2025 *Activity Report*

RESEARCH CENTRE: **Inria Centre at Université Côte d'Azur**
IN PARTNERSHIP WITH: **CNRS, Université Côte d'Azur**

Team

MCTAO

Mathematics for Control, Transport and Applications

In collaboration with Laboratoire Jean-Alexandre Dieudonné (JAD)

Team MCTAO

Creation of the Team: 2025 January 01

Each year, Inria research teams publish an Activity Report presenting their work and results over the reporting period. These reports follow a common structure, with some optional sections depending on the specific team. They typically begin by outlining the overall objectives and research programme, including the main research themes, goals, and methodological approaches. They also describe the application domains targeted by the team, highlighting the scientific or societal contexts in which their work is situated. The reports then present the highlights of the year, covering major scientific achievements, software developments, or teaching contributions. When relevant, they include sections on software, platforms, and open data, detailing the tools developed and how they are shared. A substantial part is dedicated to new results, where scientific contributions are described in detail, often with subsections specifying participants and associated keywords. Finally, the Activity Report addresses funding, contracts, partnerships, and collaborations at various levels, from industrial agreements to international cooperations. It also covers dissemination and teaching activities, such as participation in scientific events, outreach, and supervision. The document concludes with a presentation of scientific production, including major publications and those produced during the year.

Keywords

Computer sciences and digital sciences

A2.1.1. – Semantics of programming languages
A2.2.1. – Static analysis
A2.5. – Software engineering
A5.10.3. – Planning
A5.10.4. – Robot control
A6.1.1. – Continuous Modeling (PDE, ODE)
A6.1.5. – Multiphysics modeling
A6.2.1. – Numerical analysis of PDE and ODE
A6.2.6. – Optimization
A6.4. – Automatic control
A6.4.1. – Deterministic control
A6.4.3. – Observability and Controlability
A6.4.4. – Stability and Stabilization
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A6.4.6. – Optimal control
A6.5. – Mathematical modeling for physical sciences
A8.2.3. – Calculus of variations

Other research topics and application domains

B4.3.3. – Wind energy
B5.2.3. – Aviation
B5.2.4. – Aerospace
B5.6. – Robotic systems
B5.11. – Quantum systems

Contents

1 Team members, visitors, external collaborators

Research Scientists

- Jean-Baptiste Pomet [Team leader, INRIA, Senior Researcher, HDR]
- Ivan Beschastnyi [INRIA, Researcher]
- Jean-Luc Bouchot [INRIA, Advanced Research Position, from Feb 2025, HDR]
- Lamberto Dell'Elce [INRIA, Researcher, HDR]
- Laurent Hascoët [INRIA, Emeritus, from Feb 2025, HDR]
- Ludovic Sacchelli [INRIA, Researcher]

Faculty Members

- Jean-Baptiste Caillau [UNIV COTE D'AZUR, Professor, HDR]
- Olivier Cots [TOULOUSE INP, Associate Professor Delegation, from Sep 2025]

Post-Doctoral Fellows

- Jean Jacques Godeme [INRIA, Post-Doctoral Fellow, until Feb 2025]
- Marco Rando [INRIA, Post-Doctoral Fellow, from Nov 2025]

PhD Students

- Adel Malik Annabi [UNIV COTE D'AZUR]
- Antonin Bavoil [CNRS]
- David Da Silva Tinoco [INRIA]
- Riccardo Daluiso [UNIV COTE D'AZUR]
- Frank De Veld [INRIA]
- Eliot Stein [ONERA]

Interns and Apprentices

- Ghaieth Aloui [UNIV COTE D'AZUR, Intern, from Apr 2025 until Jul 2025]
- Matthieu Estines [CESI, Intern, from Sep 2025 until Oct 2025]
- Marwan Ksiks [UNIV COTE D'AZUR, Intern, from Apr 2025 until Jul 2025]
- Laurent Moinet [INRIA, Intern, from Jun 2025 until Sep 2025]
- Mohamad Samman [UNIV COTE D'AZUR, Intern, from Apr 2025 until Aug 2025]

Administrative Assistant

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External Collaborator

- Bernard Bonnard [UNIV BOURGOGNE, HDR]

2 Overall objectives

Our goal is to develop methods in geometric control theory for nonlinear systems, mostly finite dimensional, and to transfer our expertise through real applications of these methods. The methodological developments range from feedback control and observers to optimal control, extending to fields like sub-Riemannian geometry. Optimal control leads to developments in Hamiltonian dynamics, and also requires sophisticated numerics, to which the team contributes too. In addition, dynamical systems and modeling are also a part of the background of the team.

Our primary domain of industrial applications in the past years has been space engineering, in particular using optimal control and stabilization techniques for mission design: orbit transfer or rendez-vous problems in the gravity field of a single body (typically satellites around the earth), interplanetary missions and multi body problems, or control design of solar sails, where propulsion is drastically constrained.

The team also has continued involvement with applications regarding human bio-mechanics (muscle stimulation), and various modeling and control questions in biology (Lotka-Volterra models, bacterial growth, microbiome models, networks of chemical reaction...) The list is not exhaustive. Past domains of application include swimming at low Reynolds number (micro-swimmers) and control for Magnetic Resonance Imaging.

3 Research program

3.1 Control Problems

McTAO's major field of expertise is control theory in the broad sense. Let us give an overview of this field.

Modeling. Our effort is directed toward efficient methods for the control of real (physical) *systems*, based on a *model* of the system to be controlled. Choosing accurate models yet simple enough to allow control design is in itself a key issue. The typical continuous-time model is of the form

$$\frac{\mathrm{d}x}{\mathrm{d}t} = f(x,u)$$

where x is the *state*, ideally finite dimensional, and u the *control*. The control is left free to be a function of time, or a function of the state, or obtained as the solution of another dynamical system that takes x as an input. Modeling amounts to deciding the nature and dimension of x, as well as the dynamics (roughly speaking the function f). We are in general not involved in the modeling phase, but sometimes in identification, where the structure of "f" is known but it may contain a certain number of parameters that are not a priori known, and are to be identified from measurements.

Controllability, path planning. Controllability is a property of a control system (in fact of a model) that two states, say $x_{\text{init.}}$ and x_{final}, in the state space can be connected by a trajectory generated by some control $t \mapsto u(t)$ on a time-interval. In most cases, controllability can be decided by linear approximation, or non-controllability by "physical" first integrals that the control does not affect. For some critically actuated systems, it is quite difficult to decide local or global controllability, and the general problem is open. Path planning is the problem of constructing the control that actually steers one state to another.

Optimal control. In optimal control, one wants to find, among the controls that satisfy some constraints at initial and final time (for instance given initial and final state as in path planning), the ones that minimize some criterion. This is important in many control engineering problems, because minimizing a cost is often very relevant. Mathematically speaking, optimal control is the modern branch of the calculus of variations, rather well established and mature [83, 44, 32], but still displaying important and hard open questions. In the end, in order to actually compute these controls, ad-hoc numerical schemes have to be derived for effective computations of the optimal solutions. See more about our research program in optimal control in section 3.2.

Feedback control. In the above two paragraphs, the control is an explicit function of time. To address in particular the stability issues (sensitivity to errors in the model or the initial conditions for example), the control has to be taken as a function of the (measured) state, or part of it. This is known as closed-loop control. It must be combined with optimal control in many real problems. On the problem of stabilization, there is longstanding research record from members of the team, in particular on the construction of “Control Lyapunov Functions”, see [66, 84]. It may happen that only part of the state is measured at any one time, because of physical or engineering constraints, so that the control cannot be assigned as a function of the whole state. In that case, a popular strategy is to pair feedback methods with dynamic estimation of the state, creating so-called output feedback loops. Simultaneous feedback control and estimation can become a major hurdle for nonlinear systems, see [54, 87].

Classification of control systems. One may perform various classes of transformations acting on systems, or rather on models. The simpler ones come from point-to-point transformations (changes of variables) on the state and control (new state and control (z, v) given as a smooth function of (x, u) in a diffeomorphic manner yields a system $\mathrm{d}z/\mathrm{d}t = g(z, v)$ from the above control system). More intricate ones consist in embedding an extraneous dynamical system into the model. These are dynamic feedback transformations that change the dimension of the state. In most problems, choosing the proper coordinates, or the right quantities that describe a phenomenon, sheds light on a path to the solution. These proper choices may sometimes be found from an understanding of the modeled phenomena, or it can come from the study of the geometry of the equations and the transformation acting on them. This justifies the investigation of these transformations. These topics are central in control theory and they are present in the team, see for instance the classification aspect in [47] or —although this research has not been active very recently— the study [82] of dynamic feedback and the so-called “flatness” property [69]. Likewise, classification tools such as feedback invariants [38] are still currently in use in the team (see, for instance, [49]).

3.2 Optimal Control and its Geometry

Let us detail our research program concerning optimal control. Relying on Hamiltonian dynamics is now prevalent, instead of the Lagrangian formalism in classical calculus of variations. The two points of view run parallel when computing geodesics and shortest path in Riemannian Geometry for instance, in that there is a clear one-to-one correspondence between the solutions of the geodesic equation in the tangent bundle and the solution of the Pontryagin Maximum Principle in the cotangent bundle. In most optimal control problems, on the contrary, due to the differential constraints (velocities of feasible trajectories do not cover all directions in the state space), the Lagrangian formalism becomes more involved, while the Pontryagin Maximum Principle keeps the same form, its solutions still live in the cotangent bundle, their projections are the extremals, and a minimizing curve must be the projection of such a solution.

