

2025 Activity Report

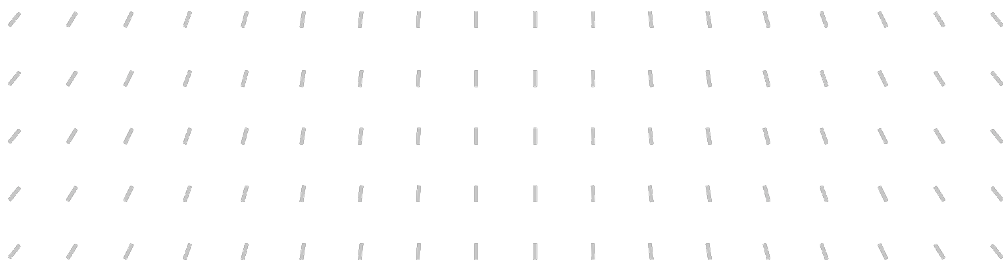
RESEARCH CENTRE: Inria Paris Centre at Sorbonne University
IN PARTNERSHIP WITH: CNRS, Sorbonne Université

Project-Team

ALPINES

Algorithms and parallel tools for integrated numerical
simulations

In collaboration with Laboratoire Jacques-Louis Lions (LJLL)



Project-Team ALPINES

Creation of the Project-Team: 2025 June 01

Each year, Inria research teams publish an Activity Report presenting their work and results over the reporting period. These reports follow a common structure, with some optional sections depending on the specific team. They typically begin by outlining the overall objectives and research programme, including the main research themes, goals, and methodological approaches. They also describe the application domains targeted by the team, highlighting the scientific or societal contexts in which their work is situated. The reports then present the highlights of the year, covering major scientific achievements, software developments, or teaching contributions. When relevant, they include sections on software, platforms, and open data, detailing the tools developed and how they are shared. A substantial part is dedicated to new results, where scientific contributions are described in detail, often with subsections specifying participants and associated keywords. Finally, the Activity Report addresses funding, contracts, partnerships, and collaborations at various levels, from industrial agreements to international cooperations. It also covers dissemination and teaching activities, such as participation in scientific events, outreach, and supervision. The document concludes with a presentation of scientific production, including major publications and those produced during the year.

Keywords

Computer sciences and digital sciences

- A6.1.1. – Continuous Modeling (PDE, ODE)
- A6.1.4. – Multiscale modeling
- A6.1.5. – Multiphysics modeling
- A6.2.1. – Numerical analysis of PDE and ODE
- A6.2.5. – Numerical Linear Algebra
- A6.2.6. – Optimization
- A6.2.7. – HPC for machine learning
- A6.3. – Computation-data interaction
 - A6.3.1. – Inverse problems
- A7.1. – Algorithms

Other research topics and application domains

- B3.3.1. – Earth and subsoil
- B9.5.2. – Mathematics
- B9.5.3. – Physics

Contents

Project-Team ALPINES	1
1 Team members, visitors, external collaborators	5
2 Overall objectives	6
2.1 Introduction	6
3 Research program	6
3.1 Overview	6
3.2 Domain specific language - parallel FreeFem	6
3.3 Solvers for numerical linear algebra	7
3.4 Computational kernels for numerical linear and multilinear algebra	7
4 Application domains	8
4.1 Radiative transfer in the atmosphere	8
4.2 Inverse problems	8
4.2.1 Time Reversal techniques	8
4.2.2 Seismic imaging	8
4.2.3 Electroencephalography	9
4.2.4 Medical imaging with microwave tomography	9
4.3 Numerical methods for time harmonic wave propagation problems	9
4.4 Molecular simulations	10
5 Social and environmental responsibility	10
5.1 Impact of research results	10
6 Highlights of the year	10
7 Latest software developments, platforms, open data	10
7.1 Latest software developments	10
7.1.1 FreeFem++	10
7.1.2 HPDDM	11
7.1.3 Htool-DDM	11
7.1.4 scatensor	11
7.2 Open data	12
8 New results	12
8.1 Numerical shape and topology optimization of regions supporting the boundary conditions of a physical problem	12
8.2 Distributionally robust shape and topology optimization	12
8.3 A shape optimization approach for inferring sources of volcano ground deformation	13
8.4 Domain decomposition preconditioners for efficient parallel simulations of single-phase flow in three-dimensional fractured porous media with a very large number of fractures	13
8.5 A scalable Domain Decomposition method for Saddle Point problems with GenEO coarse spaces	13
8.6 A robust and adaptive GenEO-type domain decomposition preconditioner for $H(\text{curl})$ problems in general non-convex three-dimensional geometries	14
8.7 Coarse spaces for non-symmetric two-level preconditioners based on local generalized eigenproblems	14
8.8 Coarse spaces using extended generalized eigenproblems for heterogeneous Helmholtz problems	14
8.9 Numerical comparison of coarse spaces for heterogeneous Helmholtz problems	14
8.10 Modal analysis of a domain decomposition method for Maxwell's equations in a waveguide	15
8.11 A Robust Two-Level Schwarz Preconditioner For Sparse Matrices	15
8.12 Microwave tomographic imaging of shoulder injury	15

8.13	New developments in FreeFEM	16
8.14	Reduced models for highly oscillating differential equations	16
8.15	Stable Trefftz approximation of Helmholtz solutions using evanescent plane waves	16
8.16	A Trefftz Continuous Galerkin method for Helmholtz problems	17
8.17	Parallel approximation of the exponential of Hermitian matrices	17
8.18	A finite element toolbox for the Bogoliubov-de Gennes stability analysis of Bose-Einstein condensates	17
8.19	Mathematics and Finite Element Discretizations of Incompressible Navier-Stokes Flows	18
8.20	Randomized Householder QR	18
8.21	Randomized Krylov-Schur eigensolver with deflation	18
8.22	PDE-constrained optimization within FreeFEM	18
9	Bilateral contracts and grants with industry	18
9.1	Bilateral contracts with industry	18
9.2	Bilateral training with industry	19
10	Partnerships and cooperations	19
10.1	European initiatives	19
10.1.1	Horizon Europe	19
10.1.2	H2020 projects	20
10.2	National initiatives	21
10.2.1	ANR	21
11	Dissemination	22
11.1	Promoting scientific activities	22
11.1.1	Journal	22
11.1.2	Invited talks	22
11.1.3	Research administration	23
11.2	Teaching - Supervision - Juries - Educational and pedagogical outreach	23
11.2.1	Supervision	23
11.2.2	Juries	23
12	Scientific production	23
12.1	Major publications	23
12.2	Publications of the year	24
12.3	Cited publications	25

1 Team members, visitors, external collaborators

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Technical Staff

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Interns and Apprentices

- Aminata Diarra [INRIA, Intern, from May 2025 until Sep 2025]

Administrative Assistants

- Laurence Bourcier [INRIA]
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2 Overall objectives

2.1 Introduction

The focus of our research is on the development of novel parallel numerical algorithms and tools appropriate for state-of-the-art mathematical models used in complex scientific applications, and in particular numerical simulations. The proposed research program is by nature multi-disciplinary, interweaving aspects of applied mathematics, computer science, as well as those of several specific applications, as porous media flows, elasticity, wave propagation in multi-scale media, molecular simulations, and inverse problems.

Our first objective is to develop numerical methods and tools for complex scientific and industrial applications that will enhance their scalable execution on the emergent heterogeneous hierarchical models of massively parallel machines. Our second objective is to integrate the novel numerical algorithms into a middle-layer that will hide as much as possible the complexity of massively parallel machines from the users of these machines.

