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# 2025 *Activity Report*

RESEARCH CENTRE: **Inria Centre at Université Côte d'Azur**
IN PARTNERSHIP WITH: **National & Kapodistrian University of Athens**

**Project-Team**

# AROMATH

## AlgebRa, geOmetry, Modeling and AlgoriTHms

# Project-Team AROMATH

*Creation of the Project-Team: 2016 July 01*

Each year, Inria research teams publish an Activity Report presenting their work and results over the reporting period. These reports follow a common structure, with some optional sections depending on the specific team. They typically begin by outlining the overall objectives and research programme, including the main research themes, goals, and methodological approaches. They also describe the application domains targeted by the team, highlighting the scientific or societal contexts in which their work is situated. The reports then present the highlights of the year, covering major scientific achievements, software developments, or teaching contributions. When relevant, they include sections on software, platforms, and open data, detailing the tools developed and how they are shared. A substantial part is dedicated to new results, where scientific contributions are described in detail, often with subsections specifying participants and associated keywords. Finally, the Activity Report addresses funding, contracts, partnerships, and collaborations at various levels, from industrial agreements to international cooperations. It also covers dissemination and teaching activities, such as participation in scientific events, outreach, and supervision. The document concludes with a presentation of scientific production, including major publications and those produced during the year.

# Keywords


## Computer sciences and digital sciences

A5.5.1. – Geometrical modeling

A6.1. – Methods in mathematical modeling

A8.3. – Geometry, Topology

A8.4. – Computer Algebra

## Other research topics and application domains

B9.5.1. – Computer science

B9.5.2. – Mathematics

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# 1 Team members, visitors, external collaborators

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# 2 Overall objectives

Our daily life environment is increasingly interacting with digital information. An important amount of this information is of geometric nature. It concerns the representation of our environment, the analysis and understanding of “real” phenomena, the control of physical mechanisms or processes. The interaction between physical and digital worlds is two-way. Sensors are producing digital data related to measurements or observations of our environment. Digital models are also used to “act” on the physical world. Objects that we use at home, at work, to travel, such as furniture, cars, planes, . . . are nowadays produced by industrial processes which are based on digital representation of shapes. CAD-CAM (Computer Aided Design – Computer Aided Manufacturing) software is used to represent the geometry of these objects and to control the manufacturing processes which create them. The construction capabilities themselves are also expanding, with the development of 3D printers and the possibility to create daily-life objects “at home” from digital models.

The impact of geometry is also important in the analysis and understanding of phenomena. The 3D conformation of a molecule explains its biological interaction with other molecules. The profile of a wing determines its aeronautic behavior, while the shape of a bulbous bow can decrease significantly the wave resistance of a ship. Understanding such a behavior or analyzing a physical phenomenon can nowadays be achieved for many problems by numerical simulation. The precise representation of the geometry and the link between the geometric models and the numerical computation tools are closely related to the quality of these simulations. This also plays an important role in optimization loops where the numerical simulation results are used to improve the “performance” of a model.

Geometry deals with structured and efficient representations of information and with methods to treat it. Its impact in animation, games and VAMR (Virtual, Augmented and Mixed Reality) is important. It also has a growing influence in e-trade where a consumer can evaluate, test and buy a product from its digital description. Geometric data produced for instance by 3D scanners and reconstructed models are nowadays used to memorize old works in cultural or industrial domains.

Geometry is involved in many domains (manufacturing, simulation, communication, virtual world. . . ), raising many challenging questions related to the representations of shapes, to the analysis of their properties and to the computation with these models. The stakes are multiple: the accuracy in numerical engineering, in simulation, in optimization, the quality in design and manufacturing processes, the capacity of modeling and analysis of physical problems.

# 3 Research program

## 3.1 High order geometric modeling

The accurate description of shapes is a long standing problem in mathematics, with an important impact in many domains, inducing strong interactions between geometry and computation. Developing precise geometric modeling techniques is a critical issue in CAD-CAM. Constructing accurate models, that can be exploited in geometric applications, from digital data produced by cameras, laser scanners, observations or simulations is also a major issue in geometry processing. A main challenge is to construct models that can capture the geometry of complex shapes, using few parameters while being precise.

Our first objective is to develop methods, which are able to describe accurately and in an efficient way, objects or phenomena of geometric nature, using algebraic representations.

The approach followed in Computer Aided Geometric Design (CAGD) to describe complex geometry is based on parametric representations called NURBS (Non Uniform Rational B-Spline). The models are constructed by trimming and gluing together high order patches of algebraic surfaces. These models are built from the so-called B-Spline functions that encode a piecewise algebraic function with a prescribed regularity at knots. Although these models have many advantages and have become the standard for designing nowadays CAD models, they also have important drawbacks. Among them, the difficulty to locally refine a NURBS surface and also the topological rigidity of NURBS patches that imposes to use many such patches with trims for designing complex models, with the consequence of the appearing of cracks at the seams. To overcome these difficulties, an active area of research is to look for new blending functions for the representation of CAD models. Some examples are the so-called T-Splines, LR-Spline blending functions, or hierarchical splines, that have been recently devised in order to perform efficiently local refinement. An important problem is to analyze spline spaces associated to general subdivisions, which is of particular interest in higher order Finite Element Methods. Another challenge in geometric modeling is the efficient representation and/or reconstruction of complex objects, and the description of computational domains in numerical simulation. To construct models that can represent efficiently the geometry of complex shapes, we are interested in developing modeling methods, based on alternative constructions such as skeleton-based representations. The change of representation, in particular between parametric and implicit representations, is of particular interest in geometric computations and in its applications in CAGD.

We also plan to investigate adaptive hierarchical techniques, which can locally improve the approximation of a shape or a function. They shall be exploited to transform digital data produced by cameras, laser scanners, observations or simulations into accurate and structured algebraic models.

The precise and efficient representation of shapes also leads to the problem of extracting and exploiting characteristic properties of shapes such as symmetry, which is very frequent in geometry. Reflecting the symmetry of the intended shape in the representation appears as a natural requirement for visual quality, but also as a possible source of sparsity of the representation. Recognizing, encoding and exploiting symmetry requires new paradigms of representation and further algebraic developments. Algebraic foundations for the exploitation of symmetry in the context of non linear differential and polynomial equations are addressed. The intent is to bring this expertise with symmetry to the geometric models and computations developed by AROMATH.

## 3.2 Robust algebraic-geometric computation

In many problems, digital data are approximated and cannot just be used as if they were exact. In the context of geometric modeling, polynomial equations appear naturally as a way to describe constraints between the unknown variables of a problem. *An important challenge is to take into account the input error in order to develop robust methods for solving these algebraic constraints.* Robustness means that a small perturbation of the input should produce a controlled variation of the output, that is forward stability, when the input-output map is regular. In non-regular cases, robustness also means that the output is an exact solution, or the most coherent solution, of a problem with input data in a given neighborhood, that is backward stability.

Our second long term objective is to develop methods to robustly and efficiently solve algebraic problems that occur in geometric modeling.