Cut and conjugate loci. The cut locus —made of the points where the extremals lose optimality— is obviously crucial in optimal control, but usually out of reach (even in low dimensions), and anyway does not have an analytic characterization because it is a non-local object. Fortunately, conjugate points —where the extremals lose *local* optimality— can be effectively computed with high accuracy for many control systems. Elaborating on the seminal work of the Russian and French schools (see [86, 31, 33] and [48] among others), efficient algorithms were designed to treat the smooth case. This was the starting point of a series of papers of members of the team culminating in the outcome of the *cotcot* software [43], followed by the *HamPath* [56] code and *control toolbox*. Over the years, these codes have allowed for the computation of conjugate loci in a wealth of situations including applications to space mechanics, quantum control, and more recently swimming at low Reynolds number. With in mind the two-dimensional analytic Riemannian framework, a heuristic approach to the global issue of determining cut points is to search for singularities of the conjugate loci. This line is, however, very delicate to follow on problems stemming from applications in three or more dimensions (see *e.g.* [57] and [40]). In all these situations, the fundamental object underlying the analysis is the curvature tensor. In Hamiltonian terms, one considers the dynamics of subspaces (spanned by Jacobi fields) in the Lagrangian Grassmannian [30]. This point of view withstands generalizations far beyond the smooth case. In L^1-minimization, for instance, discontinuous curves in the Grassmannian have to be considered (instantaneous rotations of Lagrangian subspaces still obeying symplectic rules [64]). The cut locus is a central object in Riemannian geometry, control and optimal transport. This was the motivation for a series of conferences on “The cut locus: A bridge over differential geometry, optimal control, and transport”, co-organized by team members and Japanese colleagues.

Riemann and Finsler geometry. Studying the distance and minimizing geodesics in Riemannian Geometry or Finsler Geometry is a particular case of optimal control, simply because there are no differential constraints.

It is studied in the team for the following two reasons. On one hand, after some transformations, like averaging or reduction, some more difficult optimal control problems lead to a Riemann or Finsler geometry problem. On the other hand, optimal control, mostly the Hamiltonian setting, brings a fresh viewpoint on problems in Riemann and Finsler geometry. On Riemannian ellipsoids of revolution, the optimal control approach allowed to decide on the convexity of the injectivity domain, which, associated with non-negativity of the Ma-Trudinger-Wang curvature tensor, ensures continuity of the optimal transport on the ambient Riemannian manifold [67, 68]. The analysis in the oblate geometry [41] was completed in [61] in the prolate one, including a preliminary analysis of non-focal domains associated with conjugate loci. Averaging in systems coming from space mechanics control with L^2-minimization yields a Riemannian metric, thoroughly computed in [39] together with its geodesic flow. In reduced dimension, its conjugate and cut loci were computed in [42] with Japanese Riemannian geometers. Averaging the same systems for minimum time yields a Finsler Metric, as noted in [37]. In [46], the geodesic convexity properties of these two types of metrics were compared. When perturbations (other than the control) are considered, they introduce a “drift”, *i.e.* the Finsler metric is no longer symmetric.

Sub-Riemannian Geometry. Optimal control problems that pertain to sub-Riemannian Geometry bear all the difficulties of optimal control, like the role of singular/abnormal trajectories, while having some useful structure. They lead to many open problems, see the monograph [80] for an introduction. The sub-Riemannian problem can be encoded by a non-linear control system with no drift, subjected to a quadratic energy minimization objective. This allows the sub-Riemannian problem to serve as rich model spaces for optimal control. The interest of sub-Riemannian geometry can go beyond these aspects however. It was proved by Hormander in 1967 [75] that local controllability of the system (given in terms of Lie-brackets of vector fields) is equivalent to sub-ellipticity of a second order differential operator associated with the vector fields. In this way, sub-Riemannian geometry acts as a bridge between elements of analysis of PDEs and geometric control theory. For instance, many recent works focus on framing properties of sub-elliptic operators in terms of minimizers of the optimal control problem (such as the influence of cut and conjugate points on diffusion asymptotics [36]). This link even allowed to successfully introduce concepts of sub-elliptic diffusions in computer vision algorithms thanks to sub-Riemannian geometric structures identified in mammal visual mechanisms [51].

Small controls and conservative systems, averaging. Using averaging techniques to study small perturbations of integrable Hamiltonian systems is as old an idea as celestial mechanics. It is very subtle in the case of multiple periods but more elementary in the single period case, here it boils down to taking the average of the perturbation along each periodic orbit [34, 85]. This line of research stemmed out of applications to space engineering (see Section 4.1): the control of the super-integrable Keplerian motion of a spacecraft orbiting around the Earth is an example of a slow-fast controlled system. Since weak propulsion is used, the control itself acts as a perturbation, among other perturbations of similar magnitudes: higher order terms of the Earth potential (including J_2 effect, first), potential of more distant celestial bodies (such as the Sun and the Moon), atmospheric drag, or even radiation pressure. Properly qualifying the convergence properties (when the small parameter goes to zero) is important and is made difficult by the presence of control. In [37], convergence is seen as convergence to a differential inclusion; this applies to minimum time; a contribution of this work is to put forward the metric character of the averaged system by yielding a Finsler metric (see Section 3.2). Proving convergence of the extremals (solutions of the Pontryagin Maximum Principle) is more intricate. In [60], standard averaging ([34, 85]) is performed on the minimum time extremal flow after carefully identifying slow variables of the system thanks to a symplectic reduction. This alternative approach allows to retrieve the previous metric approximation, and to partly address the question of convergence. Under suitable assumptions on a given geodesic of the averaged system (disconjugacy conditions, namely), one proves existence of a family of quasi-extremals for the original system that converge towards the geodesic when the small perturbation parameter goes to zero. This needs to be improved, but convergence of all extremals to extremals of an “averaged Pontryagin Maximum Principle” certainly fails. In particular, one cannot hope for C^1-regularity on the value function when the small parameter goes to zero as swallowtail-like singularities due to the structure of local minima in the problem are expected (a preliminary analysis has been made in [58]).

Optimality of periodic solutions/periodic controls. When seeking to minimize a cost with the constraint that the controls and/or part of the states are periodic (and with other initial and final conditions), the notion of conjugate points is more difficult than with straightforward fixed initial point. In [45], for the problem of optimizing the efficiency of the displacement of some micro-swimmers with periodic deformations, we used the sufficient optimality conditions established by R. Vinter's group [91, 71] for systems with non unique minimizers due to the existence of a group of symmetry (always present with a periodic minimizer-candidate control). This takes place in a long term collaboration with P. Bettiol (Univ. Bretagne Ouest) on second order sufficient optimality conditions for periodic solutions, or in the presence of higher dimensional symmetry groups, following [91, 71]. Another question relevant to locomotion is the following. Observing animals (or humans), or numerically solving the optimal control problem associated with driftless micro-swimmers for various initial and final conditions, we remark that the optimal strategies of deformation seem to be periodic, at least asymptotically for large distances. This observation is the starting point for characterizing dynamics for which some optimal solutions are periodic, and asymptotically attract other solutions as the final time grows large; this is reminiscent of the "turnpike theorem" (classical, recently applied to nonlinear situations in [88]).

In a completely different setting, namely the spectral analysis of Sturm-Liouville operators, periodic control has also been considered in the recent work [27]. For periodic conditions, the boundary problem treated in this paper can be considered as problem on the circle, so that the control is by design periodic. The problem is moreover well posed only in this category (translations of a non-constant solution would otherwise provide infinitely many solutions).

3.3 Software

Optimal control applications (but also the development of theory where numerical experiments can be very enlightening) require many algorithmic and numerical developments that are an important side of the team activity. We develop on-demand algorithms and pieces of software, for instance we have to interact with a production software developed by Thales Alenia Space. A strong asset of the team is the interplay of its expertise in geometric control theory with applications and algorithms, and the team has a long-lasting commitment to the development of numerical codes for the efficient resolution of optimal control problems. Methods for solving optimal control problems with ordinary differential equations more or less fall into three main categories. Dynamic Programming (or Hamilton Jacobi Bellman method) computes the global optimum but suffers from high computational costs, the so-called *curse of dimensionality*. Indirect methods based on Pontryagin Maximum Principle are extremely fast and accurate but often require more work to be applied, in terms of mathematical analysis and a priori knowledge of the solution; this kind of fine geometrical analysis is one of the strong know-how of McTAO. Direct transcription methods offer a good tradeoff between robustness and accuracy and are widely used for industrial applications. For challenging problems, an effective strategy is to start with a direct method to find a first rough solution, then refine it through an indirect method. We develop this further in a recent book chapter [59]. Such a combined approach has been for instance used between McTAO, the former COMMANDS team (Inria Saclay), and CNRS team APO (Université Toulouse, CNRS, ENSEEIHT) for the optimization of contrast in medical imaging (MRI), and fuel-effective trajectories for airplanes. This combination of direct and indirect methods has a lot of interest to solve optimal control problems that contain state or control constraints. In the collaborations mentioned above, the interfacing between the two solvers BOCOP and HamPath were done manually by *ad hoc* `python` or `matlab` layers. In collaboration with COMMANDS and colleagues from ENSEEIHT, McTAO leads the ct: control toolbox project whose goal is to interoperate these solvers using a high level common interface. The project is an Inria Sophia ADT[1] (2019-2023) in AMDT[1] mode supported by Inria Sophia SED. While the ADT ended in 2023, regular interaction between the project members and the SED team still take place. For instance, SED colleagues recently helped us to set up the CI infrastructure of control-toolbox.org on github.

3.4 Algorithmic differentiation (AD)

Although relevant to the previous section, this topic is only in the research program of the team *per se* since the end of February, 2025, when Jean-Luc Bouchot and Laurent Hascoët joined the team, due to the ending of the ECUADOR team (in fact, Jean-Luc Bouchot will be in the GAMMA-O team as of 2026).

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[1] ADT is the name of software development actions supported by the service "SED", by devoting some engineers to these projects. AMDT means that a group of engineers works on the project part time, rather than sending one engineer in the team.

Algorithmic differentiation (AD), also known as Automatic Differentiation or Differentiable Programming, and the related method of backpropagation, have received growing interest for their many uses in optimization, uncertainty quantification, and machine learning. Given a program that implements some mathematical function, AD creates a new program that computes derivatives of that function. While many approaches and tools have been developed to this end, we focus on reverse AD by source-transformation (ST-AD) which, compared to other approaches, often results in the highest efficiency to obtain gradients. ST-AD transforms the source code that computes the function into a new source code in the same language, that computes the derivatives. Reverse AD, specifically, computes the gradients in the reverse of the original computation order. As a consequence, the complexity of derivative computation becomes independent of the number of inputs. This is a key advantage when computing the gradient of simulations that go from millions of inputs (or more) to a handful of outputs of interest.

