

2025 Activity Report

RESEARCH CENTRE: Inria Centre at Rennes University

IN PARTNERSHIP WITH: École normale supérieure de Rennes, CNRS, Université de Rennes


Project-Team

MINGUS

Multi-scale Numerical Geometric Schemes



In collaboration with Institut de recherche mathématique de Rennes (IRMAR)



Project-Team MINGUS

Creation of the Project-Team: 2018 August 01

Each year, Inria research teams publish an Activity Report presenting their work and results over the reporting period. These reports follow a common structure, with some optional sections depending on the specific team. They typically begin by outlining the overall objectives and research programme, including the main research themes, goals, and methodological approaches. They also describe the application domains targeted by the team, highlighting the scientific or societal contexts in which their work is situated. The reports then present the highlights of the year, covering major scientific achievements, software developments, or teaching contributions. When relevant, they include sections on software, platforms, and open data, detailing the tools developed and how they are shared. A substantial part is dedicated to new results, where scientific contributions are described in detail, often with subsections specifying participants and associated keywords. Finally, the Activity Report addresses funding, contracts, partnerships, and collaborations at various levels, from industrial agreements to international cooperations. It also covers dissemination and teaching activities, such as participation in scientific events, outreach, and supervision. The document concludes with a presentation of scientific production, including major publications and those produced during the year.

Keywords

Computer sciences and digital sciences

- A6. – Modeling, simulation and control
 - A6.1. – Methods in mathematical modeling
 - A6.1.1. – Continuous Modeling (PDE, ODE)
 - A6.1.2. – Stochastic Modeling
 - A6.1.4. – Multiscale modeling
 - A6.2. – Scientific computing, Numerical Analysis & Optimization
 - A6.2.1. – Numerical analysis of PDE and ODE
 - A6.2.3. – Probabilistic methods
 - A6.2.7. – HPC for machine learning
 - A6.5. – Mathematical modeling for physical sciences
 - A6.5.2. – Fluid mechanics
 - A6.5.3. – Transport
 - A6.5.4. – Waves

Other research topics and application domains

- B4.2.2. – Fusion
- B4.3. – Renewable energy production
- B5.11. – Quantum systems
- B9.5.2. – Mathematics

Contents

Project-Team MINGUS	1
1 Team members, visitors, external collaborators	5
2 Overall objectives	6
2.1 Presentation	6
3 Research program	7
3.1 Dissipative problems	8
3.1.1 Asymptotic analysis of dissipative PDEs	8
3.1.2 Numerical schemes for dissipative problems	9
3.2 Highly-oscillatory problems	9
3.2.1 Asymptotic analysis of highly-oscillatory PDEs	10
3.2.2 Numerical schemes for highly-oscillatory problems	11
4 Application domains	12
4.1 Application domains	12
4.2 Plasmas problems	13
4.3 Quantum problems	13
4.4 Population dynamics	13
5 Social and environmental responsibility	14
5.1 Footprint of research activities	14
6 Highlights of the year	14
6.1 Awards	14
7 Latest software developments, platforms, open data	14
7.1 Latest software developments	14
7.1.1 Selalib	14
7.1.2 HOODESolver.jl	15
7.1.3 PhaseLifting	15
7.2 Open data	15
8 New results	15
8.1 Multiscale numerical schemes	15
8.2 Geometric numerical schemes	17
8.3 Analysis of SPDEs	18
9 Bilateral contracts and grants with industry	20
9.1 Bilateral contracts with industry	20
10 Partnerships and cooperations	21
10.1 International initiatives	21
10.1.1 Inria associate team not involved in an ILL or an international program	21
10.1.2 Participation in other International Programs	21
10.2 International research visitors	21
10.2.1 Visits of international scientists	21
10.2.2 Visits to international teams	22
10.3 National initiatives	22
10.4 Regional initiatives	24

11 Dissemination	24
11.1 Promoting scientific activities	24
11.1.1 Scientific events: organisation	24
11.1.2 Journal	24
11.1.3 Invited talks	25
11.1.4 Scientific expertise	25
11.1.5 Research administration	26
11.2 Teaching - Supervision - Juries - Educational and pedagogical outreach	26
11.2.1 Supervision	26
11.2.2 Juries	27
11.3 Popularization	27
11.3.1 Specific official responsibilities in science outreach structures	27
11.3.2 Participation in Live events	27
12 Scientific production	27
12.1 Major publications	27
12.2 Publications of the year	28
12.3 Cited publications	30

1 Team members, visitors, external collaborators

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2 Overall objectives

2.1 Presentation

In applications involving complex physics, such as plasmas and nanotechnologies, numerical simulations serve as a prediction tool supplementing real experiments and are largely endorsed by engineers or researchers. Their performances rely not only on computational power, but also on the efficiency of the underlying numerical method and the complexity of the underlying models. The contribution of applied mathematics is then required, on the one hand for a better understanding of qualitative properties and a better identification of the different regimes present in the model, and on the other hand, for a more sounded construction of new models based on asymptotic analysis. This mathematical analysis is expected to greatly impact the design of *multiscale* numerical schemes.

The proposed research group MINGUS will be dedicated to the mathematical and numerical analysis of (possibly stochastic) partial differential equations (PDEs), originating from plasma physics and nanotechnologies, with emphasis on *multiscale* phenomena either of **highly-oscillatory**, of **dissipative** or **stochastic** types. These equations can be also encountered in applications to rarefied gas dynamics, radiative transfer, population dynamics or laser propagation, for which the multiscale character is modelled by a scale physical parameter ε .

Producing accurate solutions of multiscale equations is extremely challenging owing to severe restrictions to the numerical methods imposed by fast (or stiff) dynamics. *Ad-hoc* numerical methods should aim at capturing the slow dynamics solely, instead of resolving finely the stiff dynamics at a formidable computational cost. At the other end of the spectrum, the separation of scales -as required for numerical efficiency- is envisaged in asymptotic techniques, whose purpose is to describe the model in the limit where the small parameter ε tends to zero. MINGUS aspires to accommodate sophisticated tools of mathematical analysis and heuristic numerical methods in order to produce simultaneously rich asymptotic models and efficient numerical methods.

To be more specific, MINGUS aims at finding, implementing and analysing new multiscale numerical schemes for the following physically relevant multiscale problems:

- **Highly-oscillatory Schrödinger equation for nanoscale physics:** In quantum mechanics, the Schrödinger equation describes how the quantum state of some physical system changes with time. Its mathematical and numerical study is of paramount importance to fundamental and applied physics in general. We wish to specifically contribute to the mathematical modeling and the numerical simulation of confined quantum mechanical systems (in one or more space dimensions) possibly involving stochastic terms. Such systems are involved in quantum semi-conductors or atom-chips, as well as in cold atom physics (Bose-Einstein condensates) or laser propagation in optical fibers.

The prototypical equation is written

$$i\varepsilon\partial_t\psi^\varepsilon = \frac{\varepsilon^2}{\beta}\Delta\psi^\varepsilon + |\psi^\varepsilon|^2\psi^\varepsilon + \psi^\varepsilon\xi \quad (1)$$

where the function $\psi^\varepsilon = \psi^\varepsilon(t, x) \in \mathbb{C}$ depends on time $t \geq 0$ and position $x \in \mathbb{R}^3$, $\xi = \xi(x, t)$ is a white noise and where the small parameter ε is the Planck's constant describing the microscopic/macroscopic ratio. The limit $\varepsilon \rightarrow 0$ is referred to as the semi-classical limit. The regime $\varepsilon = 1$ and $\beta \rightarrow 0$ (this can be for instance the relative length of the optical fiber) is highly-oscillatory. The noise ξ acts as a potential, it may represent several external perturbations. For instance temperature effects in Bose-Einstein condensation or amplification in optical fibers. The highly oscillatory regime combined with noise introduces new challenges in the design of efficient schemes.

- **Highly-oscillatory or highly-dissipative kinetic equations:** Plasma is sometimes considered as the fourth state of matter, obtained for example by bringing a gas to a very high temperature. A globally

neutral gas of neutral and charged particles, called plasma, is then obtained and is described by a kinetic equation as soon as collective effects dominate as compared to binary collisions. A situation of major importance is magnetic fusion in which collisions are not predominant. In order to confine such a plasma in devices like tokamaks (ITER project) or stellarators, a large magnetic field is used to endow the charged particles with a cyclotronic motion around field lines. Note that kinetic models are also widely used for modeling plasmas in earth magnetosphere or in rarefied gas dynamics.

Denoting $f^\varepsilon = f^\varepsilon(t, x, v) \in \mathbb{R}^+$ the distribution function of charged particles at time $t \geq 0$, position $x \in \mathbb{R}^3$ and velocity $v \in \mathbb{R}^3$, a typical kinetic equation for f^ε reads

$$\partial_t f^\varepsilon + v \cdot \nabla_x f^\varepsilon + \left(E + \frac{1}{\varepsilon} (v \times B) \right) \cdot \nabla_v f^\varepsilon = \frac{1}{\beta} Q(f^\varepsilon) + f^\varepsilon m^\varepsilon \quad (2)$$

where (E, B) is the electro-magnetic field (which may itself depend on f through Maxwell's equations), m^ε is a random process (which may describe absorption or creation of particles) and Q is a collision operator. The dimensionless parameters ε, β are related to the cyclotronic frequency and the mean free path. Limits $\varepsilon \rightarrow 0$ and $\beta \rightarrow 0$ do not share the same character (the former is oscillatory and the latter is dissipative) and lead respectively to gyrokinetic and hydrodynamic models. The noise term m^ε is correlated in space and time. At the limit $\varepsilon \rightarrow 0$, it converges formally to a white noise and stochastic PDEs are obtained.

The objective of MINGUS is twofold: the construction and the analysis of numerical schemes for multiscale (S)PDEs originating from physics. In turn, this requires (i) a deep mathematical understanding of the (S)PDEs under consideration and (ii) a strong involvement into increasingly realistic problems, possibly resorting to parallel computing. For this aspect, we intend to benefit from the Inria Selalib software library which turns out to be the ideal complement of our activities.

During the last period, some results have been obtained by the members of the team [2, 3, 4, 5, 6].

3 Research program

The MINGUS project is devoted to the mathematical and numerical analysis of models arising in plasma physics and nanotechnology. The main goal is to construct and analyze numerical methods for the approximation of PDEs containing multiscale phenomena. Specific multiscale numerical schemes will be proposed and analyzed in different regimes (namely highly-oscillatory and dissipative). The ultimate goal is to dissociate the physical parameters (generically denoted by ε) from the numerical parameters (generically denoted by h) with a uniform accuracy. Such a task requires mathematical prerequisite of the PDEs.

