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2025 *Activity Report*

RESEARCH CENTRE: **Inria Centre at the University of Bordeaux**
IN PARTNERSHIP WITH: **Université de Bordeaux, Bordeaux INP, Naval Group, CNRS**

Project-Team

ASTRAL

Advanced StatisTical infeRence And controL

In collaboration with Institut de Mathématiques de Bordeaux (IMB)

Project-Team ASTRAL

Creation of the Project-Team: 2021 January 01

Each year, Inria research teams publish an Activity Report presenting their work and results over the reporting period. These reports follow a common structure, with some optional sections depending on the specific team. They typically begin by outlining the overall objectives and research programme, including the main research themes, goals, and methodological approaches. They also describe the application domains targeted by the team, highlighting the scientific or societal contexts in which their work is situated. The reports then present the highlights of the year, covering major scientific achievements, software developments, or teaching contributions. When relevant, they include sections on software, platforms, and open data, detailing the tools developed and how they are shared. A substantial part is dedicated to new results, where scientific contributions are described in detail, often with subsections specifying participants and associated keywords. Finally, the Activity Report addresses funding, contracts, partnerships, and collaborations at various levels, from industrial agreements to international cooperations. It also covers dissemination and teaching activities, such as participation in scientific events, outreach, and supervision. The document concludes with a presentation of scientific production, including major publications and those produced during the year.

Keywords

Computer sciences and digital sciences

A3.4. – Machine learning and statistics

A6.1.2. – Stochastic Modeling

A6.1.3. – Discrete Modeling (multi-agent, people centered)

A6.2.2. – Numerical probability

A6.2.3. – Probabilistic methods

A6.2.4. – Statistical methods

A6.2.6. – Optimization

A6.3.3. – Data processing

A6.3.4. – Model reduction

A6.3.5. – Uncertainty Quantification

A6.4. – Automatic control

A6.4.1. – Deterministic control

A6.4.2. – Stochastic control

A6.4.3. – Observability and Controlability

A6.4.4. – Stability and Stabilization

A6.4.5. – Control of distributed parameter systems

A6.4.6. – Optimal control

A8.2.1. – Operations research

A8.2.2. – Evolutionary algorithms

A8.2.4. – Mathematical programming

A8.11. – Game Theory

A9.2. – Machine learning

A9.2.1. – Supervised learning

A9.2.2. – Unsupervised learning

A9.2.3. – Reinforcement learning

A9.2.4. – Optimization and learning

A9.2.5. – Bayesian methods

A9.2.6. – Neural networks

A9.2.7. – Kernel methods

A9.2.8. – Deep learning

A9.3. – Signal processing

A9.6. – Decision support

A9.7. – AI algorithmics

Other research topics and application domains

B1.1.2. – Molecular and cellular biology

B1.2.3. – Computational neurosciences

B2.5.1. – Sensorimotor disabilities

B4.2.1. – Fission

Contents

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2 Overall objectives

2.1 Outline of the research project

The highly interconnected contemporary world is faced with an immense range of serious challenges in statistical learning, engineering and information sciences which make the development of statistical and stochastic methods for complex estimation problems and decision making critical. The most significant challenges arise in risk analysis, in environmental and statistical analysis of massive data sets, as well as in defense systems. From both the numerical and the theoretical viewpoints, there is a need for unconventional statistical and stochastic methods that go beyond the current frontier of knowledge.

Our approach to this interdisciplinary challenge is based on recent developments in statistics and stochastic computational methods. We propose a work programme which will lead to significant breakthroughs in fundamental and applied mathematical research, as well as in advanced engineering and information sciences with industrial applications with a particular focus on defence applications, in collaboration with Naval Group.

Many real-world systems and processes are dynamic and essentially random. Examples can be found in many areas like communication and information systems, biology, geophysics, finance, economics, production systems, maintenance, logistics and transportation. These systems require dynamic and stochastic mathematical representations with discrete and/or continuous state variables in possibly infinite dimensional space. Their dynamics can be modeled in discrete or continuous time according to different time scales and are governed by different types of processes such as stochastic differential equations, piecewise deterministic processes, jump-diffusion processes, branching and mean field type interacting processes, reinforced processes and self-interacting Markov processes, to name a few. Our interdisciplinary project draws knowledge from information science, signal processing, control theory, statistics and applied probability including numerical and mathematical analysis. The idea is to work across these scientific fields in order to enhance their understandings and to offer an original theory or concept.

Our group mainly focuses on the development of advanced statistical and probabilistic methods for the analysis and the control of complex stochastic systems, as outlined in the following three topics.

- ***Statistical and Stochastic modeling:*** Design and analysis of realistic and tractable statistical and stochastic models, including measurement models, for complex real-life systems taking into account various random phenomena. Refined qualitative and quantitative mathematical analysis of the stability and the robustness of statistical models and stochastic processes.
- ***Estimation/Calibration:*** Theoretical methods and advanced computational methodologies to estimate the parameters and the random states of the model given partial and noisy measurements as well as statistical data sets. Refined mathematical analysis of the performance and the convergence of statistical and stochastic learning algorithms.
- ***Decision and Control:*** Theoretical methods and advanced computational methodologies for solving regulation and stochastic optimal control problems, including optimal stopping problems and partially observed models. Refined mathematical analysis of the long time behavior and the robustness of decision and control systems.

These three items are by no means independent.

- Regarding the interdependence between the modeling aspects and the estimation/calibration/control aspects, it must be emphasized that when optimizing the performance of a partially observed/known stochastic system, the involved mathematical techniques will heavily depend on the underlying mathematical characteristics and complexity of the model of the state process and the model of the observation process. The main difficulty here is to find a balance between complexity and feasibility/solvability. The more sophisticated a model is, the more complicated the statistical inference and optimization problems will be to solve.
- The interdependence that arises between estimation/calibration and the optimal control can be summarised as follows. When the decision-maker has only partial information on the state process, it is necessary to assume that the admissible control policies will depend on the filtration generated by the

observation process. This is a particularly difficult optimisation problem to solve. Roughly speaking, by introducing the conditional distribution of the state process, the problem can then be reformulated in terms of a fully observed control problem. This leads to a separation of estimation and control principle, i.e. the estimation step is carried out first and then the optimisation. The price to be paid for this new formulation is an enlarged state space of infinite dimension. More precisely, in addition to the observable part of the state, a probability distribution enters the new state space which defines the conditional distribution of the unobserved part of the state given the history of the observations.

Solving such global optimization problems remain an open problem and is recognized in the literature as a very difficult challenge to meet.

One of the fundamental challenges we will address is to develop estimation/calibration and optimal control techniques related to general classes of stochastic processes in order to deal with real-world problems. Our research results will combine, mathematical rigour (through the application of advanced tools from probability, statistics, measure theory, functional analysis and optimization) with computational efficiency (providing accurate and applicable numerical methods with a refined analysis of the convergence). Thus, the results that we will obtain in this research programme will be of interest to researchers in the fields of stochastic modeling, statistics and control theory both for the theoretical and the applied communities. Moreover, the topics studied by Naval Group, such as target detection, nonlinear filtering, multi-object tracking, trajectory optimization and navigation systems, provide a diverse range of application domains in which to implement and test the methodologies we wish to develop.

The final goal is to develop a series of reliable and robust softwares dedicated to statistical and stochastic learning, as well as automated decision and optimal control processes. The numerical codes are required to be both accurate and fast since they are often elements of real time estimation and control loops in automation systems. In this regard, the research topics proposed by Naval Group will provide a natural framework for testing the efficiency and robustness of the algorithms developed by the team.

From our point of view, this collaboration between the INRIA project team and Naval Group offers new opportunities and strategies to design advanced cutting-edge estimation and control methodologies.

2.2 Approach and methodologies

The types of learning and control methodologies developed by the team differ in their approach as well as in the problems that they are intended to solve. They can be summarised by the following three sets of interdependent methodologies.

- ***Statistical learning:*** Regression, clustering, volume and dimensionality reduction, classification, data mining, training sets analysis, supervised and unsupervised learning, active and online learning, reinforcement learning, identification, calibration, Bayesian inference, likelihood optimisation, information processing and computational data modeling.

- ***Stochastic learning:*** Advanced Monte Carlo methods, reinforcement learning, local random searches, stochastic optimisation algorithms, stochastic gradients, genetic programming and evolutionary algorithms, interacting particle and ensemble methodologies, uncertainty propagation, black box inversion tools, uncertainty propagation in numerical codes, rare event and default tree simulation, nonlinear and high dimensional filtering, prediction and smoothing.

- ***Decision and control:*** Markov decision processes, piecewise deterministic Markov processes, stability, robustness, regulation, optimal stopping, impulse control, stochastic optimal control including partially observed problems, games, linear programming approaches, dynamic programming techniques.

All team members of the project work at the interface of the these three areas. This joint research project between INRIA and Naval Group is a natural and unprecedented opportunity to embrace and push the frontiers of the applied and theoretical sides of these research topics in a common research team.

Despite some recent advances, the design and the mathematical analysis of statistical and stochastic learning tools, as well as automated decision processes, is still a significant challenge. For example, since the mid-1970s nonlinear filtering problems and stochastic optimal control problems with partial observations have been the subject of several mathematical studies, however very few numerical solutions have been proposed in the literature.

Conversely, since the mid-1990s, there has been a virtual explosion in the use of stochastic particle methods as powerful tools in real-word applications of Monte Carlo simulation; to name a few, particle filters, evolutionary and genetic algorithms and ensemble Kalman filters. Most of the applied research in statistics, information theory and engineering sciences seems to be developed in a completely blind way with no apparent connections to the mathematical counterparts.

This lack of communication between the fields often produces a series of heuristic techniques often tested on reduced or toy models. In addition, most of these methodologies do not have a single concrete industrial application nor do they have any connection with physical problems.

As such, there exists a plethora of open mathematical research problems related to the analysis of statistical learning and decision processes. For instance, a variety of theoretical studies on particle algorithms, including particle filters and sequential Monte Carlo models are often based on ad-hoc and practically unrealistic assumptions for the kinds of complex models that are increasingly emerging in applications.

The aim of this project is to fill these gaps with an ambitious programme at the intersection of probability, statistics, engineering and information sciences.

One key advantage of the project is its interdisciplinary nature. Combining techniques from pure and applied mathematics, applied probability and statistics, as well as computer science, machine learning, artificial intelligence and advanced engineering sciences enables us to consider these topics holistically, in order to deal with real industrial problems in the context of risk management, data assimilation, tracking applications and automated control systems. The overarching aim of this ambitious programme is to make a breakthrough in both the mathematical analysis and the numerical aspects of statistical learning and stochastic estimation and control.

2.3 Innovation and industrial transfer

Fundamentally, our team is not driven by a single application. The reasons are three-fold. Firstly, the robustness and transferability of our approaches means that the same statistical or stochastic learning algorithms can be used in a variety of application areas. On the other hand every application domain offers a series of different perspectives that can be used to improve the design and performances of our techniques and algorithms. Last but not least, industrial applications, including those that arise in defence, require specific attention. As such, we use a broad set of stochastic and statistical algorithms to meet these demands.

This research programme is oriented towards concrete applications with significant potential industrial transfers on three central problems arising in engineering and information and data sciences, namely, risk management and uncertainty propagation, process automation, and data assimilation, tracking and guidance. Our ultimate goal is to bring cutting edge algorithms and advanced statistical tools to industry and defence. The main application domains developed by the team are outlined below:

- ***Risk management and uncertainty propagation:*** Industrial and environmental risks, fault diagnostics, phase changes, epidemiology, nuclear plants, financial ruin, systemic risk, satellite debris collisions.

- ***Process automation:*** Production maintenance and manufacturing planing, default detection, integrated dynamics and control of distributed dynamical systems, multi-object coordination, automatic tuning of cochlear implants, classification of EGG signals.

- ***Data assimilation, tracking and guidance:*** Target detection and classification, nonlinear filtering and multi-object tracking, multiple sensor fusions, motion planning, trajectory optimization, design of navigation systems.

The main objectives and challenges related to the three application domains discussed above will be developed in section 4. The latter application domain will be developed in collaboration with Naval Group. The reader is refereed to section 4.1 for a description of this collaboration and to sections 3.2 and 3.3 for the theoretical aspects that will be carried out by the team in relation to these topics. Specific details on the particular techniques used to tackle the estimation and tracking problems in the context of the collaboration with Naval Group will remain confidential.

3 Research program

This section describes the different challenges we intend to address in the theoretical and numerical aspects of statistical/stochastic learning and optimal control. It will be difficult to convey the full complexity of the various topics and to provide a complete overview through a detailed timetable. Nevertheless, we will explain our motivation and why we think it is imperative to address these challenges. We will also highlight the technical issues inherent to these challenges, as well as the difficulties we might expect.

We are confident that the outcomes of this scientific project will lead to significant breakthroughs in statistical/stochastic learning and optimal control with a special emphasis on applications in the defence industry in collaboration with Naval Group. In this respect, we would like to quote Hervé Guillou, CEO of Naval Group, on the occasion of the signing of the partnership agreement between INRIA and Naval Group on December 10, 2019: *"This partnership will enable Naval Group to accelerate its innovation process in the fields of artificial intelligence, intelligence applied to cyber and signal processing. This is a necessity given the French Navy's need for technological superiority in combat and the heightened international competition in the naval defence field..."*

One of our greatest achievements would undoubtedly be to meet these challenges with Naval Group, particularly those related to the fields of statistical/stochastic learning and control. We could not dream of a better outcome for our project.