Providing a ST-AD tool is a demanding development, comparable to developing a compiler. The required techniques involve code analysis, mostly static data-flow analysis and semantic analysis. The AD tool Tapenade that we distribute results from a development of more than 20 person-year.

Application of AD to large codes is always a challenge, and often requires interaction between the developers of the application code and those of the AD tool. Even if the AD tools progressively become more reliable, interaction and new development are all the more necessary when performance is sought. All this is a source of new research questions for AD.

The evolution of application languages is also a source of new questions. While AD of Fortran or C has been studied for a long time, the novel idioms found in new popular languages (Python, Julia . . .), especially their dynamic or interactive aspects, pose plenty of challenging questions to the model of AD, and to the related AD tools.

4 Application domains

4.1 Aerospace Engineering

Participants: Jean-Baptiste Caillau, Thierry Dargent, Lamberto Dell'Elce, Frank de Veld, Jean-Baptiste Pomet.

Space engineering is very demanding in terms of safe and high-performance control laws. It is therefore prone to fruitful industrial collaborations. McTAO now has an established expertise in space and celestial mechanics. Our collaborations with industry are mostly on orbit transfer problems with low-thrust propulsion. It can be orbit transfer to put a commercial satellite on station, in which case the dynamics are a Newtonian force field plus perturbations and the small control. There is also, currently, a renewed interest in low-thrust missions such as Lisa Pathfinder (ESA mission towards a Lagrange point of the Sun-Earth system) or BepiColombo (joint ESA-JAXA mission towards Mercury). Such missions look more like a controlled multibody system. In all cases the problem involves long orbit transfers, typically with many revolutions around the primary celestial body. When minimizing time, averaging techniques provide a good approximation. Another important criterion in practice is fuel consumption minimization (crucial because only a finite amount of fuel is onboard a satellite for all its “life”), which amounts to L^1-minimization. Both topics are studied by the team. We have a steady relationship with CNES and Thales Alenia Space (Cannes), that have financed or co-financed 4 PhDs and 2 post-docs in the decade and are a source of inspiration even at the methodological level. Team members also have connections with Airbus-Safran (Les Mureaux) on launchers.

Some of the authoritative papers in the field were written by team members, with an emphasis on the geometric analysis and on algorithms (coupling of shooting and continuation methods). There are also connections with peers more on the applied side, like D. Scheeres (Colorado Center for Astrodynamics Research at Boulder), the group of F. Bernelli (Politecnico Milano), and colleagues from University of Barcelona (A. Farrès, A. Jorba).

Two new directions have been taken recently. The first one is about the control of solar sails, the second one about collision avoidance for spacecrafts (see Section 7.12). Collision avoidance is becoming very important in nowadays space missions due to the growing number of various bodies (garbage, micro-satellites. . .) orbiting around the earth. A PhD (Frank de Veld), defended in December [11], supported by Thales Alenia

Space. Solar sailing has been actively studied for two decades and recent missions have demonstrated its interest for "zero-fuel" missions; it poses delicate control questions due to drastic constraints on the control direction. It was the topic of Alesia Herasimenka's PhD, selected by ESA for a three-year research co-sponsorship, and defended in September, 2023.

4.2 Quantum control

Participants: Charles Babin *(Univ. Bourgogne Europe, CNRS, ICB)*, Ivan Beschastnyi, Jean-Baptiste Caillau, Lamberto Dell'Elce, Jean-Baptiste Pomet, Ludovic Sacchelli, Dominique Sugny *(Univ. Bourgogne Europe, CNRS, ICB)*, David Tinoco.

Quantum systems are increasingly used in various applications, from metrology to computation. However, their manipulation is very challenging due to the effects of quantum theory itself as well as interactions with the environment. Therefore, creating efficient and robust control strategies is one of the central tasks of quantum engineering.

At MCTAO we are interested in two research directions. The first is the development of control tools for general quantum control systems. This includes optimal control, adiabatic control, qualitative and quantitative controllability, and the identification of quantum systems. Our goal is to develop a broad range of methods that can be applied to general classes of quantum systems without directly specifying the type of technology used.

The second direction involves working with experimental physicists on concrete problems, where the specific nature of a physical system suggests additional tools. Currently, we have established a collaboration with physicists (C. Babin, D. Sugny) from the Laboratoire Interdisciplinaire Carnot de Bourgogne in Dijon on a project related to the control of Nitrogen-Vacancy centers. Our PhD student, David Tinoco, is working on mathematical aspects relevant to the control of such systems. At INPHYNI (Institut de Physique de Nice), we are currently working with J. Etesse on control strategies for quantum memory.

4.3 Neural dynamics

Participants: Adel Annabi, Dario Prandi *(CNRS, CentraleSupélec)*, Jean-Baptiste Pomet, Ludovic Sacchelli.

Neural fields serve as integro-differential dynamical models for the transmission of activity within cortical areas [53]. Originating in the 1970s, these models prove particularly advantageous when exploring the mesoscopic scale. At this level, the neuronal clusters under examination are sufficiently large to be understood as a continuum, yet compact enough to enable a targeted investigation of specific cortical functions. A significant appeal of these models lies in their efficacy in describing phenomena within the perceptual mechanisms of vision and audition. Notably, they have paved the way for sub-Riemannian-inspired geometric models addressing the anisotropic diffusion of information [50, 52].

Given their successes in characterizing cortical areas, their interplay and their scale, these models also offer valuable insights into experiments involving the measurement and stimulation of neural activity via electrodes. Consequently, substantial interest has been directed toward these models from the point of view of control, where the input-output formalism provides strategic avenues for deep-brain stimulation techniques. This interest has manifested in recent applications, including the treatment of Parkinson's disease [63]. The exploration of this perspective is the topic of A. Annabi's PhD research, which delves into the visual cortex, specifically concentrating on observability and observer design for low-dimensional models within the V1 cortical area.

5 Highlights of the year

5.1 Awards

- Jean-Baptiste Caillau received a 2025 Prix d'Excellence from Université Côte d'Azur

6 Latest software developments, platforms, open data

6.1 Latest software developments

6.1.1 ct

Name: control-toolbox

Keywords: Optimal control, Ordinary differential equations, Mathematical Optimization, Differential homotopy, Automatic differentiation

Scientific Description: Numerical resolution of optimal control problems

Functional Description: The project gathers and allows to interoperate tools designed to solve numerically optimal control problems on ordinary differential equations. The available approaches include direct methods (based on a transcription of optimal control problems into mathematical programs) as well as indirect ones (based on Pontrjagin maximum principle, like the shooting method). The latter can be coupled to differential continuation. Automatic differentiation (aka Differentiable Programming) plays a crucial a role in all these algorithms. The project strongly leverages on SED Sophia support.

Release Contributions: - solve on GPU - benchmarking

News of the Year: - solving both on CPU and GPU - benchmarking on the collection defined in OptimalControlProblems.jl

URL: `https://control-toolbox.org`

Contact: Jean-Baptiste Caillau

Participant: 11 anonymous participants

Partners: Université de Toulouse, CNRS, IRIT, ENSEEIHT

7 New results

7.1 Control templates for output feedback stabilization of non-uniformly observable systems

Participants: Ludovic Sacchelli, Lucas Brivadis *(CentraleSupélec, Gif-sur-Yvette)*, Ulysse Serres *(Université Claude Bernard Lyon 1)*, Vincent Andrieu *(Université Claude Bernard Lyon 1)*, Jean-Paul Gauthier *(Université de Toulon)*, Itaï Ben Yacoov *(Université Claude Bernard Lyon 1)*.

Stabilizing a system by feedback is a fundamental problem in control theory. When only partial measurements are available, output feedback control relies on an observer to reconstruct the state. In nonlinear systems, however, state estimation may depend on the control itself, a phenomenon known as non-uniform observability, which can severely hinder stabilization. In particular, the presence of controls that are singular for observability creates intrinsic obstructions to output feedback stabilization, for which no general solution is known. It has long been recognized that purely state-based feedback laws may fail in this context, motivating the introduction of time-varying or hybrid mechanisms. One approach consists in perturbing a state-feedback design to restore observability, a direction the group has been exploring through hybrid control techniques.

In [1], published this year, we introduced a new method based on control templates. The feedback is replaced by a piecewise-defined approximation, where each segment follows a control known to ensure observability, generalizing the classical sample-and-hold paradigm. Building on recent results by Lin et al. on hybrid output feedback under observability for all inputs (see, e.g. [78]), [1] extends these guarantees to analytic systems observable under a null input, and proves that control templates are generic among analytic controls.

7.2 Singular arcs in L^1-optimal control problems

Participants: Ivan Beschastnyi, Andrei Agrachev *(SISSA Trieste, Italy)*, Michele Motta *(SISSA Trieste, Italy)*.

L^1-optimal control problems play an important role in engineering sciences. They appear frequently when we want to minimize work or total cost. For this reason they appear frequently in applications, for example, in space mechanics [55] or in economic epidemiology [79]. A notable feature of such minimal solutions to such problems is the presence of long intervals of zero control (sparse control strategy).

This year, we have found sufficient conditions for local minimality of singular arcs using the methods of over-maximized Hamiltonians. This solves completely the problem of their minimality. The results are in preprint [12].

7.3 Optimal experiment design for parameter identification

Participants: Ludovic Sacchelli, Alessandro Scagliotti *(Technical University of Munich, Germany)*.

In parameter identification, experiment design aims to plan data acquisition so that the resulting estimation is as informative as possible. A key difficulty is that the outcome of an experiment typically depends on the very parameter being estimated, so that the optimization objective itself depends on the unknown parameter. Standard approaches replace the unknown value with a nominal estimate, such as the prior mean, while more robust strategies account explicitly for parameter uncertainty by averaging the objective over the prior distribution, yielding an infinite-dimensional formulation.

In [22], we considered a linear system with noisy measurements in a Bayesian Gaussian setting and formulated an input-design problem balancing posterior uncertainty reduction with constraints on state magnitude. We first analyzed the classical formulation from the perspective of optimal control, then extended it to the ensemble-control framework. Existing results on the ensemble Pontryagin maximum principle did not cover unbounded parameter ensembles, such as our Gaussian-distributed case, so we introduced a generalization of the Pontryagin Maximum Principle (PMP) to this setting and characterized the resulting optimal flow.