Robustness is a major issue in geometric modeling and algebraic computation. Classical methods in computer algebra, based on the paradigm of exact computation, cannot be applied directly in this context. They are not designed for stability against input perturbations. New investigations are needed to develop methods which integrate this additional dimension of the problem. Several approaches are investigated to tackle these difficulties.

One relies on linearization of algebraic problems based on “elimination of variables” or projection into a space of smaller dimension. Resultant theory provides a strong foundation for these methods, connecting the geometric properties of the solutions with explicit linear algebra on polynomial vector spaces, for families of polynomial systems (e.g., homogeneous, multi-homogeneous, sparse). Important progress has been made in the last two decades to extend this theory to new families of problems with specific geometric properties. Additional advances have been achieved more recently to exploit the syzygies between the input equations. This approach provides matrix based representations, which are particularly powerful for approximate geometric computation on parametrized curves and surfaces. They are tuned to certain classes of problems and an important issue is to detect and analyze degeneracies and to adapt them to these cases.

A more adaptive approach involves linear algebra computation in a hierarchy of polynomial vector spaces. It produces a description of quotient algebra structures, from which the solutions of polynomial systems can be recovered. This family of methods includes Gröbner Basis, which provides general tools for solving polynomial equations. Border Basis is an alternative approach, offering numerically stable methods for solving polynomial equations with approximate coefficients. An important issue is to understand and control the numerical behavior of these methods as well as their complexity and to exploit the structure of the input system.

In order to compute “only” the (real) solutions of a polynomial system in a given domain, duality techniques can also be employed. They consist in analyzing and adding constraints on the space of linear forms which vanish on the polynomial equations. Combined with semi-definite programming techniques, they provide efficient methods to compute the real solutions of algebraic equations or to solve polynomial optimization problems. The main issues are the completness of the approach, their scalability with the degree and dimension and the certification of bounds.

Singular solutions of polynomial systems can be analyzed by computing differentials, which vanish at these points. This leads to efficient deflation techniques, which transform a singular solution of a given problem into a regular solution of the transformed problem. These local methods need to be combined with more global root localization methods.

Subdivision methods are another type of methods which are interesting for robust geometric computation. They are based on exclusion tests which certify that no solution exists in a domain and inclusion tests, which certify the uniqueness of a solution in a domain. They have shown their strength in addressing many algebraic problems, such as isolating real roots of polynomial equations or computing the topology of algebraic curves and surfaces. The main issues in these approaches is to deal with singularities and degenerate solutions.

# 4 Application domains

## 4.1 Geometric modeling for Design and Manufacturing.

The main domain of applications that we consider for the methods we develop is Computer Aided Design and Manufacturing.

Computer-Aided Design (CAD) involves creating digital models defined by mathematical constructions, from geometric, functional or aesthetic considerations. Computer-aided manufacturing (CAM) uses the geometrical design data to control the tools and processes, which lead to the production of real objects from their numerical descriptions.

CAD-CAM systems provide tools for visualizing, understanding, manipulating, and editing virtual shapes. They are extensively used in many applications, including automotive, shipbuilding, aerospace industries, industrial and architectural design, prosthetics, and many more. They are also widely used to produce computer animation for special effects in movies, advertising and technical manuals, or for digital content creation. Their economic importance is enormous. Their importance in education is also growing, as they are more and more used in schools and educational purposes.

CAD-CAM has been a major driving force for research developments in geometric modeling, which leads to very large software, produced and sold by big companies, capable of assisting engineers in all the steps from design to manufacturing.

Nevertheless, many challenges still need to be addressed. Many problems remain open, related to the use of efficient shape representations, of geometric models specific to some application domains, such as in architecture, naval engineering, mechanical constructions, manufacturing . . . Important questions on the robustness and the certification of geometric computation are not yet answered. The complexity of the models which are used nowadays also appeals for the development of new approaches. The manufacturing environment is also increasingly complex, with new type of machine tools including: turning, 5-axes machining and wire EDM (Electrical Discharge Machining), 3D printer. It cannot be properly used without computer assistance, which raises methodological and algorithmic questions. There is an increasing need to combine design and simulation, for analyzing the physical behavior of a model and for optimal design.

The field has deeply changed over the last decades, with the emergence of new geometric modeling tools built on dedicated packages, which are mixing different scientific areas to address specific applications. It is providing new opportunities to apply new geometric modeling methods, output from research activities.

## 4.2 Geometric modeling for Numerical Simulation and Optimization

A major bottleneck in the CAD-CAM developments is the lack of interoperability of modeling systems and simulation systems. This is strongly influenced by their development history, as they have been following different paths.

The geometric tools have evolved from supporting a limited number of tasks at separate stages in product development and manufacturing, to being essential in all phases from initial design through manufacturing.

Current Finite Element Analysis (FEA) technology was already well established 40 years ago, when CAD-systems just started to appear, and its success stems from using approximations of both the geometry and the analysis model with low order finite elements (most often of degree $\leq 2$).

There has been no requirement between CAD and numerical simulation, based on Finite Element Analysis, leading to incompatible mathematical representations in CAD and FEA. This incompatibility makes interoperability of CAD/CAM and FEA very challenging. In the general case today, this challenge is addressed by expensive and time-consuming human intervention and software developments.

Improving this interaction by using adequate geometric and functional descriptions should boost the interaction between numerical analysis and geometric modeling, with important implications in shape optimization. In particular, it could provide a better feedback of numerical simulations on the geometric model in a design optimization loop, which incorporates iterative analysis steps.

The situation is evolving. In the past decade, a new paradigm has emerged to replace the traditional Finite Elements by B-Spline basis element of any polynomial degree, thus in principle enabling exact representation of all shapes that can be modeled in CAD. It has been demonstrated that the so-called isogeometric analysis approach can be far more accurate than traditional FEA.

It opens new perspectives for the interoperability between geometric modeling and numerical simulation. The development of numerical methods of high order using a precise description of the shapes raises questions on piecewise polynomial elements, on the description of computational domains and of their interfaces, on the construction of good function spaces to approximate physical solutions. All these problems involve geometric considerations and are closely related to the theory of splines and to the geometric methods we are investigating. We plan to apply our work to the development of new interactions between geometric modeling and numerical solvers.

# 5 Latest software developments, platforms, open data

## 5.1 Latest software developments

### 5.1.1 G+Smo

**Name:** Geometry plus Simulation Modules

**Keyword:** Isogeometric analysis

**Scientific Description:** G+Smo (Geometry + Simulation Modules, pronounced "gismo") is an open-source C++ library that brings together mathematical tools for geometric design and numerical simulation. It implements the relatively new paradigm of isogeometric analysis, which suggests the use of a unified framework in the design and analysis pipeline.

