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2025 *Activity Report*

RESEARCH CENTRE: **Inria Centre at Université Grenoble Alpes**

Project-Team

ELAN

modELing the Appearance of Nonlinear phenomena

In collaboration with Laboratoire Jean Kuntzmann (LJK)

Project-Team ELAN

Creation of the Project-Team: 2019 August 01

Each year, Inria research teams publish an Activity Report presenting their work and results over the reporting period. These reports follow a common structure, with some optional sections depending on the specific team. They typically begin by outlining the overall objectives and research programme, including the main research themes, goals, and methodological approaches. They also describe the application domains targeted by the team, highlighting the scientific or societal contexts in which their work is situated. The reports then present the highlights of the year, covering major scientific achievements, software developments, or teaching contributions. When relevant, they include sections on software, platforms, and open data, detailing the tools developed and how they are shared. A substantial part is dedicated to new results, where scientific contributions are described in detail, often with subsections specifying participants and associated keywords. Finally, the Activity Report addresses funding, contracts, partnerships, and collaborations at various levels, from industrial agreements to international cooperations. It also covers dissemination and teaching activities, such as participation in scientific events, outreach, and supervision. The document concludes with a presentation of scientific production, including major publications and those produced during the year.

Keywords

Computer sciences and digital sciences

A2.5. – Software engineering

A5.5.4. – Animation

A6.1.1. – Continuous Modeling (PDE, ODE)

A6.1.4. – Multiscale modeling

A6.1.5. – Multiphysics modeling

A6.2.1. – Numerical analysis of PDE and ODE

A6.2.5. – Numerical Linear Algebra

A6.2.6. – Optimization

A6.2.7. – HPC for machine learning

A6.2.8. – Computational geometry and meshes

A6.3.1. – Inverse problems

A6.5. – Mathematical modeling for physical sciences

A6.5.1. – Solid mechanics

A6.5.2. – Fluid mechanics

A6.5.3. – Transport

Other research topics and application domains

B1.1.2. – Molecular and cellular biology

B3.3.1. – Earth and subsoil

B5.5. – Materials

B9.2.2. – Cinema, Television

B9.5.3. – Physics

B9.5.5. – Mechanics

Contents

1 Team members, visitors, external collaborators

Research Scientists

- Florence Descoubes [Team leader, INRIA, Senior Researcher, HDR]
- Thibaut Metivet [INRIA, Researcher]
- Victor Romero Gramegna [INRIA, ISFP]

PhD Students

- Emile Hohnadel [INRIA, until Aug 2025]
- Sergio Murillo Garcia [SORBONNE UNIVERSITE, from Oct 2025]

Technical Staff

- Octave Crespel [INRIA, Engineer, until Aug 2025]
- Emile Hohnadel [INRIA, Engineer, from Sep 2025]
- Arun Kumar [INRIA, Engineer, until Jul 2025]
- Alice Teixeira Da Silva [INRIA, Engineer, from Feb 2025 until Apr 2025]

Interns and Apprentices

- Sergio Murillo Garcia [LJK, Intern, from Feb 2025 until Jul 2025]
- Brice Peres [INRIA, Intern, from Sep 2025]
- Dimitri Sircat [LJK, Intern, from May 2025 until Nov 2025]

Administrative Assistant

- Julia Di Toro [INRIA]

Visiting Scientist

- David Breen [Univ Drexel , from Feb 2025 until Jul 2025]

2 Overall objectives

Elan is a modelling research team of Inria and Laboratoire Jean Kuntzmann (UMR 5224), with an original positioning across Computer Graphics and Computational Mechanics. The team is focussed on the design of predictive, robust, efficient, and controllable numerical models for capturing the shape and motion of visually rich mechanical phenomena, such as the buckling of an elastic ribbon, the flowing of sand, or the entangling of large fiber assemblies. Target applications encompass the digital entertainment industry (e.g., feature film animation, special effects), as well as virtual prototyping for the mechanical engineering industry (e.g., direct and inverse design of textiles and metamaterials, sport performance optimisation, cosmetology); though very different, these two application fields require predictive and scalable models for capturing complex mechanical phenomena at the macroscopic scale. An orthogonal objective is the improvement of our understanding of natural physical and biological processes involving slender structures and frictional contact, through active collaborations with soft matter physicists. To achieve its goals, the team strives to master as finely as possible the entire modeling pipeline, involving a pluridisciplinary combination of scientific skills across Mechanics and Physics, Applied Mathematics, and Computer Science.

3 Research program

Thanks to an original and transverse positioning across Computer Graphics and Computational Mechanics, complemented by tight connections with physicists, our goal is to tackle some challenging numerical modelling issues related to complex macroscopic phenomena characterised by a nonlinear mechanical behaviour and rich geometrical deformations. One major ambition of the Elan team is to favour interactions between all the relevant communities, with two objectives: 1/ significantly improve our understanding and modelling capabilities of complex mechanical phenomena, in tight connection with physicists, and 2/ better anticipate practical solutions for the wide diversity of exciting applications to come in the near future. We propose in particular to focus on three research axes, detailed below.

3.1 Discrete modelling of slender elastic structures

For the last 15 years, we have investigated new discrete models for solving the Kirchhoff dynamic equations for thin elastic rods [23, 25, 28]. All our models share a curvature-based spatial discretisation, allowing them to capture inextensibility of the rod intrinsically, without the need for adding any kinematic constraint. Moreover, elastic forces boil down to linear terms in the dynamic equations, making them well-suited for implicit integration. Interestingly, our discretisation methodology can be interpreted from two different points-of-views. From the finite-elements point-of-view, our strain-based discrete schemes can be seen as discontinuous Galerkin methods of zero and first orders. From the multibody system dynamics point of view, our discrete models can be interpreted as deformable Lagrangian systems in finite dimension, for which a dedicated community has started to grow recently [49]. We note that adopting the multibody system dynamics point of view helped us formulate a linear-time integration scheme [22], which had only be investigated in the case of multibody rigid bodies dynamics so far.