7.4 Controllability of fast-oscillating system with control constraints

Participants: Jean-Baptiste Caillau, Lamberto Dell'Elce, Alesia Herasimenka *(Univ. of Surrey)*, Jean-Baptiste Pomet.

Control of solar sails requires a comprehensive understanding of controllability, that is a difficult problem because the controls (forces) are drastically constrained. This motivated a controllability study of a class of systems that have periodic behavior without control and where the control is constrained in a convex set that has zero in its boundary. Condition for controllability of these systems are in the paper [5], published this year. They have to be formulated in terms of pushforwards along the flow of the drift, rather than in terms of

Lie brackets, and they take into account not only the vector fields associated to the system but also the shape of the control set; it turns out that they amount to local controllability of a time-varying linear approximation with constrained controls. These conditions are harder to check in practice than these formulated in terms of the rank of a family of vector fields; methods to check these conditions in the case of solar sails are given in [74] (published earlier): these are based on convex optimization; the approach leverages fine properties of trigonometric polynomials as well as Nesterov's technique of sum of squares relaxation.

7.5 Stability of linear time-varying time-delay systems

Participants: Laurent Baratchart *(FACTAS project-team)*, Sébastien Fueyo *(DANCE project-team)*, Jean-Baptiste Pomet.

A linear time-periodic difference-delay systems (periodic LDDS for short) is a dynamical system of the form $z(t) = A_1(t)z(t-\tau_1) + \cdots + A_N(t)z(t-\tau_N)$, where z is finite dimensional and the matrices A_j depend periodically on time; the state of this dynamical system is infinite dimensional. S. Fueyo's doctoral work [70] was about testing the stability of nonlinear amplifiers for high frequency signals by frequency domain methods; linearizing along an internal periodic solution yields a model based on networks of 1-D hyperbolic PDEs, and a periodic LDDS appears as its 'high frequency limit"; the stability of this LDDS conditions stability of the PDE dynamical system (see *e.g.* the long introduction of [35]). We give in the paper [3], published this year, a necessary and sufficient condition for hyperbolic stability of periodic time-varying LDDS, generalizing the well-kown one for the time-invariant case, due to Hale and Henry [73, 72].

7.6 Random walks in sub-Riemannian manifolds

Participants: Ludovic Sacchelli, Robert W. Neel *(Lehigh University, USA)*.

Laplacian-type operators on sub-Riemannian manifolds encode the non-holonomic constraints of the underlying control structure. Their associated heat diffusion links the geometry of sub-Riemannian geodesics to the behavior of random paths on the manifold. Since Varadhan's seminal work in the Riemannian setting [90], it has been known that short-time heat kernel asymptotics recover the distance between two points, reflecting the exponentially rare event that a diffusion connects them in very small time.

This program has been actively developed in the sub-Riemannian setting, and [21] brings together and completes several strands of this line of inquiry. The paper provides a unified treatment of small-time asymptotics of the heat kernel, its derivatives, and its logarithmic derivatives, with new results on localization that make the analysis applicable to incomplete manifolds under essentially optimal assumptions. Away from abnormal minimizers, the asymptotics are governed by the geometry of minimizing geodesics, including subtle behavior near the cut locus, leading to uniform bounds on compacts and full asymptotic expansions in many cases. A genuinely new component concerns logarithmic derivatives: using a law of large numbers for diffusion bridges, we show that the leading behavior of the log-Hessian characterizes the non-abnormal cut locus. The length and scope of the paper reflect several years of work, and this final revision consolidates the results into a coherent framework.

7.7 Nonregularity of abnormal sub-Riemannian geodesics

Participants: Ludovic Sacchelli, Yacine Chitour *(CentraleSupélec, Gif-sur-Yvette)*, Frederic Jean *(ENSTA, Palaiseau)*, Roberto Monti *(University of Padova, Italy)*, Ludovic Rifford *(Université Côte d'Azur, Nice)*, Mario Sigalotti *(Inria Paris)*, Alessandro Socionovo *(Inria Paris)*.

A distinctive feature of sub-Riemannian geometry is the existence of abnormal minimizing geodesics, whose smoothness is not guaranteed. While normal geodesics are smooth and can be characterized via the

Pontryagin maximum principle, abnormal minimizers escape this regular framework, making their analysis particularly challenging. Previous results have established partial regularity only in low-dimensional or analytic cases, leaving open the general question of whether abnormal minimizers can fail to be smooth.

In [17], the above consortium constructed an explicit example of a strictly singular minimizing geodesic that is C^3 but not C^4. Establishing this result required precise asymptotic analysis to rule out the possibility that nearby normal trajectories could be shorter, and relied on numerical simulations to identify key qualitative features among short normal competitors to the abnormal curve. The construction provides a definitive instance of a non-smooth minimizing geodesic, contrasting with other recent results on the regularity of abnormal minimizers, and answers a long-standing open question in sub-Riemannian geometry.

7.8 Geometry and optimal control for navigation problem

Participants: Bernard Bonnard, Olivier Cots, Jérémy Rouot *(Université de Bretagne Occidentale, Brest)*.

This is a long term research program that revisits and generalizes the Navigation Problem set by Carathéodory and Zermelo (see [92, 62]) of a ship navigating on a river with a linear current and aiming to reach the opposite shore in minimum time. This work was motivated by the displacement of particles in a two dimensional fluid, in presence of a vortex (initially, a singularity in the Helhmoltz-Kirchhoff equations) inducing a strong current that hampers local controllability. To define a minimum time Zermelo navigation problem, we consider the particle as the ship of the navigation problem and the control is defined as the heading angle of the ship axis.

This year, a new application to spin magnetization reversing was considered. The geometric interpretation of this phenomenon in micromagnetism as a Zermelo navigation problem is a novel point of view. It allowed us to fit the problem in a L^∞ accessibility framework, which can be solved by combining algebraic and numerical simulations. The problem can be sorted in two cases (Randers vs Finsler) according to the 4 physical parameters. In the Finsler case, the numerical framework developed by Cots et al. permitted us to completely solve the problem on the 2d-sphere and determine the cut locus (simple branch with two extremities) [4].

7.9 Pulse control of affine systems with applications to quantum control

Participants: Ivan Beschastnyi, Lamberto Dell'Elce, Jean-Baptiste Pomet, Ludovic Sacchelli, David Tinoco.

Time-optimal control problems with unbounded controls naturally arise in quantum engineering, where fast operations are required to mitigate decoherence. In this work, we study time-minimization for affine control systems allowing impulsive (pulse-like) controls. Such problems are typically ill-posed in classical control spaces, as arbitrarily large control amplitudes lead to instantaneous state transitions.

In [8], we introduce a relaxation framework based on a time-rescaling technique that provides a rigorous treatment of impulsive controls without resorting to geometric reduction procedures. The resulting extended system is defined on the original state manifold and admits optimal solutions, albeit with a high degree of symmetry and degeneracy. Using Pontryagin's Maximum Principle, we characterize the structure of extremal trajectories and recover, in a simplified way, several known results on optimal syntheses.

The approach is illustrated on concrete quantum control problems, including time-optimal gate generation for closed and open two-level systems and population transfer in dissipative dynamics. In particular, we analyze the role of pulse and singular arcs and discuss numerical issues arising from the degeneracy of the relaxed problem.

7.10 Open qubit parameter identification with bounded pulses

Participants: Ghaieth Aloui, Ivan Beschastnyi, Ludovic Sacchelli.

Open quantum systems are inherently affected by environmental interactions, which manifest as relaxation and dephasing, and fundamentally limit the performance of quantum devices. Accurate identification of the parameters governing these processes is therefore essential for reliable control and calibration of qubit-based technologies. In [13], we study the problem of parameter identification for a single open qubit subject to dissipation. Our approach relies on a small number of carefully designed qubit configurations generated by saturating control pulses and measured through simple projective readout, leading to an explicit and interpretable identification protocol.

In an idealized regime of infinite-amplitude pulses, we showed that all parameters can be reconstructed analytically from a minimal set of observables. We then analyzed the effect of realistic, finite-amplitude pulses as a perturbation of this ideal setting and derive bounds on the resulting estimation error. This allowed us to separate statistical uncertainty due to finite measurements from modeling errors induced by bounded controls, providing quantitative guarantees for practical implementations.

7.11 Control of an nitrogen-vacancy center as a two-qubit system

Participants: David Tinoco, Ivan Beschastnyi, Jean-Baptiste Caillau, Charles Babin *(Université Bourgogne Europe, CNRS, ICB)*, Dominique Sugny *(Université Bourgogne Europe, CNRS, ICB)*.

Nitrogen-vacancy centers constitute a leading platform for solid-state quantum technologies, where high-fidelity control is limited by partial accessibility and environmental constraints. A collaboration with physicists from ICB (Institut Carnot de Bourgogne) has started, with CNRS funding (CONV project, for Control of nitrogen-vacancy centers), that targets at modeling, simulating and optimizing such systems, and also at making the experiments on small instances (nitrogen-vacancy centers inside diamonds). We study a benchmark control problem consisting of an electronic spin coupled to a nuclear spin, in which only the electronic degree of freedom can be directly driven by microwave fields.

Starting from physically motivated approximations, in paper [23], we analyze the controllability of the resulting two-qubit model using standard tools from geometric control theory. We identified the reachable sets for this approximation and computed the quantum speed limit for two-qubit gate generation by exploiting Lie algebraic and group-theoretic techniques. Finally, numerical optimal control simulations demonstrate that this fundamental limit can be closely approached while maintaining bounded control amplitudes, illustrating the practical relevance of the theoretical bounds.

7.12 Low-thrust satellite collision avoidance maneuvers

Participants: Lamberto Dell'Elce, Frank de Veld, Jean-Baptiste Pomet, Joel Amalric *(Thales Alenia Space)*, Roberto Armellin *(University of Auckland)*.

