2025 *Activity Report*

RESEARCH CENTRE: **Inria Centre at Université Grenoble Alpes**
IN PARTNERSHIP WITH: **Université de Grenoble Alpes, CNRS**

Project-Team

AIRSEA

Mathematics and computing applied to oceanic and atmospheric flows

In collaboration with Laboratoire Jean Kuntzmann (LJK)

Project-Team AIRSEA

Creation of the Project-Team: 2016 April 01

Each year, Inria research teams publish an Activity Report presenting their work and results over the reporting period. These reports follow a common structure, with some optional sections depending on the specific team. They typically begin by outlining the overall objectives and research programme, including the main research themes, goals, and methodological approaches. They also describe the application domains targeted by the team, highlighting the scientific or societal contexts in which their work is situated. The reports then present the highlights of the year, covering major scientific achievements, software developments, or teaching contributions. When relevant, they include sections on software, platforms, and open data, detailing the tools developed and how they are shared. A substantial part is dedicated to new results, where scientific contributions are described in detail, often with subsections specifying participants and associated keywords. Finally, the Activity Report addresses funding, contracts, partnerships, and collaborations at various levels, from industrial agreements to international cooperations. It also covers dissemination and teaching activities, such as participation in scientific events, outreach, and supervision. The document concludes with a presentation of scientific production, including major publications and those produced during the year.

Keywords

Computer sciences and digital sciences

A3.1.8. – Big data (production, storage, transfer)
A6.1.1. – Continuous Modeling (PDE, ODE)
A6.1.2. – Stochastic Modeling
A6.1.4. – Multiscale modeling
A6.1.5. – Multiphysics modeling
A6.2.1. – Numerical analysis of PDE and ODE
A6.2.4. – Statistical methods
A6.2.6. – Optimization
A6.2.7. – HPC for machine learning
A6.3.1. – Inverse problems
A6.3.2. – Data assimilation
A6.3.4. – Model reduction
A6.3.5. – Uncertainty Quantification
A6.4.6. – Optimal control
A6.5.2. – Fluid mechanics
A6.5.4. – Waves
A9.2.1. – Supervised learning
A9.2.2. – Unsupervised learning
A9.2.5. – Bayesian methods
A9.2.6. – Neural networks
A9.2.7. – Kernel methods
A9.2.8. – Deep learning

Other research topics and application domains

B3.2. – Climate and meteorology
B3.3.2. – Water: sea & ocean, lake & river
B3.3.4. – Atmosphere
B3.4.1. – Natural risks
B4.3.2. – Hydro-energy
B4.3.3. – Wind energy
B9.11.1. – Environmental risks

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2 Overall objectives

The general scope of the AIRSEA project-team is to develop *mathematical and computational methods for the modeling of oceanic and atmospheric flows*. The mathematical tools used involve both *deterministic and statistical approaches*. The main research topics cover a) modeling and coupling b) model reduction for sensitivity analysis, coupling and multiscale optimizations c) sensitivity analysis, parameter estimation and risk assessment d) algorithms for high performance computing. The range of application is from climate modeling to the prediction of extreme events.

3 Research program

Recent events have raised questions regarding the social and economic implications of anthropic alterations of the Earth system, i.e. climate change and the associated risks of increasing extreme events. Ocean and atmosphere, coupled with other components (continent and ice) are the building blocks of the Earth system. A better understanding of the ocean atmosphere system is a key ingredient for improving prediction of such events. Numerical models are essential tools to understand processes, and simulate and forecast events at various space and time scales. Geophysical flows generally have a number of characteristics that make it difficult to model them. This justifies the development of specifically adapted mathematical methods:

- Geophysical flows are strongly non-linear. Therefore, they exhibit interactions between different scales, and unresolved small scales (smaller than mesh size) of the flows have to be **parameterized** in the equations.

- Geophysical fluids are non closed systems. They are open-ended in their scope for including and dynamically coupling different physical processes (e.g., atmosphere, ocean, continental water, etc). **Coupling** algorithms are thus of primary importance to account for potentially significant feedback.
- Numerical models contain parameters which cannot be estimated accurately either because they are difficult to measure or because they represent some poorly known subgrid phenomena. There is thus a need for **dealing with uncertainties**. This is further complicated by the turbulent nature of geophysical fluids.
- The computational cost of geophysical flow simulations is huge, thus requiring the use of **reduced models, multiscale methods** and the design of algorithms ready for **high performance computing** platforms.

Our scientific objectives are divided into four major points. The first objective focuses on developing advanced mathematical methods for both the ocean and atmosphere, and the coupling of these two components. The second objective is to investigate the derivation and use of model reduction to face problems associated with the numerical cost of our applications. The third objective is directed toward the management of uncertainty in numerical simulations. The last objective deals with efficient numerical algorithms for new computing platforms. As mentioned above, the targeted applications cover oceanic and atmospheric modeling and related extreme events using a hierarchy of models of increasing complexity.

3.1 Modeling for oceanic and atmospheric flows

Current numerical oceanic and atmospheric models suffer from a number of well-identified problems. These problems are mainly related to lack of horizontal and vertical resolution, thus requiring the parameterization of unresolved (subgrid scale) processes and control of discretization errors in order to fulfill criteria related to the particular underlying physics of rotating and strongly stratified flows. Oceanic and atmospheric coupled models are increasingly used in a wide range of applications from global to regional scales. Assessment of the reliability of those coupled models is an emerging topic as the spread among the solutions of existing models (e.g., for climate change predictions) has not been reduced with the new generation models when compared to the older ones.

Advanced methods for modeling 3D rotating and stratified flows The continuous increase of computational power and the resulting finer grid resolutions have triggered a recent regain of interest in numerical methods and their relation to physical processes. Going beyond present knowledge requires a better understanding of numerical dispersion/dissipation ranges and their connection to model fine scales. Removing the leading order truncation error of numerical schemes is thus an active topic of research and each mathematical tool has to adapt to the characteristics of three dimensional stratified and rotating flows. Studying the link between discretization errors and subgrid scale parameterizations is also arguably one of the main challenges.

Complexity of the geometry, boundary layers, strong stratification and lack of resolution are the main sources of discretization errors in the numerical simulation of geophysical flows. This emphasizes the importance of the definition of the computational grids (and coordinate systems) both in horizontal and vertical directions, and the necessity of truly multi resolution approaches. At the same time, the role of the small scale dynamics on large scale circulation has to be taken into account. Such parameterizations may be of deterministic as well as stochastic nature and both approaches are taken by the AIRSEA team. The design of numerical schemes consistent with the parameterizations is also arguably one of the main challenges for the coming years. This work is complementary and linked to that on parameters estimation described in 3.3.

Ocean Atmosphere interactions and formulation of coupled models State-of-the-art climate models (CMs) are complex systems under continuous development. A fundamental aspect of climate modeling is the representation of air-sea interactions. This covers a large range of issues: parameterizations of atmospheric and oceanic boundary layers, estimation of air-sea fluxes, time-space numerical schemes, non conforming grids, coupling algorithms . . . Many developments related to these different aspects were performed over the last 10-15 years, but were in general conducted independently of each other.

The aim of our work is to revisit and enrich several aspects of the representation of air-sea interactions in CMs, paying special attention to their overall consistency with appropriate mathematical tools. We intend to work consistently on the physics and numerics. Using the theoretical framework of global-in-time

Schwarz methods, our aim is to analyze the mathematical formulation of the parameterizations in a coupling perspective. From this study, we expect improved predictability in coupled models (this aspect will be studied using techniques described in 3.3). Complementary work on space-time nonconformities and acceleration of convergence of Schwarz-like iterative methods (see 8.1.2) are also conducted.

3.2 Model reduction / multiscale algorithms

The high computational cost of the applications is a common and major concern to have in mind when deriving new methodological approaches. This cost increases dramatically with the use of sensitivity analysis or parameter estimation methods, and more generally with methods that require a potentially large number of model integrations.

A dimension reduction, using either stochastic or deterministic methods, is a way to reduce significantly the number of degrees of freedom, and therefore the calculation time, of a numerical model.

Model reduction Reduction methods can be deterministic (proper orthogonal decomposition, other reduced bases) or stochastic (polynomial chaos, Gaussian processes, kriging), and both fields of research are very active. Choosing one method over another strongly depends on the targeted application, which can be as varied as real-time computation, sensitivity analysis (see e.g., Section 8.3.1) or optimisation for parameter estimation (see below).

Our goals are multiple, but they share a common need for certified error bounds on the output. Our team has a 4-year history of working on certified reduction methods and has a unique positioning at the interface between deterministic and stochastic approaches. Thus, it seems interesting to conduct a thorough comparison of the two alternatives in the context of sensitivity analysis. Efforts will also be directed toward the development of efficient greedy algorithms for the reduction, and the derivation of goal-oriented sharp error bounds for non linear models and/or non linear outputs of interest. This will be complementary to our work on the deterministic reduction of parametrized viscous Burgers and Shallow Water equations where the objective is to obtain sharp error bounds to provide confidence intervals for the estimation of sensitivity indices.

Reduced models for coupling applications Global and regional high-resolution oceanic models are either coupled to an atmospheric model or forced at the air-sea interface by fluxes computed empirically preventing proper physical feedback between the two media. Thanks to high-resolution observational studies, the existence of air-sea interactions at oceanic mesoscales (i.e., at $O(1km)$ scales) have been unambiguously shown. Those interactions can be represented in coupled models only if the oceanic and atmospheric models are run on the same high-resolution computational grid, and are absent in a forced mode. Fully coupled models at high-resolution are seldom used because of their prohibitive computational cost. The derivation of a reduced model as an alternative between a forced mode and the use of a full atmospheric model is an open problem.

Multiphysics coupling often requires iterative methods to obtain a mathematically correct numerical solution. To mitigate the cost of the iterations, we will investigate the possibility of using reduced-order models for the iterative process. We will consider different ways of deriving a reduced model: coarsening of the resolution, degradation of the physics and/or numerical schemes, or simplification of the governing equations. At a mathematical level, we will strive to study the well-posedness and the convergence properties when reduced models are used. Indeed, running an atmospheric model at the same resolution as the ocean model is generally too expensive to be manageable, even for moderate resolution applications. To account for important fine-scale interactions in the computation of the air-sea boundary condition, the objective is to derive a simplified boundary layer model that is able to represent important 3D turbulent features in the marine atmospheric boundary layer.

Reduced models for multiscale optimization The field of multigrid methods for optimisation has known a tremendous development over the past few decades. However, it has not been applied to oceanic and atmospheric problems apart from some crude (non-converging) approximations or applications to simplified and low dimensional models. This is mainly due to the high complexity of such models and to the difficulty in handling several grids at the same time. Moreover, due to complex boundaries and physical phenomena, the grid interactions and transfer operators are not trivial to define.

