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2025 *Activity Report*

RESEARCH CENTRE: **Inria Centre at Université Côte d'Azur**
IN PARTNERSHIP WITH: **Université de Montpellier**

Project-Team

ANGUS

Adaptive modelling and numerical simulations for governing equations with underlying structures

In collaboration with Institut Montpelliérain Alexander Grothendieck (IMAG)

Project-Team ANGUS

Creation of the Project-Team: 2025 January 01

Each year, Inria research teams publish an Activity Report presenting their work and results over the reporting period. These reports follow a common structure, with some optional sections depending on the specific team. They typically begin by outlining the overall objectives and research programme, including the main research themes, goals, and methodological approaches. They also describe the application domains targeted by the team, highlighting the scientific or societal contexts in which their work is situated. The reports then present the highlights of the year, covering major scientific achievements, software developments, or teaching contributions. When relevant, they include sections on software, platforms, and open data, detailing the tools developed and how they are shared. A substantial part is dedicated to new results, where scientific contributions are described in detail, often with subsections specifying participants and associated keywords. Finally, the Activity Report addresses funding, contracts, partnerships, and collaborations at various levels, from industrial agreements to international cooperations. It also covers dissemination and teaching activities, such as participation in scientific events, outreach, and supervision. The document concludes with a presentation of scientific production, including major publications and those produced during the year.

Keywords

Computer sciences and digital sciences

A6.1.1. – Continuous Modeling (PDE, ODE)

A6.1.4. – Multiscale modeling

A6.1.5. – Multiphysics modeling

A6.2.1. – Numerical analysis of PDE and ODE

A6.2.6. – Optimization

A6.5.1. – Solid mechanics

A6.5.2. – Fluid mechanics

A6.5.3. – Transport

A6.5.4. – Waves

Other research topics and application domains

B1.1.8. – Mathematical biology

B3.3.1. – Earth and subsoil

B3.3.2. – Water: sea & ocean, lake & river

B3.4.1. – Natural risks

B3.4.2. – Industrial risks and waste

B4.2.1. – Fission

B5.2.4. – Aerospace

Contents

1 Team members, visitors, external collaborators

Research Scientist

- Nicolas Seguin [Team leader, INRIA, Senior Researcher]

Faculty Members

- Matthieu Hillairet [UNIV MONTPELLIER, Professor]
- Helene Mathis [UNIV MONTPELLIER, Professor]
- François Vilar [UNIV MONTPELLIER, Associate Professor]

Post-Doctoral Fellow

- Fabien Lespagnol [UNIV MONTPELLIER, Post-Doctoral Fellow]

PhD Students

- Pierrick Le Vourc'H [UNIV MONTPELLIER]
- Christina Mahmoud [UNIV MONTPELLIER]
- Nicolas Roblet [UNIV MONTPELLIER]
- Giscard Wouadji [UNIV MONTPELLIER]

Interns and Apprentices

- Raphael Bigey [INRIA, Intern, from Jun 2025 until Jul 2025]
- Ronan Choquert [INRIA, Intern, from Jun 2025 until Jul 2025]

Administrative Assistant

- Sandrine Boute [INRIA]

2 Overall objectives

The objectives of Angus focus on **modeling and simulation of complex fluid flows**. The aim is to introduce the most advanced mathematical techniques in this field, both for the construction and analysis of original physical models, and for their approximation by innovative numerical methods. Direct applications are related to nuclear energy, through collaborations with colleagues at CEA and EDF, as well as fast fluid dynamics issues. Other applications should be impacted too by our works, such as solid mechanics, biology, and porous media flows.

Our work addresses two major axes. The first concerns the **mathematical modeling of multiphase flows**, such as the atomization of liquids in gases inside cryogenic injectors of rocket engines, or liquid water behavior at high temperature and pressure conditions in pressurized water reactors. These flows exhibit different types of mixtures ranging from clearly separated phases to dispersed droplets, requiring nonlinear systems of partial differential equations with appropriate constitutive laws to describe the thermodynamic behavior of the phases. Multiphase models are by nature multiscale, since scales and structure of droplets strongly depend on the flow and interfacial coupling relations. The physical effects to include remain debated, with no consensus model available, see for instance [30, 19, 26].

Our approach is comprehensive, returning to the fundamentals of constructing these models based on mathematically sound reasoning. We work in three interconnected areas: the construction and analysis of mesoscale models where phases are continuous and well separated; multiscale modeling and asymptotic derivation to transition from separated phases to averaged mixing descriptions; and mathematical re-examination of existing multiphase models from the perspective of well-posedness, stability, and possible relationships between models. We emphasize the importance of **underlying structures** generally associated with fundamental physical characteristics, such as entropy, dissipation of stabilizing terms, and the principle of stationary action. This approach yields models with better mathematical and physical properties.

The second major axis addresses the **numerical approximation of multiphase models**. Numerical methods must provide accurate approximations at reasonable computational cost while preserving discrete versions of stability properties such as maximum principles, positivity of densities and temperatures, and underlying structures. Furthermore, they must remain stable and accurate with respect to strong variations of physical parameters. We favor high-order approximations using discontinuous Galerkin methods, which have proven compatible with stability requirements and invariance under parameter variations. The presence of interfaces in our models presents particular challenges, especially for mesoscale models requiring sharp reconstructions to avoid artificial mixing of separated phases. To address this, we employ the arbitrary-Lagrange-Euler framework, though its appropriate implementation in several space dimensions with high-order methods remains a significant challenge.

