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2025 *Activity Report*

RESEARCH CENTRE: **Inria Paris Centre**

IN PARTNERSHIP WITH: **Ecole Nationale des Ponts et Chaussées, CNRS, Université Gustave Eiffel**

Team

MATHRISK

Mathematical Risk handling

In collaboration with Centre d'Enseignement et de Recherche en Mathématiques et Calcul Scientifique (CERMICS)

Team MATHRISK

Creation of the Team: 2025 January 01

Each year, Inria research teams publish an Activity Report presenting their work and results over the reporting period. These reports follow a common structure, with some optional sections depending on the specific team. They typically begin by outlining the overall objectives and research programme, including the main research themes, goals, and methodological approaches. They also describe the application domains targeted by the team, highlighting the scientific or societal contexts in which their work is situated. The reports then present the highlights of the year, covering major scientific achievements, software developments, or teaching contributions. When relevant, they include sections on software, platforms, and open data, detailing the tools developed and how they are shared. A substantial part is dedicated to new results, where scientific contributions are described in detail, often with subsections specifying participants and associated keywords. Finally, the Activity Report addresses funding, contracts, partnerships, and collaborations at various levels, from industrial agreements to international cooperations. It also covers dissemination and teaching activities, such as participation in scientific events, outreach, and supervision. The document concludes with a presentation of scientific production, including major publications and those produced during the year.

Keywords

Computer sciences and digital sciences

A6. – Modeling, simulation and control

A6.1. – Methods in mathematical modeling

A6.1.2. – Stochastic Modeling

A6.2.1. – Numerical analysis of PDE and ODE

A6.2.2. – Numerical probability

A6.2.3. – Probabilistic methods

A6.4.2. – Stochastic control

A8.7. – Graph theory

A8.12. – Optimal transport

Other research topics and application domains

B3.1. – Sustainable development

B3.2. – Climate and meteorology

B3.4. – Risks

B4. – Energy

B9.4. – Sports

B9.5.2. – Mathematics

B9.6.3. – Economy, Finance

B9.11. – Risk management

B9.11.1. – Environmental risks

B9.11.2. – Financial risks

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2 Overall objectives

The Inria project team **MathRisk** team was created in 2013. It is the follow-up of the MathFi project team founded in 2000. MathFi was focused on financial mathematics, in particular on computational methods for pricing and hedging increasingly complex financial products. The 2007 global financial crisis and its "aftermath crisis" has abruptly highlighted the critical importance of a better understanding and management of risk.

The project team MathRisk addresses broad research topics embracing risk management in quantitative finance and insurance and in other related domains as economy and sustainable development. In these contexts, the management of risk appears at different time scales, from high frequency data to long term life insurance management, raising challenging renewed modeling and numerical issues. We aim at both producing advanced mathematical tools, models, algorithms, and software in these domains, and developing collaborations with various institutions involved in risk control. The scientific issues we consider include:

Option pricing and hedging, and risk-management of portfolios in finance and insurance. These remain crucial issues in finance and insurance, with the development of increasingly complex products and various regulatory legislations. Models must take into account the multidimensional features, incompleteness issues, model uncertainties and various market imperfections and defaults. It is also important to understand and capture the joint dynamics of the underlying assets and their volatilities. The insurance activity faces a large class of risk, including financial risk, and is submitted to strict regulatory requirements. We aim at proposing modelling frameworks which catch the main specificity of life insurance contracts.

Systemic risk and contagion modeling. These last years have been shaped by ever more interconnectedness among all aspects of human life. Globalization and economics growth as well as technological progress have led to more complex dependencies worldwide. While these complex networks facilitate physical, capital and informational transmission, they have an inherent potential to create and propagate distress and risk. The financial crisis 2007-2009 has illustrated the significance of network structure on the amplification of initial shocks in the banking system to the level of the global financial system, leading to an economic recession. We are contributing on the issues of systemic risk and financial networks, aiming at developing adequate tools for monitoring financial stability which capture accurately the risks due to a variety of interconnections in the financial system.

(Martingale) Optimal transport. Optimal transport problems arise in a wide range of topics, from economics to physics. In mathematical finance, an additional martingale constraint is considered to take the absence of arbitrage opportunities into account. The minimal and maximal costs provide price bounds robust to model risk, i.e. the risk of using an inadequate model. On the other hand, optimal transport is also useful to analyse mean-field interactions. We are in particular interested in particle approximations of McKean-Vlasov stochastic differential equations (SDEs) and the study of mean-field backward SDEs with applications to systemic risk quantization.

Advanced numerical probability methods and Computational finance. Our project team is very much involved in numerical probability, aiming at pushing numerical methods towards the effective implementation. This numerical orientation is supported by a mathematical expertise which permits a rigorous analysis of the algorithms and provides theoretical support for the study of rates of convergence and the introduction of new tools for the improvement of numerical methods. Financial institutions and insurance companies, submitted to more and more stringent regulatory legislations, such as FRTB or XVA computation, are facing numerical implementation challenges and research focused on numerical efficiency is strongly needed. Overcoming the curse of dimensionality in computational finance is a crucial issue that we address by developing advanced stochastic algorithms and deep learning techniques.

The **MathRisk** project is strongly devoted to the development of new mathematical methods and numerical algorithms. Mathematical tools include stochastic modeling, stochastic analysis, in particular various aspects of stochastic control and optimal stopping with nonlinear expectations, Malliavin calculus, stochastic optimization, random graphs, (martingale) optimal transport, mean-field systems, numerical probability and generally advanced numerical methods for effective solutions. The numerical platform Premia that MathRisk is developing in collaboration with a consortium of financial institutions, focuses on the computational challenges the recent developments in financial mathematics encompass, in particular risk control in large dimensions.

3 Research program

3.1 Systemic risk in financial networks

After the recent financial crisis, systemic risk has emerged as one of the major research topics in mathematical finance. Interconnected systems are subject to contagion in time of distress. The scope is to understand and model how the bankruptcy of a bank (or a large company) may or not induce other bankruptcies. By contrast with the traditional approach in risk management, the focus is no longer on modeling the risks faced by a single financial institution, but on modeling the complex interrelations between financial institutions and the mechanisms of distress propagation among these.

The mathematical modeling of default contagion, by which an economic shock causing initial losses and default of a few institutions is amplified due to complex linkages, leading to large scale defaults, can be addressed by various techniques, such as network approaches or mean field interaction models.

The goal of our project is to develop a model that captures the dynamics of a complex financial network and to provide methods for the control of default contagion, both by a regulator and by the institutions themselves.

We have contributed in the last years to the research on the control of contagion in financial systems in the framework of random graph models (see PhD thesis of R. Chen [80] and Z. Cao [54]).

In [64, 108], [9], we consider a financial network described as a weighted directed graph, in which nodes represent financial institutions and edges the exposures between them. The distress propagation is modeled as an epidemics on this graph. We study the optimal intervention of a lender of last resort who seeks to make equity infusions in a banking system prone to insolvency and to bank runs, under complete and incomplete information of the failure cluster, in order to minimize the contagion effects. The paper [9] provides in particular important insight on the relation between the value of a financial system, connectivity and optimal intervention.

The results show that up to a certain connectivity, the value of the financial system increases with connectivity. However, this is no longer the case if connectivity becomes too large. The natural question remains how to create incentives for the banks to attain an optimal level of connectivity. This is studied in [81], where network formation for a large set of financial institutions represented as nodes is investigated. Linkages are source of income, and at the same time they bear the risk of contagion, which is endogeneous and depends on the strategies of all nodes in the system. The optimal connectivity of the nodes results from a game. Existence of an equilibrium in the system and stability properties is studied. The results suggest that financial stability is best described in terms of the mechanism of network formation than in terms of simple statistics of the network topology like the average connectivity.

In [8], H. Amini (University of Florida), A. Minca (Cornell University) and A. Sulem study Dynamic Contagion Risk Model With Recovery Features. We introduce threshold growth in the classical threshold contagion model, in which nodes have downward jumps when there is a failure of a neighboring node. We are motivated by the application to financial and insurance-reinsurance networks, in which thresholds represent either capital or liquidity. An initial set of nodes fail exogenously and affect the nodes connected to them as they default on financial obligations. If those nodes' capital or liquidity is insufficient to absorb the losses, they will fail in turn. In other terms, if the number of failed neighbors reaches a node's threshold, then this node will fail as well, and so on. Since contagion takes time, there is the potential for the capital to recover before the next failure. It is therefore important to introduce a notion of growth. Choosing the configuration model as underlying graph, we prove fluid limits for the baseline model, as well as extensions to the directed case, state-dependent inter-arrival times and the case of growth driven by upward jumps. We then allow nodes to choose their connectivity by trading off link benefits and contagion risk. Existence of an asymptotic equilibrium is shown as well as convergence of the sequence of equilibria on the finite networks. In particular, these results show that systems with higher overall growth may have higher failure probability in equilibrium.

3.2 Stochastic Control, optimal stopping and non-linear backward stochastic differential equations (BSDEs) with jumps

Option pricing in incomplete and nonlinear financial market models with default. A. Sulem with M.C. Quenez and M. Grigorova have studied option pricing and hedging in nonlinear incomplete financial markets model with default. The underlying market model consists of a risk-free asset and a risky asset driven by a Brownian motion and a compensated default martingale. The portfolio processes follow nonlinear dynamics with a nonlinear driver f, which encodes the imperfections or constraints of the market. A large class of imperfect market models can fit in this framework, including imperfections coming from different borrowing and lending interest rates, taxes on profits from risky investments, or from the trading impact of a large investor seller on the market prices and the default probability. Our market is *incomplete*, in the sense that not every contingent claim can be replicated by a portfolio. In this framework, we address in [14] the problem of pricing and (super)hedging of European options. By using a dynamic programming approach, we provide a dual formulation of the seller's superhedging price as the supremum over a suitable set of equivalent probability measures $Q \in \mathcal{Q}$ of the non-linear $\mathcal{E}^f_Q$-expectation under Q of the payoff. We also provide a characterization of this price as the minimal supersolution of a *constrained* BSDE with default. In [92], we study the superhedging problem for American options with irregular payoffs. We establish a dual formulation of the seller's price in terms of the value of a non-linear mixed optimal control/stopping problem. We also characterize the seller's price process as the minimal supersolution of a *reflected* BSDE with *constraints*. We then prove a duality result for the *buyer's* price in terms of the value of a non-linear optimal control/stopping *game* problem. A crucial step in the proofs is to establish a non-linear optional and a non-linear predictable decomposition for processes which are $\mathcal{E}^f_Q$-strong supermartingales under Q, for all $Q \in \mathcal{Q}$. American option pricing in a non-linear *complete* market model with default is previously studied in [83]. A complete analysis of BSDEs driven by a Brownian motion and a compensated default jump process with intensity process (λ_t) is achieved in [82]. Note that these equations do not correspond to a particular case of BSDEs with Poisson random measure, and are particularly useful in default risk modeling in finance.