High-order spatial discretisation schemes for rods, ribbons and shells Our goal is to investigate similar high-order modelling strategies for surfaces, in particular for the case of inextensible ribbons and shells. Elastic ribbons have been scarcely studied in the past, but they are nowadays drawing more and more the attention from physicists [38, 47]. Their numerical modelling remains an open challenge. In contrast to ribbons, a huge litterature exists for shells, both from a theoretical and numerical viewpoints (see, e.g., [42, 29]). However, no real consensus has been obtained so far about a unified nonlinear shell theory able to support large displacements. In [27] we have started building an inextensible shell patch by taking as degrees of freedom the curvatures of its mid-surface, expressed in the local frame. As in the super-helix model, we show that when taking curvatures uniform over the element, each term of the equations of motion may be computed in closed-form; besides, the geometry of the element corresponds to a cylinder patch at each time step. Compared to the 1D (rod) case however, some difficulties arise in the 2D (plate/shell) case, where compatibility conditions are to be treated carefully. In [2] we have proposed a new, curvature-based discretisation for a developable ribbon (i.e., a narrow plate), which we plan to extend for building an inextensible plate model.

Numerical continuation of rod equilibria in the presence of unilateral constraints In Alejandro Blumentals' PhD thesis [26], we have adopted an optimal control point of view on the static problem of thin elastic rods, and we have shown that direct discretisation methods [1] are particularly well-suited for dealing with scenarios involving both bilateral and unilateral constraints (such as contact). We would like to investigate how our formulations extend to continuation problems, where the goal is to follow a certain branch of equilibria when the rod is subject to some varying constraints (such as one fixed end being applied a constant rotation). To the best of our knowledge, classical continuation methods used for rods [39] are not able to deal with non-persistent or sliding contact.

3.2 Discrete and continuous modelling of frictional contact

Most popular approaches in Computer Graphics and Mechanical Engineering consist in assuming that the objects in contact are locally compliant, allowing them to slightly penetrate each other. This is the principle of

[1]Within this optimal control framework, our previous curvature-based methods can actually be interpreted as a special case of direct single shooting methods.

penalty-based methods (or molecular dynamics), which consists in adding mutual repulsive forces of the form $k\,f(\delta)$, where δ is the penetration depth detected at current time step [30, 46]. Though simple to implement and computationally efficient, the penalty-based method often fails to prevent excessive penetration of the contacting objects, which may prove fatal in the case of thin objects as those may just end up traversing each other. One solution might be to set the stiffness factor k to a large enough value, however this causes the introduction of parasitical high frequencies and calls for very small integration steps [21]. Penalty-based approaches are thus generally not satisfying for ensuring robust contact handling.

In the same vein, the friction law between solid objects, or within a yield-stress fluid (used to model foam, sand, or cement, which, unlike water, cannot flow beyond a certain threshold), is commonly modeled using a regularised friction law (sometimes even with simple viscous forces), for the sake of simplicity and numerical tractability (see e.g., [48, 41]). Such a model cannot capture the threshold effect that characterises friction between contacting solids or within a yield-stress fluid. The nonsmooth transition between sticking and sliding is however responsible for significant visual features, such as the complex patterns resting on the outer surface of hair, the stable formation of sand piles, or typical stick-slip instabilities occurring during motion.

The search for a realistic, robust and stable frictional contact method encouraged us to depart from those, and instead to focus on rigid contact models coupled to the exact nonsmooth Coulomb law for friction (and respectively, to the exact nonsmooth Drucker-Prager law in the case of a fluid), which better integrate the effects of frictional contact at the macroscopic scale. This motivation was the sense of the hiring of F. Bertails-Descoubes in 2007 in the Inria/LJK Bipop team, specialised in nonsmooth mechanics and related convex optimisation methods. In the line of F. Bertails-Descoubes's work performed in the Bipop team, the Elan team keeps on including some active research on the finding of robust frictional contact algorithms specialised for slender deformable structures.

Optimised algorithms for large nodal systems in frictional contact In the fibre assembly case, the resulting mass matrix M is block-diagonal, so that the Delassus operator can be computed in an efficient way by leveraging sparse-block computations [32]. This justifies solving the reduced discrete frictional contact problem where primary unknowns are forces, as usually advocated in nonsmooth mechanics [44]. For cloth however, where primal variables (nodal velocities of the cloth mesh) are all interconnected via elasticity through implicit forces, the method developed above is computationally inefficient. Indeed, the matrix M (only block-sparse, but not block-diagonal) is costly to invert for large systems and its inverse is dense. Recently, we have leveraged the fact that generalised velocities of the system are 3D velocities, which simplifies the discrete contact problem when contacts occur at the nodes. Combined with a multiresolution strategy, we have devised an algorithm able to capture exact Coulomb friction constraints at contact, while retaining computational efficiency [45]. This work also supports cloth self-contact and cloth multilayering. How to enrich the interaction model with, e.g., cohesion, remains an open question. The experimental validation of our frictional contact model is also one of our goals in the medium run.

Continuum modelling of granular and fibrous media Though we have recently made progress on the continuum formulation and solving of granular materials in Gilles Daviet's PhD thesis [31, 34, 33], we are still far from a continuum description of a macroscopic dry fibrous medium such as hair. One key ingredient that we have not been considering in our previous models is the influence of air inside divided materials. Typically, air plays a considerable role in hair motion. To advance in that direction, we have started to look at a diphasic fluid representation of granular matter, where a Newtonian fluid and the solid phase are fully coupled, while the nonsmooth Drucker-Prager rheology for the solid phase is enforced implicitly [35]. This first approach could be a starting point for modelling immersed granulars in a liquid, or ash clouds, for instance.

