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2025 *Activity Report*

RESEARCH CENTRE: **Inria Centre at the University of Bordeaux**

IN PARTNERSHIP WITH: **Bordeaux INP, Université de Pau et des Pays de l'Adour, CNRS, TotalEnergies**

Project-Team

MAKUTU

Experimental-based modeling and simulation of wave propagation to characterize geophysical and heliophysical media and to design complex objects

In collaboration with Laboratoire de mathématiques et de leurs applications (LMAP)

Project-Team MAKUTU

Creation of the Project-Team: 2021 February 01

Each year, Inria research teams publish an Activity Report presenting their work and results over the reporting period. These reports follow a common structure, with some optional sections depending on the specific team. They typically begin by outlining the overall objectives and research programme, including the main research themes, goals, and methodological approaches. They also describe the application domains targeted by the team, highlighting the scientific or societal contexts in which their work is situated. The reports then present the highlights of the year, covering major scientific achievements, software developments, or teaching contributions. When relevant, they include sections on software, platforms, and open data, detailing the tools developed and how they are shared. A substantial part is dedicated to new results, where scientific contributions are described in detail, often with subsections specifying participants and associated keywords. Finally, the Activity Report addresses funding, contracts, partnerships, and collaborations at various levels, from industrial agreements to international cooperations. It also covers dissemination and teaching activities, such as participation in scientific events, outreach, and supervision. The document concludes with a presentation of scientific production, including major publications and those produced during the year.

Keywords

Computer sciences and digital sciences

A6.1. – Methods in mathematical modeling
A6.1.1. – Continuous Modeling (PDE, ODE)
A6.1.4. – Multiscale modeling
A6.1.5. – Multiphysics modeling
A6.2. – Scientific computing, Numerical Analysis & Optimization
A6.2.1. – Numerical analysis of PDE and ODE
A6.2.5. – Numerical Linear Algebra
A6.2.6. – Optimization
A6.2.7. – HPC for machine learning
A6.3.1. – Inverse problems
A6.3.4. – Model reduction
A6.5. – Mathematical modeling for physical sciences
A6.5.1. – Solid mechanics
A6.5.4. – Waves
A9.2.5. – Bayesian methods
A9.2.6. – Neural networks

Other research topics and application domains

B3. – Environment and planet
B3.3. – Geosciences
B3.3.1. – Earth and subsoil
B4. – Energy
B9. – Society and Knowledge
B9.5. – Sciences
B9.5.2. – Mathematics
B9.5.3. – Physics
B9.5.5. – Mechanics
B9.5.6. – Data science

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1 Team members, visitors, external collaborators

Research Scientists

- Helene Barucq [Team leader, Inria, Senior Researcher, HDR]
- Henri Calandra [TotalEnergies, Industrial member]
- Ruiyang Dai [Inria, Starting Research Position, from Dec 2025]
- Julien Diaz [Inria, Senior Researcher, HDR]
- Florian Faucher [Inria, Researcher]
- Stefano Frambati [TotalEnergies, Industrial member]
- Ha Ngoc Pham Howard Faucher [Inria, Researcher]
- Chengyi Shen [Inria, Starting Research Position, until Jun 2025]

Faculty Members

- Marc Durufle [BORDEAUX INP, Associate Professor]
- Sebastien Tordeux [UPPA, Professor Delegation, from Sep 2025, HDR]
- Sebastien Tordeux [UPPA, Associate Professor, until Aug 2025, HDR]

Post-Doctoral Fellows

- Victor Martins Gomes [Inria, Post-Doctoral Fellow]
- Janosch Preuss [Inria, Post-Doctoral Fellow]
- Ishak Tifouti [Inria, Post-Doctoral Fellow, from Dec 2025]

PhD Students

- Julien Besset [Inria, until Jun 2025]
- Lola Chabat [Inria, from Oct 2025]
- Lola Chabat [UPPA, until Sep 2025]
- Florian Delprat [Inria, from Apr 2025 until Sep 2025]
- Ibrahima Djiba [Inria, until Mar 2025]
- Pierre Dubois [CEA]
- Jean Dutheil [Inria]
- Arjeta Heta [UPPA, until Mar 2025]
- Andrea Lagardere [AIRBUS]
- Mailys Lassale [ONERA, from Dec 2025]
- Matthias Rivet [Inria, from Nov 2025]
- Matthias Rivet [UPPA, until Sep 2025]
- Manon Sarrouilhe [Inria]

Technical Staff

- Alexis Bandet [Inria, Engineer, from Feb 2025]
- Aurélien Citrain [Inria, Engineer]

Interns and Apprentices

- Aurelia Bergeret [Inria, Intern, from Aug 2025 until Sep 2025]
- Aurelia Bergeret [Inria, Intern, until Jan 2025]
- Chloe Garcia [Inria, Intern, from Jun 2025 until Aug 2025]
- Basile Mouret [Inria, Intern, from Jun 2025 until Aug 2025]
- Eduard Occhipinti [Inria, Intern, from Apr 2025 until Sep 2025]
- Julien Royer [Inria, Intern, from Jun 2025 until Aug 2025]
- Pape Farba Seck [Inria, from Nov 2025]

Administrative Assistants

- Fabienne Cuyollaa [Inria]
- Nathalie Robin [Inria]

External Collaborators

- Patrick Amestoy [Mumps Tech, HDR]
- Damien Fournier [Max Planck Institute]
- Laurent Gizon [Max Planck Institute, HDR]
- Jean-Yves L'Excellent [Mumps Tech, HDR]
- Mamadou Ndiaye [UNIV VALENCIENNES]
- Chiara Puglisi [Mumps Tech]
- Nathan Rouxelin [INSA ROUEN NORMANDIE]
- Maarten V. de Hoop [Rice University, HDR]

2 Overall objectives

Imagine trying to describe a place with exactness from more or less numerous and precise memories, or guessing the content and internal structures of an object after having observed it only partially, without ever touching it because it is inaccessible or very fragile? These are the objectives of Makutu team, where recordings of reflected waves correspond to the memories. Waves can be seismic, electromagnetic or acoustic and Makutu focuses its research on the characterization of Earth's subsurface and the internal dynamic of the Sun. An important component of Makutu's work is the improvement of the resolution methods for direct problems, in order to simulate the propagation of waves in complex media. The characterization and reconstruction of objects using non-invasive approaches then need the resolution of an inverse problem, with efficient forward modeling at the center.

Makutu is an industrial Inria project-team joint with TotalEnergies, in partnership with University of Pau and Pays de l'Adour, Institut Polytechnique de Bordeaux and CNRS. The team is bi-located, one part is hosted by UPPA and the other is hosted by Inria on the University of Bordeaux campus. It is a follow-up of

Magique-3D (Advanced Modeling in 3D Geophysics) and its research topics have expanded are summarized as "*Experimental-based modeling and simulation of wave propagation to characterize geophysical and heliophysical media and to design complex objects*" with the name Makutu (magicians in Maori).

The numerical simulation of waves propagating in complex media (the direct problems solved by Makutu) requires the development of advanced numerical methods but the research does not stop there. Indeed, to probe a medium or reconstruct an object from reflected waves measurements (the inverse problems addressed by Makutu), it is important to accurately solve systems of partial differential equations that model all the waves that can be measured. As the complexity of the physical models goes hand in hand with the complexity of the calculations, Makutu is particularly committed in the development and analysis of appropriate mathematical models as well as in the design and study of advanced numerical methods taking into account the characteristics of the physics considered.

Makutu's research has many facets, with final goals ranging from the development of open-source prototype codes written to assess new ideas, to software packages to be ported to an industrial environment. Makutu's research activities are inspired by a strong interdisciplinary industrial and academic partnership. The team's contributions are at the interface of applied analysis, numerical analysis, and scientific computing. The size of some of our problems projects us into the High Performance Computing (HPC) environment and dictates the choices we make for certain approximation spaces that are conducive to massive parallelism.

Makutu has important contributions in the field of high-order discretization methods along with high-order time schemes. Whatever the application is, numerical schemes are all designed with a view to reduce computational time or limit memory consumption, while maintaining a high level of accuracy. Sometimes, it is also necessary to work on the mathematical models themselves whose brute complexity can be a source of difficulty or even blockage for the numerical simulations. One of the originality of the team is to collaborate with experimenters to compare measurements and numerical data in order to calibrate the models. All these contributions are significant steps to reach the final team's objectives which are expressed as the resolution of complex inverse problems. Large-scale computing is then an important part of our activity, which we carry out taking into account the three pillars that make up HPC, i.e. computing time, storage and precision. Recently, the team has started to work on the use of machine learning to assist the numerical schemes they develop in order to control the numerical pollution (or dispersion) which becomes very strong in large-scale computations. This is particularly the case in geophysics where several hundred wavelengths can be propagated. It is worth noting that we take particular care in developing our numerical methods so that they can be used for a wide range of applications, whether the calculations are done in an HPC environment or on a simple laptop.

3 Research program

Makutu's research program decomposes itself into three axes that are: (1) Methodological contributions to the simulation of mechanical and electromagnetic waves in complex media; (2) Seismic imaging; (3) Helioseismology. Each axis shares the same objective to realize simulations of real phénomena. To achieve this, one needs real data and advanced mathematical models and high-order numerical schemes that are compatible with high-performance computing architectures.

To obtain real data, in addition to its current collaborations with scientists both from Academia and Industry, Makutu is developing a new branch of research activities by carrying out its own laboratory measurements. For instance, in order to take into account porosity, parameters such as viscosity, attenuation, thermodynamic effects, etc., must be integrated, and their impact must be properly analyzed before considering using them to characterize the propagation media. This constitutes a clear step ahead for Makutu, and opens up new prospects of contributing to the characterization of very complex media based on wave field measurements.

Regarding the development of numerical schemes, Makutu is developing and analyzing high-order finite elements like Discontinuous Galerkin (DG) methods and spectral element methods possibly coupled with high-order time schemes in the time-dependent regime. The coupling of DG methods with other techniques of discretization is also under consideration. Trefftz-DG and Hybridizable DG methods have been developed both for poro-elastic waves and electromagnetic waves. HDG and HDG+ formulations are also under study for helioseismology with with coupling to integral equation methods.

The research activities of Makutu members have in common the use of wave field measurements. These data are either real or synthetic, produced in the latter case by a different numerical method. The medium can be reconstructed by identifying the physical or geometric parameters that characterize it. In each case, the aim is to solve a non-linear, ill-posed inverse problem. To solve it, Makutu focuses on full waveform inversion (FWI), which is a high-definition imaging method widely used in geophysics.