Multigrid solvers (or multigrid preconditioners) are efficient methods for the solution of variational data assimilation problems. We would like to take advantage of these methods to tackle the optimization problem in high dimensional space. High dimensional control space is obtained when dealing with parameter fields estimation, or with control of the full 4D (space time) trajectory. It is important since it enables us to take into account model errors. In that case, multigrid methods can be used to solve the large scales of the problem at a lower cost, this being potentially coupled with a scale decomposition of the variables themselves.

3.3 Dealing with uncertainties

There are many sources of uncertainties in numerical models. They are due to imperfect external forcing, poorly known parameters, missing physics and discretization errors. Studying these uncertainties and their impact on the simulations is a challenge, mostly because of the high dimensionality and non-linear nature of the systems. To deal with these uncertainties we work on three axes of research, which are linked: sensitivity analysis, parameter estimation and risk assessment. They are based on either stochastic or deterministic methods.

Sensitivity analysis Sensitivity analysis (SA), which links uncertainty in the model inputs to uncertainty in the model outputs, is a powerful tool for model design and validation. First, it can be a pre-stage for parameter estimation (see 3.3), allowing for the selection of the more significant parameters. Second, SA permits understanding and quantifying (possibly non-linear) interactions induced by the different processes defining e.g., realistic ocean atmosphere models. Finally SA allows for validation of models, checking that the estimated sensitivities are consistent with what is expected by the theory. On ocean, atmosphere and coupled systems, only first order deterministic SA are performed, neglecting the initialization process (data assimilation). AIRSEA members and collaborators proposed to use second order information to provide consistent sensitivity measures, but so far it has only been applied to simple academic systems. Metamodels are now commonly used, due to the cost induced by each evaluation of complex numerical models: mostly Gaussian processes, whose probabilistic framework allows for the development of specific adaptive designs, and polynomial chaos not only in the context of intrusive Galerkin approaches but also in a black-box approach. Until recently, global SA was based primarily on a set of engineering practices. New mathematical and methodological developments have led to the numerical computation of Sobol' indices, with confidence intervals assessing for both metamodel and estimation errors. Approaches have also been extended to the case of dependent entries, functional inputs and/or output and stochastic numerical codes. Other types of indices and generalizations of Sobol' indices have also been introduced.

Concerning the stochastic approach to SA we plan to work with parameters that show spatio-temporal dependencies and to continue toward more realistic applications where the input space is of huge dimension with highly correlated components. Sensitivity analysis for dependent inputs also introduces new challenges. In our applicative context, it would seem prudent to carefully learn the spatio-temporal dependences before running a global SA. In the deterministic framework we focus on second order approaches where the sought sensitivities are related to the optimality system rather than to the model; i.e., we consider the whole forecasting system (model plus initialization through data assimilation).

All these methods allow for computing sensitivities and more importantly a posteriori error statistics.

Parameter estimation Advanced parameter estimation methods are barely used in ocean, atmosphere and coupled systems, mostly due to a difficulty of deriving adequate response functions, a lack of knowledge of these methods in the ocean-atmosphere community, and also to the huge associated computing costs. In the presence of strong uncertainties on the model but also on parameter values, simulation and inference are closely associated. Filtering for data assimilation and Approximate Bayesian Computation (ABC) are two examples of such association.

The stochastic approach can be compared with the deterministic approach, which allows to determine the sensitivity of the flow to parameters and optimize their values relying on data assimilation. This approach is already shown to be capable of selecting a reduced space of the most influent parameters in the local parameter space and to adapt their values in view of correcting errors committed by the numerical approximation. This approach assumes the use of automatic differentiation of the source code with respect to the model parameters, and optimization of the obtained raw code.

AIRSEA assembles all the required expertise to tackle these difficulties. As mentioned previously, the choice of parameterization schemes and their tuning has a significant impact on the result of model simulations. Our research will focus on parameter estimation for parameterized Partial Differential Equations (PDEs) and also for parameterized Stochastic Differential Equations (SDEs). Deterministic approaches are based on optimal control methods and are local in the parameter space (i.e., the result depends on the starting point of the estimation) but thanks to adjoint methods they can cope with a large number of unknowns that can also vary in space and time. Multiscale optimization techniques as described in 8.2 will be one of the tools used. This in turn can be used either to propose a better (and smaller) parameter set or as a criterion for discriminating parameterization schemes. Statistical methods are global in the parameter state but may suffer from the curse of dimensionality. However, the notion of parameter can also be extended to functional parameters. We may consider as parameter a functional entity such as a boundary condition on time, or a probability density function in a stationary regime. For these purposes, non-parametric estimation will also be considered as an alternative.

Risk assessment Risk assessment in the multivariate setting suffers from a lack of consensus on the choice of indicators. Moreover, once the indicators are designed, it still remains to develop estimation procedures, efficient even for high risk levels. Recent developments for the assessment of financial risk have to be considered with caution as methods may differ pertaining to general financial decisions or environmental risk assessment. Modeling and quantifying uncertainties related to extreme events is of central interest in environmental sciences. In relation to our scientific targets, risk assessment is very important in several areas: hydrological extreme events, cyclone intensity, storm surges... Environmental risks most of the time involve several aspects which are often correlated. Moreover, even in the ideal case where the focus is on a single risk source, we have to face the temporal and spatial nature of environmental extreme events. The study of extremes within a spatio-temporal framework remains an emerging field where the development of adapted statistical methods could lead to major progress in terms of geophysical understanding and risk assessment thus coupling data and model information for risk assessment.

Based on the above considerations we aim to answer the following scientific questions: how to measure risk in a multivariate/spatial framework? How to estimate risk in a non stationary context? How to reduce dimension (see 3.2) for a better estimation of spatial risk?

Extreme events are rare, which means there is little data available to make inferences of risk measures. Risk assessment based on observation therefore relies on multivariate extreme value theory. Interacting particle systems for the analysis of rare events is commonly used in the community of computer experiments. An open question is the pertinence of such tools for the evaluation of environmental risk.
Most numerical models are unable to accurately reproduce extreme events. There is therefore a real need to develop efficient assimilation methods for the coupling of numerical models and extreme data.

3.4 High performance computing

Methods for sensitivity analysis, parameter estimation and risk assessment are extremely costly due to the necessary number of model evaluations. This number of simulations require considerable computational resources, depends on the complexity of the application, the number of input variables and desired quality of approximations. To this aim, the AIRSEA team is an intensive user of HPC computing platforms, particularly grid computing platforms. The associated grid deployment has to take into account the scheduling of a huge number of computational requests and the links with data-management between these requests, all of these as automatically as possible. In addition, there is an increasing need to propose efficient numerical algorithms specifically designed for new (or future) computing architectures and this is part of our scientific objectives. According to the computational cost of our applications, the evolution of high performance computing platforms has to be taken into account for several reasons. While our applications are able to exploit space parallelism to its full extent (oceanic and atmospheric models are traditionally based on a spatial domain decomposition method), the spatial discretization step size limits the efficiency of traditional parallel methods. Thus the inherent parallelism is modest, particularly for the case of relative coarse resolution but with very long integration time (e.g., climate modeling). Paths toward new programming paradigms are thus needed. As a step in that direction, we plan to focus our research on parallel in time methods.

New numerical algorithms for high performance computing Parallel in time methods can be classified into three main groups. In the first group, we find methods using parallelism across the method, such as parallel integrators for ordinary differential equations. The second group considers parallelism across the problem. Falling into this category are methods such as waveform relaxation where the space-time system is decomposed into a set of subsystems which can then be solved independently using some form of relaxation techniques or multigrid reduction in time. The third group of methods focuses on parallelism across the steps. One of the best known algorithms in this family is parareal. Other methods combining the strengths of those listed above (e.g., PFASST) are currently under investigation in the community.

Parallel in time methods are iterative methods that may require a large number of iteration before convergence. Our first focus will be on the convergence analysis of parallel in time (Parareal / Schwarz) methods for the equation systems of oceanic and atmospheric models. Our second objective will be on the construction of fast (approximate) integrators for these systems. This part is naturally linked to the model reduction methods of Section 8.2.1. Fast approximate integrators are required both in the Schwarz algorithm (where a first guess of the boundary conditions is required) and in the Parareal algorithm (where the fast integrator is used to connect the different time windows). Our main application of these methods will be on climate (i.e., very long time) simulations. Our second application of parallel in time methods will be in the context of optimization methods. In fact, one of the major drawbacks of the optimal control techniques used in 3.3 is a lack of intrinsic parallelism in comparison with ensemble methods. Here, parallel in time methods also offer ways to better efficiency. The mathematical key point is centered on how to efficiently couple two iterative methods (i.e., parallel in time and optimization methods).

4 Application domains

The Ocean-Atmosphere System The evolution of natural systems, in the short, mid, or long term, has extremely important consequences for both the global Earth system and humanity. Forecasting this evolution is thus a major challenge from the scientific, economic, and human viewpoints.

Humanity has to face the problem of **global warming**, brought on by the emission of greenhouse gases from human activities. This warming will probably cause huge changes at global and regional scales, in terms of climate, vegetation and biodiversity, with major consequences for local populations. Research has therefore been conducted over the past 15 to 20 years in an effort to model the Earth's climate and forecast its evolution in the 21st century in response to anthropic action.

With regard to short-term forecasts, the best and oldest example is of course **weather forecasting**. Meteorological services have been providing daily short-term forecasts for several decades which are of crucial importance for numerous human activities.

Numerous other problems can also be mentioned, like **seasonal weather forecasting** (to enable powerful phenomena like an El Niño event or a drought period to be anticipated a few months in advance), **operational oceanography** (short-term forecasts of the evolution of the ocean system to provide services for the fishing industry, ship routing, defense, or the fight against marine pollution) or the prediction of **floods**.

As mentioned previously, mathematical and numerical tools are omnipresent and play a fundamental role in these areas of research. In this context, the vocation of AIRSEA is not to carry out numerical prediction, but to address mathematical issues raised by the development of prediction systems for these application fields, in close collaboration with geophysicists.

5 Social and environmental responsibility

Most of the research activities of the AIRSEA team are directed towards the improvement of numerical systems of the ocean and the atmosphere. This includes the development of appropriate numerical methods, model/parameter calibration using observational data and uncertainty quantification for decision making. The AIRSEA team members work in close collaboration with the researchers in the field of geophyscial fluid and are partners of several interdisciplinary projects. They also strongly contribute to the development of state of the art numerical systems, like NEMO and CROCO in the ocean community.

6 Highlights of the year

- Clémentine Prieur was a Kirk Distinguished Visiting Fellow in the Isaac Newton Institute for Mathematical Sciences, Cambridge. She participated in the Programme "Representing, calibrating & leveraging prediction uncertainty, from statistics to machine learning" and delivered a lecture titled "On feasible set estimation with Bayesian active learning" www.newton.ac.uk.

- E. Blayo and A. Vidard have produced a series of pedagogical videos on numerical ocean modeling, in collaboration with L'Esprit Sorcier TV, available here).