3 Research program

3.1 Mathematical Modeling of Multiphase Flows

To illustrate the diversity and complexity of multiphase flows, consider the atomization process shown in Figure 1. In cryogenic rocket injectors, a liquid jet atomizes into a surrounding gas, creating distinct

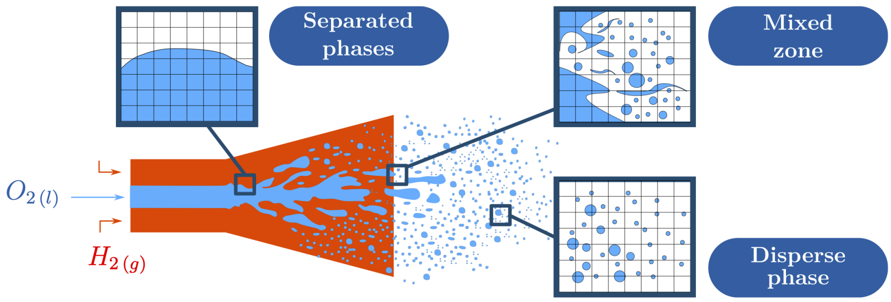

Figure 1: Jet atomization in cryogenic injectors of rocket engines (credits: [18]). According to the distance with the injector, different two-phase flow configurations appear: separated phases, mixed zone and disperse phase.

flow regimes within different zones: clearly separated phases with sharp interfaces, mixed transition zones, and fully dispersed droplets. Another critical application involves the primary circuit of pressurized water reactors, where liquid water operates at high temperature and pressure. During a loss-of-coolant accident, rapid depressurization triggers partial vaporization, requiring careful modeling of thermodynamic phase transitions alongside hydrodynamic coupling.

Given the complexity and lack of consensus models for such phenomena, our work is organized into three complementary areas:

1. **Construction and analysis of mesoscale models.** The phases are assumed to be continuous and well separated from each other. We analyze the relevance of the different modeling choices and the properties of the solutions according to specific interfacial conditions and the assumptions on the phases (presence of viscosity, compressibility, geometry of inclusions. . .). See [24] for instance.

2. **Multiscale modeling and asymptotic derivation.** This topic concerns the transition from a description of well separated continuous phases to an averaged description where the interfaces are no longer precisely localized. This results in averaged mixing multiphase models, obtained by different limit transitions in the physical parameters of the mesoscale models: droplet size and number, viscosity, etc. See for instance the pioneer works [12] and [14], and later [13] and [25].

3. **Mathematical re-examination of existing multiphase models.** We study the averaged physical models found in the literature from a mathematical perspective. In particular, we examine their well-posedness, the stability of their solutions, and the possible relationships between them. We may also propose corrections to these models based on these perspectives.

Beyond developing new mathematical tools for multiphase models, we emphasize the identification and exploitation of **underlying structures** inherent to physical models. These structures include the existence of entropy, dissipation of various stabilizing terms (viscosity, damping, friction, etc.), and the principle of stationary action. Our experience demonstrates that leveraging these fundamental properties yields models with superior mathematical and physical characteristics. Illustrative examples of this approach include [10, 23, 22, 27] and [20, 17].

3.2 Numerical Approximation of Multiphase Models

Developing numerical methods for the models described above requires addressing specific challenges beyond standard concerns. The methods must satisfy three key requirements:

- providing accurate approximations at a reasonable computational cost,
- preserving discrete versions of stability properties (maximum principle, positivity of densities and temperatures) and the underlying structures identified in our models,
- maintaining stability and accuracy across strong variations of physical parameters.

While the first two requirements are standard in the numerical analysis of partial differential equations, they prove particularly challenging in our nonlinear contexts [16]. The third requirement directly connects to our multiscale modeling efforts (item 2), as it enables discrete-level reproduction of the asymptotic derivations linking different model descriptions.

To achieve high accuracy, we employ **discontinuous Galerkin methods**, which have demonstrated compatibility with both stability requirements and parameter-variation invariance [34, 31]. The interfacial nature of our models presents an additional challenge: mesoscale models require sharp interface reconstruction to prevent artificial phase mixing. We address this through the **arbitrary-Lagrange-Euler** framework, though extending this approach to multiple space dimensions while maintaining high-order accuracy remains an open and significant challenge.

In this project, we aim at studying complex phenomena in Continuum Mechanics. Even if the following presentation is more focused on multiphase flows, other problems with similar constraints and difficulties may be studied.

4 Application domains

Our approach is comprehensive from a mathematical point of view. The structures of the models we are interested in are widespread enough to appear in a wide variety of fields of application. Nevertheless, the non-linear aspects that concern us mean that the applications often correspond to extreme situations.

The applications we have in mind, particularly through our collaborations with colleagues from other sciences or our industrial partners, are as follows.

- Nuclear energy: accidental situations in pressurized water reactors, such as the loss-of-coolant-accident, with strong pressure drop in the primary circuit.
- Aerospace: multiphase flow in injectors (before combustion), hypersonic flows in the upper atmosphere. . .
- Rapid phase transition in compressible fluids of materials, for instance in case of high-speed impacts.

5 Highlights of the year

The ANGUS team was created on January 1, 2025!

6 New results

6.1 Mathematical modeling of multiphase flows

Participants: Matthieu Hillairet, Pierrick Le Vourc'h, Fabien Lespagnol, Hélène Mathis, Nicolas Seguin, Giscard Leonel Zouakeu Wouadji, Hervé Jourdren *(CEA DAM)*, Khaled Saleh *(Aix-Marseille Université)*, Corentin Stéphan *(Université de Montpellier)*.

In [4], Hélène Mathis presents the derivation of a two-phase flow model that incorporates surface tension effects using Hamilton's principle of stationary action. The Lagrangian functional, which defines the action consists of kinetic energy — accounting for interface characteristics — and potential energy. A key feature of the model is the assumption that the interface separating the two phases possesses its own internal energy, which satisfies a Gibbs form that includes both surface tension and interfacial area. Consequently, surface tension is considered in both the kinetic and potential energy terms that define the Lagrangian functional. By applying the stationary action principle, a set of partial differential equations (PDEs) governing the dynamics of the two-phase flow is derived. This includes evolution equations for the volume fraction and interfacial area, incorporating mechanical relaxation terms. The final model is proven to be well-posed, demonstrating hyperbolicity and satisfying Lax entropy conditions.

The article [9] is taken from Corentin Stéphan's PhD thesis [29], co-advised by Nicolas Seguin and Hervé Jourdren, who defended in July 2025 — the PhD grant ended in November 2024. A mathematical framework is proposed to construct three-phase equations of state at thermodynamic equilibrium. Independent equations of state are used for each phase and the phase transition is modeled by maximizing the mixture entropy. Under appropriate geometric assumptions, phase transitions and the triple point are rigorously described. This process is applied to the case of tin, which can be found in solid form as either β-tin or γ-tin, or in liquid form. Different exact solutions of symmetric Riemann problem are constructed and standard numerical schemes are compared. In the PhD manuscript, validation tests are performed. It turns out that thermodynamic equilibrium is not always instantaneous. As a result, a kinetic approach of phase transition is proposed and calibrated. Numerical results are in very good agreement with experimental data.