4 Application domains

Makutu research program is organized around three principal domains of applications: geophysical exploration, solar imaging, and music. Each of them requires a relevant panel of significant contributions requiring achievements in laboratory measurements, modeling, mathematical analysis, advanced numerical schemes and massively parallel software development. Recently, the team has added experimental contributions to feed simulations with real data and also improve modeling through better calibration. Makutu's application domains can be regrouped into a long-standing activity dedicated to subsurface imaging, and two more recent activities dedicated to solar imaging and the development of numerical wind instruments. Each field of application is not compartmentalized in the methodological sense of the term: equations, numerical schemes and programming practices are shared and possibly adapted to the underlying application.

4.1 Geophysical exploration

Geophysical exploration is a historical field for the team (see e.g [39, 41, 42, 43].Geophysical exploration has been driven for a very long time by the goal of finding hydrocarbons. Today, it is evolving towards a very proactive direction in favor of renewable energies and Makutu commits part of its research activities in this direction, in the framework of industrial and international collaborations. Industrial partnership with TotalEnergies has evolved to the transformation of Makutu into an industrial project-team since January 2022. The dedicated research project targets monitoring of CO2 storage through the development of a new numerical branch in GEOSX for seismic propagation and inversion. As far as geothermical energy is concerned, Makutu is member of the international project SEE4GEO lead by C. Morency from Lawrence Livermore National Laboratory. The project combines experimental research in the field and in laboratory with numerical developements in the continuity of CHICkPEA project previously funded by UPPA (2018-2021).

Inversion is central for geophysical exploration and Makutu focuses on Full Waveform Inversion (FWI) as a high-fidelity solution methodology for reconstructing the physical parameters from observed data. FWI can be carried out in time-domain [40, 53, 60, 59] or in frequency domain [56, 57, 55]. Its main feature is to avoid the formation of the large Jacobian matrix by computing the gradient of the misfit functional using the adjoint-state method [44]. A detailed review of FWI for geophysical applications can be found in [54].

4.1.1 Deep geothermal energy

Obtaining accurate images of natural reservoirs is critical for their management and exploitation and seismic imaging is an efficient tool (see [52, 51] and their references therein. One example is with deep geothermal energy which requires precise imaging of deep fractured reservoirs filled with geothermal fluids. Standard seismic imaging is based upon inverting mechanical waves which have difficulties to detect them, whereas electromagnetic waves are more sensitive. We see here a clear interest of coupling seismic with electromagnetic methods and this is what Makutu began developing with CHICkPEA project ended in 2021. The team is now involved in project **SEE4GEO** funded by ADEME, in the framework of Geothermica call.

4.1.2 CO2 injection monitoring

The reduction of greenhouse gases in the atmosphere is a societal topic of the utmost importance, with the Paris Agreement setting ambitious goals for many countries. One fundamental pillar of greenhouse emission management is Carbon Capture Utilisation and Storage (CCUS) [61]. With this strategy, carbon dioxide produced on- or off-site is sequestered and injected into depleted reservoirs, thus offsetting an important portion of current CO2 emissions. The successful and safe implementation of this strategy requires the prediction, monitoring and surveillance of stored CO2 over long periods, which presents significant challenges in terms of seismic acquisition, seismic inversion and numerical simulation. These tools, coupled with state-of-the-art flow simulations, are vital in order to support the injection operations with vital real-time and long-term information. Moreover, specific challenges related to the physics of injected CO2, such as viscosity, temperature and multi-phase fluid conditions push to the limits our current numerical models, and require ambitious new multi-physics simulations to support safe and cost-effective CO2 injection operations. For example, some recent publications like [58, 62] have shown that the combination of CO2-brine flow with wave propagation provides efficient simulations for the monitoring of sequestered CO2. Makutu is currently developing numerical methods for this new application, in collaboration with TotalEnergies, as a new computational branch of the open-source multiphysics simulator GEOSX.

4.2 Solar imaging

Helioseismology studies the interior and dynamics of the Sun based on the observation of wave oscillation in the solar photosphere. These movements can be observed at the surface by the Dopplergrams given by ground-based or satellite-borne observatories. In recent years, methods for understanding Earth subsurfaces have opened up new ways to study the interior of the Sun as in the case with helioseismology and the interior of stars with aesteroseismology from oscillation observed at their surface. Techniques in helioseisomolgy is generally divided into global and local helioseismology. The first approach studies frequencies of oscillations modes, cf. [45]. On the other hand, local helioseimology, which adapts techniques of geophysical seismic interferometry studies, measures local wave propagation and works with the full 3D observed wavefield, and is thus more adapted to study additional features such large-scale flows in active region, sun spots and plage, cf. [50, 49].

Makutu extends its activity on terrestrial seismology to studying the Sun, for the latter offers a vast wealth of problems to be explored both for direct modeling as well as inversion. The collaboration between Makutu and the solar group at the Max Planck institute for Solar research MPS at MPS brings together the expertise of MPS in solar physics and seismology and that of Makutu in numerical simulation of wave propagation and large-scale inversion in geophysics. This ongoing collaboration dating from 2016 with the creation of associated team ANTS which started in 2019 and ended in 2022. The main goal of the collaboration is the creation of a computational framework for accurate and efficient simulation of solar oscillation to be used in full wave-form inversion, e.g. for 3D solar flow.

The stochastic nature of solar oscillation is described by random right-hand source term, and in using statistical analysis, under appropriate assumptions (e.g. the convenient source assumption), power spectrums and time-distance diagrams can obtained from the deterministic Green kernel of modeling wave equation, cf. [48]. In this way, the Green kernel becomes a crucial object in local helioseismology, and its accurate and efficient computation is the main goal of forward modeling. In addition to appropriate numerical schemes, investigation of radiation boundary conditions is required in order to describe accurately waves above cut-off frequencies.

Up until 2021, the focus has been put on acoustic waves which are identified with p-ridges in observed solar power spectrum. Acoustic waves at low frequencies can be adequately described by a scalar equation. Recent and ongoing works extend the investigation to vector wave equation to include gravity and differential rotation. The latter is particularly of interest due to the recent discovery of inertial waves in the Sun. This is subject of the thesis of Lola Chabat which starts in October 2022, and the goal of which is to create an in-house software to compare accurately eigenvalues for the solar wave equation with differential rotation. The remaining challenge is to include full 3d flow to the vector equation.

The above works lay the necessary foundation for inversion of solar parameters such as flow and active region sound speed. Current state-of-the art tools in these references is linear inversion using Born approximation [47]. In additional they are carried out in 1D or 2D. It is thus interesting to apply nonlinear inversion such as Full Waveform Inversion in 3D cf. [46] to these problems.

4.2.1 Aeroacoustics

The development of numerical simulations for aircraft is a major challenge for the aeronautical industry. These simulations make it possible to predict the noise generated by aircraft, which has a significant impact on passengers, and above all on airline employees and residents living near airports.

The idea here is to rely on solvers based on realistic linear models, whose computational costs are much lower than those associated with nonlinear models from fluid mechanics. These methods are then coupled in order to perform localized computations in very specific regions where nonlinear effects must be taken into account.

The design of new aircrafts, in connection with emerging propulsion technologies (hydrogen, electric, or sustainable aviation fuels), makes these developments particularly timely. The objective is to propose viable solutions for the reduction of aeronautical noise pollution.

4.2.2 Electromagnetism

The work carried out falls within the scope of numerical simulation of electromagnetic waves in large-scale complex environments, particularly in regimes where the size of the propagation domains is very large compared to the wavelength. These issues lie at the core of many industrial and strategic applications that require robust, accurate simulation tools compatible with high-performance computing constraints.

A major target application concerns the planning and optimization of telecommunication networks in highly connected urban and peri-urban environments. In such contexts, electromagnetic wave propagation is governed by complex phenomena involving multiple scattering paths, reflections and diffractions, as well as shadowing effects induced by buildings, infrastructures, and various obstacles. The developed simulation tools make it possible to model these environments at large scale and to optimally determine the placement of communication relays, in scenarios involving fixed antennas, connected vehicles, drones, and low-power personal devices.

These works also have direct implications for defense applications, particularly in the context of electromagnetic environment control in urban areas. They contribute to the analysis and optimization of tactical communications, the assessment of radio coverage under degraded conditions, and the simulation of complex scenarios involving heterogeneous and dynamic environments.

5 Social and environmental responsibility

Makutu recognizes the importance of conducting research in a responsible and sustainable way. We are committed to ensuring that our work has a positive impact on society and the environment.

In terms of social responsibility, Makutu members ensure that their research is inclusive and accessible to all members of society. The team prides itself on bringing together researchers from diverse social and cultural backgrounds. It makes its results and publications available to the general public and is involved in scientific dissemination activities.

In terms of environmental responsibility, Makutu strives to minimize the environmental impact of its research. Wherever possible, the team works to reduce its carbon footprint by implementing environmentally friendly practices and maintaining remote collaborations to limit international travel. It is also engaged in a research program dedicated to sustainable energy. In particular, it is contributing to the development of advanced software for monitoring CO2 storage and is studying complex models that can assist in the development of geothermal drilling by avoiding the devastating creation of micro-earthquakes.

Overall, Makutu is committed to conducting research in a responsible and sustainable manner and is committed to having a positive impact on society and the environment.

6 Highlights of the year

6.1 HR News

- Sébastien Tordeux has been promoted University professor
- Ibrahima Djiba, former PhD student who defended his PhD thesis in March 2025, has been warded a fixed-term contract with CS Group in Pau.
- Matthias Rivet, former PhD student who defended his PhD thesis in March 2025, has been warded a permanent contract with Dassault systems.

6.2 Scientific events

- Makutu organized the conference JOSO (Journées Ondes du Sud-Ouest) which has been held in Pau,
- The work of Makutu with the Max Planck Institute on the modeling of 3D Solar oscillations has been presented at the Solar Dynamis Obersvatory (SDO) 2025 conference “A Gathering of the Helio Hive”.
- As part of the joint ANR-DFG project Butterfly with the University of Göttingen, Makutu participated in the International Workshop on Low-Frequency Oscillations of the Sun and Star at the Harnack House in Berlin.

7 Latest software developments, platforms, open data

7.1 Latest software developments

7.1.1 OpenWind

Name: Open Wind Instrument Design

Keywords: Wave propagation, Inverse problem, Experimental mechanics, Time Domain, Physical simulation

Scientific Description: Implementation of first order finite elements for wind musical instrument simulation. Implementation of the Full Waveform inversion method for wind musical instrument inversion. Implementation of energy consistent numerical schemes for time domain simulation of reed-type wind musical instrument.

Functional Description: Simulation and inversion of wind musical instruments using one-dimensional finite element method with tonholes or valves and fingering chart. The software has three functionnalities. First, the software takes the shape of a wind instrument and computes the acoustical response (answer to a given frequential excitation). Second, the software takes the instrument shape and the control parameters of a musician, and computes the produced sound and the time evolution of many acoustical quantities. Last, the software takes a measured acoustical response and computes the corresponding instrument geometry (inner bore and tone holes parameters).