3 Research program

3.1 Overview

The research described here is directly relevant to several steps of the numerical simulation chain. Given a numerical simulation that was expressed as a set of differential equations, our research focuses on mesh generation methods for parallel computation, novel numerical algorithms for linear algebra, as well as algorithms and tools for their efficient and scalable implementation on high performance computers. The validation and the exploitation of the results is performed with collaborators from applications and is based on the usage of existing tools. In summary, the topics studied in our group are the following:

- Numerical methods and algorithms
 - Mesh generation for parallel computation
 - Solvers for numerical linear algebra:
 - domain decomposition methods, preconditioning for iterative methods
 - Computational kernels for numerical linear algebra
 - Tensor computations for high dimensional problems
- Validation on numerical simulations and other numerical applications

3.2 Domain specific language - parallel FreeFem

In the engineering, researchers, and teachers communities, there is a strong demand for simulation frameworks that are simple to install and use, efficient, sustainable, and that solve efficiently and accurately complex problems for which there are no dedicated tools or codes available. In our group we develop **FreeFem++**, a user-dedicated language for solving PDEs. The goal of FreeFem++ is not to be a substitute for complex numerical codes, but rather to provide an efficient and relatively generic tool for:

- getting a quick answer to a specific problem,
- prototyping the resolution of a new complex problem.

The current users of FreeFem++ are mathematicians, engineers, university professors, and students. In general for these users the installation of public libraries as MPI, MUMPS, Ipopt, Blas, lapack, OpenGL, fftw, scotch, PETSc, SLEPc is a very difficult problem. For this reason, the authors of FreeFem++ have created a user friendly language, and over years have enriched its capabilities and provided tools for compiling FreeFem++ such that the users do not need to have special knowledge of computer science. This leads to an important work on porting the software on different emerging architectures.

Today, the main components of parallel FreeFem++ are:

- definition of a coarse grid,

- splitting of the coarse grid,
- mesh generation of all subdomains of the coarse grid, and construction of parallel data structures for vectors and sparse matrices from the mesh of the subdomain,
- discretization by a chosen numerical method,
- call to a linear solver,
- analysis of the result.

All these components are parallel, except for the last point which is not in the focus of our research. However for the moment, the parallel mesh generation algorithm is very simple and not sufficient, for example it addresses only polygonal geometries. Having a better parallel mesh generation algorithm is one of the goals of our project. In addition, in the current version of FreeFem++, the parallelism is not hidden from the user, it is done through direct calls to MPI. Our goal is also to hide all the MPI calls in the specific language part of FreeFem++. In addition to these in-house domain decomposition methods, FreeFem++ is also linked to PETSc solvers which enables an easy use of third parties parallel multigrid methods.

3.3 Solvers for numerical linear algebra

Iterative methods are widely used in industrial applications, and preconditioning is the most important research subject here. Our research considers domain decomposition methods [6] and iterative methods and its goal is to develop solvers that are suitable for parallelism and that exploit the fact that the matrices are arising from the discretization of a system of PDEs on unstructured grids.

One of the main challenges that we address is the lack of robustness and scalability of existing methods as incomplete LU factorizations or Schwarz-based approaches, for which the number of iterations increases significantly with the problem size or with the number of processors. This is often due to the presence of several low frequency modes that hinder the convergence of the iterative method. To address this problem, we study different approaches for dealing with the low frequency modes as coarse space correction in domain decomposition or deflation techniques. We minimize the memory footprint and communication avoiding algorithms by leveraging mixed precision arithmetic.

We also focus on developing boundary integral equation methods that would be adapted to the simulation of wave propagation in complex physical situations, and that would lend themselves to the use of parallel architectures. The final objective is to bring the state of the art on boundary integral equations closer to contemporary industrial needs. From this perspective, we investigate domain decomposition strategies in conjunction with boundary element method as well as acceleration techniques (H-matrices, FMM and the like) that would appear relevant in multi-material and/or multi-domain configurations. Our work on this topic also includes numerical implementation on large scale problems, which appears as a challenge due to the peculiarities of boundary integral equations.

3.4 Computational kernels for numerical linear and multilinear algebra

The design of new numerical methods that are robust and that have well proven convergence properties is one of the challenges addressed in Alpines. Another important challenge is the design of parallel algorithms for the novel numerical methods and the underlying building blocks from numerical linear algebra. The goal is to enable their efficient execution on a diverse set of node architectures and their scaling to emerging high-performance clusters with an increasing number of nodes.

Increased communication cost is one of the main challenges in high performance computing that we address in our research by investigating algorithms that minimize communication, as communication avoiding algorithms. The communication avoiding algorithmic design is an approach originally developed in our group since more than ten years (initially in collaboration with researchers from UC Berkeley and CU Denver). While our first results concerned direct methods of factorization as LU or QR factorizations, our recent work focuses on designing robust algorithms for computing the low rank approximation of a matrix or a tensor. We focus on both deterministic and randomized approaches.

Our research also focuses on solving problems of large size that feature high dimensions as in molecular simulations. The data in this case is represented by objects called tensors, or multilinear arrays. The goal is

to design novel tensor techniques to allow their effective compression, i.e. their representation by simpler objects in small dimensions, while controlling the loss of information. The algorithms are aiming to being highly parallel to allow to deal with the large number of dimensions and large data sets, while preserving the required information for obtaining the solution of the problem.

4 Application domains

Our work is by design as general purpose as possible in the field of scientific computing based on partial differential equations modeling including multiphysics and multiscale modeling. Moreover, since our methods are available in standalone open sources libraries and in the free finite element specific language FreeFEM which is developed in our team, they have many potential users that we are not necessarily aware of. We give here some recent works performed in collaboration with team members.

4.1 Radiative transfer in the atmosphere

The Hierarchical matrix acceleration techniques implemented in our parallel Htool 7.1.3 library, which is interfaced with FreeFEM, allows to efficiently solve the radiative transfer equations with complexity $n \log(n)$, where n is the number of vertices of the physical domain. The method can be used to study the change of temperature in the atmosphere, and in particular the effect of clouds and greenhouse gases [7, 13].

4.2 Inverse problems

4.2.1 Time Reversal techniques

We focus on methods related to the blend of time reversal techniques and absorbing boundary conditions (ABC) used in a non standard way. Since the seminal paper by [M. Fink et al., Imaging through inhomogeneous media using time reversal mirrors. *Ultrasonic Imaging*, 13(2):199, 1991.], time reversal is a subject of very active research. The principle is to back-propagate signals to the sources that emitted them. The initial experiment was to refocus, very precisely, a recorded signal after passing through a barrier consisting of randomly distributed metal rods. In [de Rosny and Fink. Overcoming the diffraction limit in wave physics using a time-reversal mirror and a novel acoustic sink. *Phys. Rev. Lett.*, 89 (12), 2002], the source that created the signal is time reversed in order to have a perfect time reversal experiment. In [32], we improve this result from a numerical point of view by showing that it can be done numerically without knowing the source. This is done at the expense of not being able to recover the signal in the vicinity of the source. In [33], we have extended these methods to non uniform media.

4.2.2 Seismic imaging

In the context of seismic imaging, frequency-domain full-waveform inversion (FWI) is suitable for long-offset stationary-recording acquisition, since reliable subsurface models can be reconstructed with a few frequencies and attenuation is easily implemented without computational overhead. In the frequency domain, wave modeling is a Helmholtz-type boundary-value problem which requires to solve a large and sparse system of linear equations per frequency with multiple right-hand sides (sources).