**Functional Description:** G+Smo (pronounced gismo or gizmo) is a C++ library for isogeometric analysis (IGA).

G+Smo (Geometry + Simulation Modules, pronounced "gismo") is an open-source C++ library that brings together mathematical tools for geometric design and numerical simulation. It implements the relatively new paradigm of isogeometric analysis, which suggests the use of a unified framework in the design and analysis pipeline. G+Smo is an object-oriented, cross-platform, template C++ library and follows the generic programming principle, with a focus on both efficiency and ease of use. The library aims at providing access to high quality, open-source software to the forming isogeometric numerical simulation community and beyond. Geometry plus simulation modules aims at the seamless integration of Computer-aided Design (CAD) and high order Finite Element Analysis (FEA).

The library and its documentation are available at https://gismo.github.io/

**News of the Year:** The new release v25.7 introduces several major features, including more robust analysis codes, new file formatos (JSON file support), better variational expressions module, Zenodo DOI integration, and enhanced spline tools (improved evaluation of rational splines). Organizationally, future releases will now come from the main branch, with the stable branch serving as a pre-release development area. Numerous improvements were made across documentation, code clarity, CMake configuration, iterator design, new CPUs support, XML utilities, and general code modernization. The release also delivers a wide range of fixes addressing build issues, namespace exposure, assembly bugs, spline truncation behavior, quadrature node handling, spline curve modeling, and missing variational expressions. Finally, API updates refine and extend domain-iteration capabilities for more robust and efficient usage.

**URL:** https://github.com/gismo

**Contact:** Angelos Mantzaflaris

#### 5.1.2 MomentPolynomialOpt

**Name:** MomentPolynomialOpt

**Keywords:** Global optimization, Moment, Polynomial equations, Semi-algebraic set, Convex relaxation

**Functional Description:** The package provides efficient tools to build convex relaxations of moment sequences and their dual Sum-of-Squares relaxations, to optimize vectors of moment sequences that satisfy positivity constraints or mass constraints, to compute global minimizers of polynomial and moment optimization problems from moment sequences, polar ideals, approximate real radical. It also provides tools for computing minimum enclosing ellipsoids of basic semi-algebraic sets. It uses a connection with SDP solvers via the JuMP interface.

The documentation is available at https://algebraicgeometricmodeling.github.io/MomentPolynomialOpt.jl/

**Release Contributions:** New functionalities for the construction of SDP relaxations of Generalized Moment Problems (GMP), in particular for applications in Tensor Decomposition Problems.

**URL:** https://github.com/AlgebraicGeometricModeling/MomentPolynomialOpt.jl

**Contact:** Bernard Mourrain

**Participants:** Lorenzo Baldi, Lucas Gamertsfelder

#### 5.1.3 TensorDec

**Keywords:** Tensor decomposition, Multivariate series, Low rank models, Hankel

**Functional Description:** TensorDec is a Julia package for the decomposition of tensors and polynomial-exponential series. It provides tools to compute rank decomposition or Waring decomposition of symmetric tensors or multivariate homogeneous, of multilinear tensors.

It also allows computing low rank tensor approximations of given tensors, using Riemannian optimization techniques, with well-chosen initial start. It also provides tools to compute catalecticant or Hankel operators associated to tensors and their apolar ideal.

The documentation is available at https://algebraicgeometricmodeling.github.io/TensorDec.jl/

**Release Contributions:** New functions for the decomposition of symmetric and multilinear tensors. Improved documentation. Tutorials.

**URL:** https://github.com/AlgebraicGeometricModeling/TensorDec.jl

**Contact:** Bernard Mourrain

**Participant:** Rima Khouja

#### 5.1.4 G1ACC

**Name:** G1 Approximate Catmull-Clark

**Keywords:** Splines, Subdivision surfaces

**Functional Description:** This module provides implementation of the G1ACC scheme for obtaining surfaces with G1 continuity starting from a quad mesh. The surfaces approximate Catmull-Clark subdivision surfaces and converge quadratically to them.

**Contact:** Angelos Mantzaflaris

**Participants:** Bernard Mourrain, Michelangelo Marsala

# 6 New results

## 6.1 An Effective Positivstellensatz over the Rational Numbers for Finite Semialgebraic Sets

**Participant:** Lorenzo Baldi *(Max-Planck-Institut für Mathematik in den Naturwissenschaften, Germany)*, Térésa Krick *(Universidad de Buenos Aires, Argentina)*, Bernard Mourrain.

In [21], we study the problem of representing multivariate polynomials with rational coefficients, which are nonnegative and strictly positive on finite semialgebraic sets, using rational sums of squares.

We focus on the case of finite semialgebraic sets S defined by equality constraints, generating a zero-dimensional ideal I, and by nonnegative sign constraints. First, we obtain existential results. We prove that a strictly positive polynomial f with coefficients in a subfield K of R has a representation in terms of weighted Sums-of-Squares with coefficients in this field, even if the ideal I is not radical. We generalize this result to the case where f is nonnegative on S and $(f) + (I : f) = 1$. We deduce that nonnegative polynomials with coefficients in K can be represented in terms of Sum-of-Squares of polynomials with coefficients in K, when the ideal is radical. Second, we obtain degree bounds for such Sums-of-Squares representations, which depend linearly on the regularity of the ideal and the degree of the defining equations, when they form a graded basis. Finally, we analyze the bit complexity of the Sums-of-Squares representations for polynomials with coefficients in Q, in the case where the ideal is radical. The bitsize bounds are quadratic or cubic in the Bezout bound, and linear in the regularity, generalizing and improving previous results obtained for special zero dimensional ideals. As an application in the context of polynomial optimization, we retrieve and improve results on the finite convergence and exactness of the moment/Sums-of-Squares hierarchy.

## 6.2 Singularity, approximation and representation

**Participant:** Adam Parusiński, Laurentiu Päunescu *(Univ. Sidney, Australia)*, Guillaume Rond *(Univ. Aix-Marseille, France)*.

The aim of the paper doi:10.4064/ap241218-2-6 is to review how some approximation results in commutative algebra are being used to construct equisingular deformations of singularities. The first example of such an approximation result appeared in A. Płoski's PhD thesis.

A result of Teissier says that the cone over one of classical polygon examples in the real projective space gives, by complexification, a surface singularity which is not Whitney equisingular to a singularity defined over the field of rational numbers $\mathbb{Q}$. In doi:10.4064/ap241208-25-4, we correct the example and give a complete proof of Teissier's result.

## 6.3 Separation of the orbits in representations of $SO_2$ and $O_2$ over $\mathbb{R}$ and $\mathbb{C}$

**Participant:** Martin Jalard.

[29] provides a minimal set of invariant polynomials separating the orbits for representations of $SO_2$ and $O_2$ over $\mathbb{R}$ and $\mathbb{C}$. The idea is to select only polynomials of support of size 2 for $SO_2$ and 4 for $O_2$ . Cardinalities in respectively $O(\dim(V)^2)$ and $O(\dim(V)^4)$ are thus obtained. These cardinalities are much smaller than for generating sets, which require polynomials of arbitrary large supports. Yet a separating set is

sufficient for most of the applications. It appears also that real separating sets are smaller than the complex ones, which helps significatively for applications over $\mathbb{R}$. The obtained separating set are used to stratify the real representations by isotropy classes.

