optics, in particular fiber optics. It deals with fields of mathematics such as algebraic geometry, modular forms, operator algebras, harmonic analysis, and quantum groups, that have promising interactions with several branches of theoretical physics.

8 Dissemination

Participants: André De Laire Peirano, Olivier Goubet, Guillaume Dujardin, Stephan De Bièvre, Guillaume Ferrière.

8.1 Promoting scientific activities

8.1.1 Scientific events: organisation

Member of the organizing committees

- A. de Laire was one of the organizers of the 1st PANDA Workshop in Santiago, Chile, from Sep. 5 to Sep. 6, 2024.
- A. de Laire was one of the organizers of the 2nd PANDA Workshop in Inria Lille, from Nov. 7 to Nov. 8, 2024.
- A. de Laire was one of the organizers of the conference Applied Analysis and Modeling: a conference in honor of Olivier Goubet in Lille, from Nov. 4 to Nov. 6, 2024.

8.1.2 Journal

Member of the editorial boards

- S. De Bièvre is associate editor of the Journal of Mathematical Physics (since January 2019).
- O. Goubet is the editor-in-chief of the North-Western European Journal of Mathematics.
- O. Goubet is associate editor of ANONA (Advances in Nonlinear Analysis).
- O. Goubet is associate editor of the Journal of Mathematical Study.
- O. Goubet is associate editor of the Bulletin of Mathematical Sciences.

Reviewer - reviewing activities

All permanent members of the PARADYSE team work as referees for many of the main scientific publications in analysis, partial differential equations and statistical physics, depending on their respective fields of expertise.

8.1.3 Invited talks

All PARADYSE team members take active part in numerous scientific conferences, workshops and seminars, and in particular give frequent talks both in France and abroad.

8.1.4 Research administration

- S. De Bièvre and A. de Laire are both members of the “Conseil de Laboratoire Paul Painlevé” at Université de Lille.
- S. De Bièvre is a member of the executive committee of the LabEx CEMPI.
- A. de Laire is a member of the “Fédération de Recherche Mathématique des Hauts-de-France”.
- A. de Laire is a member of the “Domain Board” of the Graduate School MADIS.
- G. Dujardin is a member of the Executive Committee of the CPER WaveTech.
- G. Dujardin is the deputy head for science of the Inria center at the University of Lille. In particular, he is a member of the Inria Evaluation Committee.
- O. Goubet is a member of the "Conseil de département de mathématiques" at Université de Lille.
- O. Goubet is a member of the "Bureau du HUB numérique" of the I-Site U-Lille.
- O. Goubet, former President of SMAI, is a member of the "Conseil d’Administration de la SMAI".

8.2 Teaching - Supervision - Juries

8.2.1 Teaching

The PARADYSE team teaches various undergraduate level courses in several partner universities. We only make explicit mention here of the Master courses (level M1-M2) and the doctoral courses.

- Master: O. Goubet "Ondes à la surface de l’eau", M2 (Université de Lille, 40h).
- Master: O. Goubet, "Analyse numérique pour les EDP", M1 (Université de Lille, 36h).
- Master: O. Goubet, "Méthodes spectrales et Fourier", M1 (Université de Lille, 27h).
- Master: O. Goubet, "Méthodes spectrales et Fourier", M1 (Université de Lille, 27h).
- Master: S. De Bièvre, "Modeling", M2 (Université de Lille, 40h).
- Master: O. Goubet, "Exemples de problèmes elliptiques et paraboliques", M1 (Université de Lille, 24h).
- Master: A. de Laire, "Etude de problèmes elliptiques et paraboliques", M1 (Université de Lille, 66h).
- Master: A. de Laire and G. Ferriere, "Nonlinear PDEs", M2 (Université de Lille, 66h).
- Doctoral School: S. De Bièvre, "Quantum information" (Université de Lille, 24h).