For sparse node acquisitions covering large domains (more than 50 millions of unknowns), an iterative solver should be the method of choice. However, fast convergence remains challenging in the high frequency regime due to the non-definiteness of the operator and to the discretization constraints needed to minimize dispersion, hence requiring efficient preconditioners. For such wave propagation problems, our Schwarz domain decomposition preconditioners are good candidates to accelerate the time to solution and to provide robustness with respect to heterogeneities and frequency, for both finite difference and finite element discretizations [42].

4.2.3 Electroencephalography

The inverse source problem in electroencephalography (EEG) is usually solved using a numerical head model composed of different layers of tissues (scalp, skull, brain, ...) having constant electric conductivity. However, the conductivity of the skull can hardly be assumed to be constant: (i) hard and spongy bones, especially for neonates (fontanels), (ii) conductivity models vary between individuals, and through time for a single person.

In [5], we consider a new numerical method for the EEG inverse source problem which takes into account the heterogeneity of the skull. In particular, in the first step of source reconstruction, the so-called "cortical mapping" procedure which deals with (i) data completion on the scalp knowing partial measurements of the electric potential from electrode measurements and (ii) data transmission from the scalp to the brain surface, we use the quasi-reversibility method which provides a regularized solution of Cauchy problems in a bounded domain. The problem is solved using the finite element method in the scalp and skull layers.

4.2.4 Medical imaging with microwave tomography

Microwave tomography is a novel imaging modality with a large number of potential attractive medical applications, and is based on the difference between the dielectric properties of normal and diseased tissues. Microwave tomography features rapid data acquisition time, and together with rapid tomographic reconstructions allows detecting, identifying and monitoring injuries continuously.

From a computational point of view, microwave imaging requires the solution of an inverse problem based on a minimisation algorithm. Reconstruction algorithms are computationally intensive with successive solutions of the forward problem needing efficient numerical modelling and high-performance parallel computing. This methodology involves distinct research fields: optimisation, inverse problems, approximation and solution methods for the simulation of the forward problem modelled by Maxwell's equations. The latter is challenging in itself as the modelling must accurately take account of the high heterogeneity and complexity of the different tissues.

Our numerical framework for microwave imaging is based on the FreeFEM 7.1.1 finite element software, and the forward problem is solved efficiently in parallel using Domain Decomposition methods implemented in the HPDDM 7.1.2 library.

Detection and imaging of rotator cuff tears

One of the most challenging shoulder injuries is rotator cuff tear, which increases with aging and particularly happens among athletes. These tears cause pain and highly affect the functionality of the shoulder.

In [3, 2], we propose to study the feasibility of microwave tomographic imaging to detect these tears. We introduce a wearable imaging system design, adapted to the real shoulder structure and designed to surround it partially. We investigate the efficiency of the imaging system numerically using realistic CAD models for shoulder profile and bones, including humerus and scapula, and taking into account the dielectric properties of the different tissues (bone, tendon, muscle, skin, synovial fluid). The reconstructed images of the shoulder joint obtained using noisy synthetic data show that the proposed imaging system is capable of accurately detecting and localizing large (5mL) and partial (1mL) rotator cuff tears.

4.3 Numerical methods for time harmonic wave propagation problems

Acoustic, electromagnetic and elastic linear waves are ubiquitous phenomena in science and engineering. The numerical simulation of their propagation and interaction is a core task in areas like medical imaging, seismic remote sensing, design of electronic devices, atmospheric particle scattering, radar and sonar modelling, etc. The design of appropriate numerical schemes is challenging because of approximation issues arising from the fact that the solutions are highly oscillatory, which usually require a large computational effort to produce a meaningful numerical approximation. In addition, common formulations suffer from stability issues, such as the numerical dispersion (also known as pollution effect), and sign-indefiniteness, which typically limit the achievable accuracy. The variety of different applications and the challenges of their numerical simulations produces a great research effort in trying to design new efficient schemes: new approximation spaces using degrees of freedom sparingly (wave based and Trefftz methods), preconditioners for solving large linear systems (e.g. by domain decomposition), new formulations, acceleration techniques, etc.

4.4 Molecular simulations

Molecular simulation is one of the most dynamic areas of scientific computing. Its field of application is very broad, ranging from theoretical chemistry and drug design to materials science and nanotechnology. It provides many challenging problems to mathematicians, and computer scientists.

In the context of the ERC Synergy Grant EMC2 we address several important limitations of state of the art molecular simulation. In particular, the simulation of very large molecular systems, or smaller systems in which electrons interact strongly with each other, remains out of reach today. In an interdisciplinary collaboration between chemists, mathematicians and computer scientists, we focus on developing a new generation of reliable molecular simulation algorithms and software.

5 Social and environmental responsibility

5.1 Impact of research results

Our activities in scientific computing help in the design and optimization of non fossil energy resources. Here are three examples we are aware of and there are many others:

- the startup **Airthium** specialized in decarbonized energy uses our free parallel finite element software **FreeFEM**.
- The French electricity company EDF has integrated our domain decomposition methods GenEO via the open source library **HPDDM** in its simulation software Salomé in order to be able to perform large scale computations related to nuclear reactors.
- In the context of climate change, our work on radiative transfer studies the change of temperature in the atmosphere, in particular due to the effect of clouds and greenhouse gases.

6 Highlights of the year

Emile Parolin, together with Laure Giovangigli (Inria Saclay), obtained an Exploratory Action funding from Inria: AEx QUI (Quantitative Ultrasound Imaging).

Summary of the AEx projet: Conventional ultrasound imaging algorithms rely on the assumption that the speed of sound is constant in the medium of propagation. Can we overcome this limitation and produce reliable ultrasound images through a fat layer or the skull? This is the question that this project intends to answer. In collaboration with the Langevin Institute, we are developing innovative algorithms capable of reconstructing a local map of the speed of sound directly from measurements. In parallel, we are developing a cutting-edge numerical simulation tool that generates realistic ultrasound propagation data in soft tissues. Through high-performance computing, we can simulate these models on a real scale, thus ensuring the validity and effectiveness of our algorithms.

7 Latest software developments, platforms, open data

7.1 Latest software developments

7.1.1 FreeFem++

Name: FreeFem

Keywords: Scientific computing, High performance computing, Boundary element method

Scientific Description: FreeFem++ is a partial differential equation solver. It has its own language. freefem scripts can solve multiphysics non linear systems in 2D and 3D.

Problems involving PDE (2d, 3d) from several branches of physics such as fluid-structure interactions require interpolations of data on several meshes and their manipulation within one program. FreeFem++

includes a fast $2\hat{d}$ -tree-based interpolation algorithm and a language for the manipulation of data on multiple meshes (as a follow up of bamg (now a part of FreeFem++)).

FreeFem++ is written in C++ and the FreeFem++ language is a C++ idiom. It runs on Macs, Windows, Unix machines. FreeFem++ replaces the older freefem and freefem+.

Functional Description: FreeFem is a PDE (partial differential equation) solver based on a flexible language that allows a large number of problems to be expressed (elasticity, fluids, etc) with different finite element approximations on different meshes.

URL: <https://freefem.org/>

Contact: Frédéric Hecht

7.1.2 HPDDM

Keywords: High performance computing, Parallel computing

Scientific Description: HPDDM is an efficient implementation of various domain decomposition methods (DDM) such as one- and two-level Restricted Additive Schwarz methods, the Finite Element Tearing and Interconnecting (FETI) method, and the Balancing Domain Decomposition (BDD) method. This code has been proven to be efficient for solving various elliptic problems such as scalar diffusion equations, the system of linear elasticity, but also frequency domain problems like the Helmholtz equation. A comparison with modern multigrid methods can be found in the thesis of Pierre Jolivet.