7 Latest software developments, platforms, open data

7.1 Latest software developments

7.1.1 AGRIF

Name: Adaptive Grid Refinement In Fortran

Keyword: Mesh refinement

Scientific Description: AGRIF is a Fortran 90 package for the integration of full adaptive mesh refinement (AMR) features within a multidimensional finite difference model written in Fortran. Its main objective is to simplify the integration of AMR potentialities within an existing model with minimal changes. Capabilities of this package include the management of an arbitrary number of grids, horizontal and/or vertical refinements, dynamic regridding, parallelization of the grids interactions on distributed memory computers. AGRIF requires the model to be discretized on a structured grid, like it is typically done in ocean or atmosphere modelling.

Functional Description: AGRIF is a Fortran 90 package for the integration of full adaptive mesh refinement (AMR) features within a multidimensional finite difference model written in Fortran. Its main objective is to simplify the integration of AMR potentialities within an existing model with minimal changes. Capabilities of this package include the management of an arbitrary number of grids, horizontal and/or vertical refinements, dynamic regridding, parallelization of the grids interactions on distributed memory computers. AGRIF requires the model to be discretized on a structured grid, like it is typically done in ocean or atmosphere modelling.

News of the Year: Within the framework of a European Copernicus contract, improvements have been made to the management of parallelization (assignment of processors to computational grids).

URL: `https://gitlab.inria.fr/ldebreu/agrif`

Publications: tel-01546328, hal-00387435

Contact: Laurent Debreu

Participant: Laurent Debreu

7.1.2 NEMOVAR

Name: Variational data assimilation for NEMO

Keywords: Oceanography, Data assimilation, Adjoint method, Optimal control

Functional Description: NEMOVAR is a state-of-the-art multi-incremental variational data assimilation system with both 3D and 4D var capabilities, and which is designed to work with NEMO on the native ORCA grids. The background error covariance matrix is modelled using balance operators for the multivariate component and a diffusion operator for the univariate component. It can also be formulated as a linear combination of covariance models to take into account multiple correlation length scales associated with ocean variability on different scales. NEMOVAR has recently been enhanced with the addition of ensemble data assimilation and multi-grid assimilation capabilities. It is used operationnaly in both ECMWF and the Met Office (UK)

Contact: Patrick Vidard

Partners: CERFACS, ECMWF, Met Office

7.1.3 SWEET

Name: Shallow Water Equation Environment for Tests, Awesome!

Keywords: High-Performance Computing, Time integration methods

Functional Description: Solver for various kinds of PDEs (not only Shallow Water) in 1D, the bi-periodic plane or sphere based on using global spectral methods (Fourier / spherical harmonics).

SWEET supports periodic boundary conditions for - the bi-periodic plane (2D torus) - the sphere

Space discretization - PLANE: Spectral methods based on Fourier space - PLANE: Finite differences - SPHERE: Spherical Harmonics

Time discretization - Explicit RK - Implicit RK - Crank-Nicolson - Semi-Lagrangian - Parallel-in-time - Parareal - PFASST - Rational approximation of exponential Integrators (REXI) ...and many more time steppers...

Special features - Graphical user interface - Fast Helmholtz solver in spectral space - Easy-to-code in C++ ...

There's support for various applications - Shallow-water equations on plane/sphere - Advection - Burgers' ...

URL: `https://sweet.gitlabpages.inria.fr/sweet-www/`

Contact: Martin Schreiber

Partners: University of São Paulo, Technical University of Munich (TUM)

7.2 Open data

8 New results

8.1 Modeling for Oceanic and Atmospheric flows

8.1.1 Numerical Schemes for Ocean Modeling

Participants: Eric Blayo, Laurent Debreu, Gabriel Derrida, Florian Lemarié, Gurvan Madec, Pierre Lozano.

Beyond the hydrostatic assumption Most large scale ocean models are based on the so-called "primitive equations", which use the hydrostatic and incompressibility assumptions. However, with the increase of resolution, a systematic use of the hydrostatic assumption becomes less valid. The French regional oceanic modeling system CROCO (Coastal and Regional Ocean COmmunity model) developed these last years allows for the use of either the hydrostatic incompressible (HI) equations and the non-hydrostatic compressible (NHC) equations, the latter being much more computationally expensive. A natural idea is thus to limit the use of the NHC version to some particular regions of interest where the hydrostatic assumption is not relevant, and to nest such local NHC zooms within a larger model using the HI version. However such a coupling is quite delicate from a mathematical point of view, due to the different nature of hydrostatic and nonhydrostatic equations (where the vertical velocity is either a diagnostic or a prognostic variable). In his PhD ([20], defended in December 2025), P. Lozano has pointed out and analyzed the fundamental problems occurring when coupling hydrostatic and nonhydrostatic models [16]. He also performed numerical simulations in a simplified configuration to illustrate their impact, and proposed avenues toward improved coupling procedures, in particular through the definition of an adapted transition zone.

Optimized vertical coordinates for ocean modeling In the PhD work of G. Derrida (started in October 2023), we are investigating new vertical coordinate systems for ocean modeling, with the objective of improving the representation of key physical processes while maintaining numerical accuracy and efficiency. The idea is to design flexible vertical coordinates that can adapt to the specific characteristics of the oceanic regions being modeled, such as areas with strong stratification or complex topography. The first part of the work focuses on an improved representation of the vertical normal modes in the ocean, which are crucial for accurately capturing internal wave dynamics. In a submitted papier, we show that the vertical discretization can be improved two ways: first by developing a new original scheme (based on error compensation for discrete eigenvalues systems) in the vertical, and second by optimizing the vertical grid point distribution to better capture the structure of the modes.
This work is carried out in collaboration with Mercator Ocean International.

8.1.2 Coupling Methods for Oceanic and Atmospheric Models and representation of the Air-Sea Interface

Participants: Eric Blayo, Emile Deléage, Florian Lemarié.

The Airsea team is involved in the modeling and algorithmic aspects of ocean-atmosphere (OA) coupling. We have been actively working on the analysis of such coupling both in terms of continuous and numerical formulations. Particular attention is paid to the inclusion of physical parameterizations in our theoretical framework. Our activities have led to practical implementations in state-of-the-art oceanic and Earth system models. Our focus during the last few years has been on the following topics:

1. *Mathematical analysis of OA coupling formulation* Coupling problems arising in Earth system modeling involve turbulent boundary layers, for which parameterizations induce non-standard interface conditions and nonlinear diffusion operators. The complexity of these coupled models is increasing far more rapidly than their mathematical formalization and theoretical analysis. Notably, rigorous studies that explicitly incorporate boundary layer parameterizations remain scarce in the literature. As part of the postdoctoral project of Emile Deléage (funded by the PEPR MathsVives program), the mathematical analysis of a coupling problem between an air column and a water column is being conducted, incorporating turbulent boundary layer parameterizations based on turbulent kinetic energy and an interface flux computation derived from Monin–Obukhov similarity theory to be representative of the formulation of realistic coupled models. One objective is to study the well-posedness of strong solutions over a short time interval.

2. *Impact of the coupling formulation in a realistic context* Building on preliminary work carried out by members of the airsea team, a Schwarz-like iterative method has been applied in a state-of-the-art Earth-System model (IPSL-CM6) to evaluate the consequences of inaccuracies in the usual ad-hoc ocean-atmosphere coupling algorithms used in realistic models [48, 49]. Numerical results obtained with an iterative process show large differences at sunrise and sunset compared to usual ad-hoc algorithms, thus showing that synchrony errors inherent to ad-hoc coupling methods can be large. As part of V. Schüller's thesis in collaboration with Lund University, a single-column version of the EC-Earth climate model has been used to further our study of coupling algorithms. The goal was to extend the analysis of [48] using a less complex model that remains representative of the parameterization schemes employed in 3D models. This single-column model has made it possible to focus on ocean-atmosphere coupling in the presence of sea ice. It was identified, among other findings, that the convergence of an iterative coupling in this context was compromised due to non-differentiabilities (jumps) in the parameterization of albedo. We showed that a regularized albedo parameterization solved the convergence issues. This work is summarized in a publication [1]. An important conclusion of this work is that global-in-time Schwarz algorithms provide a general framework for performing sanity checks during the development of model physics. This approach is expected to offer a means to better understand the interactions between parameterizations developed independently and in an uncoupled framework.

3. *A simplified atmospheric boundary layer model for oceanic purposes* Our activities within the ongoing ENMASSE project is dedicated to the development of a simplified model of the marine atmospheric boundary layer of intermediate complexity between a bulk parameterization and a full three-dimensional atmospheric model, and to its integration into the NEMO general circulation model (based on [46]). A constraint in the conception of such a simplified model is to allow an apt representation of the main air-sea feedbacks while keeping the computational efficiency and flexibility inherent to ocean-only modeling. The simplified model, called ABL3d, has been derived using multiple scales asymptotic techniques to cast the equations in terms of perturbations around an ambient state given by a large-scale dataset. Such simplified model leads to good results for academic semi-idealized cases [45]. The objective is now to extend the analysis to realistic cases in the framework of the ENMASSE project funded by the Copernicus Marine Environment Monitoring Service (CMEMS). Over the course of 2025, the ABL3d approach was implemented in the NEMO model, and preprocessing tools were developed to support the preparation of ambient state input datasets. In parallel, in the framework of the AIRSEA/Eviden collaboration, an objective is to design a surrogate via learning strategies of the response of the atmospheric boundary layer to anomalies in ocean surface temperatures and currents. An M2 internship was co-supervised on this topic (A. Ameziane, M2 Calcul Haute Performance, Simulation; Saclay) in 2025, and will be followed by a CIFRE PhD thesis starting in 2026.

These topics are addressed through strong collaborations between the applied mathematicians and the climate and operational community (Météo-France, Ifremer, SHOM, Mercator-Ocean, LMD, and LOCEAN). Airsea team members play a major role in the structuration of a multi-disciplinary scientific community working on ocean-atmosphere coupling spanning a broad range from mathematical theory to practical implementations in climate and operational models.

8.1.3 Physics-Dynamics coupling: Consistent subgrid-scale modeling

Participants: Eric Blayo, Florian Lemarié, Manolis Perrot.

A few years ago, the AIRSEA team has started to work on new topics around physics-dynamics coupling [43]. Schematically, numerical models consist of two blocks generally identified as "physics" and "dynamics" which are often developed separately. The "Physics" represents unresolved or under-resolved processes with typical scales below model resolution, while the "dynamics" corresponds to a discrete representation in space and time of resolved processes. Unresolved processes cannot be ignored because they directly influence the resolved part of the flow since energy is continuously transferred between scales. The interplay between resolved and unresolved scales is a large, incomplete, and complex topic for which there is still much to do within the Earth system modeling community [38]. During the last year, we worked on the following topics:

1. *Representation of penetrative convection in oceanic models* Accounting for the mean effect of subgrid-scale intermittent coherent structures like convective plumes is very challenging. Currently, this is done very crudely in ocean models (vertical diffusion is locally increased to "mix" unstable density profiles). A difficulty is that in convective conditions, turbulent fluxes are dominated by processes unrelated to local gradients, thus invalidating the usual downgradient (a.k.a. eddy-diffusion) approach. In the framework of the PhD of M. Perrot [21], a first step is to study the derivation of mass-flux convection schemes arising from a multi-fluid decomposition to extend them specifically to the oceanic context [10]. This extension is done under certain "consistency" constraints: energetic considerations and scale-awareness of the resulting model. Reference LES simulations have been developed to guide the formulation of unknown/uncertain free parameters (coefficients or functions) in the proposed extended mass-flux scheme [9]. A first calibration of these free parameters was carried out using Bayesian approaches and will be further pursued within the ANR PLUME project.