Frank de Veld defended his thesis [11] on the design of collision avoidance maneuvres for a satellite equipped with low-thrust propulsion in encounters with non-cooperative objects (e.g., space debris). The thesis addressed the challenge of optimally controlling low-thrust satellites to avoid collisions, taking into account both the satellite dynamics and operational constraints. Below, we briefly summarize the major contributions of the thesis, all of which were essentially completed during 2025.

The first contribution consisted in characterizing the latest safe time to start a maneuvre. To this end, we introduced a novel set of state variables, comprising the time of closest approach and the satellite state at that instant. This formulation allowed the closest approach to be tracked directly. Necessary conditions for optimality were derived using Pontryagin's Maximum Principle, and approximate solutions for fast encounters were obtained by backward integration from the safe set. We also performed a detailed analysis of the optimal solution structure, including the computation of cut and conjugate loci.

The second contribution was aimed at including operational constraints in the metodology. Namely, we considered a prescribed thrusting direction imposed by mission requirement and we included non-thrusting arcs (e.g., during eclipses).

The third contribution, which was developed during F. de Veld's stay at the University of Auckland, revisited the optimization of thrust initiation using Differential Algebra, employing Taylor series expansions to efficiently compute safe maneuvres under three danger metrics: Euclidean distance, Mahalanobis distance, and probability of collision.

No peer-reviewed publications were produced during the year, but several articles were prepared. Two papers discussed the minimum warning time and its extension to operational constraints. One paper presented work from Frank's secondment at the University of Auckland.

7.13 Levi-Civita regularization of collisions in the controlled case

Participants: Alain Albouy *(Observatoire de Paris, CNRS)*, Jean-Baptiste Caillau, Riccardo Daluiso.

In the n-body problem, n point masses move in space under their mutual gravitational interactions as described by Newton's theory of gravity. The forces acting between particles approach infinity when the mutual distances approach zero. Therefore, at collision, the equations of motion have singularities. Since Levi-Civita (1903) and Sundman (1907), the double collision has been "regularized", *i.e.* the singularity has been made to disappear by means of algebraic transformations. In his 1920 paper [77], Levi-Civita presented a regularization of the three-body problem which was ignored by almost all the authors with only a few exceptions. The idea of Levi-Civita was to first regularize a parabolic collision orbit of the Kepler problem by means of the Hamilton-Jacobi method, following procedures from Poincaré. These techniques surprisingly provide contact transformations which make the Hamiltonian of the three-body problem perfectly regular. More than fifty years later, Moser (1970) compared the stereographic projection of the great circles of the sphere with Keplerian odograph, following ideas from Fock. This construction gives rise to a compactification of the phase space of the Keplerian orbits for a fixed level of energy together with the same regularization found by Levi-Civita, which is thus designated to be unique and fundamental. The main issue arising from these technique is the inability for the authors in finding regularizazions which work simultaneously for all the energy surfaces. Indeed, it turns out that the Levi-Civita transformation, defined by

$$q \mapsto |p|^2 q - 2\langle p, q\rangle p, \qquad p \mapsto \frac{p}{|p|^2},$$

if applied to the two-body problem sends all the collision states to the same value of the energy (*i.e.* with the same value of the semi-major axis of the collision orbit) on the same state of the regularized system: the transformation obtained is not bijective. In a recent work, A. Albouy proposes a simple trick to avoid this issue by viewing the problem as defined on 4-manifold embedded into the 5-dimensional Euclidean space on which energy is just another coordinate.

The purpose of this work is to apply the regularization methods to optimal control. Indeed, the compactification of the state space is relevant for the existence of solutions for optimal control problems in space mechanics, and we consider the control of a spacecraft under the attraction of one or more bodies. The main question we face is the extension of the theory to the case of non-constant energy. Our first results include controllability, existence and extremal flow analysis at collisions on the regularized problem.

7.14 Powered landing of reusable space launchers

Participants: Jean-Baptiste Caillau, Lamberto Dell'Elce, Samuel Hornus *(Inria Nancy Grand Est)*, Jean-Baptiste Pomet.

We propose to extend the study of the Soft Planetary Landing problem as defined in [29] or, more recently, by [76]. This problem seeks to determine the best trajectory for landing a vehicle at a given point while consuming as little propellant as possible. We wish to provide the detailed mathematical structure of the optimal trajectories for this problem using the necessary conditions for optimality (*indirect methods* in the sense of optimal control theory). By ensuring an understanding of the structure of optimal solutions for almost any instance of the problem, we seek to pave the way for a quasi-real-time MPC (Model Predictive Control) controller design without time discretization.

Two American companies, Blue Origin and SpaceX, have demonstrated that reusing a space launcher is viable and advantageous. Unlike previous launchers developed by public agencies, these reusable launchers are developed by private companies, meaning that the expertise and theoretical and practical knowledge they have acquired remain largely inaccessible outside these companies. In the long term, we wish to contribute to the establishment and publication of technical knowledge useful for the successful development of reusable European space vehicles. This optimal control problem seeks to minimize fuel consumption for a rocket-propelled vehicle moving from a given position/velocity above the ground to a rest position on the ground. The associated trajectories must satisfy several constraints, including cone constraints both on the control (thrust angle) and on the position.

We want to describe in detail the mathematical structure of the optimal trajectories for this problem, with the help of the necessary optimality conditions given by the Pontryagin Maximum Principle. We compute indirect solutions for any feasible instance of the problem. This is possible thanks to the determination of the dynamics along constrained arcs for each type of constraint, and to the understanding of the possible sequences of arcs that can arise. For instance, it can be proved that the two constraints (on the state and on the control, see above) probably cannot be active at the same time along a positive interval of time. On top of this mathematical analysis, we develop techniques to determine the structure of the optimal trajectory for a large variety of specific instances of the problem. A crucial step for this is the use of a direct discretization coupled with an efficient nonlinear optimization solver; the resulting approximate solution allows to capture in most cases the structure of the true solution, leading to a precise "indirect" resolution via multiple shooting.

7.15 Precise orbit determination of non-cooperative objects in medium and geostationary orbits

Participants: Lamberto Dell'Elce, Eliot Stein, Frederic Cassaing *(ONERA)*, Hanae Labriji *(ONERA)*, Florian Thuillet *(ONERA)*.

The tracking of resident space objects (RSO) with high accuracy is essential for safe and sustainable space operations, enabling early conjunction warnings, improved decision-making, reduced fuel consumption, and longer satellite lifetimes. Orbit determination (OD) is limited by observation accuracy: passive optical stations for medium- and high-Earth orbits (MHO) typically provide few hundreds of milliarcseconds angular accuracy, yielding OD errors of several hundred meters between successive nights of observations.

In the framework of E. Stein's PhD thesis, we plan to exploit measurements from the CICLOPE telescope, which provides 50 milliarcseconds accuracy (5–10 m at MHO distances), sufficient to detect deviations from gravitational dynamics, including solar radiation pressure shifts of 10 m over 4 h for typical satellites and up to 35 m for debris. This precision allows reliable non-gravitational perturbation estimation and advanced orbit determination.

In 2025, we developed and algorithm for precise orbit prediction with decametric accuracy over one day for non-cooperative MHO RSOs using sparse angle-only measurements. A high-fidelity force model is built, and OD is performed with weighted least squares and a Jacobian from variational equations. Using simulated 50 milliarcseconds observations, standard gravitational models give hectometric errors, while including SRP estimation reduces errors to about 20 m.

7.16 Hardware-in-the-loop attitude simulator for the NiceCube mission

Participants: Lamberto Dell'Elce, Matthieu Estines, Laurent Moinet.

The Centre Spatial Universitaire (CSU) of the Université Côte d'Azur is planning to build its first nanosatellite, named NiceCube, which has the technological objective of demonstrating data transmission from the satellite to the ground via an optical link. McTAO is in charge of developing the attitude determination and control system (ADCS) of NiceCube. In the previous years, we developed a high-fidelity numerical simulator of the coupled orbital-attitude motion of a satellite in low-Earth orbit. This year, two internships were devoted to the integration of such simulator into the so-called "FlatSat" under development at the CSU, which will enable hardware in the loop simulations of NiceCube. Specifically, L. Moinet implemented an interface aimed at transferring data from the simulator to the on-board computer of the satellite. This work was then finalized by M. Estines, who also initiated the implementation of a simple attitude determination and control algorithm onboard the satellite.

7.17 Kite Electrical Energy Production (the KEEP project)

Participants: Antonin Bavoil, Jean-Baptiste Caillau, Alain Nême *(ENSTA)*, Christian Jochum *(ENSTA)*, Jean-Baptiste Leroux *(ENSTA)*.

This project aims at improving the performance of technologies that meet the challenges of the law concerning the energy transition. The innovative KEEP (Kite Electrical Energy Production) technology developed at ENSTA enables the production of electrical energy from a kite capturing wind energy. It is a Ground-Gen system with a moving-ground-station (rocking arm) simpler than the devices described, *e.g.*, in [65]. The first numerical models of the system show an increase of about 50% in the efficiency of this technology compared to already existing ones [89]. In the short and medium term, the onshore and shipboard applications with a power production ranging from a few tens to hundreds of kW are targeted, with an extension to an offshore production power capacity at a MW scale with 1000 m^2 kites. To this end, the following developments are required: the improvement of the mechanical model, as well as the theoretical analysis and optimization of the complete dynamical system.

In the preprint [14], we present a brief derivation of the model. Our approach consists in an implicit modeling of the control exerted on the kite: we compute the force such that the motion is indeed prescribed to an eight shaped figure, typical of the desired motion. While the resulting system is a differential algebraic equation (DAE), it is possible to make it explicit as a two-dimensional second-order ODE. We are interested in the limit cycles of this equation, and study them numerically in detail. In order to obtain the relevant periodic motion (only some limit cycles are admissible for the kite), it is convenient to start from specific equilibria on the system so we also examine them. The resulting limit cycle, that depends on several design parameters, is then optimized so as to maximize the energy generated by the motion (the kite is attached to an arm in the ground plane whose oscillations feed a generator). This optimization with respect to a set of finite dimensional parameters leads to an approximate 12% increase of the generated power.

7.18 Parameter and state estimation in neural models

Participants: Adel Annabi, Dario Prandi *(CNRS, CentraleSupélec, LSS)*, Jean-Baptiste Pomet, Ludovic Sacchelli.