3.1 Statistical learning

Permanent researchers: M. Chavent, P. Del Moral, F. Dufour, A. Genadot, P. Legrand, J. Saracco.

Regarding statistical learning, some of the objectives of the team is to develop dimension reduction models, data visualization, non-parametric estimation methods, genetic programming and artificial evolution. These models/methodologies provide a way to understand and visualize the structure of complex data sets. Furthermore, they are important tools in several different areas of research, such as data analysis and machine learning, that arise in many applications in biology, genetics, environment and recommendation systems. Of particular interest is the analysis of classification and clustering approaches and semi-parametric modeling that combines the advantages of parametric and non-parametric models, amongst others. One major challenge is to tackle both the complexity and the quantity of data when working on real-world problems that emerge in industry or other scientific fields in academia. Of particular interest is to find ways to handle high-dimensional data with irrelevant and redundant information.

Another challenging task is to take into account successive arrivals of information (data stream) and to dynamically refine the implemented estimation algorithms, whilst finding a balance between the estimation precision and the computational cost. One potential method for this is to project the available information into suitably chosen lower dimensional spaces.

For regression models, sliced inverse regression (SIR) and related approaches have proven to be highly efficient methods for modeling the link between a dependent variable (which can be multidimensional) and multivariate covariates in several frameworks (data stream, big data, etc.). The underlying regression model is semi-parametric (based on a single index or on multiple indices that allow dimension reduction). Currently, these models only deal with quantitative covariates. One of the team's goals is to extend these regression models to mixed data, i.e. models dealing with quantitative and categorical covariates. This generalization would allow one to propose discriminant analysis to deal with mixed data. Extension of sparse principal component analysis (PCA) to mixed data is also another challenge. One idea is to take inspiration from the underlying theory and method of recursive SIR and SIR approaches for data stream in order to adapt them to commonly used statistical methods in multivariate analysis (PCA, discriminant analysis, clustering, etc.). The common aim of all these approaches is to estimate lower dimensional subspaces whilst minimizing the loss of statistical information. Another important aspect of data stream is the possible evolution in time of the underlying model: we would like to study break(s) detection in semi-parametric regression model, the breakdown being susceptible to appear in the parametric part or in the functional part of the regression model. The question of selecting covariates in regression modelling when we deal with big data is a fundamental and difficult problem. We will address this challenge using genetic programming and artificial evolution. Several directions are possible: for instance, improve, via genetic algorithms, the exploration of the covariate space in closest submodel selection (CSS) method or study optimization problems that simultaneously take into account variable selection, efficiency of estimation and interpretability of the model. Another important question concerns the detection of outliers that will disturb the estimation of the model, and this is not an obvious problem to deal with when working with large, high dimensional data.

In multivariate data analysis, an objective of the team is to work on a new formulation/algorithm for group-sparse block PCA since it is always important to take into account group information when available. The advantage of the group-sparse block PCA is that, via the selection of groups of variables (based on the synthetic variables), interpretability of the results becomes easier. The underlying idea is to address the simultaneous determination of group-sparse loadings by block optimization, and the correlated problem of defining explained variance for a set of non-orthogonal components. The team is also interested in clustering of supervised variables, the idea being to construct clusters made up of variables correlated with each other, which are either well-linked or not-linked to the variable to be explained (which can be quantitative or qualitative).

Another way to study the links between variables is to consider conditional quantiles instead of conditional expectation as is the case in classical regression models. Indeed, it is often of interest to model conditional quantiles, particularly in the case where the conditional mean fails to take into account the impact of the covariates on the dependent variable. Moreover, the quantile regression function provides a much more comprehensive picture of the conditional distribution of a dependent variable than the conditional mean function. The team is interested in the non parametric estimation of conditional quantile estimation. New estimators based on quantization techniques have been introduced and studied in the literature for univariate conditional quantiles and multivariate conditional quantiles. However, there are still many open problems, such as combining information from conditional quantiles of different orders in order to refine the estimation of a conditional quantile of a given order.

Another topic of interest is genetic programming (GP) and Artificial Evolution. GP is an evolutionary computation paradigm for automatic program induction. GP has produced impressive results but there are still some practical limitations, including its high computational cost, overfitting and excessive code growth. Recently, many researchers have proposed fitness-case sampling methods to overcome some of these problems, with mixed results in several limited tests. Novelty Search (NS) is a unique approach towards search and optimization, where an explicit objective function is replaced by a measure of solution novelty. While NS has been mostly used in evolutionary robotics, the team would like to explore its usefulness in classic machine learning problems.

Another important objective of the team is to implement new R (Matlab/Python) packages or to enrich those existing in the literature with the methods we are going to develop in order to make them accessible to the scientific community.

With respect to our statistical learning research program, the objectives of the team can be divided into mid- and long-term works. Mid-term objectives focus on sparsity in SIR (via soft thresholding for instance) and group-sparse block PCA, the underlying idea being to make the selection of variables or blocks of variables in the regression model or in the data. Taking into account multi-block data in regression models via data-driven sparse partial least squares is also at the heart of our concerns. Coupling genetic algorithms and artificial evolution with statistical modeling issues is also planned. The team has several long-term projects associated with the notion of data stream. Many theoretical and practical problems arise from the possible evolution of the information contained in the data: break detection in the underlying model, balance between precision and computational cost. Another scientific challenge is to extend certain approaches such as SIR to the case of mixed data by incorporating the information provided by the qualitative variables in the associated low dimensional subspaces. Moreover, the team has already worked on clustering of variables for mixed data and the clustering of supervised variables is now planned. Finally the idea of combining information from conditional quantiles of different orders in order to refine the estimation of a given order conditional quantile is still relevant today. It should be noted that other research themes may appear or become a priority depending on the academic or industrial collaborations that may emerge during the next evaluation period.

Project-team positioning: Some topics of the INRIA project teams (STATIFY, CELESTE, MODAL, SEQUEL, CLASSIC) are close to the ASTRAL objectives such as non parametric view of high dimensional data, statistical/machine learning, model selection, clustering, sequential learning algorithms, or multivariate data analysis for complex data. While certain ASTRAL objectives are similar to those of these teams, our approaches are significantly different. For example, in multivariate data analysis of complex data including

clustering, our team mainly focuses on a geometric approach for mixed data. We also consider the case of successive arrivals of information in SIR both from the theoretical and numerical point of view. Currently there is no direct competition between our team and other INRIA project teams. However, interactions between ASTRAL and other INRIA teams exist. For instance, ASTRAL and STATIFY collaborations are fruitful with common publications, in particular with S. Girard (STATIFY project team).
In the field of multivariate data analysis, the team have interesting discussions with Agrocampus Ouest (Rennes, France) and with H.A.L. Kiers (Groningen University) on a mixed data approach for dimension reduction. Conditional and regression quantiles are very active research fields in France (University of Toulouse, Toulouse School of Economics, University of Montpellier) and around the world (ULB, Belgium; University of Illinois Urbana-Champaign, USA; Open University, UK; Brunel University, UK). The ASTRAL team has for the last four-year period collaborated with D. Paindaveine (ULB, Belgium). In the dimension reduction framework, there is a large international community in Europe, America or Asia working on SIR and related methods. However, to our knowledge, the ASTRAL team was the first to introduce importance of variables and recursive methods in SIR, and the first to adapt the SIR approach to data stream.

3.2 Stochastic learning

Permanent researchers: M. Chavent, P. Del Moral, F. Dufour, A. Genadot, D. Laneuville, P. Legrand, A. Nègre, J. Saracco.

Stochastic particle methodologies have become one of the most active intersections between pure and applied probability theory, Bayesian inference, statistical machine learning, information theory, theoretical chemistry, quantum physics, financial mathematics, signal processing, risk analysis, and several other domains in engineering and computer sciences.

Since the mid-1990s, rapid developments in computer science, probability and statistics have led to new generations of interacting particle learning/sampling type algorithms, such as:

Particle and bootstrap filters, sequential Monte Carlo methods, self-interacting and reinforced learning schemss, sequentially interacting Markov chain Monte Carlo, genetic type search algorithms, island particle models, Gibbs cloning search techniques, interacting simulated annealing algorithms, importance sampling methods, branching and splitting particle algorithms, rare event simulations, quantum and diffusion Monte Carlo models, adaptive population Monte Carlo sampling models, Ensemble Kalman filters and interacting Kalman filters.

Since computations are nowadays much more affordable, the aforementioned particle methods have become revolutionary for solving complex estimation and optimization problems arising in engineering, risk analysis, Bayesian statistics and information sciences. The books [40], [41], [47],[59] provide a rather complete review on these application domains.

These topics have constituted some of the main research axes of several of the ASTRAL team members since the beginning of the 1990s. To the best of our knowledge, the first rigorous study on particle filters and the convergence of genetic algorithms as the size of the population tends to infinity seems to be the article [49], published in 1996 in the journal Markov Processes and Related Fields. This paper has opened an avenue of research questions in stochastic analysis and particle methods applications. The uniform convergence of particle filters and ensemble Kalman filters with respect to the time horizon was first seen in [44, 43, 46] and in the more recent article [48]. The first use of particle algorithms and Approximated Bayesian Computation type methodologies in nonlinear filtering seems to have started in [45]. Last but not least, the development of sequential Monte Carlo methodology in statistics was introduced in the seminal article [42].

Despite some recent advances, the mathematical foundation and the design and the numerical analysis of stochastic particle methods is still a significant challenge. For instance, particle filter technology is often combined with Metropolis-Hastings type techniques, or with Expectation Maximization type algorithms. The resulting algorithms are intended to solve high dimensional hidden Markov chain problems with fixed parameters. In this context (despite some recent attempts) the refined convergence analysis of the resulting particle algorithms, including exponential concentration estimates, remains to be developed.

Last but not least, the expectations of their performances are constantly rising in a variety of application domains. These particle methodologies are now expected to deal with increasingly sophisticated models in high dimensions, whilst also allowing for the variables to evolve at different scales. *The overarching aim of* *this aspect of the programme is to make a breakthrough in both the mathematical analysis and the numerical simulation of stochastic and interacting particle algorithms.*

Today, partly because of the emergence of new mean field simulation methodologies and partly because of the importance of new and challenging high-dimensional problems arising in statistical machine learning, engineering sciences and molecular chemistry, we are observing the following trends:

• A need to better calibrate their performance with respect to the size of the systems and other tuning parameters, including cooling decay rates, local random search strategies, interacting and adaptive search criteria, and population size parameters. One of the main and central objectives is to obtain uniform and non asymptotic precision estimates with respect to the time parameter. These types of uniform estimates need to be developed, supporting industrial goals of enhanced design and confidence of algorithms, risk reduction and improved safety.

• A need for new stochastic and adaptive particle methods for solving complex estimation models. Such models arise in a range of application areas including forecasting, data assimilation, financial risk management and analysis of critical events. This subject is also crucial in environmental studies and in the reliability analysis of engineering automated systems. The complexity of realistic stochastic models in advanced risk analysis requires the use of sophisticated and powerful stochastic particle models. These models go far beyond Gaussian models, taking into account abrupt random changes, as well as non nonlinear dynamics in high dimensional state spaces.

• A need to find new mathematical tools to analyze the stability and robustness properties of sophisticated, nonlinear stochastic models involving space-time interaction mechanisms. Most of the theory on the stability of Markov chains is based on the analysis of the regularity properties of linear integral semigroups. To handle these questions, the interface between the theory of nonlinear dynamical systems and the analysis of measure valued processes needs to be further developed.

From a purely probabilistic point of view, the fundamental and the theoretical aspects of our research projects are essentially based on the stochastic analysis of the following three classes of interacting stochastic processes: *Spatial branching processes and mean-field type interacting particle systems, reinforced and self-interacting processes, and finally random tree based search/smoothing learning processes.*

The first class of particle models includes interacting jump-diffusions, discrete generation models, particle ensemble Kalman filters and evolutionary algorithms. This class of models refers to mean field type interaction processes with respect to the occupation measure of the population. For instance genetic-type branching-selection algorithms are built on the following paradigm: when exploring a state space with many particles, we duplicate better fitted individuals while particles with poor fitness die. The selection is made by choosing randomly better fitted individuals in the population. Our final aim is to develop a complete mean-field particle theory combining the stability properties of the limiting processes as the size of the system tends to infinity with the performance analysis of these particle sampling tools.

The second class of particle models refers to mean field type interaction processes with respect to the occupation measure of the past visited sites. This type of reinforcement is observed frequently in nature and society, where "beneficial" interactions with the past history tend to be repeated. Self interaction gives the opportunity to build new stochastic search algorithms with the ability to, in a sense, re-initialize their exploration from the past, re-starting from some better fitted previously visited initial value. In this context, we plan to explore the theoretical foundations and the numerical analysis of continuous time or discrete generation self-organized systems by combining spatial and temporal mean field interaction mechanisms.