Different works are in progress to extend the results of [24, 25]. One the one hand, Giscard Leonel Zouakeu Wouadji's PhD thesis (advised by Matthieu Hillairet, Hélène Mathis and Nicolas Seguin) focuses on the well-posedness and asymptotic analysis of compressible bubbly flows. First results are obtained in this direction. On the other hand, Fabien Lespagnol's postdoctoral work is dedicated to the analysis of the persistence of compressible spherical bubbles in a compressible fluid. A publication will be submitted soon.

The preprint [8] by Nicolas Seguin, Khaled Saleh and Pierrick Le Vourc'h is dedicated to the derivation of two-phase flow models. In a thin domain configuration similar to the one studied in [30], a mesoscale model based on compressible Navier-Stokes equations for each phase with appropriate coupling conditions at the interface is considered. Using asymptotic expansions, it is proved that the thin domain regime imposes the viscosities tend to zero. As a result, averaged models with only one common pressure can be constructed, with different averaged phase velocities if Navier type interfacial conditions are prescribed.

6.2 Numerical analysis and scientific computing

Participants: Christina Mahmoud, Hélène Mathis, Nicolas Seguin, François Vilar, Pierrick Le Vourc'h, Pierre Le Barbenchon *(Université de Rennes)*, Marianne Bessemoulin *(CNRS Nantes)*, Benjamin Boutin *(Université de Rennes)*, Saroj Chhatoi *(Université de Toulouse)*, Didier Henrion *(Université de Toulouse)*, Swann Marx *(CNRS Nantes)*.

François Vilar, in [32], proposes a new local subcell monolithic Discontinuous-Galerkin/Finite-Volume (DG/FV) convex property preserving scheme solving system of conservation laws on 2D unstructured grids. DG method is known to need some sort of nonlinear limiting to avoid spurious oscillations or nonlinear instabilities which may lead to the crash of the code. The main idea motivating the present work is to improve the robustness of DG schemes, while preserving as much as possible their high accuracy and very precise subcell resolution. To do so, a convex blending of high-order DG and first-order FV scheme will be locally performed, at the subcell scale, where it is needed. To this end, by means of the theory developed in [31, 33], it is recalled that it is possible to rewrite DG scheme as a subcell FV scheme on a subgrid provided with some specific numerical fluxes referred to as DG reconstructed fluxes. Then, the monolithic DG/FV method is defined as following: each face of each subcell will be assigned with two fluxes, a 1st-order FV one and a high-order reconstructed one, that in the end will be blended in a convex way. The goal is then to determine, through analysis, optimal blending coefficients to achieve the desired properties. Numerical results on various problems are presented to assess the very good performance of the design method. A particular emphasis is put on entropy consideration. By means of this subcell monolithic framework, several questions are addressed: is it possible through this monolithic framework to ensure any entropy stability? What is the mean of entropy stability? What is the cost of such constraints? Is this absolutely needed while aiming for high-order accuracy?

The article [7], by Christina Mahmoud and Hélène Mathis, presents the construction of two numerical schemes for the solution of hyperbolic systems with relaxation source terms. The methods are built by considering the relaxation system as a whole, without separating the resolution of the convective part from that of the source term. The first scheme combines the centered FORCE approach of Toro and co-authors with the unsplit strategy proposed by Béreux and Sainsaulieu. The second scheme consists of an approximate Riemann solver which carefully handles the source term approximation. The two schemes are built to be asymptotic preserving, in the sense that their limit schemes are consistent with the equilibrium model as the relaxation parameter tends to zero, without any restriction on the time step. For specific models, it is possible to prove that they preserve invariant domains and admit a discrete entropy inequality.

The article [5], by Marianne Bessemoulin *(CNRS Nantes)* and Hélène Mathis, deals with the diffusive limit of the Jin and Xin model and its approximation by an asymptotic preserving finite volume scheme. At the continuous level, they determine a convergence rate to the diffusive limit by means of a relative entropy method. Considering a semi-discrete approximation (discrete in space and continuous in time), they adapt the method to this semi-discrete framework and establish that the approximated solutions converge towards the discrete convection-diffusion limit with the same convergence rate.

The article [1] is the latest work from Pierre Le Barbenchon's PhD thesis [28], co-advised by Nicolas Seguin and Benjamin Boutin *(Université de Rennes)* and defended in 2023. It deals with the numerical approximation of the transport equation with an incoming boundary condition. In the case of extended stencils, particularly for high-order schemes, the numerical approximation of the boundary condition at the discrete level can take very different forms. The associated theory [21] provides a sufficient framework for the stability of these approximations, but remains impractical in concrete terms. They propose an intrinsic reformulation of the usual stability condition, leading to improved robustness. Consequently, drawing on previous work on the totally upwind case [11], they show how a simple numerical analysis of a winding number can be used to conclude whether or not the numerical discretization of the mixed problem is stable. This work has also been incorporated into the Python package boundaryscheme.

In [2], Nicolas Seguin, with Saroj Chhatoi, Didier Henrion, Swann Marx, extended the classical moment-SOS (sum of square) hierarchy to an infinite setting, associated with quasi-dissipative nonlinear partial differential equations. This method of non-convex optimization uses the very weak notion of measure-valued solutions. In order to be able to apply it, exact identification of classical solutions with measure-valued solutions is needed — called in this topic the absence of relaxation gap. Using its linear Liouville equation reformulation on probability measures, such result is proved using tools from optimal transport theory. This work is related to the article [15] where similar techniques were applied to parameter-dependent conservation laws.

6.3 Other models

Participants: Matthieu Hillairet, Jessica Guerand *(Université de Montpellier)*, Sepideh Mirrahimi *(CNRS Toulouse)*.

