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2025 *Activity Report*

RESEARCH CENTRE: **Inria Paris Centre**
IN PARTNERSHIP WITH: **Université Paris-Dauphine, CNRS**

Project-Team

MOKAPLAN

Advances in Numerical Calculus of Variations

In collaboration with CEREMADE

Project-Team MOKAPLAN

Creation of the Project-Team: 2015 December 01

Each year, Inria research teams publish an Activity Report presenting their work and results over the reporting period. These reports follow a common structure, with some optional sections depending on the specific team. They typically begin by outlining the overall objectives and research programme, including the main research themes, goals, and methodological approaches. They also describe the application domains targeted by the team, highlighting the scientific or societal contexts in which their work is situated. The reports then present the highlights of the year, covering major scientific achievements, software developments, or teaching contributions. When relevant, they include sections on software, platforms, and open data, detailing the tools developed and how they are shared. A substantial part is dedicated to new results, where scientific contributions are described in detail, often with subsections specifying participants and associated keywords. Finally, the Activity Report addresses funding, contracts, partnerships, and collaborations at various levels, from industrial agreements to international cooperations. It also covers dissemination and teaching activities, such as participation in scientific events, outreach, and supervision. The document concludes with a presentation of scientific production, including major publications and those produced during the year.

Keywords

Computer sciences and digital sciences

A5.3. – Image processing and analysis

A5.9. – Signal processing

A6.1.1. – Continuous Modeling (PDE, ODE)

A6.1.2. – Stochastic Modeling

A6.2.1. – Numerical analysis of PDE and ODE

A6.2.6. – Optimization

A6.3.1. – Inverse problems

A8.2.3. – Calculus of variations

A8.2.6. – Numerical methods for optimization

A8.12. – Optimal transport

A9. – Artificial intelligence

Other research topics and application domains

B9.5.2. – Mathematics

B9.5.3. – Physics

B9.6.3. – Economy, Finance

Contents

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- Vincent Duval [Team leader, INRIA, Senior Researcher, On leave from Corps des Mines, HDR]
- Jean-David Benamou [INRIA, Senior Researcher, HDR]
- Antonin Chambolle [CNRS, Senior Researcher, HDR]
- Flavien Leger [INRIA, Researcher]
- Irene Waldspurger [CEREMADE, Researcher, until Feb 2025]

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- Christian Leonard [UNIV PARIS NANTERRE, Professor Delegation, until Aug 2025, HDR]
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- Guillaume Chazareix [DAUPHINE PSL , DAUPHINE PSL University]
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- Hugo Malamut [DAUPHINE PSL, until Aug 2025]
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2 Overall objectives

The fundamental observation at the origin of Mokaplan is that, in the last 15 years, a remarkable convergence occurred between several sub-domains of the calculus of variations, namely optimal transport (and its many generalizations), diffeomorphic registration (computational geometry over infinite dimensional groups) and inverse problems in imaging (in particular sparsity-based regularization). This convergence is mostly due to:

1. the mathematical objects manipulated in these problems, namely sparse measures (e.g. coupling in transport, edge location in imaging, displacement fields for diffeomorphisms) and
2. the use of similar numerical tools from non-smooth optimization and geometric discretization schemes.

Such sparse and non-smooth objects are particularly difficult to handle with the standard tools of numerical analysis, and call for efficient numerical strategies.

Optimal Transport (OT) is a rich and powerful theory that has drawn the attention of first-class mathematicians [106, 84, 43] since the renewal of interest triggered by the work of Brenier [59]. Moreover, in the last decade, the number and the diversity of applications of OT has dramatically increased: machine learning [101, 93], computer graphics [104, 53], genomics [103], economics [86], chemistry [79]... The case of physics is remarkable, with the connection to fluid dynamics pointed out by Brenier [60] (see also [49]), and the interpretation proposed by Jordan, Kinderlehrer and Otto [91], of several partial differential equations (PDE) as a gradient flow in the Wasserstein metric (i.e. in the metric determined by optimal transport). Some questions have both theoretical and practical interest, such as the existence and regularity of a transport map (which are related, e.g., to mode collapse in generative models, or the connectedness of barycenters in data analysis), and of course the need of fast and accurate solvers. In particular, entropic regularization and Sinkhorn's algorithm [77], was a breakthrough in the numerical resolution of OT, but it induces blur in the transport plan. Quantifying this error and possibly reducing it is crucial for applications.

Besides, the study of OT has led mathematicians to introduce new concepts and tools such as c-convexity, generalized geodesics, or the Kim-McCann metric, that are potentially of interest in other settings such as mean-field games, equilibrium problems, or even in the study of optimization algorithms [96].

More generally, the class of variational problems in the space of measures, of which OT is a particular instance, raises significant challenges. Such variational problems appear naturally in recent works in inverse problems [68, 57, 63], where the goal is to recover point sources, modelled as Dirac masses. There is a strong interest in solving these problems "in the continuum", or *off-the-grid*, that is, without relying on a predefined grid. On the theoretical side, it simplifies the analysis [80], and on the numerical side, as it prevents discretization artifacts as well as the instabilities and computational and memory cost of using thin grids [81]. The corresponding numerical methods involve greedy approaches [55] which require nonconvex optimization steps, and possibly moment matrices which have low-rank [69] when the measure is sparse. There is therefore a crucial need to better understand nonconvex problems, especially those which have a form of sparsity (e.g. as low rank matrices). Not only are they interesting *per se*, but they are also the key to memory efficient implementation, as the Burer-Monteiro factorization [61] typically illustrates.