The last generation of stochastic random tree models is concerned with biology-inspired algorithms on paths and excursions spaces. These genealogical adaptive search algorithms coincide with genetic type particle models in excursion spaces. They have been successfully applied in generating the excursion distributions of Markov processes evolving in critical and rare event regimes, as well as in path estimation and related smoothing problems arising in advanced signal processing. The complete mathematical analysis of these random tree models, including their long time behavior, their propagation of chaos properties, as well as their combinatorial structures are far from complete.

Our research agenda on stochastic learning is developed around the applied mathematical axis as well as the numerical perspective, including concrete industrial transfers with a special focus on Naval Group. From the theoretical side, mid-term objectives are centered around non asymptotic performance analysis and long time behavior of Monte Carlo methods and stochastic learning algorithms. We also plan to further develop the links with Bayesian statistical learning methodologies and artificial intelligence techniques, including the analysis of genetic programming discussed in section 3.1. We also have several long term projects. The first one is to develop new particle type methodologies to solve high dimensional data assimilation problems arising in forecasting and fluid mechanics, as well as in statistical machine learning. We also plan to design stochastic filtering-type algorithms to solve partially observed control problems such as those discussed in section 3.3.

Project-team positioning: In the last three decades, the use of Feynman-Kac type particle models has been developed in variety of scientific disciplines, including in molecular chemistry, risk analysis, biology, signal processing, Bayesian inference and data assimilation.

The design and the mathematical analysis of Feynman-Kac particle methodologies has been one of the main research topics of P. Del Moral since the late 1990's [49, 45, 44], see also the books [46, 40, 41, 47] and references therein. These mean field particle sampling techniques encapsulate particle filters, sequential Monte Carlo methods, spatial branching and evolutionary algorithms, Fleming-Viot genetic type particles methods arising in the computation of quasi-invariant measures and simulation of non absorbed processes, as well as diffusion Monte Carlo methods arising in numerical physics and molecular chemistry. The term "particle filters" was first coined in the article [49] published in 1996 in reference to branching and mean field interacting particle methods used in fluid mechanics since the beginning of the 1960s. This article presents the first rigorous analysis of these mean field type particle algorithms.

The INRIA project teams applying the particle methodology developed by ASTRAL include the INRIA project team SIMSMART (rare event simulation as well as particle filters) and the INRIA project team Matherials (applications in molecular chemistry). The project team ASTRAL also has several collaborative research projects with these, teams as well as with researchers from international universities working in this subject, including Oxford, Cambridge, New South Wales Sydney, UTS, Bath, Warwick and Singapore Universities.

3.3 Decision and stochastic control

Permanent researchers: P. Del Moral, F. Dufour, A. Genadot, D. Laneuville, O. Marceau, A. Nègre, J. Saracco.

Part of this research project is devoted to the analysis of stochastic decision models. Many real applications in dynamic optimization can be, roughly speaking, described in the following way: a certain system evolves randomly under the control of a sequence of actions with the objective to optimize a performance function. Stochastic decision processes have been introduced in the literature to model such situations and it is undoubtedly their generic capacity to model real life applications that leads to and continues to contribute to their success in many fields such as engineering, medicine and finance.

In this project we will focus on specific families of models that can be identified according to the following elements: the nature of the time variable (discrete or continuous), the type of dynamics (piecewise deterministic trajectories) and the numbers of decision makers. For one player, the system will be called a *stochastic control process* and for the case of several decision-makers, the name *(stochastic) game* will be used. For ease of understanding, we now provide an informal description of the classes of stochastic processes we are interested in, according to the nature of the time variable.

Discrete-time models. In this framework, the basic model can be described by a state space where the system evolves, an action space, a stochastic kernel governing the dynamic and, depending on the state and action variables, a one-step cost (reward) function. The distribution of the controlled stochastic process is defined through the control policy which is then selected in order to optimize the objective function. This is a very general model for dynamic optimization in discrete-time, which also goes by the name of *stochastic dynamic programming*. For references, the interested reader may consult the following books [35, 37, 50, 51, 53, 54, 55, 56, 61, 60, 63] and the references therein (this list of references is, of course, not exhaustive).

Continuous-time models. Most of the continuous-time stochastic processes consist of a combination of the following three different ingredients: stochastic jumps, diffusion and deterministic motions. In this project, we will focus on **non-diffusive models**, in other words, stochastic models for which the randomness appears only at fixed or random times, *i.e.* those combining deterministic motions and random jumps. These stochastic processes are the so-called piecewise deterministic Markov processes (PDMPs) [36, 38, 39, 52, 58, 57, 62]. This family of models plays a central role in applied probability because it forms the bulk of models in many research fields such as, *e.g.* operational research, management science and economy and covers an enormous variety of applications.

These models can be framed in several different forms of generality, depending on their mathematical properties such as the type of performance criterion, full or incomplete state information, with or without constraints, adaptative or not, but more importantly, the nature of the boundary of the state space, the type of dynamic between two jumps and on the number of decision-makers. These last three characteristics make the analysis of the controlled process much more involved.

Part of this project will cover both theoretical and numerical aspects of stochastic optimal control. It is clear that stochastic problems and control games have been extensively studied in the literature. Nevertheless, important challenges remain to be addressed. From the theoretical side, there are still many technical issues that are, for the moment, still unanswered or at most have received partial answers. This is precisely what makes them difficult and requires either the creative transposition of pre-existing methodologies or the development of new approaches. It is interesting to note that one of the feature of these theoretical problems is that they are closely related to practical issues. Solving such problems not only gives rise to challenging mathematical questions, but also allow a better insight into the structure and properties of real practical problems. Theory for applications will be for us the thrust that will guide us in this project. From the numerical perspective, solving a stochastic decision model remains a critical issue. Indeed, except for very few specific models, the determination of an optimal policy and the associated value function is an extremely difficult problem to tackle. The development of computational and numerical methods to get quasi-optimal solutions is, therefore, of crucial importance to demonstrate the practical interest of stochastic decision model as a powerful modeling tool. During the International Conference on Dynamic Programming and Its Applications held at the University of British Columbia, Canada in April 1977, Karl Hinderer, a pioneer in the field of stochastic dynamic programming emphasized that "*whether or not our field will have a lasting impact on science beyond academic circles depends heavily on the success of implemented applications*". We believe that this statement is still in force some forty years later.

The objective of this project is to address these important challenges. They are mainly related to models with general state/action spaces and with continuous time variables covering a large field of applications. Here is a list of topics we would like to study: games, constrained control problems, non additive types of criteria, numerical and computational challenges, analysis of partially observed/known stochastic decision processes. This list is not necessarily exhaustive and may of course evolve over time.

Our research agenda on optimal stochastic control is developed around the applied mathematical axis as well as the numerical perspective, including concrete industrial transfers with a special focus on Naval Group. Our mid-term objectives will focus on the following themes described above: properties of control policies in continuous-time control problems, non additive types of criteria, numerical and computational challenges. Our long-term objectives will focus on the analysis of partially observed/known stochastic control problems, constrained control problems and games.

Project-team positioning: There exists a large national/international community working on PDMPs and MDPs both on the theoretical, numerical and practical aspects. One may cite A. Almudevar (University of Rochester, USA), E. Altman (INRIA Team NEO, France), K. Avrachenkov (INRIA Team NEO, France), N. Bauerle (Karlsruhe University, Germany), D. Bertsekas (Massachusetts Institute of Technology, USA), O. Costa (Sao Paulo University, Brazil), M. Davis (Imperial College London, England), E. Feinberg (Stony Brook University, USA), D. Goreac (Université Paris-Est Marne-la-Vallée, France), X. Guo (Zhongshan University, China), O. Hernandez-Lerma (National Polytechnic Institute, Mexico), S. Marcus (University of Maryland, USA), T. Prieto-Rumeau (Facultad de Ciencias, UNED, Spain), A. Piunovskiy (University of Liverpool, England), U. Rieder (Universität Ulm, Germany), J. Tsitsiklis (Massachusetts Institute of Technology, USA), B. Van Roy (Stanford University, USA), O. Vega-Amaya (Universidad de Sonora, Mexico), Y. Zhang (University of Liverpool, England) to name just a few. Many of the colleagues cited above are at the head of research groups which have not been described in detail due to space limitation and so, this list is far from being exhaustive.

To some extent, our team is in competition wit the colleagues and teams mentioned above. We emphasize that there exists a long standing collaboration between our group and O. Costa (Sao Paulo University, Brazil) since 1998. In the last 10 years, we have established very fruitful collaborations with T. Prieto-Rumeau (Facultad de Ciencias, UNED, Spain) and A. Piunovskiy (University of Liverpool, England).

Inside INRIA, the team NEO and in particular E. Altman and K. Avrachenkov work on discrete-time MDPs but they are mainly focused on the case of countable (finite) state/action spaces MDPs. From this point of view, our results on this theme may appear complementary to theirs.

4 Application domains

It is important to point out that (for the time being) only a sub-group of the academic part of the team collaborates with Naval Group. Initially the topics of interest for Naval Group was focused on filtering and control problems. The academic members of this sub-group are P. Del Moral, F. Dufour, A. Genadot. It is also important to emphasize that Naval Group is undoubtedly our privileged industrial partner. This collaboration is described in section 4.1. For reasons of confidentiality, this section is not very detailed, in particular it does not mention the timetable and does not detail the technical solutions that will be considered. Our aim in the short term is to integrate the remaining academic team members into the group to work on the themes of interest to NG. A seminar was organized for this purpose in August 2020. The academic members of the team who are not involved in collaboration with NG (M. Chavent, P. Legrand and J. Saracco) have their own industrial collaborations that are described in section 4.2.

4.1 Naval Group research activities

Permanent researchers: D. Arrivault, P. Del Moral, F. Dufour, A. Genadot, E. Iglesis, D. Laneuville, A. Nègre.
An important line of research of the team is submarine passive target tracking. This is a very complicated practical problem that combines both filtering and stochastic control topics. In the context of passive underwater acoustic warfare, let us consider a submarine, called the observer, equipped with passive sonars collecting noisy bearing-only measurements of the target(s). The trajectory of the observer has to be controlled in order to satisfy some given mission objectives. These can be, for example, finding the best trajectory to optimize the state estimation (position and velocity) of the targets, maximize the different targets' detection range and/or minimize its own acoustic indiscretion with respect to these targets, and reaching a way-point without being detected. Let us now describe in more detail some of the topics we intend to work on.

In the case of passive tracking problems, one of the main issues is that the observer must manoeuvre in order to generate observability. It turns out that these manoeuvres are actually necessary but not sufficient to guarantee that the problem becomes observable. In fact, a significant body of the literature pertains to attempting to understand whether this type of problem is solvable. Despite this observability analysis, the following practical questions, which we would like to address in this project, remain challenging: What kind of trajectory should the observer follow to optimize the estimation of the target's motion? What is the accuracy of that estimate? How to deal with a multitarget environment? How to take into account some physical constraints related to the sonar?

Another aspect of target tracking is to take into account both the uncertainties on the target's measurement and also the signal attenuation due to acoustic propagation. To the best of our knowledge, there are few works focusing on the computation of optimal trajectories of underwater vehicles based on signal attenuation. In this context, we would like address the problem of optimizing the trajectory of the observer to maximize the detection of the acoustic signals issued by the targets. Conversely, given that the targets are also equipped with sonars, how can one optimize the trajectory of the observer itself to keep its own acoustic indiscretion as low as possible with respect to those targets.

It must be emphasized that a human operator can find a suitable trajectory for either of these objectives in the context of a single target. However, if both criteria and/or several targets are taken into account simultaneously, it is hardly possible for a human operator to find such trajectories. From an operational point of view, these questions are therefore of great importance.

Such practical problems are strongly connected to the mathematical topics described in sections 3.2 and 3.3. For example it is clearly related to partially observed stochastic control problems. The algorithmic solutions that we will develop in the framework of submarine passive target tracking will be evaluated on the basis of case studies proposed by Naval Group. Our short-term aim is to obtain explicit results and to develop efficient algorithms to solve the various problems described above.

4.2 Other collaborations

Permanent researchers: M. Chavent, P. Legrand and J. Saracco.
For several years, the team has also had strong collaborations with INRAE which is the French National Research Institute for Agriculture, Food and Environment. More precisely, consumer satisfaction when eating beef is a complex response based on subjective and emotional assessments. Safety and health are very important in addition to taste and convenience but many other parameters are also extremely important for breeders. Many models were recently developed in order to predict each quality trait and to evaluate the possible trade-off that could be accepted in order to satisfy all the operators of the beef chain at the same time. However, in none of these quality prediction systems are issues of joint management of the different expectations addressed. Thus, it is vital to develop a model that integrates the sensory quality of meat but also its nutritional and environmental quality, which are expectations clearly expressed by consumers. Our team are currently developing statistical models and machine learning tools in order to simultaneously manage and optimize the different sets of expectations. Combining dimension reduction methodologies, nonparametric quantiles estimation and “Pareto front” approaches could provide an interesting way to address this complex problem. These different aspects are currently in progress.