A long path then remains to be achieved, if one wants to take into account long fibres instead of isotropic grains in the solid phase. How to couple the fibre elasticity with our current formulation remains a challenging problem.

3.3 Inverse design of slender elastic structures [ERC Gem]

With the considerable advance of automatic image-based capture in Computer Vision and Computer Graphics these latest years, it becomes now affordable to acquire quickly and precisely the full 3D geometry of many mechanical objects featuring intricate shapes. Yet, while more and more geometrical data get collected and shared among the communities, there is currently very little study about how to infer the underlying mechanical properties of the captured objects merely from their geometrical configurations.

An important challenge consists in developing a non-invasive method for inferring the mechanical properties of complex objects from a minimal set of geometrical poses, in order to predict their dynamics. In contrast to classical inverse reconstruction methods, our claim is that 1/ the mere geometrical shape of physical objects reveals a lot about their underlying mechanical properties and 2/ this property can be fully leveraged for a wide range of objects featuring rich geometrical configurations, such as slender structures subject to contact and friction (e.g., folded cloth or twined filaments).

In addition to significant advances in fast image-based measurement of diverse mechanical materials stemming from physics, biology, or manufacturing, this research is expected in the long run to ease considerably the design of physically realistic virtual worlds, as well as to boost the creation of dynamic human doubles.

To achieve this goal, we shall develop an original inverse modelling strategy based upon the following research topics:

Design of well-suited discrete models for slender structures We believe that the quality of the upstream, reference physics-based model is essential to the effective connection between geometry and mechanics. Typically, such a model should properly account for the nonlinearities due to large displacements of the structures, as well as to the nonsmooth effects typical of contact and friction.

It should also be parametrised and discretised in such a way that inversion gets simplified mathematically, possibly avoiding the huge cost of large and nonconvex optimisation. In that sense, unlike concurrent methods which impose inverse methods to be compatible with a generic physics-based model, we instead advocate the design of specific physics-based models which are tailored for the inversion process.

More precisely, from our experience on fibre modelling, we believe that reduced Lagrangian models, based on a minimal set of coordinates and physical parameters (as opposed to maximal coordinates models such as mass-springs), are particularly well-suited for inversion and physical interpretation of geometrical data [37, 36]. Furthermore, choosing a high-order coordinate system (e.g., curvatures instead of angles) allows for a precise handling of curved boundaries and contact geometry, as well as the simplification of constitutive laws (which are transformed into a linear equation in the case of rods). We are currently investigating high-order discretisation schemes for elastic ribbons and developable shells [27, 2].

Static inversion of physical objects from geometrical poses We believe that pure static inversion may by itself reveal many insights regarding a range of parameters such as the undeformed configuration of the object, some material parameters or contact forces.

The typical settings that we consider is composed of, on the one hand, a reference mechanical model of the object of interest, and on the other hand a single or a series of complete geometrical poses corresponding each to a static equilibrium. The core challenge consists in analyzing theoretically and practically the amount of information that can be gained from one or several geometrical poses, and to understand how the fundamental under-determinacy of the inverse problem can be reduced, for each unknown quantity (parameter or force) at play. Both the equilibrium condition and the stability criterion of the equilibrium are leveraged towards this goal. On the theoretical side, we have recently shown that a given 3D curve always matches the centerline of an isotropic suspended Kirchhoff rod at equilibrium under gravity, and that the natural configuration of the rod is unique once material parameters (mass, Young modulus) are fixed [1]. On the practical side, we have recently devised a robust algorithm to find a valid natural configuration for a discrete shell to match a given surface under gravity and frictional contact forces [6]. Unlike rods however, shells can have multiple inverse (natural) configurations. Choosing among the multiple solutions based on some selection criteria is an open challenge. Another open issue, in all cases, is the theoretical characterisation of material parameters allowing the equilibrium to be stable.

Dynamic inversion of physical objects from geometrical poses To refine the solution subspaces searched for in the static case and estimate dynamic parameters (e.g., some damping coefficients), a dynamic inversion process accounting for the motion of the object of interest is necessary.

In contrast to the static case where we can afford to rely on exact geometrical poses, our analysis in the dynamic case will have to take into account the imperfect quality of input data with possible missing parts or outliers. One interesting challenge will be to combine our high-order discretised physics-based model together with the acquisition process in order to refine both the parameter estimation and the geometrical acquisition. Our pluridisciplinary work [8] gives encouraging results regarding the ability to recover material parameters and friction coefficient from merely visual observations of elastic bodies in motion.

Experimental validation with respect to real data The goal will be to confront the theories developed above to real experiments. Compared to the statics, the dynamic case will be particularly involving as it will be highly dependent on the quality of input data as well as the accuracy of the motion predicted by our physics-based simulators. Such experiments will not only serve to refine our direct and inverse models, but will also be leveraged to improve the 3D geometrical acquisition of moving objects. Besides, once validation will be performed, we shall work on the setting up of new non-invasive measurement protocols to acquire physical parameters of slender structures from a minimal amount of geometrical configurations. Our recent publication on validation benchmarks [9] represents a first important milestone towards this research direction.

4 Application domains

4.1 Mechanical Engineering

Many physicists and mathematicians have strived for centuries to understand the principles governing those complex mechanical phenomena, providing a number of continuous models for slender structures, granular matter, and frictional contact. In the XXth century, industrial applications such as process automatization and new ways of transportation have boosted the fields of Mechanical Engineering and Computer-Aided Design, where material strength, reliability of mechanisms, and safety, standed for the main priorities. Instead, large displacements of structures, buckling, tearing, or entanglement, and even dynamics, were long considered as undesirable behaviors, thus restraining the search for corresponding numerical models.