Functional Description: HPDDM is an efficient implementation of various domain decomposition methods (DDM) such as one- and two-level Restricted Additive Schwarz methods, the Finite Element Tearing and Interconnecting (FETI) method, and the Balancing Domain Decomposition (BDD) method. It is interfaced, among others, via PETSc, FEM, Feel++.

URL: <https://github.com/hpddm>

Contact: Pierre Jolivet

Participant: 2 anonymous participants

7.1.3 Htool-DDM

Keywords: Hierarchical matrices, Domain decomposition, Preconditioner

Functional Description: Htool-DDM is a lightweight, header-only C++14 library that provides parallel iterative solvers with domain decomposition preconditioners, relying on an in-house hierarchical matrix compression for dense/compressed linear systems.

URL: <https://github.com/htool-ddm/htool>

Contact: Pierre Marchand

7.1.4 scatensor

Name: Scalable and Communication Avoiding Tensor Library

Keywords: Tensor decomposition, Linear algebra, Matrix calculation, Distributed computing, MPI, Communication avoiding

Scientific Description: Matrix and tensor decomposition in parallel, to obtain a compressed version.

Functional Description: - Reading distributed matrices from a file. - Generating random matrices. - Distributed QR decomposition with tournament pivoting (QRTP) of a matrix. - Reading/Generating a tensor. - Tucker decomposition of a tensor, using QRTP.

Publications: [hal-02947991](#), [hal-03079236](#)

Contact: Laura Grigori

Participant: 3 anonymous participants

7.2 Open data

Non applicable

8 New results

Participants: Alpines members.

8.1 Numerical shape and topology optimization of regions supporting the boundary conditions of a physical problem

With: Eric Bonnetier, Carlos Brito-Pacheco, Charles Dapogny, Rafael Estevez.

The article [34] deals with a particular class of shape and topology optimization problems: the optimized design is a region G of the boundary $\partial\Omega$ of a given domain Ω , which supports a particular type of boundary conditions in the state problem characterizing the physical situation. In our analyses, we develop adapted versions of the notions of shape and topological derivatives, which are classically tailored to functions of a “bulk” domain. This leads to two complementary notions of derivatives for a quantity of interest $J(G)$ depending on a region $G \subset \partial\Omega$ on the one hand, we elaborate on the boundary variation method of Hadamard for evaluating the sensitivity of $J(G)$ with respect to “small” perturbations of the boundary of G within $\partial\Omega$. On the other hand, we use techniques from asymptotic analysis to appraise the sensitivity of $J(G)$ with respect to the addition of a new connected component to the region G , shaped as a “small” surface disk. The calculation of both types of derivatives raises original difficulties, which are closely related to the weakly singular behavior of the solution to a boundary value problem at the points of $\partial\Omega$ where the boundary conditions change types. These aspects are carefully detailed in a simple mathematical setting based on the conductivity equation. We notably propose formal arguments to calculate our derivatives with a minimum amount of technicality, and we show how they can be generalized to handle more intricate problems, arising for instance in the physical contexts of acoustics and structural mechanics, respectively governed by the Helmholtz equation and the linear elasticity system. In numerical applications, our derivatives are incorporated into a recent algorithmic framework for tracking arbitrarily dramatic motions of a region G within a fixed ambient surface, which combines the level set method with remeshing techniques to offer a clear, body-fitted discretization of the evolving region. Finally, various 3d numerical examples are presented to illustrate the salient features of our analysis.

8.2 Distributionally robust shape and topology optimization

With: Charles Dapogny, Julien Prando, Boris Thibert.

In [24], we aim to introduce the paradigm of distributional robustness from the field of convex optimization to tackle optimal design problems under uncertainty. We consider realistic situations where the physical model, and thereby the cost function of the design to be minimized depend on uncertain parameters. The probability distribution of the latter is itself known imperfectly, through a nominal law, reconstructed from a few observed samples. The distributionally robust optimal design problem is an intricate bilevel program which consists in minimizing the worst value of a statistical quantity of the cost function (typically, its expectation) when the law of the uncertain parameters belongs to a certain “ambiguity set”. We address three classes of such problems: firstly, this ambiguity set is made of the probability laws whose Wasserstein distance to the nominal law is less than a given threshold; secondly, the ambiguity set is based on the first- and second-order moments of the actual and nominal probability laws. Eventually, a statistical quantity of the cost other than its expectation is made robust with respect to the law of the parameters, namely its conditional value at risk. Using techniques from convex duality, we derive tractable, single-level reformulations of these

problems, framed over augmented sets of variables. Our methods are essentially agnostic of the optimal design framework; they are described in a unifying abstract framework, before being applied to multiple situations in density-based topology optimization and in geometric shape optimization. Several numerical examples are discussed in two and three space dimensions to appraise the features of the proposed techniques.

8.3 A shape optimization approach for inferring sources of volcano ground deformation

With: Théo Perrot, Freysteinn Sigmundsson, Charles Dapogny.

One of the main goals of volcano geodesy is to improve the understanding of how an increase in pressure related to magma accumulation causes ground deformation in order to evaluate volcanic unrest. The inversion methods used for this purpose rely on a parametrization of the shape of the crustal volume in which pressure changes due to magma inflow/outflow (the magma domain), to search for the optimal parameters that minimize the difference between model predicted and measured ground displacements. However, these methods assume a predefined shape of the magma domain, which limits their applicability.

@unpublishedperrot:hal-05373455, In [40], we propose a new shape optimization framework that can invert these sources without such prior, formulating a reconstruction problem to infer the complete shape of the magma domain. First, we validate this approach using a synthetic test case and then apply it to observations of the Svartsengi volcanic system in Iceland.

8.4 Domain decomposition preconditioners for efficient parallel simulations of single-phase flow in three-dimensional fractured porous media with a very large number of fractures

With: Pierre Jolivet, Michel Kern, Frédéric Nataf, Géraldine Pichot, Daniel Zegarra Vasquez.

In [30], three-dimensional fractured-porous media are classically described by the Discrete Fracture Matrix (DFM) model, where the rock matrix remains three-dimensional, while the fracture network is of co-dimension one. This study deals with efficient solutions of the linear system arising in single-phase flow in large DFMs using a mixed-hybrid finite element method. It demonstrates the effectiveness of two-level domain decomposition methods based on the GenEO framework for preconditioning iterative solvers applied to such flow problems. Due to the mixed-dimensional nature of the geometry, a specific partitioning strategy is required. The coarse spaces for these preconditioners are built by locally solving spectral problems. This paper presents simulations with very large DFMs, with up to hundreds of thousands of fractures, achieving a low iteration count and short total computation time. Furthermore, this work confirms the scalability of these preconditioners.

8.5 A scalable Domain Decomposition method for Saddle Point problems with GenEO coarse spaces

With: Filippo Brunelli, Guillaume Delay, Frédéric Nataf, Emile Parolin, Pierre-Henri Tournier.

In [22], we present an adaptive domain decomposition (DD) preconditioning technique for the solution of saddle point problems with a 2×2 blocks structure. This work utilises the GenEO theory for symmetric positive definite (SPD) problems (Spillane et al., 2014) to tackle the solution of saddle point linear systems with iterative methods. The latter are preferred over direct methods due to the typically large size of these problems. Under the same assumptions of the GenEO method we can develop a robust and scalable preconditioning technique for the saddle point linear system. We present a preconditioner for the primal-primal block and one for the Schur complement matrix, both based on the DD paradigm. Then, these ingredients are used to precondition the 2×2 block saddle point matrix. Numerical tests are performed on a benchmark systems of partial differential equations (PDEs) discretized with finite elements, for which the assumptions are easy to verify. The results emphasize preconditioner's robustness with respect to the specific domain's partitioning.