Then, for a given stiff (highly-oscillatory or dissipative) PDE, the methodology of the MINGUS team will be the following

- Mathematical study of the asymptotic behavior of multiscale models.
This part involves averaging and asymptotic analysis theory to derive asymptotic models, but also long-time behavior of the considered models.
- Construction and analysis of multiscale numerical schemes.
This part is the core of the project and will be deeply inspired from the mathematical prerequisite. In particular, our ultimate goal is the design of *Uniformly Accurate* (UA) schemes, whose accuracy is independent from ε .
- Validation on physically relevant problems.
The last goal of the MINGUS project is to validate the new numerical methods, not only on toy problems, but also on realistic models arising in physics of plasmas and nanotechnologies. We will benefit from the Selalib software library which will help us to scale-up our new numerical methods to complex physics.

3.1 Dissipative problems

In the dissipative context, the asymptotic analysis is quite well understood in the deterministic case and multiscale numerical methods have been developed in the last decades. Indeed, the so-called Asymptotic-Preserving schemes has retained a lot of attention, in particular in the context of collisional kinetic equations. But, there is still a lot to do if one is interested in deriving high order asymptotic models, which enable to capture the original solution for all time. Moreover, this analysis is still misunderstood when more complex systems are considered, involving non homogeneous relaxation rates or stochastic terms for instance. Following the methodology we aim at using, we first address the mathematical analysis before deriving multiscale efficient numerical methods.

A simple model of dissipative systems is governed by the following differential equation

$$\begin{cases} \frac{dx^\varepsilon(t)}{dt} &= \mathcal{G}(x^\varepsilon(t), y^\varepsilon(t)), & x^\varepsilon(0) = x_0, \\ \frac{dy^\varepsilon(t)}{dt} &= -\frac{y^\varepsilon(t)}{\varepsilon} + \mathcal{H}(x^\varepsilon(t), y^\varepsilon(t)), & y^\varepsilon(0) = y_0, \end{cases} \quad (3)$$

for given initial condition $(x_0, y_0) \in \mathbb{R}^2$ and given smooth functions \mathcal{G}, \mathcal{H} which possibly involve stochastic terms.

3.1.1 Asymptotic analysis of dissipative PDEs

Derivation of asymptotic problems

Our main goal is to analyze the asymptotic behavior of dissipative systems of the form (3) when ε goes to zero. The *center manifold theorem* [38] is of great interest but is largely unsatisfactory from the following points of view

- a constructive approach of h and x_0^ε is clearly important to identify the high-order asymptotic models: this would require expansions of the solution by means of B-series or word-series [40] allowing the derivation of error estimates between the original solution and the asymptotic one.
- a better approximation of the transient phase is strongly required to capture the solution for small time: extending the tools developed in averaging theory, the main goal is to construct a suitable change of variable which enables to approximate the original solution for all time.

Obviously, even at the ODE level, a deep mathematical analysis has to be performed to understand the asymptotic behavior of the solution of (3). But, the same questions arise at the PDE level. Indeed, one certainly expects that dissipative terms occurring in collisional kinetic equations (2) may be treated theoretically along this perspective. The key new point indeed is to see the center manifold theorem as a change of variable in the space of unknowns, while the standard point of view leads to considering the center manifold as an asymptotic object.

Stochastic PDEs

We aim at analyzing the asymptotic behavior of stochastic collisional kinetic problems, that is equation of the type (2). The noise can describe creation or absorption (as in (2)), but it may also be a forcing term or a random magnetic field. In the parabolic scaling, one expects to obtain parabolic SPDEs at the limit. More precisely, we want to understand the fluid limits of kinetic equations in the presence of noise. The noise is smooth and non delta correlated. It contains also a small parameter and after the rescaling, it converges formally to white noise. Thus, this adds another scale in the multiscale analysis. Following the pioneering work by Debussche and Vovelle [41], substantial progresses have been done in this topic.

More realistic problems may be addressed such as high field limit describing sprays, or even hydrodynamic limit. The full Boltzmann equation is a very long term project and we wish to address simpler problems such as convergence of BGK models to a stochastic Stokes equation.

The main difficulty is that when the noise acts as a forcing term, which is a physically relevant situation, the equilibria are affected by the noise and we face difficulties similar to that of high field limit problems. Also, a good theory of averaging lemma in the presence of noise is lacking. The methods we use are generalization of the perturbed test function method to the infinite dimensional setting. We work at the level of the generator of the infinite dimensional process and prove convergence in the sense of the martingale problems. A further step is to analyse the speed of convergence. This is a prerequisite if one wants to design efficient schemes. This requires more refined tools and a good understanding of the Kolmogorov equation.

3.1.2 Numerical schemes for dissipative problems

The design of numerical schemes able to reproduce the transition from the microscopic to macroscopic scales largely matured with the emergence of Asymptotic Preserving schemes which have been developed initially for collisional kinetic equations (actually, for solving (2) when $\beta \rightarrow 0$). Several techniques have flourished in the last decades. As said before, AP schemes entail limitations which we aim at overcoming by deriving

- AP numerical schemes whose numerical cost diminishes as $\beta \rightarrow 0$,
- Uniformly accurate numerical schemes, whose accuracy is independent of β .

Time diminishing methods

The main goal consists in merging Monte-Carlo techniques [36] with AP methods for handling *automatically* multiscale phenomena. As a result, we expect that the cost of the so-obtained method decreases when the asymptotic regime is approached; indeed, in the collisional (i.e. dissipative) regime, the deviational part becomes negligible so that a very few number of particles will be generated to sample it. A work in this direction has been done by members of the team.

We propose to build up a method which enables to realize the transition from the microscopic to the macroscopic description without domain decomposition strategies which normally requires to fix and tune an interface in the physical space and some threshold parameters. Since it will permit to go over domain decomposition and AP techniques, this approach is a very promising research direction in the numerical approximation of multiscale kinetic problems arising in physics and engineering.

Uniformly accurate methods

To overcome the accuracy reduction observed in AP schemes for intermediate regimes, we intend to construct and analyse multiscale numerical schemes for (3) whose error is uniform with respect to ε . The construction of such a scheme requires a deep mathematical analysis as described above. Ideally one would like to develop schemes that preserve the center manifold (without computing the latter!) as well as schemes that resolve numerically the stiffness induced by the fast convergence to equilibrium (the so-called transient phase). First, our goal is to extend the strategy inspired by the central manifold theorem in the ODE case to the PDE context, in particular for collisional kinetic equations (2) when $\beta \rightarrow 0$. The design of Uniformly Accurate numerical schemes in this context would require to generalize two-scale techniques introduced by members of the team in the framework of highly-oscillatory problems [39].

Multiscale numerical methods for stochastic PDEs

AP schemes have been developed recently for kinetic equations with noise in the context of Uncertainty Quantification UQ [44]. These two aspects (multiscale and UQ) are two domains developed in separate communities. UQ has drawn a lot of attention recently to control the propagation of data pollution; undoubtedly UQ has a lot of applications and one of our goals will be to study how sources of uncertainty are amplified or not by the multiscale character of the model. We also wish to go much further in developing AP schemes when the noise is also rescaled and the limit is a white noise driven SPDE, as described in section (3.1.1). For simple nonlinear problem, this should not present much difficulties but new ideas will definitely be necessary for more complicated problems when noise deeply changes the asymptotic equation.

3.2 Highly-oscillatory problems

As a generic model for highly-oscillatory systems, we will consider the equation

$$\frac{du^\varepsilon(t)}{dt} = \mathcal{F}(t/\varepsilon, u^\varepsilon(t)), \quad u^\varepsilon(0) = u_0, \quad (4)$$

for a given u_0 and a given periodic function \mathcal{F} (of period P w.r.t. its first variable) which possibly involves stochastic terms. Solution u^ε exhibits high-oscillations in time superimposed to a slow dynamics. Asymptotic techniques -resorting in the present context to *averaging* theory [48]- allow to decompose

$$u^\varepsilon(t) = \Phi_{t/\varepsilon} \circ \Psi_t \circ \Phi_0^{-1}(u_0), \quad (5)$$

into a fast solution component, the εP -periodic change of variable $\Phi_{t/\varepsilon}$, and a slow component, the flow Ψ_t of a non-stiff *averaged* differential equation. Although equation (5) can be satisfied only up to a small

remainder, various methods have been recently introduced in situations where (4) is posed in \mathbb{R}^n or for the Schrödinger equation (1).

In the asymptotic behavior $\varepsilon \rightarrow 0$, it can be advantageous to replace the original singularly perturbed model (for instance (1) or (2)) by an approximate model which does not contain stiffness any longer. Such reduced models can be derived using asymptotic analysis, namely averaging methods in the case of highly-oscillatory problems. In this project, we also plan to go beyond the mere derivation of limit models, by searching for better approximations of the original problem. This step is of mathematical interest *per se* but it also paves the way of the construction of multiscale numerical methods.

3.2.1 Asymptotic analysis of highly-oscillatory PDEs

Derivation of asymptotic problems

We intend to study the asymptotic behavior of highly-oscillatory evolution equations of the form (4) posed in an infinite dimensional Banach space.

Recently, the stroboscopic averaging has been extended to the PDE context, considering nonlinear Schrödinger equation (1) in the highly-oscillatory regime. A very exciting way would be to use this averaging strategy for highly-oscillatory kinetic problem (2) as those encountered in strongly magnetized plasmas. This turns out to be a very promising way to re-derive gyrokinetic models which are the basis of tokamak simulations in the physicists community. In contrast with models derived in the literature (see [37]) which only capture the average with respect to the oscillations, this strategy allows for the complete recovery of the exact solution from the asymptotic (non stiff) model. This can be done by solving companion transport equation that stems naturally from the decomposition (5).

Long-time behavior of Hamiltonian systems

The study of long-time behavior of nonlinear Hamiltonian systems have received a lot of interest during the last decades. It enables to put in light some characteristic phenomena in complex situations, which are beyond the reach of numerical simulations. This kind of analysis is of great interest since it can provide very precise properties of the solution. In particular, we will focus on the dynamics of nonlinear PDEs when the initial condition is close to a stationary solution. Then, the long-time behavior of the solution is studied through mainly three axes

- *linear stability*: considering the linearized PDE, do we have stability of a stationary solution ? Do we have linear Landau damping around stable non homogeneous stationary states?
- *nonlinear stability*: under a criteria, do we have stability of a stationary solution in energy norm like in [45], and does this stability persist under numerical discretization? For example one of our goals is to address the question of the existence and stability of discrete travelling wave in space and time.
- do we have existence of damped solutions for the full nonlinear problem ? Around homogeneous stationary states, solutions scatter towards a modified stationary state (see [46, 42]). The question of existence of Landau damping effects around non homogeneous states is still open and is one of our main goal in the near future.