Only recently, the engineering industry has shown some new and growing interest into the modeling of dynamic phenomena prone to large displacements, contact and friction. For instance, the cosmetology industry is more and more interested in understanding the nonlinear deformation of hair and skin, with the help of simulation. Likewise, auto and aircraft manufacturers are facing new challenges involving buckling or entanglement of thin structures such as carbon or optical fibers; they clearly lack predictive, robust and efficient numerical tools for simulating and optimizing their new manufacturing process, which share many common features with the large-scale simulation scenarii traditionally studied in Computer Graphics applications.

4.2 Computer Graphics

In contrast, Computer Graphics, which has emerged in the 60's with the advent of modern computers, was from the very beginning eager to capture such peculiar phenomena, with the sole aim to produce spectacular images and create astonishing stories. At the origin, Computer Graphics thus drastically departed from other scientific fields. Everyday-life phenomena such as cloth buckling, paper tearing, or hair fluttering in the wind, mostly ignored by other scientists at that time, became actual topics of interest, involving a large set of new research directions to be explored, both in terms of modelling and simulation. Nowadays, although the image production still remains the core activity of the Computer Graphics community, more and more research studies are directed through the virtual and real prototyping of mechanical systems, notably driven by a myriad of new applications in the virtual try on industry (e.g., hairstyling and garment fitting). Furthermore, the advent of additive fabrication is currently boosting research in the free design of new mechanisms or systems for various applications, from architecture design and fabrication of metamaterials to the creation of new locomotion modes in robotics. Some obvious common interests and approaches are thus emerging between Computer Graphics and Mechanical Engineering, yet the two communities remain desperately compartmentalized.

4.3 Soft Matter Physics

From the physics-based viewpoint, since a few decades a new generation of physicists became interested again in the understanding of such visually fascinating phenomena, and started investigating the tight links between geometry and elasticity [2]. Common objects such as folded or torn paper, twined plants, coiled honey threads, or human hair have thus regained some popularity among the community in Nonlinear Physics [3]. In consequence, phenomena of interest have become remarkably close to those of Computer Graphics, since scientists in both places share the common goal to model complex and integrated mechanical phenomena at the macroscopic scale. Of course, the goals and employed methodologies differ substantially from one community to the other, but showcase some evident complementarity: while computer scientists are eager to learn and understand new physical models, physicists get more and more interested in the numerical tools, in which they perceive not only a means to confirm predictions afterwards, but also a support for testing new hypothesis and exploring scenarios that would be too cumbersome or even impossible to investigate experimentally. Besides, numerical exploration starts becoming a valuable tool for getting insights into the search for analytic solutions, thus fully participating to the modeling stage and physical understanding. However, physicists may be limited to a blind usage of numerical black boxes, which may furthermore not be dedicated to their specific needs. According to us, promoting a science of modeling in numerical physics would thus be a promising and rich avenue for the two research fields. Unfortunately, very scarce cooperation currently exists between the two communities, and large networks of collaboration still need to be set up.

5 Social and environmental responsibility

5.1 Footprint of research activities

The Elan team is environment-sensitive. Since its creation in 2017, 100% of its research staff moves daily from home to the lab using soft transportation means (biking, public transportation). Intercontinental missions are limited while train is the preferred mode of transportation in Europe.

5.2 Impact of research results

A large part of the research conducted in the team is of fundamental level. Direct applications lie in numerical arts, cloth design, sports, and environmental studies, all of these being of limited negative impact for the environment. Collaborations with industry leading specially harmful activities to the environment are avoided.

6 Highlights of the year

6.1 Defenses

Emile Hohnadel defended his PhD thesis the first of July 2025 [19].

6.2 Visits

David Breen, Associate Professor at Drexel University (USA), was visiting our Elan team in 2025 for a sabbatical of 6 months. D. Breen has been awarded a prestigious Fulbright grant to visit and collaborate with our team. This collaboration (still ongoing remotely) deals with the modelling of knitted cloth. D. Breen has applied to UGA to come back to the Elan team next year.

[2] In France this new trend was particularly stimulated by the work of Yves Pomeau, who convinced many young scientists to study the nonlinear physics of common objects such as paper, plants, or hair [40].

[3] It is however amusing to observe that research in these areas is quite successful in obtaining the IG Nobel prize [20, 43], thus still being considered as an exotic research topic by physicists.

7 Latest software developments, platforms, open data

7.1 Latest software developments

7.1.1 Feel++

Keywords: High order finite elements, Discontinuous Galerkin, High-Performance Computing

Functional Description: Feel++ is a high-performance C++ library for the resolution of general variational formulations, including continuous and discontinuous Galerkin methods, finite element or spectral element methods, reduced basis formulations, etc. It features a high-level domain specific embedded language (DSEL) for Galerkin methods, space dimension-agnostic computation kernels and seamless and automatic parallelism. It also includes applicative toolboxes to solve physics problems in fluid mechanics, solid mechanics, thermal conduction, and the corresponding multi-physics coupling.

Release Contributions: - Support of distance-based contact forces between immersed bodies - BVH implementation for contact pruning - Various improvements in expression support

URL: https://docs.feelpp.org/home/index.html

Contact: Thibaut Metivet

Partners: Université de Strasbourg, UGA, Inria

7.1.2 ElanFab

Keywords: Experimental mechanics, Experimental design, Thin elastic ribbon, Thin elastic rod, Thin elastic shell, Frictional contact

Functional Description: Experimental platform of the ELAN team.