## 6.4 Orbit separation and stratification by isotropy classes of piezoelectricity tensors.

**Participant:** Evelyne Hubert, Martin Jalard.

In [27], we explore an innovative method to compute separating invariants in a real G-variety $\mathcal{V}$.

A refinement of Seshadri slice Lemma enables us to decompose $\mathcal{V}$ into a union of stable subsets $\mathrm{G}\cdot\mathcal{Z}_1 \cup \ldots \mathrm{G}\cdot\mathcal{Z}_r$. This reduces the problem to separating the orbits in the slices $\mathcal{Z}_i$ for their normalizers $\mathrm{N}_i < \mathrm{G}$. This sequencing allows also to identify efficiently the isotropy class of any point. After the presentation of three types of Seshadri slices for representations of $\mathrm{SO}_3(\mathbb{R})$, we apply the method to the space of piezoelectricity tensors. This provides a separating set of low cardinality and a complete stratification of this space by isotropy classes.

## 6.5 Rationality of the invariant field for a class of representations of the real orthogonal groups.

**Participants:** Evelyne Hubert, Martin Jalard.

In [28], we give a criterion for the invariant field of a representation of $\mathrm{SO}_n(\mathbb{R})$ to be rational. We define a length $\lambda_n$ on representations of $\mathrm{SO}_n(\mathbb{R})$, depending on their weights. This length is at most $\frac{n}{2}$ . If a representation of $\mathrm{SO}_n(\mathbb{R})$ or $\mathrm{O}_n(\mathbb{R})$ on $\mathcal{V}$ contains the standard representation $\mathbb{R}^n$ with multiplicity greater than this length, its invariant field is rational. To prove it, we construct a sequence of Seshadri slices, each reducing the problem to a repesentation of a subgroup of codimension one preserving the inequality between the multiplicity of the standard representation and the length. The case of length $\lambda_n(V) = 0$ corresponds to the natural action of $\mathrm{SO}_n(\mathbb{R})$ on matrices $\mathrm{M}_{n,k}(\mathbb{R})$, for which we construct a minimal basis.

## 6.6 Bigraded Castelnuovo-Mumford regularity and Groebner bases.

**Participant:** Matias Bender *(Inria, Tropical)*, Laurent Busé, Carlès Checa *(Univ. Copenhague)*, Elias Tsigaridas *(Inria, Ouragan)*.

In [22], we study the relation between the bigraded Castelnuovo-Mumford regularity of a bihomogeneous ideal $I$ in the coordinate ring of the product of two projective spaces and the bidegrees of a Groebner basis of $I$ with respect to the degree reverse lexicographical monomial order in generic coordinates. For the single-graded case, Bayer and Stillman unraveled all aspects of this relationship forty years ago and these results led to complexity estimates for computations with Groebner bases. We build on this work to introduce a bounding region of the bidegrees of minimal generators of bihomogeneous Groebner bases for $I$. We also use this region to certify the presence of some minimal generators close to its boundary. Finally, we show that, up to a certain shift, this region is related to the bigraded Castelnuovo-Mumford regularity of $I$.

## 6.7 Solving bihomogeneous polynomial systems with a zero-dimensional projection.

**Participants:** Matias Bender *(Inria, Tropical)*, Laurent Busé, Carlès Checa *(Univ. Copenhague)*, Elias Tsigaridas *(Inria, Ouragan)*.

In [32], we study bihomogeneous systems defining, non-zero dimensional, biprojective varieties for which the projection onto the first group of variables results in a finite set of points. To compute (with) the 0-dimensional projection and the corresponding quotient ring, we introduce linear maps that greatly extend the classical multiplication maps for zero-dimensional systems, but are not those associated to the elimination ideal; we also call them multiplication maps. We construct them using linear algebra on the restriction of the ideal to a carefully chosen bidegree or, if available, from an arbitrary Gröbner bases. The multiplication maps allow us to compute the elimination ideal of the projection, by generalizing FGLM algorithm to bihomogenous, non-zero dimensional, varieties. We also study their properties, like their minimal polynomials and the multiplicities of their eigenvalues, and show that we can use the eigenvalues to compute numerical approximations of the zero-dimensional projection. Finally, we establish a single exponential complexity bound for computing multiplication maps and Gröbner bases, that we express in terms of the bidegrees of the generators of the corresponding bihomogeneous ideal. This work is a collaboration with Matias Bender, Carlès Checa and Elias Tsigaridas.

## 6.8 Construction of birational trilinear volumes via tensor rank criteria.

**Participant:** Laurent Busé, Pablo Mazon *(Univ. CUNEF, Madrid)*.

In [23], in collaboration with Pablo Mazon (former PhD student of the team), we provide effective methods to construct and manipulate trilinear birational maps $\phi : (\mathbb{P}^1)^3 \to \mathbb{P}^3$ by establishing a novel connection between birationality and tensor rank. These yield four families of nonlinear birational transformations between 3D spaces that can be operated with enough flexibility for applications in computer-aided geometric design. More precisely, we describe the geometric constraints on the defining control points of the map that are necessary for birationality, and present constructions for such configurations. For adequately constrained control points, we prove that birationality is achieved if and only if a certain $2 \times 2 \times 2$ tensor has rank one. As a corollary, we prove that the locus of weights that ensure birationality is $\mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^1$. Additionally, we provide formulas for the inverse $\phi^{-1}$ as well as the explicit defining equations of the irreducible components of the base loci. Finally, we introduce a notion of "distance to birationality" for trilinear rational maps, and explain how to continuously deform birational maps.

## 6.9 An algebraic framework for geometrically continuous splines

**Participants:** Angelos Mantzaflaris, Bernard Mourrain, Nelly Villamizar *(Swansea University, UK)*, Beihui Yuan *(Swansea University, UK)*.

Geometrically continuous splines are piecewise polynomial functions defined on a collection of patches which are stitched together through transition maps. They are called $G^r$-splines if, after composition with the transition maps, they are continuously differentiable functions to order $r$ on each pair of patches with stitched boundaries. This type of splines has been used to represent smooth shapes with complex topology for which (parametric) spline functions on fixed partitions are not sufficient. In [31], we develop new algebraic tools to analyze $G^r$-spline spaces. We define $G^r$-domains and transition maps using an algebraic approach, and establish an algebraic criterion to determine whether a piecewise function is $G^r$-continuous on the given domain. In the proposed framework, we construct a chain complex whose top homology is isomorphic to the $G^r$-spline space. This complex generalizes Billera-Schenck-Stillman homological complex used to study parametric splines. Additionally, we show how previous constructions of $G^r$-splines fit into this new algebraic framework, and present an algorithm to construct a basis for $G^r$-spline spaces. We illustrate how our algebraic approach works with concrete examples, and prove a dimension formula for the $G^r$-spline space in terms of invariants to the chain complex. In some special cases, explicit dimension formulas in terms of the degree of splines are also given.