2. *Link between stochastic grid perturbation and Location Uncertainty (LU) framework* Recent oceanic parameterizations "under Location Uncertainty" are based on the hypothesis that the small-scale

processes are uncorrelated in time. Our work on this topic investigates the theoretical connection between Stochastic Grid Perturbation (SGP) and LU in ocean modeling. The LU framework, based on random velocity fluctuations, has proven effective in organizing large-scale flow and reproducing long-term statistical properties. SGP offers a simpler alternative by perturbing the computational grid across ensemble members to represent small-scale uncertainties in high-resolution predictability studies. We derive SGP from the LU formalism, introduce time-correlated noise to preserve grid structure, and demonstrate that a compensating advection term maintains LU properties and enables strict equivalence between both approaches. Numerical experiments using a 3-layer Quasi-Geostrophic model confirm that the compensating advection term is essential to achieve exact consistency between SGP and LU implementations. This work is summarized in a publication [4].

Those topics are addressed through collaborations with the climate and operational community (Météo-France, SHOM, Mercator-Ocean, and IGE). The AIRSEA team is involved in the PLUME ANR project. One of the objectives of this project is to use LES numerical simulations and laboratory experiments of deep convection to calibrate and evaluate physical parameterizations like the one developped in [10] and [9].

8.2 Model Reduction and Multifidelity Methods

8.2.1 Model Reduction

Participants: Clémentine Prieur, Katarina Radišić, Romain Verdière, Arthur Vidard, Olivier Zahm.

When numerical models are too costly to evaluate, it is common to address the task of uncertainty quantification using an approximate model, which is faster to compute. However, constructing such a reduced model (or surrogate) is challenging due to the high number of variables involved. It is therefore crucial to identify the input variables that are most important for building the reduced model.

In [23], we propose a nonlinear dimensionality reduction method that leverages gradient evaluations of the model. The objective is to align the Jacobian of the feature map (a nonlinear function that extracts the key components of the parameters) with the model's gradients. Our main contribution is to use feature maps defined as the first components of a diffeomorphism from $\mathbb{R}^d$ to $\mathbb{R}^d$, parameterized by a Coupling Flows neural network. This architecture preserves essential properties of the feature map, notably ensuring that its level sets remain simply connected. In addition, we propose a dimension augmentation trick to increase the approximation power of feature detection. A generalization to vector-valued functions demonstrate that our methodology directly applies to learning autoencoders, showing the versatility of our proposed framework.

On a related topic, in her PhD work, Katarina Radišić conducted an in-depth investigation into the use of stochastic polynomial chaos expansion, where the coefficients of the polynomial basis are themselves treated as random variables. This approach allows for an efficient representation of external uncertainties while retaining the computational efficiency of a surrogate emulator. Such a framework proves particularly advantageous for problems involving complex systems with significant variability in external conditions. This methodology was adaptated to a hydrological and pesticide transfer model, where the primary external uncertainty arose from variability in rainfall. By incorporating this uncertainty directly into the stochastic framework, the model achieved a flexible representation of system behavior under varying environmental conditions. This was in turn used extensively in the context of sensitivity analysis and for robust parameter estimation, see (8.3.1, 8.3.4).

8.2.2 Multifidelity

Participants: Elise Arnaud, Eric Blayo, Angélique Saillet, Jean-Baptiste Seby, Arthur Vidard, Hélène Hénon.

Multifidelity methods seek to balance the computational load across a hierarchy of models with varying accuracy and evaluation cost, using lower-fidelity models for inexpensive approximations and higher-fidelity ones only when necessary. By integrating information across fidelities, it ensures better performance for complex tasks. The efficiency of such an approach, however, relies on our ability to decide on how to allocate resources on the different level of accuracy in order to reduce overall computation while maintaining accuracy.

1. *Variational data assimilation*

   Incremental Variational Data Assimilation addresses the non-linear least-square optimization challenges inherent in variational data assimilation by minimizing a sequence of linear least-squares cost functions iteratively. However, due to the high dimensionality and ill-conditioning of the associated problems, the computational burden can become prohibitive. To address these challenges, we propose leveraging multifidelity methods and machine learning to improve efficiency and reduce computational costs.

   This is explored in the context of Hélène Hénon's PhD research, investigating the application of multifidelity strategies to tackle the inner loop optimization problem. The research focuses on leveraging a low-fidelity model and randomization techniques to construct a limited-memory preconditioner, while ensuring an improvement in the conditioning of the Hessian. In particular, the goal is to achieve a reduced condition number even when the preconditioner is derived from a low-fidelity model and thus an approximation of the true problem. A key advantage of this approach lies in the massively parallel nature of the preconditioner construction, which enables efficient parallelization of the 4D-Var method. Her work will be presented in two international conferences early 2026

   In a related work, we propose utilizing Deep Neural Networks to construct a preconditioner. This preconditioner is trained on properties derived from the singular value decomposition of the matrices, ensuring it effectively mitigates the effects of ill-conditioning. To further optimize resource utilization, the training dataset is designed to be constructed dynamically during the optimization process, thereby reducing storage requirements. This work is detailed in a submitted paper [34]

2. *Exploration of climate scenarios*

   Numerical models are important tools for predicting climate change and helping policy-makers to make decisions (e.g. in terms of protecting marine areas, land use or defining fishing quotas). The huge complexity of models and the generally very high cost of numerical simulations make an exhaustive exploration of the parameter space, corresponding to all possible scenarios and all the model's internal options, completely illusory. The idea is therefore to use statistical tools for the design of experiments. These tools enable us the parameter combinations that provide the most information on a given quantity of interest (QoI) calculated from the simulation carried out. The design of experiments also has the advantage of being able to be built adaptively, in order to take into account the results of pre-existing simulations, performed with various models, under various scenarios.

   In Angelique Saillet's PhD, to address these issues, we use multi-fidelity Gaussian process regression in the context of a marine biogeochemistry model, in collaboration with M. Baklouti (MIO Marseille). A sensitivity analysis has been performed on a 1D (vertical) version of the model, in order to identify the parameters that are most influential for certain quantities of interest (QoI). This enables the construction of meta-models for these QoI, thanks to the use of Gaussian processes. The current step consists in evaluating how the use of additional "low-fidelity" simulations performed with one or several degraded versions of the model can improve these metamodels. Simpler methods but on a more realistic configuration have also been presented in [14]

   In Jean-Baptiste Seby's PhD (started on October 2025, funded by Institut des Mathématiques pour la Planète Terre), we aim to explore these methodologies for regional climate models, to try to make the best use of archived simulations. New theoretical developments will probably be necessary. This work is done in collaboration with Pierre Nabat (Meteo France) and Céline Helbert (Ecole Centrale Lyon).

8.3 Dealing with Uncertainties

8.3.1 Sensitivity Analysis

Participants: Alexis Anagostakis, Elise Arnaud, Clémentine Prieur, Arthur Vidard, Ri Wang, Olivier Zahm, Mohamed Doumbouya, Katarina Radišić.

Sensitivity analysis is a crucial step in uncertainty quantification as it helps identifying which input variables most influence the variability in a model's output. This understanding guides model simplification, parameter prioritization, and robust decision-making. Our research results this year can be organized around two main axes :

1. *Gradient-based methods*

   When model gradients are available, gradient-based sensitivity methods offer a convenient and efficient alternative to traditional approaches. Over the past year, we have proposed several improvement of such gradient-based sensitivity analysis:

   - The Active Subspaces (AS) method are quite effective for reducing input dimensions of a smooth model, but its performance suffers when models have high-frequency, low-amplitude oscillations, leading to poor feature selection. The work [35] introduces the Mollified Active Subspace (MAS) method, which smoothen (or mollify) the model gradients in order to improve feature selection and surrogate accuracy, offering a better error bound and practical guidelines for balancing smoothing and approximation quality. [50].
   - The optimal sensor placement problem can be understood as a sensitivity analysis of an optimality system with respect to the data. One canonical approach is to maximize the Expected Information Gain (EIG) associated with a given system of obervable quantities. In the PhD project of Mohamed Doumbouya, we consider the case of computationally expensive ocean models. Our approach is to optimize a tractable gradient-based bound of the EIG instead of the EIG itself. We plan to compare different bound- based and EIG-based solutions, and also to accelerate computations via randomized linear algebra.
   - *Quantile oriented sensitivity analysis* Quantiles provide a rich information on model output. Quantile oriented sensitivity analysis measures the sensitivity of the quantiles of model outputs to inputs. In the framework of Ri Wang we investigated random forest based inference of sensitivity measures (QOSA and QOSE). A first paper is in minor revision [51] and a second one should be submitted in 2026.
   - *Sensitivity analysis for stochastic models* We proposed in [6] new algorithms for global sensitivity analysis of non Markovian stochastic compartmental models. The algorithms were implemented on a model designed to the COVID19 pandemics.
   - *Sensitivity Analysis in a Game-Theoretic Approach to an Environmental Management Problem* In [5], we analyzed the added value of sensitivity analysis to solve environmental management problems.

2. *Given data inference for sensitivity analysis* In [41], we introduce two semi-parametric estimators of Sobol' indices of any order. These estimators can be computed from a single input-output sample. We prove assymptotic normality and efficiency.

3. *GSA for hydrological models*

   - Traditional global sensitivity analysis (GSA) often neglects natural variability in forcing conditions, limiting result validity. In [12], we treat Sobol' indices as random variables influenced by forcing variability and estimate them efficiently using stochastic polynomial chaos expansions. Applying this to a hydrological model, we found parameter rankings vary with forcing, and proposed an aggregated sensitivity index to enhance GSA robustness and decision reliability.
   - During Mélanie Villain's internship we performed a Sobol' based GSA on Nihm, a realistic ground water hydrological model of the Strengbach catchment area. It enabled the identification of the parameters that most influence the model outputs and provided additional insight into the sources of variability and uncertainty. This work form the basis for Mélanie's PhD (started November 2025) and open the way to parameter estimation. This work is our main contribution to the ANR CASH project.

8.3.2 Bayesian inversion

Participants: Elise Arnaud, Arthur Vidard, Adama Barry, Clément Duhamel, Clémentine Prieur, Olivier Zahm, Hippolyte Signargout.

Bayesian inverse problems become challenging when computational models are expensive, as the repeated evaluations required for inference (e.g., in Markov Chain Monte Carlo) become infeasible. Additionally, complex priors, such as those with heavy tails or multimodal distributions, complicate sampling and convergence, making efficient exploration of the posterior space significantly harder.