The aim of this platform is to experimentally explore the mechanics and geometry of highly deformable elastic objects of low dimensions (rods, fibers, plates, shells).

The platform allow us to fabricate with controlled materials and geometries elastic objects. By means of state of the art manufacturing techniques we are able to make curved elastic objects, with a controlled target geometry. For the moment we use elastomeric materials to remain in the elastic regime, however we are interested in exploring new materials to include viscous and plastic effects.

Our platform has a modular mechanical testing device that allow load and tensile testing in multiple configurations for a wide range of force magnitudes, from 1e-3 to 100 Newtons. In this setup we have tested highly compliant, as well as, very stiff materials, for example we study the tensile response of feathers and elastic knotted rods.

The platform is constantly undergoing new improvements to allow us to obtain geometrical information by means of a combination of image analysis and computer vision techniques. Multiple views are obtained by using multiple cameras and mirrors or by using one single camera that moves in a highly controlled manner. Furthermore we are implementing the use of a semi fast camera to study dynamic phenomena of complex elastic objects assemblies. We are also implementing structured light into our setup to improve the accuracy of our measurements.

Currently, we are expanding our capabilities to the segmentation and reconstruction of fibrous materias from 3D volumetric tomographic data.

Contact: Victor Romero Gramegna

7.1.3 circonflex

Keywords: 2D, Thin elastic rod, Frictional contact

Functional Description: Dynamic simulation of 2D Kirchhoff rods based on the mixed super-circle model.

Reference code for the paper: Hohnadel, E. and Métivet, T. and Bertails-Descoubes, F., Mixed Super Circles, conditionally accepted to Eurographics 2026

URL: https://gitlab.inria.fr/elan-public-code/circonflex

Contact: Florence Bertails Descoubes

8 New results

8.1 A Noetherian approach to invariants for the statics and dynamics of elastic rods

Participants: Florence Bertails-Descoubes, Sébastien Neukirch.

The static-dynamic analogy discovered by Gustav Kirchhoff in 1859 shows that the statics of an elastic beam is equivalent to the dynamics of a spinning top. This analogy, where time and angular velocity are, for example, equivalent to arclength and curvatures, allows the use of Emmy Noether's 1918 theorem on continuous symmetries to unravel a quantity that is invariant along elastic rods at equilibrium. A spinning top having a Lagrangian independent of time will have its mechanical energy constant in time. In the same manner, an elastic rod with uniform elastic properties will have the sum of its curvature energy and its tension force uniform along the structure. This arclength invariance property is known in simple cases, but the present approach generalises it to more complex scenarios where extensibility, shear, conservative loads (e.g. gravity), and contact are involved. Moreover, still using Noether's theorem and bringing to light the continuous symmetries of the Lagrangian of the variational approach, we recover all known invariants for the statics and dynamics of rods and ribbons, including coordinate invariants. Furthermore, we show how the arclength invariant may be used to obtain pivotal information on some landmark elastic rod problems, including confinement buckling, sliding sleeves, or plectonemes. Finally, we extend the approach to vibrations. Overall, this paper is an attempt to explain, unify and extend all previous results on rod invariants thanks to the beautiful Noetherian formalism, and to show its practical use on a few relevant applications.

The corresponding work will be published at the International Journal of Non-linear Mechanics in 2026 [11]. It was also presented at several international conferences in 2025 [14, 13], and selected as a talk ("exposé long") at Rencontres du Non-Linéaire 2025, Paris (Florence Bertails-Descoubes, in duet with Sébastien Neukirch).

8.2 Mixed super-circles, a mixed position-curvature discretisation scheme for 2D Kirchhoff rods

Participants: Emile Hohnadel, Florence Bertails-Descoubes, Thibaut Metivet.

Following our mixed position-curvature based strategy successfully developed for the statics of thin elastic ribbons [3], we introduce mixed *super-circles*, a position-curvature formulation for the *dynamics* of thin elastic rods. Compared to our former purely curvature-based model – the so-called 2D super-helix model [24] –, the mixed formulation that we propose here drastically reduces the algorithmic complexity of the solving scheme – from quadratic to quasi-linear – and simplifies the handling of positional constraints, including contacts. As such, it recovers the main advantages of classical position-based models, while at the same time preserving the high-order convergence of curvature-based models, hence offering an interesting trade-off. Furthermore, the smooth, piecewise circular arc representation of super-circles allows to avoid the spurious jumps in contact forces that are difficult to get rid of with position-based models. Our model is validated quantitatively against demanding mechanical tests involving contact, friction and snapping. Moreover, its versatility, robustness and efficiency are demonstrated through several dynamic scenarios involving, in real-time, multiple planar elastic rods subject to various types of boundary conditions and constraints.

This work has been conditionally accepted for publication at Eurographics 2026. Moreoever, the corresponding source code, Circonflex, is freely delivered to the research community under the GNU GPL v3 licence.

8.3 Hydrodynamic model for fish locomotion

Participants: Thibaut Métivet.

In collaboration with Bruno Ventéjou (co-supervised post-doc at LIPhy, UGA), Philippe Peyla (LIPhy, UGA) and Aurélie Dupont (LIPhy, CNRS), we study the respective roles of hydrodynamic and social interactions within schools of fish, in the context of the FISHSIF ANR project. As a first step toward the simulation of large assemblies of swimming fish, we have developed a simplified hydrodynamic model of a swimmer, able to account for individual fish swimming and stigmergy, in particular regarding the generation of vortices wakes, without the need to introduce deformation of the body of the fish. We have performed detailed hydrodynamic scaling analyses of the velocity of a moving immersed body, and shown that the motion of swimmers obeys a universal scaling law expressed in terms of only two dimensionless quantities describing the relative importance of inertia, viscosity and swimming forces. Using extensive numerical simulations, we have shown excellent agreement between our theoretical scaling laws and the swimming behaviour of our model fish. The validity of our scaling laws notably extend across a wide range of hydrodynamic regimes (from the Stokes to the turbulent regime), and demonstrates the ubiquitous decrease in swimming efficiency as the velocity increases. We have further compared our results to experimental data collected among many aquatic species, with very different body shapes, deformations, and swimming velocities. The overall collapse of swimmers' data onto our single master curve supports the robustness and genericity of our analysis and model. This work has been published at Physical Review Letters [12], and presented at several international events [18, 17].