8.6 A robust and adaptive GenEO-type domain decomposition preconditioner for $H(\text{curl})$ problems in general non-convex three-dimensional geometries

With: Niall Bootland, Victorita Dolean, Frédéric Nataf, Pierre-Henri Tournier.

In [17], we develop and analyse domain decomposition methods for linear systems of equations arising from conforming finite element discretisations of positive Maxwell-type equations, namely for $H(\text{curl})$ problems. It is well known that convergence of domain decomposition methods rely heavily on the efficiency of the coarse space used in the second level. We design adaptive coarse spaces that complement a near-kernel space made from the gradient of scalar functions. The new class of preconditioner is inspired by the idea of subspace decomposition, but based on spectral coarse spaces, and is specially designed for curl-conforming discretisations of Maxwell's equations in heterogeneous media on general domains which may have holes. Our approach has wider applicability and theoretical justification than the well-known Hiptmair-Xu auxiliary space preconditioner, with results extending to the variable coefficient case and non-convex domains at the expense of a larger coarse space.

8.7 Coarse spaces for non-symmetric two-level preconditioners based on local generalized eigenproblems

With: Frédéric Nataf, Emile Parolin.

In [39], we present an analysis of adaptive coarse spaces which is quite general since it applies to symmetric and non symmetric problems, to symmetric preconditioners such the additive Schwarz method (ASM) and to the non-symmetric preconditioner restricted additive Schwarz (RAS), as well as to exact or inexact subdomain solves. The coarse space is built by solving generalized eigenvalues in the subdomains and applying a well-chosen operator to the selected eigenvectors.

8.8 Coarse spaces using extended generalized eigenproblems for heterogeneous Helmholtz problems

With: Emile Parolin, Frédéric Nataf, Aminata Diarra.

An abstract construction of coarse spaces for non-Hermitian problems and non-Hermitian domain decomposition preconditioners based on extended generalized eigenproblems was proposed in § 8.7 and analyzed on the matrix formulation. Building upon this work, in [31], we consider instead the specific case of heterogeneous Helmholtz problems, and the derivation and analysis is performed at the continuous level.

Additional numerical investigations and a computational cheaper alternative construction of the coarse space have been obtained during the Master thesis of Aminata Diarra.

8.9 Numerical comparison of coarse spaces for heterogeneous Helmholtz problems

With: Victorita Dolean, Mark Fry, Matthias Langer, Emile Parolin, Pierre-Henri Tournier.

Solving time-harmonic wave propagation problems in the frequency domain within heterogeneous media poses significant mathematical and computational challenges, particularly in the high-frequency regime. Among the available numerical approaches, domain decomposition methods can achieve near-constant time-to-solution as the wavenumber increases, though often at the expense of a computationally intensive coarse correction step. In [25], we focus on identifying the best algorithms and numerical strategies for benchmark problems modelled by the Helmholtz equation. Specifically, we examine and compare several coarse spaces which are part of different families, e.g. GenEO (Generalised Eigenvalue Overlap) type coarse spaces and harmonic coarse spaces (including the one in §8.8), that underpin two-level domain decomposition methods. By leveraging spectral information and multiscale approaches, we aim to provide a comprehensive overview of the strengths and weaknesses of these methods. Numerical experiments demonstrate that the effectiveness of these coarse spaces depends on the specific problem and numerical configuration, highlighting the trade-offs between computational cost, robustness, and practical applicability.

8.10 Modal analysis of a domain decomposition method for Maxwell's equations in a waveguide

With: Victorita Dolean, Antoine Tonnoir, Pierre-Henri Tournier.

Time-harmonic wave propagation problems, especially those governed by Maxwell's equations, pose significant computational challenges due to the non-self-adjoint nature of the operators and the large, non-Hermitian linear systems resulting from discretization. Domain decomposition methods, particularly one-level Schwarz methods, offer a promising framework to tackle these challenges, with recent advancements showing the potential for weak scalability under certain conditions. In [26], we analyze the weak scalability of one-level Schwarz methods for Maxwell's equations in strip-wise domain decompositions, focusing on waveguides with general cross sections and different types of transmission conditions such as impedance or perfectly matched layers (PMLs). By combining techniques from the limiting spectrum analysis of Toeplitz matrices and the modal decomposition of Maxwell's solutions, we provide a novel theoretical framework that extends previous work to more complex geometries and transmission conditions. Numerical experiments confirm that the limiting spectrum effectively predicts practical behavior even with a modest number of subdomains. Furthermore, we demonstrate that the one-level Schwarz method can achieve robustness with respect to the wave number under specific domain decomposition parameters, offering new insights into its applicability for large-scale electromagnetic wave problems.

8.11 A Robust Two-Level Schwarz Preconditioner For Sparse Matrices

With: Hussam Al Daas, Pierre Jolivet, Frédéric Nataf, Pierre-Henri Tournier.

In [16], we introduce a fully algebraic two-level additive Schwarz preconditioner for general sparse large-scale matrices. The preconditioner is analyzed for symmetric positive definite (SPD) matrices. For those matrices, the coarse space is constructed based on approximating two local subspaces in each subdomain. These subspaces are obtained by approximating a number of eigenvectors corresponding to dominant eigenvalues of two judiciously posed generalized eigenvalue problems. The number of eigenvectors can be chosen to control the condition number. For general sparse matrices, the coarse space is constructed by approximating the image of a local operator that can be defined from information in the coefficient matrix. The connection between the coarse spaces for SPD and general matrices is also discussed. Numerical experiments show the great effectiveness of the proposed preconditioners on matrices arising from a wide range of applications. The set of matrices includes SPD, symmetric indefinite, nonsymmetric, and saddle-point matrices. In addition, we compare the proposed preconditioners to the state-of-the-art domain decomposition preconditioners.

8.12 Microwave tomographic imaging of shoulder injury

One of the most challenging shoulder injuries is rotator cuff tear, which increases with aging and particularly happens among athletes. These tears cause pain and highly affect the functionality of the shoulder. The motivation of this work [37] is to detect these tears by microwave tomographic imaging. This imaging method requires the solution of an inverse problem based on a minimization algorithm, with successive solutions of a direct problem. The direct problem consists in solving the time-harmonic Maxwell's equations discretized with a Nédélec edge finite element method, and we make use of an ORAS Schwarz domain decomposition preconditioner to solve it efficiently in parallel. We propose a wearable imaging system design with 96 antennas distributed over two fully-circular and two half-circular layers, adapted to the real shoulder structure and designed to surround it partially. We test the imaging system numerically using realistic CAD models for shoulder profile and bones, including humerus and scapula, and taking into account the dielectric properties of the different tissues (bone, tendon, muscle, skin, synovial fluid). The reconstructed images of the shoulder joint obtained using noisy synthetic data show that the proposed imaging system is capable of accurately detecting and localizing large (5mL) and partial (1mL) rotator cuff tears. The reconstruction takes 20 minutes on 480 computing cores and shows great promise for rapid diagnosis or medical monitoring. In a second step, we take advantage of numerical modeling to try and optimize the number of antennas in the imaging system, achieving a drastic reduction from 96 to 32 antennas while still being able to detect the injury in the difficult partial tear case, reducing the computing time from 20 to 12 minutes.

In [36, 35], we use the previous forward numerical modeling techniques to generate a generalizable dataset of scattering parameters in order to bypass real-world data collection challenges. We then explore the use of machine learning (ML) algorithms for the fast detection of shoulder tendon injuries. The corresponding data of various healthy and injured models are categorized into two classes. We use a support vector machine (SVM) to differentiate between injured and healthy shoulder models. This approach is more efficient in terms of required memory resources and computing time compared with traditional imaging methods, and we manage to achieve an accuracy of 100%.