Optimal stopping. The theory of optimal stopping in connection with American option pricing has been extensively studied in recent years. Our contributions in this area concern:

(i) *The analysis of the binomial approximation of the American put price in the Black-Scholes model.* We proved that the rate of convergence is, up to a logarithmic factor, of the order $1/n$, where n is the number of discretization time points [104]; (ii) *The American put in the Heston stochastic volatility model.* We have results about existence and uniqueness for the associated variational inequality, in suitable weighted Sobolev spaces, following up on the work of P. Feehan et al. (2011, 2015, 2016) (cf [106]). We also established some qualitative properties of the value function (monotonicity, strict convexity, smoothness) [105]. (iii) *A probabilistic approach to the smoothness of the free boundary in the optimal stopping of a one-dimensional diffusion* (work in collaboration with T. De Angelis)(University of Torino) (see [65]),

Stochastic control with jumps. The 3rd edition of the book *Applied Stochastic Control of Jump diffusions* (Springer, 2019) by B. Øksendal and A. Sulem [16] contains recent developments within stochastic control and its applications. In particular, there is a new chapter devoted to a comprehensive presentation of financial markets modelled by jump diffusions, one on backward stochastic differential equations and risk measures, and an advanced stochastic control chapter including optimal control of mean-field systems, stochastic differential games and stochastic Hamilton-Jacobi-Bellman equations.

3.3 Volatility Modeling

J. Guyon and co-authors have investigated the modeling of the volatility of financial markets [51, 50, 52]. In particular, the (mostly) path-dependent nature of volatility has been shown in [51], an article that has been downloaded 8,500+ times on SSRN and has already been cited in 100+ articles. Path-dependent volatility (PDV) provides a new paradigm of volatility modeling, which can be mixed with stochastic volatility (PDSV) to account for the exogenous part of volatility. In [93], J. Guyon has uncovered a remarkable property of the S&P 500 and VIX markets, which he called inversion of convex ordering. In [50], M. El Amrani and J. Guyon have shown that, contrary to a common belief in the mathematical finance community, the term-structure of the at-the-money skew does not follow a power law. In [52], J. Guyon and S. Mustapha have calibrated neural stochastic differential equations jointly to S&P 500 smiles, VIX futures, and VIX smiles.

3.4 Insurance modeling

Asset Liability Management. Life insurance contracts are popular and involve very large portfolios, for a total amount of trillions of euros in Europe. To manage them in a long run, insurance companies perform Asset and Liability Management (ALM) : it consists in investing the deposit of policyholders in different asset classes such as equity, sovereign bonds, corporate bonds, real estate, while respecting a performance warranty with a profit sharing mechanism for the policyholders. A typical question is how to determine an allocation strategy which maximizes the rewards and satisfies the regulatory constraints. The management of these portfolios is quite involved: the different cash reserves imposed by the regulator, the profit sharing mechanisms, and the way the insurance company determines the crediting rate to its policyholders make the whole dynamics path-dependent and rather intricate. A. Alfonsi et al. have developed in [58] a synthetic model that takes into account the main features of the life insurance business. This model is then used to determine the allocation that minimizes the Solvency Capital Requirement (SCR). In [59], numerical methods based on Multilevel Monte-Carlo algorithms are proposed to calculate the SCR at future dates, which is of practical importance for insurance companies. The standard formula prescribed by the regulator is basically obtained from conditional expected losses given standard shocks that occur in the future.

3.5 (Martingale) Optimal Transport and Mean-field systems

3.5.1 Numerical methods for Optimal transport

Optimal transport problems arise in a wide range of topics, from economics to physics. There exists different methods to solve numerically optimal transport problems. A popular one is the Sinkhorn algorithm which uses an entropy regularization of the cost function and then iterative Bregman projections. Alfonsi et al. [61] have proposed an alternative relaxation that consists in replacing the constraint of matching exactly the marginal laws by constraints of matching some moments. Using Tchakaloff's theorem, it is shown that the optimum is reached by a discrete measure, and the optimal transport is found by using a (stochastic) gradient descent that determines the weights and the points of the discrete measure. The number of points only depends of the number of moments considered, and therefore does not depend on the dimension of the problem. The method has then been developed in [60] in the case of symmetric multimarginal optimal transport problems. These problems arise in quantum chemistry with the Coulomb interaction cost. The problem is in dimension $(\mathbb{R}^3)^M$ where M is the number of electrons, and the method is particularly relevant since the optimal discrete measure weights only $N + 2$ points, where N is the number of moments constraint on the distribution of each electron. Numerical examples up to $M = 100$ can be thus investigated while existing methods could not go beyond $M \approx 10$.

3.5.2 Mean-field systems

Mean-field systems and optimal transport. In [78], O.Bencheikh and B. Jourdain prove that the weak error between a stochastic differential equation with nonlinearity in the sense of McKean given by moments and its approximation by the Euler discretization with time-step h of a system of N interacting particles is $O(N^{-1} + h)$. The challenge was to improve the $O(N^{-1/2})$ strong rate of convergence in the number of particles. In [79], they prove the same estimation for the Euler discretization of a system interacting particles with mean-field rank based interaction in the drift coefficient. To deal with the initialization error, they

investigate in [77] the approximation rate in Wasserstein distance with index $\rho \geq 1$ of a probability measure μ on the real line with finite moment of order ρ by the empirical measure of N deterministic points.
In [102], B. Jourdain and A. Tse propose a generalized version of the central limit theorem for nonlinear functionals of the empirical measure of i.i.d. random variables, provided that the functional satisfies some regularity assumptions for the associated linear functional derivatives of various orders. Using this result to deal with the contribution of the initialization, they check the convergence of fluctuations between the empirical measure of particles in an interacting particle system and its mean-field limiting measure. In [87], R. Flenghi and B. Jourdain pursue their study of the central limit theorem for nonlinear functionals of the empirical measure of random variables by relaxing the i.i.d. assumption to deal with the successive values of an ergodic Markov chain. In [62], A. Alfonsi and B. Jourdain show that any optimal coupling for the quadratic Wasserstein distance $\mathcal{W}_2^2(\mu,\nu)$ between two probability measures μ and ν on $\mathbf{R}^d$ is the composition of a martingale coupling with an optimal transport map. They prove that $\sigma \mapsto \mathcal{W}_2^2(\sigma,\nu)$ is differentiable at μ in both Lions and the geometric senses iff there is a unique optimal coupling between μ and ν and this coupling is given by a map.

3.5.3 Martingale Optimal Transport

In mathematical finance, optimal transport problems with an additional martingale constraint are considered to handle the model risk, i.e. the risk of using an inadequate model. The Martingale Optimal Transport (MOT) problem introduced in [76] provides model-free hedges and bounds on the prices of exotic options. The market prices of liquid call and put options give the marginal distributions of the underlying asset at each traded maturity. Under the simplifying assumption that the risk-free rate is zero, these probability measures are in increasing convex order, since by Strassen's theorem this property is equivalent to the existence of a martingale measure with the right marginal distributions. For an exotic payoff function of the values of the underlying on the time-grid given by these maturities, the model-free upper-bound (resp. lower-bound) for the price consistent with these marginal distributions is given by the following martingale optimal transport problem : maximize (resp. minimize) the integral of the payoff with respect to the martingale measure over all martingale measures with the right marginal distributions. Super-hedging (resp. sub-hedging) strategies are obtained by solving the dual problem. With J. Corbetta, A. Alfonsi and B. Jourdain [6] have studied sampling methods preserving the convex order for two probability measures μ and ν on $\mathbf{R}^d$, with ν dominating μ. Their method is the first generic approach to tackle the martingale optimal transport problem numerically and it can also be applied to several marginals.

Martingale Optimal Transport provides thus bounds for the prices of exotic options that take into account the risk neutral marginal distributions of the underlying assets deduced from the market prices of vanilla options. For these bounds to be robust, the stability of the optimal value with respect to these marginal distributions is needed. Because of the global martingale constraint, stability is far less obvious than in optimal transport (it even fails in multiple dimensions). B. Jourdain has advised the PhD of W. Margheriti devoted to this issue and related problems. He also initiated a collaboration on this topic with M. Beiglböck, one of the founders of MOT theory. In [96], B. Jourdain and W. Margheriti exhibit a new family of martingale couplings between two one-dimensional probability measures μ and ν in the convex order. The integral of $|x-y|$ with respect to each of these couplings is smaller than twice the $\mathcal{W}^1$ distance between μ and ν. Moreover, for $\rho > 1$, replacing $|x-y|$ and $\mathcal{W}_1$ respectively with $|x-y|^\rho$ and $\mathcal{W}_\rho^\rho$ does not lead to a finite multiplicative constant. In [97], they show that a finite constant is recovered when replacing $\mathcal{W}_\rho^\rho$ with the product of $\mathcal{W}_\rho$ times the centred ρ-th moment of the second marginal to the power $\rho-1$ and they study the generalisation of this stability inequality to higher dimension. In [98], they give a direct construction of the projection in adapted Wasserstein distance onto the set of martingale couplings of a coupling between two probability measures on the real line in the convex order which satisfies the barycentre dispersion assumption. Under this assumption, Wiesel had given a clear algorithmic construction of the projection for finitely supported marginals before getting rid of the finite support condition by a rather messy limiting procedure. In [75], with M. Beiglböck and G. Pammer they establish stability of martingale couplings in dimension one : when approximating in Wasserstein distance the two marginals of a martingale coupling by probability measures in the convex order, it is possible to construct a sequence of martingale couplings between these probability measures converging in adapted Wasserstein distance to the original coupling. In [49], they deduce the stability of the Weak Martingale Optimal Transport Problem with respect to the marginal distributions in dimension one which is important since financial data can give only imprecise information on these marginals. As application, this yields the stability of the superreplication bound for VIX futures and of the stretched Brownian motion. In [53], B. Jourdain et al. prove that, in dimension one, contrary to the minimum and maximum in the convex order, the Wasserstein projections of μ (resp. ν) on the set of probability measures dominated by ν (resp. dominating μ) in the convex order are Lipschitz continuous in (μ, ν) for the Wasserstein distance. The thesis of K. Shao (advisers: B. Jourdain, A. Sulem) focuses so far on optimal couplings for costs $|y - x|^\rho$ in dimension one.