9 Bilateral contracts and grants with industry

9.1 Bilateral contracts with industry

Participants: Florence Bertails-Descoubes, Emile Hohnadel.

Since March 2024, the Elan team collaborates through a bilateral contract with L'Oréal research. The contract has been extended in September 2025. Topic: fun- damental understanding of the mechanical properties of highly curly hair (E. Hohnadel, B. Peres, V. Romero and F. Bertails-Descoubes). A 3-year extension (with the co-supervision of a PhD student and code licensing) is planned in January 2026.

10 Partnerships and cooperations

10.1 International initiatives

10.1.1 International collaborations

- Scientific collaboration with Theodore Kim (Yale University, USA) on the modelling of highly curly hair.

10.2 International research visitors

10.2.1 Visits of international scientists

David Breen

Status Full Professor

Institution of origin: Drexel University

Country: USA

Dates: 01/02/2025 - 01/07/2025

Context of the visit: Scientific collaboration on the modelling of knitted cloth

Mobility program/type of mobility: Fulbright sabbatical grant

10.3 National initiatives

FISHSIF ANR Project

Participants: Thibaut Métivet.

Title: FISHSIF: Fish In Silico with Hydrodynamics and Social Forces

Duration: 01/10/2021 - 31/12/2025

Summary: The FISHSIF project has received a four-year funding from the ANR (Agence Nationale pour la Recherche). The goal of this project is to introduce dynamical cognition models within full hydrodynamic simulations in order to understand the role played by social or flow interactions in the organisation and behaviour of schools of fish. The project will be led in a collaboration between the ELAN team, the Laboratoire Interdisciplinaire de Physique (LIPhy, UGA/CNRS) and the Laboratoire de Psychologie et NeuroCognition (LPNC, UGA/CNRS).

Partners:

- Elan Inria project-team
- Laboratoire Interdisciplinaire de Physique (LIPhy), Université Grenoble-Alpes (UGA)
- Laboratoire de Psychologie et NeuroCognition (LPNC), Université Grenoble-Alpes (UGA)

National collaborations

- Collaboration with Philippe Peyla, Aurélie Dupont (LIPhy, UGA/CNRS) and Christian Graff (LPNC, UGA/CNRS) within the FISHSIF project.
- Collaboration with Baptiste Darbois-Texier (FAST, Univ. Paris Saclay/CNRS).
- Long-term collaboration with Sébastien Neukirch (Institut Jean le Rond d'Alembert, Sorbonne Université).
- Long-term collaboration with Christophe Prud'homme and Vincent Chabannes (Université de Strasbourg and Centre de modélisation et de simulation de Strasbourg).

11 Dissemination

11.1 Promoting scientific activities

11.1.1 Journal

Reviewer - reviewing activities

- Siggraph (ACM, accepted papers published in Transactions on Graphics)
- Siggraph Asia (ACM, accepted papers published in Transactions on Graphics)
- Physical Review X (PRX, American Physical Society)

11.1.2 Invited talks

Florence Bertails-Descoubes:

- **October 2025**: Invited seminar at Laboratoire de Physique et Mécanique des Milieux Hétérogènes (PMMH), Paris.
- **March 2025**: Invited seminar at Laboratoire d'Acoustique de l'Université du Mans, Le Mans.

11.2 Teaching - Supervision - Juries - Educational and pedagogical outreach

Licence

- Victor Romero: TD Electromagnétisme et optique pour la chimie, PHY405, 33h, DLST, Université Grenoble Alpes, Grenoble.
- Victor Romero: TP La physique par l'expérience, PHY408, 32h, PHYTEM, Université Grenoble Alpes, Grenoble.

Master

- Florence Bertails-Descoubes: Master 2 ENSIMAG, Grenoble INP, (13h), Mécanique Numérique pour l'Informatique Graphique Course co-founded by F. Bertails-Descoubes, M. Skouras (EPI Anima) and M. Ly in 2019 (initially as a special course for ENS Lyon), delivered with M. Skouras and T. Métivet in 2024/2025 (36h in total).
- Thibaut Métivet: Master 2 ENSIMAG, Grenoble INP, Mécanique Numérique pour l'Informatique Graphique (16h).

11.2.1 Supervision

Post-doctorate

- Arun Kumar: 01/09/2021-31/08/2025, co-supervised by Florence Betails-Descoubes et Victor Romero.
- Emile Hohnadel: 01/09/2025-to date, supervised by Florence Bertails-Descoubes.