Release Contributions: - Access to the energy field along the instrument in the frequency domain - possibility to partially closed the embouchure hole of flute-like instrument - Macro to generate 3D file of instruments - New elements in documentation and example - fix some issues

URL: `https://openwind.inria.fr`

Publications: hal-02984478, hal-02996142, hal-03132474, hal-02917351, hal-04898462, hal-02432750, hal-04563493, hal-02019515, hal-04563493, hal-03231946, hal-03328715, hal-03794474, hal-01963674, hal-04008847, hal-04217988

Contact: Augustin Ernoult

Participants: Augustin Ernoult, 6 anonymous participants

Partner: Sorbonne Université

7.1.2 Hou10ni

Keywords: 2D, 3D, Elastodynamic equations, Acoustic equation, Elastoacoustic, Frequency Domain, Time Domain, Discontinuous Galerkin

Scientific Description: Hou10ni simulates acoustic and elastic wave propagation in time domain and in harmonic domain, in 2D and in 3D. It is also able to model elasto acoustic coupling. The time domain solver is based on the second order formulation of the wave equation and the space discretization is achieved using Interior Penalty Discontinuous Galerkin (IPDG) Method. Both IPDG and Hybridizable Discontinuous Galerkin (HDG) Methods are implemented in the frequency domain solver. Recently, the

HDG version has been extender to poroelastic and conducting poroelastic (poroelastic+electromagnetic) media.

Functional Description: This software simulates the propagation of waves in heterogeneous 2D and 3D media in time-domain and in frequency domain. It is based on an Interior Penalty Discontinuous Galerkin Method (IPDGM) and allows for the use of meshes composed of cells of various order (p-adaptivity in space).

URL: `https://team.inria.fr/magique3d/software/hou10ni/`

Publications: hal-04394440, hal-03948879, hal-01513597, hal-01957131, hal-01388195, hal-01972134, hal-01957147, hal-02152117, hal-02486942, hal-02408315, hal-02911686, hal-03464413, tel-03442300, tel-03014772, hal-01656440, hal-01662677, hal-01623953, hal-01623952, hal-01513597, hal-01519168, hal-01254194, hal-01400663, hal-01400656, hal-01400643, hal-01313013, hal-01303391, hal-01408981, tel-01304349, hal-01184090, hal-01223344, hal-01207897, hal-01184111, hal-01184110, hal-01184107, hal-01207906, hal-01184104, hal-01207886, hal-01176854, hal-01408705, hal-01408700, tel-01292824, hal-01656440, hal-00931852, hal-01096390, hal-01096392, hal-01096385, hal-01096324, hal-01096318, tel-01133713, tel-00880628

Contact: Julien Diaz

Participant: 9 anonymous participants

7.1.3 Hawen

Name: time-HArmonic waVe modEling and INversion using Hybridizable Discontinuous Galerkin Discretization

Keywords: Digital twin, Inverse problems, 3D modeling, Wave Equations, Wave propagation, Helioseismology, Geophysics, Medical imaging

Scientific Description: Many applications such as seismic and medical imaging, material sciences, helioseismology, and planetary science, aim to reconstruct properties of a non-directly accessible or non-visible interior. For this purpose, they rely on waves whose propagation through a medium interrelates with the physical properties (density, sound speed, etc.) of this medium. Hawen is a software designed to perform imaging with waves, following an algorithm that comprises of two main stages: In the data acquisition stage, the medium response to probing waves is recorded (e.g., seismic waves from Earthquakes recorded by ground network). In the second stage, we rely on a reconstruction procedure that iteratively updates an initial model of physical parameters, so that numerical simulations approach the measurements. This procedure is employed, for instance, for seismic (reconstruction of subsurface layers) and medical (disease diagnostic) imaging.

Functional Description: The software allows the reconstruction of the physical properties of a media using waves propagating therein. For instance, Hawen allows the recovery of the physical properties of the Earth and the Sun for the observations of surface oscillations. Such applications are of interest in geophysics and helioseismology.

Release Contributions: Compared with the 10/2024 version (v1.3.0 -> v1.5.0) - Implementation of cross-correlation modeling - Implementation of cross-correlation inversion - Include additional options for saving formats - Include latest MUMPS options - Improve accuracy of HDG local operations

URL: `https://ffaucher.gitlab.io/hawen-website/`

Publications: hal-04336798, hal-04356602, hal-04087228, hal-04503374v1, hal-04503407v1, hal-03871831, hal-03877239, hal-03406861, hal-02982650, hal-03101659, hal-03101642, hal-02982619

Contact: Florian Faucher

Participant: 3 anonymous participants

7.1.4 MONTJOIE

Keywords: High order finite elements, Edge elements, Aeroacoustics, High order time schemes

Scientific Description: Montjoie is designed for the efficient solution of time-domain and time-harmonic linear partial differential equations using high-order finite element methods. This code is mainly written for quadrilateral/hexahedral finite elements, partial implementations of triangular/tetrahedral elements are provided. The equations solved by this code, come from the ”wave propagation” problems, particularly acoustic, electromagnetic, aeroacoustic, elastodynamic problems.

Functional Description: Montjoie is a code that provides a C++ framework for solving partial differential equations on unstructured meshes with finite element-like methods (continuous finite element, discontinuous Galerkin formulation, edge elements and facet elements). The handling of mixed elements (tetrahedra, prisms, pyramids and hexahedra) has been implemented for these different types of finite elements methods. Several applications are currently available : wave equation, elastodynamics, aeroacoustics, Maxwell’s equations.

URL: `https://www.math.u-bordeaux.fr/~durufle/montjoie`

Contact: Marc Durufle

Participant: 3 anonymous participants

7.1.5 GEOSX

Keywords: Physical simulation, Multiphysics modelling

Functional Description: GEOSX is an open-source, multiphysics simulator developed cooperatively by Lawrence Livermore National Laboratory, Stanford University, and TotalEnergies. Its goal is to open up new horizons in modeling carbon storage and other subsurface energy systems. This includes: - taking advantage of the ongoing revolution in high-performance computing hardware, which is enabling orders-of-magnitude gains in performance, but also forcing a fundamental rethink of our software designs, - enriching the physics used in industrial simulations, allowing complex fluid flow, thermal, and geomechanical effects to be handled in a seamless manner, - developing highly-scalable algorithms for solving these coupled systems, - and improving workflows for modeling faults, fractures, and complex geologic formations. Inria contributes to the seismic wave propagators of GEOSX, and to its python interface. Inria also contributes advanced workflows for seismic inversion, and CO2 storage an monitoring.

URL: `http://www.geosx.org/`

Contact: Randolph Settgast

7.1.6 Gar6more2D

Keywords: Validation, Wave propagation

Functional Description: This code computes the analytical solution of problems of waves propagation in two layered 3D media such as- acoustic/acoustic- acoustic/elastodynamic- acoustic/porous- porous/porous,based on the Cagniard-de Hoop method.

URL: `https://gitlab.inria.fr/jdiaz/gar6more2d`

Publications: inria-00274136, inria-00404224, inria-00305395

Contact: Julien Diaz

Participant: 2 anonymous participants

Partner: Université de Pau et des Pays de l’Adour

7.1.7 GoTem3

Keywords: Trefftz, Computational electromagnetics, HPC, Domain decomposition

Functional Description: GoTem3 is domain decomposition platform based on the ultra-weak formulation of Cessenat and Després for the solution of diffraction problems posed on regular grids. It uses matrix free strategies as well as local and global preconditioners to solve cases involving more than a billion degrees of freedom on a single computational core.

News of the Year: The code has been endowed with the ability to account for basic quasiTrefftz functions derived from an auxiliary code for solving electromagnetic wave equations using the flux reconstruction and spectral difference methods. This work was implemented as part of Matthias Rivet's thesis.

Publications: hal-03945383, tel-04172930, hal-03642116

Contact: Sebastien Tordeux

Participant: 4 anonymous participants

7.2 New platforms

FUnTiDES: Fast Unstructured Time Dynamic Equation Solver

Participants: Alexis Bandet, Henri Calandra, Aurélien Citrain, Stefano Frambati, Jie Meng.

FUnTiDES is a collection of simplified codes that represent real scientific applications. It serves as a standard tool for evaluating and comparing the performance of various high-performance computing (HPC) systems, particularly those used for scientific simulations. Included Applications

The current implementation includes two proxy applications for solving the 2nd-order acoustic wave equation in 2D and 3D:

SEM (Spectral Element Method) A benchmark designed to simulate wave propagation using SEM, a Galerkin-based finite element method for solving partial differential equations (PDEs).

FD (Finite Difference Method) A benchmark that uses finite-difference stencil operators to simulate wave propagation and solve PDEs.

A key feature of these proxy applications is their adaptability to different programming models and HPC architectures. They are also easy to build and run, making them accessible to both researchers and developers.

8 New results

8.1 Methodological contributions to the simulation of mechanical and electromagnetic waves in complex media

8.1.1 Enhanced finite element methods using neural networks

Participants: Hélène Barucq, Florian Faucher.

In this work, we present a study combining two approaches in the context of solving PDEs: the continuous finite element method (FEM) and more recent techniques based on neural networks. In recent years, physics-informed neural networks (PINNs) have become particularly interesting for rapidly solving PDEs, especially in high dimensions. However, their lack of accuracy can be a significant drawback in this context, hence the interest in combining them with FEM, for which error estimates are already known. The complete pipeline proposed here consists in modifying the classical FEM approximation spaces by taking information from a prior, chosen as the prediction of a neural network. On the one hand, this combination improves and certifies the prediction of neural networks, to obtain a fast and accurate solution. On the other hand, error estimates are proven, showing that such strategies outperform classical ones by a factor that depends only on the quality of the prior. We validate our approach with numerical results performed on parametric problems with 1D, 2D and 3D geometries. These experiments demonstrate that to achieve a given accuracy, a coarser mesh can be used with our enriched FEM compared to the standard FEM, leading to reduced computational time, particularly for parametric problems.

This is a joint work with Frédérique Lecourtier, Michel Duprez, Emmanuel Franck, Vanessa Lleras, Victor Michel-Dansac and Nicolas Victorion. A preprint is available online: [27].

8.1.2 A Model Order Reduction Strategy for Parametrized PDEs: A New Paradigm for Efficient Subsurface Imaging

Participants: Hélène Barucq, Julien Besset, Stefano Frambati.