Asymptotic behavior of stochastic PDEs

The study of SPDEs has known a growing interest recently, in particular with the fields medal of M. Hairer in 2014. In many applications such as radiative transfer, molecular dynamics or simulation of optical fibers, part of the physical interactions are naturally modeled by adding supplementary random terms (the noise) to the initial deterministic equations. From the mathematical point of view, such terms change drastically the behavior of the system.

- In the presence of noise, highly-oscillatory dispersive equations present new problems. In particular, to study stochastic averaging of the solution, the analysis of the long time behavior of stochastic dispersive equations is required, which is known to be a difficult problem in the general case. In some cases (for instance highly-oscillatory Schrödinger equation (1) with a time white noise in the regime $\varepsilon \ll 1$), it is however possible to perform the analysis and to obtain averaged stochastic equations. We plan to go further by considering more difficult problems, such as the convergence of a stochastic Klein-Gordon-Zakharov system to a stochastic nonlinear Schrödinger equation.

- The long-time behavior of stochastic Schrödinger equations is of great interest to analyze mathematically the validity of the Zakharov theory for wave turbulence (see [47]). The problem of wave turbulence can be viewed either as a deterministic Hamiltonian PDE with random initial data or a randomly forced PDEs where the stochastic forcing is concentrated in some part of the spectrum (in this sense it is expected to be a hypoelliptic problem). One of our goals is to test the validity the Zakharov equation, or at least to make rigorous the spectrum distribution spreading observed in the numerical experiments.

3.2.2 Numerical schemes for highly-oscillatory problems

This section proposes to explore numerical issues raised by highly-oscillatory nonlinear PDEs for which (4) is a prototype. Simulating a highly-oscillatory phenomenon usually requires to adapt the numerical parameters in order to solve the period of size ε so as to accurately simulate the solution over each period, resulting in an unacceptable execution cost. Then, it is highly desirable to derive numerical schemes able to advance the solution by a time step independent of ε . To do so, our goal is to construct *Uniformly Accurate* (UA) numerical schemes, for which the numerical error can be estimated by Ch^p (h being any numerical parameters) with C independent of ε and p the order of the numerical scheme.

Recently, such numerical methods have been proposed by members of the team in the highly-oscillatory context [39]. They are mainly based on a separation of the fast and slow variables, as suggested by the decomposition (5). An additional ingredient to prove the uniform accuracy of the method for (4) relies on the search for an appropriate initial data which enables to make the problem smooth with respect to ε .

Such an approach is assuredly powerful since it provides a numerical method which enables to capture the high oscillations in time of the solution (and not only its average) even with a large time step. Moreover, in the asymptotic regime, the potential gain is of order $1/\varepsilon$ in comparison with standard methods, and finally averaged models are not able to capture the intermediate regime since they miss important information of the original problem. We are strongly convinced that this strategy should be further studied and extended to cope with some other problems. The ultimate goal being to construct a scheme for the original equation which degenerates automatically into a consistent approximation of the averaged model, without resolving it, the latter can be very difficult to solve.

- **Space oscillations:**
When rapidly oscillating coefficients in **space** (*i.e.* terms of the form $a(x, x/\varepsilon)$) occur in elliptic or parabolic equations, homogenization theory and numerical homogenization are usually employed to handle the stiffness. However, these strategies are in general not accurate for all $\varepsilon \in]0, 1]$. Then, the construction of numerical schemes which are able to handle both regimes in an uniform way is of great interest. Separating fast and slow *spatial* scales merits to be explored in this context. The delicate issue is then to extend the choice of suitable initial condition to an *appropriate choice of boundary conditions* of the augmented problem.
- **Space-time oscillations:**
For more complex problems however, the recent proposed approaches fail since the main oscillations cannot be identified explicitly. This is the case for instance when the magnetic field B depends on t or x in (2) but also for many other physical problems. We then have to deal with the delicate issue of space-time oscillations, which is known to be a very difficult problem from a mathematical and a numerical points of view. To take into account the space-time mixing, a periodic motion has to be detected together with a phase S which possibly depends on the time and space variables. These techniques originate from **geometric optics** which is a very popular technique to handle high-frequency waves.
- **Geometrical properties:**
The questions related to the geometric aspects of multiscale numerical schemes are of crucial importance, in particular when long-time simulations are addressed (see [43]). Indeed, one of the main questions of geometric integration is whether intrinsic properties of the solution may be passed onto its numerical approximation. For instance, if the model under study is Hamiltonian, then the exact flow is symplectic, which motivates the design of symplectic numerical approximation. For practical simulations of Hamiltonian systems, symplectic methods are known to possess very nice properties (see [43]). It is important to combine multiscale techniques to geometric numerical integration. All

the problems and equations we intend to deal with will be addressed with a view to preserve intrinsic geometric properties of the exact solutions and/or to approach the asymptotic limit of the system in presence of a small parameter. An example of a numerical method developed by members of the team is the multi-revolution method.

- Quasi-periodic case:

So far, numerical methods have been proposed for the periodic case with single frequency. However, the quasi-periodic case ¹ is still misunderstood although many complex problems involve multi-frequencies. Even if the quasi-periodic averaging is doable from a theoretical point of view in the ODE case, (see [48]), it is unclear how it can be extended to PDEs. One of the main obstacle being the requirement, usual for ODEs like (4), for \mathcal{F} to be analytic in the periodic variables, an assumption which is clearly impossible to meet in the PDE setting. An even more challenging problem is then the design of numerical methods for this problem.

- Extension to stochastic PDEs:

All these questions will be revisited within the stochastic context. The mathematical study opens the way to the derivation of efficient multiscale numerical schemes for this kind of problems. We believe that the theory is now sufficiently well understood to address the derivation and numerical analysis of multiscale numerical schemes. Multi-revolution composition methods have been recently extended to highly-oscillatory stochastic differential equations. The generalization of such multiscale numerical methods to SPDEs is of great interest. The analysis and simulation of numerical schemes for highly-oscillatory nonlinear stochastic Schrödinger equation under diffusion-approximation for instance will be one important objective for us. Finally, an important aspect concerns the quantification of uncertainties in highly-oscillatory kinetic or quantum models (due to an incomplete knowledge of coefficients or imprecise measurement of data). The construction of efficient multiscale numerical methods which can handle multiple scales as well as random inputs have important engineering applications.

4 Application domains

4.1 Application domains

The MINGuS project aims at applying the new numerical methods on realistic problems arising for instance in physics of nanotechnology and physics of plasmas. Therefore, in addition to efforts devoted to the design and the analysis of numerical methods, the inherent large size of the problems at hand requires advanced mathematical and computational methods which are hard to implement. Another application is concerned with population dynamics for which the main goal is to understand how the spatial propagation phenomena affects the demography of a population (plankton, parasite fungi, ...). Our activity is mostly at an early stage in the process of transfer to industry. However, all the models we use are physically relevant and all have applications in many areas (ITER, Bose-Einstein condensate, wave turbulence, optical tomography, transport phenomena, population dynamics, ...). As a consequence, our research aims at reaching theoretical physicists or computational scientists in various fields who have strong links with industrial applications. In order to tackle as realistic physical problems as possible, a fundamental aspect will consist in working on the realization of numerical methods and algorithms which are able to make an efficient use of a large number of processors. Then, it is essential for the numerical methods developed in the MINGuS project to be thought through this prism. We will benefit from the strong expertise of P. Navaro in scientific computing and more precisely on the Selalib software library (see description below). Below, we detail our main applications: first, the modeling and numerical approximation of magnetized plasmas is our major application and will require important efforts in terms of software developments to scale-up our multiscale methods; second, the transport of charged particles in nanostructures has very interesting applications (like graphene material), for which our contributions will mainly focus on dedicated problems; lastly, applications on population dynamics will be dedicated to mathematical modeling and some numerical validations.

¹replacing t/ε by $t\omega/\varepsilon$ in (4), with $\omega \in \mathbb{R}^d$ a vector of non-resonant frequencies

4.2 Plasmas problems

The **SeLaLib** (Semi-Lagrangian Library) software library is a modular library for kinetic and gyrokinetic simulations of plasmas in fusion energy devices. Selalib is a collection of fortran modules aimed at facilitating the development of kinetic simulations, particularly in the study of turbulence in fusion plasmas. Selalib offers basic capabilities and modules to help parallelization (both MPI and OpenMP), as well as pre-packaged simulations. Its main objective is to develop a documented library implementing several numerical methods for the numerical approximation of kinetic models. Another objective of the library is to provide physicists with easy-to-use gyrokinetic solvers. It has been originally developed by E. Sonnendrücker and his collaborators in the past CALVI Inria project, and has played an important role in the activities of the IPL FRATRES. P. Navaro is one of the main software engineer of this library and as such he played an important daily role in its development and its portability on supercomputers. Though Selalib has reached a certain maturity, additional work is needed to make it available to the community. There are currently discussions for a possible evolution of Selalib, namely the writing of a new release which will be available for free download. At the scientific level, Selalib is of great interest for us since it provides a powerful tool with which we can test, validate and compare our new methods and algorithms (user level). Besides numerical algorithms the library provides low-level utilities, input-output modules as well as parallelization strategies dedicated to kinetic problems. Moreover, a collection of simulations for typical test cases (of increasing difficulties) with various discretization schemes supplements the library. This library turns out to be the ideal complement of our activities and it will help us to scale-up our numerical methods to high-dimensional kinetic problems. During the last years, several experiments have been successfully performed in this direction (especially with PhD students) and it is important for us that this approach remains thorough. Then, we intend to integrate several of the numerical methods developed by the team within the Selalib library, with the strong help of P. Navaro (contributor level). This work has important advantages: (i) it will improve our research codes (in terms of efficiency software maintenance); (ii) it will help us to promote our research by making our methods available to the research community.