8.13 New developments in FreeFEM

Work has been started to ease the parallelization of a user's sequential code. In this setting, the parallelization of the assembly and solution steps is entirely hidden to the user, and a sequential code can be parallelized with only very few changes in the script. For example, PETSc can easily be used to solve the linear system stemming from a coupled problem with a fieldsplit preconditioner.

The FreeFEM software now includes the GenEO for saddle-point [11] examples with PCHPDDM in PETSc.

FreeFEM can now run Markdown (.md) files as well as .edp files. The Markdown syntax can be used to document FreeFEM scripts. When running a .md file, FreeFEM will execute the code contained in the FreeFEM code blocks. Documented Markdown examples in the distribution can be viewed on the documentation website. Markdown can also be previewed with Visual Studio Code, with the FreeFEM VS Code extension providing syntax highlighting for FreeFEM code blocks.

8.14 Reduced models for highly oscillating differential equations

With: Sever Hirstoaga

In this part we are concerned with solving Vlasov equation involving several time scales. This work shows that finding and solving reduced models, though complicated to derive, turns out to be very useful for reducing the computational cost of an equation. In addition, using the parareal algorithm, a method performing parallel computing in time, allows to accelerate the computations.

The aim in this context is to derive reduced models of first-order from a two-scale asymptotic expansion in a small parameter, in order to approximate the solution of a stiff differential equation. The problem of interest is a multi-scale Newton-Lorentz equation modeling the dynamics of a charged particle under the influence of a linear electric field and of a perturbed strong magnetic field (see [38]). In [27] we extend this idea to a more general equation, which models the motion of a charged particle in a tokamak-like magnetic field, dependent on space and time. Thus, by symbolic computations, we obtain first-order equations approximating stiff differential equations in this general case. We develop numerical simulations to support the approximation results and the efficiency in terms of computational times. More precisely, we demonstrate on several numerical examples that the first-order two-scale solution (i) is much less costly than the reference solution of the initial model and (ii) is much more accurate than the zero-order two-scale solution.

In view of these encouraging results, we continue working on the derivation of first-order models by homogenization in time, in the case of partial differential equations as Vlasov-Poisson, by expressing its solution using characteristics. The implementation in this framework have yielded very good numerical results, emphasizing the efficiency of the first-order reduced model. The numerical framework is the particle-in-cell approach, widely used for kinetic simulations. Developed in 3D+3V (six dimensions plus time), this code is intended to be used in a parallel frame with distributed and shared memory paradigms, in order to achieve high performance computing. This is ongoing work.

8.15 Stable Trefftz approximation of Helmholtz solutions using evanescent plane waves

With: Nicola Galante, Andrea Moiola, Emile Parolin.

Superpositions of plane waves are known to approximate well the solutions of the Helmholtz equation. Their use in discretizations is typical of Trefftz methods for Helmholtz problems, aiming to achieve high accuracy with a small number of degrees of freedom. However, Trefftz methods lead to ill-conditioned linear systems, and it is often impossible to obtain the desired accuracy in floating-point arithmetic.

In [18] we extend the analysis of [12] from two to three space dimensions. We show that a judicious choice of plane waves can ensure high-accuracy solutions in a numerically stable way, in spite of having to solve such ill-conditioned systems. Numerical accuracy of plane wave methods is linked not only to the approximation space, but also to the size of the coefficients in the plane wave expansion. We show that the use of plane waves can lead to exponentially large coefficients, regardless of the orientations and the number of plane waves, and this causes numerical instability. We prove that all Helmholtz fields are continuous superposition of evanescent plane waves, i.e., plane waves with complex propagation vectors associated with exponential decay, and show that this leads to bounded representations. We provide a constructive scheme to select a set of real and complex-valued propagation vectors numerically. This results in an explicit selection of plane waves and an associated Trefftz method that achieves accuracy and stability. A non-trivial challenge in 3D was the parametrization of the complex direction set. Our approach involves defining a complex-valued reference direction and then consider its rigid-body rotations via Euler angles. Then, by generalizing the Jacobi–Anger identity to complex-valued directions, we prove that any solution of the Helmholtz equation on a three-dimensional ball can be written as a continuous superposition of evanescent plane waves in a stable way. Our numerical results showcase a great improvement of our proposed method on the achievable accuracy compared to standard method.

8.16 A Trefftz Continuous Galerkin method for Helmholtz problems

With: Nicola Galante, Bruno Després, Emile Parolin.

In [28], we introduce a novel Trefftz Continuous Galerkin (TCG) method for 2D Helmholtz problems based on evanescent plane waves (EPWs). We construct a new globally-conforming discrete space, departing from standard discontinuous Trefftz formulations, and investigate its approximation properties, providing wavenumber-explicit best-approximation error estimates. The mesh is defined by intersecting the domain with a Cartesian grid, and the basis functions are continuous in the whole computational domain, compactly supported, and can be expressed as simple linear combinations of EPWs within each element. This ensures they remain local solutions to the Helmholtz equation and allows the system matrix to be assembled in closed form for polygonal domains. The discrete space provides stable approximations with bounded coefficients and spectral accuracy for analytic Helmholtz solutions. The approximation error is proved to decay exponentially both at a fixed frequency, with respect to the discretization parameters, and along suitable sequences of increasing wavenumbers, with the number of degrees of freedom scaling linearly with the frequency. Numerical results confirm these theoretical estimates for the full Galerkin error.

8.17 Parallel approximation of the exponential of Hermitian matrices

In [10], we consider a rational approximation of the exponential function to design an algorithm for computing matrix exponential in the Hermitian case. Using partial fraction decomposition, we obtain a parallelizable method, where the computation reduces to independent resolutions of linear systems. We analyze the effects of rounding errors on the accuracy of our algorithm. We complete this work with numerical tests showing the efficiency of our method and a comparison of its performances with Krylov algorithms.

8.18 A finite element toolbox for the Bogoliubov-de Gennes stability analysis of Bose-Einstein condensates

In [41], we present a finite element toolbox for the computation of Bogoliubov-de Gennes modes used to assess the linear stability of stationary solutions of the Gross-Pitaevskii (GP) equation. Applications concern one (single GP equation) or two-component (a system of coupled GP equations) Bose-Einstein condensates in one, two and three dimensions of space. An implementation using the free software FreeFem++ is distributed with this paper. For the computation of the GP stationary (complex or real) solutions we use a Newton algorithm coupled with a continuation method exploring the parameter space (the chemical potential or the interaction constant). Bogoliubov-de Gennes equations are then solved using dedicated libraries for the associated eigenvalue problem. Mesh adaptivity is proved to considerably reduce the computational time for cases implying complex vortex states. Programs are validated through comparisons with known theoretical results for simple cases and 97 numerical results reported in the literature.

8.19 Mathematics and Finite Element Discretizations of Incompressible Navier-Stokes Flows

With : C. Bernardi, V. Girault, F. Hecht, P.-A. Raviart, B. Riviere.

This book in final preparation (2024) is a revised, updated, and augmented version of the out-of-print classic book “Finite Element Methods for Navier–Stokes Equations” by Girault and Raviart published by Springer in 1986 [GR]. The incompressible Navier–Stokes equations model the flow of incompressible Newtonian fluids and are used in many practical applications, including computational fluid dynamics. In addition to the basic theoretical analysis, this book presents a fairly exhaustive treatment of the up-to-date finite element discretizations of incompressible Navier–Stokes equations and a variety of numerical algorithms used in their computer implementation. It covers the cases of standard and non-standard boundary conditions and their numerical discretizations via the finite element methods. Both conforming and non-conforming finite elements are examined in detail, as well as their stability or instability. The topic of time-dependent Navier–Stokes equations, which was missing from [GR], is now presented in several chapters. In the same spirit as [GR], we have tried as much as possible to make this book self-contained and therefore we have either proved or recalled all the theoretical results required. This book can be used as a textbook for advanced graduate students.