Subsurface exploration plays a crucial role in many fields, ranging from energy production (oil, gas, geothermal) to civil engineering and environmental issues such as CO2 storage. This thesis is situated within the framework of subsurface imaging, where the goal is to reconstruct the internal properties of the subsurface from recordings of artificially generated wavefields. This process, known as Full Waveform Inversion (FWI), relies on the repeated solution of wave equations, resulting in high computational costs, particularly in high-resolution and multi-parameter contexts. To address these challenges, this thesis explores model order reduction (MOR) approaches, which aim to reduce the dimensionality of the systems to be solved while preserving their essential dynamics. After presenting the foundations of FWI in the context of acoustic wave propagation, as well as the spectral element method used for discretization, the study focuses on the Proper Orthogonal Decomposition (POD) method and its application to the acoustic wave equation problem. It then introduces a variant using a QR decomposition, designed to mitigate the memory costs of the classical POD approach while maintaining a similar level of accuracy. One of the major challenges of Reduced Order Models (ROMs) lies in their sensitivity to parameter variations. To address this, the thesis proposes a method based on the Fréchet derivatives of the problem, enabling the construction of reduced bases that are more robust to parameter changes. This method is validated on 2D and 3D acoustic problems and then integrated into an FWI framework via the GEOS platform. This work makes an original contribution to the efficient solution of inverse problems in geophysics by combining advanced numerical methods with model reduction, paving the way for large-scale applications with reduced computational costs.

This is the topic of Julien Besset Ph.D. thesis, [24] which has been defended in June, the 23th.

8.1.3 Dynamic seismo-electric coupling : from frequency to time domain models

Participants: Hélène Barucq, Julien Diaz, Arjeta Heta.

This project deals with the mathematical modelling of seismo-electric effects which occur in porous media comprised of charged fluid within an oppositely-charged solid matrix. As such, the medium is neutral at a macroscopic scale, but relative displacements between the solid and fluid induce electrical currents (displacement of charges). Seismic waves propagating in these media induce local fluid flow, hence charge displacements, leading to the creation of electromagnetic waves. Mathematically, this is modelled by Biot's equations of poro-elasticity coupled to Maxwell's equations. The coupling theory derived by S. Pride in 1994 is carried out in frequency domain, and the coupling quantities (coupling coefficient and dynamic permeability) depend on the frequency, in a manner that leads to difficulties in time domain. By interpreting these parameters as symbols of pseudo-differential operators that are global in time, it becomes clear that the model in time domain involves operators whose discretization will lead to very high computational cost. This explains why simulations carried out in time domain always involve a very low frequency model in which the frequency dependence of these parameters is masked.Two main developments are carried out. The first deals with the approximation of the frequency-dependent coupling operators, in order to obtain higher fidelity time domain seismo-electric equations, i.e. valid for a larger frequency range than the very low frequency approximations. This opens the perspective of using time domain equations in a laboratory scale. This thesis introduces several polynomial and rational approximation types and studies them numerically, leading to new time domain equations, which take into account the dynamic nature of the coupling. At low frequencies, local approximants such as Padé or Taylor (very low frequency) can be used. Global approximations — Legendre and Chebyshev — are used with high frequencies, or a wide frequency band. These approximations are tested numerically using analytical solutions to compute, as well as with a numerical code based on the hydridizable discontinuous Galerkin method in an HPC context.The second development deals with the mathematical analysis of Pride's equations in time domain with strong (bidirectional) coupling. Considering this bidirectional coupling introduces difficulties to show the well-posedness of the problem. As such, only one-way coupling has been considered to show well-posedness of the equations, that is electro-seismic coupling only. Two approaches are introduced in a low frequency approximation of the system. Firstly, an application of the Hille-Yosida Theorem to the full system. Secondly, we use a fixed-point technique, to show there exists a unique solution to the fully coupled system in a weak sense.

This is the topic of Arjeta Heta Ph.D. thesis, [26] which has been defended in January, the 24th.

8.1.4 MOR-T L: A Novel Model Order Reduction Method for Parametrized Problems with Application to Seismic Wave Propagation

Participants: Hélène Barucq, Julien Besset, Stefano Frambati.

This work presents an efficient strategy for constructing Reduced-Order Model (ROM) bases using Taylor polynomial expansions and Fréchet derivatives with respect to model parameters. The proposed approach enables the construction of ROM bases with minimal additional computational cost. By exploiting Fréchet derivatives -solution to the same problem with distinct right-hand sides -the method introduces a streamlined multiple-right-hand-side (RHS) strategy for ROM bases construction. This approach not only reduces overall computational expenses but also improves accuracy during model parameter updates. Numerical experiments on a two-dimensional wave problem demonstrate significant efficiency gains and enhanced performance, highlighting the potential of the proposed method to advance computational cost-effectiveness, particularly in seismic inversion applications.

This is a joint work with Rabia Djellouli (Northridge University), a preprint has been written, [28] and is currentlt under revision for publication.

8.1.5 Quasi-Trefftz Method for Aeroacoustics: Part I - Model Problem and Application to the Helmholtz Equation

Participants: Andréa Lagardère, Sébastien Tordeux.

This work presents a quasi-Trefftz Discontinuous Galerkin method designed for the numerical resolution of problems governed by the Helmholtz equation. The method relies on local approximate solutions of the equation as test and trial functions. The formulation is based on a first-order system expressed within the Friedrichs framework. Three families of basis functions are considered: plane waves modulated in phase or amplitude, and polynomials. These are extended to the vector-valued setting, marking a key novelty in this work, and used within a variational formulation compatible with the Friedrichs structure. Numerical experiments confirm the expected convergence rate and demonstrate that the method achieves comparable accuracy with significantly fewer degrees of freedom compared to classical Discontinuous Galerkin approaches. This work lays the foundation for further developments, including the extension to the convected Helmholtz equation for the design of efficient numerical tools for aeroacoustic simulations.

This is a joint work with Lise-Marie Imbert-Gérard (University of Arizona) and Guillaume Sylvand (Airbus), preprints have been written, [32, 33] and a talk has been given in Tucson [13] and in a SIAM conference [19].

8.1.6 Trefftz Method for a Class of Time-Harmonic Two-Fields Friedrichs Systems

Participants: Matthias Rivet, Sébastien Tordeux.

This work presents a class of two-fields Friedrichs systems, allowing to encompass classic time-harmonic wave propagation problems into a unique formalism: notions of incoming and outgoing traces, in addition to a normal flux decomposition and a consistent numerical flux expression are defined in this general setting. Then, a Trefftz method is introduced for this class of problems: formulations based on the reciprocity formula or the Ultra Weak Variational Formulation are defined, allowing to prove weak-coercivity and contraction properties of the preconditioned system. Finally, we discuss the interpretation of the method from the point of view of Domain Decomposition methods, and recall classic error estimates results and discretisation by plane waves.

This is a joint work with Sébastien Pernet (Onera), preprints has been written,[34] and the work has been presented in conferences, [16, 36, 15].

8.1.7 Trefftz methods for solving large-scale time-harmonic wave problems

Participants: Hélène Barucq, Ibrahima Djiba, Sébastien Tordeux.

Wave propagation is a complex physical phenomenon that makes the invisible visible by solving an inverse problem. The underlying mathematical model can be formulated in either the time or frequency regime, each with its own advantages and disadvantages. Here, we prefer the frequency domain, which makes it easier to take into account physical parameters such as attenuation. In this case, direct problem solving, crucial in the inversion algorithm, is very costly and the size of the system to be solved quickly reaches the limits of direct linear solvers. Here, we propose a numerical method that relaxes memory constraints by adopting an iterative approach. To this end, we construct an iterative method that belongs to the class of Discontinuous Galerkin Trefftz methods. By reducing calculations to the level of the mesh skeleton, these are known to use less memory than conventional finite element methods. The method is explained in detail in the 1D case, describing its main features, which are a legacy of Trefftz's idea of using approximation spaces composed of special functions, in this case plane waves. This leads to a wellposed discrete problem that can be solved by an iterative block Jacobi method. The method's performance is illustrated in 1D and 3D.

This is a joint work with Abderrahmane Bendali (Toulouse), and a book chapter has been published, [22].

Ibrahima Djiba has also defended his PhD [25]

8.1.8 Discretization error analysis of a high-order unfitted space–time method for moving domain problems

Participants: Janosch Preuss.

We present a numerical analysis of a higher-order unfitted space-time finite element method applied to a convection-diffusion model problem posed on a moving bulk domain. The method uses isoparametric space-time mappings for the geometry approximation of level set domains and has been presented and investigated computationally in Heimann, Lehrenfeld and Preuss (2023, SIAM J. Sci. Comp. 45(2), B139 - B165). Recently, in Heimann and Lehrenfeld (2025, IMA J. Numer. Anal. 45(6):3643-3697) error bounds for the geometry approximation have been proven. In this paper we prove stability and accuracy including the influence of the geometry approximation.

This is a joint work with Fabian Heimann and Christoph Lehrenfeld (University of Göttingen), it is published in the IMA Journal of Numerical Analysis, [10].

8.1.9 Unique continuation for the wave equation: the stability landscape

Participants: Janosch Preuss.

We consider a unique continuation problem for the wave equation given data in a volumetric subset of the space time domain. In the absence of data on the lateral boundary of the space-time cylinder we prove that the solution can be continued with Hölder stability into a certain proper subset of the space-time domain. Additionally, we show that unique continuation of the solution to the entire space-time cylinder with Lipschitz stability is possible given the knowledge of a suitable finite dimensional space in which the trace of the solution on the lateral boundary is contained. These results allow us to design a finite element method that provably converges to the exact solution at a rate that mirrors the stability properties of the continuous problem.

This is a joint work with Lauri Oksana and Ziyao Zhao (University of Helsinki), and Erik Burmann (University College London), a preprint has been written, [29] and it has been presented in conference, [17].

8.1.10 Variational data assimilation for the wave equation in heterogeneous media

Participants: Janosch Preuss.

In recent years, several numerical methods for solving the unique continuation problem for the wave equation in a homogeneous medium with given data on the lateral boundary of the space-time cylinder have been proposed. This problem enjoys Lipschitz stability if the geometric control condition is fulfilled, which allows devising optimally convergent numerical methods. In this article, we investigate whether these results carry over to the case in which the medium exhibits a jump discontinuity. Our numerical experiments suggest a positive answer. However, we also observe that the presence of discontinuities in the medium renders the computations far more demanding than in the homogeneous case.

This is a joint work with Erik Burmann (University College London), a preprint has been written, [30].

8.1.11 Performance Analysis and CUDA Acceleration of the Open Source Software Hawen

Participants: Florian Faucher, Marc Fuentes, Eduard Occhipinti.