4.3 Quantum problems

Nowadays, a great challenge consists in the downscaling at the nanometer scale of electronic components in order to improve speed and efficiency of semiconductor materials. In this task, modeling and numerical simulations play an important role in the determination of the limit size of the nanotransistors. At the nanoscale, quantum effects have to be considered and the Schrödinger equation is a prominent equation in this context. In the so-called semiclassical regime or when the transport is strongly confined, the solution endows space-time high oscillations which are very difficult to capture numerically. An important application is the modeling of charged particles transport in graphene. Graphene is a sheet of carbone made of a single layer of molecule, organized in a bidimensional honeycomb crystal. The transport of charged particles in this structure is usually performed by Dirac equation (which is the relativistic counterpart of the Schrödinger equation). Due to the unusual properties of graphene -at room temperature, electrons moving in graphene behave as massless relativistic particles- physicists and compagnies are nowadays actively studying this material. In this context, predicting how the material properties are affected by the uncertainties in the hexagonal lattice structure or in external potentials, is a major issue.

4.4 Population dynamics

The main goal is to characterize how spatial propagation phenomena (diffusion, transport, advection, . . .) affect the time evolution of the demography of a population. In collaboration with Y. Lagadeuc (ECOBIO, Rennes), this question has been studied for plankton. In this context, mathematical models have been proposed and it has been shown that the spatial dynamic (in this context, due to the marine current) which is fast compared to demographic scales, can strongly modify the demographic evolution of the plankton. In collaboration with Ecole d'Agronomie de Rennes, a mathematical study on the demography of a parasite fungi of plants has been performed. In this context, the demography is specific: the fungi can proliferate through sexual reproduction or through parthenogenesis. These two ways of reproduction give rise mathematically to quadratic and linear growth rates with respect to the population variable. The demography is then coupled with transport (transport of fungi spore by wind). Here, the goal is to characterize the propagation of the fungi

population by finding travelling waves solutions which are well adapted to describe the evolution of invasive fronts. Moreover, this approach enables to recover with a good agreement realistic examples (infection of ash or banana tree) for which experimental data are available. In these contexts, mathematical models are a powerful tool for biologists since measurements are very complicated to obtain and laboratory experiments hardly reproduce reality. The models derived are multiscale due to the nature of the underlying phenomena and the next step is to provide efficient numerical schemes.

5 Social and environmental responsibility

5.1 Footprint of research activities

A group called ECO-IRMAR has been created in the IRMAR laboratory to inform about the footprint of research activities at the level of the laboratory. The members of the team follow the advices proposed by this group.

6 Highlights of the year

6.1 Awards

No highlight this year.

7 Latest software developments, platforms, open data

7.1 Latest software developments

7.1.1 Selalib

Name: SEmi-LAgrangian LIBrary

Keywords: Plasma physics, Semilagrangian method, Parallel computing, Plasma turbulence

Scientific Description: The objective of the Selalib project (SEmi-LAgrangian LIBrary) is to develop a well-designed, organized and documented library implementing several numerical methods for kinetic models of plasma physics. Its ultimate goal is to produce gyrokinetic simulations.

Another objective of the library is to provide to physicists easy-to-use gyrokinetic solvers, based on the semi-lagrangian techniques developed by Eric Sonnendrücker and his collaborators in the past CALVI project. The new models and schemes from TONUS are also intended to be incorporated into Selalib.

Functional Description: Selalib is a collection of modules conceived to aid in the development of plasma physics simulations, particularly in the study of turbulence in fusion plasmas. Selalib offers basic capabilities from general and mathematical utilities and modules to aid in parallelization, up to pre-packaged simulations.

URL: <https://selalib.github.io>

Contact: Philippe Helluy

Participant: 5 anonymous participants

Partners: Max Planck Insitute - Garching, Université de Strasbourg, CNRS, Université de Rennes 1

7.1.2 HOODESolver.jl

Name: Julia package for solving numerically highly-oscillatory ODE problems

Keywords: Ordinary differential equations, Numerical solver

Functional Description: Julia is a programming language for scientists with a syntax and functionality similar to MATLAB, R and Python. HOODESolver.jl is a julia package allowing to solve ordinary differential equations with sophisticated numerical techniques resulting from research within the MINGUS project team. To use it, just install Julia on your workstation.

Release Contributions: This is the first version of the package. It will evolve further because we want to have a better integration with the Julia organization on differential equations. This one already includes a lot of methods to numerically solve differential equations. This integration will allow us to have a larger audience and thus more feedback and possibly external collaborations.

Contact: Nicolas Crouseilles

Participant: 3 anonymous participants

Partners: Université de Rennes 1, CNRS

7.1.3 PhaseLifting

Keyword: Python

Functional Description: Performs the phase bearing of a complex function defined on a grid, taking into account two criteria based on a non uniform weight map.

Contact: Erwan Faou

7.2 Open data

8 New results

Our results of the year are presented in the following three sections:

- multiscale numerical schemes (for dissipative or highly-oscillatory problems),
- numerical schemes for Hamiltonian problems,
- analysis of PDEs and SPDEs.

8.1 Multiscale numerical schemes

Participants: M. Badsì, A. Busnot Laurent, N. Crouseilles, A. Debussche.

Multiscale (ie highly oscillatory or dissipative) ordinary differential equations (ODEs) have a long history since they are ubiquitous to describe dynamical multiscale physical phenomena in physics or chemistry. They can be obtained by appropriate spatial discretization of the partial differential equation or can directly describe the behavior of dynamical quantities. In addition to the standard difficulties coming from their numerical resolution, multiscale ODEs involve a stiffness (characterized by a parameter $\varepsilon \in]0, 1[$).

Creating strong gradients in the solution. Hence, to capture these small scales, conventional methods have to consider a time step smaller than ε leading to unacceptable computational cost. The team members proposed several strategies to overcome this stiffness.

This work [7] deals with the numerical approximation of plasmas which are confined by the effect of a fast oscillating magnetic field in the Vlasov model. The presence of this magnetic field induces oscillations

(in time) to the solution of the characteristic equations. Due to its multiscale character, a standard time discretization would lead to an inefficient solver. In this work, time integrators are derived and analyzed for a class of highly oscillatory differential systems. We prove the uniform accuracy property of these time integrators, meaning that the accuracy does not depend on the small parameter ε . Moreover, we construct an extension of the scheme which degenerates towards an energy preserving numerical scheme for the averaged model, when $\varepsilon \rightarrow 0$. Several numerical results illustrate the capabilities of the method.

In [18], we propose and study a fully implicit finite volume scheme for the pressureless Euler-Poisson-Boltzmann equations on the one dimensional torus. Especially, we design a consistent and dissipative discretization of the force term which yields an unconditional energy decay. In addition, we establish a discrete analogue of the modulated energy estimate around constant states with a small velocity. Numerical experiments are carried to illustrate our theoretical results and to assess the accuracy of our scheme. A test case of the literature is also illustrated.

In [8], we construct and study a mean-field model that describes the nonlinear dynamics of a spin-polarized electron gas interacting with fixed, positively-charged ions possessing a magnetic moment that evolves in time. The mobile electrons are modeled by a four-component distribution function in the two-dimensional phase space (x, v) , obeying a Vlasov-Poisson set of equations. The ions are modeled by a Landau-Lifshitz equation for their spin density, which contains ion-ion and electron-ion magnetic exchange terms. We perform a linear response study of the coupled Vlasov-Poisson-Landau-Lifshitz (VPLL) equations for the case of a Maxwell-Boltzmann equilibrium, focussing in particular on the spin dispersion relation. Condition of stability or instability for the spin modes are identified, which essentially depend on the electron spin polarization rate η and the electron-ion magnetic coupling constant K . We also develop an Eulerian grid-based computational code for the fully nonlinear VPLL equations, based on the geometric Hamiltonian method recently developed previously. This technique allows us to achieve great accuracy for the conserved quantities, such as the modulus of the ion spin vector and the total energy. Numerical tests in the linear regime are in accordance with the estimations of the linear response theory. For two-stream equilibria, we study the interplay of instabilities occurring in both the charge and the spin sectors. The set of parameters used in the simulations, with densities close to those of solids ($\approx 10^{29} m^{-3}$) and temperatures of the order of 10 eV, may be relevant to the warm dense matter regime appearing in some inertial fusion experiments.

In [19], we study the interplay between spin waves (magnons) and plasma waves (plasmons) in a ferromagnetic material, using an augmented Vlasov-Poisson model that includes the electron spin dynamics. The ions are fixed, but their spins can evolve in time on the unit sphere according to the Landau-Lifshitz equation, which includes nearest-neighbor magnetic interactions. The two components interact not only through the electrostatic Coulomb force, but also via magnetic-exchange interaction terms. The linear response analysis reveals the existence of a wave-particle resonance occurring at the frequency of the magnons. This resonance gives rise to significant energy exchanges between the magnons and the electrons, resulting in a rapid loss of the localized magnetism akin to the ultrafast demagnetisation observed in experiments on thin ferromagnetic films. Depending on the initial electronic spin polarization, the resonance can lead to either damping or instability of the wave. These results show that wave-particle effects, similar to those frequently encountered in plasmas physics, may play a key role in spin-polarized plasmas and electron beams.

In [9], we study the convergence of a Zakharov system driven by a time white noise, colored in space, to a multiplicative stochastic nonlinear Schrödinger equation, as the ion-sound speed tends to infinity. In the absence of noise, the conservation of energy gives bounds on the solutions, but this evolution becomes singular in the presence of the noise. To overcome this difficulty, we show that the problem may be recasted in the diffusion-approximation framework, and make use of the perturbed test-function method. We also obtain convergence in probability. The result is limited to dimension one, to avoid too much technicalities. As a prerequisite, we prove the existence and uniqueness of regular solutions of the stochastic Zakharov system.

The unified gas kinetic scheme (UGKS) was initially designed to address multiscale challenges in rarefied gas dynamics and then extended to radiative transfert theory, as described by BGK like relaxation models. In this work [33], we extend its application to linear kinetic models with non isotropic scattering collision operators, as well as Fokker-Planck models. These problems typically exhibit a fully diffusive nature in the optically thick limit (corresponding to a small Knudsen number). It still leads to an asymptotic preserving (AP) property not only in this diffusive regime but also in the free transport limit. A series of numerical experiments confirm the effectiveness of the approach.

8.2 Geometric numerical schemes

Participants: G. Beck, A. Busnot Laurent, N. Crouseilles, E. Faou, L. Martaud.

The MINGUS team has a long history in the design and study of numerical schemes for Hamiltonian PDEs. The main examples are Schroedinger or Vlasov equations.