The team is currently initiating scientific collaboration with the Advanced Data Analytics Group of Sartorius Corporate Research which is an international pharmaceutical and laboratory equipment supplier, covering the segments of Bioprocess Solutions and Lab Products & Services. The current work concerns the development of a partial least squares (PLS) inspired method in the context of multiblocks of covariates (corresponding to different technologies and/or different sampling techniques and statistical procedures) and high dimensional datasets (with the sample size n much smaller than the number of variables in the different blocks). The proposed method allows variable selection in the X and in the Y components thanks to interpretable parameters associated with the soft-thresholding of the empirical correlation matrices (between the X ’s blocks and the Y block) decomposed using singular values decomposition (SVD). In addition, the method is able to handle specific missing values (i.e. “missing samples” in some covariate blocks). The suggested `ddsPLS + Koh Lanta` methodology is computationally fast. Some technical and/or theoretical work on this methodology must be naturally pursued in order to further refine this approach. Moreover, another aspect of the future research with Sartorius consists of associating the structures of datasets with the real biological dynamics described, until now, by differential equations and for which the most advanced solutions do not merge with both high dimensional multiblock analysis and these differential equations. Combining these two approaches in a unified framework will certainly have many applications in industry and especially in the biopharmaceutical production.

Within the framework of the GIS ALBATROS, the team has initiated a scientific collaboration with IMS and THALES. The first topic is focused on the measurement of the cognitive load of a pilot through the development of methods for measuring the regularity of biological signals (Hölderian regularity, Detrended Fluctuation Analysis, etc.). The second topic is dedicated to the development of classification techniques of vessels. The different methods we proposed are based on deep learning, evolutionary algorithms and signal processing techniques and are compared to the approaches in the literature.

5 Highlights of the year

5.1 Awards

The paper [23] "Segmentation and lossless compression of SAR data: a new approach to ensure transmission robustness" has won a Best of Session (BOS) award at 44th Digital Avionics Systems Conference (DASC 2025).

5.2 Nomination

- Pierrick Legrand has been appointed as AI Mission Officer for Bordeaux INP.
- Jérôme Saracco was elected to the Bordeaux INP Board of Directors in June 2025.
- Jérôme Saracco is a guest member of the Bordeaux INP Academic Council in his capacity as Director of Studies at ENSC.

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6 Latest software developments, platforms, open data

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Participants: Denis Arrivault, Pierre del Moral, Francois Dufour, Alexandre Genadot, Enzo Iglesis, Pierrick Legrand, Marie Chavent, Jérôme Saracco.

6.1 Latest software developments

6.1.1 FracLab

Keyword: Stochastic process

Functional Description: FracLab is a general purpose signal and image processing toolbox based on fractal, multifractal and local regularity methods. FracLab can be approached from two different perspectives: - (multi-) fractal and local regularity analysis: A large number of procedures allow to compute various quantities associated with 1D or 2D signals, such as dimensions, Hölder and 2-microlocal exponents or multifractal spectra.

- Signal/Image processing: Alternatively, one can use FracLab directly to perform many basic tasks in signal processing, including estimation, detection, denoising, modeling, segmentation, classification, and synthesis.

URL: https://project.inria.fr/fraclab/

Contact: Jacques Levy-Vehel

Participant: 7 anonymous participants

Partners: Centrale Paris, Mas

6.1.2 PCAmixdata

Keyword: Statistic analysis

Functional Description: Mixed data type arise when observations are described by a mixture of numerical and categorical variables. The R package PCAmixdata extends standard multivariate analysis methods to incorporate this type of data. The key techniques included in the package are PCAmix (PCA of a mixture of numerical and categorical variables), PCArot (rotation in PCAmix) and MFAmix (multiple factor analysis with mixed data within a dataset). The MFAmix procedure handles a mixture of numerical and categorical variables within a group - something which was not possible in the standard MFA procedure. We also included techniques to project new observations onto the principal components of the three methods in the new version of the package.

URL: https://cran.r-project.org/web/packages/PCAmixdata/index.html

Contact: Marie Chavent

6.1.3 vimplclust

Keywords: Clustering, Fair and ethical machine learning

Functional Description: vimpclust is an R package that implements methods related to sparse clustering and variable importance. The package currently allows to perform sparse k-means clustering with a group penalty, so that it automatically selects groups of numerical features. It also allows to perform sparse clustering and variable selection on mixed data (categorical and numerical features), by preprocessing each categorical feature as a group of numerical features. Several methods for visualizing and exploring the results are also provided.

URL: https://CRAN.R-project.org/package=vimpclust

Contact: Marie Chavent

6.1.4 divdiss

Name: divisive monothetic clustering on dissimilarity matrix

Keywords: Clustering, Machine learning

Functional Description: The div_diss function implements a divisive monotopic hierarchical classification algorithm.

URL: https://github.com/chavent/divdiss

Contact: Marie Chavent

6.1.5 pybellhop

Name: Pybellhop

Keywords: 3D, Python, Hydroacoustics

Scientific Description: Hydroacoustic waves simulator.

Functional Description: Pybellhob is exposing some bellhopcuda functionalities in python in order to make them callable from any python software without any disk access to write and read configuration and result files.

Release Contributions: First version

News of the Year: Développement de la version v0.1.

Contact: Denis Arrivault

Participant: Denis Arrivault

6.1.6 pyastral

Name: PyAstral

Keyword: Python

Scientific Description: Trajectography

Functional Description: Trajectography software

News of the Year: Developments, tests, documentation pour la version 0.1.

Contact: Enzo Iglesis

Participants: Enzo Iglesis, Denis Arrivault

Partner: Naval Group

6.1.7 estimation_filters

Name: Estimation Filters

Keywords: Kalman filter, Particular filter, Bayesian estimation

Scientific Description: Python library offering several stochastic estimation filters for estimating and tracking the state of a dynamic system.

Functional Description: Python library offering several stochastic estimation filters for estimating and tracking the state of a dynamic system.

News of the Year: 27 commits cette année. Divers corrections et adaptations plus l'ajout de comentaires et de tests.

Contact: Enzo Iglesis

Participants: Enzo Iglesis, Denis Arrivault, an anonymous participant

7 New results

7.1 Statistical learning

7.1.1 Analysis of electrical features for detection of subjects at risk for sudden cardiac death

Sudden cardiac death (SCD) accounts for 30% of adult mortality in industrialized countries. The majority of SCD cases are the result of an arrhythmia called ventricular fibrillation, which itself results from structural abnormalities in the heart muscle. Despite the existence of effective therapies, most individuals at risk for SCD are not identified preventively due to the lack of available testing. Developing specific markers on electrocardiographic recordings would enable the identification and stratification of SCD risk. Over the past six years, the Liryc Institute has recorded surface electrical signals from over 800 individuals (both healthy and pathological) using a high-resolution 128-electrode device. Features were calculated from these signals (signal duration per electrode, frequency, amplitude fractionation, etc.). In total, more than 1,500 electrical features are available per patient. During the acquisition process using the 128-electrode system in a hospital setting, noise or poor positioning of specific electrodes sometimes prevents calculating the intended features, leading to an incomplete database. This thesis [24] is organized around two main axes. First, we developed a method for imputing missing data to address the problem of faulty electrodes. Then, we developed a risk score for the sudden death risk stratification. The most commonly used family of methods for handling missing data is imputation, ranging from simple completion by averaging to local aggregation methods, local regressions, optimal transport, or even modifications of generative models. Recently, Autoencoders (AE) and, more specifically, Denoising AutoEncoders (DAE) have performed well in this task. AEs are neural networks used to learn a representation of data in a reduced-dimensional space. DAEs are AEs that have been proposed to reconstruct original data from noisy data. In this work, we propose a new methodology based on DAEs called the modified Denoising AutoEncoder (mDAE) to allow for the imputation of missing data. The second research axis of the thesis focused on developing a risk score for sudden cardiac death. DAEs can model and reconstruct complex data. We trained DAEs to model the distribution of healthy individuals based on a selected subset of electrical features. Then, we used these DAEs to discriminate pathological patients from healthy individuals by analyzing the imputation quality of the DAE on partially masked features. We also compared different classification methods to establish a risk score for sudden death.

Participants: Mariette Dupuy (ASTRAL).

7.1.2 ClimLoco1.0: CLimate variable confidence Interval of Multivariate Linear Observational COnstraint

Projections of future climate are key to society's adaptation and mitigation plans in response to climate change. Numerical climate models provide projections, but the large dispersion between them makes future climate very uncertain. To refine it, approaches called observational constraints (OC) have been developed. They constrain an ensemble of climate projections by some real-world observations. However, there are many difficulties in dealing with the large literature on OC: the methods are diverse, the mathematical formulation and underlying assumptions used are not always clear, and the methods are often limited to the use of the observation of only one variable. To address these challenges, we propose in [18] a new statistical model called ClimLoco1.0, which stands for "CLimate variable confidence Interval of Multivariate Linear Observational COnstraint". It describes, in a rigorous way, the confidence interval of a projected variable (its best guess associated with an uncertainty at a confidence level) obtained using a multivariate linear OC. The article is built up in increasing complexity by expressing in three different cases, the last one being ClimLoco1.0, the confidence interval of a projected variable: unconstrained, constrained by multiple real-world observations assumed to be noiseless, and constrained by multiple real-world observations assumed to be noisy. ClimLoco1.0 thus accounts for observational noise (instrumental error and climate-internal variability), which is sometimes neglected in the literature but is important as it reduces the impact of the OC. Furthermore, ClimLoco1.0 accounts for uncertainty rigorously by taking into account the quality of the estimators, which depends, for example, on the number of climate models considered. In addition to providing an interpretation of the mathematical results, this article provides graphical interpretations based on synthetic data.

Participants: Valentin Portmann (ASTRAL), Marie Chavent (ASTRAL), Didier Swingedouw.

7.1.3 Détection et quantification de manipulations comptables autour d'un seuil psychologique avec R.

In this work [21], we study the implementation in R of a new statistical method for identifying/detecting and quantifying manipulations of accounting data around psychological thresholds such as zero earnings, zero earnings variation, or earnings forecasted by analysts' and investors' consensus. The method should also allow the analysis and comparison of manipulations across different sub-populations. Note that although the current illustration concerns accounting-related problems and data, the code and the developed methodology can be used in other fields to study behaviors in the presence of psychological threshold effects. More specifically, an EM-type algorithm is proposed to estimate the underlying parameters of the considered model, which is a mixture model (involving an Exponential distribution to model manipulators' behavior, and a mixture of two Gaussian distributions to model the behavior of non-manipulating agents). A goodness-of-fit test for assessing the adequacy of the model to the data is provided. The method also delivers Bootstrap confidence intervals for the estimated model parameters. In addition, several graphical representations are available via the plot function. After evaluating the numerical performance of the methodology on simulated data, the proposed approach is illustrated on real-world data to study earnings management. The various R functions are easy to apply to financial or extra-financial performance benchmarks, or beyond the field of accounting. An R package will be publicly available soon.

Participants: Marie Chavent (ASTRAL), Delphine Féral, Jérôme Saracco (ASTRAL), Véronique Darmendrail, Frédéric Pourtier.

7.1.4 Algorithme de partitionnement en ligne pour une compression incrémentale sans perte des données vibratoires

Lossless data compression is a strategic research area and a major challenge for data storage. In the context of continuous monitoring, data volumes constantly increase, requiring efficient processing. For example, continuous compression for various applications such as predictive maintenance may prove to be an effective strategy. In this work [22] we present an online compression method that divides the signal into several segments in order to minimize a specific criterion based on Shannon entropy weighted by the number of values in each partition. For the compression step, the well-established Huffman coding method is used. Furthermore, our approach introduces an iterative process to enable online monitoring of the compression rate. Thus, for each new acquisition, previously computed results are reused for entropy calculation, which reduces computational cost by avoiding the need to recompute the criterion over the entire signal at each iteration. In this way, the computation time of the proposed threshold value remains below the sampling period, allowing real-time control of the compression rate and guaranteeing that the compressed file does not exceed a maximum size. The existence of an optimal value for the online criterion is theoretically proven. To demonstrate the efficiency of the proposed method, the approach is finally applied to real flight data, enabling an optimal partitioning of the signal with the smallest compressed file size obtained at the end of the process. The results are then compared to a classical Huffman-based method to assess the relevance of the approach.

Participants: Guillaume Cottin, Franck Cazaurang, Jérôme Saracco (ASTRAL), Loïc Lavigne, Vincent Corretja, Benoît Souyri, Franck Tailliez.

7.1.5 Segmentation and lossless compression of SAR data: a new approach to ensure transmission robustness

Efficient lossless data compression is essential in many application areas, such as synthetic aperture radar (SAR) imaging. In addition, controlling data integrity during the transmission process makes data segmentation an attractive option, particularly in critical applications like UAV, because, no preview is available. In this work [23] we present an innovative method for lossless online data segmentation and compression, based on recursive weighted entropy calculations and Huffman coding. First, the image is converted into a 1D signal following a relevant direction inspired by principal component analysis. Next, segmentation is performed by minimizing a weighted Shannon entropy criterion, achieving a slightly higher compression ratio than without segmentation. The approach is validated using opensource SAR satellite images, demonstrating efficient signal partitioning and compression while ensuring a final limit on compressed file size. The proposed method offers a robust solution compatible with sequential transmission and is particularly suited for scenarios requiring reliable data flow.

Participants: Guillaume Cottin, Franck Cazaurang, Jérôme Saracco (ASTRAL), Loïc Lavigne, Vincent Corretja, Benoît Souyri, Franck Tailliez.