In this work, we present improvements on open-source software Hawen, used to solve the wave problem in the frequency domain. The software can be used to both model the propagation of waves and solve the inverse problem, which consists in the reconstruction of the characteristics of the media in which the waves propagated. Hawen relies on a hybridizable discontinuous Galerkin discretization which makes heavy usage of operations on dense matrices. In this work, we will focus on improving the performance of the code by ensuring that these operations are executed efficiently. We will analyze the current state of libraries and compilers for the Fortran language. In particular, we will work with NVIDIA's NVHPC Toolkit and explore effective strategies for parallelization on GPU whilst maintaining the current combination of MPI and OpenMP parallelism. Another part will concern introducing a recently released GPU accelerated sparse solver, cuDSS, as alternative to the current one, MUMPS, to address the global linear system.

This work is the topic of Eduard Occhipinti Master thesis.

8.1.12 A partitioned thermoelastic coupling using displacement-dependent effective conductivity: application to an HPC framework

Participants: Hélène Barucq, Pierre Dubois.

In partitioned thermomechanical couplings, the influence of mechanical displacement on the thermal problem during the iterative solution process is generally neglected or handled by solving the thermal model on the deformed geometry. As this strategy can be limiting, we investigate an approach based on an effective conductivity that accounts for the displacement, allowing the thermal computations to be performed on the reference configuration. This approach is first validated in Cast3M and then implemented in the MFEM library, relying on distributed-memory parallelization. This is a collaboration with Isabelle Ramière and Raphaël Prat from CEA Cadarache, within the project Exa-MA of Numpex and in the context of Pierre Dubois PhD thesis. An extended abstract has been submitted to the next national conference "17ème Colloque National en Calcul des Structures" which will be held in the Presqu'île de Giens, in May 2026. The elasto-acoustic case is now under study.

8.2 Seismic imaging

8.2.1 A numerical study on the sensitivity of DAS and geophone signals to a thin CO2 plume

Participants: Hélène Barucq, Henri Calandra, Florian Faucher, Stefano Frambati, Chengyi Shen.

One of the main concerns in CO2 monitoring during a CCUS (Carbon Capture, Utilization, and Storage) project is the detectability of changes in petrophysical properties induced by the substitution of the initial fluids by CO2 . Seismic attributes are thought to have the potential for directly tracking changes in geophysical properties such as the wave velocities. The emerging Distributed Acoustic Sensing (DAS), in addition to the traditional sensors such as geophones, is bringing new perspectives in seismic acquisitions and attribute analyses. We study the sensitivity of different seismic attributes to a CO2 plume with numerical simulations of wave propagation. An efficient visco-elastic wave problem solver featuring the Spectral Element Method and memory variables is built inside the GEOS platform to calculate DAS and geophone responses in the context of a Vertical Seismic Profile (VSP) acquisition. The synthetic data allow us to compare quantitatively the VSP records from DAS and geophones for the purpose of discussing the CO2 plume detectability with different sensors, for example the DAS vertical normal strain EZZ versus the geophone horizontal (UX) and vertical (UZ) displacements.

This is a joint-work with Estelle Rebel (Total Energies) and a preprint has been submitted.

8.2.2 Time-harmonic cross-correlation inversion for passive imaging

Participants: Jean Dutheil, Florian Faucher.

The objective of passive imaging is to use these ambient data that come from the superposition of stochastic events, to reconstruct the inner properties of the medium. This approach is also referred to as 'ambient noise imaging'. In order to be able to reconstruct the properties of the medium, one must first connect to some deterministic objects that we can analyze with the wave equations. This task can be achieved by using the cross-correlation of the measured signals, which gives us the relation with the deterministic solution of the wave equation which is the Green's function. Then we study the quantitative inversion algorithm based upon an iterative minimization for passive imaging. Contrary to active-source imaging, additional operations have to be added, in particular, to compute the gradient of the misfit function. Eventually, we carry out numerical experiments of inversion.

This work is the topic of Jean Dutheil Ph.D. thesis.

8.2.3 Quantitative inverse problem in ultrasound imaging for viscoelastic anisotropy

Participants: Florian Faucher.

We consider the quantitative inverse problem for reconstructing physical properties in viscoelastic anisotropic media using wave data-sets. The time-harmonic formulation of the anisotropic elastic wave equations is used to facilitate handling different models of viscosity. The system is discretized with the hybridizable discontinuous Galerkin (HDG) method which employs static condensation to reduce the computational cost, although requiring non-trivial stabilization term for efficiency. The nonlinear inversion algorithm is performed following a minimization process in which the model parameters are iteratively updated. We carry out reconstructions with attenuation model uncertainty, and emphasize the importance of considering anisotropy in the model with synthetic experiments for ultrasound imaging.

This work is joint with Otmar Scherzer (University of Vienna), it has been presented at SFB conference in Strobl, [11].

8.2.4 Accelerating Full Waveform Inversion with Reduced Ordre Modeling

Participants: Hélène Barucq, Julien Besset, Victor Martins Gomes.

In this work, we propose to use Model Order Reduction to accelerate the acoustic Full Waveform Inversion. Our approach differs from existing works as it is based upon a MOR technique that reduces the update number of basis functions as compared to the update of the velocity model during the inversion. The potential of the approach comes from the fact that it follows the same optimization procedure as the inversion algorithm. Here, we consider the 2D case, adopt the line search algorithm and show that the FWI can be accelerated by a factor two at least. We use the Marmousi and the statoil tests cases for illustrating the performance of the method.

8.2.5 Machine Learning Approaches For CO2 Geological Storage Monitoring And Repeatability of Acquisition.

Participants: Hélène Barucq, Henri Calandra, Stefano Frambati, Manon Sarrouilhe.

CO2 geological storage is an important goal for reaching long-term carbon neutrality and seismic monitoring will play a crucial role in making sure that storage is safe, effective and permanent. Traditional technologies for the monitoring of CO2 injection rely on geophysical techniques such as seismic surveying, gravimetry and electromagnetic methods. Due to limitations of current numerical methods, new approaches are needed for the monitoring operations. Some recent Machine Learning techniques could help us to face these challenges and complete traditional approaches rooted in numerical analysis. In this work, we present a method to face the problem of repeatability of acquisitions, which requires to accurately treat systematic errors arising from repeated measurements done at different points in time. To do so, we use the technique of Machine Learning developed by Bharadwaj, Li and Demanet based on autoencoders. This technique also allows to take into account the natural sismicity of the ground without knowing many characteristics of the source, such as its position. This method is potentially very impactful for CO2 geological monitoring because it allows to reduce the cost of continuous monitoring. Realistic data generated with Geos, an open-source code developed by TotalEnergies, Chevron, LLNL and Stanford can be used to illsutrate the potetial of the method. It is worth noting that the method can substantially be improved by changing convolutions by transformers. This work has been presented at the conference Mathematics to Product 2025 [23], in Mathias days [21] and Journées des Ondes du Sud-Ouest [38].

8.3 Helioseismology

8.3.1 Wave and spectral solvers with self-gravitation for radially symmetric adiabatic backgrounds in helioseismology

Participants: Lola Chabat, Hélène Barucq, Florian Faucher, Ha Pham.

Numerical simulations play an important role in helioseismology, which aims to understand the interior of the Sun. With the Sun being the star closest to Earth, its study not only helps monitor the influence of the Sun on Earth and the solar system, for example in the context of space weather, space communication, and exploration, but also provides a unique laboratory to study distant stars. Numerical solvers simulate the different types of waves propagating inside the Sun, which are then compared with observations to constrain its internal structure. The types of waves under investigation are acoustic, gravity and inertial modes. Their computation requires solving large-scale eigenvalue problems and wave propagation problem that exceed the memory capacity of clusters when classical numerical methods are used. The objective of this work is to develop efficient and accurate numerical solvers for the computation of solar oscillation modes, including the full effects of self-gravity by removing the so-called Cowling's approximation. To achieve this, we employ high-order methods from the Discontinuous Galerkin family, in particular the Hybridizable Discontinuous Galerkin (HDG) method for wave propagation and the Local Discontinuous Galerkin (LDG) method for the computation of eigenvalues. The new solvers are implemented on top of the open-source software platform hawen, and are used at the Max-Planck Institute for Solar System Research in Göttingen in ongoing collaborations

This is the Ph.D. thesis of Lola Chabat, and it has been presented at the JOSO conference, [18].

8.3.2 Numerical simulations of oscillations for axisymmetric solar backgrounds with differential rotation and gravity

Participants: Hélène Barucq, Florian Faucher, Ha Pham.

Local helioseismology comprises of imaging and inversion techniques employed to reconstruct the dynamic and interior of the Sun from correlations of oscillations observed on the surface, all of which require modeling solar oscillations and computing Green's kernels. In this context, we implement and investigate the robustness of the Hybridizable Discontinuous Galerkin (HDG) method in solving the equation modeling stellar oscillations for realistic solar backgrounds containing gravity and differential rotation. While a common choice for modeling stellar oscillations is the Galbrun's equation, our working equations are derived from an equivalent variant, involving less regularity in its coefficients, working with Lagrangian displacement and pressure perturbation as unknowns. Under differential rotation and axisymmetric assumption, the system is solved in azimuthal decomposition with the HDG method. Compared to no-gravity approximations, the mathematical nature of the wave operator is now linked to the profile of the solar buoyancy frequency N which encodes gravity, and leads to distinction into regions of elliptic or hyperbolic behavior of the wave operator at zero attenuation. While small attenuation is systematically included to guarantee theoretical well-posedness, the above phenomenon affects the numerical solutions in terms of amplitude and oscillation pattern, and requires a judicious choice of stabilization. We investigate the stabilization of the HDG discretization scheme, and demonstrate its importance to ensure the accuracy of numerical results, which is shown to depend on frequencies relative to N, and on the position of the Dirac source. As validations, the numerical power spectra reproduce accurately the observed effects of the solar rotation on acoustic waves.

This work is joint with Laurent Gizon and Damien Fournier (Max Planck Institute in Göttingen), a preprint is online, cf. [35] and it has been presented at the LFOSS conference, [12].

8.3.3 3D Modeling of Solar Oscillations with Hybridizable Discontinuous Galerkin Method

Participants: Hélène Barucq, Florian Faucher, Ha Pham.

With increasing quantity and quality of solar observations, it becomes essential to account for three-dimensional heterogeneities in wave modeling for seismic data interpretation. In this context, we present a 3D solver of the time-harmonic adiabatic stellar oscillation equations without background flows on a domain consisting of the Sun and its photosphere. The background medium consists of 3D heterogeneities on top of a radial strongly-stratified standard solar model. The oscillation equations are solved with the Hybridizable Discontinuous Galerkin (HDG) method, considering a first-order formulation in terms of the vector displacement and the pressure perturbation. This method combines the high-order accuracy and the parallelism of DG methods while yielding smaller linear systems. These are solved with a direct solver, with block low-rank compression and mixed-precision arithmetic to reduce memory footprint. The trade-off between compression and solution accuracy is investigated, and our 3D solver is validated by comparing with resolution under axial symmetry for solar backgrounds. The capacity of the solver is illustrated with wave speed heterogeneities characteristic of two physical phenomena: active regions and convection. We show the importance of global 3D gravito-acoustic wave simulations, in particular when the amplitudes of the perturbations are strong and their effect on the wavefield cannot be estimated by linear approximations.