The goal of this work [10] is to study waves interacting with partially immersed objects allowed to move freely in the vertical direction, and in a regime in which the propagation of the waves is described by the one dimensional Boussinesq-Abbott system. The problem can be reduced to a transmission problem for this Boussinesq system, in which the transmission conditions between the components of the domain at the left and at the right of the object are determined through the resolution of coupled forced ODEs in time satisfied by the vertical displacement of the object and the average discharge in the portion of the fluid located under the object. We propose a new extended formulation in which these ODEs are complemented by two other forced ODEs satisfied by the trace of the surface elevation at the contact points. The interest of this new extended formulation is that the forcing terms are easy to compute numerically and that the surface elevation at the contact points is furnished for free. Based on this formulation, we propose a second order scheme that involves a generalization of the MacCormack scheme with nonlocal flux and a source term, which is coupled to a second order Heun scheme for the ODEs. In order to validate this scheme, several explicit solutions for this wave-structure interaction problem are derived and can serve as benchmark for future codes. As a byproduct, our method provides a second order scheme for the generation of waves at the entrance of the numerical domain for the Boussinesq-Abbott system.

This work [22] concerns the numerical approximations of the boundary conditions for linear hyperbolic systems. Discrete boundary procedure computations and an artificial viscosity are proposed to corrected any three-points finite volume schemes. The fully discrete stability of the resulting numerical schemes is established. Numerical test cases are performed to illustrate the relevancy of the proposed procedure.

This paper [23] presents a simple and robust method to enforce discrete entropy stability in firstorder well-balanced finite volume schemes for systems of balance laws, including non-conservative terms. By leveraging an artificial viscosity technique originally introduced by Tadmor and extended in recent works, the authors propose an entropy-preserving modification applicable to a wide class of three-point schemes. The core contribution lies in the construction of a viscosity coefficient that maintains the well-balanced property while ensuring entropy dissipation at the discrete level. A theoretical framework is developed to guarantee robustness, well-balancing, and entropy stability under a suitable CFL condition. The approach is validated through numerical experiments, including applications to shallow water systems and two-layer flows, demonstrating accuracy and stability.

This work [24] concerns the numerical approximations of the weak solutions of scalar hyperbolic conservation laws. After showing how to bypass the barrier theorems for the linear advection, the derivation of a second-order entropy-satisfying scheme is presented for non-linear equations. The fully discrete stability result is established for regular strictly convex entropy and under a parabolic CFL-like condition. Some numerical experiments are done to assess the accuracy and the stability of the proposed scheme.

For a class of ergodic parabolic semilinear stochastic partial differential equations (SPDEs) with gradient structure, we introduce in [25] a preconditioning technique and design high-order integrators for the approximation of the invariant distribution. The preconditioning yields improved temporal regularity of the dynamics while preserving the invariant distribution and allows the application of postprocessed integrators. For the semilinear heat equation driven by space-time white noise in dimension 1, we obtain new temporal integrators with orders 1 and 2 for sampling the invariant distribution with a minor overcost compared to the standard semilinear implicit Euler method of order 1/2. Numerical experiments confirm the theoretical findings and illustrate the efficiency of the approach.

In [26], we present a new class of numerical methods for solving stochastic differential equations with additive noise on general Riemannian manifolds with high weak order of accuracy. In opposition to the popular approach with projection methods, the proposed methods are intrinsic: they only rely on geometric operations and avoid coordinates and embeddings. We provide a robust and general convergence analysis and an algebraic formalism of exotic planar Butcher series for the computation of order conditions at any high order. To illustrate the methodology, an explicit method of second weak order is introduced, and several

numerical experiments confirm the theoretical findings and extend the approach for the sampling of the invariant measure of Riemannian Langevin dynamics.

In [28], we introduce the notion of post-Hopf algebroids, generalizing the pre-Hopf algebroids introduced recently in the study of exotic aromatic S-series. We construct action post-Hopf algebroids through actions of post-Hopf algebras. We show that the universal enveloping algebra of a post-Lie-Rinehart algebra (post-Lie algebroid) is naturally a post-Hopf algebroid. As a byproduct, we construct the free post-Lie-Rinehart algebra using a magma algebra with a linear map to the derivation Lie algebra of a commutative associative algebra. Applications in geometric numerical integration on manifolds are given.

In [27], we exhibit a new pre-Lie algebra in the framework of symplectic groupoids and, in turn, introduce a pre-Lie formalism of Butcher trees for the approximation of Hamilton-Jacobi solutions on any symplectic groupoid $G\tilde{N}M$. The impact of this new algebraic approach is twofold. On the geometric side, it yields algebraic operations to approximate Lagrangian bisections of G using the Butcher-Connes-Kreimer Hopf algebra and, in turn, aims at a better understanding of the group of Hamiltonian diffeomorphisms of M . On the computational side, we define a new class of Poisson integrators for Hamiltonian dynamics on Poisson manifolds.

In [32], a new numerical method to approximate the solution of the Vlasov-Maxwell equations is presented. The method uses a phase space discretization and its main properties are: energy and charge conservation thanks to a semi-implicit treatment of the Maxwell equations, but allowing for an explicit and efficient update of the unknown. One of the main ingredients lies in the introduction of an auxiliary scalar variable inspired from the Scalar Auxiliary Variable (SAV) approach together with a suitable splitting inspired from previous works which enables the use of a semi-Lagrangian method.

The use of symplectic numerical schemes on Hamiltonian systems is widely known to lead to favorable long-time behaviour. While this phenomenon is thoroughly understood in the context of finite-dimensional Hamiltonian systems, much less is known in the context of Hamiltonian PDEs. In this work [35], we provide the first dimension-independent backward error analysis for a Runge-Kutta-type method, the midpoint rule, which shows the existence of a modified energy for this method when applied to nonlinear Schroedinger equations regardless of the level of spatial discretisation. We use this to establish long-time stability of the numerical flow for the midpoint rule.

This work [17] concerns the design of well-balanced entropy-stable numerical schemes for the shallow water equations. The fully discrete entropy inequality is reached by introducing a local entropy condition incorporated in the scheme design. The source term is discretized to preserve both steady states and entropy stability. The method yields explicit schemes which are relevantly illustrated with several test cases.

8.3 Analysis of SPDEs

Participants: G. Beck, F. Castella, A. Debussche, E. Faou.

In view of the construction of efficient multiscale numerical schemes, the study and analysis of PDEs or SPDEs is of great importance. Below is a list of some contributions of the team on this aspect.

In [21], we deal with the interactions of waves governed by a non-linear dispersive Boussinesq type system with the vertical displacement of a cylindrical floating structure in an axisymmetric without swirl situation. The Boussinesq regime is a good approximation of free surface Euler's equations when the non-linear parameter and the shallowness parameter are small. The vertical motion of the floating body is governed by the Newton equation. The full coupled wave-structure interaction problem under consideration is reduced to a boundary problem. The boundary condition satisfied by the discharge is given in terms of the vertical displacement of the floating cylinder. The latter is calculated using an ODE, which requires knowledge of the trace of the surface elevation and its second-time derivative. We use the dispersion in order to exhibit a hidden second order ODE on the trace of the surface elevation. This finally allows us to rewrite the waves-structure interaction problem as a system of non-local conservative PDEs plus bounded radial terms with a dispersive boundary layer, combined with an ODE at the boundary. This is what we call the Augmented formulation. Afterwards we showed that this formulation is well-posed with two different methods. In our proof, we have tracked down the explicit dependence on the shallowness parameter, which is the small parameter representing dispersion. The first method gives a continuous solution with a small

existence time, i.e. proportional to the product of the inverse of the non-linear parameter and the square of the shallowness parameter, whereas the second one requires higher regularity of the solution but with a larger existence time, i.e. proportional to the product of the inverse of the non-linear parameter and the shallowness parameter. Finally, we study the return to equilibrium situation in the linear regime. In particular, we have improved previous results on the explicit time decay. We have shown that the center mass of the floating body cannot converge to its equilibrium faster than $O(t^{-1/2})$ in 2D without viscosity and faster than $O(t^{-3/2})$ with viscosity.

In [20], we propose linear dynamics that can generate a given sea wavenumber spectrum via a linear partial differential equation stirred by an additive random forcing term that is δ -correlated in time. In particular, the correlation structure of the solution to this linear dynamics converges towards the target spectrum as time passes. The main linear mechanism is a transport in Fourier space, which models the transfer of energy from large scales towards small scales. The proposed linear dynamics generalize previous works for power-law spectra in the context of hydrodynamic turbulence to more general spectra, possibly non-radial, including sea wavenumber spectra such as the JONSWAP spectrum. Finally, we present simulations of the correlation structure of the solution to these dynamics in 2D, whose spectrum converges towards the JONSWAP spectrum. These simulations are based on a finite volume method in the Fourier domain and a splitting method in time, following a recently proposed numerical method by the same authors which improves on pseudospectral simulations.

In [29], we present an elementary approach to observe frequency cascade on forced nonlinear Schrödinger equations. The forcing term consists of a constant term, perturbed by a modulated Gaussian well. Algebraic computations provide an explicit frequency cascade when time and space derivatives are discarded from the nonlinear Schrödinger equation. We provide stability results, showing that when derivatives are incorporated in the model, the initial algebraic solution may be little affected, possibly over long time intervals. Numerical simulations are provided, which support the analysis.

In [12], we consider the moments and the distribution of the hitting and cover times of a random walk in the complete graph. We study both the time needed to reach any subset of states and the time needed to visit all the states of a subset at least once. We obtain recurrence relations for the moments of all orders and we use these relations to analyze the asymptotic behavior of the hitting and cover times distributions when the number of states tends to infinity.

In [11], we consider the moments and the distribution of hitting times on the lollipop graph which is the graph exhibiting the maximum expected hitting time among all the graphs having the same number of nodes. We obtain recurrence relations for the moments of all order and we use these relations to analyze the asymptotic behavior of the hitting time distribution when the number of nodes tends to infinity.

In [30], we consider the moments and the distribution of the hitting times and cover time of the star graph composed of m arms each of length n . We obtain recurrence relations for the distribution and the moments of all order of both the hitting times of the leaves and the cover times for various initial states. We use the recurrence relations on the moments to analyze the asymptotic behavior of the cover times distribution when the number m of arms, which may depend on depends on the length n of each arm, tends to infinity when n tends to infinity. We call this new graph, composed of an infinite number of arms each of infinite length, the sun graph.

The stochastic ordering is useful to compare random variables in terms of their probability distribution: if X and Y are two real random variables, we say X is less than Y in the usual stochastic ordering whenever $P(X > x) \leq P(Y > x)$ for all $x \in \mathbb{R}$. We then write $X \leq_{st} Y$. For instance if X and Y are two durations then $X \leq_{st} Y$ means that for all values of x , the duration X has less chances to exceed x than duration Y . In [31], we consider the case of sums of independent and exponentially distributed random variables: if X_i , resp. Y_i ($i = 1, \dots, n$), are independent random variables that are exponentially distributed with rates λ_i , resp. μ_i , we examine under which conditions on the λ_i 's and μ_i 's one can ensure $\sum_{i=1}^n X_i \leq_{st} \sum_{i=1}^n Y_i$. Adopting a geometric point of view in the space of the parameters λ_i and μ_i , we come up with close to optimal conditions, improving previous results on this question, as well as shedding a geometric light on known results.