Quantization. In order to exploit the natural links between quantization and convex order in view of numerical methods for (Weak) Martingale Optimal Transport, B. Jourdain has initiated a fruitful collaboration with G. Pagès, one of the leading experts of quantization. For two compactly supported probability measures in the convex order, any stationary quadratic primal quantization of the smaller remains dominated by any dual quantization of the larger. B. Jourdain and G. Pagès prove in [101] that any martingale coupling between the original probability measures can be approximated by a martingale coupling between their quantizations in Wassertein distance with a rate given by the quantization errors but also in the much finer adapted Wassertein distance. In [99], in order to approximate a sequence of more than two probability measures in the convex order by finitely supported probability measures still in the convex order, they propose to alternate transitions according to a martingale Markov kernel mapping a probability measure in the sequence to the next and dual quantization steps. In the case of ARCH models, the noise has to be truncated to enable the dual quantization steps. They exhibit conditions under which the ARCH model with truncated noise is dominated by the original ARCH model in the convex order and also analyse the error of the scheme combining truncation of the noise according to primal quantization with the dual quantization steps. In [100], they prove that for compactly supported one dimensional probability distributions having a log-concave density, L^r-optimal dual quantizers are unique at each level N. In the quadratic $r = 2$ case, they propose an algorithm which computes this unique optimal dual quantizer with geometric rate of convergence.

3.5.4 Martingale Schrödinger problems

Calibration problems in finance can be cast as Schrödinger problems. Due to the no-arbitrage condition, martingale Schrödinger problems must be considered. To jointly calibrate S&P 500 (SPX) and VIX options, J. Guyon has introduced dispersion-constrained martingale Schrödinger problems. In [50], he solved for the first time this longstanding puzzle of quantitative finance that has often been described as the Holy Grail of volatility modeling: build a model that jointly and exactly calibrates to the prices of SPX options, VIX futures, and VIX options. He did so using a nonparametric, discrete-time, minimum-entropy approach. He established a strong duality theorem and characterized the absence of joint SPX/VIX arbitrage. The minimum entropy jointly calibrating model is explicit in terms of the dual Schrödinger portfolio, i.e., the maximizer of the dual problems, should it exist, and is numerically computed using an extension of the Sinkhorn algorithm. Numerical experiments show that the algorithm performs very well in both low and high volatility regimes.

3.6 Deep learning for large dimensional financial problems

Neural networks and Machine Learning techniques for high dimensional American options. The pricing of American option or its Bermudan approximation amounts to solving a backward dynamic programming equation, in which the main difficulty comes from the conditional expectation involved in the computation of the continuation value.

In [107], B. Lapeyre and J. Lelong study neural networks approximations of conditional expectations. They prove the convergence of the well-known Longstaff and Schwartz algorithm when the standard least-square regression on a finite-dimensional vector space is replaced by a neural network approximation, and illustrate the numerical efficiency of the method on several numerical examples. Its stability with respect to a change of parameters as interest rate and volatility is shown. The numerical study proves that training neural network with only a few chosen points in the grid of parameters permits to price efficiently for a whole range of parameters.

In [89], two efficient techniques, called GPR Tree (GRP-Tree) and GPR Exact Integration (GPR-EI), are proposed to compute the price of American basket options. Both techniques are based on Machine Learning, exploited together with binomial trees or with a closed formula for integration. On the exercise dates, the value of the option is first computed as the maximum between the exercise value and the continuation value and then approximated by means of Gaussian Process Regression. In [91], an efficient method is provided to compute the price of multi-asset American options, based on Machine Learning, Monte Carlo simulations and variance reduction techniques. Numerical tests show that the proposed algorithm is fast and reliable, and can handle American options on very large baskets of assets, overcoming the curse of dimensionality issue.

• *Machine Learning in the Energy and Commodity Market.* Evaluating moving average options is a computational challenge for the energy and commodity market, as the payoff of the option depends on the prices of underlying assets observed on a moving window. An efficient method for pricing Bermudan style moving average options is presented in [90], based on Gaussian Process Regression and Gauss-Hermite quadrature. This method is tested in the Clewlow-Strickland model, the reference framework for modeling prices of energy commodities, the Heston (non-Gaussian) model and the rough-Bergomi model, which involves a double non-Markovian feature, since the whole history of the volatility process impacts the future distribution of the process.

3.7 Advanced numerical probability methods and Computational finance

Our project team is very much involved in numerical probability, aiming at pushing numerical methods towards the effective implementation. This numerical orientation is supported by a mathematical expertise which permits a rigorous analysis of the algorithms and provides theoretical support for the study of rates of convergence and the introduction of new tools for the improvement of numerical methods. This activity in the MathRisk team is strongly related to the development of the Premia software.

3.7.1 Approximation of stochastic differential equations

High order schemes. The approximation of SDEs and more general Markovian processes is a very active field. One important axis of research is the analysis of the weak error, that is the error between the law of the process and the law of its approximation. A standard way to analyse this is to focus on marginal laws, which boils down to the approximation of semigroups. The weak error of standard approximation schemes such as the Euler scheme has been widely studied, as well as higher order approximations such as those obtained with the Richardson-Romberg extrapolation method.

Stochastic Volterra Equations. Stochastic Volterra Equations (SVE) provide a wide family of non-Markovian stochastic processes. They have been introduced in the early 80's by Berger and Mizel and have received a recent attention in mathematical finance to model the volatility : it has been noticed that SVEs with a fractional convolution kernel $G(t) = c_H t^{H-1/2}$ reproduce some important empirical features. The problem of approximating these equations has been tackled by Zhang [114] and Richard et al. [113] who show under suitable conditions a strong convergence rate of $O(n^{-H-})$ for the Euler scheme, where n is the number of time steps. We almost recover the rate for classical SDEs when $H \to 1/2$. However, an important drawback is that the required computation time is proportional to n^2.

Abstract Malliavin calculus and convergence in total variation. In collaboration with L. Caramellino and G. Poly, V. Bally has settled a Malliavin type calculus for a general class of random variables, which are not supposed to be Gaussian (as it is the case in the standard Malliavin calculus). This is an alternative to the Γ-calculus settled by Bakry, Gentile and Ledoux. The main application is the estimate in total variation distance of the error in general convergence theorems. This is done in [72].

Invariance principles. As an application of the above methodology, V. Bally et al. have studied several limit theorems of Central Limit type (see [67] and [71]). In particular they estimate the total variation distance between random polynomials, and prove a universality principle for the variance of the number of roots of trigonometric polynomials with random coefficients [73]).

Analysis of jump type SDEs. V. Bally, L. Caramellino and A. Kohatsu Higa, study the regularity properties of the law of the solutions of jump type SDE's [69]. They use an interpolation criterion (proved in [66]) combined with Malliavin calculus for jump processes. They also use a Gaussian approximation of the solution combined with Malliavin calculus for Gaussian random variables. Another approach to the same regularity property, based on a semigroup method has been developed by Bally and Caramellino in [68]. An application for the Bolzmann equation is given by V. Bally in [66]. In the same line but with different application, the total variation distance between a jump equation and its Gaussian approximation is studied by V. Bally and his PhD student Y. Qin [74] and by V. Bally, V. Rabiet, D. Goreac [73]. A general discussion on the link between total variation distance and integration by parts is done in [72]. Finally V. Bally et al. estimate in [70] the probability that a diffusion process remains in a tube around a smooth function.

3.7.2 Monte-Carlo and Multi-level Monte-Carlo methods

Error bounds of MLMC. In [95], B. Jourdain and A. Kebaier are interested in deriving non-asymptotic error bounds for the multilevel Monte Carlo method. As a first step, they deal with the explicit Euler discretization of stochastic differential equations with a constant diffusion coefficient. As long as the deviation is below an explicit threshold, they check that the multilevel estimator satisfies a Gaussian-type concentration inequality optimal in terms of the variance.

Approximation of conditional expectations. The approximation of conditional expectations and the computation of expectations involving nested conditional expectations are important topics with a broad range of applications. In risk management, such quantities typically occur in the computation of the regulatory capital such as future Value-at-Risk or CVA. A. Alfonsi et al. [59] have developed a Multilevel Monte-Carlo (MLMC) method to calculate the Solvency Capital Ratio of insurance companies at future dates. The main advantage of the method is that it avoids regression issues and has the same computational complexity as a plain Monte-Carlo method (i.e. a computational time in $O(\varepsilon^{-2})$ to reach a precision of order ε). In other contexts, one may be interested in approximating conditional expectations. To do so, the classical method consists in considering a parametrized family $\varphi(\alpha,\cdot)$ of functions, and to minimize the empirical L^2-distance $\frac{1}{M}\sum_{k=1}^{M}(Y_i-\varphi(\alpha,X_i))^2$ between the observations and their prediction. In general, it is assumed to have as many observations as explanatory variables. However, when these variables are sampled, it may be possible to sample K values of Y's for a given X_i and to minimize $\frac{1}{M}\sum_{k=1}^{M}(\frac{1}{K}\sum_{k=1}^{K}Y_i^k-\varphi(\alpha,X_i))^2$. A. Alfonsi, J. Lelong and B. Lapeyre [48] have determined the optimal value of K which minimizes the computation time for a given precision. They show that K is large when the family approximates well the conditional expectation. The computational gain can be important, especially if the computational cost of sampling Y given X is small with respect to the cost of sampling X.

3.8 Remarks

We have focused above on the research program of the last four years. We refer to the previous MathRisk activity report for a description of the research done earlier, in particular on Liquidity and Market Microstructure [63, 57], [4], dependence modelling [103], interest rate modeling [55], Robust option pricing in financial markets with imperfections [82, 112], [13, 12], Mean field control and Stochastic Differential Games [109, 94, 111], Stochastic control and optimal stopping (games) under nonlinear expectation [83, 86, 84, 85], robust utility maximization [110, 111, 88], Generalized Malliavin calculus and numerical probability.

4 Application domains

4.1 Quantitative Finance

One of the domains of application is quantitative finance, with emphasis on risk modeling and control. In particular, the project-team Mathrisk focuses on financial modeling and calibration, systemic risk, option pricing and hedging, portfolio optimization, risk measures.

4.2 Insurance

There are some specificity of the insurance business that raises major challenges, in particular for life insurance. Special issues deal with regulation constraints, climate risk, long time horizon management.

4.3 Electricy power system management

Power system management is an important source of applicative challenges, in particular with the growing share of renewable energies in the electricity mix, which are essentially non-controllable and power grid constraints on the capacity limits of the network.

4.4 Sports

J. Guyon has a strong expertise in quantitative analysis in football (soccer), focusing on fairness and efficiency of competition formats, ranking systems, seeding systems, draw procedures, match schedules.

5 Social and environmental responsibility

Our work aims to contribute to a better management of risk in the banking and insurance systems, in particular by the study of systemic risk, climate risk, asset price modeling, stability of financial markets.

Our applications to energy power systems modeling and optimization are addressed in the context of increasing penetration of renewables.