This work is joint with Laurent Gizon and Damien Fournier (Max Planck Institute in Göttingen), a preprint is online, cf. [31] and it has been presented at the SDO and LFOSS conference, [12, 37].

8.3.4 Computational aspects of Green's kernels in local helioseismology

Participants: Hélène Barucq, Florian Faucher, Ha Pham.

Helioseismology infers the interior of the Sun from oscillations which are continuously excited by near-surface turbulent convection and observed in the photosphere. Global helioseismology reconstructs global structures from free oscillations which manifest as ridges in power spectrums of Dopplergrams. On the other hand, techniques in local helioseismology, e.g. time-distance helioseismology and far-side helioseismic holography, are based on correlation of Dopplergrams to reconstruct local perturbations (e.g. in flow and sound speed). After choosing a mathematical equation together with boundary conditions to model oscillations, for global seismology, an eigensolver needs to be constructed, while for correlation-based techniques, be this qualitative or quantitative, Born approximation or iterative, a wave solver to compute Green's kernel is required. In this talk, I will give an overview of various Green's kernels we have obtained. These results are achieved with in-house software Hawen, which solves scalar and vector equations modeling solar waves, for backgrounds ranging from radially symmetric standard models with or without Cowling approximation, to differential rotation. The equations with radially symmetric backgrounds are solved in 1D via spherical harmonics decomposition, the ones with differential rotation in 2D via azimuthal decomposition, and general backgrounds without flow and rotation in 3D. These direct solvers serve as essential steps towards inversion, in particular full-wave form inversion. As a first validation, our power spectrums computed for model S show ridges in agreement with HMI observation. Secondly, in simulations with realistic differential rotation, our spectrums reproduce the observed splitting in azimuthal modes due to rotation.

This work is joint with Laurent Gizon and Damien Fournier (Max Planck Institute in Göttingen), it has been presented at SFB conference in Strobl, [14].

8.3.5 HDG-BEM coupling in helioseismology

Participants: Hélène Barucq, Florian Faucher, Ha Pham, Janosch Preuss, Sébastien Tordeux.

We develop an efficient solver for the Green's function in a setting where the background model is perturbed by compactly supported inhomogeneities representing active regions. To this end, the computation is reduced to these active regions and their boundary by means of coupling the volumetric problem to a boundary integral formulation. While our in-house code `hawen` is designed to solve the volumetric problem in the active regions efficiently using an HDG method, it was so far not able to deal with the boundary integral terms. Consequently, a significant part of the year was spent in enriching `hawen` with the functionality of boundary elements and establishing a coupling to the existing HDG discretization. This development has been carried out in the two-dimensional setting for the Helmholtz equation, which allowed for comprehensive studies of convergence and computational efficiency of different HDG-BEM coupling strategies. Note that - in contrast to the Helmholtz equation - the Green's function for the solar background model is not analytically known. Therefore, it is essentiel to study whether accucary and efficiency of the HDG-BEM solver can be preserved if the analytic Green's function is replaced by a Green's function that has been obtained numerically. The investigation of this question is ongoing and promises to yield interesting insights not only for helioseismology but for the field of boundary elements as a whole.

This is part the ANR-DFG project Butterfly joint with the University of Göttinger, [20].

9 Bilateral contracts and grants with industry

- Makutu research agreement

  Participants: Hélène Barucq, Henri Calandra, Florian Faucher, Stefano Frambati, Sébastien Tordeux.

  Period: January 2022 – December 2025; Management: INRIA Bordeaux Sud-Ouest, Amount: 350000 euro per year.

- CIFRE thesis support contract with Airbus

  Participants: Andrea Lagardère, Sébastien Tordeux.

  Period: May 2024 - April 2027,
  PhD scholarship, management Airbus,
  Research Support contract Management: INRIA Bordeaux Sud-Ouest Amount: 15000 euros per year

10 Partnerships and cooperations

10.1 European initiatives

10.1.1 Horizon Europe

INCORWAVE INCORWAVE project on cordis.europa.eu

Title: Nonlinear inversion of correlation waveforms with hierarchical reconstructions

Duration: From January 1, 2024 to December 31, 2028

Partners:

- INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET AUTOMATIQUE (INRIA), France

Inria contact: Florian Faucher

Summary: Waves propagating through a complex medium provide a non-invasive way to probe its interior structures. In ambient noise imaging, the input data are the cross-correlation of the stochastic wavefields. To reconstruct the properties of the medium, the waveform inversion is formulated as an optimization problem involving a misfit function whose convexity plays a critical role in the achievable spatial resolution of the inversion results, especially in the absence of a priori information about the medium. Current inversions are often limited by computational cost, cross-talk between the physical quantities, and the use of single-scattering approximations. Project INCORWAVE proposes to create a new mathematical and computational framework for nonlinear inversion of full waveform cross-correlation. Two specific problems are considered: first, for the reconstruction of geophysical visco-elasticity tensors with applications to Earth's subsurface monitoring; secondly, for the reconstruction of three-dimensional flows in the Sun to characterize the poorly understood properties of deep solar convection. To improve the convexity of misfit functions, the inversion procedure of project INCORWAVE will follow a hierarchical progression which is established by selecting subsets of input data, unknown parameters, and frequencies. The choice of each of these subsets, as well as the associated misfit function, is controlled by criteria in form of convergence estimates. Indispensable to meaningful inversion is accurate modeling operators that describe the physics under consideration and that are adapted to the treatment of real data. For the reconstruction of the elasticity tensor, the project will develop a solver in terms of P- and S-potentials for heterogeneous media. A 3D global Sun vector-wave solver is created for the inversion of the convection component of the solar flow that does not bear symmetry.

10.1.2 Other european programs/initiatives

ANR-DFG program

Title: Stellar butterfly diagrams

Duration: From May 1, 2024 to April 30, 2027

Participants Hélène Barucq, Florian Faucher, Ha Pham, Janoss Preuss, Sébastien Tordeux.

Partners:
- INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET AUTOMATIQUE (INRIA), France
- University of Göttingen.

Inria contact: Hélène Barucq

Coordinator: Hélène Barucq and Laurent Gizon

Summary: This project has emerged out of an ongoing collaboration started in 2016 between the INRIA- Makutu team in Pau led by Hélène Barucq and the Göttingen helioseismology group led by Laurent Gizon. This collaboration focuses on the development of computational and theoretical techniques to study the forward and inverse problems of helioseismology using advanced discretization methods and high-performance computing. In project Butterfly we propose to further develop the computational models and tools that are required to recover information about stellar activity from observations of stellar oscillations (asteroseismology).

Geothermica program

Title: SeismoElectric Effects for GEOthermal resource assessment and monitoring (SEE4GEO)

Duration: From November 1, 2021 to June 30, 2026 Partners:

Participants: Hélène Barucq, Julien Diaz, Arjeta Heta.

Partners:
- University of Hawaii at Mänoa (USA);
- University of Pau and the Pays de l'Adour, UPPA (France);

- TLS Geothermics, TLS (France),
- NORCE (Norway)

Inria contact: Hélène Barucq

Coordinator: Christina Morency (morency1@llnl.gov) LLNL (Lawrence Livermore National Laboratory, US department of Energy)

Summary: Geothermal systems involve the injection of large amounts of fluid into the subsurface. Identifying fracture networks is of great importance to assess geothermal resources. Traditional seismic imaging techniques fail to resolve fluid-phase properties, while purely electromagnetic (EM) approaches provide limited, low-resolution constraints on the rock structure. Seismoelectric effects (SEE) arise from the seismic-to-electromagnetic conversion in naturally charged porous media with a certain degree of fluid saturation. With SEE, we leverage seismic and EM technique sensitivities. In this project, we offer an integrated SEE assessment for geothermal systems relying on numerical modelling, laboratory experiments and field surveys. The project is managed by UPPA (University of Pau and Pays de l'Adour).

10.2 National initiatives

PEPR Numpex - Focused project Exa-MA (Methods and Algorithms for Exascale)

Participants: Hélène Barucq, Henri Calandra, Pierre Dubois, Florian Faucher, Stefano Frambati, Sébastien Tordeux.

The French exascale program aims at designing and developing software bricks that will equip the future exascale computers. Makutu contributes to the topic of advanced discretization and to the design of demonstrators with a focus on large scale inverse problems as demonstrators. Members of Makutu participate in the targeted project Exa-MA working on Mathematical Methods and Algorithms for Exascale. A PhD thesis started in November 2024 in collaboration with CEA Cadarache. A new project has been lauched in collaboration with Mark Asch (Picardie University) on bayesian inversion.

11 Dissemination

11.1 Promoting scientific activities

11.1.1 Scientific events: organisation

- Hélène Barucq and Ha Pham have organized the Journées Ondes du Sud-Ouest (JOSO). The conference took place at the University of Pau and Pays de l'Adour from March 18th to 20th, see link.
- Florian Faucher took part in the organization of the conference on Inverse Problems in Milano. It took place at the University of Milano from Jun 9th to June 13th, see link.
- Hélène Barucq, Florian Faucher, and Ha Pham organized the kick-off workshop meeting for the ANR-DFG project Butterfly which is a joint project with the Univeristy of Göttingen. It was organized at Inria Bordeaux from September 8th to September 10th.

11.1.2 Journal

Reviewer - reviewing activities Members of Makutu are regular reviewers for Journal of Computational Physics, Inverse Problems, Geophysics, SIAM Journal of applied mathematics.

Some members of Makutu have served as reviewers for ERC.

Hélène Barucq is official reviewer for Research Foundation Flanders' (FWO) through the European Science Foundation (www.esf.org) which has been appointed by FWO to implement the independent evaluation process of applications.

11.1.3 Invited talks

- Ha Pham was invited to give a talk on helioseismology at the SFB conference “Tomography across the scales” organized in Strobl, Austria, see link.
- Florian Faucher was invited to give a talk on anisotropic inversion at the SFB conference “Tomography across the scales” organized in Strobl, Austria, see link.
- Florian Faucher was invited to give a talk on solar Green’s kernel computation at the conference “Low-frequency oscillations of the Sun and Stars” organized in Berlin, Germany, see link.
- Hélène Barucq was invited to give a talk on Radiation boundary conditions for truncating the atmosphere of the Sun, joint work with Ibrahima Djiba and Sébastien Tordeux, in honor of Bruno Desprès’s sixtieth birthday, Ecole des Mines de Paris, Juin 2025.
- Hélène Barucq was invited to give a talk on Radiation boundary conditions for truncating the atmosphere of the Sun, joint work with Florian Faucher, Damien Fournier, Laurent Gizon and Ha Pham, in honor of Olivier Lafitte’s sixtieth birthday, Ecole des Mines de Paris, Juillet 2025.