In [13], we consider systems of damped wave equations with a state-dependent damping coefficient and perturbed by a Gaussian multiplicative noise. Initially, we investigate their well-posedness, under quite general conditions on the friction. Subsequently, we study the validity of the so-called Smoluchowski-Kramers diffusion approximation. We show that, under more stringent conditions on the friction, in the small-mass limit the solution of the system of stochastic damped wave equations converges to the solution of a system of stochastic quasi-linear parabolic equations. In this convergence, an additional drift emerges as a result of the

interaction between the noise and the state-dependent friction. The identification of this limit is achieved by using a suitable generalization of the classical method of perturbed test functions, tailored to the current infinite dimensional setting.

In [16], we study slow-fast systems of coupled equations from fluid dynamics, where the fast component is perturbed by additive noise. We prove that, under a suitable limit of infinite separation of scales, the slow component of the system converges in law to a solution of the initial equation perturbed with transport noise, and subject to the influence of an additional Itô-Stokes drift. The obtained limit equation is very similar to turbulent models derived heuristically. Our results apply to the Navier-Stokes equations in dimension $d = 2, 3$.

This work [15] investigates variational frameworks for modeling stochastic dynamics in incompressible fluids, focusing on large-scale fluid behavior alongside small-scale stochastic processes. The authors aim to develop a coupled system of equations that captures both scales, using a variational principle formulated with Lagrangians defined on the full flow, and incorporating stochastic transport constraints. The approach smooths the noise term along time, leading to stochastic dynamics as a regularization parameter approaches zero. Initially, fixed noise terms are considered, resulting in a generalized stochastic Euler equation, which becomes problematic as the regularization parameter diminishes. The study then examines connections with existing stochastic frameworks and proposes a new variational principle that couples noise dynamics with large-scale fluid motion. This comprehensive framework provides a stochastic representation of large-scale dynamics while accounting for fine-scale components. Our main result is that the evolution of the small-scale velocity component is governed by a linearized Euler equation with random coefficients, influenced by large-scale transport, stretching, and pressure forcing.

In [34], we investigate how weakening the classical hydrostatic balance hypothesis impacts the well-posedness of the stochastic LU primitive equations. The models we consider are intermediate between the incompressible 3D LU Navier-Stokes equations and the LU primitive equations with standard hydrostatic balance. As such, they are expected to be numerically tractable, while accounting well for phenomena within the grey zone between hydrostatic balance and non-hydrostatic processes. Our main result is the well-posedness of a low-pass filtering-based stochastic interpretation of the LU primitive equations, with rigid-lid type boundary conditions, in the limit of "quasi-barotropic" flow. This assumption is linked to the structure assumption proposed previously, which can be related to the dynamical regime where the primitive equations remain valid. Furthermore, we present and study two eddy-(hyper)viscosity-based models.

In [14], we address a slow-fast system of coupled three dimensional Navier-Stokes equations where the fast component is perturbed by an additive Brownian noise. By means of the rough path theory, we establish the convergence in law of the slow component towards a Navier-Stokes system with an Itô-Stokes drift and a rough path driven transport noise. This gives an alternative, more general and direct proof compared to the literature. Notably, the limiting rough path is identified as a geometric rough path, which does not necessarily coincide with the Stratonovich lift of the Brownian motion.

9 Bilateral contracts and grants with industry

Participants: Erwan Faou.

9.1 Bilateral contracts with industry

Participants: Erwan Faou.

- Contract with the Cailabs compagny.

A long standing collaboration has emerged between MINGuS and the company **CAILABS** whose main aim is the conception and construction of optical fibers. Most of the main objectives of this collaboration are strictly confidential. However they have strong common point with the scientific

goals of the MINGUS project, for instance the development of efficient numerical methods for quantum simulation and many aspects of mathematical and physical analysis of quantum systems. The impact of this collaboration are very important both from the transfer of technological points of view and from the interaction with a very active startup providing very practical problems that are often very close to hot academic topics. We believe that this interaction will last long and continue to feed the scientific activity of the whole project with problem directly coming from the industrial and economical worlds.

10 Partnerships and cooperations

10.1 International initiatives

10.1.1 Inria associate team not involved in an ILL or an international program

We obtain grant from the call Inria associate teams to develop collaborations with the university of Cambridge (UK)

Bubbles 2022-2026: Bubbles and modulations for solving Schrödinger equations.
PI: E. Faou. Partner: P. Raphaël (university of Cambridge, UK).

Participants: Erwan Faou.

10.1.2 Participation in other International Programs

- **SIMONS** collaboration project. E. Faou is PI of the Simons collaboration on wave turbulence gathering mathematicians and physicists from New-York University, ENS Paris, ENS Lyon and Torino. This collaboration is the first attempt for a systematic coordinated study of Wave Turbulence Theory in a large-scale project, bringing together state-of-the-art skills in the areas of mathematics and physics, with theoretical, experimental, and numerical expertise. It is a joint effort of several groups of researchers who are ready to collectively collaborate, question all assumptions and approximations, and coordinate the progress on an interdisciplinary set of problems.

Participants: Erwan Faou.

- 2025-2030: L. Martaud is member of the International Research Project (IRP) Picasso headed in France by C. Berthon (université Nantes, France), C. Chalons (université de Versailles Saint- Quentin, France) and R. Loubère (université de Bordeaux, France). This project involves partners from University of Málaga (Spain) and Coimbra University (Portugal) and the purpose concerns the development, the analysis of models, numerical methods and simulation codes devoted to solve fluid flow problems arising from geophysical and environmental applications.

Participants: Ludovic Martaud.

10.2 International research visitors

10.2.1 Visits of international scientists

Other international visits to the team

Invited by N. Crouseilles

K. Arun**Status** (researcher)**Institution of origin:** IISER Thiruvananthapuram**Country:** India**Dates:** 2 weeks in november-december 2025**Context of the visit:** granted by project "Actions internationales" from university of Rennes**Mobility program/type of mobility:** research stay**S. Samantaray****Status** (postdoc)**Institution of origin:** university of Mainz**Country:** Germany**Dates:** 2 weeks in november-december 2025**Context of the visit:** granted by project "Actions internationales" from university of Rennes**Mobility program/type of mobility:** research stay**10.2.2 Visits to international teams****Research stays abroad****M. Badsì, N. Crouseilles****Visited institution:** CUHK**Country:** Hong-Kong**Dates:** April 2025**Context of the visit:****Mobility program/type of mobility:** research stay**10.3 National initiatives**

- 2025: project funded by Fédération de Recherche Fusion par Confinement Magnétique, headed by N. Crouseilles. Budget 10 keuros.

Participants: Mehdi Badsì, Nicolas Crouseilles.

This project is focused on the design of numerical schemes for tokamak plasmas and involve members of the team but also colleagues from university of Nantes.

- 2023-2027: E. Faou is the PI of the ANR project KEN (Kinetic, PDE and Numerics). The partners are R. Krikorian (Ecole Polytechnique) and B. Grébert (University Nantes). Budget 391 keuros.

Participants: Nicolas Crouseilles, Erwan Faou.

The project involved a group in Nantes, Ecole Polytechnique and some MINGUS members. It gathers people from the Dynamical system community, specialists of the analysis of Partial Differential Equations, as well as people coming from the numerical analysis and scientific computing worlds.

- 2023-2027: F. Castella is a member of the ANR project BEEP (Behavioural epidemiology and evolution of plant pathogens) headed by F. Hamelin. The partners are Inrae Rennes and Sophia-Antipolis, CIRAD Montpellier, Univ. Cambridge and Univ. Osnabrueck.

Participants: Francois Castella.

The project involved a group in Nantes, Ecole Polytechnique and some MINGUS members. It gathers people from the Dynamical system community, specialists of the analysis of Partial Differential Equations, as well as people coming from the numerical analysis and scientific computing worlds.

- 2025-2029: A. Busnot Laurent is the PI of the ANR JCJC MaStoC (Manifolds and Stochastic Computations). Budget 225 keuros. This project will allow for the recruitment of one or two postdocs and will fathom local collaborations in the team. The aim of MaStoC is the design of new numerical methods in the spirit of Lie-group methods for solving SDEs on Riemannian manifolds, possibly with multiscale features.

Participants: Adrien Busnot Laurent.

- 2025-2029: N. Crouseilles and L. Martaud are members of the ANR project Cookie (COMputing and apprOXimating KInetic Equations) headed by F. Filbet (université Toulouse, France).

Participants: Nicolas Crouseilles, Ludovic Martaud.

- 2023-2027: G. Beck is a member of the ANR project BOURGEONS, headed by A.-L. Dalibard (Sorbonne university).

Participants: Geoffrey Beck.

- 2023-2027: A. Debussche is a member of the CNRS-MITI project SpatialBioNet.

Participants: Arnaud Debussche.

This project focuses on some theoretical challenges and environmental applications on boundary, congestion and vorticity in fluids.

10.4 Regional initiatives

- 2025: Actions internationales (université Rennes): between IRMAR and IISER Thiruvananthapuram (India). In this project, we aim at deriving and analysing Asymptotic Preserving schemes for fluid models arising in plasma physics.

Participants: Nicolas Crouseilles.

- AIS (selective grant from Rennes metropole for young researcher) obtained by A. Busnot Laurent (2025). Budget 10 keuros.

Participants: Adrien Busnot Laurent.

11 Dissemination

11.1 Promoting scientific activities

11.1.1 Scientific events: organisation

- G. Beck co-organized the **advanced summer school mathematical fluid dynamics** (Cargèse, France, april 2025)
- N. Crouseilles and E. Faou organized the **workshop KEN** (Rennes, June 2025).
- A. Debussche co-organized the conference **Conference Climath : Coastal flows, extreme waves and wave-structure interaction** (Bordeaux, november 2025).
- A. Debussche co-organized the conference **Theoretical and numerical analysis of multi-scale equations** (Lyon, may 2025).

On top of this, G. Beck organizes the **numerical analysis seminar** and of the **analysis seminar** at IRMAR laboratory and A. Busnot Laurent organizes the **seminar** at ENS Rennes.

Finally, E. Faou is a member of the scientific council of the SCICADE-2025 conference.