In 2025, we launched a collaboration with glaciologists at IGE to estimate Antarctic ice-sheet deep temperatures profiles using satellite data of the radiative emission of the ice. Together with Ghislain Picard (IGE), Olivier Zahm co-supervises Hippolyte Signargout's postdoctoral research, which focuses on developing efficient numerical methods for Bayesian inversion. The challenge lies in the problem's ill-posed nature (weakly informative data) compounded by the sheer volume of satellite data available (several measurement per image pixels).

The PhD thesis of Adama Barry [18] was focused on metamodel based calibration of complex computer codes. A first paper on the design of physical and simulation experiments is in revision [39].

The PhD thesis of Clément Duhamel was focused on set inversion with application to the calibration of wind turbines. In [28] we investigate active learning strategies for set inversion.

8.3.3 Sampling algorithms

Participants: Olivier Zahm, Benjamin Zanger, Lorenzo Calzolari, Clémentine Prieur.

Sampling from high-dimensional distributions that are multi-modal and/or heavy- tailed poses significant challenges across various fields. This is particularly true in large-scale Bayesian inference with sparsity-inducing priors, but also, for example, in molecular dynamics, where the Boltzmann distribution of a molecular system is highly multimodal, each of the mode corresponding to a distinct physical conformation. Conventional sampling methods such as Markov Chain Monte Carlo (MCMC) face difficulties in accurately exploring the entire landscape (modes, tails, etc) of the target distribution, resulting in poor numerical performances. We have made diverse contributions which aim at addressing these challenges, focusing on a range of applications including imaging, epidemiology, and also on providing proof-of-concepts for certain advanced pioneering algorithms.

These contributions involve the development of:

1. dimension reduction techniques [47]
2. transport maps methods [36]
3. preconditioners for stochastic differential equations (SDE) in order to enhance the convergence properties of sampling algorithms [40]
4. sampling algorithms for functional data in a RKHS

8.3.4 Robust inversion

Participants: Elise Arnaud, Exaucé Luweh Adjim Ngarti, Katarina Radišić, Arthur Vidard.

Estimating key parameters in numerical models is a crucial aspect in numerical simulation, particularly when these parameters are not directly observable. Traditional estimation methods infer parameters indirectly from their effects on observable variables, introducing inherent uncertainties. In addition to the parameters to be estimated, numerical models often include uncertain and uncontrollable nuisance parameters, which can further complicate the estimation process.

In Exaucé Ngarti's PhD research, we investigate extending variational inference to account for the presence of nuisance parameters. Ignoring the stochastic nature of these nuisance parameters can lead to suboptimal parameter estimation due to error compensation effects. To address this, we model nuisance parameters as random variables, redefining the numerical model itself as a random variable. This problem is formulated within a Bayesian framework, where the goal is to estimate the posterior distribution by minimizing the Kullback-Leibler divergence over a family of parameterized distributions. To increase the flexibility of this approach, we integrate generative neural networks, such as normalizing flows, to enhance the expressiveness of the posterior approximation. These methods are currently applied to a 1DV ocean model to estimate the subgrid scale convection parametrization.

In Katarina Radišić's PhD research, we explored an alternative approach based on optimal control. Here, the parameter estimation problem is addressed by minimising an objective function. When nuisance parameters are present, the objective function becomes a random variable, adding a layer of complexity. To efficiently handle this uncertainty, we employ stochastic polynomial chaos expansion as a surrogate for the "random" objective function. This technique enables effective exploration of the uncertain parameter space and facilitates the computation of robust parameter estimates. The methodology has been successfully applied to a hydrology and pesticide transfer model, demonstrating its feasibility. This work is described in [22] and is under revision in RESS [31].

8.4 Analysis of reflected Langevin processes

Participants: Clémentine Prieur.

In collaboration with Jose R. Leon (Universidad de la República de Uruguay) and Pierre Etoré (LJK/DATA) we study ergodicity properties of reflected Langevin processes [29]. Our aim is then to propose statistical inference for such models with environmental applications.

8.5 High performance computing

8.5.1 Dynamic compute-resource utilization

Participants: Martin Schreiber.

The way how applications are executed on supercomputers still follows a traditional static resource allocation pattern: Computing resources are allocated at the start of a job which executes the application and are only released at the end of the job's runtime. This still follows the way of running jobs since decades where a dynamic resource allocation over the application's runtime would lead to several benefits: higher utilization of the computing resources, ad-hoc allocation of AI accelerator cards, less energy consumption, faster response for interactive jobs, improved data locality and I/O over the full runtime, support of urgent computing without necessarily killing running jobs, etc. Various attempts have been conducted under different terminologies used such as "evolving" jobs (application-driven dynamic resource changes) and "malleability" (system-driven dynamic resource changes) where we see a hybridization of them required for reaching optimal results.

We continued our roadmap to further develop our new approach called "Dynamic Processes with PSets (DPP)":

- We assessed the feasibility of DPP to be even applied to asynchronous Many-Task (AMT) Runtime System, [11], special issue in SN Computer Science Journal, in Springer Nature.
- Large-scale studies with DPP in collaboration with Barcelona Supercomputing Center and the DataMOVE Inria team, see www.martin-schreiber.info/data/publications, accepted at HiPC 25 conference.
- Design Principles of Dynamic Resource Management for High-Performance Parallel Programming Models, see [44], accepted in DynResHPC'25 workshop at EuroPAR 2025
- Bridging the Gap Between Genericity and Programmability of Dynamic Resources in HPC, see [15], accepted at ISC'25 in Hamburg
- Forming the “Dynamic Resources for HPC (DynResHPC) Consortium” to provide a platform for including a variety of experts.

External collaborators: Dominik Huber (TUM, DataMOVE), Pierre-François Dutot (DataMOVE), Olivier Richard (DataMOVE), Howard Pritchard (LANL), Martin Schulz (TUM)

8.5.2 Hardware-aware numerics

Participants: Hugo Brunie, Laurent Debreu, Julien Remy, Martin Schreiber.

The Poseidon project advanced further, working on the vision to push the performance of the NEMO and CROCO ocean simulation models to the HPC limit with in-depth optimizations that can't be done with currently existing compilers and to simplify the development of highly performing code for model developers.

The underlying idea is to uplift the fluid dynamics equation solver to a DSL-like intermediate representation (IR). This IR is based on a hypergraph with nodes representing computations and (hyper)edges the data flow. The strong formalism of the IR representation forms the foundation for performing the required HPC optimization. Poseidon supports writing back code to the original CROCO ocean model, and two research codes TPDesHoughes and Schweinshaxe; hence, it doesn't require using a different development which would require disruptive changes.

Based on the extracted barotropic solver of the CROCO model, our first **HPC results** are to perform a fully automatic kernel and loop fusion. This already led to a substantial reduction of memory access, leading to speedups of over 2 for GPU code that was considered to be highly optimized by HPC engineers. Our current work is under review: [32].

Due to the strong formalism, further work was conducted in collaboration with Anna Mittermair & Martin Schulz (Technical University of Munich) on **optimizing the communication for distributed memory systems**. Based on Poseidon and the data flow, we can automatically inject nodes into the hypergraph to perform automatic MPI communication. Our current work is under review with a preprint available here: [30].

As part of the Poseidon project, an overlapping Schwarz method for latency hiding has been integrated to the barotropic solver of the Croco ocean model with substantial speedups of around on larger-scale studies on Jean Zay (unpublished).

External collaborators: Anna Mittermair (TUM), Andrew Porter (STFC), Sergi Siso (STFC), Jörg Heinrichs (ABOM)

8.5.3 New time-integration methods

Participants: Martin Schreiber.

We investigated the parallel performance of parallel spectral deferred corrections, a numerical method that enables fine-grained parallelism for the solution of initial value problems. The scheme was applied to the shallow water equations and employs an IMEX splitting, treating fast modes implicitly and slow modes explicitly to ensure efficiency. We present OpenMP-based parallel implementations of parallel SDC in two well-established simulation codes: the finite-volume–based operational ocean model ICON-O and the spherical-harmonics-based research code SWEET. The implementations were benchmarked on a single node of the JUSUF (SWEET) and JUWELS (ICON-O) systems at the Jülich Supercomputing Centre. We demonstrate a reduction in time-to-solution over a range of accuracy levels. For ICON-O, we show speedup compared to the currently used Adams-Bashforth-2 integrator with OpenMP loop parallelization. For SWEET, we demonstrate speedup over serial spectral deferred corrections and a second-order implicit-explicit integrator. See [42] for more information.

We explored and extended semi-Lagrangian exponential methods, which integrate stiff linear terms with exponential time integration and handle nonlinear advection using a semi-Lagrangian approach. These techniques are relevant for partial differential equations found in atmospheric models. A truncation error analysis reveals that existing methods are limited to first-order accuracy due to linear term discretization. To address this, we develop a second-order scheme. Stability comparisons between various Eulerian and semi-Lagrangian exponential methods and a widely used semi-Lagrangian semi-implicit method are conducted. Numerical tests on shallow-water equations confirm the proposed method's improved stability and accuracy, albeit with higher computational costs. However, its stability and cost are comparable to the semi-implicit method, making it a competitive option for atmospheric modeling [3]. This forms an extremely important part for future work on parallel-in-time methods.

Solving partial differential equations is a central task in scientific computing, and this work focuses on the numerical solution of initial value problems governed partly or entirely by linear PDEs using Rational Approximation of Exponential Integration (REXI). REXI replaces sequential time-stepping with a sum of rational terms, enabling parallelization across these terms and thereby offering additional scalability for problems limited by spatial parallelism. We introduce a unified REXI framework, showing its algebraic equivalence to several classical methods, including diagonalized implicit Runge–Kutta schemes, Cauchy-contour integration approaches, and direct approximations, and provide the first comprehensive numerical comparison of these techniques for challenging hyperbolic problems. Performance is demonstrated for the nonlinear shallow-water equations on the rotating sphere, showing that diagonalized low-order Gauss Runge–Kutta methods formulated as REXI achieve up to a 64-fold reduction in computational resources at fixed accuracy compared to existing approaches. See [33].

External collaborators: Pedro S. Peixoto (USP), João C. Steinstraesser (USP), Elizaveta Boriskova (TUM)

9 Bilateral contracts and grants with industry

Participants: Clémentine Prieur, Elise Arnaud, Clément Duhamel, Exaucé Luweh Adjim Ngarti, Lorenzo Calzolari.

9.1 Bilateral contracts with industry

- Consortium CIROQUO – Consortium Industrie Recherche pour l'Optimisation et la QUantification d'incertitude pour les données Onéreuses – gathers academical and technological partners to work on problems related to the exploitation of numerical simulators. This Consortium, created in January 2021, is the continuation of the projects DICE, ReDICE and OQUAIDO which respectively covered the periods 2006-2009, 2011-2015 and 2015-2020. CIROQUO will be continued from 2025 as CIROQUO 2 with new industrial partners such as EDF or Michelin (cf ciroquo.ec-lyon.fr).