11.1.4 Leadership within the scientific community

- Hélène Barucq is co-heading with Christophe Prud’homme (UNISTRA) the targeted project Exa-MA which is part of the PEPR Numpex.
- Hélène Barucq is member of the board of GAMNI (Group for the Advancement of Numerical Methods in Engineering), a thematic group of SMAI (Society for Applied and Industrial Mathematics) whose overall objective is to promote relations between research and industry in the field of numerical methods.
- Hélène Barucq chairs the GENCI (National High-Performance Computing Infrastructure) CT6 technical committee for applied mathematics, computer science, scientific machine learning, and quantum computing.

11.1.5 Scientific expertise

- Hélène Barucq is member of the scientific committee of BRGM.
- Hélène Barucq is member of the scientific committee of CERFACS.
- Hélène Barucq is member of the scientific committee of LMA2S (Laboratoire de Mathématiques Appliquées à l’Aéronautique et au Spatial) at Onera.
- Sébastien Tordeux is a permanent collaborator of Onera-Toulouse.

11.1.6 Research administration

Julien Diaz is elected member of the "Comité Social d’Administration" of Inria. He is member of the local "Formation Spécialisée en Santé et Sécurité au Travail" of Inria Bordeaux, of the national "Formation Spécialisée en Santé et Sécurité au Travail" of Inria and of "Formation Spécialisée en Santé et Sécurité au Travail" of the Ministry of Higher Education and Research . He as been elected member of the administrative board of Université de Pau et des Pays de l’Adour since february 2025. He is appointed member of the Bureau du Comité des Projets (BCP) of Inria Bordeaux Sud-Ouest.

11.2 Teaching - Supervision - Juries - Educational and pedagogical outreach

11.2.1 Supervision of PhD theses

- Julien Besset. Supervisor: Hélène Barucq
- Lola Chabat, Supervisors: Hélène Barucq, Ha Pham. Additional supervisor: Florian Faucher.

- Florian Delprat. Supervisors: Hélène Barucq, Stefano Frambati
- Ibrahima Djiba. Supervisors: Hélène Barucq and Sébastien Tordeux
- Pierre Dubois. Supervisors: Hélène Barucq and Isabelle Ramière. Additional supervisor: Raphaël Prat.
- Jean Dutheil. Supervisor: Florian Faucher.
- Arjeta Heta. Supervisors: Hélène Barucq and Julien Diaz
- Andrea Lagardère. Supervisors: Sébastien Tordeux and Guillaume Sylvand
- Maylis Lassalle. Supervisors: Sébastien Tordeux and Sébastien Pernet
- Manon Sarrouilhe. Supervisors: Hélène Barucq and Stefano Frambati
- Mathias Rivet. Supervisors: Sébastien Tordeux and Sébastien Pernet

11.2.2 Participation in PhD defense committee

- Hélène Barucq, chair of the jury, PhD defense on Transdimensional inversion of flow data in geomodeling, defended on May 14th, 2025 for the degree of Doctor of the Université de Lorraine Geosciences Specialization, by Julien Herrero
- Hélène Barucq, chair of the jury, PhD defense on Hybrid high-order methods for the numerical simulation of elasto-acoustic wave propagation, defended on July, 23th for the degree of Doctor of École nationale des ponts et chaussées, Applied Mathematics specialization, by Romain Mottier.
- Hélène Barucq, examiner, PhD defense on Quantum algorithms for partial differential equations: quantum circuits and physical applications, defended on September 11th, for the degree of Doctor of Sorbonne Université, Physics specialization, by Julien Zylberman.
- Hélène Barucq, examiner, PhD defense on Impact of Deep Generative Models for solving non-linear Inverse Problems: application to Seismic Imaging, defended on November 3th, for the degree of Doctor of Université Paris PSL (Mines Paris), Geosciences and Geoengineering specialization, by Yuke Xie.
- Hélène Barucq, chair of the jury, PhD defense on Optimisation of Trefftz and quasi-Trefftz methods for the iterative solution of wave problems in time-harmonic regime, defended in december the 10th for the degree of Doctor of the Université de Pau et des Pays de l'Adour, Mathematics specialization, by Matthias Rivet.
- Hélène Barucq, examiner, PhD defense on "Wave and spectral solvers with self-gravitation for radially symmetric adiabatic backgrounds in helioseismology", defended december 12th by Lola Chabat at the Université de Pau et des Pays de l'Adour, Mathematics specialization.
- Ha Pham, examiner, PhD defense on "Wave and spectral solvers with self-gravitation for radially symmetric adiabatic backgrounds in helioseismology", defended december 12th by Lola Chabat at the Université de Pau et des Pays de l'Adour, Mathematics specialization.
- Florian Faucher invited, PhD defense on "Wave and spectral solvers with self-gravitation for radially symmetric adiabatic backgrounds in helioseismology", defended december 12th by Lola Chabat at the Université de Pau et des Pays de l'Adour, Mathematics specialization.
- Sébastien Tordeux examiner, PhD defense on "Wave and spectral solvers with self-gravitation for radially symmetric adiabatic backgrounds in helioseismology", defended december 12th by Lola Chabat at the Université de Pau et des Pays de l'Adour, Mathematics specialization.

11.2.3 Reviews on PhD dissertations

- Hélène Barucq, PhD dissertation entitled Can One Hear the Shape of a Room ? Room Geometry Reconstruction from Acoustic Measurements using Super-Resolution and Shape Optimization, by Tom Sprunck, PhD delivered by Université de Strasbourg, Spécialité Mathématiques Appliquées
- Sébastien Tordeux, PhD defense dissertation entitled “Discontinuous Galerkin finite element methods with transmission variables for time-harmonic wave propagation”, defended on december 10th by SIMONE PESCUMA at ENSTA Paris-tech

11.2.4 Supervision of interships

- Hélène Barucq and Florian Faucher co-supervised Chloé Garcia on the topic “Etude d’une famille de conditions de troncature pour l’équation d’Helmholtz résolue avec une méthode d’éléments finis dans le code Hawen” in the context of a third-year internship for the Bachelor’s degree program in the Master’s in Engineering program at the University of Pau and the Pays de l’Adour.
- Hélène Barucq and Florian Faucher supervised Aurélia Bergeret on the topic “Étude de la modélisation fréquentielle dans un milieu visco-acoustique” in the context of a third-year internship for the Bachelor’s degree program in the Master’s in Engineering program at the University of Pau and the Pays de l’Adour.
- Florian Faucher and Marc Fuentes supervised the Master thesis of Eduard Occhipinti (Grenoble-INP) on “Performance Analysis and CUDA Acceleration of the Open Source Software Hawen”.
- Florian Faucher and Marc Fuentes supervised the L3 internship of Basile Mouret (Grenoble-INP) on “Numerical studies of quadrature formulas for high-order discretization”.
- Sébastien Tordeux supervised the M1 internship of Bruno Latre (UPPA) on “a domain decomposition method”.

11.2.5 Participation in recruitment committees

- Hélène Barucq chaired the admissions panel for the Inria center at the University of Paris Saclay

11.3 Popularization

Participants: Hélène Barucq, Manon Sarrouilhe.

“TimeLock: The Age of Discovery” Lycée d’Orthez, October the 8th

12 Scientific production

12.1 Major publications

[1] H. Barucq, N. Rouxelin and S. Tordeux. ‘Construction and analysis of a HDG solution for the total-flux-formulation of the convected Helmholtz equation’. In: *Mathematics of Computation* 92.343 (2023), pp. 2097–2131. DOI: `10.1090/mcom/3850`. URL: `https://hal.science/hal-04006555`.

[2] S. Frambati, H. Barucq, H. Calandra and J. Diaz. ‘Practical unstructured splines: Algorithms, multi-patch spline spaces, and some applications to numerical analysis’. In: *Journal of Computational Physics* 471 (15th Dec. 2022), p. 111625. DOI: `10.1016/j.jcp.2022.111625`. URL: `https://hal.science/hal-03788980`.

[3] D. Gregor, P. Moczo, J. Kristek, A. Mesgouez, G. Lefeuve-Mesgouez, C. Morency, J. Diaz and M. Kristekova. ‘Seismic waves in medium with poroelastic/elastic interfaces: a two-dimensional P-SV finite-difference modelling’. In: *Geophysical Journal International* 228.1 (24th Jan. 2022), pp. 551–588. DOI: `10.1093/gji/ggab357`. URL: `https://hal.inrae.fr/hal-03471065`.

[4] P. Lalanne, T. Wu, D. Arrivault, M. Duruflé, A. Gras, F. Binkowski, S. Burger and W. Yan. ‘Efficient hybrid method for the modal analysis of optical microcavities and nanoresonators’. In: *Journal of the Optical Society of America. A Optics, Image Science, and Vision* 38.8 (26th July 2021), p. 1224. DOI: `10.1364/JOSAA.428224`. URL: `https://hal.archives-ouvertes.fr/hal-03358012`.

[5] J. A. Lara Benitez, T. Furuya, F. Faucher, A. Kratsios, X. Tricoche and M. de Hoop. ‘Out-of-distributional risk bounds for neural operators with applications to the Helmholtz equation’. In: *Journal of Computational Physics* (6th June 2024), p. 113168. DOI: `10.1016/j.jcp.2024.113168`. URL: `https://hal.science/hal-03963324`.

[6] V. Martins Gomes, D. Brito, S. Garambois, M. Dietrich, C. Bordes and H. Barucq. ‘Seismoelectric wave conversions at an interface: a quantitative comparison between laboratory data and full-waveform modelling’. In: *Geophysical Journal International* 235.3 (18th Dec. 2023), pp. 2992–3011. DOI: `10.1093/gji/ggad409`. URL: `https://inria.hal.science/hal-04396286`.

[7] S. Pernet, M. Sirdey and S. Tordeux. ‘Ultra-weak variational formulation for heterogeneous maxwell problem in the context of high performance computing’. In: *ESAIM: Proceedings and Surveys* 75 (2023), pp. 96–121. DOI: `10.1051/proc/202375096`. URL: `https://hal.science/hal-03642116`.

[8] H. Pham, F. Faucher, D. Fournier, H. Barucq and L. Gizon. ‘Assembling algorithm for Green’s tensors and absorbing boundary conditions for Galbrun’s equation in radial symmetry’. In: *Journal of Computational Physics* 519 (15th Dec. 2024), p. 113444. DOI: `10.1016/j.jcp.2024.113444`. URL: `https://hal.science/hal-04503374`.