7.2 Stochastic learning

7.2.1 On the Particle Approximation of Lagged Feynman-Kac Formulae

In [8], we examine the numerical approximation of the limiting invariant measure associated with Feynman-Kac formulae. These are expressed in a discrete time formulation and are associated with a Markov chain and a potential function. The typical application considered here is the computation of eigenvalues associated with non-negative operators as found, for example, in physics or particle simulation of rare-events. We focus on a novel *lagged* approximation of this invariant measure, based upon the introduction of a ratio of time-averaged Feynman-Kac marginals associated with a positive operator iterated $l \in \mathbb{N}$ times; a lagged Feynman-Kac formula. This estimator and its approximation using Diffusion Monte Carlo (DMC) have been extensively employed in the physics literature. In short, DMC is an iterative algorithm involving $N \in \mathbb{N}$ particles or walkers simulated in parallel, that undergo sampling and resampling operations. In this work, it is shown that for the DMC approximation of the lagged Feynman-Kac formula, one has an almost sure characterization of the $\mathbb{L}_1$-error as the time parameter (iteration) goes to infinity and this is at most of $O(\exp\{-\kappa l\}/N)$, for $\kappa > 0$. In addition a non-asymptotic in time, and time uniform $\mathbb{L}_1-$bound is proved which is $O(l/\sqrt{N})$. We also prove a novel central limit theorem to give a characterization of the exact asymptotic in time variance. This analysis demonstrates that the strategy used in physics, namely, to run DMC with N and l small and, for long time enough, is mathematically justified. Our results also suggest how one should choose N and l in practice. We emphasize that these results are not restricted to physical applications; they have broad relevance to the general problem of particle simulation of the Feynman-Kac formula, which is utilized in a great variety of scientific and engineering fields.

Participants: Elsiddig Awadelkarim, Michel Caffarel, Pierre del Moral (ASTRAL), Ajay Jasra.

7.2.2 On the Mathematical foundations of Diffusion Monte Carlo

The Diffusion Monte Carlo method with constant number of walkers, also called Stochastic Reconfiguration as well as Sequential Monte Carlo, is a widely used Monte Carlo methodology for computing the ground-state energy and wave function of quantum systems. In [9], we present the first mathematically rigorous analysis of this class of stochastic methods on non necessarily compact state spaces, including linear diffusions evolving in quadratic absorbing potentials, yielding what seems to be the first result of this type for this class of models. We present a novel and general mathematical framework with easily checked Lyapunov stability conditions that ensure the uniform-in-time convergence of Diffusion Monte Carlo estimates towards the top of the spectrum of Schrödinger operators. For transient free evolutions, we also present a divergence blow up of the estimates w.r.t. the time horizon even when the asymptotic fluctuation variances are uniformly bounded. We also illustrate the impact of these results in the context of generalized coupled quantum harmonic oscillators with non necessarily reversible nor stable diffusive particle and a quadratic energy absorbing well associated with a semi-definite positive matrix force.

Participants: Michel Caffarel, Pierre del Moral (ASTRAL), Luc de Montella (ASTRAL).

7.2.3 On time uniform Wong-Zakai approximation theorems

In [17], we consider the long time behavior of Wong-Zakai approximations of stochastic differential equations. These piecewise smooth diffusion approximations are of great importance in many areas, such as those with ordinary differential equations associated to random smooth fluctuations; e.g. robust filtering problems. In many examples, the mean error estimate bounds that have been derived in the literature can grow exponentially with respect to the time horizon. We show in a simple example that indeed mean error estimates do explode exponentially in the time parameter, i.e. in that case a Wong-Zakai approximation is only useful for extremely short time intervals. Under spectral conditions, we present some quantitative time-uniform convergence theorems, i.e. time-uniform mean error bounds, yielding what seems to be the first results of this type for Wong-Zakai diffusion approximations.

Participants: Pierre del Moral (ASTRAL), Shulan Hu, Ajay Jasra, Hamza Ruzayqat, Xinyu Wang.

7.2.4 Study of particle methods in nonlinear filtering : application to passive trajectography

This thesis [25] presents a theoretical and applied study of particle filtering methods, with the aim of increasing confidence in their use for critical applications, such as in the military or underwater domains. First, we focus on the Diffusion Monte Carlo (DMC) method, a variant of particle methods used in physics to compute the ground state of quantum systems. We establish assumptions that guarantee the uniform-in-time convergence of this method on non-compact state spaces while ensuring that the conditions remain flexible enough to include Gaussian linear models. This work is the first result of its kind for particle methods. In order to provide a concrete example that satisfies our assumptions and to study the implications of our theorem, we perform a detailed analysis of the coupled harmonic oscillator. This study allows us to highlight cases in which the DMC exhibits asymptotic convergence properties, even though its error diverges for any finite number of particles. This result underscores the importance of establishing uniform-in-time convergence guarantees. Furthermore, we show that this divergence is not inevitable : a modification of DMC can be sufficient to ensure its convergence, thereby opening new perspectives for its application to more complex systems. Following on from our theoretical work, we explore the question of passive trajec- tography, with the aim of improving the performance of particle-based methods. To this end, we propose first to integrate sound pressure level measurements with the angle measurements traditionally used for tracking. Our numerical studies show that this approach significantly improves tracking accuracy. In addition, we demonstrate that combining this new data with a more realistic conical angle measurement and a Doppler frequency shift measurement enhances the robustness and efficiency of our method. This encouraging result opens promising prospects for future applications, but the study presented here is only a first step before the method can be fully operational in real-world conditions. In order to propose an immediately applicable solution, we perform an in-depth analysis of the quality of information in the context of passive bearing-based localization. During this analysis, we study the Fisher information associated with the bearing-based localization problem, as well as the convergence properties of the position estimators. This work leads to the definition of a maneuvering protocol that allows an observer to localize a target with precision and efficiency.

Participants: Luc de Montella (ASTRAL).

7.2.5 On the contraction properties of Sinkhorn semigroups

We develop in [26] a novel semigroup stability analysis based on Lyapunov techniques and contraction coefficients to prove exponential convergence of Sinkhorn equations on weighted Banach spaces. This operator-theoretic framework yields exponential decays of Sinkhorn iterates towards Schrödinger bridges with respect to general classes of Φ-divergences and Kantorovich-type criteria, including the relative entropy, squared Hellinger integrals, α-divergences as well as weighted total variation norms and Wasserstein distances. To the best of our knowledge, these contraction inequalities are the first results of this type in the literature on entropic transport and the Sinkhorn algorithm. We also provide Lyapunov contractions principles under minimal regularity conditions that allow to provide quantitative exponential stability estimates for a large class of Sinkhorn semigroups. We apply this novel framework in a variety of situations, ranging from polynomial growth potentials and heavy tailed marginals on general normed spaces to more sophisticated boundary state space models, including semi-circle transitions, Beta, Weibull, exponential marginals as well as semi-compact models. Last but not least, our approach also allows to consider statistical finite mixture of the above models, including kernel-type density estimators of complex data distributions arising in generative modeling.

Participants: Deniz Akyildiz, Pierre del Moral (ASTRAL), Joaquin Miguez.

7.2.6 Particle Filtering for Non-Deterministic Electrocardiographic Imaging

Electrocardiographic imaging (ECGI) aims to non-invasively reconstruct activation maps of the heart from temporal body surface potentials. While most existing approaches rely on inverse and optimization techniques that may yield satisfactory reconstructions, they typically provide a single deterministic solution, overlooking the inherent uncertainty of the problem stemming from its very ill-posed nature, the poor knowledge of biophysical features and the unavoidable presence of noise in the measurements. The Bayesian framework, which naturally incorporates uncertainty while also accounting for temporal correlations across time steps, can be used to address this limitation. In [30], we propose a low-dimensional representation of the activation sequence that enables the use of particle filtering, a Bayesian filtering method that does not rely on predefined assumptions regarding the shape of the posterior distribution, in contrast to approaches like the Kalman filter. This allows to produce not only activation maps but also probabilistic maps indicating the likelihood of activation at each point on the heart over time, as well as pseudo-probability maps reflecting the likelihood of a point being part of an earliest activation site. Additionally, we introduce a method to estimate the probability of the presence of a conduction lines of block on the heart surface. Combined with classical reconstruction techniques, this could help discriminate artificial from true lines of block in activation maps. We support our approach with a numerical study based on simulated data, demonstrating the potential of our method.

Participants: Emma Lagracie, Luc de Montella (ASTRAL).

7.2.7 Entropic continuity bounds for conditional covariances with applications to Schrödinger and Sinkhorn bridges

In [31], we present new entropic continuity bounds for conditional expectations and conditional covariance matrices. These bounds are expressed in terms of the relative entropy between different coupling distributions. Our approach combines Wasserstein coupling with quadratic transportation cost inequalities. We illustrate the impact of these results in the context of entropic optimal transport problems. The entropic continuity theorem presented in the article allows to estimate the conditional expectations and the conditional covariances of Schrödinger and Sinkhorn transitions in terms of the relative entropy between the corresponding bridges. These entropic continuity bounds turns out to be a very useful tool for obtaining remarkably simple proofs of the exponential decays of the gradient and the Hessian of Schrödinger and Sinkhorn bridge potentials.

Participants: Pierre del Moral (ASTRAL).

7.2.8 On the Kantorovich contraction of Markov semigroups

In [33], we develop a novel operator theoretic framework to study the contraction properties of Markov semigroups with respect to a general class of Kantorovich semi-distances, which notably includes Wasserstein distances. The rather simple contraction cost framework developed in this article, which combines standard Lyapunov techniques with local contraction conditions, helps to unifying and simplifying many arguments in the stability of Markov semigroups, as well as to improve upon some existing results. Our results can be applied to both discrete time and continuous time Markov semigroups, and we illustrate their wide applicability in the context of (i) Markov transitions on models with boundary states, including bounded domains with entrance boundaries, (ii) operator products of a Markov kernel and its adjoint, including two-block-type Gibbs samplers, (iii) iterated random functions and (iv) diffusion models, including overdampted Langevin diffusion with convex at infinity potentials.

Participants: Pierre del Moral (ASTRAL), Mathieu Gerber.

7.2.9 Stability of Schrödinger bridges and Sinkhorn semigroups for log-concave models

In [32] we obtain several new results and developments in the study of entropic optimal transport problems (a.k.a. Schrödinger problems) with general reference distributions and log-concave target marginal measures.

Our approach combines transportation cost inequalities with the theory of Riccati matrix difference equations arising in filtering and optimal control theory. This methodology is partly based on a novel entropic stability of Schrödinger bridges and closed form expressions of a class of discrete time algebraic Riccati equations. In the context of regularized entropic transport these techniques provide new sharp entropic map estimates. When applied to the stability of Sinkhorn semigroups, they also yield a series of novel contraction estimates in terms of the fixed point of Riccati equations. The strength of our approach is that it is applicable to a large class of models arising in machine learning and artificial intelligence algorithms. We illustrate the impact of our results in the context of regularized entropic transport, proximal samplers and diffusion generative models as well as diffusion flow matching models

Participants: Pierre del Moral (ASTRAL).

7.2.10 Target motion analysis using angular measurements and underwater sound pressure levels

In [16] we address the problem of target motion analysis using data acquired by sonars operating in passive mode. Most theoretical and applied studies rely solely on bearing information, and various filtering algorithms, especially particle filter methods, have demonstrated strong performance in this context. To improve upon existing approaches, bearing information is combined with received underwater sound pressure levels. This method requires no additional sensors compared to bearings-only trackings, as hydrophones—already used for beamforming—can also measure sound pressure levels. Additionally, underwater transmission losses can be estimated through simulations based on commonly available environmental data. The effectiveness of the proposed method is demonstrated using simulated data, showing improved performance—especially in scenarios without observer maneuvers. To increase realism, the impact of replacing bearing measurements with conical angle measurements is also investigated. These measurements account for both elevation and bearing angles, thereby providing a fairly comprehensive characterization of bottom-bounce paths commonly encountered under real-world conditions. This enables exploration of scenarios with different immersion levels for both target and observer. The results further emphasize the benefits of incorporating sound pressure levels measurements into the tracking process.

Participants: Enzo Iglésis, Luc de Montella (ASTRAL).

7.3 Stochastic control and games

7.3.1 Non-stationary value iteration for adaptive average control of piecewise deterministic Markov processes

The main goal of [10] is to present a non-stationary value iteration adaptive average control for Piecewise Deterministic Markov Processes (PDMPs), introduced by M.H.A. Davis, as a family of continuous-time Markov processes punctuated by random jumps and with inter-jump movement driven by a deterministic flow. It is assumed in this paper that there are no boundary jumps. We study the adaptive average optimal control problem of PDMPs, considering that the jump intensity λ, the post-jump transition kernel Q, as well as the cost C depend on an unknown parameter β^*. For a sequence of strongly consistent estimators $\{\beta_n^*\}$ of β^* (that is, β_n^* converge to β^* almost surely) a non-stationary value iteration (depending on the current estimate β_n^*) is shown to be optimal for the long-run average control problem. We assume a total variation norm condition on the parameters λ and Q of the process (which generalizes the minorization condition considered in the literature), resulting in a span-contraction operator. The paper concludes with a numerical example.

Participants: O.L.V Costa, François Dufour (ASTRAL), Alexandre Genadot (ASTRAL).