- The project "LOMIS" (started in 2025) is a project funded by ESA on ice-sheet temperature retriveal. The partners are IGE-UGA, University of Trento and CNR-IFAC. It will fund the postdoc of Hippolyte Signargout in 2026.

9.2 Bilateral grants with industry

- Funding of Exaucé Luweh Adjim Ngarti's PhD with a CIFRE contract with Eviden. PhD subject: Deep learning for inverse problem in geophysics.
- Funding of Clément Duhamel's PhD (sept. 2020-nov. 2024) by IFP Energies Nouvelles (IFPEN) within the framework of IFPEN and Inria strategic partnership. PhD subject: Gaussian processes-based excursion set estimation for scalar or vector black box functions. Application to the calibration of a numerical wind turbine simulator.
- Funding of Lorenzo Calzolari's PhD (nov. 2024-. . .) by IFP Energies Nouvelles (IFPEN). PhD subject: Active learning with functional inputs: application to wind turbine reliability design.

10 Partnerships and cooperations

10.1 International initiatives

10.1.1 Associate Teams in the framework of an Inria International Lab or in the framework of an Inria International Program

Crocodiles (team.inria.fr/crocodiles/):

Optimization of PDE solvers is one of the big challenges in High-Performance Computing (HPC). This requires not only skills and a deeper understanding of HPC from all the hardware and software layers but also research on software solutions that are sustainable and accepted by the developers and users of these solvers.

This associate team brings together members of ANL and the Inria AIRSEA team who are both currently working on the HPC modernization of models under the aforementioned constraints. This allows us to share, on the one hand, our experience and plans with the model developments. On the other hand, we can strongly benefit from the experience of all the current developments, which share many similarities.

OSCAR

The associate team OSCAR with the National University of Singapore started in 2025. It is concerned with the design of new algorithms to address these issues, building on recent advances in dimension reduction, transport map methods, and preconditioning strategies for stochastic differential equations. These developments are motivated by applications in imaging, epidemiology, geophysics, and molecular dynamics.

10.1.2 STIC/MATH/CLIMAT AmSud projects

SMILE

Participants: Clémentine Prieur, Alexis Anagnostakis.

Title: Statistical modeling, nonparametric inference and model selection for complex data

Program: MATH-AmSud

Duration: January 1, 2024 – December 31, 2025

Local supervisor: Clementine Prieur

Partners:

- Meza Becerra (Chili)
- Jose R. Leon (Uruguay)

Inria contact: Clementine Prieur

Summary: Statistical modelling for complex data is an important framework for analyzing data in fields such as ecology, meteorology, health, and telecommunications. These models are used to model population dynamics, animal movement, longitudinal data, spatial-temporal analysis, or Poisson processes. In this proposal, we are interested in to propose novel estimation procedures in this kind of complex data, considering restricted data (for instance, data on compact domain or longitudinal compositional data), spatial weighted regression, and model selection with weakly dependent observations and non-homogeneous Poisson processes. We will use parametric and nonparametric strategies.

10.1.3 Participation in other International Programs

A Comprehensive Software Stack for Dynamic Resources Management

Participants: Sergio Iserte, Dominik Huber, Martin Schreiber, Pierre-François Dutot, Olivier Richard, Antonio J. Peña.

Title: A Comprehensive Software Stack for Dynamic Resources Management

Partner Institution(s): BSC, Inria

Date/Duration: 2024-

Additionnal info/keywords: dynamic resource management

10.2 International research visitors

10.2.1 Visits of international scientists

Other international visits to the team

Jose R. Leon

Status researcher

Institution of origin: Universidad de la República de Uruguay

Country: Uruguay

Dates: January and December 2025 (2 visits of 2 weeks)

Context of the visit: collaboration with Clémentine Prieur

Mobility program/type of mobility: SMILE project

Sergio Iserte

Status researcher

Institution of origin: Barcelona Supercomputing Center

Country: Spain

Dates: February 2025

Context of the visit: collaboration with Martin Schreiber

10.2.2 Visits to international teams

Research stays abroad

Clémentine Prieur

Visited institution: Isaac Newton Institute Cambridge

Country: UK

Dates: July

Context of the visit: Kirk Distinguished Visiting Fellow for the Representing, calibrating & leveraging prediction uncertainty from statistics to machine learning programme

Mobility program/type of mobility: research stay, plenary lecture

10.3 European initiatives

10.3.1 Horizon Europe

DARE

Participants: Maurice Brémond, Laurent Debreu, Martin Schreiber.

Title: DARE, Digital Autonomy with RISC-V in Europe, dare-riscv.eu

Duration: 2025-2030

Abstract: The AIRSEA team is involved in the DARE project, which aims to develop an open and secure European processor architecture based on RISC-V. The AIRSEA team's role is to explore the application of RISC-V architecture in high-performance computing for environmental modeling, particularly in oceanographic simulations. This involves adapting existing models to run efficiently on RISC-V processors. The focus is on source to source code translation and optimization techniques to ensure that the models can leverage the capabilities of RISC-V architecture effectively. NEMO is one of the ocean models targeted in this project.

10.3.2 Other european programs/initiatives

- **Program: CMEMS**

Project acronym: ENMASSE

Project title: Enhancing Nemo for Marine Applications and Services

Coordinator: F. Lemarié

Duration: Dec. 2024 - Dec. 2027.

Other partners: CMCC (Italy), Sorbonne Université, MetOffice (UK), National Oceanography Center (UK), STFC Hartree Centre (UK), Datlas (FR).

Abstract: The Enhancing NEMO for Marine Applications and Services (ENMASSE) project represents a pivotal initiative aimed at advancing the capabilities of the NEMO (Nucleus for European Modelling of the Ocean) modelling platform. This enhancement is designed to address specific scientific and operational requirements set by the Copernicus Marine Service (CMS) program for the development and delivery of more precise and sophisticated ocean modelling products. These products are intended to support a wide range of applications, including marine safety, climate prediction, and ecosystem monitoring, ultimately contributing to informed decision-making and sustainable ocean management.

- **Program: C3S2**

Project acronym: ERGO2

Project title: Advancing ocean data assimilation methodology for climate applications

Duration: August 2022 - December 2025

Coordinator: Arthur Vidard

Other partners: Cerfacs (France), CNR (Italy)

Abstract: The scope of this contract is to improve ocean data assimilation capabilities at ECMWF, used in both initialization of seasonal forecasts and generation of coupled Earth System reanalyses. In particular it shall focus on i) improving ensemble capabilities in NEMO and NEMOVAR and the use of their information to represent background error statistics; ii) extend NEMOVAR capabilities to allow for multiple resolution in multi-incremental 3D-Var; iii) make better use of ocean surface observations. It shall also involve performing scout experiments and providing relevant diagnostics to evaluate the benefit coming from the proposed developments.

10.4 National initiatives

10.4.1 ANR

- **A 4-year contract: ANR MOTIONS** (Multiscale Oceanic simulaTIONS based on mesh refinement strategies with local adaptation of dynamics and physics).

  PI: F. Lemarié

  Duration: Jan. 2024 - Dec. 2027.

  Other partners:
  - ∗ Laboratoire d'Aérologie, UMR 5560 (LAERO),
  - ∗ Service Hydrographique et Océanographique de la Marine (SHOM),
  - ∗ Institut Camille Jordan,
  - ∗ UMR5208 (ICJ),
  - ∗ Laboratoire d'Etudes en Géophysique et Océanographie Spatiales,
  - ∗ UMR5566 (LEGOS).

  Abstract: The MOTIONS project aims at delivering robust and efficient numerical algorithms allowing an innovative multiscale modeling strategy based on block-structured mesh refinement with local adaptation of model equations, numerics and physics in selected areas of interest. The target application to evaluate numerical developments is the simulation of important fine-scale non-hydrostatic processes and their feedback to larger scales within the Mediterranean / North-East Atlantic dynamical continuum.

- **A 4-year contract: ANR PLUME** (Observation and Parameterization of Oceanic Convection).

  PI: B. Deremble (CNRS), Inria PI: F. Lemarié.

  Objectives:
  1. build a consistent database of convective events (both in the lab and with a numerical model) in order to calibrate free parameters of parameterizations of deep convection.
  2. characterize the structure of the thermal plumes in a well defined parameter space that characterizes the rotating/non rotating state and forced vs free convection
  3. use a data driven approach to formulate a model of convection without any preconceived bias about the mathematical formulation.

- **A 5-year contract: ANR MEDIATION:** Methodological developments for a robust and efficient digital twin of the ocean.

  Duration: 2022-2027

  Funding: French priority research program (PPR) ”Ocean and Climate"

  Partners:
  - Inria (DATAMOVE and ODYSSEY teams)
  - CNRS
  - IFREMER
  - IMT-Atlantique
  - IRD

  - Metéo-France
  - SHOM
  - Univ. Grenoble Alpes
  - Univ. Aix-Marseille.

Inria contact: Laurent Debreu

Coordinator: Laurent Debreu

Summary: The MEDIATION project targets two questions: how will global change impact the functioning of regional marine ecosystems, and how to evaluate the effect of measures to preserve the environment? With two main demonstrators on the French coasts (Atlantic and Mediterranean), MEDIATION combines methodological developments in numerical sciences (taking into account uncertainties, high performance computing and artificial intelligence) with advances in the modeling of physical, biogeochemical and biological processes in the ocean. It aims at setting up a modeling chain, integrating data and allowing to significantly increase the number of scenarios (climate change, human activities) evaluated. The digital tools developed will also contribute to a better science-society-policy interaction

10.4.2 Other Initiatives

- E. Blayo is co-advising the PhD thesis of Valentin Bellemin-Laponnaz with IGE Lab, in the framework of the NASA-CNES working group on the SWOT satellite.
- MIAM (Multi-fIdelity Approach for climate Models): Institut des Mathématiques pour la Planète Terre, PI: Elise Arnaud, Pierre Nabat (Météo- France).Collaboration with Météo France and Ecole Centrale Lyon

11 Dissemination

11.1 Promoting scientific activities

11.1.1 Scientific events: organisation

- In 2025, the AIRSEA team has organised the SAMO conference in Grenoble. SAMO is a cross-disciplinary conference, held every three years, related to the fields of sensitivity analysis, design of experiments, model calibration and validation, structural reliability, uncertainty quantification, machine learning interpretability, explainable AI, and related application areas (engineering, environment, agronomy, finance, etc.).
- Since 2024, the AIRSEA team has been organizing an annual workshop on the topic of Digital Twins. The two first editions (2024-2025) were held at CIRM and brought together an international audience of mathematicians and computational scientists, fostering interdisciplinary exchanges in this field. In 2026, this workshop will be organised at the Institut d'Études Scientifiques of Cargèse.

General chair, scientific chair

- Clémentine Prieur was the general chair of SAMO 2025 conference.