In [3], Matthieu Hillairet with Sepideh Mirrahimi *(CNRS Toulouse)* and Jessica Guerand *(Université de Montpellier)* discuss the validity of Gaussian approximation for solutions to the Fisher's infinitesimal model in the regime of small variance. In evolutionary biology, the Fisher's infinitesimal model is based on a reproduction kernel similar to collision kernel arising in Boltzmann-like kinetic equations describing multiphase flows. The tools that are developed (moments approach, Wasserstein distance estimate, Tanaka inequality) contribute to the knowledge on the mathematical treatment of such equations.

7 Partnerships and cooperations

7.1 National initiatives

7.1.1 France 2030

Hélène Mathis was awarded in October 2022 the Défis Mathématiques 2030, for her project, which is implicated within ten major challenges of the Plan France 2030. This project aims at providing new models and numerical methods for the safety of small modular nuclear reactors.

- Title: Modélisation et simulation numérique pour la sûreté des Réacteurs Nucléaires de petite taille
- Budget: one PhD grant (Christina Mahmoud) and 25,000 euros
- Beginning and duration of the scientific project: October 2022 - up to 2030
- Project coordinator: Hélène Mathis

Participants: Matthieu Hillairet, Christina Mahmoud, Hélène Mathis, Nicolas Seguin, François Vilar.

7.1.2 ANR

MSMΦ – Modélisation de systèmes multiphysiques et multiéchelles par équations aux dérivées partielles. The ANR PRME project MSMΦ gathers several members of ACSIOM/MACS team of IMAG. It intends to address different problems of coupling of phases (solid, liquid, gas...), from the theoretical point of view to scientific computing. Multiscale approaches are investigated as well as methods with sharp interfaces.

- Type: Projet de recherche mono-équipe (CE40 - Mathématiques)
- Budget: 263,268 euros
- Beginning and duration of the scientific project: February 2024 - 48 Months
- Project coordinator: Matthieu Hillairet

Participants: Matthieu Hillairet, Fabien Lespagnol, Hélène Mathis, Nicolas Seguin, François Vilar.

HEAD – Hyperbolic Equations, Approximations & Dynamics. The ANR project HEAD is focused on the analysis of the long-time dynamics of first-order hyperbolic systems of nonlinear partial differential equations and their approximations by numerical schemes, vanishing viscosity or in the dispersionless limit. It contributes to three general aims: the development of a stability theory applicable to singular waves including discontinuous and/or characteristic ones, uniform stability results when the long-time limit and the approximation process commute, and a refined description of obstructions when they do not. Its concrete applications are focused on models from fluid mechanics and plasma dynamics.

- Type: Projet de recherche collaborative (CE40 - Mathématiques)

- Budget: 422,820 euros
- Beginning and duration of the scientific project: September 2024 - 60 Months
- Project coordinator: Luis Miguel Rodrigues *(Université de Rennes)*

Participants: Nicolas Seguin.

COSS – COntrol on Stratified Structures. The ANR project COSS adresses control theory and partial differential equations (in particular Hamilton-Jacobi equations), posed on stratified structures and networks. These equations appear very naturally in several applications like traffic flow modeling, energy management in smart-grids networks or sea-land trajectories with different dynamics. These control problems can be studied within the framework of Hamilton Jacobi equations theory. Recently, significant results have been obtained, leading to a good understanding of the notion of viscosity solutions (in particular the questions of existence and uniqueness) on some specific stratified structures. This base of results will be further developed in different directions. It will first be necessary to complete the analysis for more general problems under weaker hypotheses than the one used so far (nature of the stratification, hypotheses on the Hamiltonians. . .). On the other hand, it is necessary to use the already existing base to advance research in other active areas such as homogenization or mean field games. Moreover, all of the theoretical results will be used to achieve progress in the modeling and numerical resolution of some control problems on stratified domains.

- Type: Projet de recherche collaborative (CE40 - Mathématiques)
- Budget: 597,826 euros
- Beginning and duration of the scientific project: December 2022 - 48 Months
- Project coordinator: Nicolas Forcadel *(Insa de Rouen)*

Participants: Nicolas Seguin.

7.1.3 Other initiatives

The collaboration between Pierrick Le Vourc'h and Nora Boulerie (Université Claude Bernard-Lyon I) has been founded by a Projet BOUM of the SMAI. The subject is the analysis and the approximation of a one-pressure two-phase flow model. The support has been used for travel expenses for their meetings.

8 Dissemination

8.1 Promoting scientific activities

8.1.1 Scientific events: organization

- As head of the federation OcciMath, Matthieu Hillairet participated to the organization of several scientific events. In spring, he organized the annual conference of the federation (May 22-23) and a conference towards master students (June 2-4)
- In autumn, he co-organized a working session on mathematical biology in Toulouse (September 18-19) and, in collaboration with several colleagues including Fabien Lespagnol, a regional session of PhD-student seminar (November 28).
- Beyond these events, Matthieu also organized a workshop at CIRM on fluid/solid interactions (April 21-25) and the Maths-bio-santé days (November 5-7).

- As Déléguée scientifique à l'innovation at CNRS Mathématiques, Hélène Mathis co-organized the PEP'S MATHéO, which was held on May 14, in Paris. She also is organizing two upcoming events: 'La journée Cryptographie et Mathématiques, Mathématiques du risque' and the 'Action Nationale de Formation Mathématique et Innovation'.
- Nicolas Seguin co-organized with B. Després (Sorbonne Université), F. Madiot (CEA) and A. Zoia (CEA) the Journée Mathématiques pour la neutronique, which was held on March 27 in Paris. This event was jointly sponsored by the RT Terre & Énergies and the CEA. He also co-organized with Ph. Helluy (Strasbourg) and J.-M. Hérard (EDF), the Seventh Workshop on Compressible Multiphase Flows, in Strasbourg, from June 26 to June 28.
- Pierrick Le Vourc'h co-organizes the 'séminaire doctoral', with a special session dedicated to women in science (with funding and invitations to speakers).

8.1.2 Scientific events: selection

- Hélène Mathis is member of the scientific committees of the following events:
  - Séminaire de Mécanique des Fluides Numérique du CEA-GAMNI (two days at the end of January each year),
  - 12e biennale de la SMAI, June 2-6,
  - Workshop on Theoretical aspects of geophysical flows and wave dynamics, Les Houches, January 24-29, 2027.
- François Vilar is member of the scientific committee of annual Shark-FV Conferences (Minho, Portugal).