## 6.10 Geometric tools in structural bioinformatics

**Participant:** Ioannis Emiris.

In a couple of papers, with two distinct groups of co-authors, we employ various tools from computational geometry to model the 3d shape of molecules and their spatial interactions. In [30], we focus on the 4 human Argonaute (AGO) proteins, critical in RNA interference and gene regulation. We investigated the underexplored structural relationships of these paralogs through microsecond-scale molecular dynamics simulations. Our findings reveal that AGO proteins adopt similar, yet unsynchronized, open-close states. We observed similar and unique local conformations, interdomain distances and intramolecular interactions. By combining simulation data with large datasets of experimental structures and AlphaFold's predictions, we identified proteins with genomic and proteomic similarities. In [24] we study the plant root microbiome, vital in plant health, nutrient uptake, and environmental resilience. We present the MetagRoot database that integrates metagenomic, metatranscriptomic, and reference genome-derived protein data to characterize 71,091 enriched protein families. These families are annotated with multiple sequence alignments, hidden Markov models, taxonomic and functional classifications, and predicted 3D structures using AlphaFold2.

## 6.11 O-GEST: Overground gait events detector using B-Spline-based geometric models for marker-based and markerless analysis.

**Participant:** Laurent Busé, Mehran Hatamzadeh *(Univ. Gustave Eiffel, Lyon)*, Katia Turcot *(Univ. Laval, Canada)*, Raphaël Zory *(Univ. Côte d'Azur)*.

Accurate gait events detection is imperative for reliable assessment of normal and pathological gaits. However, this detection becomes challenging in the absence of force plates. In [26], we introduce two geometric models integrated into an automatic algorithm (O-GEST) for overground gait events detection using kinematic data. O-GEST employs B-Spline-based geometric models to represent the horizontal trajectory of foot landmarks. It leverages gait-dependent thresholds, along with optimal coefficients to detect events and compute spatiotemporal parameters on healthy and pathological gaits. To validate the proposed algorithm, timing differences in the detected events using the force plates and O-GEST were calculated and also compared between different methods on the gait data of 390 subjects. This dataset includes 200 healthy subjects, 100 subjects with unilateral hip osteoarthritis, 50 stroke survivors, 26 individuals diagnosed with Parkinson's disease, and 14 children with cerebral palsy. The validation results show that O-GEST detects gait events with an overall accuracy of 13.5 ms for foot-strike and 12.6 ms for foot-off. It also demonstrates significantly more accurate results than the common deep learning-based and kinematic-based methods. O-GEST offers several advantages, including its applicability for events detection across various pathologies, capability to handle noisy trajectories, and usability in the absence of certain foot landmarks. Development of such algorithms could lead to enhanced accuracy and reliability of gait analysis in force-plate-less environments, especially in markerless gait analysis setups. This work is a collaboration with Mehran Hatamzadeh (former PhD student of the team), Katia Turcot and Raphael Zory.

## 6.12 Efficient alternating and joint distance minimization methods for adaptive spline surface fitting.

**Participants:** Mantzaflaris Angelos, Giannelli Carlotta *(Univ. Florence, Italy)*, Imperatore Sofia *(Univ. Florence, Italy)*, Mokris Mokriš *(MTU Aero Engines)*.

In [25] we propose a new paradigm for scattered data fitting with adaptive spline constructions based on the key interplay between parameterization and adaptivity. Specifically, we introduce two novel adaptive fitting schemes that combine moving parameterizations with adaptive spline refinement, for highly accurate CAD models reconstruction from real-world scattered point clouds. The first scheme alternates surface fitting and data parameter optimization. The second scheme jointly optimizes the parameters and the surface control points. To combine the proposed fitting methods with adaptive spline constructions, we present a key treatment of boundary points. Industrial examples show that updating the parameterization, within an adaptive spline approximation framework, significantly reduces the number of degrees of freedom needed for a certain accuracy, especially if spline adaptivity is driven by suitably graded hierarchical meshes. The numerical experiments employ Truncated Hierarchical B-splines, thus exploiting the existing CAD integration within the considered industrial setting, nevertheless, any adaptive spline construction can be chosen.

# 7 Bilateral contracts and grants with industry

## 7.1 Bilateral grants with industry

- **G1 Splines in TopSolid**

**Participants:** Angelos Mantzaflaris, Michelangelo Marsala *(Post-Doctorate, Univ. Firenze)*, Bernard Mourrain.

The transfer concerns the integration of new techniques for constructing $G^1$ splines from quadrangular meshes into the products of the company TopSolid. The technology enables the real-time generation of smooth spline surfaces or surfaces with specific sharp edges from a quadrangular control mesh. It provides new features in `TopSolid` software that enable the simple and intuitive creation of free forms with complex geometries. It takes into account the treatment of sharp and evanescent edges. Based on a Julia prototype, a C++ package `G1ACC` was produced, intensive and integrated in `TopSolid` software. The technique used, relying on the computation of dedicated masks for spline surface control points, is based on the thesis work of Mr. Marsala [tel-04459277].

# 8 Partnerships and cooperations

## 8.1 International research visitors

### 8.1.1 Visits of international scientists

**Manfred Buchacher**

**Status:** post-Doc

**Institution of origin:** Johannnes Kepler University of Linz

**Country:** Austria

**Dates:** November 19 – December 6

**Context of the visit:** collaboration with Laurent Busé, Angelos Mantzaflaris and Bernard Mourrain, on resultant techniques for enumerative combinatorics + seminar talk.

**Mobility program/type of mobility:** research stay

**Alberto Biliotti**

**Status:** PhD student

**Institution of origin:** University of Florence

**Country:** Italy

**Dates:** October 1 – December 23

**Context of the visit:** collaboration with A. Mantzaflaris, working on spline volume approximation.

**Mobility program/type of mobility:** research stay

## 8.2 European initiatives

### 8.2.1 Horizon Europe

**TENORS**

**Participants:** Enrica Barrilli *(PhD)*, Matteo Bechere *(PhD)*, Evelyne Hubert, Yassine Koubaa *(PhD)*, Angelos Mantzaflaris, Mattia Matucci *(PhD)*, Bernard Mourrain *(Scientific Coordinator)*, Linh Nguyen *(Administrative coordinator)*, Oriol Reig Fite *(PhD)*.