8.20 Randomized Householder QR

With Laura Grigori, Edouard Timsit

In [8], we introduce a randomized Householder QR factorization (RHQR). This factorization can be used to obtain a well conditioned basis of a set of vectors and thus can be employed in a variety of applications. We discuss in particular the usage of this randomized Householder factorization in the Arnoldi process. Numerical experiments show that RHQR produces a well conditioned basis and an accurate factorization. We observe that for some cases, it can be more stable than Randomized Gram-Schmidt (RGS) in both single and double precision.

8.21 Randomized Krylov-Schur eigensolver with deflation

With: Jean-Guillaume De Damas, Laura Grigori.

In [23], we introduce a novel algorithm to solve large-scale eigenvalue problems and seek a small set of eigenpairs. The method, called randomized Krylov-Schur (rKS), has a simple implementation and benefits from fast and efficient operations in low-dimensional spaces, such as sketch-orthogonalization processes and stable reordering of Schur factorizations. It also includes a practical deflation technique for converged eigenpairs, enabling the computation of the eigenspace associated with a given part of the spectrum. Numerical experiments are provided to demonstrate the scalability and accuracy of the method.

8.22 PDE-constrained optimization within FreeFEM

With Gontran Lance, and Emmanuel Trélat

The book [20] is aimed at students and researchers who want to learn how to efficiently solve constrained optimization problems involving partial differential equations (PDE) using the FreeFEM software. PDE-constrained optimization problems are frequently encountered in many academic and industrial contexts. Readers should have a basic knowledge of the analysis and numerical solution of partial differential equations using finite element methods, optimization, algorithms and numerical implementation.

9 Bilateral contracts and grants with industry

9.1 Bilateral contracts with industry

Participants: F. Nataf, B. Antoine dit Urban.

- Contract with IFPEN and ONERA, January 2025 - December 2027, that funds half of the Phd thesis of Brieuc Antoine dit Urban on "Méthodes d'apprentissage pour les solveurs linéaires et préconditionneurs pour matrices creuses" . Supervisor Frédéric Nataf, T. Faney and E. Martin.

Participants: C. Dapogny.

- Contract with Schneider Electric, April 2025 - April 2028, that funds the Phd thesis of Stéphane Gaydier on "Shape and topology optimization of current sensors and actuators" . Supervisors Charles Dapogny, Olivier Chadebec, Carlos Valdivieso.

9.2 Bilateral training with industry

We organized a three-day training session on the finite element toolbox FreeFEM followed by a specialized training on problems from the company Alice & Bob at a pace of 2 hours per week online for two months.

10 Partnerships and cooperations

10.1 European initiatives

10.1.1 Horizon Europe

ERC Synergy: PSINumScat [PSINumScat project on cordis.europa.eu](https://cordis.europa.eu/PSINumScat)

Participants: Pierre-Henri Tournier, Frédéric Nataf.

Title: Phase-space-inspired Numerical Methods for High Frequency Wave Scattering

Duration: From May 1, 2025 to 2031

Partners:

- UNIVERSITY OF BATH, UK
- UNIVERSITY COLLEGE LONDON, UK
- CENTRE NATIONAL DE LA RECHERCHE SCIENTIFIQUE CNRS (CNRS), France

Inria contact: Pierre-Henri Tournier (Alpines)

Coordinators: Euan Spence (U. Bath), Jeffrey Galkowski (UCL), Pierre-Henri Tournier (Alpines)

Summary: Phase-space-inspired Numerical Methods for High Frequency Wave Scattering (PSINumScat) is an ERC Synergy funded project. Designing fast and reliable algorithms to numerically simulate the behaviour of high-frequency acoustic and electromagnetic waves is a longstanding open problem in computational mathematics. These waves underpin a plethora of communication and imaging technologies; therefore any progress towards solving this problem will have wide impact. By exploiting techniques from pure mathematics specifically designed to study high-frequency problems, PSINumScat aims to design, analyse, and implement in open-source software new methods for the numerical solution of high-frequency acoustic and electromagnetic wave scattering problems.

Inno4Scale [Inno4Scale project on cordis.europa.eu](https://cordis.europa.eu/project/309204)**Title:** Innovative Algorithms for Applications on European Exascale Supercomputers**Duration:** From July 1, 2023 to June 30, 2025**Partners:**

- SCAPOS AG (SCAPOS), Germany
- PARTNERSHIP FOR ADVANCED COMPUTING IN EUROPE AISBL (PRACE), Belgium
- UNIVERSITY OF STUTTGART (USTUTT), Germany
- BARCELONA SUPERCOMPUTING CENTER CENTRO NACIONAL DE SUPERCOMPUTACION (BSC CNS), Spain

Inria contact: Frédéric Nataf**Coordinator:**

Summary: New architectures for Exascale and post-Exascale computers will have massively parallel and heterogeneous processing capabilities that will require the complete redesign and reimplementations of the used algorithms to fully exploit the possibilities of these supercomputers. The objective of the Inno4scale project is to promote the efficient use of European HPC systems by identifying and funding the most promising novel algorithms for applications. Through this action, the project will facilitate the design and development of highly efficient application software for European Exascale computers, the most successful of which should later be taken up by science and industry, resulting in important performance gains. The project will design and implement a call for proposals based on the cascade funding mechanism, financing the most promising ideas on novel algorithms for applications to fully exploit and benefit from these valuable European Exascale resources. Proposals received will be evaluated by external experts based on their potential innovation and impact on the upcoming Exascale era, and the CASTIEL/EuroCC2 and PRACE networks will be used to ensure the widest possible reach of the call. The Inno4scale consortium, made up of two major European HPC research centres and two experienced partners in managing HPC-related programmes, is in the ideal position to carry out the project in an effective and efficient manner. The results of the selected innovation studies that will receive more than 4 million euros in total, include the development and publication of the novel algorithms to reduce the time-to-solution or energy-to-solution of existing computing problems, as well as to provide novel approaches for large-scale projects. In this way, the Inno4scale project will contribute to the realisation of the EuroHPC JU overall objectives and those specific for this call, as well as enable an optimal exploitation of European Exascale and Post-Exascale HPC systems.

10.1.2 H2020 projects**EMC2** [EMC2 project on cordis.europa.eu](https://cordis.europa.eu/project/101017781)**Title:** Extreme-scale Mathematically-based Computational Chemistry**Duration:** From September 1, 2019 to August 31, 2026**Partners:**

- INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET AUTOMATIQUE (INRIA), France
- ECOLE POLYTECHNIQUE FEDERALE DE LAUSANNE (EPFL), Switzerland
- ECOLE NATIONALE DES PONTS ET CHAUSSEES (ENPC), France
- CENTRE NATIONAL DE LA RECHERCHE SCIENTIFIQUE CNRS (CNRS), France
- SORBONNE UNIVERSITE, France

Inria contact: Laura GRIGORI (Alpines)

Coordinator:

Summary: Molecular simulation has become an instrumental tool in chemistry, condensed matter physics, molecular biology, materials science, and nanosciences. It will allow to propose de novo design of e.g. new drugs or materials provided that the efficiency of underlying software is accelerated by several orders of magnitude.