8.2.2 Supervision

- S. De Bièvre is supervising the PhD thesis of Christopher Langrenez on "Kirkwood-Dirac nonclassicality", during 2022-2025.
- S. De Bièvre is supervising the PhD thesis of Matéo Spriet "Mesures opérationnels de nonclassicalité" during 2024-2027.
- A. de Laire and O. Goubet are supervising the PhD thesis of Erwan Le Quiniou on the "Study of a quasilinear Gross-Pitaevskii equation", during 2022-2025.

- A. de Laire and G. Dujardin are supervising the PhD thesis of Sebastian Tapia on the "Theorical and numerical study of dark solitons for nonlinear Schrödinger equations" during 2024-2027. They also supervised his internship from January to March 2024.
- G. Dujardin is supervising the PhD thesis of Abbas El Hajj during 2024-2027.
- O. Goubet is supervising the PhD thesis of Céline Wang during 2023-2026.
- A. de Laire and A. Natale supervised the internship of Alonso Salvador Carrasco Urbina from January to March 2024.

8.2.3 Juries

G. Dujardin is a member of the jury of the agrégation externe de mathématiques, in charge with Frédérique Charles of the "Scientific Computing" option.

9 Scientific production

9.1 Major publications

[1] R. Ahmed, C. Bernardin, P. Gonçalves and M. Simon. 'A Microscopic Derivation of Coupled SPDE's with a KPZ Flavor'. In: *Annales de l'Institut Henri Poincaré* 58.2 (2022). DOI: 10.1214/21-AIHP1196. URL: https://hal.archives-ouvertes.fr/hal-02307963.

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[3] C. Besse, S. Descombes, G. Dujardin and I. Lacroix-Violet. 'Energy preserving methods for nonlinear Schrödinger equations'. In: *IMA Journal of Numerical Analysis* 41.1 (Jan. 2021), pp. 618–653. DOI: 10.1093/imanum/drz067. URL: https://hal.archives-ouvertes.fr/hal-01951527.

[4] O. Blondel, C. Erignoux and M. Simon. 'Stefan problem for a non-ergodic facilitated exclusion process'. In: *Probability and Mathematical Physics* 2.1 (2021). DOI: 10.2140/pmp.2021.2.127. URL: https://hal.inria.fr/hal-02482922 (cit. on p. 7).

[5] Q. Chauleur. 'Growth of Sobolev norms and strong convergence for the discrete nonlinear Schrödinger equation'. In: *Nonlinear Analysis: Theory, Methods and Applications* 242 (May 2024), p. 113517. DOI: 10.1016/j.na.2024.113517. URL: https://hal.science/hal-04142120.

[6] R. Côte and G. Ferriere. 'Asymptotic stability of 2-domain walls for the Landau-Lifshitz-Gilbert equation in a nanowire with Dzyaloshinskii-Moriya interaction'. In: *International Mathematics Research Notices* 2024.4 (21st Feb. 2024), pp. 3551–3600. DOI: 10.1093/imrn/rnad249. URL: https://hal.science/hal-03913853.

[7] S. De Bièvre. 'Complete Incompatibility, Support Uncertainty, and Kirkwood-Dirac Nonclassicality'. In: *Physical Review Letters* (2021). DOI: 10.1103/PhysRevLett.127.190404. URL: https://hal.archives-ouvertes.fr/hal-03464950.

[8] S. Dumont, O. Goubet and Y. Mammeri. 'Decay of solutions to one dimensional nonlinear Schrödinger equations with white noise dispersion'. In: *Discrete and Continuous Dynamical Systems - Series S* 14.8 (2021), pp. 2877–2891. DOI: 10.3934/dcdss.2020456. URL: https://hal.archives-ouvertes.fr/hal-02944262.

[9] C. Erignoux. 'Hydrodynamic limit for an active exclusion process'. In: *Mémoires de la Société Mathématique de France* 169 (May 2021). DOI: 10.24033/msmf.477. URL: https://hal.science/hal-01350532.