8.1.3 Editorial responsibilities

- Nicolas Seguin is member of the editorial board of Annales Henri Lebesgue.
- Nicolas Seguin, with Philippe Helluy (Université de Strasbourg) and Jean-Marc Hérard (EDF), is editor of the proceedings of the Sixth Workshop on Compressible Multiphase Flows - Derivation, Closure laws, Thermodynamics. See the editorial introduction [6] and the associated volume in *ESAIM: Proceedings and Surveys*.

8.1.4 Invited talks

- At the 7th Workshop on Compressible Multiphase Flows which was held on May 26-28 at Strasbourg:
  - Pierrick Le Vourc'h gave a talk entitled 'Formal derivation of a stratified compressible two-phase flow model'
  - Hélène Mathis gave a talk entitled 'Derivation of a two-phase flow model accounting for surface tension'
- Matthieu Hillairet was invited to give a talk entitled "Variational methods for Stokes asymptotics" at the special topic school Particles in Flow June 23-27 and also a talk at the closing workshop of the ARC "Equations aux dérivées partielles en interaction" organized at Spa (Decembrer 2-5).
- Fabien Lespagnol gave a talk entitled 'Existence of solutions for a compressible fluid-bubbles interaction problem' at the closing workshop of the ARC "Equations aux dérivées partielles en interaction" organized at Spa (December 2-5). In the same workshop, Nicolas Roblet gave a talk entitled 'Concentrated suspension analysis'.
- Hélène Mathis gave a talk entitled 'Modélisation d'écoulements avec transition de phase' at the Rencontres doctorales Lebesgue, which was held on April 28-30 in Rennes, and a talk entitled 'Derivation of compressible two-phase flow models with surface tension', at the Journées annuelles de la fédération Occimath, which was held on May 22-23, in Nîmes.

- Nicolas Seguin gave a talk entitled 'Thermodynamically coherent models for three-phase transition' at the conference HyPNuT : Hyperbolic Problems - Numerics and Theory, which was held on November 5–7 in Amiens, and a talk entitled 'Stabilité de solutions stationnaires discontinues de systèmes hyperboliques avec terme source' at the kick-off conference of the ANR project HEAD, which was held on February 17-20, in Rennes.

- François Vilar presented his work entitled 'Schémas monolithiques GD/VF de sous-mailles : préservation des propriétés convexes et stabilités entropiques' at the Séminaire EDPs2 of the LAMA, Université Savoie Mont Blanc (February 2025), and at the Séminaire de Mécanique des Fluides Numérique du CEA-GAMNI (Paris, January 27).

- François Vilar and Nicolas Seguin gave a talk at the Laboratoire en Informatique Haute Performance pour le Calcul et la Simulation of the CEA and Université Paris-Saclay, respectively entitled 'Schémas monolithiques GD/VF de sous-mailles : préservation des propriétés convexes et stabilités entropiques' (February 6) and 'Dérivation asymptotique de modèles diphasiques' (March 31).

8.1.5 Leadership within the scientific community

- Hélène Mathis is the coordinator of the PTL - Colors of the Pôle MIPS (Université de Montpellier), which aims to develop and analyze methods and techniques related to the optimization and use of resources and knowledge. As resources can be digital, material, energy-related, etc., the methods developed will be designed to respond in the long term to the challenges of digital sobriety and sustainable development. This task includes managing calls for projects (drafting and selection) and organizing monthly seminars for the Pôle MIPS.

- Nicolas Seguin is at the head of the axis 'Nuclear Energy' of the RT Terre & Énergies.

8.1.6 Scientific expertise

- Nicolas Seguin was member of the *comité de Sélection* for a *maître.sse de conférences* position at Nantes Université (May).

- Nicolas Seguin, Federica Fanoni and Barbara Schapira have written an open letter aimed at minimizing bias in mathematics selection committees. The aim is to identify a number of good practices and reverse the balance of power: the signatories of this letter agree to participate in committees only if these good practices are guaranteed. This letter appeared in the Gazette de la Société Mathématique de France. It was signed by 244 colleagues ont the associated web page.

8.1.7 Research administration

- Matthieu Hillairet is director of the fédération occitane de mathématiques OcciMath.

- Hélène Mathis is Déléguée scientifique à l'innovation at CNRS Mathématiques.

- Hélène Mathis, in the Pôle MIPS (Université de Montpellier), is a member of the Bureau de direction and of the Commission des Ressources Humaines.

- Nicolas Seguin is the head of the ACSIOM team at IMAG. In this capacity, he is a member of the IMAG management committee, alongside the other team leaders, the director Marc Herzlich and administrative manager Nathalie Collain.

- Matthieu Hillairet and Nicolas Seguin are members of the Conseil de Laboratoire of IMAG.

8.2 Teaching - Supervision - Juries - Educational and pedagogical outreach

8.2.1 Teaching responsibilities

- Matthieu Hillairet is the head of the Département de Mathématiques of the Université de Montpellier.

- Hélène Mathis is the head of the 2nd year of the Master MANU (Modélisation et analyse numérique) of the Université de Montpellier.
- François Vilar is the head of the 1st year of the Master MANU (Modélisation et analyse numérique) of the Université de Montpellier.