6 Highlights of the year

6.1 Awards

Julien Guyon: Quant of the year 2025

7 Latest software developments, platforms, open data

7.1 Latest software developments

7.1.1 PREMIA

Keywords: Computational finance, Quantum Finance, Monte-Carlo methods, Option pricing, Numerical probability, Machine learning, Numerical algorithm

Scientific Description: Premia is a numerical platform for computational finance. It is designed for option pricing, hedging and financial model calibration. Premia is developed by the MathRisk project team in collaboration with a consortium of financial institutions. The Premia project keeps track of the most recent advances in the field of computational finance in a well-documented way. It focuses on the implementation of numerical analysis techniques for both probabilistic and deterministic numerical methods. An important feature of the platform Premia is the detailed documentation which provides extended references in option pricing. Premia contains various numerical algorithms: deterministic methods (Finite difference and finite element algorithms for partial differential equations, wavelets, Galerkin, sparse grids ...), stochastic algorithms (Monte-Carlo simulations, quantization methods, Malliavin calculus based methods), tree methods, approximation methods (Laplace transforms, Fast Fourier transforms...) These algorithms are implemented for the evaluation of vanilla and exotic options on equities, interest rate, credit, energy and insurance products. Moreover Premia provides a calibration toolbox for Libor Market model and a toolbox for pricing Credit derivatives. The latest developments of the software address evaluation of financial derivative products, risk management and computations of risk measures required by new financial regulation. They include the implementation of advanced numerical algorithms taking into account model dependence, counterparty credit risk, hybrid features, rough volatility and various nonlinear effects. A big effort has been put these last years on the development and implementation of deep learning techniques using neural network approximations, and Machine Learning algorithms in finance, in particular for high-dimensional American option pricing, high-dimensional PDEs, deep hedging. Moreover Quantum computing in Finance is explored, in particular option pricing using quantum computers.

Functional Description: Premia is a software designed for quantitative finance, developed by the MathRisk project team in collaboration with a consortium of financial institutions presently composed of Crédit Agricole CIB and NATIXIS. The Premia project keeps track of the most recent advances in computational finance and focuses on the implementation of numerical techniques to solve financial problems. An important feature of the platform Premia is its detailed documentation which provides extended references in computational finance. Premia is a powerful tool to assist Research and Development professional teams in their day-to-day duty. It is also a useful support for academics who wish to perform tests on new algorithms or pricing methods. Besides being a single entry point for accessible overviews and basic implementations of various numerical methods, the aim of the Premia project is: - to elaborate a powerful testing platform for comparing different numerical methods between each other, - to build a link between professional financial teams and academic researchers, - to provide a useful teaching support for Master and PhD students in mathematical finance. The project Premia has started in 1999 and is now considered as a standard reference platform for quantitative finance among the academic mathematical finance community.

Release Contributions: A big effort has been put these last years on the development and implementation of deep learning techniques using neural network approximations, and Machine Learning algorithms in finance, in particular for high-dimensional American option pricing, high-dimensional PDEs, deep hedging.The latest developments of the software address also the evaluation of financial derivative products, risk management and computations of risk measures by advanced numerical algorithms taking into account model dependence, counterparty credit risk (computations of XVA), hybrid features, rough stochastic volatility models and various new regulations. Nested Monte Carlo strategies with GPU optimizations, and Chebyshev Interpolation method for Parametric Option Pricing have been implemented. We have also developed our activity on insurance contracts, in particular on the computation of risk measures (Value at Risk, Condition Tail Expectation) of variable annuities contracts like GMWB (guaranteed minimum withdrawal benefit) including taxation and customers mortality modeling.

News of the Year: The new release Premia 27 has been delivered to the Consortium on June 3 2025. It contains the following new implemented algorithms in Machine Learning, Neural networks, Risk Management:

- Optimal stopping with signatures. C. Bayer, P. Hager,S. Riedel, J. Schoenmakers, The Annals of Applied Probability 33-1, 2023 - Primal and dual optimal stopping with signature. C. Bayer, L. Pelizzari, J. Schoenmakers arxiv.org/abs/2312.03444 - Pricing American options under rough volatility using deep-signatures and signature-kernels. C. Bayer, L. Pelizzari, Zhou arxiv.org/abs/2501.06758 - Deep calibration of rough stochastic volatility models. C.Bayer B.Stemper - Optimal Damping with Hierarchical Adaptive Quadrature for Efficient Fourier Pricing of Multi-Asset Options in Levy Models, C. Bayer, C. Ben Hammouda, A. Papapantoleon, M. Samet and R. Tempone Journal of Computational Finance, Volume 27-3, 2023. - Leveraging Machine Learning for High-Dimensional Option Pricing within the Uncertain Volatility Model. L. Goudenege A. Molent, A. Zanette arxiv.org/abs/2407.13213 - Robust Pricing of Equity-Indexed Annuities under Uncertain Volatility and Stochastic Interest Rate, L. Goudenege A. Molent, A. Zanette arxiv.org/abs/2407.13213 - Neural Optimal Stopping Boundary. A. Max Reppen, H. Mete Soner, Valentin Tissot-Daguette, Mathematical Finance, Volume 35-2, 2025. - Estimating risks of option books using neural-SDE market models. S. N. Cohen, C. Reisinger and S. Wang, Journal of Computational Finance, Volume 26-3, 2022. - Hedging option books using neural-SDE market models. S. N. Cohen, C. Reisinger and S. Wang, Applied Mathematical Finance 29-5, 2022. - Arbitrage-free neural-SDE market models. Arbitrage-free neural-SDE market models. S. N. Cohen, C. Reisinger and S. Wang, Appl. Math. Finance 30-1, 2023. - Neural variance reduction for stochastic differential equations, P. D. Hinds and M. V. Tretyakov Journal of Computational Finance, Volume 27-3, 2023.

URL: `http://www.premia.fr`

Publications: hal-03791594, hal-03436046, hal-01940715, hal-01873346, hal-03810106, hal-05421581, hal-03526905, hal-03013606, hal-02183587

Contact: Agnes Sulem

Participants: Agnes Sulem, Antonino Zanette, Aurélien Alfonsi, Benjamin Jourdain, Jerome Lelong, Bernard Lapeyre, Ahmed Kebaier, Ludovic Goudenège

Partners: Ecole des Ponts ParisTech, Université d'Udine

7.2 New platforms

Participants: Julien Guyon.

- UEFA draws: draw simulator of the league phase of the UEFA Champions League.

Available here

7.3 Open data

8 New results

Participants: MathRisk Members.

8.1 Fire Sales, Default Cascades and Complex Financial Networks

Participants: A. Sulem, Z. Cao, H. Amini.

In [20], we present a general tractable framework to understand the joint impact of fire sales and default cascades on systemic risk in complex financial networks. Our limit theorems quantify how price-mediated contagion across institutions with common asset holdings can worsen cascades of insolvencies in a heterogeneous financial network during a financial crisis. For given prices of illiquid assets, we show that, under some regularity assumptions, the default cascade model can be transferred to a death process problem. We model the price impact using a specified inverse demand function and state limit theorems concerning the total shares sold and the equilibrium price of illiquid assets in a stylized fire sales model. In the numerical studies we investigate the effect of heterogeneity in network structure and price impact function on the final size of the default cascade and fire sales loss.

8.2 Graphon Mean-field games

Participants: A. Sulem, Z. Cao, H. Amini.

8.2.1 Stochastic Graphon Mean-field Games and approximate Nash Equilibria

The use of graphons has emerged recently in order to analyze heterogeneous interaction in mean-field systems and game theory. *Graphon BSDEs* with jumps and associated dynamic global risk measures are studied by H. Amini, A. Sulem, and Z. Cao in [21]. Existence, uniqueness and stability of solutions under some regularity assumptions are established. We also prove convergence results for finite interacting mean-field particle systems with heterogeneous interactions to graphon mean-field BSDE systems. Finally, we introduce the graphon dynamic risk measure induced by the solution of a graphon mean-field BSDE system and study its properties. In particular, a dual representation theorem is provided in the convex case.

In [23] we study continuous stochastic games with heterogeneous mean field interactions and jumps on large networks and explore their limit counterparts. We introduce the graphon game model based on a controlled graphon mean field stochastic differential equation system with jumps, which can be regarded as the limiting case of a finite game dynamic system as the number of players goes to infinity. We examine the case of controlled dynamics, with control terms present in the drift, diffusion, and jump components. We focus on the study of Markovian controls and concentrate on the limit theory. We provide convergence results on the state trajectories and their laws, transitioning from finite game systems to graphon systems. We also study approximate equilibria for finite games on large networks, using the graphon equilibrium as a benchmark. The rates of convergence are analyzed under various underlying graphon models and regularity assumptions.

In [22], we study continuous stochastic graphon games with heterogeneous mean field interactions and jumps. We consider a continuum of players, where each player's dynamics involve graphon mean field interactions and individual jumps induced by a Poisson random measure. We investigate the case when the drift, diffusion, and jump terms of the dynamics of the graphon system are controlled by a stochastic process. The graphon objective function presents non-linear mean field interactions in the running and final rewards. We prove the existence of a relaxed equilibrium of the graphon mean field game via controlled martingale problems. Under convexity assumptions and using measurable selection arguments, strict Markovian equilibria are constructed from relaxed equilibria. Under some additional monotonicity assumptions, we obtain the uniqueness of the graphon equilibrium. We provide an explicit solution for the graphon mean field equilibrium for two simple examples in the linear-quadratic case.

8.2.2 Extended Graphon Mean Field Games in Discrete Time

Participants: A. Sulem, Z. Cao, H. Amini, K. Shao.

This is an ongoing work in collaboration with Mathieu Laurière (NYU Shangai) and Gökçe Dayanıklı (University of Illinois). In this paper, we study games involving a continuum of heterogeneous players in the discrete-time setting with finite state spaces and continuous action spaces. We introduce a new model that incorporates joint state-action interactions within the graphon-weighted aggregate, described by a coupled forward-backward system. We rigorously establish the existence of solutions to this system and demonstrate a one-to-one correspondence between graphon Nash equilibria (GNE) and the solutions of the forward-backward system. Additionally, we prove the uniqueness of the GNE under various structural assumptions. To illustrate the practical relevance of our framework, we provide an example of optimal investment with a relative performance objective and solve it numerically using fixed-point iterations. See the phD thesis of K. Shao [32].

This is an ongoing work in collaboration with Mathieu Laurière (NYU Shanghai). We develop theoretical and numerical analysis of extended Graphon Mean Field Games (GMFG) in a discrete-time setting. On the theoretical side, we provide rigorous analysis on the existence of approximated Nash equilibrium of the GMFG system by considering joined state-action distribution, we also refined the proof of existence by categorizing pure policies and mixed policies. On the numerical side, we explore some learning schemes (i.e. reinforcement learning) to study graphon mean field equilibrium.

8.3 High-imensional Stochastic control problems with pairwise interaction through controls

Participants: E. Devey, P. Cardaliaguet.

The project aims at investigating large-population stochastic control problems in which agents share their state information and cooperate to minimize a convex cost functional. The latter is decomposed into individual and coupling costs, with the distinctive feature that the coupling term is a pairwise interaction function between the controls. To address this setting, we follow closely (Jackson & Lacker, 2025): we introduce a related problem where each agent observes only its own state. We then establish a quantitative bound on the difference between the value functions associated with these two problems. We obtain this result by reformulating the problems analytically as Hamilton-Jacobi type equations and comparing their associated Hamiltonians. The main difficulty of our approach lies in establishing a precise comparison between the distributions of the corresponding optimal controls. See [41].