7.3.2 Minimum Contrast Estimators for Piecewise Deterministic Markov Processes

The main goal of this paper [11] is to study the minimum contrast estimator (MCE) approach for the parameter estimation problem of piecewise deterministic Markov processes (PDMPs), associated to adaptive control problems. It is assumed that the control acts continuously on the jump intensity λ and on the transition measure Q of the process, as well as on the costs, and that these parameters depend on an unknown parameter β^*. One of our objective is to introduce a minimum contrast estimator $(\beta_n)_{n\in\mathbb{N}}$ for the family of PDMPs. Sufficient conditions are then presented to ensure that $(\beta_n)_{n\in\mathbb{N}}$ is a strongly consistent estimator of β^*. It should be noticed that PDMPs are characterized by a deterministic motion punctuated by random jumps (either spontaneous or due to the flow touching a boundary), which brings new challenges in the analysis of the problem. The paper is concluded with a numerical example for the adaptive discounted control of PDMPs.

Participants: O.L.V Costa, François Dufour (ASTRAL), Alexandre Genadot (ASTRAL).

7.3.3 Absorbing Markov decision processes and their occupation measures.

In [13], we consider an absorbing Markov decision process with Borel state and action spaces. We study conditions under which the MDP is uniformly absorbing and the set of occupation measures of the MDP is compact in the usual weak topology. These include suitable continuity requirements on the transition kernel and conditions on the dynamics of the system at the boundary of the absorbing set. We generalize previously known results and give an answer to some conjectures that have been mentioned in the related literature.

Participants: François Dufour (ASTRAL), Tomas Prieto-Rumeau.

7.3.4 The bearing only localization problem via partially observed Markov decision process

In [12], We consider the classical problem of localization of a target from an observer from bearing measurements. We reformulate this problem within the framework of the theory of partially observed Markov decision processes and propose a method for numerically solving this problem. Theoretical convergence of this numerical solution scheme is obtained and numerical investigations are also carried out, enabling us to recover optimal curves already suggested in the literature via other techniques.

Participants: François Dufour (ASTRAL), Alexandre Genadot (ASTRAL), Romain Namyst.

7.3.5 Asymptotic optimality of a class of controlled non-Markov processes

Motivated by applications in power systems and problems arising in simulation of large scale complex system optimizations, this work is concerned with controlled stochastic switching systems. The system of interest displays a multi-time scale structure. In contrast to the so-called singularly perturbed diffusions and multi-scale Markov decision processes, controlled non-Markov processes (also known as non-Markov decision processes) are treated. The novelty of this work [14] is the treatment of the non-Markov controlled processes and the time-scale used. The fast and slow processes are coupled through a stochastic differential equation. Using averaging, it is first shown that the non-Markov switching process has a weak limit that is a Markov decision process. Then asymptotic optimal control of the non-Markov process is obtained by using the limit process.

Participants: François Dufour (ASTRAL), Ky Tran, Le Yi Wang, George Yin.

7.3.6 About Fisher Information Matrix and Self-Optimizing Control in Bearing Only Localization

In [29], we revisit the results of Optimal observer motion for localization with bearing measurements from Hammel, Liu, Hilliard, and Gong concerning the lower bound associated with the determinant of the Fisher information matrix for bearing only localization. We specify this bound and use the associated optimality curves to construct an adaptive control for locating the position of a target using only noisy bearing measurements. For this self-optimizing control, the target position estimate is updated using a contrast miminmum estimator whose strong consistency is proved. Numerical simulations allow us to discuss the effectiveness of the proposed method.

Participants: Alexandre Genadot (ASTRAL), Enzo Iglésis (ASTRAL), Luc de Montella (ASTRAL).

7.4 Signal processing, artifical evolution and neural networks

7.4.1 Multiscale entropy rates: a study on different stochastic processes

In [15], we propose to analyze the behavior of the entropy rate (ER) when applied to a signal and its coarse-grained versions. The "multiscale entropy rate" (MSER) is deduced by storing in a vector the resulting ERs. Our contribution consists in studying the MSER calculated on different stochastic processes. When dealing with Gaussian complex or real moving average (MA) processes or autoregressive (AR) processes, which can be seen as the filtering of a white Gaussian driving process, the MSER depends on the variances of the driving processes of the corresponding minimum-phase ARMA process at each scale. More particularly, we derive the analytical expression of the MSER for 1^{st} -order MA or AR processes using different approaches. This study allows us to better understand what each scale brings in and to describe the behavior of the MSER for these types of processes. We also show that there is a mapping between the stochastic-process parameters and the ER computed at different scales. Finally, we show that the multiscale procedure is not relevant for a sum of complex exponentials disturbed by an additive white Gaussian noise.

Participants: Eric Grivel, Pierrick Legrand (ASTRAL), Bastien Berthelot.

7.4.2 Empirical Results for Adjusting Truncated Backpropagation Through Time while Training Neural Audio Effects

In [19] we investigate the optimization of Truncated Backpropagation Through Time (TBPTT) for training neural networks in digital audio effect modeling, with a focus on dynamic range compression. The study evaluates key TBPTT hyperparameters – sequence number, batch size, and sequence length – and their influence on model performance. Using a convolutional-recurrent architecture, we conduct extensive experiments across datasets with and without conditionning by user controls. Results demonstrate that carefully tuning these parameters enhances model accuracy and training stability, while also reducing computational demands. Objective evaluations confirm improved performance with optimized settings, while subjective listening tests indicate that the revised TBPTT configuration maintains high perceptual quality.

Participants: Yann Bourdin (ASTRAL), Pierrick Legrand (ASTRAL), Fanny Roche.

7.4.3 Multiscale cross entropy rate as a way to compare signals: application to Gaussian ARMA processes

The entropy rate of a stochastic process corresponds to the asymptotic difference between the entropies of consecutive sample blocks as their size increases. Widely used in information theory, it also serves as a key marker for signal characterization in classification tasks. Recently, we studied the entropy rate of a signal at different scales using a multiscale approach. The latter generates a set of signals from the original one either (i) by applying a coarse-graining (CG) procedure —where the signal is filtered with an average filter of order equal to the scale and then decimated by a factor equal to the scale— or (ii) by directly decimating the original signal. In [20], we extend the multiscale framework to the cross-entropy rate, introducing the multiscale cross-entropy rate (MCER). MCER can be defined either as the sum of cross-entropy rates across scales or as a vector storing these values. By applying it to Gaussian ARMA processes, we aim to understand the insights provided by the multiscale procedure and to define the influence of the process parameters on the cross-entropy rate at each scale. To this end, we present the properties of ARMA processes after applying the multiscale procedure, provide analytical expressions for the MCER, and outline a practical method for deriving it. The MCER is a potential alternative to multiscale cross-sample entropy and its variants, which have been used in biomedical applications and finance to quantify joint synchrony between signals.

Participants: Eric Grivel, Pierrick Legrand (ASTRAL), Bastien Berthelot.

7.4.4 Time-Varying Audio Effect Modeling by End-to-End Adversarial Training

Deep learning has become a standard approach for the modeling of audio effects, yet strictly black-box modeling remains problematic for time-varying systems. Unlike time-invariant effects, training models on devices with internal modulation typically requires the recording or extraction of control signals to ensure the time-alignment required by standard loss functions. In [27] we introduce a Generative Adversarial Network (GAN) framework to model such effects using only input-output audio recordings, removing the need for modulation signal extraction. We propose a convolutional-recurrent architecture trained via a two-stage strategy: an initial adversarial phase allows the model to learn the distribution of the modulation behavior without strict phase constraints, followed by a supervised fine-tuning phase where a State Prediction Network (SPN) estimates the initial internal states required to synchronize the model with the target. Additionally, a new objective metric based on chirp-train signals is developed to quantify modulation accuracy. Experiments modeling a vintage hardware phaser demonstrate the method's ability to capture time-varying dynamics in a fully black-box context.

Participants: Yann Bourdin (ASTRAL), Pierrick Legrand (ASTRAL), Fanny Roche.

7.4.5 Epidemic Forecasting: Lessons Learned from the SARS-CoV-2 Pandemic to Balance Accuracy, Feasibility, and Impact

The COVID-19 pandemic highlighted the importance of reliable, real-time hospital forecasting. At Bordeaux University Hospital, we developed models to predict SARS-CoV-2-related hospitalizations 14 days in advance using integrated data sources. We identified six key lessons to guide future epidemic response: (1) Multimodal data improves accuracy; (2) Simple baseline models are essential for benchmarking and building trust; (3) Model and metric choices must align with decision goals which often means prioritizing absolute over relative metrics and beginning with simple models; (4) Prediction intervals should be provided to communicate the uncertainty associated with forecasts; (5) Real-world constraints such as computational cost, maintainability, and required expertise should guide model selection; (6) Forecasts must be contextualized and commu- nicated carefully to policymakers. In [28] we advocate for a systems-level forecasting approach that balances accuracy, feasibility, and impact.

Participants: Thomas Ferté, Vincent Thevenet, Xavier Hinaut, Pierrick Legrand (ASTRAL), Dan Dutartre, Romain Griffier, Viannet Jouhet, Boris Hejblum, Rodolphe Thiébaut.

7.4.6 L'IA se met au rock.

Advances in computing power and AI now make it possible to faithfully reproduce the sound of the legendary tube amplifiers that delighted generations of guitarists. In this paper [34], published in french, we examine how these digital breakthroughs are already reshaping the landscape of music.

Participants: Hugo Leroux, Tara Vanhatalo (ASTRAL), Pierrick Legrand (ASTRAL).

8 Bilateral contracts and grants with industry

8.1 Bilateral contracts with industry

Naval Group

Participants: Denis Arrivault, Pierre del Moral, François Dufour, Alexandre Genadot, Enzo Iglésis, Dann Laneuville, Adrien Nègre.

In the application domain, an important research focus of the team is the tracking of passive underwater targets in the context of passive underwater acoustic warfare. This is a very complicated practical problem that combines both filtering and stochastic control issues. This research topic is addressed in collaboration with Naval Group. We refer the reader to the section 4.1 for a more detailed description of this theme.

Thales AVS

Participants: Bastien Berthelot, Pierrick Legrand.

The collaboration is centered around some contributions to the estimation of the Hurst coefficient and his application on biosignals in the domain of crew monitoring.

Case Law Analytics

Participants: Pierrick Legrand.

Pierrick Legrand is a consultant for the startup Pyxiscience. The object of the consulting is confidential.

Thales DMS

Participants: Jérôme Saracco.

The aim of this collaboration is to develop lossless data compression methodologies for vibration data from Rafale flights and SAR radar images.

It has resulted in a patent entitled "Procédé de prise de décision rapide utilisant une décomposition en plans de bits pour une compression sans perte des images radar".

The results obtained were presented at two conferences (DASC 2025 and SAGIP 2025).

8.2 Bilateral Grants with Industry

Participants: Denis Arrivault, Pierre del Moral, Francois Dufour, Alexandre Genadot, Enzo Iglesis, Pierrick Legrand, Marie Chavent, Jérôme Saracco.

Orosys

Participants: Tara Vanhatalo, Pierrick Legrand.

Within the framework of Tara Vanhatalo's Cifre PhD thesis on the stochastic modeling of guitar amplifiers, a strong collaboration was established between the company Orosys and the ASTRAL team.

Arturia

Participants: Yann Bourdin, Pierrick Legrand.

Within the framework of Yann Bourdin's Cifre PhD thesis on the stochastic modeling of audio compressor and nonlinear audio effects, a strong collaboration was established between the company Arturia and the ASTRAL team.

9 Partnerships and cooperations

Participants: Denis Arrivault, Pierre del Moral, Francois Dufour, Alexandre Genadot, Enzo Iglesis, Pierrick Legrand, Marie Chavent, Jérôme Saracco.

9.1 International initiatives

9.1.1 Participation in other International Programs

Scientific cooperation with Spain funded by the Spanish Ministry of Science and Innovation (reference number PID2021-122442NB-I00). This project focuses on the analysis and control of deterministic/stochastic dynamic systems and on game theory (2022-2025).

Participants: Francois Dufour.

9.2 International research visitors

9.2.1 Visits of international scientists

Other international visits to the team

Prof Oswaldo Costa

Status researcher

Institution of origin: Escola Politecnica da Universidade Sao-Paulo.

Country: Brasil

Dates: July 2025

Context of the visit: Research collaboration

Mobility program/type of mobility: research stay

9.3 National initiatives

Naval Group Astral is a joint INRIA team project with Naval Group. The topic of this collaboration is described in section 4.1.

QuAMProcs of the program *Project Blanc* of the ANR The mathematical analysis of metastable processes started 75 years ago with the seminal works of Kramers on Fokker-Planck equation. Although the original motivation of Kramers was to « elucidate some points in the theory of the velocity of chemical reactions », it turns out that Kramers' law is observed to hold in many scientific fields: molecular biology (molecular dynamics), economics (modelization of financial bubbles), climate modeling, etc. Moreover, several widely used efficient numerical methods are justified by the mathematical description of this phenomenon.
Recently, the theory has witnessed some spectacular progress thanks to the insight of new tools coming from Spectral and Partial Differential Equations theory.
Semiclassical methods together with spectral analysis of Witten Laplacian gave very precise results on reversible processes. From a theoretical point of view, the semiclassical approach allowed to prove a complete asymptotic expansion of the small eigen values of Witten Laplacian in various situations (global problems, boundary problems, degenerate diffusions, etc.). The interest in the analysis of boundary problems was rejuvenated by recent works establishing links between the Dirichlet problem on a bounded domain and the analysis of exit event of the domain. These results open numerous perspectives of applications. Recent progress also occurred on the analysis of irreversible processes (e.g. on overdamped Langevin equation in irreversible context or full (inertial) Langevin equation).
The above progresses pave the way for several research tracks motivating our project: overdamped Langevin equations in degenerate situations, general boundary problems in reversible and irreversible case, non-local problems, etc.