PhD

- Emile Hohnadel: 01/09/2021-01/07/2025, co-supervised by Florence Bertails-Descoubes and Thibaut Métivet
- Sergio Murillo-Garcia: 01/09/2025-, co-supervised by Florence Bertails-Descoubes and Sébastien Neukirch (Institut D'Alembert, Sorbonne Université)

Pre-PhD

- Brice Peres: 01/09/2025-to date, co-supervised by Florence Betails-Descoubes and Victor Romero

Research Engineer

- Alice Teixeira Da Silva (formerly PhD student): until 30/06/2024, co-supervised by Thibaut Métivet, Florence Bertails-Descoubes, and Mélina Skouras (Anima, Inria GRE)

Internship

- Sergio Murillo-Garcia: 01/02/2025-31/07/2025, co-supervised by Florence Bertails-Descoubes and Sébastien Neukirch (Institut D'Alembert, Sorbonne Université)
- Dimitri Sircat: 01/05/2021-19/11/2025, co-supervised by Thibaut Métivet, Florence Bertails-Descoubes, and Baptiste Darbois-Texier (FAST, Université Paris-Saclay)

11.2.2 Juries

F. Bertails-Descoubes was a member of the PhD committees of Uday Kusupati (EPFL, Laussane 2025), as a **reviewer**, of Joo-Won Hong Soft Matter Physics, (PMMH Paris, 2025), as an examiner, of Jiayu Wang (Sorbonne Université Paris, 2025), as an examiner, of Karim Aït Ammar (ENS Paris Saclay, 2025), as a **reviewer**, and of Siyuan He (École des Ponts Paris, 2025), as a president.

11.3 Popularization

2025: Interview of F. Bertails-Descoubes by the Epsiloon magazine for the special hors-serie magazine entitled “Les créateurs de l’eau: dans les coulisses des effets spéciaux”.

12 Scientific production

12.1 Major publications

[1] F. Bertails-Descoubes, A. Derouet-Jourdan, V. Romero and A. Lazarus. ‘Inverse design of an isotropic suspended Kirchhoff rod: theoretical and numerical results on the uniqueness of the natural shape’. In: *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences* 474.2212 (Apr. 2018), pp. 1–26. DOI: 10.1098/rspa.2017.0837. URL: https://hal.inria.fr/hal-01827887 (cit. on p. 8).

[2] R. Charrondière, F. Bertails-Descoubes, S. Neukirch and V. Romero. ‘Numerical modeling of inextensible elastic ribbons with curvature-based elements’. In: *Computer Methods in Applied Mechanics and Engineering* 364 (1st June 2020), pp. 1–32. DOI: 10.1016/j.cma.2020.112922. URL: https://hal.inria.fr/hal-02515877 (cit. on pp. 6, 8).

[3] R. Charrondière, S. Neukirch and F. Bertails-Descoubes. ‘Merci: Mixed curvature-based elements for computing equilibria of thin elastic ribbons’. In: *ACM Transactions on Graphics* 43.5 (9th Aug. 2024), art. 160. DOI: 10.1145/3674502. URL: https://hal.science/hal-04601301 (cit. on p. 12).

[4] O. Crespel, E. Hohnadel, T. Métivet and F. Bertails-Descoubes. ‘Contact detection between curved fibres: high order makes a difference’. In: *ACM Transactions on Graphics* 43.4 (19th Aug. 2024), 132:1–23. DOI: 10.1145/3658191. URL: https://hal.science/hal-04364565.

[5] J. Li, G. Daviet, R. Narain, F. Bertails-Descoubes, M. Overby, G. Brown and L. Boissieux. ‘An Implicit Frictional Contact Solver for Adaptive Cloth Simulation’. In: *ACM Transactions on Graphics*. Proceedings SIGGRAPH 2018 37.4 (Aug. 2018), pp. 1–15. DOI: 10.1145/3197517.3201308. URL: https://hal.inria.fr/hal-01834705.

[6] M. Ly, R. Casati, F. Bertails-Descoubes, M. Skouras and L. Boissieux. ‘Inverse Elastic Shell Design with Contact and Friction’. In: *ACM Transactions on Graphics* 37.6 (Nov. 2018), pp. 1–16. DOI: 10.1145/3272127.3275036. URL: https://hal.inria.fr/hal-01883655 (cit. on p. 8).

[7] T. Métivet, A. Sengers, M. Ismail and E. Maitre. ‘Diffusion-redistanciation schemes for 2D and 3D constrained Willmore flow: application to the equilibrium shapes of vesicles’. In: *Journal of Computational Physics* 436 (2021), p. 110288. DOI: 10.1016/j.jcp.2021.110288. URL: https://hal.science/hal-02905870.

[8] A.-H. Rasheed, V. Romero, F. Bertails-Descoubes, S. Wuhrer, J.-S. Franco and A. Lazarus. ‘A Visual Approach to Measure Cloth-Body and Cloth-Cloth Friction’. In: *IEEE Transactions on Pattern Analysis and Machine Intelligence* (July 2021). DOI: 10.1109/TPAMI.2021.3097547. URL: https://hal.inria.fr/hal-03285624 (cit. on p. 9).

[9] V. Romero, M. Ly, A.-H. Rasheed, R. Charrondière, A. Lazarus, S. Neukirch and F. Bertails-Descoubes. ‘Physical validation of simulators in Computer Graphics: A new framework dedicated to slender elastic structures and frictional contact’. In: *ACM Transactions on Graphics* 40.4 (1st Aug. 2021), Article 66: 1–19. DOI: 10.1145/3450626.3459931. URL: https://hal.inria.fr/hal-03217459 (cit. on p. 9).

[10] G. Rousseau, T. Métivet, H. Rousseau, G. Daviet and F. Bertails-Descoubes. 'Revisiting the role of friction coefficients in granular collapses: confrontation of 3-D non-smooth simulations with experiments'. In: *Journal of Fluid Mechanics* 975 (15th Nov. 2023), A14. DOI: 10.1017/jfm.2023.835. URL: https://inria.hal.science/hal-03845323.

12.2 Publications of the year

International journals

[11] S. Neukirch and F. Bertails-Descoubes. 'A Noetherian approach to invariants for the statics and dynamics of elastic rods'. In: *International Journal of Non-Linear Mechanics* 181 (Feb. 2026), p. 105269. DOI: 10.1016/j.ijnonlinmec.2025.105269. URL: https://hal.science/hal-05149317 (cit. on p. 12).