This research focuses on dynamic estimation and observer synthesis for neural field equations, the general topic of A. Annabi's PhD. In the visual cortex, neural field models can describe activity dynamics related to orientation sensitivity of neurons. This allows mapping neural fields onto the orientation domain, enabling a Fourier representation that can be truncated to a simplified three-dimensional model of the V1 area. In [2], we investigated the observability of this model, highlighting the system's symmetries and proposing hybrid elements to counteract their effects. The study shows the role of nonlinearity in achieving observability and identifies persistence conditions required for accurate state estimation.

In the conference paper [7], presented this year at CDC 2025, we addressed parameter identification in networks of interconnected neural populations, with measurements available from only one. We proposed an online approach that exploits the system's nonlinear characteristics, particularly saturation functions,

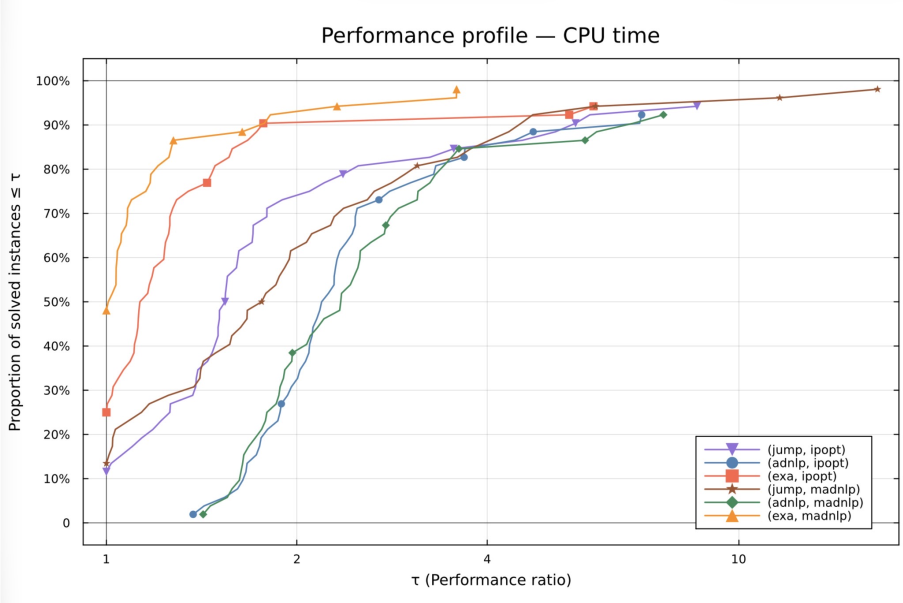

Figure 1: Performance profile of several modeler / solver combinations to solve 10+ instances of optimal control problems. Best combos are top left. More info on CTBenchmarks.jl

to recover parameters. By designing targeted control inputs that adapt to unknown states and parameters, parameters can be retrieved online.

7.19 control-toolbox

Participants: Jean-Baptiste Caillau, Olivier Cots, Joseph Gergaud, Pierre Martinon *(CAGE project-team, on leave)*.

The ADT `ct: control-toolbox` had its final sprint in 2023. The focus was on initiating new developments in Julia to take advantage of the powerful features of the language. Julia is indeed a perfect match for our needs in scientific computing for numerical optimal control; the language has a high level of abstraction well suited for mathematical descriptions, but still makes no compromise when it comes to performance thanks to efficient just-in-time compilation. Moreover, it currently has several efficient backends for AD / DP (automatic differentiation / differentiable programming), including ForwarDiff, Zygote or Enzyme: this is a crucial step for our project, both for direct and indirect methods. (Some examples of the project gallery require up to five levels of nested automatic differentiation.) The toolbox is now a full ecosystem available at control-toolbox.org. These achievements and the use of Julia have recently been presented in conferences [26, 25]. In 2024, the effort has been concentrated on the Julia package OptimalControl.jl, at the heart of the control-toolbox.org ecosystem. This development is strongly tied to an efficient use of sparse linear algebra, numerical optimization and automatic differentiation. Fruitful exchanges with colleagues from the former Ecuador team and from Argonne National Lab are key to this task and benefited in 2025 of visits from and to Argonne (J.-B. Caillau visited Argonne for one week with JLESC fundind during JLESC 2025 conference). OptimalControl.jl is currently one of the very few packages to offer state-of-the-art and out-of-the-box high level and super fast optimal control problem solving both on CPU and GPU. To this end, we leverage several modelers and optimization solvers (including ExaModels / MadNLP from our colleagues at MIT, Argonne and Mines Paris). Detailed benchmarks are on their way, as displayed in Figure 1 below for our first results. In December 2025, the project also benefited of a sprint session dedicated to BifurcationKit.jl by R. Veltz (Cronos team) that could be a key tool to add path following methods for shooting in OptimalControl.jl.

7.20 Finite differences for two-level optimization and applications in learning

Participants: Marco Rando, Samuel Vaiter *(Univ. Côte d'Azur, CNRS, LJAD)*.

This work focuses on two-level optimization, where the aim is to minimize a function whose evaluation depends on the solution to an internal optimization problem. Traditional methods are based on calculating the hypergradient (gradient of the value function), but they become inapplicable when first-order information is not available, as in the zero-order (black box) setting. In the framework of Marco Rando's postdoc (started November 2025, PDE-AI project), Marco and Samuel Vaiter propose ZOBA, the first single-loop algorithm based on finite differences for two-level optimization, avoiding costly and difficult-to-parallelize double-loop schemes. The method uses an approximation of the hypergradient that exploits delayed information, eliminating the need for nested loops. A theoretical analysis establishes convergence guarantees in a non-convex context, with complexity in $O(p(d+p)2\varepsilon-2)$, which is better than some previous approaches based on Hessian approximation. HF-ZOBA, a Hessian-free variant offering additional complexity gains, is also introduced. Experiments on synthetic functions and a real-world application in black-box adversarial learning confirm that these methods achieve state-of-the-art accuracy while reducing computation time.

7.21 Algorithmic Differentiation support and collaboration with Tapenade users

Participants: Laurent Hascoët, Jean-Luc Bouchot, Michael Vossbeck *(The Inversion Lab, Hamburg, Germany)*, Sri Hari Krishna Narayanan *(Argonne National Lab. (Illinois, USA))*, Shreyas Gaikwad *(U. of Texas, Austin, USA)*.

We support both academic and industrial users of the AD tool Tapenade for their own applications. This involves AD on codes of all sizes, providing us with suggestions for improvement and possibly new research.

This year's main applications are on:

- the global circulation model MIT GCM, in collaboration with Shreyas Gaikwad and Patrick Heimbach (University of Texas in Austin) for end-users in all fields of climatology including glaciology (Dan Goldberg, University of Edinburgh, UK). The goal is to blend MIT GCM with an approved tool for adjoint differentiation that is open-source and free of cost for end-users. This effort has run over several years and now results in Tapenade AD officially adopted by the maintainers of the GCM.

- the Biosphere model NUCAS (The Inversion Lab, Hamburg, Germany) and closely related codes BEPS and BETHY, in collaboration with Michael Vossbeck (The Inversion Lab). NUCAS was already differentiated (tangent and adjoint) with Tapenade, in its sequential version. This year, Tapenade differentiated the MPI-parallel version of NUCAS. This required a few improvements to Tapenade handling of MPI codes, resulting in adMPI: a differentiable wrapper around MPI maintained and shipped with the Tapenade distribution.

- the nuclear physics code HFBTHO (Lawrence Livermore National Lab, USA), in collaboration with Krishna Narayanan (Argonne National Lab). HFBTHO is a quantum physics code that models the orbits of particles inside the atomic nucleus. HFBTHO approximates the underlying complex physics through a number of hidden parameters, that must be guessed by parameter estimation. This uses AD-computed derivatives. Only tangent derivatives are required at present. This work is discussed in an article [6] published in "Computer Physics Communications". In parallel, Krishna Narayanan differentiates the BLAS library (Linear Algebra) by AD with Tapenade.

7.22 Jacobian Sparsity

Participants: Laurent Hascoët, Jean-Luc Bouchot, Alexis Montoison *(Argonne National Laboratory, Illinois, USA)*.

Jacobian Matrices (matrix of the first-order derivatives of each output with respect to each input) are one key derivative object. As they are often very large, it is profitable to exploit their sparsity, in particular with compression models that use coloring. The sparse structure of the Jacobian, a matrix of boolean values describing the structural zeros of the Jacobian, can be computed by source-transformation of the original model with an AD tool. This requires a special mode of AD, "sparsity-AD", which is already know in the literature, but almost always implemented in Overloading-based AD tools, with the associated performance loss. We instead study sparsity-AD for source-transformation AD models.

Sparsity-AD exhibits interesting properties. In particular, activity analysis for this mode is able to simplify the derivative code further, detecting and removing many more unnecessary instructions of the original model. Performance of the "sparsity-AD" code is thus improved. Moreover, propagation of the sparsity structure in the form of bitvectors is well adapted to vectorization on GPUs. We implemented this special "sparsity-AD" mode in Tapenade, in both tangent and adjoint modes. We validated this extension on large test cases. We presented our first implementation at the 27th EuroAD workshop in Kaiserslautern, Germany April 3-4. An article is in preparation.

In the future, we will continue sparsity-AD for Jacobian matrices, exploring alternative storage strategies of the boolean matrix, for instance replacing bit vectors with sets of non-zero ranks when sparsity exceeds some threshold. We will also study extension to higher-order derivative objects such as Hessians, that deserve different sparsity propagation rules.

7.23 Algorithmic Differentiation for Julia

Participants: Laurent Hascoët, Jean-Luc Bouchot.

We develop an extension of the AD tool Tapenade devoted to the Julia language. Tapenade can so far handle languages Fortran (77, 90, 2003) and C. Our development principle is that Tapenade internal representation is language independent, and manipulates programming constructs (types, variables, control structures, procedures, packages. . .) independently of the particular source language. This principle facilitated many aspects of the extension to C, and we expect the same kind of advantage in extending to Julia. We developed a Julia parser that targets Tapenade's internal imperative language, together with the reciprocal unparser that generates Julia source from Tapenade's internal representation.