[9] M. Rivet, S. Pernet and S. Tordeux. ‘Optimised Correction Polynomial Functions for the Flux Reconstruction Method in Time-Harmonic Electromagnetism’. In: *Applied Mathematics Letters* 157 (Nov. 2024), p. 109187. DOI: `10.1016/j.aml.2024.109187`. URL: `https://inria.hal.science/hal-04517554`.

12.2 Publications of the year

International journals

[10] F. Heimann, C. Lehrenfeld and J. Preuß. ‘Discretization error analysis of a high-order unfitted space–time method for moving domain problems’. In: *IMA Journal of Numerical Analysis* (9th Dec. 2025). DOI: `10.1093/imanum/draf084`. URL: `https://hal.science/hal-05444496` (cit. on p. 18).

Invited conferences

[11] F. Faucher. ‘Quantitative inverse problem in ultrasound imaging for viscoelastic anisotropy’. In: SFB Conference 2025 - Tomography Across the Scales. Strobl, Austria, 16th June 2025. URL: `https://hal.science/hal-05435711` (cit. on p. 20).

[12] F. Faucher, H. Pham, D. Fournier, P. Amestoy, H. Barucq and L. Gizon. ‘Accurate Green’s kernels for solar oscillations in 2.5D and 3D with Hawen’. In: LFOSS 2025 - International Workshop on low-frequency oscillations of the Sun and stars (Harnack House, Berlin). Berlin, Germany, 15th Dec. 2025. URL: `https://hal.science/hal-05435710` (cit. on pp. 21, 22).

[13] A. Lagardère, L.-M. Imbert-Gérard, G. Sylvand and S. Tordeux. ‘Quasi-Trefftz method for aeroacoustic’. In: Modeling & Computation Seminar. Tucson, United States, 20th Feb. 2025. URL: `https://inria.hal.science/hal-05454135` (cit. on p. 17).

[14] H. Pham, F. Faucher, D. Fournier, H. Barucq and L. Gizon. ‘Computational aspects of Green’s kernels in local helioseismology.’ In: SFB Tomography across the Scales scale conference.Austria. Bifeb, Strobl, Austria, 20th June 2025. URL: `https://inria.hal.science/hal-05443555` (cit. on p. 22).

International peer-reviewed conferences

[15] S. Pernet, M. Rivet and S. Tordeux. 'A Quasi-Trefftz Domain Decomposition Method for the Iterative Solution of Time-Harmonic Wave Problems'. In: DD 2025 - 29th International Conference on Domain Decomposition Methods. Milan, Italy, 23rd June 2025. URL: https://inria.hal.science/hal-05317539 (cit. on p. 17).

[16] S. Pernet, M. Rivet and S. Tordeux. 'An Optimised Quasi-Trefftz Method for the Iterative Solution of Time-Harmonic Wave problems'. In: ICOSAHOM 2025 - 15th International Conference on Spectral and High Order Methods. Montréal, Canada, 13th July 2025. URL: https://inria.hal.science/hal-05317562 (cit. on p. 17).

Conferences without proceedings

[17] E. Burman, L. Oksanen, J. Preuss and Z. Zhao. 'Unique continuation for the wave equation: the stability landscape'. In: PICOF 2025 - 11th International Conference on Inverse Problems, Control and Shape Optimization. Tunis, Tunisia, 28th Oct. 2025. URL: https://hal.science/hal-05443659 (cit. on p. 18).

[18] L. Chabat, H. Pham, F. Faucher, H. Barucq and D. Fournier. 'Wave and spectral solvers with self-gravitation for radially symmetric adiabatic backgrounds in helioseismology'. In: Journées Ondes Sud-Ouest (JOSO). Pau, France, 18th Mar. 2025. URL: https://hal.science/hal-05457023 (cit. on p. 21).

[19] A. Lagardère, G. Sylvand, S. Tordeux and L.-M. Imbert-Gérard. 'Quasi-Trefftz Method for Solving Aeroacoustic Problem'. In: SIAM CSE 25. Forth Worth, TX, United States, 3rd Mar. 2025. URL: https://inria.hal.science/hal-05454074 (cit. on p. 17).

[20] J. Preuss. 'Update on WP2: BEM-HDG coupling'. In: Workshop project Butterfly. Bordeaux, France, 8th Sept. 2025. URL: https://hal.science/hal-05443777 (cit. on p. 23).

[21] M. Sarrouilhe, H. Barucq, H. Calandra and S. Frambati. 'Machine learning approaches for CO2 geological storage monitoring: autoencoders for solving the problems of acquisitions repeatability'. In: Mathias Days 2025. Magny-le-Hongre, France, 2nd Dec. 2025. URL: https://hal.science/hal-05454110 (cit. on p. 20).

Scientific book chapters

[22] H. Barucq, A. Bendali, I. Djiba and S. Tordeux. 'Trefftz methods for solving large-scale time-harmonic wave problems'. In: *Mathematical and Computational Modeling Across the Scales*. Vol. 39. Springer International Publishing, 25th May 2025, pp. 77–100. URL: https://hal.science/hal-04877519 (cit. on p. 17).

Edition (books, proceedings, special issue of a journal)

[23] *Symmetric Autoencoder for Seismic Acquisition Repeatability in 4D CO2 Monitoring*. IMAGE 2026 - International Meeting for Applied Geoscience and Energy. Houston (Texas), United States, 17th Aug. 2025. URL: https://hal.science/hal-05454060 (cit. on p. 20).

Doctoral dissertations and habilitation theses

[24] J. Besset. 'A Model Order Reduction Strategy for Parametrized PDEs: A New Paradigm for Efficient Subsurface Imaging.' Université de Pau et des Pays de l'Adour, 23rd June 2025. URL: https://hal.science/tel-05189613 (cit. on p. 15).

[25] I. Djiba. 'Domain decomposition method based upon a Trefftz formulation for anisotropic acoustics'. Université de Pau et des Pays de l 'Adour, 27th Mar. 2025. URL: https://inria.hal.science/tel-05452196 (cit. on p. 17).

[26] A. Heta. 'Dynamic seismo-electric coupling : from frequency to time domain models'. Université de Pau et des Pays de l'Adour, 24th Jan. 2025. URL: https://theses.hal.science/tel-05362806 (cit. on p. 16).

Reports & preprints

[27] H. Barucq, M. Duprez, F. Faucher, E. Franck, F. Lecourtier, V. Lleras, V. Michel-Dansac and N. Victorion. *Enriching continuous Lagrange finite element approximation spaces using neural networks*. 7th Feb. 2025. URL: https://hal.science/hal-04935072 (cit. on p. 15).

[28] J. Besset, H. Barucq, R. Djellouli and S. Frambati. *MOR-T L : A Novel Model Order Reduction Method for Parametrized Problems with Application to Seismic Wave Propagation*. RR-9583. Inria Bordeaux - Sud Ouest, Mar. 2025. URL: https://inria.hal.science/hal-05002365 (cit. on p. 16).

[29] E. Burman, L. Oksanen, J. Preuss and Z. Zhao. *Unique continuation for the wave equation: the stability landscape*. 23rd Oct. 2025. URL: https://hal.science/hal-05329239 (cit. on p. 18).

[30] E. Burman, J. Preuss and T. van Beeck. *Variational data assimilation for the wave equation in heterogeneous media: Numerical investigation of stability*. 16th Sept. 2025. URL: https://hal.science/hal-05444361 (cit. on p. 18).

[31] F. Faucher, H. Pham, D. Fournier, P. Amestoy, H. Barucq, J.-Y. L'Excellent, T. Mary and L. Gizon. *3D Modeling of Solar Oscillations with Hybridizable Discontinuous Galerkin Method*. 2025. DOI: 10.48550/arXiv.2511.22530. URL: https://hal.science/hal-05393820 (cit. on p. 22).

[32] L.-M. Imbert-Gérard, A. Lagardère, G. Sylvand and S. Tordeux. *Quasi-Trefftz Method for Aeroacoustics: Part I - Model Problem and Application to the Helmholtz Equation*. RR-9587. Inria Bordeaux - Sud Ouest, Apr. 2025. URL: https://inria.hal.science/hal-05053699 (cit. on p. 17).

[33] L.-M. Imbert-Gérard, A. Lagardère, G. Sylvand and S. Tordeux. *Quasi-Trefftz spaces for a first-order formulation of the Helmholtz equation*. 10th Sept. 2025. URL: https://inria.hal.science/hal-05454158 (cit. on p. 17).

[34] S. Pernet, M. Rivet and S. Tordeux. *Trefftz Method for a Class of Time-Harmonic Two-Fields Friedrichs Systems*. RR-9602. INRIA Bordeaux - Sud-Ouest, 17th Nov. 2025. URL: https://inria.hal.science/hal-05368909 (cit. on p. 17).

[35] H. Pham, F. Faucher, D. Fournier, H. Barucq and L. Gizon. *Numerical simulations of oscillations for axisymmetric solar backgrounds with differential rotation and gravity*. 2025. URL: https://hal.science/hal-05283612 (cit. on p. 21).

[36] M. Rivet, S. Pernet and S. Tordeux. *A Quasi-Trefftz Method Based on Local Polynomial Impedance Boundary Conditions for Time-Harmonic Wave Propagation Problems*. 10th Jan. 2026. URL: https://inria.hal.science/hal-05452348 (cit. on p. 17).

Other scientific publications

[37] D. Fournier, F. Faucher, H. Pham, H. Barucq and L. Gizon. 'A Scalable 3D Solver for Modeling High-Resolution Solar Oscillations'. In: SDO 2025 Science Workshop: A Gathering of the Helio-hive! Boulder (CO), United States, 17th Feb. 2025. URL: https://hal.science/hal-05435715 (cit. on p. 22).

[38] M. Sarrouilhe, H. Barucq, S. Frambati and H. Calandra. 'Machine Learning approaches for CO2 geological storage monitoring: repeatability via autoencoders'. In: JOSO 2025 - Journées Ondes Sud-Ouest. Pau, France, 18th Mar. 2025. URL: https://hal.science/hal-05454168 (cit. on p. 20).

12.3 Cited publications

[39] G. Alessandrini, M. V. de Hoop, F. Faucher, R. Gaburro and E. Sincich. 'Inverse problem for the Helmholtz equation with Cauchy data: reconstruction with conditional well-posedness driven iterative regularization'. In: *ESAIM: Mathematical Modelling and Numerical Analysis* 53.3 (May 2019), pp. 1005–1030. DOI: 10.1051/m2an/2019009. URL: https://hal.archives-ouvertes.fr/hal-02168474 (cit. on p. 8).

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