TENORS project on cordis.europa.eu

**Title:** Tensor modEliNg, geOmetRy and optimiSation

**Duration:** From January 1, 2024 to December 31, 2027

**Partners:**

- INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET AUTOMATIQUE (INRIA), France
- UNIVERSITETET I TROMSOE - NORGES ARKTISKE UNIVERSITET (UiT), Norway
- BLUETENSOR S.R.L. (BLUETENSOR S.R.L.), Italy
- HSBC BANK PLC, United Kingdom
- UNIVERSITE COTE D'AZUR, France
- Arva AS (Arva), Norway
- FUNDACIO INSTITUT DE CIENCIES FOTONIQUES (ICFO-CERCA), Spain
- QUANTINUUM LTD (QUANTINUUM LTD), United Kingdom
- UNIVERSITA DEGLI STUDI DI FIRENZE (UNIFI), Italy
- UNIVERSITAET LEIPZIG (ULEI), Germany
- UNIVERSITAT POLITECNICA DE CATALUNYA (UPC), Spain
- CESKE VYSOKE UCENI TECHNICKE V PRAZE (CVUT), Czechia
- QUANDELA, France
- UNIVERSITA DEGLI STUDI DI TRENTO (UNITN), Italy
- UNIVERSITAT KONSTANZ (UKON), Germany
- TILBURG UNIVERSITY- UNIVERSITEIT VAN TILBURG (TILBURG UNIVERSITY), Netherlands
- MAX-PLANCK-GESELLSCHAFT ZUR FORDERUNG DER WISSENSCHAFTEN EV (MPG), Germany
- ARTELYS, France
- CENTRE NATIONAL DE LA RECHERCHE SCIENTIFIQUE CNRS (CNRS), France
- UNIVERSITE DE TOULOUSE (UNIVERSITE DE TOULOUSE), France
- STICHTING NEDERLANDSE WETENSCHAPPELIJK ONDERZOEK INSTITUTEN (NWO-I), Netherlands

**Inria contact:** Bernard Mourrain

**Coordinator:** Bernard Mourrain

**Summary:** TENORS aims to form the next generation of researchers and engineers in scientific computing and data analysis, disrupting the current paradigms of tensor calculus by exploiting cutting-edge research in geometry and optimization. Tensors are nowadays ubiquitous in many domains of applied mathematics, computer science, signal processing, data processing, machine learning and in the emerging area of quantum computing. The demand for highly trained scientists with a deep understanding of tensor methods and with advanced knowledge in the geometry of tensor spaces, with skills on the design of efficient algorithms and software handling tensor computation and in the applications of high performance tensor computation is raising, in many fields including machine learning and quantum computation, which are nowadays expanding very quickly.

TENORS contributes to satisfy this demand by fostering scientific and technological advances in the area of tensor sciences, stimulating interdisciplinary and intersectoriality knowledge exchange between algebraists, geometers, computer scientists, numerical analysts, data analysts, physicists, quantum scientists, and industrial actors facing real-life tensor-based problems, in a network of PhD students at its core.

A unique strength of the network is to gather top-researchers of these different domains.

TENORS will train young scientists in academy or industry in how to exploit the best of these techniques efficiently and disseminate this knowledge to industry. As a truly multidisciplinary network, TENORS will at the same time seek to apply these new techniques to real-life applications thanks to the industrial actors involved in the network.

## 8.3 National initiatives

**Participants:** Angelos Mantzaflaris, Bernard Mourrain, Régis Duvigneau *(ACUMES)*.

**Title:** ANR PRCI: *"RFF-Splines: High-order Isogeometric simulation with geometrically continuous functions"*

**Duration:** 2025–2028, administrative start October 2025

**Coordinator:** Angelos Mantzaflaris

**Partners:** University of Linz, Austria

**Summary:** Our project aims at the development of a novel framework for high-order discretization of partial differential equations on general domains. Smoothness requirements and superior approximation power are paramount for efficient simulations. We focus on the paradigm of *isogeometric analysis* that uses spline functions for design and analysis on real-world (both man-made and occurring in nature) geometries. General domains pose challenges due to their topology, in particular in the vicinity of so-called extraordinary vertices, which are essentially artifacts created by the discretization (i.e., meshing) procedures. We propose a framework of geometrically continuous splines, called Refinable FreeForm (RFF-) Splines, to enable numerical schemes for *topologically unrestricted design and analysis.*

The novelty of the construction is based on three main points: First, we will establish *theoretical guarantees for approximation power*. These will be derived based on the property of local polynomial reproduction, using an adapted construction for spline projectors on domain manifolds. Second, we will focus on the *efficient construction* of the basis functions, notably concerning the evaluation and the matrix assembly for simulation via numerical integration. Third, we will emphasize *adaptivity and approximate evaluation through local refinement*. We will employ the truncation mechanism to reduce the support of the basis functions (thereby increasing sparsity) and to preserve the partition of unity property, again with theoretical guarantees regarding the approximation power.

The starting point of our work is provided by a now classic work of Prautzsch in 1997, which focused on the fundamental property of polynomial reproduction. Obviously, this property is essential for the derivation of theoretical guarantees for the approximation performance. The project goes all the way from the design of the construction and the theory development to the algorithmic aspects and the efficient implementation in C++, as well as experimental evaluation in demanding applications that involve high order partial differential equations.

# 9 Dissemination

## 9.1 Promoting scientific activities

### 9.1.1 Scientific events: organization

**General chair, scientific chair**

Evelyne Hubert is the scientific chair for the conference Foundations of Computational Mathematics in Vienna 2026.

Bernard Mourrain is the general chair for the International Conference on Mathematical Software in Waterloo 2026.

**Member of the organizing committees**

Evelyne Hubert has been part of the funding committee for the conference Foundations of Computational Mathematics in Vienna 2026.

Bernard Mourrain co-organized with Michele Ancona and Khazhgali Kozhasov (University Côte d'Azur), the conference "Real Algebraic Geometry and Interactions" October 6-10, Nice.

Angelos Mantzaflaris is in the organizing committee of the 2026 Thematic Semester Machine Learning + Simulation, funded by the Excellence Academy “Complex Systems” at Université Côte d'Azur.

### 9.1.2 Scientific events: selection

**Chair of conference program committees**

Evelyne Hubert has chaired the Plenary Speakers Committee for the conference Foundations of Computational Mathematics in Vienna 2026. She has also chaired the selection committee for the 6th Steven Smale Prize.

**Member of the conference program committees**

Evelyne Hubert was part of the Workshops Committee for the conference Foundations of Computational Mathematics in Vienna 2026. She was also part of the selection committee for the Agnes Szanto Medal. Evelyne Hubert was part of the scientific committee of the conference *Enumerative combinatorics and effective aspects of differential equations* at CIRM (France).

Laurent Busé and Bernard Mourrain were members of the program committees of the Symposium on Solid and Physical Modeling SPM2025 and of the International Conference on Geometric Modeling and Processing GMP2025. Angelos Mantzafalris was also in the program committee of GMP.

Angelos Mantzaflaris co-organized the workshop G+smo Developer Days related to the open-source library Geometry plus Simulation Modules, at the University of Florence, Italy. He was also a member of the program committee of Isogeometric Analysis (IGA) conference, co-organizing the mini-symposium “Fast formation and solution techniques for large-scale IGA”' in Eindhoven, the Netherlands.

### 9.1.3 Journal

**Member of the editorial boards**

Evelyne Hubert is on the editorial board of the journal of *Foundations of Computational Mathematics* and the *Journal of Symbolic Computation*. She was a co-editor of the Special Issue dedicated to the Foundations of Computational Mathematics 2023 Conference, which should be published in 2026.