8.4 Decentraized control problems in energy networks

Participants: E. Devey, N. Oudjane, A. Sulem, H. Amini.

This research, in collaboration with Nadia Oudjane (EDF R&D) is motivated by an optimisation challenge faced by EDF, which aims to manage flexible producers and consumers by regulating the electricity they supply to or draw from the grid. The problem incorporates power constraints on each line of the electricity grid, leading to graphical constraints.
The project aims at solving optimal control problems in which many agents cooperate to minimize a convex cost functional.This functional includes a heterogeneous pairwise interaction term between the controls, which prevents the use of Mean Field Control theory.
The main goal of the study is to propose a new perspective on the construction of near-optimal distributed controls, which is non-asymptotic in nature and imposes no structural assumptions. Unlike methods based on Mean Field Control theory, we preserve the heterogeneity of the agents.

We design a Frank-Wolfe type optimsation algorithm that constructs near-optimal distributed solutions to the problem and provide a quantitative analysis of its convergence rate.

8.5 Optimal stopping and American option pricing

8.5.1 Differentiability of optimal stopping boundaries

Participants: D. Lamberton, T. De Angelis.

D. Lamberton and Tiziano De Angelis (University of Torino) are working on the optimal stopping problem of a one dimensional diffusion in finite horizon. They develop a probabilistic approach to the regularity of the associated free boundary problem, and derive a probabilistic proof of the differentiability of the free boundary for the optimal stopping problem of a one-dimensional diffusion in [65]. They have new results concerning the second order mixed derivative of the value function (work in progress).

8.5.2 Dual approach for hedging Bermudan options

Participants: A. Alfonsi, J. Lelong, A. Kebaier.

A. Alfonsi, J. Lelong and A. Kebaier develop in [18] a numerical method to price and hedge American and Bermudean options based on the dual representation introduced by Rogers (2010). The key idea is to rewrite the dual formula as an excess reward representation and to combine it with a strict convexification technique. The hedging strategy is then obtained by using a Monte Carlo method, solving backward a sequence of least square problems. Convergence results for the algorithm are obtained and tests on various Bermudan options are provided. Beyond giving directly the hedging portfolio, the strength of the algorithm is to assess both the relevance of including financial instruments in the hedging portfolio and the effect of the rebalancing frequency.

8.5.3 Computing XVA for American basket derivatives by machine learning techniques

Participants: A. Zanette, L. Goudenege.

In [26], L. Goudenège, A. Molent (Univ Udine) and A. Zanette study American option pricing taking into account the default risk. Total value adjustment (XVA) is the change in value to be added to the price of a derivative to account for the bilateral default risk and the funding costs. In this paper, we compute such a premium for American basket derivatives whose payoff depends on multiple underlyings. In particular, in our model, those underlyings are supposed to follow the multidimensional Black-Scholes stochastic model. In order to determine the XVA, we follow the approach introduced by (Burgard and Kjaer in SSRN Electronic J 7:1–19, 2010) and afterward applied by (Arregui et al. in Appl Math Comput 308:31–53, 2017), (Arregui et al. in Int J Comput Math 96:2157–2176, 2019) for the one-dimensional American derivatives. The evaluation of the XVA for basket derivatives is particularly challenging as the presence of several underlings leads to a high-dimensional control problem. We tackle such an obstacle by resorting to Gaussian Process Regression, a machine learning technique that allows one to address the curse of dimensionality effectively. Moreover, the use of numerical techniques, such as control variates, turns out to be a powerful tool to improve the accuracy of the proposed methods. The paper includes the results of several numerical experiments that confirm the goodness of the proposed methodologies.

8.6 Optimal transport

8.6.1 Study of the Wasserstein projections in the convex order

Participants: A. Alfonsi, B. Jourdain.

In [36], A. Alfonsi and B. Jourdain first show continuity of both Wasserstein projections in the convex order when they are unique. They also check that, in arbitrary dimension d, the quadratic Wasserstein projection of a probability measure μ on the set of probability measures dominated by ν in the convex order is non-expansive in μ and Hölder continuous with exponent 1/2 in ν. When μ and ν are Gaussian, they show that this projection is Gaussian and also consider the quadratic Wasserstein projection of ν on the set of probability measures dominating μ in the convex order. In the case when $d \geq 2$ and ν is not absolutely continuous with respect to the Lebesgue measure where uniqueness of the latter projection was not known, they check that there is always a unique Gaussian projection and characterize when non Gaussian projections with the same covariance matrix also exist. Still for Gaussian distributions, they characterize the covariance matrices of the two projections. It turns out that there exists an orthogonal transformation of space under which the computations are similar to the easy case when the covariance matrices of μ and ν are diagonal.

8.6.2 Convex comparison of Gaussian mixtures

Participants: B. Jourdain, G. Pagès.

Motivated by the study of the propagation of convexity by semi-groups of stochastic differential equations and convex comparison between the distributions of solutions of two such equations, G. Pagès (LPSM) and B. Jourdain study the comparison for the convex order between a Gaussian distribution and a Gaussian mixture. We give and discuss intrinsic necessary and sufficient conditions for convex ordering. On the examples that we have worked out, the two conditions appear to be closely related (see [27]).

8.6.3 Quadratic Wasserstein distance between Gaussian laws revisited with correlation

Participants: A. Alfonsi, B. Jourdain.

In [35], A. Alfonsi and B. Jourdain give a simple derivation of the formula obtained in the eighties for the quadratic Wasserstein distance between two Gaussian distributions on $\mathbf{R}^d$ with respective covariance matrices Σ_μ and Σ_ν. This derivation relies on the existence of an orthogonal matrix O such that $O^*\Sigma_\mu O$ and $O^*\Sigma_\nu O$ share the same correlation matrix and on the simplicity of optimal couplings in the case with the same correlation matrix and therefore the same copula.

8.6.4 Martingale Optimal Transport

Participants: B. Jourdain, K. Shao.

In [29], B. Jourdain and K. Shao investigate non-decreasing martingale couplings. See also the phD thesis of K. Shao [32].

8.7 Volatility modeling

Participants: J. Guyon, B. Jourdain, H. Andrès.

Path-dependent volatility is a new paradigm for volatility modeling that has attracted a lot of attention in the markets, both as a risk-neutral pricing model and as a model able to generate realistic real-world scenarios [38].

In [25], J. Guyon and G. Gazzani (Univ Verona) consider the path-dependent volatility (PDV) model of Guyon and Lekeufack (2023) [51], where the instantaneous volatility is a linear combination of a weighted sum of past returns and the square root of a weighted sum of past squared returns. They discuss the influence of an additional parameter that unlocks enough volatility on the upside to reproduce the implied volatility smiles of S&P 500 and VIX options. This PDV model, motivated by empirical studies, comes with computational challenges, especially in relation to VIX options pricing and calibration. They propose an accurate neural network approximation of the VIX which leverages on the Markovianity of the 4-factor version of the model. The VIX is learned as a function of the Markovian factors and the model parameters. They use this approximation to tackle the joint calibration of S&P 500 and VIX options. In [45], J. Guyon and L. Parent examine calibration "under P" and "under Q" for a discrete-time version of the 4-factor path-dependent volatility (PDV) model introduced by Guyon and Lekeufack. They show that combining path-dependent volatility with non-Gaussian innovations allows to reconcile estimation on time series of asset prices and calibration to option prices.

Local stochastic volatility refers to a popular model class in applied mathematical finance that allows for "calibration-on-the-fly", typically via a particle method, derived from a formal McKean-Vlasov equation. Well-posedness of this limit is a well-known problem in the field; the general case is largely open, despite recent progress in Markovian situations. In [42], P. Friz (Weierstrass Institute, Berlin) , B. Jourdain, T. Wagenhofer (TU Berlin) and A. Zhou start with a well-defined Euler approximation to the formal McKean-Vlasov equation, followed by a newly established half-step-scheme, allowing for good approximations of conditional expectations. In a sense, they do Euler first, particle second in contrast to previous works that start with the particle approximation. They show weak order one for the Euler discretization, plus error terms that account for the said approximation. The case of particle approximation is discussed in detail and the error rate is given in dependence of all parameters used.

In [44], J. Guyon, T. Jeannin and B. Jourdain investigate the distributions of random couples (X, Y) with X real-valued such that any non-negative integrable random variable $f(X)$ can be represented as a conditional expectation, $f(X) = \mathbb{E}[g(Y)|X]$, for some non-negative measurable function g. It turns out that this representation property is related to the smallness of the support of the conditional law of X given Y, and in particular fails when this conditional law almost surely has a non-zero absolutely continuous component with respect to the Lebesgue measure. They give a sufficient condition for the representation property and check that it is also necessary under some additional assumptions (for instance when X or Y are discrete). They also exhibit a rather involved example where the representation property holds but the sufficient condition does not. Finally, they discuss a weakened representation property where the non-negativity of g is relaxed. This study is motivated by the calibration of time-discretized path-dependent volatility models to the implied volatility surface.

8.8 Numerical probability

8.8.1 Nonlinear weak error expansion of McKean-Vlasov Stochastic differential equations

Participants: B. Jourdain, A. Lê.

According to Talay and Tubaro, the weak error between the solution to a stochastic differential equation with smooth coefficients and its Euler-Maruyama scheme can be expanded in powers of the time-step. In [46], B. Jourdain and A.-D. Lê generalize this result to the case when the error is measured by a smooth functional on the Wasserstein space of probability measures in place of the linear functional given by the expectation of a smooth function. Since this does not complicate their analysis based on the master partial differential equation, they even deal with the McKean-Vlasov case when the coefficients of the stochastic differential equation may depend on its current marginal distribution.

8.8.2 Stochastic Volterra Equations

Participants: A. Alfonsi, E. Abi Jaber, G. Szulda, B. Jourdain, G. Pagès.

Weak solutions of stochastic Volterra Equations in convex domains with general kernels. A. Alfonsi, E. Abi-Jaber (CMAP) and G. Szulda establish new weak existence results for d-dimensional Stochastic Volterra Equations (SVEs) with continuous coefficients and possibly singular one-dimensional non- convolution kernels. These results are obtained by introducing an approximation scheme and showing its convergence. A particular emphasis is made on the stochastic invariance of the solution in a closed convex set. To do so, we extend the notion of kernels that preserve nonnegativity introduced in [56] to non-convolution kernels and show that, under suitable stochastic invariance property of a closed convex set by the corresponding Stochastic Differential Equation, there exists a weak solution of the SVE that stays in this convex set. We present a family of non-convolution kernels that satisfy our assumptions, including a non-convolution extension of the well-known fractional kernel. We apply our results to SVEs with square-root diffusion coefficients and non-convolution kernels, for which we prove the weak existence and uniqueness of a solution that stays within the nonnegative orthant. We derive a representation of the Laplace transform in terms of a non-convolution Riccati equation, for which we establish an existence result (see [33].