8.2.2 Teaching

- Licence 1 Portail Mathématiques : Pierrick Le Vourc'h, Analyse 1 (TD : 25,5h), Université de Montpellier
- Licence 1 PEIP : Nicolas Roblet, Maths (TD : 54h), Polytech Montpellier
- Licence 1 PEIP : Matthieu Hillairet, Maths 1ère année PEIP S1 (TD : 54h), Polytech Montpellier
- Licence 1 PEIP : Matthieu Hillairet, Maths 1ère année PEIP S2 (CM : 36h, TD : 54h), Polytech Montpellier
- Licence 1 PEIP : Hélène Mathis, Maths 1ère année S2 (CM : 36h, TD : 54h), Polytech Montpellier
- Licence 1 PCSI : Christina Mahmoud, Outils mathématiques 3 (TD : 42h), Université de Montpellier
- Licence 1 SVSE : Christina Mahmoud, Raisonnement scientifique (TD : 21h), Université de Montpellier
- Licence 1 SVSE : Christina Mahmoud, Remédiation en mathématiques (TD : 13,5h), Université de Montpellier
- Licence 2 PEIP : Matthieu Hillairet, Maths 2ème année PEIP S1 (TD : 54h), Polytech Montpellier
- Licence 2 Mathématiques : Fabien Lespagnol, Suites de fonctions, séries, Fourier (TD : 39h), Université de Montpellier
- Licence 2 PEIP : Fabien Lespagnol, Mathématiques pour le parcours Polytech (TD : 33h), Polytech Montpellier
- Licence 2 EEA : François Vilar, Outils mathématiques pour l'EEA (TD : 42h), Université de Montpellier
- Licence 2 Mathématiques : François Vilar, Analyse numérique élémentaire (TP : 9h), Université de Montpellier
- Master 1 MANU : Hélène Mathis, Analyse numérique 3, méthodes des éléments finis pour l'elliptique (CM : 24h, TD : 15h), Université de Montpellier
- Master 1 MANU et MF : François Vilar, Analyse Numérique 1 (CM : 21h, TD : 15h, TP : 6h), Université de Montpellier
- Master 1 MANU : François Vilar, Analyse Numérique 2 (CM : 12h, TP : 12h), Université de Montpellier
- Master 2 MANU : François Vilar, Modélisation Numérique (CM : 12h), Université de Montpellier

8.2.3 Supervision

M1 Students:

- Nicolas Seguin and François Vilar supervised Ronan Choquert's eight-week (June-July) internship on 'Discontinuous Galerkin methods for gas dynamics equations'
- Nicolas Seguin and Jessica Guérand supervised Raphaël Bigey's six-week (June-July) internship on 'Numerical equivalence of Hamilton-Jacobi equations and conservation laws with pointwise limitation'

M2 students:

- Matthieu Hillairet supervised Sofia Agosteo for her master internship of Politecnico di Milano on the construction of periodic solutions to a system modeling the motion of an elliptical body immersed in a Poiseuille flow.

PhD students:

- Matthieu Hillairet, Hélène Mathis and Nicolas Seguin supervise Leonel Zouakeu-Wouadji for his PhD on multiphase flow models.
- Matthieu Hillairet co-supervises with David Gérard-Varet (Université Paris-Cité) Nicolas Roblet for his PhD on the modeling of suspensions with polyhedral particles.
- Matthieu Hillairet co-supervises with Sepideh Mirrahimi (CNRS-Université Paul Sabatier Toulouse III) Theotime Brun for his PhD on the Fisher's infinitesimal model.
- Hélène Mathis supervises Christina Mahmoud for her PhD on asymptotic analysis in the modeling of multiphase flows.
- Nicolas Seguin co-supervises with Khaled Saleh (Aix-Marseille Université) Pierrick Le Vourc'h for his PhD on the modeling of compressible multiphase flows.

Postdoctoral students:

- Matthieu Hillairet, Hélène Mathis and Nicolas Seguin are supervisors for the postdoctoral position of Fabien Lespagnol on bubbly flow models.

8.2.4 Juries

Matthieu Hillairet:

- President of the PhD defense of Emile Deléage (Aix-Marseille Université) 'Modélisation et analyse mathématique d'écoulements complexes : Modèles moyennés et milieux granulaires' (September 30)
- President of the PhD defense of Diego Gajardo (Université Paul Sabatier Toulouse III) 'Analyse d'un modèle d'interaction fluide-structure : caractère bien posé, stabilisation et simulations numériques' (November 27)

Hélène Mathis:

- Member of the jury of the PhD of Yen Chung (Université Savoie Mont Blanc) 'Le modèle amélioré de vague déferlante et le couplage morphodynamique' (March 27)
- Reviewer and member of the jury of the PhD of Yu-Hsi Lin (Aix-Marseille Université) 'Long waves model of ideal fluids' (June 6)
- Member of the jury of the PhD of Corentin Stefan (Université de Montpellier) 'Schémas hydrodynamiques d'ordre élevé avec cinétique de changement de phase' (July 17)
- Reviewer and member of the jury of the PhD of Gauthier Lazare (Université de Strasbourg) 'Développement d'une méthode numérique performante pour la résolution d'un modèle diphasique homogène partiellement déséquilibré en milieu poreux hétérogène' (October 16)

Nicolas Seguin:

- Reviewer and member of the jury of the PhD of Théo Girard (Université de Tours) 'Discontinuités dans les équations d'Hamilton-Jacobi-Bellman et dans les lois de conservation scalaires rencontrées dans les modèles de dynamique piétonnière' (July 4)
- Member of the jury of the PhD of Corentin Stéphan (Université de Montpellier) 'Schémas hydrodynamiques d'ordre élevé avec cinétique de changement de phase' (July 17)

- Reviewer and member of the jury of the PhD of Esteban Coiffier (Université de Pau et des pays de l'Adour) 'Numerical analysis and simulation of staggered schemes for low Mach number flows' (December 8)
- Member of the jury of the PhD of Clément Cardoën (Université de Nantes) 'Réduction de modèle par des approches basées sur les mesures' (December 18)

François Vilar was member of the jury of the PhD of Alexis Tardieu (Université de Bordeaux) 'Approches ADER-DG pour l'advection-diffusion non linéaire : Application aux équations de Navier-Stokes incompressibles' (November 6)

8.2.5 Educational and pedagogical outreach

Hélène Mathis gave a talk entitled 'Modélisation d'écoulements avec transition de phase' at the MOME seminar of the IREM Montpellier. This seminar focuses on teaching modeling in mathematics through accounts of experiences and historical, epistemological or didactic analyses. The aim is to consider the mathematical perspective on modeling and its teaching. Its objective is to encourage reflection on what the teaching of models and the modeling process in mathematics is/should be/could be in secondary school, at university or in the training of secondary school mathematics teachers.

8.3 Popularization

8.3.1 Productions (articles, videos, podcasts, serious games, ...)

- Nicolas Seguin helped design the poster Ah, un paysage. . . mathématique ! which promotes mathematics through a landscape illustrating the diversity of mathematics and their links to our environment.
- Nicolas Roblet supervised two secondary school pupils for the EloquenscienceS event organized by the Les Maths en Scène association.