The ambition of the EMC2 project is to achieve scientific breakthroughs in this field by gathering the expertise of a multidisciplinary community at the interfaces of four disciplines: mathematics, chemistry, physics, and computer science. It is motivated by the twofold observation that, i) building upon our collaborative work, we have recently been able to gain efficiency factors of up to 3 orders of magnitude for polarizable molecular dynamics in solution of multi-million atom systems, but this is not enough since ii) even larger or more complex systems of major practical interest (such as solvated biosystems or molecules with strongly-correlated electrons) are currently mostly intractable in reasonable clock time. The only way to further improve the efficiency of the solvers, while preserving accuracy, is to develop physically and chemically sound models, mathematically certified and numerically efficient algorithms, and implement them in a robust and scalable way on various architectures (from standard academic or industrial clusters to emerging heterogeneous and exascale architectures).

EMC2 has no equivalent in the world: there is nowhere such a critical number of interdisciplinary researchers already collaborating with the required track records to address this challenge. Under the leadership of the 4 PIs, supported by highly recognized teams from three major institutions in the Paris area, EMC2 will develop disruptive methodological approaches and publicly available simulation tools, and apply them to challenging molecular systems. The project will strongly strengthen the local teams and their synergy enabling decisive progress in the field.

10.2 National initiatives

10.2.1 ANR

PEPR Numpex - PC Exama

Participants: Brieuc Antoine Dit Urban, Tom Caruso, Frédéric Hecht, Frédéric Nataf, Emile Parolin, Pierre-Henri Tournier.

Numpex is a French program dedicated to Exascale which is divided into five Research Projects. The first one is **Exa-MA** (Methods and Algorithms of Exascale) aimed at advancing scientific simulations and modeling capabilities to reach and surpass the exascale barrier. The project is organized into several work packages (WP) that focus on different aspects of the research objectives. Frédéric Nataf is in charge of the Work Package 3 (WP3): Numerical Kernels and Coupled Solvers Focuses on designing and implementing efficient and possibly provable numerical kernels and solvers for largescale problems. Tasks include domain decomposition methods, data sparsity techniques, multiple precision, adaptive solution strategies, and efficient coupling of multiphysics simulations.

Partners in WP3: LIP6 Sorbonne University, INRIA Bordeaux, CEA.

STOIQUES : ANR appel à projet générique 2025-2029.

Participants: Charles Dapogny.

C. Dapogny is a member of the project STOIQUES, whose objective is to investigate modern problems in shape optimization, and to federate the community of researchers interested in this field, in order to develop new methods and tools for tackling new models or considering new constraints.

Principal Investigator: Y. Privat (Ecole des Mines de Nancy).

StableProxies : ANR appel à projet générique 2025-2029.

Participants: Charles Dapogny.

C. Dapogny is a member of the project STOIQUES, whose objective is to propose new models for the numerical treatment of data with heterogeneous formats (polygonal surfaces, cloud points, collections of voxels, etc.). This ambition relies on a geometric model of “proxy”, representing these data on a regular grid, which allows to obtain mathematical guarantees, and to construct robust algorithms. Two applications are targeted, in material science, and in imaging (analysis of LIDAR or photogrammetric data).

Principal Investigator: D. Coeurjolly (Université de Lyon).

11 Dissemination

11.1 Promoting scientific activities

11.1.1 Journal

Member of the editorial boards

- Frédéric Nataf is Associate Editor at Journal of Numerical Mathematics.
- Frédéric Nataf is Associate Editor on the Communications of the American Mathematical Society Editorial Committee.

Reviewer - reviewing activities

- Frédéric Nataf reviewed three articles in Journal of Computational Physics, CMAME and Computational Geosciences.
- Emile Parolin reviewed 4 articles in Journal of Computational Physics, Journal of Scientific Computing, ESAIM: Mathematical Modelling and Numerical Analysis, SIAM Journal on Scientific Computing.
- Charles Dapogny reviewed 14 articles in Journal of Computational Physics, Numerische Mathematik, ESAIM: Mathematical Modelling and Numerical Analysis, Structural and Multidisciplinary Optimization, ... He was also in the paper selection committee of the 16th World Congress on Structural and Multidisciplinary Optimization (WCSMO 16).

11.1.2 Invited talks

Emile Parolin was invited to:

- a minisymposium at the Conference on Mathematics of Wave Phenomena, Karlsruhe, Germany.
- the workshop Waves in Complex Media: theoretical aspects, numerical methods, and applications, Paris, France.
- a minisymposium at the 29th International Domain Decomposition Conference, Milan, Italy.

Charles Dapogny was invited to:

- a workshop dedicated to Front-tracking methods in Chania (Crete).
- a workshop dedicated to shape and topology optimization for electric machines in Linz (Austria).
- several colloquia: Wurzburg, Essen, CEA Cadarache...

Sever Hirstoaga was invited to:

- the minisymposium "New advances in parallel-in-time method" at the 29th International Domain Decomposition Conference, Milan, Italy.

11.1.3 Research administration

Sever Hirstoaga was member of the CRCN/ISFP recruitment committee for the Inria Paris center.

11.2 Teaching - Supervision - Juries - Educational and pedagogical outreach

- Master 2: Frédéric Nataf, Course on Domain Decomposition Methods, Sorbonne University, 22h.
- Master 2: Emile Parolin, High Performance Computing, Sorbonne Université, 15h.
- Master 1: Emile Parolin, Numerical simulation and approximation of solutions to elliptic PDEs, Sorbonne Université, 20h.
- Licence 3: Sever Hirstoaga Méthodes numériques, cours et TP, Polytech Sorbonne, EISE 3, 37 htd.
- Master 2: Sever Hirstoaga, Cours sur les méthodes numériques pour les EDO, Sorbonne université, 15h.
- Pierre-Henri Tournier, Numerical simulation in physics using the finite element method, PSL advanced training, Mines Paris - PSL, 12h.

11.2.1 Supervision

Emile Parolin supervised the Master thesis of Aminata Diarra.

11.2.2 Juries

- Frédéric Nataf was a member of two PhD defences.
- Charles Dapogny was referee for two Ph.D. theses, and jury member of two other Ph.D. defences.

12 Scientific production

12.1 Major publications

- [1] N. Bootland, V. Dolean, F. Nataf and P.-H. Tournier. *A robust and adaptive GenEO-type domain decomposition preconditioner for $\mathbf{H}(\text{curl})$ problems in general non-convex three-dimensional geometries*. 30th Nov. 2023. URL: <https://hal.science/hal-04321182>.
- [2] S. Borzooei, C. Migliaccio, V. Dolean, P.-H. Tournier and C. Pichot. 'High-Performance Numerical Modeling for Detection of Rotator Cuff Tear'. In: IEEE International Symposium on Antennas and Propagation and USNC-URSI Radio Science Meeting. Portland, Oregon, United States: IEEE, 7th Sept. 2023, pp. 1879–1880. DOI: [10.1109/USNC-URSI52151.2023.10238302](https://doi.org/10.1109/USNC-URSI52151.2023.10238302). URL: <https://hal.science/hal-04038201> (cit. on p. 9).

- [3] S. Borzooei, P.-H. Tournier, C. Migliaccio, V. Dolean and C. Pichot. ‘Microwave tomographic imaging of shoulder injury’. In: *Conférence EUCAP 2023 - 17th European Conference on Antennas and Propagation*. Florence, Italy, 26th Mar. 2023, p. 5. URL: <https://hal.science/hal-04056885> (cit. on p. 9).
- [4] H. A. Daas, P. Jolivet, F. Nataf and P.-H. Tournier. *A Robust Two-Level Schwarz Preconditioner For Sparse Matrices*. 8th Jan. 2024. URL: <https://hal.science/hal-04381509>.
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