11.1.2 Journal

All permanent members of the MINGUS team are regular reviewers for several of the main international journals in PDEs, numerical analysis and scientific computing. Moreover, we are also involved in editorial boards:

- N. Crouseilles: member of the editorial board of Journal of Computational Physics (2025-)
- A. Debussche
 - Editor in chief of Stochastics and Partial Differential Equations: Analysis and Computations (2013-)
 - Member of the editorial committee of ESAIM: PROCS (2012-).
 - Member of the editorial committee of Annales de l'IHP Probabilités et Statistiques (2020-).
 - Member of the editorial committee of Journal of Evolution equation (2014-).
 - Member of the editorial committee of Annales Henri Lebesgue (2018-).
 - Member of the editorial committee of the collection de monographie: Mathématiques and Applications, (SMAI).
- E. Faou: member of the editorial board of SIAM journal of numerical analysis (2025-).

11.1.3 Invited talks

During the last year, the members of MINGUS have been invited to give talks in scientific events. The main talks are listed below.

- M. Badsı talks
 - workshop CUHK (Hong-Kong, april 2025).
- G. Beck talks
 - conference on **Harmonic Analysis and Turbulence** (Bordeaux (France), april 2025).
- A. Busnot Laurent talks
 - Workshop "Numerical solutions to (S)PDEs with complex geometries" (Thessaloniki (Greece), december 2025).
 - Workshop "Operads, Symmetries for Quantum Field Theory and Singular SPDEs" (Nancy (France), June 2025).
 - Groupe de travail Statistiques et Géodésiques (Piriac-sur-mer (France), May 2025).
 - G-StAI (Lesbos (Greece), May 2025).
- N. Crouseilles talks
 - workshop CUHK (Hong-Kong, april 2025).
 - NumKin workshop (Munich (Germany), november 2025).
- A. Debussche talks
 - conference Stochastic equations and particle systems (Rome (Italy), april 2025).
 - conference Deterministic and Stochastic Evolution Equations (SNS Pisa (Italy), january 2025).
- E. Faou talks
 - Simons Foundations workshop on wave turbulence (New-York, (US), december 2025).
- L. Martaud talks
 - workshop PICASSO (Málaga (Spain), mars 2025).
 - workshop HypNuT (Amiens, november 2025).

11.1.4 Scientific expertise

The members of the MINGUS team are involved in scientific expertise duties (participation to hiring or evaluation committees, councils). Some of them are quite time consuming and are listed below

- M. Badsı
 - elected member of CNU (2022-).
 - hiring committees: assistant professor (Lille, Nantes).
 - member of LMJL council.
- N. Crouseilles
 - hiring committee of assistant professor position (Toulouse).
 - member of the IRMAR council and Inria Rennes council.
- A. Debussche
 - head of the Labex Lebesgue (2023-).

- hiring committees of an assistant professor position (ENS Rennes).
 - member of the scientific committee of the French-Japanese conferences: Probability and Interactions, IHES, France, March 2024 and Waseda, Japan, September 2025.
 - member of the HCERES committee for the evaluation of the Laboratory Jacques Louis Lions.
 - member of the administration council of ENS Paris Saclay.
 - member of the research committee of university Rennes.
 - member of the scientific council of the Institut Denis Poisson (Orléans, Tours and Amiens).
 - member of the External Advisory Board du projet ERC STUOD.
 - member of the steering committee of the EUR "Digisport" (Rennes) and of the cofund "MathPhD-InFrance" (FSMP).
- E. Faou
 - president of the HCERES committee for the evaluation of the Laboratory CERMICS (Marne-la-Vallée).
 - member of the scientific council the SCICADE-2025 conference.
 - P. Navaro: hiring committee of CNRS research engineer position.

11.1.5 Research administration

- M. Badsì: as an assistant professor in delegation within MINGUS, he taught about 100 hours. He is the responsible of the second year of master MACS (université Nantes).
- G. Beck: he taught about 50 hours, in the master of universit  Rennes, with a master course entitled "Wave propagation in complex media".
- A. Busnot Laurent: he taught about 50 hours, in the master of universit  Rennes and at ENS Rennes, with a master course entitled "Algebraic and geometric foundations of deterministic and stochastic numerics".
- F. Castella: as a professor, he taught about 200 hours at universit  Rennes (Licence and master).
- N. Crouseilles: he taught about 50 hours at ENS Rennes (Licence and master).
- A. Debussche: as a professor and IUF, he taught about 100 hours per year at ENS Rennes and a master course entitled "stochastic calculus". He is responsible of the fourth year of ENS math department.
- Navaro Pierre: he taught about 50 hours at INSA.

11.2 Teaching - Supervision - Juries - Educational and pedagogical outreach

11.2.1 Supervision

- Contentin Ewan, since september 2024, universit  de Rennes. Supervision: G. Beck and E. Faou.
- Mac  S bastien, since september 2025, universit  de Rennes. Supervision: A. Busnot Laurent and N. Crouseilles.
- Morimoto Takato, since June 2025, university Nagoya. Supervision: N. Crouseilles and J. Mathiaud (universit  de Rennes).
- Seetohul Tooryanand, since september 2024, universit  de Rennes. Supervision: E. Faou.

11.2.2 Juries

- N. Crouseilles participated to the jury of the PhD defense of G. Steimer (2025, universit  Strasbourg).
- N. Crouseilles was referee of the PhD of M. Antoine (2025, universit  Lorraine).
- N. Crouseilles was referee of the PhD of G. Gros (2025, universit  C te d'Azur).

11.3 Popularization

11.3.1 Specific official responsibilities in science outreach structures

- A. Busnot Laurent co-organised the mathematics department participation at the yearly event  te de la science, and participates to most of the other local outreach initiatives. This includes giving talks to the club Rennes en Maths for highschool students, animating a stand at the festival Rennes en science, and the creation of geometric objects with 3D printers with M. Theill re. Adrien started a collaboration with the EESAB (national art school in Rennes) for the representation of mathematics with different artistic techniques.
- A. Busnot Laurent is also invested in outreach for diversity events. He is an invited animator every year to the workshop "Les Marmottes" in Switzerland in collaboration with the Mathscope of the university of Geneva and he co-organised the similar workshop "Math C pour L". A. Busnot Laurent is a member of the gender-equality/diversity commission of the Mathematics laboratory.
- Thomas Menuet, musician and composer, spent two years at the IRMAR laboratory. The research-creation residency was born from the meeting of Thomas Menuet with researchers from IRMAR wishing to develop cultural actions intended to meet audiences and to promote scientific research. Some members of the team (G. Beck, N. Crouseilles, A. Busnot Laurent, P. Navaro) collaborated with Thomas Menuet to create enigmas combining music and mathematics. More informations can be found R sidence Thomas Menuet on this [link](#). We also participated to radio interviews to communicate about [the enigmas exhibition](#) that can be found on the following [links](#).
- As the outreach of numerical analysis is difficult, G. Beck, A. Busnot Laurent, and S. Mac  participated to the conference "From research to outreach" for kickstarting the creation of activities and events for the outreach of our research to a wider audience, that includes the colleagues in applied and pure mathematics, scientists in general, and the general public. It is expected that such involvement will increase the visibility of the team and may lead to surprising collaborations.

11.3.2 Participation in Live events

A. Busnot Laurent participated to the livestorm (organized by Inria) dedicated to PhD students on the following topic: "scientific outreach, why ?".

12 Scientific production

12.1 Major publications

- [1] J. Bernier, E. Faou and B. Grebert. 'Long time behavior of the solutions of NLW on the d-dimensional torus'. In: *Forum of Mathematics, Sigma* 8 (2020), E12. DOI: [10 . 1017 / fms . 2020 . 8](https://doi.org/10.1017/fms.2020.8). URL: <https://hal.archives-ouvertes.fr/hal-02151338>.
- [2] F. Casas, N. Crouseilles, E. Faou and M. Mehrenberger. 'High-order Hamiltonian splitting for Vlasov-Poisson equations'. In: *Numerische Mathematik* 135.3 (2017), pp. 769–801. DOI: [10 . 1007 / s00211 - 016 - 0816 - z](https://doi.org/10.1007/s00211-016-0816-z). URL: <https://hal.inria.fr/hal-01206164> (cit. on p. 7).
- [3] P. Chartier, N. Crouseilles, M. Lemou, F. M hats and X. Zhao. 'Uniformly accurate methods for three dimensional Vlasov equations under strong magnetic field with varying direction'. In: *SIAM Journal on Scientific Computing* 42.2 (2020), B520–B547. DOI: [10 . 1137 / 19M127402X](https://doi.org/10.1137/19M127402X). URL: <https://hal.archives-ouvertes.fr/hal-02179534> (cit. on p. 7).

- [4] N. Crouseilles, P.-A. Hervieux, Y. Li, G. Manfredi and Y. Sun. ‘Geometric Particle-in-Cell methods for the Vlasov-Maxwell equations with spin effects’. In: *Journal of Plasma Physics* 87.3 (28th May 2021), article n° 825870301. DOI: [10.1017/S0022377821000532](https://doi.org/10.1017/S0022377821000532). URL: <https://hal.inria.fr/hal-03148534> (cit. on p. 7).
- [5] A. Debussche and J. Vovelle. ‘Diffusion-approximation in stochastically forced kinetic equations’. In: *Tunisian Journal of Mathematics* 3.1 (2021), pp. 1–53. DOI: [10.2140/tunis.2021.3.1](https://doi.org/10.2140/tunis.2021.3.1). URL: <https://hal.archives-ouvertes.fr/hal-01567138> (cit. on p. 7).
- [6] E. Faou. ‘Linearized wave turbulence convergence results for three-wave systems’. In: *Communications in Mathematical Physics* 378.2 (Sept. 2020), pp. 807–849. DOI: [10.1007/s00220-020-03799-w](https://doi.org/10.1007/s00220-020-03799-w). URL: <https://hal.inria.fr/hal-01801810> (cit. on p. 7).