[10] C. Griffet, M. Arnhem, S. De Bièvre and N. Cerf. 'Interferometric measurement of the quadrature coherence scale using two replicas of a quantum optical state'. In: *Physical Review A* 108.2 (31st Aug. 2023), p. 023730. DOI: 10.1103/PhysRevA.108.023730. URL: https://hal.science/hal-03911306.

[11] A. Hertz and S. de Bièvre. 'Quadrature coherence scale driven fast decoherence of bosonic quantum field states'. In: *Physical Review Letters* (5th Mar. 2020). DOI: 10.1103/PhysRevLett.124.090402. URL: https://hal.archives-ouvertes.fr/hal-02394344.

[12] A. de Laire and P. Gravejat. 'The cubic Schrödinger regime of the Landau-Lifshitz equation with a strong easy-axis anisotropy'. In: *Revista Matemática Iberoamericana* 37.1 (2021), pp. 95–128. DOI: 10.4171/rmi/1202. URL: https://hal.archives-ouvertes.fr/hal-01954762.

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[14] A. de Laire and S. López-Martínez. 'Existence and decay of traveling waves for the nonlocal Gross-Pitaevskii equation'. In: *Communications in Partial Differential Equations* 47.9 (2022), pp. 1732–1794. DOI: 10.1080/03605302.2022.2070853. URL: https://hal.archives-ouvertes.fr/hal-03422447 (cit. on p. 10).

9.2 Publications of the year

International journals

[15] D. Arvidsson-Shukur, W. Braasch Jr, S. De Bièvre, J. Dressel, A. Jordan, C. Langrenez, M. Lostaglio, J. Lundeen and N. Y. Halpern. 'Properties and Applications of the Kirkwood-Dirac Distribution'. In: *New Journal of Physics* 26.12 (27th Dec. 2024), p. 121201. DOI: 10.1088/1367-2630/ada05d. URL: https://hal.science/hal-04652540 (cit. on p. 12).

[16] R. Carles and G. Ferriere. 'Logarithmic Gross-Pitaevskii equation'. In: *Communications in Partial Differential Equations* 49.1-2 (6th Feb. 2024), pp. 88–120. DOI: 10.1080/03605302.2023.2296924. URL: https://hal.science/hal-03790977 (cit. on p. 9).

[17] Q. Chauleur. 'Growth of Sobolev norms and strong convergence for the discrete nonlinear Schrödinger equation'. In: *Nonlinear Analysis: Theory, Methods and Applications* 242 (May 2024), p. 113517. DOI: 10.1016/j.na.2024.113517. URL: https://hal.science/hal-04142120 (cit. on p. 13).

[18] Q. Chauleur and E. Faou. 'Discrete quantum harmonic oscillator and kravchuk transform'. In: *ESAIM: Mathematical Modelling and Numerical Analysis* 58.6 (4th Dec. 2024), pp. 2155–2186. DOI: 10.1051/m2an/2024001. URL: https://hal.science/hal-03885282 (cit. on p. 14).

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[20] R. Côte and G. Ferriere. 'Asymptotic stability of 2-domain walls for the Landau-Lifshitz-Gilbert equation in a nanowire with Dzyaloshinskii-Moriya interaction'. In: *International Mathematics Research Notices* 2024.4 (21st Feb. 2024), pp. 3551–3600. DOI: 10.1093/imrn/rnad249. URL: https://hal.science/hal-03913853 (cit. on p. 9).

[21] S. De Bievre, M. Merkli and P. Parris. 'Rigorous results on approach to thermal equilibrium, entanglement, and nonclassicality of an optical quantum field mode scattering from the elements of a non-equilibrium quantum reservoir'. In: *Quantum* 8 (24th May 2024), p. 1360. DOI: 10.48550/arXiv.2312.14290. URL: https://hal.science/hal-04374710 (cit. on p. 13).

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