10 Dissemination

Participants: Denis Arrivault, Marie Chavent, Pierre del Moral, François Dufour, Alexandre Genadot, Enzo Iglésis, Pierrick Legrand, Jérôme Saracco.

10.1 Promoting scientific activities

Participants: Denis Arrivault, Marie Chavent, Pierre del Moral, François Dufour, Alexandre Genadot, Enzo Iglésis, Pierrick Legrand, Jérôme Saracco.

10.1.1 Journal

Member of the editorial boards

- Pierrick legrand is a Board Member for the journal.
- Pierrick Legrand is the main editor of the LNCS volumes artificial evolution. Genetic Programming and Evolvable Machines

- François Dufour is a Corresponding Editor for the journal *SIAM Journal on Control Optimization* (SIAM-SICON) since April 2018.
- François Dufour is an Associate Editor for the journal *Applied Mathematics and Optimization* (AMO) since January 2018.
- François Dufour is an Associate Editor for the journal *Stochastics: An International Journal of Probability and Stochastic Processes* since July 2018.
- François Dufour is an Associate Editor for the journal *Mathematical Control and Related Fields* (Math. Control Related Fields) since January 2023.

Reviewer - reviewing activities Each year, the members of the ASTRAL team review articles submitted to international journals and conferences.

10.2 Teaching - Supervision - Juries - Educational and pedagogical outreach

Participants: Denis Arrivault, Marie Chavent, Pierre del Moral, François Dufour, Alexandre Genadot, Enzo Iglésis, Pierrick Legrand, Jérôme Saracco.

10.2.1 Teaching

- J. Saracco is the head of the engineering department of ENSC, Graduate School of Cognitics (applied cognitive science and technology), which is a Bordeaux INP engineering school.
- Alexandre Genadot is the head of the MIASHS Licence of the Université de Bordeaux.
- Pierrick Legrand is in charge of the IBM Chair at ENSC.
- Licence: P. Legrand, *Espaces Euclidiens*, 46.5h, L2, Université de Bordeaux, France.
- Licence: P. Legrand, *Informatique pour les mathématiques*, 30h, L3, Université de Bordeaux, France.
- DU: P. Legrand, *Évolution Artificielle, Big Data*, 8h, DU, Bordeaux INP, France.
- Engineer School: *Signal Processing*, ENSC, Bordeaux, 1A, France.
- Engineer School: *Signal Processing*, 54 hours, ENSC, Bordeaux, 2A, France.
- Master: Scientific courses, 10 hours, Université de Bordeaux, France.
- Licence: A. Genadot, *Bases en Probabilités*, 18h, L1, Université de Bordeaux, France.
- Licence: A. Genadot, *Projet Professionnel de l'Étudiant*, 8h, L1, Université de Bordeaux, France.
- Licence: A. Genadot, *Probabilité*, 30h, L2, Université de Bordeaux, France.
- Licence: A. Genadot, *Techniques d'Enquêtes*, 10h, L2, Université de Bordeaux, France.
- Licence: A. Genadot, *Modélisation Statistique*, 16.5h, L3, Université de Bordeaux, France.
- Licence: A. Genadot, *Préparation Stage*, 15h, L3, Université de Bordeaux, France.
- Licence: A. Genadot, *TER*, 5h, L3, Université de Bordeaux, France.
- Licence: A. Genadot, *Processus*, 16.5h, L3, Université de Bordeaux, France.
- Licence: A. Genadot, *Statistiques*, 20h, L3, Bordeaux INP, France.
- Master: A. Genadot, *Savoirs Mathématiques*, 81h, M1, Université de Bordeaux and ESPE, France.

- Master: A. Genadot, *Martingales*, 29h, M1, Université de Bordeaux, France.
- Licence: F. Dufour, *Probabilités et Statistiques*, 70h, first year of ENSEIRB-MATMECA engineering school, Bordeaux INP, France.
- Master: F. Dufour, *Probabilistic Approach and Monte Carlo Methods*, 24h, third year of ENSEIRB-MATMECA engineering school, Bordeaux INP, France.
- Licence: J. Saracco, *Probabilités et Statistique*, 27h, first year of ENSC Graduate School of Engineering, Bordeaux INP, France.
- Licence: J. Saracco, *Statistique inférentielle et analyse des données*, 45h, first year of ENSC Graduate School of Engineering, Bordeaux INP, Institut Polytechnique de Bordeaux, France.
- Licence: J. Saracco, *Statistique pour l'ingénieur*, 16h, first year of ENSPIMA Graduate School of Engineering, Bordeaux INP, Institut Polytechnique de Bordeaux, France.
- Master: J. Saracco, *Modélisation statistique*, 81h, second year of ENSC Graduate School of Engineering, Bordeaux INP, Institut Polytechnique de Bordeaux, France.
- DU: J. Saracco, *Statistique et Big Data*, 45h, DU BDSI (Big Data et Statistique pour l'Ingénieur), Bordeaux INP, France.
- Licence: M. Chavent, *Statistique inférentielle*, 18h, L2, Université de Bordeaux, France.
- Licence: M. Chavent, *Techniques d'Enquêtes*, 10h, L2, Université de Bordeaux, France.
- Master: M. Chavent, *Data Mining*, 43h, M2, Université de Bordeaux, France.
- Master: M. Chavent, *Machine Learning*, 58h, Université de Bordeaux, France.
- DU: M. Chavent, *Apprentissage*, 12h, DU BDSI, Bordeaux INP, France.

10.2.2 Supervision

- The members of the team have been involved in the supervision of various Master's internships, final-year projects (PFE), individual computer science projects, etc.
- The members of the team supervise PhD theses, Master theses and engineers internships

10.2.3 Juries

- Alexandre Génadot served as a reviewer for Orlane Rossini's PhD thesis entitled "Model-based reinforcement learning for the control of partially observable piecewise deterministic semi-Markov decision processes", which was defended on November 28, 2025, at the University of Montpellier II.
- Jérôme Saracco was chair of the thesis committee for Daphné AUROUET's PhD thesis entitled Predictive recursive nonparametric methods for modelling banknotes circulation, defended at ENSAI in Rennes in October 2025.
- Jérôme Saracco was chair of the thesis committee for Valentin Portmann's PhD thesis entitled "Développement de nouveaux algorithmes d'apprentissage statistique pour coupler projections climatiques et observations passées en vue de réduire les incertitudes du changement climatique à venir" defended at Bordeaux University in November 2025.
- Jérôme Saracco was a member of the jury for the John ALBECHAALANY's PhD thesis, entitled "Modulation et optimisation des pratiques d'élevage pour améliorer la qualité de la viande bovine et de volaille," defended in March 2025 at INRAE in Clermont-Ferrand, as co-supervisor of the thesis work.
- Pierrick Legrand was chair of the thesis committee for Luc De Montella's entitled "Etude de méthodes particulaires en filtrage non linéaire : application à la trajectographie passive" defended in May 2025 at Bordeaux University.

- Pierrick Legrand was chair of the thesis committee for Alexis Boffet's PhD thesis entiled "Évaluation de la charge mentale du pilote en condition opérationnelle induisant des niveaux élevés de vigilance et/ou stress par intégration du bio signal d'activité électrodermale dans un système intégrateur multi signaux" defended in November 2025 at Bordeaux University.
- Pierrick Legrand was chair of the thesis committee for Zheng Fang's PhD thesis entiled "Study of functional changes in cerebral networks associated with cognitive control dysfunctions in Parkinson's disease" defended in June 2025 at Rennes University.

11 Scientific production

11.1 Major publications

[1] E. Awadelkarim, M. Caffarel, P. D. Moral and A. Jasra. 'On the Particle Approximation of Lagged Feynman-Kac Formulae'. In: *Stochastic Processes and their Applications* 188 (Oct. 2025), p. 104690. DOI: 10.1016/j.spa.2025.104690. URL: https://hal.science/hal-04705748.

[2] Y. Bourdin, P. Legrand and F. Roche. 'Empirical Results for Adjusting Truncated Backpropagation Through Time while Training Neural Audio Effects'. In: DAFx 2025 - 28th International Conference on Digital Audio Effects. Ancona, Italy, 2nd Sept. 2025. URL: https://inria.hal.science/hal-05394988.

[3] G. Cottin, F. Cazaurang, J. Saracco, L. Lavigne, V. Corretja, B. Souyri and F. Tailliez. 'Segmentation and lossless compression of SAR data: a new approach to ensure transmission robustness'. In: 44th Digital Avionics Systems Conference (DASC 2025). Montréal (Québec), Canada, 14th Sept. 2025. URL: https://hal.inrae.fr/hal-05466867.

[4] F. Dufour, A. Genadot and O. L. D. V. Costa. 'Minimum Contrast Estimators for Piecewise Deterministic Markov Processes'. In: *Numerical Algebra, Control and Optimization* 15.1 (2025), pp. 1–14. DOI: 10.3934/naco.2024023. URL: https://hal.science/hal-04337828.

[5] E. Grivel, P. Legrand and B. Berthelot. 'Multiscale entropy rates: a study on different stochastic processes'. In: *Digital Signal Processing* (9th May 2025). DOI: 10.1016/j.dsp.2025.105303. URL: https://hal.science/hal-05081514.

[6] E. Iglésis and L. de Montella. 'Target motion analysis using angular measurements and underwater sound pressure levels'. In: *Journal of the Acoustical Society of America* 158.3 (25th Sept. 2025), pp. 2590–2601. DOI: 10.1121/10.0039424. URL: https://inria.hal.science/hal-04796331.

[7] V. Portmann, M. Chavent and D. Swingedouw. 'ClimLoco1.0: CLimate variable confidence Interval of Multivariate Linear Observational COnstraint'. In: *Geoscientific Model Development* 18.22 (25th Nov. 2025), pp. 9015–9038. DOI: 10.5194/gmd-18-9015-2025. URL: https://inria.hal.science/hal-05448012.

11.2 Publications of the year

International journals

[8] E. Awadelkarim, M. Caffarel, P. D. Moral and A. Jasra. 'On the Particle Approximation of Lagged Feynman-Kac Formulae'. In: *Stochastic Processes and their Applications* 188 (Oct. 2025), p. 104690. DOI: 10.1016/j.spa.2025.104690. URL: https://hal.science/hal-04705748 (cit. on p. 21).

[9] M. Caffarel, P. del Moral and L. de Montella. 'On the Mathematical foundations of Diffusion Monte Carlo'. In: *Journal of Mathematical Physics* 66.1 (Jan. 2025), p. 013301. DOI: 10.1063/5.0202800. URL: https://hal.science/hal-04409602 (cit. on p. 22).

[10] O. Costa, F. Dufour and A. Genadot. 'Non-stationary value iteration for adaptive average control of piecewise deterministic Markov processes'. In: *Nonlinear Analysis: Hybrid Systems* 58 (Nov. 2025), p. 101622. DOI: 10.1016/j.nahs.2025.101622. URL: https://hal.science/hal-05441485 (cit. on p. 25).

[11] F. Dufour, A. Genadot and O. L. D. V. Costa. 'Minimum Contrast Estimators for Piecewise Deterministic Markov Processes'. In: *Numerical Algebra, Control and Optimization* 15.1 (2025), pp. 1–14. DOI: 10.3934/naco.2024023. URL: https://hal.science/hal-04337828 (cit. on p. 26).

[12] F. Dufour, A. Genadot and R. Namyst. 'The bearing only localization problem via partially observed Markov decision process'. In: *Mathematical Methods of Operations Research* 101.2 (4th Mar. 2025), pp. 219–257. DOI: 10.1007/s00186-025-00890-7. URL: https://hal.science/hal-05441419 (cit. on p. 26).

[13] F. Dufour and T. Prieto-Rumeau. 'Absorbing Markov decision processes and their occupation measures'. In: *SIAM Journal on Control and Optimization* 63.1 (2025), pp. 676–698. DOI: 10.1137/24M1661959. URL: https://hal.science/hal-04876371. In press (cit. on p. 26).

[14] F. Dufour, K. Tran, L. Y. Wang and G. Yin. 'Asymptotic optimality of a class of controlled non-Markov processes'. In: *Applicable Analysis* 104.11 (2025), pp. 2005–2017. DOI: 10.1080/00036811.2024.2368078. URL: https://hal.science/hal-05441461 (cit. on p. 26).

[15] E. Grivel, P. Legrand and B. Berthelot. 'Multiscale entropy rates: a study on different stochastic processes'. In: *Digital Signal Processing* (9th May 2025). DOI: 10.1016/j.dsp.2025.105303. URL: https://hal.science/hal-05081514 (cit. on p. 27).