In addition to the standard programming constructs, every language has a few specific ones with original semantics, that require some research or development. In Julia, the novel constructs that required some research for their differentiation contain: the type system and the multiple dispatch semantics of call, the tuple construct, or the more interpreter-like form of programs. These constructs (and a few others) required to extend the source-transformation AD model that we promote. Consequently, we implemented and tested these extensions inside our tool Tapenade. A first Julia example code (a solver for the Burgers equation) has been successfully differentiated in tangent mode. In the future, we will progressively extend the dialect of Julia that we can differentiate, also in adjoint mode, in a call-by-need fashion guided by applications. We presented this effort towards AD of Julia at the 27th EuroAD workshop in Kaiserslautern, Germany April 3-4.

8 Bilateral contracts and grants with industry

8.1 Méthodes de contrôle pour l'évitement de collisions entre satellites (Control Methods for satellite collision avoidance), Thales Alenia Space

Participants: Thierry Dargent, Lamberto Dell'Elce, Frank de Veld, Jean-Baptiste Pomet.

This contract with Thales Alenia Space is co-funding the thesis of Frank de Veld entitled “Méthodes de Contrôle pour l’évitement de collisions entre satellites”; the other source of funding is the grant from Région Provence-Alpes-Côte d’Azur mentioned in Section 9.4

- Partners: McTAO and Thales Alenia Space.
- Period: 2022–2025
- Total amount: 75k€
- Inria reference: 022-0674

9 Partnerships and cooperations

9.1 International initiatives

9.1.1 Participation in International Programs

Participants: Jean-Baptiste Caillau, Laurent Hascoët, Jean-Luc Bouchot.

- FACCTS / France-Chicago funding (U. Chicago), ”Detecting Sparsity Patterns in Tapenade for Optimal Quantum Control Applications” (with A. Montoison and M. Anitescu)
- JLESC (Argonne Nat. Lab. and Inria), ”Shared Infrastructure for Source Transformation Automatic Differentiation” (with S. H. K. Narayanan)

9.1.2 Visits to international teams

Participants: Jean-Baptiste Caillau, Jean-Luc Bouchot.

- J.-B. Caillau was invited to the Space mechanics dept. of Zhejiang Univ. in April 2025, visited Argonne Nat. Lab. (JLESC funding) in May 2025, and was invited to the Math. dept. of Univ. Varanasi in November 2025 (French-Indian CEFIPRA funding).
- J.-L. Bouchot visited twice the Argonne National Lab (MCS): a week in March and a week in October; he was invited to give a talk at the LANS Seminar in March: "Tapenade old and new".

9.2 European initiatives

Participants: Laurent Hascoët, Jean-Luc Bouchot.

L. Hascoët and J.-L. Bouchot participate regularly to the EuroAD Workshops (bi-annual) and to the attached community. They went to Kaiserslautern in April (talk: "Source transformation AD for Julia with Tapenade") and to CERN, Geneva in December.

9.3 National initiatives

9.3.1 ANR

Participants: Jean-Baptiste Caillau, Ludovic Sacchelli.

- PDE-AI: partial differential equations for AI. This project on "Numerical analysis, optimal control and optimal transport for AI", funded by **PEPR IA** from 2023 to December 2028, is led by A. Chambolle (CNRS / Dauphine) and involves 10 French nodes, including a node in Nice / Sophia supervised by J.-B. Caillau. Total amount for the node 390 k€.

- MAD (Mathematics for Automatic Differentiation). Project funded by ANR (2024-28, 200 K€), PI S. Vaiter (CNRS). J.-B. Caillau is a member.

- OCARINA (Optimal Control frAmework for Robust Iterative NonlineAr experimental design). This project was successfully submitted this year to the JCJC 2025 ANR call. Its topic is experiment design from the point of view of optimal and ensemble control. Funding was notified in July, 2025, and the project itself is set to start officially in February, 2026; it involves several local and international researchers, and is coordinated by L. Sacchelli. Total amount 248 k€.

9.3.2 Other

Participants: Jean-Baptiste Caillau, Antonin Bavoil, Ivan Beschastnyi.

- KEEP project funded by AID and CNRS (grant + PhD funding of A. Bavoil), in collaboration with ENSTA Paristech

- CNRS project CONV (Control of NV centers) with physicists from Institut Carnot de Bourgogne (CNRS), funding 20k€ on two years, P.I. J.-B. Caillau.

- I. Beschastnyi received a CALISTA COST action travel grant for a two week research visit at the University of Coimbra, to work on developing Lie groupoid techniques for sub-Riemannian geometry.

- McTAO project-team participates in the Réseau Thématique (RT) Optimisation, a CNRS network on Mathematics of Optimization and Applications (it replaces and includes the former GdR MOA).

9.4 Regional initiatives

Participants: Lamberto Dell'Elce, Jean-Baptiste Pomet, Frank de Veld, Ivan Beschastnyi, Ludovic Sacchelli.

- Grant from Région SUD – Provence Alpes Côte d'Azur "Emplois jeunes Doctorants", 2022-2025, that co-funds Frank de Veld's PhD, together with the contract with Thales Alenia Space mentioned in Section 8.1. Total amount: 54k€.

- CSI Univ. Côte d'Azur (Incentive Scientific Credits of Université Côte d'Azur): 2025 Project COEPLEX, 6k€, P.I. L. Sacchelli, on optimal control for experimental design.

- CSI Univ. Côte d'Azur 2025: Project COSQUO, 5k€, P.I. I. Beschastnyi, on quantum control.

- Idex Univ. Côte d'Azur: Welcome package of Ivan Beschastnyi, 2023-2026. Total amount: 50k€.

10 Dissemination

10.1 Promoting scientific activities

10.1.1 Scientific events: organization

- Ivan Beschastnyi was a member of the organizing committee of the workshop "Journées niçoises : Théorie spectrale et analyse sur les variétés", held on September 29-30, 2025.

- Since January 2025, Ludovic Sacchelli is co-organizer of the Séminaire Analyse & Dynamique of LJAD.

- Jean-Luc Bouchot and Laurent Hascoët organized, with U. Naumann (RWTH Aachen, Allemagne), an invited session in the SIAM CSE conference (March 3 to 7, Fort Worth, TX, USA) on "Recent advances in Algorithmic differentiation" (plus a talk: "Tapenade for Julia").

- The McTAO project team maintains a recurring seminar, Séminaire McTAO, on topics of control theory, optimization and applications (2025 organizer: Ivan Beschastnyi). The seminar has a monthly periodicity and has hosted 8 sessions in 2025.

  - **Jan. 8** - Alexis Montoison (Argonne National Lab.): Sparse Automatic Differentiation
  - **Feb. 3** - Charles Babin (Université de Bourgogne): Quantum technologies implementation exploiting spins in the solid
  - **Mar. 27** - Samuel Hornus (Centre Inria de l'Université de Lorraine): Structure of the “Powered Descent Landing” with state and control constraints
  - **Apr. 15** - Andrei Agrachev (SISSA, Trieste): Control of diffeomorphisms
  - **May 21** - Thierry Combot (Université Bourgogne Europe, CNRS, IMB): Non intégrabilité du problème de N corps (nonintegrability of the N body problem)
  - **Oct. 1** - François Delarue (Univ. Côte d'Azur, CNRS, LJAD): Jeux à champ moyen: un panorama (an overview of Mean Field Games)
  - **Oct 8** - Alexandra Fronville (Université de Bretagne Occidentale): Biological Shape Control: Morphogenesis through Viability
  - **Dec. 18** - Anas Bouali (UMR MISTEA – Centre INRAE Occitanie-Montpellier): Loss control regions in optimal control problems

10.1.2 Scientific events: selection

Reviewer

All team members take part in a continued effort to offer reviews in various conferences of importance to the community.

10.1.3 Journal

Member of the editorial boards

- Jean-Baptiste Caillau is an associate editor of ESAIM: M2AN (Mathematical Modelling and Numerical Analysis) and of ESAIM: COCV (Control, Optimization and Calculus of Variations)

Reviewer - reviewing activities

Just like for conferences, all team members take part in a continued effort to offer reviews in various journals of importance to the community.

10.1.4 Scientific expertise

- Jean-Baptiste Pomet was a member of “Comité d’évaluation scientifique CE48 Fondements du numérique : informatique, automatique, traitement du signal et des images” in charge of evaluation of research proposals for the national research agency (ANR).

10.2 Teaching - Supervision - Juries - Educational and pedagogical outreach

10.2.1 Teaching

Engineering school and University:

- Adel Annabi took part in teaching at Université Côte d’Azur in Nice as a “demi ATER” (96 hours).
- Jean-Luc Bouchot gave two courses at the L3 level at Polytech Nice Sophia (Linear systems; machine learning in Python) (50 hours).
- Jean-Baptiste Caillau has a full teaching duty of Professor at L (BSc) and M (Master) level at Polytech Nice Sophia and Université Côte d’Azur.
- Antonin Bavoil, Lamberto Dell’Elce, Ludovic Sacchelli and Ivan Beschastnyi each took part as teaching assistants at L1, L3 and M1 levels at Polytech Nice Sophia and Université Côte d’Azur.

10.2.2 Supervision

HDR defenses

- L. Dell’Elce defended his HDR on October 22 [10].
- J.-L. Bouchot defended his HDR on December 19 [28].

PhD students

- Frank de Veld, “Control methods for low-thrust satellite collision avoidance”, Inria, co-supervised by Jean-Baptiste Pomet and Lamberto Dell’Elce, defended December 12. See [11].
- Adel Malik Annabi, “Observability and observer synthesis for neural fields equations”, Université Côte d’Azur, co-supervised by Jean-Baptiste Pomet, Ludovic Sacchelli and D. Prandi (CentraleSupélec). Started in October, 2022.
- Antonin Bavoil, “Génération optimale d’énergie par un cerf-volant” (Optimal energy generation from a kite), Université Côte d’Azur, co-supervised by Jean-Baptiste Caillau and Alain Nême (ENSTA Bretagne). Funded by CNRS. Started in October, 2023.
- Riccardo Daluiso, “Study of collisions in celestial and space mechanics”, Univ. Côte d’Azur, co-supervised by A. Albouy (Obs. Paris, CNRS) and Jean-Baptiste Caillau. Started in October, 2024.
- Eliot Stein, “Satellite orbit determination and prediction from very accurate measurements”, Onera, co-supervised by F. Cassaign (ONERA), H. Labriji (ONERA) and Lamberto Dell’Elce. Started in October, 2024.
- David Tinoco, “Nearly optimal pulse control of quantum systems”, Inria, co-supervised by Ivan Beschastnyi, Jean-Baptiste Caillau. Started in November, 2024.