SVE equations with jumps. A. Alfonsi and G. Szulda study stochastic Volterra equations with jumps and non-Lipschitz coefficients in [19].

More precisely, they consider one-dimensional stochastic Volterra equations with jumps for which they establish conditions upon the convolution kernel and coefficients for the strong existence and pathwise uniqueness of a non-negative càdlàg solution. By using the approach recently developed by [56], they show the strong existence by using a nonnegative approximation of the equation whose convergence is proved via a variant of the Yamada–Watanabe approximation technique. They apply these results to Lévy-driven stochastic Volterra equations. In particular, they define a Volterra extension of the so-called alpha-stable Cox–Ingersoll–Ross process, which is especially used for applications in Mathematical Finance.

Convex ordering for stochastic Volterra equations and their Euler schemes. In [28], G. Pagès and B. Jourdain are interested in comparing solutions to stochastic Volterra equations for the convex order on the space of continuous R^d valued paths and for the monotonic convex order for the one-dimensional case.

8.8.3 An abstract framework for the approximation of the invariant measure

Participants: A. Alfonsi, V. Bally, A. Kohatsu Higa.

In collaboration with A. Kohatsu Higa (Ritsumeikan University, Japan), we establish a general framework to study the rate of convergence of a Euler type approximation scheme with decreasing time steps to the invari- ant measure, for a general class of stochastic systems (see [34]). The error is measured in general Wasserstein distances, which enables to encompass cases with non global contractivity conditions. Our main assumption is a coupling property which is expressed in terms of the one-step approximation. We show that the proposed set-up can be applied to a wide range of equations that may be law dependent, such as Langevin equations, reflected equations, Boltzmann type equations and for a recent McKean Vlasov type model for neuronal activity.

8.8.4 Sewing Lemma

Participants: A. Alfonsi, V. Bally, L. Caramellino.

A. Alfonsi and V. Bally have proposed a new approach based on the sewing lemma on the Wasserstein Space to study existence and uniqueness of solutions of the Boltzmann equation [47]. They are now working in [17] with L. Caramellino (Roma Univ) to extend their results by using the stochastic sewing lemma recently proposed by Khoa Lê (2020).

8.9 Deep learning for large dimensional financial problems

Participants: A. Zanette, L. Goudenège, A. Molent, A. Kebaier.

We pursue the development of Machine Learning an Deep learnig techniques in particular for McKean-Vlasov models of singular stochastic volatility, robust utility maximization, and high-dimensional optimal stopping problems. The corresponding algorithms are implemented in the Premia software.

9 Bilateral contracts and grants with industry

9.1 Bilateral contracts with industry

- Consortium PREMIA, Crédit Agricole Corporate Investment Bank (CA - CIB) - INRIA
- CIFRE agreement ENPC/EDF PhD thesis of Faten Ben Said
- CIFRE agreement ENPC/BNPParibas for the PhD thesis of Grégoire Ounnoughene.
- CIFRE agreement ENPC/Milliman for the PhD thesis of Arthur Bourdon

9.2 Bilateral grants with industry

- Chair Ecole Polytechnique-Ecole des Ponts ParisTech-Sorbonne Université-Société Générale "Financial Risks" of the Risk fondation.

  Participants: Aurélien Alfonsi, Benjamin Jourdain.

  Postdoctoral grant : Anh Dung Lê

- Chair Ecole des Ponts ParisTech - Université Paris-Cité - BNP Paribas "Futures of Quantitative Finance"

  Participants: Julien Guyon.

9.3 Research grants

- Institut Europlace de Finance Louis Bachelier and Labex Louis Bachelier grant : "Multi-Agent Reinforcement Learning in Large Financial Networks with Heterogeneous Interactions" from November 2023.

  Participants: Agnès Sulem, , Hamed Amini.

- Research grant from GMP0 (Gaspard Monge Program for Optimization, operations research, and their interactions with data science), 2025, for the project "Optimal distributed stochastic control and application to the management of electrical grid flexibility".

  Participants: Agnès Sulem, , Elise Devey.

10 Partnerships and cooperations

10.1 International initiatives

J. Guyon
Visiting Associate Professor, NYU Tandon School of Engineering, Department of Finance and Risk Engineering

10.2 International research visitors

10.2.1 Visits of international scientists

Participants: A. Zanette, H. Amini, Z. Cao.

- A. Zanette, Univ of Udine 4 August to September 4 to supervize Premia software development
- H. Amini, University of Florida, Collaboratio with A. Sulem and E. DEvey
- Z. Cao, Shanghai University, Collaboration with A. Sulem

11 Dissemination

11.1 Promoting scientific activities

- A. Alfonsi

  Co-organizer of the Mathrisk seminar “Méthodes stochastiques et finance”

  Co-organizer of the Bachelier (Mathematical Finance) seminar (IHP, Paris).

- V. Bally

  Organizer of the seminar of the LAMA laboratory, Université Gustave Eiffel.

- A. Sulem

  Co-organizer of the seminar INRIA-MathRisk /Université Paris Diderot LPSM “Numerical probability and mathematical finance”

11.1.1 Scientific events: organisation

Member of the organizing committees

- J. Guyon co-organizes the first Futures of Quantitative Finance Conference (January 22, 2026) jointly with Univ. Paris Cité and BNP Paribas
- A. Sulem and A. Zanette organized the Premia meeting for the delivery of the 27th release of the software to the Consortium. Talks by A. Zanette (Univ Udine), Luca Pelizzari (Weierstrass Institute Berlin), Chiheb Ben Hammouda (University of Utrecht), L. Goudenege (Université Paris-Saclay ÉvryCNRS), 3 June 2025, INRIA Paris.
- A. Sulem organized the joint seminar MathRisk/LPSM, January 9 2025, Université Paris-Cité, Talks by Peter Bank (Technische Universität Berlin), Olivier Guéant (Université Paris 1 Panthéon-Sorbonne), Julien GUYON (ENPC), Mathieu LAURIERE (NYU Shangai)

11.1.2 Journal

Member of the editorial boards

- A. Alfonsi

  Member of the editorial board of the Book Series "Mathématiques et Applications" of Springer.

- J. Guyon

  Associate editor of

  - Finance and Stochastics
  - Quantitative Finance
  - SIAM Journal on Financial Mathematics
  - Journal of Dynamics and Games

- B. Jourdain

  Associate editor of

  - ESAIM : Proceedings and Surveys
  - Stochastic Processes and their Applications (SPA)
  - Stochastic and Partial Differential Equations : Analysis and Computations
  - Journal of Mathematical Analysis and Applications
  - Mathematical Finance

- D. Lamberton
  Associate editor of
  - Mathematical Finance,
  - ESAIM Probability & Statistics
- A. Sulem
  Associate editor of
  - *Journal of Mathematical Analysis and Applications* (JMAA)
  - *SIAM Journal on Financial Mathematics* (SIFIN)

Reviewer - reviewing activities

- J. Guyon : Reviewer for Finance and Stochastics, SIAM Journal on Financial Mathematics, Mathematical Finance, Quantitative Finance, -International Journal of Theoretical and Applied Finance, Risk, Foundations of Mathematical Finance, International Journal of Sports Science and Coaching.
- B. Jourdain : Reviewer for *Mathematical Reviews*
- D. Lamberton Reviewer for *Journal of Mathematical Analysis and Applications*
- A. Sulem: Reviewer for *Mathematical Reviews*

11.1.3 Talks in Conferences and Workshops

- A. Alfonsi
  - 15 07 2025: "Stochastic Volterra Equations on Convex Domains", SIAM Conference on Financial Mathematics and Engineering, Miami.
  - 20 11 2025: "Wasserstein projections in the convex order: regularity and characterization in the quadratic Gaussian case", Workshop Geometry, duality and convexity in new OT problems, Orsay.
  - 03 12 2025: "Stochastic Volterra Equations on Convex Domains", SIAM Conference on Financial Mathematics and Engineering, Berlin Probability Colloquium.
- E. Devey
  - Ceremade YRD (Young Researcher Days), 03/06-05/06, Caen (France)
  - PGMO Days, 18/11-19/11, EDF Saclay (France)
- J. Guyon
  - Conferences and workshops:
    - Research In Options 2025, Fundação Getulio Vargas, Rio de Janeiro, December 2025.
    - Workshop on rough volatility, Weierstrass Institute, Berlin, November 2025.
    - QuantMinds 2025, London, November 2025.
    - CBOE RMC Quant Conference (keynote speaker), Chicago, October 2025.
    - WBS 21st Quantitative Finance Conference, Palermo, September 2025.
    - Advances in Mathematics of Randomness for Handling Risks in Finance and Insurance, Marseille, September 2025.
    - SIAM Conference on Financial Mathematics and Engineering (plenary speaker), Miami, July 2025.
    - VCMF 2025, Vienna, July 2025.
    - AMaMeF 2025, Verona, June 2025.
    - MathSport International 2025, Luxembourg, June 2025.
    - AFMathConf2025, Brussels, February 2025.

- Seminars
  - Bloomberg, New York, Keynote speaker at BBQ (Bloomberg Quant Seminar), October 2025.
  - Cornell Financial Engineering Manhattan, New York, CFEM & UBS AI & Data research seminar, October 2025.
  - University of California, Berkeley, IEOR Mathematical Finance Seminar, May 2025.
  - Columbia University, New York, Columbia Mathematical Finance Seminar Series, March 2025.

- B. Jourdain
  - LPSM working group on Financial and actuarial mathematics, numerical probability, 27 november 2025 : Wasserstein projections in the convex order
  - Workshop Optimal transport : stochastics, projections, and applications, The Fields Institute Toronto, 5 november 2025 : Wasserstein projections in the convex order
  - SINEQ Final Conference, GSSI L'Aquila, 23 october 2025 : Central Limit Theorem for the Stratified Resampling Scheme
  - Futures of Quantitative Finance Seminar, 24 september 2025 : Implied volatility (also) is path-dependent
  - Diffusions in Warsaw, University of Warsaw, 11 september 2025 : Convexity propagation and convex ordering of one-dimensional stochastic differential equations
  - Applied Probability and Statistics seminar, TU Graz, 10 april 2025 : Convex comparison of Gaussian mixtures
  - Bachelier seminar, 4 april 2025 : *Convex comparison of Gaussian mixtures*

- A. Sulem
  - Plenary speaker, Conference in honor of Alain Bensoussan for his 85th birthday, 06/2025, Shandong University: "Stochastic Graphon Mean-Field Games with Jumps and associated equilibria".
  - Plenary speaker, Conference in honor of B. Øksendal for his 80 birthday, 09/2025, Norwegian Academy of Sciences, Oslo, "Stochastic Graphon Mean-Field Games with Jumps and approximate Nash equilibria of large network games."