Member of the organizing committees

- Martin Schreiber was a co-organiser of "(EuroHPC) Workshop on Dynamic Resources in HPC", 25 – 29 Août 2025, Dresden, Germany
- Martin Schreiber was a co-organiser of "PDEs on the sphere workshop" 12 – 16 Mai 2025 in Sao Paulo, Brazil 5̃0 participants
- Martin Schreiber was a principal organiser of the "Birds of feather: Dynamic resources" at International Super Computing, 10 – 13 Juin 2025, Hambourg, Germany

11.1.2 Scientific events: selection

Member of the conference program committees

- Clémentine Prieur was a member of the conference program committee of the SMAI conference.
- Martin Schreiber was a member of the conference program committee of EuroMPI
- Martin Schreiber was a member of the conference program committee of ICCS 2025
- Martin Schreiber was a member of the conference program committee of IWOCL 2025
- Martin Schreiber was a member of the conference program committee of Supercomputing (applications, doctoral showcase, best paper award)

11.1.3 Journal

Member of the editorial boards

- F. Lemarié is associate editor of the Journal of Advances in Modeling Earth Systems (JAMES)
- Clémentine Prieur is associate editor for SIAM/ASA Journal of Uncertainty Quantification journal.
- Clémentine Prieur is a member of the reading committee of Annales Mathématiques Blaise Pascal.
- M.Schreiber is associate editor for “The International Journal of High Performance Computing Applications”.

11.1.4 Invited talks

- Clémentine Prieur was invited to iMSi (Institute for Mathematical and Statistical Innovation) to give a talk "Kernel Methods in Uncertainty Quantification and Experimental Design", Chicago, USA, March 31 - April 4, 2025.
- Clémentine Prieur was a Kirk Distinguished Visiting Fellow in the Isaac Newton Institute for Mathematical Sciences, Cambridge. She participated in the Programme "Representing, calibrating & leveraging prediction uncertainty, from statistics to machine learning" and delivered a lecture titled "On feasible set estimation with Bayesian active learning" on the July 22, 2025.
- E. Blayo was invited to give a 2-hour lecture “Introduction to data assimilation” in the Workshop on Uncertainty Quantification for Climate Science co-organized by IMPT, RT UQ and GdR Défis théoriques pour les sciences du climat (Paris, Nov. 2025)

11.1.5 Leadership within the scientific community

- In 2022-2025, C. Prieur was the president of SAMO group, international research group on sensitivity analysis.
- F. Lemarié is a member of the international CLIVAR Ocean Model Development Panel since Jan. 2024. CLIVAR (Climate and Ocean: Variability, Predictability, and Change) is a core project of the World Climate Research Programme (WCRP).
- F. Lemarié is a member of the scientific board of the GDR "Défis théorique pour les sciences du climat" (since Dec. 2024)
- F. Lemarié is the coordinator of the national inter-agency program SUN (computational sciences for Earth and Space sciences), affiliated with CNRS-INSU. The program includes a call for projects component as well as a broader focus on scientific networking and training activities (since may 2025).
- M. Schreiber is a Co-chair of the “Dynamic Resources for HPC” (DynResHPC) consortium

11.1.6 Scientific expertise

- Clémentine Prieur is advisor to the scientific council of IFPEN.
- E. Blayo is a member of the scientific committee of IMPT (Institut Mathématique pour la Planète Terre)
- E. Blayo is a member of the scientific committee of the Labex Persyval-3
- F. Lemarié is the co-leader with Sybille Téchené (CNRS) and Mike Bell (UK Met Office) of the NEMO (www.nemo-ocean.eu) Working Group on numerical kernel development.
- F. Lemarié is a member of the CROCO (www.croco-ocean.org) Development Committee.
- F. Lemarié is a member of the CE56 scientific evaluation committee, Interfaces: mathematics, computational sciences - Earth system and environmental sciences of the ANR (French National Research Agency)
- Martin Schreiber is a member of the OpenMP ARB (representing Inria).

11.1.7 Research administration

- E. Blayo is a deputy director of the Jean Kuntzmann Lab.
- F. Lemarié is the Inria local scientific correspondent for national calls for projects (work in coordination with Inria contract managers to identify national project calls and the teams that may be suited to respond to these calls. Since July 2020, as part of this mission, I provide support to (1) analyze the objectives of project calls to assess their relevance. (2) track trends and innovations in the relevant sector).
- Clémentine Prieur was a member of the jury for the "Prix de thèse - LMBP - Université Clermont Auvergne" in 2025.
- Clémentine Prieur was a member of the jury for the Blaise Pascal Prize from the Académie des Sciences in 2024 and 2025.
- Clémentine Prieur is Vice President of the french society of statistics (SFdS) since July 2024.
- Clémentine Prieur is currently a member of the Executive and Scientific Committees of the RT Quantification d'Incertitudes (RT2172 funded by INSMI @ CNRS) which she chaired during the period 2010-2017.
- Clémentine Prieur is currently a member of the Scientific Committee of the RT Terre et Energies (RT2166 funded by INSMI @ CNRS).
- Clémentine Prieur is local correspondent in Grenoble for the Mathematical Society of France (SMF).
- Clémentine Prieur was a member of the MSTIC pole council of UGA(nov. 2020-oct. 2024).
- Clémentine Prieur is responsible for the applied math specialty for doctoral school MSTII edmstii.univ-grenoble-alpes.fr
- E. Arnaud is in charge of the parity diversity commission at Jean Kuntzmann Lab

11.2 Teaching - Supervision - Juries - Educational and pedagogical outreach

- Licence: E. Arnaud, Mathematics for engineers, 50h, L2, UGA, France
- Licence: E. Arnaud, Statistics, 20h, L2, UGA, France
- Licence: E. Blayo, analysis and algebra, 107h, L1, University Grenoble Alpes, France
- Licence: C. Kazantsev, Mathématiques approfondies pour l'ingénieur, 36h, L2, UGA, France
- Licence: C. Kazantsev, Mathématiques pour les sciences de l'ingénieur, 36h, L2, UGA, France
- Licence: Martin Schreiber, Advanced Analysis & Algebra, L1, 69h, UGA, France
- Master: E. Blayo, Partial Differential Equations, 55h, M1, University Grenoble Alpes, France
- Master: E. Arnaud, Critical thinking, 30h, M1, UGA, France
- Master: E. Arnaud, Supervision of student in apprenticeship, 30h, M2, UGA, France
- Master: Martin Schreiber, High-Performance Computing, M1, 31.1h, UGA, France
- Master: Martin Schreiber, Parallel Algorithms and Programming, M1, 11.25h, UGA, France
- Master: Martin Schreiber, Object oriented programming with C++, M1, 18h, UGA, France
- Master: Martin Schreiber, Partial differential equations, M1, 34.5h, UGA, France
- E-learning: E. Arnaud is in charge of the pedagogic platform math@uga: implementation of a collaborative moodle platform to share pedagogical resources within teachers and towards students.
- E. Blayo is in charge of the Ecole des Mathématiques Appliquées: organization and coordination of pedagogical and administrative aspects related to teaching for the applied maths department.

11.2.1 Supervision

- PhD in progress: Angélique Saillet, Multi fidelity for marine biogeochemical model, Octobre 2023, Eric Blayo and Elise Arnaud.
- PhD in progress: Exaucé Luweh Adjim Ngarti, Deep learning for inverse problem in geophysics, Université Grenoble-Alpes Avril 2023, E. Arnaud, L. Nicoletti (Atos) and A. Vidard.
- PhD in progress: Lorenzo Calzolari, Active learning with functional inputs: application to wind turbine reliability design, Novembre 2024, C. Helbert (Ecole Centrale Lyon), C. Prieur, promoted by M. Munoz Zuniga, D. Sinoquet (IFPEN)
- PhD in progress : Jean-Baptise Seby, Multi fidelity for regional climate models, October 2025, Eric Blayo and Elise Arnaud
- PhD in progress : Mélanie Villain, Méthodes avancées d'assimilation de données pour l'estimation des paramètres et des états dans les modèles hydrologiques en contexte montagneux, November 2025, Arthur Vidard and Elise Arnaud
- PhD in progress: Hélène Hénon, Assimilation de données variationnelles multi-fidélité pour les prévisions océaniques , Université Grenoble-Alpes Octobre 2023, A. Vidard.
- PhD in progress: Doaa Akil, Placement optimal de capteurs pour des équations aux dérivées partielles hyperboliques de Saint-Venant par approche « Physics-Informed Machine Learning » : application à la détection de tsunamis, October 2025, co-supervised with D. Georges (Gipsa lab) and O. Millet (Université de La Rochelle)
- PhD in progress: Mohamed Doumbouya, Computational Bayesian optimal sensor placement for ocean models, October 2024, supervised by A. Vidard and O. Zahm.

- PhD in progress: Gabriel Derrida, Design of flexible and numerically-sound generalised vertical coordinates with vertical ALE (V-ALE) algorithm for operational ocean forecasting. October 2023, L. Debreu and F. Lemarié.
- Defended PhD: Adama Barry, Plans d'expériences pour la calibration et la validation d'un simulateur numérique, June 2025, supervised by F. Bachoc (Institut de Mathématiques de Toulouse), C. Prieur, promoted by M. Munoz Zuniga and S. Bouquet.
- Defended PhD: Pierre Lozano, Coupling hydrostatic and nonhydrostatic ocean circulation models. December 2025, E. Blayo and L. Debreu
- Defended PhD: Manolis Perrot, Modeling Oceanic and Atmospheric Convection : Energy, Uncertainties and Rotation [21], April 2025, supervised by E. Blayo and F. Lemarié.
- Defended PhD: Benjamin Zanger, Compositional surrogates for reduced order modeling [24], supervised by M. Schreiber, O. Zahm since 2022
- Defended PhD: Katarina Radisic, Prise en compte d'incertitudes externes dans l'estimation de paramètres d'un modèle de transfert d'eau et de pesticides à l'échelle du bassin versant [22], Université Grenoble-Alpes, supervised by C. Lauvernet (Inrae) and A. Vidard, defended in March 2025.
- Defended PhD: Romain Verdière, Nonlinear dimension reduction for uncertainty quantification problems [23], supervised by C. Prieur and O. Zahm since 2022
- Defended PhD: Ri Wang, Apprentissage statistique pour l'analyse de sensibilité globale avec entrées dépendantes, supervised by C. Prieur and V. Maume-Deschamps (Université Lyon 1) since 2021
- Internship: M. Aharmouch (co-encadrement à 50 % avec C. Helbert, Ecole Centrale de Lyon), M2 internship, Active learning for Gaussian processes with functional inputs: application to wind turbine reliability design, C. Helbert, M. Munoz Zuniga, C. Prieur and D. Sinoquet.
- Internship: M. Villain. Advanced data assimilation methods for estimating parameters and states in hydrological models in mountainous contexts. M2 internship, supervised by Arthur Vidard and Elise Arnaud