Interns

- Ghaieth Aloui, 1st year Master at Polytech Nice Sophia. Co-supervised by Ivan Beschastnyi and Ludovic Sacchelli on the topic of "Open qubit parameter identification".
- Matthieu Estinest, 1st year master at CESI Bordeaux, supervised by Lamberto Dell'Elce on the topic "Integration of the attitude determination and control simulator into a FlatSat".
- Marwan Ksiks, 1st year Master at Polytech Nice Sophia. Supervised by Ivan Beschastnyi on the topic of "Modeling and simulation of transmon qubit networks".
- Laurent Moinet, 1st year master at IPSA Ivry-Sur-Seine, supervised by Lamberto Dell'Elce on the topic "Integration of the attitude determination and control simulator into a FlatSat".
- Mohamad Samman, 2nd year master's degree at Université Côte d'Azur. Supervised by Ludovic Sacchelli on the topic of "Optimal control for the design of an experiment in neuroscience".

10.2.3 Juries

- Jean-Baptiste Caillau presided the PhD jury of Ruben Chenevat (U. Montpellier)
- Jean-Baptiste Pomet presided the HDR jury of Florentina Nicolau (Cergy Paris Univ., [81])
- Lamberto Dell'Elce was an examiner in the PhD jury of Nicolas Leclere (U. Liège, Belgique)

10.3 Popularization

10.3.1 Productions (articles, videos, podcasts, serious games, ...)

- In 2024-25, Ludovic Sacchelli conceived with Mathémarium a new pedagogical puzzle game on vector fields and singularities at a highschool level.

10.3.2 Participation in Live events

- Ivan Beschastnyi, Jean-Baptiste Caillau and Ludovic Sacchelli participated in Fête de la Science as scientific presenters at the booth hosted by LJAD and Mathémarium.
- Ludovic Sacchelli organized in June one of the MathC2+ workshops (math research discovery for motivated high school students) at LJAD.
- Ludovic Sacchelli presented a talk at Collège le Prés des Roures, Le Rouret, in December as part of the Cordées de la réussite program.
- Frank de Veld gave two popularization worshops in the framework of “Fête de la Science”, Village des sciences du numérique, Terra Numerica (Sophia Antipolis, October 2 and 5).

11 Scientific production

11.1 Publications of the year

International journals

[1] V. Andrieu, L. Brivadis, J.-P. Gauthier, L. Sacchelli and U. Serres. ‘Exponential stabilizability and observability at the target imply semiglobal exponential stabilizability by templated output feedback’. In: *Systems and Control Letters* 195 (1st Jan. 2025), p. 105971. DOI: `10.1016/j.sysconle.2024.105971`. URL: `https://hal.science/hal-04387614` (cit. on p. 13).

[2] A. M. Annabi, J.-B. Pomet, D. Prandi and L. Sacchelli. ‘Activity estimation via distributed measurements in an orientation sensitive neural fields model of the visual cortex’. In: *Mathematics of Control, Signals, and Systems* 37 (2025), pp. 737–767. DOI: `10.1007/s00498-025-00416-w`. URL: `https://hal.science/hal-04523994` (cit. on p. 19).

[3] L. Baratchart, S. Fueyo and J.-B. Pomet. ‘Exponential stability of linear periodic difference-delay equations’. In: *SIAM Journal on Mathematical Analysis* 57 (2025), pp. 3110–3145. DOI: `10.1137/23M160133X`. URL: `https://inria.hal.science/hal-03500720` (cit. on p. 14).

[4] B. Bonnard, O. Cots and Y. Privat. ‘The Zermelo Navigation Problem on the 2-Sphere of Revolution: An Optimal Control Perspective with Applications to Micromagnetism’. In: *Nonlinearity* 39.1 (2026), p. 015006. DOI: `10.1088/1361-6544/ae2cd9`. URL: `https://hal.univ-lorraine.fr/hal-05326962` (cit. on p. 15).

[5] J.-B. Caillau, L. Dell’Elce, A. Herasimenka and J.-B. Pomet. ‘On the controllability of nonlinear systems with a periodic drift’. In: *SIAM Journal on Control and Optimization* 63.5 (2025), pp. 3407–3429. DOI: `10.1137/22M1541381`. URL: `https://inria.hal.science/hal-03779482` (cit. on p. 13).

[6] L. Hascoët, M. Menickelly, S. H. K. Narayanan, J. O’Neal, N. Schunck and S. Wild. ‘HFBTHO-AD: Differentiation of a nuclear energy density functional code’. In: *Computer Physics Communications* (Dec. 2025). DOI: `10.1016/j.cpc.2025.109955`. URL: `https://inria.hal.science/hal-05456113` (cit. on p. 21).

International peer-reviewed conferences

[7] A. M. Annabi, J.-B. Pomet, D. Prandi and L. Sacchelli. ‘Identification of saturated networked systems’. In: 64th IEEE Conf. on Decision and Control (CDC). Rio de Janeiro, Brazil, 10th Dec. 2025, pp. 4208–4213. DOI: `10.1109/CDC57313.2025.11312953`. URL: `https://hal.science/hal-05236385` (cit. on p. 19).

[8] I. Beschastnyi, L. Dell’Elce, J.-B. Pomet, L. Sacchelli and D. Tinoco. ‘Pulse control of affine systems with applications to quantum control’. In: 64th IEEE Conf. on Decision and Control (CDC). Rio De Janeiro, Brazil, 10th Dec. 2025, pp. 3769–3774. DOI: `10.1109/CDC57313.2025.11312971`. URL: `https://inria.hal.science/hal-05042176` (cit. on p. 15).

Conferences without proceedings

[9] J.-B. Caillau, O. Cots, J. Gergaud and P. Martinon. ‘Solving optimal control problems on GPU with Julia’. In: JuliaCon local Paris 2025 - European conference on the Julia programming language. Paris, France, 2nd Oct. 2025. URL: `https://hal.science/hal-05295792`.

Doctoral dissertations and habilitation theses

[10] L. Dell’Elce. ‘Geometric Control of Satellite Trajectories with Low-Thrust Propulsion’. Université Côte D’Azur, 22nd Oct. 2025. URL: `https://inria.hal.science/tel-05461361` (cit. on p. 26).

[11] F. de Veld. ‘Control methods for low-thrust satellite collision avoidance’. Université Côte D’Azur, 12th Dec. 2025. URL: `https://theses.hal.science/tel-05461234` (cit. on pp. 10, 16, 26).

Reports & preprints

[12] A. Agrachev, I. Beschastnyi and M. Motta. *Singular extremals of optimal control problems with L^1 cost*. 6th Jan. 2026. URL: `https://hal.science/hal-05443565` (cit. on p. 13).

[13] G. Aloui, I. Beschastnyi and L. Sacchelli. *Open qubit parameter identification with bounded pulses*. 6th Dec. 2025. URL: `https://hal.science/hal-05401792` (cit. on p. 16).

[14] A. Bavoil, J.-B. Caillau, L. Dell’Elce and A. Nême. *Airborne wind energy: the KEEP project **. 14th Nov. 2025. URL: `https://hal.science/hal-05366391` (cit. on p. 19).

[15] I. Beschastnyi, F. Colombo, S. A. Lucas and I. Sabadini. *The S-resolvent estimates for the Dirac operator on hyperbolic and spherical spaces*. 17th Apr. 2025. URL: `https://hal.science/hal-05450210`.

[16] J.-B. Caillau and A. Montoison. *Modeling and Optimization of Control Problems on GPUs*. 3rd Oct. 2025. URL: `https://hal.science/hal-05295809`.

[17] Y. Chitour, F. Jean, R. Monti, L. Rifford, L. Sacchelli, M. Sigalotti and A. Socionovo. *Not all sub-Riemannian minimizing geodesics are smooth*. 14th Jan. 2025. URL: https://ensta.hal.science/hal-04885315 (cit. on p. 15).

[18] S. Fueyo, L. Baratchart and J.-B. Pomet. *Monodromy Structure and Harmonic Transfer Function in Lossless Transmission Line Circuits*. 2025. URL: https://hal.science/hal-05042937.

[19] J.-J. Godeme. *Inertial Bregman Proximal Gradient under Partial Smoothness*. 15th July 2025. URL: https://hal.science/hal-04918793.

[20] S. Hornus, J.-B. Caillau, L. Dell'Elce and J.-B. Pomet. *State Constraint Analysis for Powered Descent Guidance*. 9th Feb. 2026. URL: https://inria.hal.science/hal-05504905.

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[22] L. Sacchelli and A. Scagliotti. *A case study in ensemble optimal control for Bayesian input design*. 20th Nov. 2025. URL: https://hal.science/hal-05374484 (cit. on p. 13).

[23] D. Tinoco, C. Babin, I. Beschastnyi, J.-B. Caillau and D. Sugny. *Control of an NV center as a two-qubit system*. 8th Dec. 2025. URL: https://inria.hal.science/hal-05404999 (cit. on p. 16).

Software

[24] [SW] J.-B. Caillau, B. Bonnard and E. Trélat, *cotcot: conditions of order two, conjugate times* 23rd Jan. 2025. LIC: MIT License. DOI: 10.5281/zenodo.14722043, HAL: ⟨hal-04907384⟩, URL: https://hal.science/hal-04907384, SWHID: ⟨swh:1:dir:c02953bfb701d30f057c26e9619233821e63ecb9;origin=https://hal.archives-ouvertes.fr/hal-04907384;visit=swh:1:snp:8b88eae872d72a9c460aee25a3a93e4a842a4934;anchor=swh:1:rel:0573b1654a52251014b1a50c89054f2c597c2657;path=/⟩.

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