Laurent Busé is an editor of the Journal of Pure and Applied Algebra.

Bernard Mourrain is a member of the editorial board of the Journal of Symbolic Computation.

Angelos Mantzaflaris is a guest editor of Computer-Aided Geometric Design, for the "Special issue on Geometry: Theory and Applications".

**Reviewer - reviewing activities**

Laurent Busé reviewed for Transactions on Graphics, Computer-Aided Geometric Design journal, Contributions to Algebra and Geometry, Computer-Aided Design journal, Communications in Mathematics and Statistics journal and The American Mathematical Monthly.

Bernard Mourrain reviewed submissions for the following journals: Journal of Algebra and Applications, Journal of Applied Algebra and Error Correcting Codes, Journal of Computational and Applied Mathematics, Journal of Computer Aided Design, Journal of Symbolic Computation, Mathematics of Computation, Numerical Algebra, Control and Optimization, Optimization Letters, SIAM Journal on Applied Algebra and Geometry, SIAM Journal of Optimization, Transactions on Graphics, Transactions on Mathematical Software.

Angelos Mantzaflaris reviewed for Computer Aided Geometric Design, Computer Methods in Applied Mechanics and Engineering, Journal of Computational and Applied Mathematics.

### 9.1.4 Invited talks

Enrica Barrilli was invited to give a talk at "Journées Nationales de Calcul Formel", CIRM Luminy, March 10-14, 2025, at "Colloque des doctorants", May 5-8, 2025, Igesa Porquerolles and at University of Trento, September 30, 2025.

Laurent Busé was invited to give a talk at the CUNEF university, Madrid, May 21, and at the international conference RAGI in Nice, Ocotber 6-10.

Evelyne Hubert was invited to give a talk at the workshop *Randomness, Invariants and Complexity* and participate to the program *Complexity and Linear Algebra* at the Simons Institute for Theoretical Computer Science (Berkeley USA). She was also invited to give a talk at the conferences *Structures Algébriques Ordonnées* (Banyuls, France) and *Real-world Applications of Geometry and Algebra* (TU Eindhoven, The Netherlands).

Bernard Mourrain was invited to give a talk at the conference "Jordan Types of Artinian Algebras and Geometry of Punctual Hilbert Schemes" for the 80th of T. Iarrobino, Nice, June 23-27, 2025; at the conference "Geometry of tensors", November 3-7, Nancy; at the conference "Matemax: Computational Algebra, Algebraic Geometry and Applications" for the 70th of A. Dickenstein, December 15-17, Buenos Aires, Argentina.

Adam Parusiǹski was invited to give a talk at the conferences "Tame geometry and extensions of functions", Pawlucki 70 Kraków, June 23-27, 2025, at "Singularities", Oberwolfach, September 28 - October 3 2025, and at "Real Algebraic Geometry and Interactions" October 6-10, 2025, Nice.

Angelos Mantzaflaris was a Keynote Speaker at the "ECCOMAS Thematic Conference on Modern Finite Element Technologies"' (MFET 2025), August 2025, Aachen, Germany. He also delivered an invited talk at the National Institute of Higher Mathematics F. Severi (INdAM, Rome, Italy), at the workshop "Fast Methods for Isogeometric Analysis"', May 2025

### 9.1.5 Leadership within the scientific community

Evelyne Hubert is chair of the society Foundations of Computational Mathematics for the period 2023-2026.

Laurent Busé was co-chair of the Computer Algebra French Research Community (GT CF of the GDR IFM) until June.

### 9.1.6 Scientific expertise

Evelyne Hubert was an expert on the selection panel for Horizon Europe MSCA Staff Exchange (HORIZON-MSCA-2024-SE-01) programme. She was also on the hiring committee (CoS) for a Professor position in Mathematics at Université de Limoges.

Bernard Mourrain was reviewer for the ANR (Fondements du numérique : informatique, automatique, traitement du signal et des images).

### 9.1.7 Research administration

Laurent Busé is chair of the Comité de Suivi Doctoral (CSD) of the Inria centre at Université Côte d'Azur, member of the Comité NICE of the Inria centre at Université Côte d'Azur, representative member of Inria centre at Université Côte d'Azur at the Département Disciplinaires de Mathématiques of Université Côte d'Azur, member of the Comité de Pilotage of the EUR SPECTRUM of Université Côte d'Azur.

Bernard Mourrain is member of the Bureau du Comité des Equipes Projets of Inria Centre at Université Côte d'Azur.

Angelos Mantzaflaris serves in the Commission de Développement Technologique of Inria Université Côte d'Azur, since 2025. He is also a board member of AMIES (Agence pour les Mathématiques en Interaction avec l'Entreprise et la Société) as well as the correspondent at AMIES for Inria Université Côte d'Azur, since september 2022.

## 9.2 Teaching - Supervision - Juries - Educational and pedagogical outreach

### 9.2.1 Teaching

Master : Laurent Busé, Geometric Modeling, 18h (M2), Polytech Nice Sophia - Université Côte d'Azur.

Angelos Mantzaflaris, master course "Numerical Interpolation" at Polytech Nice Sophia – Université Côte d'Azur.

Bernard Mourrain, course on "Algebra for the analysis of tensors and moment sequences" (3h), Learning Week 1 of TENORS network, January 2025.

### 9.2.2 Supervision

Evelyne Hubert supervised the PhD of Martin Jalard [33] and has co-supervised the PhD work of Henri Breloer with Cordian Riener (Arctic University in Tromso, Norway).

Laurent Busé is the co-advisor of the PhD thesis of Issam Tauil, on the topic "Enumerative geometry on toric surfaces and reduced elimination theory", co-supervised with Thomas Dedieu (Toulouse).

Angelos Mantzaflaris co-advices together with Bernard Mourrain the thesis of Dimitrios Tolis on "Refinable freeform splines and applications", Université Côte d'Azur. He also supervised the post-doctorate Mustapha Bahari, *Isogeometric analysis for complex shapes*, on the use of $r$–refinement in isogeometric analysis.

Bernard Mourrain is co-advisor of the following theses:

- E. Barilli, *Geometry of extensor varieties*. Université Côte d'Azur; joint supervision with A. Bernardi, University of Trento, Marie Skłodowska-Curie Network DN-JD TENORS, on the the study and development of new approaches for tensor decomposition, based on duality and flat extension properties.
- Oriol Reig Fite, *Algorithms for Tensor Decomposition*. University of Trento; joint supervision with A. Bernardi (main supervisor at the University of Trento), Marie Skłodowska-Curie Network DN-JD TENORS, on the development and analysis of new methods and algorithms for tensor decomposition, based on the study of one-form apolar schemes.
- Matteo Bechere, *Tensor decompositions for sums of even powers of real polynomials*, University of Konstanz; joint supervision with S. Kuhlmann (main supervisor at the University of Konstanz), Marie Skłodowska-Curie Network DN-JD TENORS, on the study of quadratic modules of sums of even powers of forms of a given degree.