11.2.2 Juries

- F. Lemarié:
  - Dec 11, 2025 — PhD thesis of Adrien Marcel, Université Toulouse III — Paul Sabatier, (**reviewer**)
- M. Schreiber:
  - 2025 — PhD thesis of Arsène Marzorati, l'INSA Lyon, (**reviewer**)
  - July 2, 2025 — PhD thesis of Abdessalam BENHARI, UGA, (**jury member**)
- A. Vidard:
  - February 20, 2025 — PhD thesis of Olivier Goux, ISAE Supaero, (**jury member**)
  - June 30, 2025 — PhD thesis of Djahou Norbert Tognon, Sorbonne University, (**reviewer**)
- E. Blayo:
  - May 7, 2025 — PhD thesis of Jacopo Iollo (**president**)
  - Oct 14, 2025 — PhD thesis of Jean-Paul Travert (**reviewer**)
  - Nov 18, 2025 — Habilitation thesis of Florian Lemarié (**president**)
  - Dec 9, 2025 — PhD thesis of Arthur Grange (**president**)
- C. Prieur:

  - 27 nov. 2025 thèse de Mohamed Bahi Yahiaoui, Université Grenoble Alpes (**examinatrice**)
  - 22 oct. 2025 thèse de Benjamin Zanger, Université Grenoble Alpes (**présidente**)
  - 25 sept. 2025 thèse de Marine Dumon, Université Gustave Eiffel (**rapportrice**)
  - 15 sept. 2025 thèse de Romain Ait Abdelmalek-Lomenech, CentraleSupélec (**rapportrice**)
  - 2 juin 2025 thèse de Justin Reverdi, Université de Toulouse (**présidente**)
  - 24 mars 2025 thèse de Katarina Radisik, Université Grenoble Alpes (**présidente**)
  - 12 déc. 2025 membre du jury d'HDR de J. Chevallier, Université Grenoble Alpes (**présidente**)
  - 4 juin 2025 membre du jury d'HDR de J. Garnier, Université Savoie-Mont-Blanc (**examinatrice**)
  - 2025-... Membre du jury du prix Blaise Pascal, décerné chaque année par l'Académie des Sciences, après consultation de la SMAI et du groupe SMAI-GAMNI.

11.3 Popularization

11.3.1 Productions (articles, videos, podcasts, serious games, ...)

- Ch. Kazantsev and E. Blayo are strongly involved in the creation and dissemination of pedagogic suitcases with mathematical activities designed for primary and secondary schools, as well as an escape game. These actions are led in the context of the association La Grange des Maths.

- E. Blayo and A. Vidard have produced a series of pedagogical videos on numerical ocean modeling, in collaboration with L'Esprit Sorcier TV, available here).

11.3.2 Participation in Live events

- E. Blayo gave several outreach talks, in particular for high school students, and for more general audiences.

11.3.3 Others science outreach relevant activities

- E. Blayo is in charge of the project *Terra Numerica Grenoble*. This project brings together numerous institutional partners (Inria, UGA, Territoire de Sciences, etc.) and associations (La Grange des Maths, Info sans Ordi, Aconit, etc.). It aims to open two venues (La Casemate in Grenoble and a space on the university campus) dedicated to popularizing mathematics and computer sciences within the next two years. In addition, a program offering a range of activities (educational kits, exhibitions, escape games, conferences, etc.) in a network of partner venues (schools, colleges, high schools, media libraries, third places, etc.) across the region will be developed.

12 Scientific production

12.1 Major publications

[1] V. Schüller, F. Lemarié, P. Birken and E. Blayo. 'Quantifying coupling errors in atmosphere-ocean-sea ice models: A study of iterative and non-iterative approaches in the EC-Earth AOSCM'. In: *Geoscientific Model Development* 18.22 (27th Nov. 2025), pp. 9167–9187. DOI: `10.5194/gmd-18-9167-2025`. URL: `https://inria.hal.science/hal-05404526` (cit. on p. 14).

12.2 Publications of the year

International journals

[2] R. Bhatt, L. Debreu and A. Vidard. 'Introducing time parallelisation within data assimilation'. In: *SIAM Journal on Scientific Computing* 47.2 (9th Apr. 2025), B533–B557. DOI: `10.1137/24M1651903`. URL: `https://inria.hal.science/hal-03540480`.

[3] J. G. Caldas Steinstraesser, M. Schreiber and P. Peixoto. ‘Analysis and improvement of a semi-Lagrangian exponential scheme for the shallow-water equations on the rotating sphere’. In: *ESAIM: Mathematical Modelling and Numerical Analysis* 59.3 (4th June 2025), pp. 1531–1564. DOI: `10.1051/m2an/2025034`. URL: `https://hal.science/hal-05480030` (cit. on p. 22).

[4] S. Clement, E. Blayo, L. Debreu, J.-M. Brankart, P. Brasseur, L. Li and E. Mémin. ‘Link between stochastic grid perturbation and location uncertainty framework’. In: *Journal of Advances in Modeling Earth Systems* 17.5 (May 2025), e2024MS004528. DOI: `10.1029/2024MS004528`. URL: `https://inria.hal.science/hal-04629335` (cit. on p. 16).

[5] C. Dutang and C. Prieur. ‘On the Use of Global Sensitivity Analysis in a Game-Theoretic Approach to an Environmental Management Problem’. In: *Environmental Modeling & Assessment* (16th Jan. 2026). DOI: `10.1007/s10666-025-10098-y`. URL: `https://hal.science/hal-05461442` (cit. on p. 18).

[6] H. M. Kouye, C. Prieur and E. Vergu. ‘An extension of Sellke construction and uncertainty quantification for non-Markovian epidemic models’. In: *Mathematical Modelling of Natural Phenomena* (23rd Sept. 2025), pp. 1–31. DOI: `10.1051/mmnp/2025027`. URL: `https://hal.science/hal-04719348` (cit. on p. 18).

[7] D. Mignac, J. Waters, D. J. Lea, M. J. Martin, J. While, A. T. Weaver, A. Vidard, C. Guiavarc’H, D. Storkey, D. Ford, E. W. Blockley, J. Baker, K. Haines, M. R. Price, M. J. Bell and R. Renshaw. ‘Improvements to the Met Office’s global ocean–sea ice forecasting system including model and data assimilation changes’. In: *Geoscientific Model Development* 18.11 (13th June 2025), pp. 3405–3425. DOI: `10.5194/gmd-18-3405-2025`. URL: `https://hal.science/hal-05113966`.

[8] S. Ouala, L. Debreu, B. Chapron, F. Collard, L. Gaultier and R. Fablet. ‘Enhanced Computational Complexity in Continuous-Depth Models: Neural Ordinary Differential Equations With Trainable Numerical Schemes’. In: *IEEE Transactions on Pattern Analysis and Machine Intelligence* (2025), pp. 1–8. DOI: `10.1109/TPAMI.2025.3599629`. URL: `https://hal.science/hal-05232101`.

[9] M. Perrot and F. Lemarié. ‘Energetically Consistent Eddy-Diffusivity Mass-Flux Convective Schemes: 2. Implementation and Evaluation in an Oceanic Context’. In: *Journal of Advances in Modeling Earth Systems* 17.7 (16th July 2025), e2024MS004616. DOI: `10.1029/2024MS004616`. URL: `https://hal.science/hal-04666049` (cit. on pp. 15, 16).

[10] M. Perrot, F. Lemarié and T. Dubos. ‘Energetically Consistent Eddy-Diffusivity Mass-Flux Convective Schemes: 1. Theory and Models’. In: *Journal of Advances in Modeling Earth Systems* 17.1 (28th Jan. 2025), pp. 1–31. DOI: `10.1029/2024MS004273`. URL: `https://hal.science/hal-04439113` (cit. on pp. 15, 16).

[11] J. Posner, T. Ellersiek, N. Bietendorf, D. Huber, M. Schreiber and M. Schulz. ‘Toward Dynamic Resource Management: An Asynchronous Many-Task (AMT) Runtime System leveraging Dynamic Processes with PSets (DPP)’. In: *SN Computer Science* 6.8 (5th Nov. 2025), p. 941. DOI: `10.1007/s42979-025-04405-3`. URL: `https://hal.science/hal-05480032` (cit. on p. 21).

[12] K. Radišić, C. Lauvernet and A. Vidard. ‘Impact of input forcings variability on the global sensitivity analysis of a hydrological model’. In: *Environmental Modelling and Software* 192 (Aug. 2025), p. 106522. DOI: `10.1016/j.envsoft.2025.106522`. URL: `https://hal.inrae.fr/hal-04849856` (cit. on p. 18).

[13] R. Vaudry, C. Sonveaux, D. Georges and C. Prieur. ‘An integro-ODE-PDE model for Covid-19 pandemic with vaccination and loss of immunity: well-posedness, RBF-FD numerical scheme and simulation’. In: *IMA Journal of Mathematical Control and Information* 42.3 (Sept. 2025), dnaf020. DOI: `10.1093/imamci/dnaf020`. URL: `https://inria.hal.science/hal-05155149`.

[14] Y. Zhang, M. Baklouti, P. Brasseur and L. Debreu. ‘A semi-automated sensitivity-based approach for simplifying marine biogeochemical models for targeted applications: A case study with the Eco3M-MED model’. In: *Ecological Modelling* 514 (Apr. 2026), p. 111491. DOI: `10.1016/j.ecolmodel.2026.111491`. URL: `https://amu.hal.science/hal-05460893` (cit. on p. 17).

International peer-reviewed conferences

[15] D. Huber, S. Iserte, M. Schreiber, A. J. Peña and M. Schulz. 'Bridging the Gap Between Genericity and Programmability of Dynamic Resources in HPC'. In: ISC High Performance 2025 - 40th ISC High Performance International Conference. Hamburg, Germany, 2025, pp. 1–11. URL: `https://inria.hal.science/hal-04994828` (cit. on p. 21).

Conferences without proceedings

[16] P. Lozano, E. Blayo, L. Debreu and L. Debreu. 'Couplage de modèles hydrostatique et non-hydrostatique de circulation océanique'. In: SMAI 2025 - 12ième Biennale Française des Mathématiques Appliquées et Industrielles. Carcans-Maubuisson, France, 2025. URL: `https://inria.hal.science/hal-05344291` (cit. on p. 13).

[17] K. Radišić, C. Lauvernet and A. Vidard. 'Effect of forcing uncertainty in the sensitivity and calibration of a pesticide transfer model'. In: SAMO 2025 - 11th International Conference on Sensitivity Analysis of Model Output. Grenoble, France, 2025. URL: `https://hal.inrae.fr/hal-05076842`.

Doctoral dissertations and habilitation theses

[18] A. Barry. 'Design of experiments for the calibration of expensive computer codes'. Université de Toulouse, 25th June 2025. URL: `https://hal.science/tel-05186532` (cit. on p. 19).

[19] F. Lemarié. 'Numerical modeling of the ocean and its interaction with the atmosphere'. MSTII, 18th Nov. 2025. URL: `https://inria.hal.science/tel-05373731`.

[20] P. Lozano. 'Coupling hydrostatic and nonhydrostatic ocean circulation models'. Université Grenoble Alpes, 17th Dec. 2025. URL: `https://theses.hal.science/tel-05469608` (cit. on p. 13).

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