11.1.4 Scientific expertise

- A. Alfonsi

  Member of the council of the Bachelier Finance Society

11.1.5 Research administration

- J. Guyon

  head of the Applied Probability team at CERMICS, Ecole des Ponts

- B. Jourdain

  Deputy head of the federation Bézout

- A. Sulem

  Member of the Scientific Committee of AMIES(Agence pour les Mathématiques en Interaction avec l'Entreprise et la Société)

11.1.6 Academic responsabilities

- A. Alfonsi

  - In charge of the Master "Finance and Data" at the Ecole des Ponts (until Sept. 2025).

  -Representant of the Master "Probabilité et Finance" at Ecole des Ponts.

11.2 Teaching - Supervision - Juries - Educational and pedagogical outreach

11.2.1 Teaching

- A. Alfonsi
  - “Probabilités”, first year course at the Ecole des Ponts.
  - “Données Haute Fréquence en finance”, lecture for the Master at UPEMLV.
  - “Mesures de risque”, Master course of UPEMLV and Sorbonne Université.
  - Professeur chargé de cours at Ecole Polytechnique.
- B. Jourdain

  - course "Probability theory", 1st year ENPC

  - course "Mathematical finance", 2nd year ENPC

  - course "Monte-Carlo methods", 3rd year ENPC and Research Master Mathématiques et Application, University Gustave Eiffel

  - course "Monte-Carlo Markov chain methods and particle algorithms", Research Master Probabilités et Modèles Aléatoires, Sorbonne Université

  - course "Machine Learning 1", MSC Data Science for Business, X-HEC

  - course "Randomness", 1st year Ecole Polytechnique
- J. Guyon

  - course "Probability Theory", 1st year ENPC. Lead teacher.

  - course "Volatility Modeling", Master Mathematics for Finance and Data (MFD), 3rd year ENPC - UGE

  - course "Advanced calibration methods and VIX derivatives", joint lecture of the BNP Paribas chair Futures of Quantitative Finance, Master Probabilités et Finance, Master M2MO, Master MFD (Sorbonne Université, Université Paris Cité, and ENPC-UGE)

  - J. Guyon, B. Liang : course "Nonlinear Option Pricing", Master MAFN, Columbia University

  - course "Volatility Modeling", Master of Science in Financial Engineering, NYU
- A. Sulem

  Master of Mathematics, Université du Luxembourg, Responsible of the course on "Numerical Methods in Finance", and lectures (22 hours)

11.2.2 Supervision

- Postdoral fellows
  - Léo Parent (supervision : J. Guyon)
- PhD in progress
  - Elise Devey (started October 2023), "Graphon Mean-Field Games and Renewable Energy Systems", Supervisor: Agnès Sulem, INRIA doctoral grant
  - Thibault Jeannin (started in november 2024) "Calibration of pure path-dependent volatility models", supervised by J. Guyon and B. Jourdain
  - Arthur Bourdon (started in november 2024) "Approximation and explication of insurance valuation computations by Artificial Intelligence", supervised by B. Jourdain
  - Grégoire Ounnoughene (Oct 2025 -), "Uncertain path-dependent volatility models" (co-supervison: J. Guyon with A. Alfonsi and G. Loeper), CIFRE PhD thesis with BNP Paribas

- Rémi Surat (Oct 2025 -), "Generative modeling of financial time series" (co-supervised J. Guyon with L. Pillaud-Vivien and G. Loeper), CIFRE PhD thesis with BNP Paribas
- Faten Ben Said (CIFRE EDF, co-advisor: Julien Reygner), “Caractérisation et prise en compte des dépendances statistiques dans le cadre d’applications de dynamique sédimentaire”, started in March 2023.
- François Escolan (co-advisors: A. Alfonsi, Virginie Ehrlacher and Julien Reygner), “Mean-field limit of stochastic particle systems on manifolds”, started in November 2024.

• PhD defended

- Kexin Shao, "Martingale optimal transport and Graphon mean-field games", supervised by A. Sulem and B. Jourdain, defended on March 27 2025
- Edoardo Lombardo (International PhD, co-advisors: A. Alfonsi and Lucia Caramellino, Tor Vergata, Roma), “High order numerical approximation for some singular stochastic processes and related PDEs”, defended on January 22, 2025, ENPC.

• Internship

- Hassene Kallala, 2nd year, IMI Dept, ENPC (June-August 2025), quintic OU stochastic volatility model, supervision: J. Guyon
- Maxence Caucheteux (June to August 2025): “Espace d’état pour le processus markovien multifactoriel associé à un processus de Volterra”. supervision: A. Alfonsi
- Kevin Aoun, ENSTA, 26/05/2025 - 28/07/2025: implementation of deep pricing algorithms in the Premia software; supervision: L. Goudenege.
- Hassen Ben Jemaa, Ecole Polytechnique de Tunisie, " Pricing of Bernudean Options in high dimensions using deep learning and high order weak approximation", March 1- July 31 2025,
- Mohamed Ben Saada, "Deep learning algorithms for Linear Quadratic Mean-field games with common noise", Ecole Polytechnique de Tunisie, March 1- July 31 2025, Supervision: A. Kebaier.
- Wissai Haouami, "implementation of recent algorithms of option pricing using neural networks and implementation in the Premia software", ENSTA, June - September 2025, supervision: L. Goudenege.

• Project supervision

- Project of the ENPC course TDLOG (2024-25): Draw simulator, league phase of the New UEFA Champions League Format, supervision: J. Guyon

- Project of the ENPC course TDLOG (2025-26): Simulation of the New UEFA Champions League Format, supervision: J. Guyon

- Project of the ENPC course TDLOG (2025-26): Luck index, supervision: J. Guyon

- IMI Department Project (2024-25): Simulation of the New UEFA Champions League Format, importance of a game, supervision: J. Guyon

11.2.3 Juries

• A. Alfonsi

- President of the jury for the PhD thesis of Natascha Hey “Trading with concave (cross-) impact”.
- Examiner of the PhD thesis of Alexis Houssard “Some aspects of model risk management using singular stochastic control and model-free approaches”.

• J. Guyon

PhD thesis examination of Jules Delemotte (Ecole Polytechnique, December 10, 2025): Smile dynamics, rough volatility, volatility with memory

- B. Jourdain
  - Reviewer for the PhD of Paul Maurer defended on November 14 2025, University Côte d'Azur
- A. Sulem
  - President of the committee for the PhD thesis examination of Thomas Le Corre, *Distributed control of flexible loads in power grids*, PSL ENS, 29/10/2025
  - PhD thesis examination of Anna De Crescenzo, *"Heterogeneous mean-field systems"*, Université Paris-Cité, 13/10/2025
  - PhD thesis examination of Yadh Hafsi, *" Inference and Control of Liquidity Risk "*, Université Paris Saclay, President of the committee, 09/09/2025
  - Member of the recruitment committee for n assistant professor position in Applied Mathematics, " Mathematics for Economy, Finance and game theory ", Université Paris-Dauphine, Spring 2025.
  - Member of the committee for the tenure of Eduardo Abi Jaber, Ecole Polytechnique, 28 May 2025

11.3 Popularization

11.3.1 Productions (articles, videos, podcasts, serious games, ...)

Participants: J. Guyon.

J. Guyon has a strong expertise in quantitative analysis in football (soccer). His main reserach interest is fairness in sports, in particular the fairness and efficiency of competition formats, ranking systems, seeding systems, draw procedures, and match schedules [43].

- J. Guyon, cited in The Times for his work on the 2026 FIFA World Cup draw probabilities, December 5, 2025
- J. Guyon, Interview in the French sports daily , L'Equipe, December 5, 2025
- J. Guyon: Interview in the French sports daily L'Equipe, December 11, 2025
- Live intervention on La chaîne L'Équipe in the TV program L'Équipe du soir, January 20, 2025. L'Equipe du soir
- Interview in the French newspaper Le Télégramme, January 21, 2025. Le Télégramme
- Interview in the French newspaper L'Équipe, January 22, 2025. L'Equipe
- Live intervention on La chaîne L'Équipe in the TV program L'Équipe du soir, January 22, 2025. L'Equipe du soir
- Interview in the French newspaper L'Équipe, January 23, 2025. L'Equipe
- Interview in the French newspaper Le Télégramme, January 28, 2025. Le Télégramme
- Interview in the French newspaper L'Équipe, January 30, 2025. L'Equipe

12 Scientific production

12.1 Major publications

[1] A. Al Gerbi, B. Jourdain and E. Clément. 'Ninomiya-Victoir scheme: strong convergence, antithetic version and application to multilevel estimators'. In: *Monte Carlo Method and Applications* 22.3 (July 2016). https://arxiv.org/abs/1508.06492, pp. 197–228. URL: https://hal-enpc.archives-ouvertes.fr/hal-01188675.

[2] A. Alfonsi. *Affine Diffusions and Related Processes: Simulation, Theory and Applications*. 2015. DOI: 10.1007/978-3-319-05221-2. URL: https://hal-enpc.archives-ouvertes.fr/hal-03127212.

[3] A. Alfonsi and V. Bally. 'A generic construction for high order approximation schemes of semigroups using random grids'. In: *Numerische Mathematik* (2021). DOI: 10.1007/s00211-021-01219-2. URL: https://hal-enpc.archives-ouvertes.fr/hal-02406433.

[4] A. Alfonsi and P. Blanc. 'Dynamic optimal execution in a mixed-market-impact Hawkes price model'. In: *Finance and Stochastics* (Jan. 2016). https://arxiv.org/abs/1404.0648. DOI: 10.1007/s00780-015-0282-y. URL: https://hal-enpc.archives-ouvertes.fr/hal-00971369 (cit. on p. 13).

[5] A. Alfonsi, A. Cherchali and J. A. I. Acevedo. 'A full and synthetic model for Asset-Liability Management in life insurance, and analysis of the SCR with the standard formula'. In: *European Actuarial Journal* (2020). DOI: 10.1007/s13385-020-00240-3. URL: https://hal-enpc.archives-ouvertes.fr/hal-02406439.

[6] A. Alfonsi, J. Corbetta and B. Jourdain. 'Sampling of probability measures in the convex order by Wasserstein projection'. In: *Annales de l'Institut Henri Poincaré (B) Probabilités et Statistiques* 56.3 (2020), pp. 1706–1729. DOI: 10.1214/19-AIHP1014. URL: https://hal.archives-ouvertes.fr/hal-01589581 (cit. on p. 10).

[7] A. Alfonsi, B. Jourdain and A. Kohatsu-Higa. 'Optimal transport bounds between the time-marginals of a multidimensional diffusion and its Euler scheme'. In: *Electronic Journal of Probability* (2015). https://arxiv.org/abs/1405.7007. URL: https://hal-enpc.archives-ouvertes.fr/hal-00997301.

[8] H. Amini, A. Minca and A. Sulem. 'A dynamic contagion risk model with recovery features'. In: *Mathematics of Operations Research* (24th Nov. 2021). DOI: 10.1287/moor.2021.1174. URL: https://hal.inria.fr/hal-02421342 (cit. on p. 7).