He is also supervising the Post-doctorate Lucas Gamertsfelder, *Primal-Dual relaxations for Non-Linear Problems* on the study of primal and dual convex relaxations for polynomial optimization and Generalized Moment Problems.

#### 9.2.3 Juries

Laurent Busé was a reviewer and a member of the habilitation committee of Pierre-Jean Spaenlehauer, titled Fast algebraic algorithms for arithmetic geometry and polynomial systems. Habilitation from Université de Lorraine, defended on February 12.

Evelyne Hubert chaired the PhD committee of Alexandre Goyer (Université Paris-Saclay), and was an examiner in the PhD committee of Thomas Bouchet (Université de Nice).

Angelos Mantzaflaris was a reviewer and in the master thesis committee of Ioannis Karampinis at University of Macedonia, Greece.

### 9.3 Popularization

#### 9.3.1 Productions (articles, videos, podcasts, serious games, ...)

**Participant:** Matteo Bechere, Henri Breloer, Angelos Mantzaflaris, Mattia Matucci, Linh Nguyen, Dimitrios Tolis, Jana Vrablikova.

Aromath participated in the Inria stand at the Village des Sciences et de l'Innovation d'Antibes Juan les Pins (October 2025), presenting the mathematics and applications of tensors to the general public. In particular A. Mantzaflaris, L. Nguyen, M. Matucci, J. Vrablikova, M. Bechere and H. Breloer were present at the event. D. Tolis created educational videos for the event.

## 10 Scientific production

### 10.1 Major publications

[1] L. Baldi and B. Mourrain. 'On the Effective Putinar's Positivstellensatz and Moment Approximation'. In: *Mathematical Programming, Series A* (6th Sept. 2022). DOI: 10.1007/s10107-022-01877-6. URL: https://hal.science/hal-03437328.

[2] E. Bartzos, I. Z. Emiris and J. Schicho. 'On the multihomogeneous Bézout bound on the number of embeddings of minimally rigid graphs'. In: *Applicable Algebra in Engineering, Communication and Computing* 31.5-6 (2020), pp. 325–357. URL: https://hal.archives-ouvertes.fr/hal-02696362.

[3] E. Bartzos, I. Z. Emiris and R. Vidunas. 'New upper bounds for the number of embeddings of minimally rigid graphs'. In: *Discrete and Computational Geometry* 68.3 (2022), p. 796. DOI: 10.1007/s00454-022-00370-3. URL: https://hal.archives-ouvertes.fr/hal-03895585.

[4] L. Busé, Y. Cid-Ruiz and C. D'Andrea. 'Degree and birationality of multi-graded rational maps'. In: *Proceedings of the London Mathematical Society* 121.4 (2020), pp. 743–787. DOI: 10.1112/plms.12336. URL: https://hal.inria.fr/hal-01793578.

[5] L. Busé and A. Karasoulou. 'Resultant of an equivariant polynomial system with respect to the symmetric group'. In: *Journal of Symbolic Computation* 76 (2016), pp. 142–157. DOI: 10.1016/j.jsc.2015.12.004. URL: https://hal.inria.fr/hal-01022345.

[6] L. Calès, A. Chalkis, I. Z. Emiris and V. Fisikopoulos. 'Practical volume approximation of high-dimensional convex bodies, applied to modeling portfolio dependencies and financial crises'. In: *Computational Geometry* 109 (Feb. 2023), p. 101916. DOI: 10.1016/j.comgeo.2022.101916. URL: https://hal.science/hal-04294307.

[7] A. Chalkis, I. Z. Emiris and V. Fisikopoulos. 'A Practical Algorithm for Volume Estimation based on Billiard Trajectories and Simulated Annealing'. In: *ACM Journal of Experimental Algorithmics* 28 (11th May 2023), pp. 1–34. DOI: 10.1145/3584182. URL: https://hal.science/hal-04294298.

[8] A. Chalkis, I. Z. Emiris, V. Fisikopoulos, E. Tsigaridas and H. Zafeiropoulos. 'Geometric algorithms for sampling the flux space of metabolic networks'. In: *Journal of Computational Geometry* 14.1 (2023). DOI: 10.20382/jocg.v14i1a8. URL: https://inria.hal.science/hal-04310109.

[9] E. Christoforou, H. Leontiadou, F. Noé, J. Samios, I. Z. Emiris and Z. Cournia. 'Investigating the Bioactive Conformation of Angiotensin II Using Markov State Modeling Revisited with Web-Scale Clustering'. In: *Journal of Chemical Theory and Computation* 18.9 (13th Sept. 2022), pp. 5636–5648. DOI: 10.1021/acs.jctc.1c00881. URL: https://hal.science/hal-03895590.

[10] I. Z. Emiris and I. Psarros. 'Products of Euclidean Metrics, Applied to Proximity Problems among Curves'. In: *ACM Transactions on Spatial Algorithms and Systems* 6.4 (Aug. 2020), pp. 1–20. DOI: 10.1145/3397518. URL: https://hal.inria.fr/hal-03045134.

[11] A. J. Fuentes Suárez and E. Hubert. 'Scaffolding skeletons using spherical Voronoi diagrams: feasibility, regularity and symmetry'. In: *Computer-Aided Design* 102 (May 2018), pp. 83–93. DOI: 10.1016/j.cad.2018.04.016. URL: https://hal.inria.fr/hal-01774909.

[12] A. Giust, B. Jüttler and A. Mantzaflaris. 'Local (T)HB-spline projectors via restricted hierarchical spline fitting'. In: *Computer Aided Geometric Design* 80 (June 2020), p. 101865. DOI: 10.1016/j.cagd.2020.101865. URL: https://hal.inria.fr/hal-02985011.

[13] P. Görlach, E. Hubert and T. Papadopoulo. 'Rational invariants of even ternary forms under the orthogonal group'. In: *Foundations of Computational Mathematics* 19 (2019), pp. 1315–1361. DOI: 10.1007/s10208-018-9404-1. URL: https://hal.inria.fr/hal-01570853.

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[20] K. Tertikas, D. Paschalidou, B. Pan, J. J. Park, M. A. Uy, I. Z. Emiris, Y. Avrithis and L. Guibas. ‘PartNeRF: Generating Part-Aware Editable 3D Shapes without 3D Supervision’. In: CVPR 2023 - IEEE/CVF Conference on Computer Vision and Pattern Recognition. Vol. Proceedings of the 2023 IEEE/CVF Conference on Computer Vision and Pattern Recognition. Vancouver, Canada: IEEE, 21st Mar. 2023, pp. 4466–4478. DOI: 10.1109/CVPR52729.2023.00434. URL: https://hal.science/hal-04294277.

## 10.2 Publications of the year

### International journals

[21] L. Baldi, T. Krick and B. Mourrain. ‘An Effective Positivstellensatz over the Rational Numbers for Finite Semialgebraic Sets’. In: *Mathematics of Computation* (1st Dec. 2025). DOI: 10.1090/mcom/4153. URL: https://hal.science/hal-04723493 (cit. on p. 11).

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