[16] E. Iglésis and L. de Montella. 'Target motion analysis using angular measurements and underwater sound pressure levels'. In: *Journal of the Acoustical Society of America* 158.3 (25th Sept. 2025), pp. 2590–2601. DOI: 10.1121/10.0039424. URL: https://inria.hal.science/hal-04796331 (cit. on p. 25).

[17] P. del Moral, S. Hu, A. Jasra, H. Ruzayqat and X. Wang. 'On time uniform Wong-Zakai approximation theorems'. In: *Mathematics of Computation* 66.1 (16th June 2025). DOI: 10.1090/mcom/4112. URL: https://inria.hal.science/hal-05441306 (cit. on p. 22).

[18] V. Portmann, M. Chavent and D. Swingedouw. 'ClimLoco1.0: CLimate variable confidence Interval of Multivariate Linear Observational COnstraint'. In: *Geoscientific Model Development* 18.22 (25th Nov. 2025), pp. 9015–9038. DOI: 10.5194/gmd-18-9015-2025. URL: https://inria.hal.science/hal-05448012 (cit. on p. 20).

International peer-reviewed conferences

[19] Y. Bourdin, P. Legrand and F. Roche. 'Empirical Results for Adjusting Truncated Backpropagation Through Time while Training Neural Audio Effects'. In: DAFx 2025 - 28th International Conference on Digital Audio Effects. Ancona, Italy, 2nd Sept. 2025. URL: https://inria.hal.science/hal-05394988 (cit. on p. 27).

[20] E. Grivel, P. Legrand and B. Berthelot. 'Multiscale cross entropy rate as a way to compare signals: application to Gaussian ARMA processes'. In: *2025 33rd European Signal Processing Conference (EUSIPCO)*. EUSIPCO 2025 - 33rd European Signal Processing Conference. Palerme, Italy, 8th Sept. 2025. URL: https://hal.science/hal-05253965 (cit. on p. 28).

Conferences without proceedings

[21] M. Chavent, D. Féral, J. Saracco, V. Darmendrail and F. Pourtier. 'Détection et quantification de manipulations comptables autour d'un seuil psychologique avec R'. In: 11èmes Rencontres R. Mons, Belgium, 19th May 2025. URL: https://hal.science/hal-05466933 (cit. on p. 20).

[22] G. Cottin, F. Cazaurang, J. Saracco, L. Lavigne, V. Corretja, B. Souyri and F. Tailliez. 'Algorithme de partitionnement en ligne pour une compression incrémentale sans perte des données vibratoires'. In: 3ème congrès annuel de la SAGIP (SAGIP 2025). Mulhouse (FR), France, 21st May 2025. URL: https://hal.inrae.fr/hal-05466886 (cit. on p. 21).

[23] G. Cottin, F. Cazaurang, J. Saracco, L. Lavigne, V. Corretja, B. Souyri and F. Tailliez. 'Segmentation and lossless compression of SAR data: a new approach to ensure transmission robustness'. In: 44th Digital Avionics Systems Conference (DASC 2025). Montréal (Québec), Canada, 14th Sept. 2025. URL: https://hal.inrae.fr/hal-05466867 (cit. on pp. 17, 21).

Doctoral dissertations and habilitation theses

[24] M. Dupuy. 'Analysis of electrical features for detection of subjects at risk for sudden cardiac death'. Université de Bordeaux, 14th Jan. 2025. URL: https://theses.hal.science/tel-04953524 (cit. on p. 19).

[25] L. de Montella. 'Study of particle methods in nonlinear filtering : application to passive trajectography'. Université de Bordeaux, 28th May 2025. URL: https://theses.hal.science/tel-05164859 (cit. on p. 23).

Reports & preprints

[26] O. D. Akyildiz, P. del Moral and J. Miguez. *On the contraction properties of Sinkhorn semigroups*. 2025. URL: https://inria.hal.science/hal-05050312 (cit. on p. 23).

[27] Y. Bourdin, P. Legrand and F. Roche. *Time-Varying Audio Effect Modeling by End-to-End Adversarial Training*. 12th Dec. 2025. URL: https://inria.hal.science/hal-05413743 (cit. on p. 28).

[28] T. Ferté, V. Thevenet, X. Hinaut, P. Legrand, D. Dutartre, R. Griffier, V. Jouhet, B. P. Hejblum and R. Thiébaut. *Epidemic Forecasting: Lessons Learned from the SARS-CoV-2 Pandemic to Balance Accuracy, Feasibility, and Impact*. 6th Nov. 2025. DOI: 10.1101/2025.11.03.25339385. URL: https://hal.science/hal-05392781 (cit. on p. 28).

[29] A. Genadot, E. Iglésis and L. de Montella. *About Fisher Information Matrix and Self-Optimizing Control in Bearing Only Localization*. 17th Jan. 2025. URL: https://inria.hal.science/hal-04894826 (cit. on p. 27).

[30] E. Lagracie and L. de Montella. *Particle Filtering for Non-Deterministic Electrocardiographic Imaging*. 22nd Sept. 2025. URL: https://inria.hal.science/hal-05272068 (cit. on p. 24).

[31] P. del Moral. *Entropic continuity bounds for conditional covariances with applications to Schrödinger and Sinkhorn bridges*. 2025. URL: https://inria.hal.science/hal-05050314 (cit. on p. 24).

[32] P. del Moral. *Stability of Schrödinger bridges and Sinkhorn semigroups for log-concave models*. 2025. URL: https://inria.hal.science/hal-05050315 (cit. on p. 24).

[33] P. del Moral and M. Gerber. *On the Kantorovich contraction of Markov semigroups*. 2025. URL: https://inria.hal.science/hal-05441283 (cit. on p. 24).

Other scientific publications

[34] H. Leroux, T. Vanhatalo and P. Legrand. *L'IA se met au rock*. 26th Mar. 2025. URL: https://inria.hal.science/hal-05464131 (cit. on p. 29).

11.3 Cited publications

[35] E. Altman. *Constrained Markov decision processes*. Stochastic Modeling. Chapman & Hall/CRC, Boca Raton, FL, 1999, pp. xii+242 (cit. on p. 13).

[36] N. Bäuerle and U. Rieder. *Markov decision processes with applications to finance*. Universitext. Springer, Heidelberg, 2011, pp. xvi+388. DOI: 10.1007/978-3-642-18324-9. URL: https://doi.org/10.1007/978-3-642-18324-9 (cit. on p. 14).

[37] D. P. Bertsekas and S. E. Shreve. *Stochastic optimal control: The discrete time case*. Vol. 139. Mathematics in Science and Engineering. New York: Academic Press Inc., 1978, pp. xiii+323 (cit. on p. 13).

[38] O. L. d. V. Costa and F. Dufour. *Continuous average control of piecewise deterministic Markov processes*. SpringerBriefs in Mathematics. Springer, New York, 2013, pp. xii+116. DOI: 10.1007/978-1-4614-6983-4. URL: https://doi.org/10.1007/978-1-4614-6983-4 (cit. on p. 14).

[39] M. H. A. Davis. *Markov models and optimization*. Vol. 49. Monographs on Statistics and Applied Probability. Chapman & Hall, London, 1993, pp. xiv+295. DOI: 10.1007/978-1-4899-4483-2. URL: http://dx.doi.org/10.1007/978-1-4899-4483-2 (cit. on p. 14).

[40] P. Del Moral. *Genealogical and interacting particle systems with applications*. Probability and its Applications. Springer-Verlag, New York, 2004, p. 573 (cit. on pp. 11, 13).

[41] P. Del Moral. *Mean field simulation for Monte Carlo integration*. Monographs on Statistics and Applied Probability. Chapman and Hall, 2013. URL: http://www.crcpress.com/product/isbn/9781466504059 (cit. on pp. 11, 13).

[42] P. Del Moral, A. Doucet and J. A. 'Sequential Monte Carlo samplers'. In: Journal of the Royal Statistical Society: Series B (Statistical Methodology) 68.3 (2006), pp. 411–436 (cit. on p. 11).

[43] P. Del Moral and A. Guionnet. 'On the stability of interacting processes with applications to filtering and genetic algorithms'. In: Annales de l'Institut Henri Poincaré 37.2 (2001), pp. 155–194 (cit. on p. 11).

[44] P. Del Moral and A. Guionnet. 'On the stability of Measure Valued Processes with Applications to filtering'. In: C.R. Acad. Sci. Paris, t. 329, Serie I (1999), pp. 429–434 (cit. on pp. 11, 13).

[45] P. Del Moral, J. Jacod and P. Protter. 'The Monte-Carlo method for filtering with discrete-time observations'. In: Probability Theory and Related Fields 120.3 (2001), pp. 346–368 (cit. on pp. 11, 13).

[46] P. Del Moral and L. Miclo. *Branching and Interacting Particle Systems Approximations of Feynman-Kac Formulae with Applications to Non-Linear Filtering*. Vol. 1729. Séminaire de Probabilités XXXIV. Ed. J. Azéma et al., 2000, pp. 1–145 (cit. on pp. 11, 13).

[47] P. Del Moral and S. Penev. *Stochastic Processes: From Applications to Theory*. Chapman and Hall/CRC, 2017 (cit. on pp. 11, 13).

[48] P. Del Moral and J. Tugaut. 'On the stability and the uniform propagation of chaos properties of ensemble Kalman-Bucy filters'. In: Annals of Applied Probability 28.2 (2018), pp. 790–850 (cit. on p. 11).

[49] P. Del Moral. 'Non Linear Filtering: Interacting Particle Solution'. In: Markov Processes and Related Fields 2.4 (1996), pp. 555–580 (cit. on pp. 11, 13).

[50] E. Dynkin and A. Yushkevich. *Controlled Markov processes*. Vol. 235. Grundlehren der Mathematischen Wissenschaften. Berlin: Springer-Verlag, 1979, pp. xvii+289 (cit. on p. 13).

[51] J. Filar and K. Vrieze. *Competitive Markov decision processes*. New York: Springer-Verlag, 1997, pp. xii+393 (cit. on p. 13).

[52] X. Guo and O. Hernández-Lerma. *Continuous-time Markov decision processes*. Vol. 62. Stochastic Modelling and Applied Probability. Theory and applications. Springer-Verlag, Berlin, 2009, pp. xviii+231. DOI: 10.1007/978-3-642-02547-1. URL: https://doi.org/10.1007/978-3-642-02547-1 (cit. on p. 14).

[53] O. Hernández-Lerma. *Adaptive Markov control processes*. Vol. 79. Applied Mathematical Sciences. New York: Springer-Verlag, 1989, pp. xiv+148 (cit. on p. 13).

[54] O. Hernández-Lerma and J.-B. Lasserre. *Discrete-time Markov control processes: Basic optimality criteria*. Vol. 30. Applications of Mathematics. New York: Springer-Verlag, 1996, pp. xiv+216 (cit. on p. 13).

[55] O. Hernández-Lerma and J.-B. Lasserre. *Further topics on discrete-time Markov control processes*. Vol. 42. Applications of Mathematics. New York: Springer-Verlag, 1999, pp. xiv+276 (cit. on p. 13).

[56] K. Hinderer. *Foundations of non-stationary dynamic programming with discrete time parameter*. Lecture Notes in Operations Research and Mathematical Systems, Vol. 33. Springer-Verlag, Berlin-New York, 1970, pp. vi+160 (cit. on p. 13).

[57] A. Hordijk and F. van der Duyn Schouten. 'Markov decision drift processes: conditions for optimality obtained by discretization'. In: *Math. Oper. Res.* 10.1 (1985), pp. 160–173. DOI: 10.1287/moor.10.1.160. URL: https://doi.org/10.1287/moor.10.1.160 (cit. on p. 14).

[58] A. Hordijk and F. A. van der Duyn Schouten. 'Discretization and weak convergence in Markov decision drift processes'. In: *Math. Oper. Res.* 9.1 (1984), pp. 112–141. DOI: 10.1287/moor.9.1.112. URL: http://dx.doi.org/10.1287/moor.9.1.112 (cit. on p. 14).

[59] V. kolokoltsov. 'Nonlinear Markov Processes and Kinetic Equations'. In: Cambridge Univ. Press (2010) (cit. on p. 11).

[60] A. B. Piunovskiy. *Examples in Markov decision processes*. Vol. 2. Imperial College Press Optimization Series. Imperial College Press, London, 2013, pp. xiv+293 (cit. on p. 13).

[61] A. B. Piunovskiy. *Optimal control of random sequences in problems with constraints*. Vol. 410. Mathematics and its Applications. With a preface by V. B. Kolmanovskii and A. N. Shiryaev. Kluwer Academic Publishers, Dordrecht, 1997, pp. xii+345. DOI: 10.1007/978-94-011-5508-3. URL: https://doi.org/10.1007/978-94-011-5508-3 (cit. on p. 13).

[62] T. Prieto-Rumeau and O. Hernández-Lerma. *Selected topics on continuous-time controlled Markov chains and Markov games*. Vol. 5. ICP Advanced Texts in Mathematics. Imperial College Press, London, 2012, pp. xii+279. DOI: 10.1142/p829. URL: https://doi.org/10.1142/p829 (cit. on p. 14).

[63] M. Puterman. *Markov decision processes: discrete stochastic dynamic programming*. Wiley Series in Probability and Mathematical Statistics: Applied Probability and Statistics. A Wiley-Interscience Publication. New York: John Wiley & Sons Inc., 1994, pp. xx+649 (cit. on p. 13).