12.2 Publications of the year

International journals

- [7] M. Anandan, B. Boutin and N. Crouseilles. ‘Uniformly higher order accurate schemes for dynamics of charged particles under fast oscillating magnetic fields’. In: *IMA Journal of Numerical Analysis* 048 (2025), pp. 1–26. URL: <https://inria.hal.science/hal-04773938> (cit. on p. 15).
- [8] B. Bakri, N. Crouseilles, P.-A. Hervieux, X. Hong and G. Manfredi. ‘Ultrafast dynamics of a spin-polarized electron plasma with magnetic ions’. In: *Journal of Plasma Physics* 91.1 (7th Jan. 2025), pp. 1–38. DOI: [10.1017/S0022377824001594](https://doi.org/10.1017/S0022377824001594). URL: <https://hal.science/hal-04844507> (cit. on p. 16).
- [9] G. Barraué, A. de Bouard and A. Debussche. ‘From Stochastic Zakharov System to Multiplicative Stochastic Nonlinear Schrödinger Equation’. In: *Stochastics and Partial Differential Equations: Analysis and Computations* (2025), pp. 1–40. URL: <https://hal.science/hal-04704413>. In press (cit. on p. 16).
- [10] G. Beck, D. Lannes and L. Weynans. ‘A numerical method for wave-structure interactions in the boussinesq regime’. In: *ESAIM: Mathematical Modelling and Numerical Analysis* 59.6 (7th Nov. 2025), pp. 2895–2931. DOI: [10.1051/m2an/2025075](https://doi.org/10.1051/m2an/2025075). URL: <https://hal.science/hal-04151128> (cit. on p. 17).
- [11] F. Castella and B. Sericola. ‘Hitting times on the lollipop graph’. In: *Probability in the Engineering and Informational Sciences* 39.4 (2025), pp. 486–519. URL: <https://inria.hal.science/hal-05423122> (cit. on p. 19).
- [12] F. Castella and B. Sericola. ‘Random Walk in the Complete Graph : Hitting and Cover Times’. In: *Methodology and Computing in Applied Probability* 27.39 (28th Apr. 2025). URL: <https://inria.hal.science/hal-05423075> (cit. on p. 19).
- [13] S. Cerrai and A. Debussche. ‘Smoluchowski-Kramers diffusion approximation for systems of stochastic damped wave equations with non-constant friction’. In: *The Annals of Applied Probability* (2025). DOI: [10.48550/arXiv.2312.08925](https://doi.org/10.48550/arXiv.2312.08925). URL: <https://hal.science/hal-04383799>. In press (cit. on p. 19).
- [14] A. Debussche and M. Hofmanová. ‘Rough analysis of two scale systems’. In: *Annales de la Faculté des Sciences de Toulouse. Mathématiques.* (2025). DOI: [10.48550/arXiv.2306.15781](https://doi.org/10.48550/arXiv.2306.15781). URL: <https://hal.science/hal-04846233>. In press (cit. on p. 20).
- [15] A. Debussche and E. Mémin. ‘Variational principles for fully coupled stochastic fluid dynamics across scales’. In: *Physica D: Nonlinear Phenomena* 481 (Nov. 2025), p. 134777. DOI: [10.1016/j.physd.2025.134777](https://doi.org/10.1016/j.physd.2025.134777). URL: <https://inria.hal.science/hal-05327779> (cit. on p. 20).
- [16] A. Debussche and U. Pappalettera. ‘Second order perturbation theory of two-scale systems in fluid dynamics’. In: *Journal of the European Mathematical Society* (2025). URL: <https://hal.science/hal-04383854>. In press (cit. on p. 20).

- [17] L. Martaud and C. Berthon. ‘How to enforce an entropy inequality of (fully) well-balanced Godunov-type schemes for the shallow water equations’. In: *ESAIM: Mathematical Modelling and Numerical Analysis* 59 (Mar. 2025), pp. 955–997. DOI: [10.1051/m2an/2025012](https://doi.org/10.1051/m2an/2025012). URL: <https://hal.science/hal-05376635> (cit. on p. 18).

Reports & preprints

- [18] M. Badsì and N. Crouseilles. *Discrete stability estimates for the pressureless Euler-Poisson-Boltzmann equations in the Quasi-Neutral limit*. 2025. URL: <https://hal.science/hal-05014885> (cit. on p. 16).
- [19] B. Bakri, N. Crouseilles, P.-A. Hervieux and G. Manfredi. *Wave-particle interactions in a spin polarized plasma*. 16th Nov. 2025. URL: <https://hal.science/hal-05370955> (cit. on p. 16).
- [20] G. Beck, C.-É. Bréhier, L. Chevillard, R. Grande and W. Ruffenach. *A linear model of dynamical sea wavenumber spectra*. 25th Mar. 2025. URL: <https://hal.science/hal-05005567> (cit. on p. 19).
- [21] G. Beck, E. Contentin and L. Martaud. *Freely floating cylinder on a 3D fluid governed by the boussinesq equations in the axisymmetric without swirl case*. 29th Dec. 2025. URL: <https://hal.science/hal-05434682> (cit. on p. 18).
- [22] G. Beck, L. Martaud and N. Crouseilles. *On the stability of discrete boundaries conditions for linear hyperbolic systems*. 27th Mar. 2025. URL: <https://hal.science/hal-05008631> (cit. on p. 17).
- [23] C. Berthon, M. Castro Díaz, L. Martaud and T. Morales de Luna. *How to make fully discrete entropy stability of three-point well-balanced finite-volume schemes*. 24th July 2025. URL: <https://hal.science/hal-05378579> (cit. on p. 17).
- [24] C. Berthon and L. Martaud. *Local fully discrete entropy stability for a genuine second-order scheme for scalar hyperbolic equations*. 10th Sept. 2025. URL: <https://hal.science/hal-05248101> (cit. on p. 17).
- [25] C.-É. Bréhier, A. Busnot Laurent, A. Debussche and G. Vilmart. *Preconditioning for the high-order sampling of the invariant distribution of parabolic semilinear SPDEs*. 19th Dec. 2025. URL: <https://hal.science/hal-05426627> (cit. on p. 17).
- [26] E. Bronasco, A. Busnot Laurent and B. N. Huguet. *High order integration of stochastic dynamics on Riemannian manifolds with frozen flow methods*. 15th July 2025. URL: <https://hal.science/hal-05368742> (cit. on p. 17).
- [27] A. Busnot Laurent and O. Cosserrat. *Butcher series for Hamiltonian Poisson integrators through symplectic groupoids*. 2025. URL: <https://hal.science/hal-05368917> (cit. on p. 18).
- [28] A. Busnot Laurent, Y. Li and Y. Sheng. *Post-Hopf algebroids, post-Lie-Rinehart algebras and geometric numerical integration*. 28th Dec. 2025. URL: <https://hal.science/hal-05433426> (cit. on p. 18).
- [29] R. Carles and E. Faou. *A toy model for frequency cascade in the nonlinear Schrodinger equation*. 2025. URL: <https://hal.science/hal-05117614> (cit. on p. 19).
- [30] F. Castella and B. Sericola. *Hitting and cover times of the star graph and the sun graph*. Centre Inria de l’Université de Rennes, 14th July 2025, p. 29. URL: <https://inria.hal.science/hal-05161555> (cit. on p. 19).
- [31] F. Castella and B. Sericola. *Stochastic ordering of sums of exponential random variables. A geometric approach*. Univ Rennes, Inria, CNRS, IRISA, 23rd June 2025, pp. 1–38. URL: <https://inria.hal.science/hal-05125505> (cit. on p. 19).
- [32] N. Crouseilles, H. Liu and Y. Yue. *Semi-Lagrangian SAV method for Vlasov-Maxwell equations*. 2025. URL: <https://hal.science/hal-05273412> (cit. on p. 18).
- [33] N. Crouseilles, J. Mathiaud and L. Mieussens. *Generalized UGK scheme in the diffusive limit*. 2025. URL: <https://hal.science/hal-05137746> (cit. on p. 16).
- [34] A. Debussche, É. Mémin and A. Moneyron. *Stochastic interpretations of the oceanic primitive equations with relaxed hydrostatic assumptions*. 20th Feb. 2025. URL: <https://hal.science/hal-04958571> (cit. on p. 20).

- [35] E. Faou, G. Maierhofer and K. Schratz. *Fully discrete backward error analysis for the midpoint rule applied to the nonlinear Schroedinger equation*. 2nd May 2025. URL: <https://inria.hal.science/hal-05054444> (cit. on p. 18).

12.3 Cited publications

- [36] C. Birdsall and A. Langdon. *Plasmas physics via computer simulations*. New York: Taylor and Francis, 2005 (cit. on p. 9).
- [37] A. Brizard and T. Hahm. ‘Foundations of nonlinear gyrokinetic theory’. In: *Reviews of Modern Physics* 79 (2007) (cit. on p. 10).
- [38] J. Carr. ‘Applications of Centre Manifold Theory’. In: *Applied Mathematical Sciences Series 35* (1981) (cit. on p. 8).
- [39] P. Chartier, N. Crouseilles, M. Lemou and F. Méhats. ‘Uniformly accurate numerical schemes for highly-oscillatory Klein-Gordon and nonlinear Schrödinger equations’. In: *Numer. Math.* 129 (2015), pp. 513–536 (cit. on pp. 9, 11).
- [40] P. Chartier, A. Murua and J. Sanz-Serna. ‘Higher-order averaging, formal series and numerical integration III: error bounds’. In: *Foundation of Comput. Math.* 15 (2015), pp. 591–612 (cit. on p. 8).
- [41] A. Debussche and J. Vovelle. ‘Diffusion limit for a stochastic kinetic problem’. In: *Commun. Pure Appl. Anal.* 11 (2012), pp. 2305–2326 (cit. on p. 8).
- [42] E. Faou and F. Rousset. ‘Landau damping in Sobolev spaces for the Vlasov-HMF model’. In: *Arch. Ration. Mech. Anal.* 219 (2016), pp. 887–902 (cit. on p. 10).
- [43] E. Hairer, C. Lubich and G. Wanner. *Geometric Numerical Integration. Structure-Preserving Algorithms for Ordinary Differential Equations, Second edition*. Springer Series in Computational Mathematics 31. Berlin: Springer, 2006 (cit. on p. 11).
- [44] S. Jin and H. Lu. ‘An Asymptotic-Preserving stochastic Galerkin method for the radiative heat transfer equations with random inputs and diffusive scalings’. In: *J. Comp. Phys.* 334 (2017), pp. 182–206 (cit. on p. 9).
- [45] M. Lemou, F. Méhats and P. Raphaël. ‘Orbital stability of spherical galactic models’. In: *Invent. Math.* 187 (2012), pp. 145–194 (cit. on p. 10).
- [46] C. Mouhot and C. Villani. ‘On Landau damping’. In: *Acta Math.* 207 (2011), pp. 29–201 (cit. on p. 10).
- [47] S. Nazarenko. *Wave turbulence*. Springer-Verlag, 2011 (cit. on p. 11).
- [48] L. Perko. ‘Higher order averaging and related methods for perturbed periodic and quasi-periodic systems’. In: *SIAM J. Appl. Math.* 17 (1969), pp. 698–724 (cit. on pp. 9, 12).