[9] H. Amini, A. Minca and A. Sulem. 'Control of interbank contagion under partial information'. In: *SIAM Journal on Financial Mathematics* 6.1 (Dec. 2015), p. 24. URL: https://hal.inria.fr/hal-01027540 (cit. on p. 7).

[10] V. Bally and L. Caramellino. 'Convergence and regularity of probability laws by using an interpolation method'. In: *Annals of Probability* 45.2 (2017), pp. 1110–1159. URL: https://hal-upec-upem.archives-ouvertes.fr/hal-01109276.

[11] A. Bouselmi and D. Lamberton. 'The critical price of the American put near maturity in the jump diffusion model'. In: *SIAM Journal on Financial Mathematics* 7.1 (May 2016). https://arxiv.org/abs/1406.6615, pp. 236–272. DOI: 10.1137/140965910. URL: https://hal-upec-upem.archives-ouvertes.fr/hal-00979936.

[12] R. Dumitrescu, M.-C. Quenez and A. Sulem. 'A Weak Dynamic Programming Principle for Combined Optimal Stopping/Stochastic Control with E^f-Expectations'. In: *SIAM Journal on Control and Optimization* 54.4 (2016), pp. 2090–2115. DOI: 10.1137/15M1027012. URL: https://hal.inria.fr/hal-01370425 (cit. on p. 13).

[13] R. Dumitrescu, M.-C. Quenez and A. Sulem. 'Game Options in an Imperfect Market with Default'. In: *SIAM Journal on Financial Mathematics* 8.1 (Jan. 2017), pp. 532–559. DOI: 10.1137/16M1109102. URL: https://hal.inria.fr/hal-01614758 (cit. on p. 13).

[14] M. Grigorova, M.-C. Quenez and A. Sulem. 'European options in a non-linear incomplete market model with default'. In: *SIAM Journal on Financial Mathematics* 11.3 (2nd Sept. 2020), pp. 849–880. DOI: 10.1137/20M1318018. URL: https://hal.archives-ouvertes.fr/hal-02025833 (cit. on p. 8).

[15] B. Jourdain. *Probabilités et statistique*. seconde édition. Ellipses, 2016. URL: https://hal.archives-ouvertes.fr/hal-03133840.

[16] B. Øksendal and A. Sulem. *Applied Stochastic Control of Jump Diffusions*. 3rd edition. Springer, Universitext, 2019, p. 436. DOI: 10.1007/978-3-030-02781-0. URL: https://hal.archives-ouvertes.fr/hal-02411121 (cit. on p. 8).

12.2 Publications of the year

International journals

[17] A. Alfonsi, V. Bally and L. Caramellino. 'Stochastic sewing lemma on Wasserstein space'. In: *Electronic Journal of Probability* 30.none (1st Jan. 2025). DOI: 10.1214/25-EJP1422. URL: https://enpc.hal.science/hal-04827019 (cit. on p. 22).

[18] A. Alfonsi, A. Kebaier and J. Lelong. 'A pure dual approach for hedging Bermudan options'. In: *Mathematical Finance* 35.4 (Oct. 2025), pp. 745–759. DOI: 10.1111/mafi.12460. URL: https://enpc.hal.science/hal-04563713 (cit. on p. 18).

[19] A. Alfonsi and G. Szulda. 'On non-negative solutions of stochastic Volterra equations with jumps and non-Lipschitz coefficients'. In: *Bernoulli* 31.4 (1st Nov. 2025). DOI: 10.3150/24-BEJ1830. URL: https://enpc.hal.science/hal-04827040 (cit. on p. 21).

[20] H. Amini, Z. Cao and A. Sulem. 'Fire Sales, Default Cascades and Complex Financial Networks'. In: *Mathematics and Financial Economics* 19.2 (9th Apr. 2025), pp. 225–260. DOI: 10.2139/ssrn.3935450. URL: https://inria.hal.science/hal-03425599 (cit. on p. 16).

[21] H. Amini, Z. Cao and A. Sulem. 'Graphon Mean-Field Backward Stochastic Differential Equations With Jumps and Associated Dynamic Risk Measures'. In: *Finance and Stochastics* (2025). DOI: 10.2139/ssrn.4162616. URL: https://hal.science/hal-03830110. In press (cit. on p. 16).

[22] H. Amini, Z. Cao and A. Sulem. 'Markovian Equilibria of Stochastic Graphon Games with Jumps'. In: *Pure and Applied Functional Analysis* Special Issue on Systems Theory, Control and PDE dedicated to Professor Alain Bensoussan (2025). DOI: 10.2139/ssrn.5084740. URL: https://inria.hal.science/hal-05428710. In press (cit. on p. 17).

[23] H. Amini, Z. Cao and A. Sulem. 'Stochastic Graphon Mean Field Games with Jumps and Approximate Nash Equilibria'. In: *SIAM Journal on Control and Optimization* (2026). DOI: 10.2139/ssrn.4412999. URL: https://hal.science/hal-04872702. In press (cit. on p. 17).

[24] H. Andrès and B. Jourdain. 'Existence, uniqueness and positivity of solutions to the Guyon-Lekeufack path-dependent volatility model with general kernels'. In: *International Journal of Theoretical and Applied Finance* (12th Dec. 2025). DOI: 10.1142/S0219024925500190. URL: https://hal.science/hal-04667144.

[25] G. Gazzani and J. Guyon. 'Pricing and Calibration in the 4-Factor Path-Dependent Volatility Model'. In: *Quantitative Finance* 25.3 (2025), pp. 471–489. DOI: 10.2139/ssrn.4853419. URL: https://inria.hal.science/hal-04855305 (cit. on p. 20).

[26] L. Goudenège, A. Molent and A. Zanette. 'Computing XVA for American basket derivatives by machine learning techniques'. In: *Computational Management Science* 22 (8th Aug. 2025), pp. 541–569. DOI: 10.1007/s10287-025-00540-7. URL: https://hal.science/hal-05421581 (cit. on p. 19).

[27] B. Jourdain and G. Pagès. 'Convex comparison of Gaussian mixtures'. In: *Journal of Multivariate Analysis* 209 (2025), p. 105448. URL: https://hal.science/hal-04731943 (cit. on p. 19).

[28] B. Jourdain and G. Pagès. 'Convex ordering for stochastic Volterra equations and their Euler schemes'. In: *Finance and Stochastics* 29 (2025), pp. 1–62. URL: https://hal.science/hal-03862241 (cit. on p. 22).

[29] B. Jourdain and K. Shao. 'Non-decreasing martingale couplings'. In: *ESAIM: Probability and Statistics* 29 (3rd Jan. 2025), pp. 1–44. DOI: 10.1051/ps/2024012. URL: https://hal.science/hal-04086915 (cit. on p. 20).

International peer-reviewed conferences

[30] M. Castellano, L. Goudenège and F. Nabet. 'Energy estimate of a Discrete Duality Finite Volume scheme for a phase-field model with surfactants'. In: DD29 - 29th International Conference on Domain Decomposition Methods. Milan, Italy, 23rd June 2025. URL: https://hal.science/hal-05473939.

Doctoral dissertations and habilitation theses

[31] E. Lombardo. 'Approximation and regularity results for the Heston model and related processes'. École des Ponts ParisTech; Università degli studi di Roma "Tor Vergata" (1972-....), 22nd Jan. 2025. URL: https://theses.hal.science/tel-05172676.

[32] K. Shao. 'Martingale optimal transport and graphon mean fieldgames'. Université Paris sciences et lettres, 27th Mar. 2025. URL: https://theses.hal.science/tel-05095520 (cit. on pp. 17, 20).

Reports & preprints

[33] E. Abi Jaber, A. Alfonsi and G. Szulda. *Weak solutions of stochastic volterra equations in convex domains with general kernels*. 6th June 2025. DOI: 10.48550/arXiv.2506.04911. URL: https://hal.science/hal-05100416 (cit. on p. 21).

[34] A. Alfonsi, V. Bally and A. Kohatsu-Higa. *Euler-type approximation for the invariant measure: An abstract framework*. 4th Sept. 2025. URL: https://enpc.hal.science/hal-05441104 (cit. on p. 22).

[35] A. Alfonsi and B. Jourdain. *Quadratic Wasserstein distance between Gaussian laws revisited with correlation*. 17th Dec. 2025. URL: https://hal.science/hal-05420706 (cit. on p. 20).

[36] A. Alfonsi and B. Jourdain. *Wasserstein projections in the convex order: regularity and characterization in the quadratic Gaussian case*. 17th Dec. 2025. URL: https://hal.science/hal-05420721 (cit. on p. 19).

[37] A. Alfonsi, A. Kebaier and J. Lelong. *How can the dual martingale help solving the primal optimal stopping problem?* 10th Feb. 2026. URL: https://enpc.hal.science/hal-05504568.

[38] H. Andrès, A. Boumezoued and B. Jourdain. *The implied volatility surface (also) is path-dependent*. 13th Oct. 2025. URL: https://hal.science/hal-04362544 (cit. on p. 20).

[39] F. Ben Said, A. Alfonsi, A. Dutfoy, C. Goeury, M. Jodeau, J. Reygner and F. Zaoui. *A tree-based Polynomial Chaos expansion for surrogate modeling and sensitivity analysis of complex numerical models*. 16th Sept. 2025. URL: https://hal.science/hal-05262028.

[40] A. Bourdon, B. Jourdain and H. Andrès. *Linear independence properties of the signature components of time-augmented stochastic processes*. 15th Jan. 2026. URL: https://hal.science/hal-05462206.

[41] E. Devey. *Approximately optimal distributed controls for high-dimensional stochastic systems with pairwise interaction through controls*. 27th Oct. 2025. URL: https://hal.science/hal-05335445 (cit. on p. 18).

[42] P. K. Friz, B. Jourdain, T. Wagenhofer and A. Zhou. *On the Weak Error for Local Stochastic Volatility Models*. 5th Aug. 2025. URL: https://hal.science/hal-05199651 (cit. on p. 20).

[43] J. Guyon, A. Ben Salem, T. Buchholtzer and M. Tanré. *Drawing and Scheduling the UEFA Champions League League Phase*. 2025. DOI: 10.2139/ssrn.5413142. URL: https://inria.hal.science/hal-05446606 (cit. on p. 29).

[44] J. Guyon, T. Jeannin and B. Jourdain. *On the surjectivity of the conditional expectation given a real random variable*. 5th Aug. 2025. URL: https://hal.science/hal-05199581 (cit. on p. 20).

[45] J. Guyon and L. Parent. *The Discrete-Time 4-Factor Path-Dependent Volatility Model: Calibration under P and Q*. 2025. DOI: 10.2139/ssrn.5382733. URL: https://inria.hal.science/hal-05446613 (cit. on p. 20).

[46] B. Jourdain and A.-D. Le. *Nonlinear weak error expansion of McKean-Vlasov stochastic differential equations*. 21st Nov. 2025. URL: https://hal.science/hal-05373596 (cit. on p. 21).

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