8.3.2 Organization of Live events

As Déléguée scientifique à l'innovation at CNRS Mathématiques, Hélène Mathis co-organized Les mathématiciennes innovent, which was held on March 10-11, at Hôtel de l'Industrie in Paris. The event included a one-day public roundtable discussion and a morning dedicated to INSMI innovation networks.

8.3.3 Participation in Live events

- Pierrick Le Vourc'h participated to the Fête de la Science in Montpellier (October 4-5).
- Nicolas Seguin used the poster Ah, un paysage. . . mathématique ! with Barbara Schapira during meetings with middle and high school students at la Nuit Méditerranéenne des Chercheuses (September 26).

8.3.4 Others science outreach relevant activities

Pierrick Le Vourc'h was member of the jury of the regional tournoi français des jeunes mathématiciennes et mathématiciens.

9 Scientific production

9.1 Publications of the year

International journals

[1] B. Boutin, P. Le Barbenchon and N. Seguin. 'Stability of finite difference schemes for the hyperbolic initial boundary value problem by winding number computations'. In: *Annales de la Faculté des Sciences de Toulouse. Mathématiques.* (23rd June 2025). DOI: `10.5802/afst.1809`. URL: `https://hal.science/hal-03970775` (cit. on p. 8).

[2] S. P. Chhatoi, D. Henrion, S. Marx and N. Seguin. 'Optimizing quasi-dissipative evolution equations with the moment-SOS hierarchy'. In: *Discrete and Continuous Dynamical Systems - Series A* 45.11 (2025), pp. 4139–4159. DOI: 10.3934/dcds.2025051. URL: https://hal.science/hal-04828131 (cit. on p. 8).

[3] J. Guerand, M. Hillairet and S. Mirrahimi. 'A moment-based approach for the analysis of the infinitesimal model in the regime of small variance'. In: *Kinetic and Related Models* 18.3 (2025), pp. 389–425. DOI: 10.3934/krm.2024021. URL: https://hal.science/hal-04208328 (cit. on p. 8).

[4] H. Mathis. 'Derivation of a two-phase flow model accounting for surface tension'. In: *Journal of Applied Mathematics and Mechanics / Zeitschrift für Angewandte Mathematik und Mechanik* 105.1 (2025). DOI: 10.1002/zamm.202301019. URL: https://hal.science/hal-05080075 (cit. on p. 7).

Edition (books, proceedings, special issue of a journal)

[5] *Relative Entropy for the Numerical Diffusive Limit of the Linear Jin-Xin System*. CJC-MA 2023 - Le Congrès des Jeunes Chercheuses et Chercheurs en Mathématiques et Applications. Vol. 79. Gif-sur-Yvette, France: EDP Sciences, 1st Dec. 2025, pp. 126–138. DOI: 10.1051/proc/202579126. URL: https://hal.science/hal-04598587 (cit. on p. 8).

[6] P. Helluy, J.-M. Hérard and N. Seguin, eds. *Sixth Workshop on Compressible Multiphase Flows - Derivation, Closure laws, Thermodynamics* 78 (10th Dec. 2025). DOI: 10.1051/proc/202578001. URL: https://hal.science/hal-05409989 (cit. on p. 11).

Reports & preprints

[7] C. Mahmoud and H. Mathis. *ASYMPTOTIC PRESERVING SCHEMES FOR HYPERBOLIC SYSTEMS WITH RELAXATION*. 1st Oct. 2025. URL: https://hal.science/hal-05291431 (cit. on p. 8).

[8] N. Seguin, K. Saleh and P. Le Vourc'H. *Averaged models for compressible two-phase stratified flows on thin domains*. 4th June 2025. URL: https://hal.science/hal-05096837 (cit. on p. 7).

[9] C. Stéphan, H. Jourdren and N. Seguin. *A three-phase equation of state for shock-induced phase transitions in tin*. 18th Sept. 2025. URL: https://hal.science/hal-05254026 (cit. on p. 7).

9.2 Cited publications

[10] T. Barberon and P. Helluy. 'Finite volume simulation of cavitating flows'. English. In: *Comput. Fluids* 34.7 (2005), pp. 832–858. DOI: 10.1016/j.compfluid.2004.06.004 (cit. on p. 6).

[11] B. Boutin, P. Le Barbenchon and N. Seguin. 'On the stability of totally upwind schemes for the hyperbolic initial boundary value problem'. In: *IMA J. Numer. Anal.* 44.2 (2024), pp. 1211–1241. DOI: 10.1093/imanum/drad040. URL: https://doi.org/10.1093/imanum/drad040 (cit. on p. 8).

[12] D. Bresch and M. Hillairet. 'Note on the derivation of multi-component flow systems'. In: *Proc. Amer. Math. Soc.* 143.8 (2015), pp. 3429–3443. DOI: 10.1090/proc/12614. URL: https://doi.org/10.1090/proc/12614 (cit. on p. 5).

[13] D. Bresch, C. Burtea and F. Lagoutière. 'Mathematical justification of a compressible bi-fluid system with different pressure laws: a semi-discrete approach and numerical illustrations'. English. In: *J. Comput. Phys.* 490 (2023). Id/No 112259, p. 39. DOI: 10.1016/j.jcp.2023.112259 (cit. on p. 5).

[14] D. Bresch and M. Hillairet. 'A compressible multifluid system with new physical relaxation terms'. In: *Ann. Sci. Éc. Norm. Supér. (4)* 52.2 (2019), pp. 255–295. DOI: 10.24033/asens.2387. URL: https://doi.org/10.24033/asens.2387 (cit. on p. 5).

[15] C. Cardoen, S. Marx, A. Nouy and N. Seguin. 'A moment approach for entropy solutions of parameter-dependent hyperbolic conservation laws'. In: *Numer. Math.* 156.4 (2024), pp. 1289–1324. DOI: 10.1007/s00211-024-01428-5. URL: https://doi.org/10.1007/s00211-024-01428-5 (cit. on p. 8).

[16] F. Coquel, E. Godlewski and N. Seguin. 'Relaxation of fluid systems'. en. In: *Mathematical Models and Methods in Applied Sciences* 22.08 (Aug. 2012), p. 1250014. DOI: 10.1142/S0218202512500145. URL: https://www.worldscientific.com/doi/abs/10.1142/S0218202512500145 (visited on 27/01/2023) (cit. on p. 6).