[12] B. Ventéjou, T. Métivet, A. Dupont and P. Peyla. 'Universal Scaling Laws for a Generic Swimmer Model'. In: *Physical Review Letters* 134.13 (3rd Apr. 2025), p. 134002. DOI: 10.1103/PhysRevLett.134.134002. URL: https://hal.science/hal-04702713 (cit. on p. 13).

International peer-reviewed conferences

[13] F. Bertails-Descoubes and S. Neukirch. 'A useful invariant for elastic rods'. In: ESMC 2025 - 12th European Solid Mechanics Conference. Lyon, France, 2025. URL: https://hal.science/hal-05162593 (cit. on p. 12).

[14] F. Bertails-Descoubes and S. Neukirch. 'Noether symmetries and conserved quantities for elastic rods with contact'. In: HFSS 2025 - 2nd International Conference on Highly Flexible Slender Structures. Kaiserslautern, Germany, 2025, pp. 1–1. URL: https://hal.science/hal-05388769 (cit. on p. 12).

Conferences without proceedings

[15] É. Hohnadel, O. Crespel, T. G. Sano, T. Métivet and F. Bertails-Descoubes. 'Accurate frictional contact algorithms for the numerical exploration of the mechanics of fibrous assemblies'. In: ESMC 2025 - 12th European Solid Mechanics Conference. Lyon, France, 2025, pp. 1–2. URL: https://hal.science/hal-05228112.

[16] J. Jouve, V. Romero, R. Narain, L. Boissieux, T. Kim and F. Bertails-Descoubes. 'Modelling a Feather as a Strongly Anisotropic Elastic Shell'. In: ESMC 2025 - 12th European Solid Mechanics Conference. Lyon, France, 2025, pp. 1–29. URL: https://hal.science/hal-05230404.

[17] P. Peyla, B. Ventéjou, A. Dupont and T. Métivet. 'A Minimal Swimmer Model Unveils Universal Scaling Across Reynolds Numbers'. In: DFD 2025 - 78th Annual Meeting of the APS Division of Fluid Dynamics. Houston, TX, United States, 2025. URL: https://hal.science/hal-05390418 (cit. on p. 13).

[18] B. Ventéjou, T. Métivet, A. Dupont and P. Peyla. 'Hydrodynamics of Confined Swimming: Persistence of Stokes Flow and Emergence of Boundary Layers'. In: DFD 2025 - 78th Annual Meeting of the APS Division of Fluid Dynamics. Houston (Texas), United States, 2025, pp. 1–1. URL: https://hal.science/hal-05390358 (cit. on p. 13).

Doctoral dissertations and habilitation theses

[19] E. Hohnadel. 'High order contact detection and mixed rod model for predictive numerical simulations of tangled fibrous assemblies'. Université Grenoble Alpes, 1st July 2025. URL: https://hal.science/tel-05293793 (cit. on p. 10).

12.3 Cited publications

[20] B. Audoly and S. Neukirch. 'Fragmentation of Rods by Cascading Cracks: Why Spaghetti Does Not Break in Half'. In: *Physical Review Letters* 95.9 (2005), p. 095505 (cit. on p. 10).

[21] D. Baraff. ‘Analytical methods for dynamic simulation of non-penetrating rigid bodies’. In: *Computer Graphics Proceedings (Proc. ACM SIGGRAPH’89)*. New York, NY, USA: ACM, 1989, pp. 223–232 (cit. on p. 7).

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[23] F. Bertails, B. Audoly, M.-P. Cani, B. Querleux, F. Leroy and J.-L. Lévêque. ‘Modélisation de coiffures naturelles à partir des propriétés physiques du cheveu’. In: *Journées Francophones d’Informatique Graphique (AFIG)*. AFIG / EG-France. Strasbourg, France, Nov. 2005 (cit. on p. 6).

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[27] A. Blumentals, F. Bertails-Descoubes and R. Casati. ‘Dynamics of a developable shell with uniform curvatures’. In: *The 4th Joint International Conference on Multibody System Dynamics*. Montréal, Canada, May 2016. URL: https://hal.inria.fr/hal-01311559 (cit. on pp. 6, 8).

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[31] G. Daviet. ‘Modèles et algorithmes pour la simulation du contact frottant dans les matériaux complexes : application aux milieux fibreux et granulaires’. PhD thesis. Grenoble Alpes Universités, Dec. 2016 (cit. on p. 7).

[32] G. Daviet, F. Bertails-Descoubes and L. Boissieux. ‘A hybrid iterative solver for robustly capturing Coulomb friction in hair dynamics’. In: *ACM Transactions on Graphics (Proc. ACM SIGGRAPH Asia’11)* 30 (6 2011), 139:1–139:12. URL: http://www.inrialpes.fr/bipop/people/bertails/Papiers/hybridIterativeSolverHairDynamicsSiggraphAsia2011.html (cit. on p. 7).

[33] G. Daviet and F. Bertails-Descoubes. ‘A semi-implicit material point method for the continuum simulation of granular materials’. In: *ACM Transactions on Graphics*. SIGGRAPH ’16 Technical Papers 35.4 (July 2016), p. 13. DOI: 10.1145/2897824.2925877. URL: https://hal.inria.fr/hal-01310189 (cit. on p. 7).

[34] G. Daviet and F. Bertails-Descoubes. ‘Nonsmooth simulation of dense granular flows with pressure-dependent yield stress’. In: *Journal of Non-Newtonian Fluid Mechanics* 234 (Apr. 2016), pp. 15–35. DOI: 10.1016/j.jnnfm.2016.04.006. URL: https://hal.inria.fr/hal-01236488 (cit. on p. 7).

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