[17] F. Coquel, J.-M. Hérard, K. Saleh and N. Seguin. 'Two properties of two-velocity two-pressure models for two-phase flows'. en. In: *Communications in Mathematical Sciences* 12.3 (2014), pp. 593–600. DOI: 10.4310/CMS.2014.v12.n3.a10. URL: http://www.intlpress.com/site/pub/pages/journals/items/cms/content/vols/0012/0003/a010/ (visited on 27/01/2023) (cit. on p. 6).

[18] P. Cordesse. 'Contribution to the study of combustion instabilities in cryotechnic rocket engines: coupling diffuse interface models with kinetic-based moment methods for primary atomization simulations'. PhD thesis. Université Paris-Saclay, 2020 (cit. on p. 5).

[19] D. A. Drew and S. L. Passman. *Theory of multicomponent fluids*. Vol. 135. Applied Mathematical Sciences. Springer-Verlag, New York, 1999, pp. x+308. DOI: 10.1007/b97678. URL: https://doi.org/10.1007/b97678 (cit. on p. 4).

[20] T. Gallouët, J.-M. Hérard and N. Seguin. 'Numerical modeling of two-phase flows using the two-fluid two-pressure approach'. en. In: *Mathematical Models and Methods in Applied Sciences* 14.05 (May 2004), pp. 663–700. DOI: 10.1142/S0218202504003404. URL: https://www.worldscientific.com/doi/abs/10.1142/S0218202504003404 (visited on 27/01/2023) (cit. on p. 6).

[21] B. Gustafsson, H.-O. Kreiss and A. Sundström. 'Stability theory of difference approximations for mixed initial boundary value problems. II'. In: *Math. Comp.* 26 (1972), pp. 649–686. DOI: 10.2307/2005093. URL: https://doi.org/10.2307/2005093 (cit. on p. 8).

[22] P. Helluy and H. Mathis. 'Pressure laws and fast Legendre transform.' English. In: *Math. Models Methods Appl. Sci.* 21.4 (2011), pp. 745–775. DOI: 10.1142/S0218202511005209 (cit. on p. 6).

[23] P. Helluy and N. Seguin. 'Relaxation models of phase transition flows'. In: *M2AN Math. Model. Numer. Anal.* 40.2 (2006), pp. 331–352 (cit. on p. 6).

[24] M. Hillairet, H. Mathis and N. Seguin. 'Analysis of compressible bubbly flows. Part I: Construction of a microscopic model'. In: *ESAIM: Math. Model Numer. Anal.* 57.5 (Sept. 2023), pp. 2835–2863. URL: https://www.esaim-m2an.org/articles/m2an/abs/2023/05/m2an220200/m2an220200.html (cit. on pp. 5, 7).

[25] M. Hillairet, H. Mathis and N. Seguin. 'Analysis of compressible bubbly flows. Part II: Derivation of a macroscopic model'. In: *ESAIM: Math. Model Numer. Anal.* 57.5 (Sept. 2023), pp. 2865–2906. URL: https://doi.org/10.1051/m2an/2023046 (cit. on pp. 5, 7).

[26] M. Ishii and T. Hibiki. *Thermo-fluid dynamics of two-phase flow*. Second. With a foreword by Lefteri H. Tsoukalas. Springer, New York, 2011, pp. xviii+518. DOI: 10.1007/978-1-4419-7985-8. URL: https://doi.org/10.1007/978-1-4419-7985-8 (cit. on p. 4).

[27] F. James and H. Mathis. 'A relaxation model for liquid-vapor phase change with metastability'. In: *Commun. Math. Sci.* 14.8 (2016), pp. 2179–2214. DOI: 10.4310/CMS.2016.v14.n8.a4. URL: https://doi.org/10.4310/CMS.2016.v14.n8.a4 (cit. on p. 6).

[28] P. Le Barbenchon. 'Étude théorique et numérique de la stabilité GKS pour des schémas d'ordre élevé en présence de bords'. Theses. Université de Rennes, June 2023. URL: https://theses.hal.science/tel-04214887 (cit. on p. 8).

[29] C. Stéphan. 'Schémas hydrodynamiques d'ordre élevé et cinétique de changement de phase'. PhD thesis. Université de Montpellier, July 2025 (cit. on p. 7).

[30] H. B. Stewart and B. Wendroff. 'Two-phase flow: models and methods'. In: *J. Comput. Phys.* 56.3 (1984), pp. 363–409. DOI: 10.1016/0021-9991(84)90103-7. URL: https://doi.org/10.1016/0021-9991(84)90103-7 (cit. on pp. 4, 7).

[31] F. Vilar. 'A posteriori correction of high-order discontinuous Galerkin scheme through subcell finite volume formulation and flux reconstruction'. en. In: *Journal of Computational Physics* 387 (June 2019), pp. 245–279. DOI: 10.1016/j.jcp.2018.10.050. URL: https://linkinghub.elsevier.com/retrieve/pii/S0021999118307174 (visited on 27/01/2023) (cit. on pp. 6, 8).

[32] F. Vilar. 'Local subcell monolithic DG/FV convex property preserving scheme on unstructured grids and entropy consideration'. In: *J. Comput. Phys.* 521 (2025), Paper No. 113535, 32. DOI: 10.1016/j.jcp.2024.113535. URL: https://doi.org/10.1016/j.jcp.2024.113535 (cit. on p. 7).

[33] F. Vilar and R. Abgrall. *A Posteriori local subcell correction of high-order discontinuous Galerkin scheme for conservation laws on two-dimensional unstructured grids*. 2022. arXiv: 2212.11358 [math.NA] (cit. on p. 8).

[34] F. Vilar, C.-W. Shu and P.-H. Maire. 'Positivity-preserving cell-centered Lagrangian schemes for multi-material compressible flows: From first-order to high-orders. Part II: The two-dimensional case'. In: *Journal of Computational Physics* 312 (2016), pp. 416–442. DOI: https://doi.org/10.1016/j.jcp.2016.01.037. URL: https://www.sciencedirect.com/science/article/pii/S0021999116000802 (cit. on p. 6).