Beyond point sources, measures may have support in more complex objects such as curves, surfaces. This is typically the case of geometric variational problems. Those may arise in inverse problems (e.g. the gradient of a piecewise constant function with bounded variation is such a sparse measure), or in branched transport problems [108, 51]. That theory models communication networks satisfying an economy of scale (e.g. road systems, sewage systems, pipelines, power-grids) as well as natural networks (blood vessels, branches of trees, plant roots) where joint transportation is favored. They also appear in urban planning [56]. Understanding the structure of solutions and proposing efficient numerical approaches matters a lot for better comprehension or design of the above-mentioned networks.

Mokaplan is a joint research effort to advance the state of the art in the numerical resolution of such non-linear problems, which are challenging as they are formulated in the space of measures and/or they involve some form of sparsity. Our contribution is mainly methodological and our approach is transverse, in the sense that we do not focus on a particular application, but we are happy to contribute to applications when opportunities arise.

Our research goals may be summarized as:

1. Understand better Optimal Transport and related problems, and propose efficient numerical solvers,

2. Leverage the tools and concepts introduced in Optimal Transport in other settings (e.g. non-convex or non-variational problems, general optimization problems),
3. Provide theories and tools to solve inverse problems in a sparse or measure-theoretic framework,
4. Derive novel numerical methods and a better understanding of geometric variational problems.

3 Research program

Since its creation, the Mokaplan team has made important contributions in Optimal Transport both on the theoretical and the numerical side, together with applications such as fluid mechanics, the simulation biological systems, machine learning. We have also contributed to to the field of inverse problems in signal and image processing (super-resolution, nonconvex low rank matrix recovery). In 2022, the team was renewed with the following research program which broadens our spectrum and addresses exciting new problems.

3.1 OT and related variational problems solvers

Participants: Flavien Léger , Jean-David Benamou , Guillaume Carlier , Thomas Gallouët , Guillaume Chazareix , Adrien Vacher , Paul Pegon.

Asymptotic analysis of entropic OT for a small entropic parameter is well understood for regular data on compact manifolds and standard quadratic ground cost [74], the team will extend this study to more general settings and also establish rigorous asymptotic estimates for the transports maps. This is important to provide a sound theoretical background to efficient and useful debiasing approaches like Sinkhorn Divergences [83]. Guillaume Carlier, Paul Pegon and Luca Tamanini are investigating speed of convergence and quantitative stability results under general conditions on the cost (so that optimal maps may not be continuous or even fail to exist). Some sharp bounds have already been obtained, the next challenging goal is to extend the Laplace method to a nonsmooth setting and understand what entropic OT really selects when there are several optimal OT plans.

High dimensional - Curse of dimensionality We will continue to investigate the computation or approximation of high-dimensional OT losses and the associated transports [105] in particular in relation with their use in ML. In particular for Wasserstein 2 metric but also the repulsive Density Functional theory cost [85].

Back-and-forth The back-and-forth method [90, 89] is a state-of-the-art solver to compute optimal transport with convex costs and 2-Wasserstein gradient flows on grids. Based on simple but new ideas it has great potential to be useful for related problems. We plan to investigate: OT on point clouds in low dimension, the principal-agent problem in economics and more generally optimization under convex constraints [94, 97].

Transport and diffusion The diffusion induced by the entropic regularization is fixed and now well understood. For recent variations of the OT problem (Martingale OT, Weak OT see [45]) the diffusion becomes an explicit constraint or the control itself [87]. The entropic regularisation of these problems can then be understood as metric/ground cost learning [64] (see also [100]) and offers a tractable numerical method.

Wasserstein Hamiltonian systems We started to investigate the use of modern OT solvers for the SG equation [76, 48] Semi-Discrete and entropic regularization. This is a special instance Hamiltonian Systems in the sense of [42]. with an OT component in the Energy.

Nonlinear fourth-order diffusion equations such as thin-films or (the more involved) DLSS quantum drift equations are WGF. Such WGF are challenging both in terms of mathematical analysis (lack of maximum principle...) and of numerics. They are currently investigated by Jean-David Benamou, Guillaume Carlier in collaboration with Daniel Matthes. Note also that Mokaplan already contributed to a related topic through the TV-JKO scheme [66].

Lagrangian approaches for fluid mechanics More generally we want to extend the design and implementation of Lagrangian numerical scheme for a large class of problem coming from fluids mechanics (WHS or WGF) using semi-discrete OT or entropic regularization. We will also take a special attention to link this approaches with problems in machine/statistical learning. To achieve this part of the project we will join forces with colleagues in Orsay University: Y. Brenier, H. Leclerc, Q. Mérigot, L. Nenna.

L^∞ **optimal transport** is a variant of OT where we want to minimize the maximal displacement of the transport plan, instead of the average distance. Following the seminal work of [73], and more recent developments [78], Guillaume Carlier, Paul Pegon and Luigi De Pascale are working on the description of *restrictable* solutions (which are cyclically ∞-monotone) through some potential maps, in the spirit of Mange-Kantorovich potentials provided by a duality theory. Some progress has been made to partially describe cyclically quasi-motonone maps (related in some sense to cyclically ∞-monotone maps), through quasi-convex potentials.

3.2 Application of OT numerics to non-variational and non convex problems

Participants: Flavien Léger , Guillaume Carlier , Jean-David Benamou.

Market design Z-mappings form a theory of non-variational problems initiated in the '70s but that has been for the most part overlooked by mathematicians. We are developing a new theory of the algorithms associated with convergent regular splitting of Z-mappings. Various well-established algorithms for matching models can be grouped under this point of view (Sinkhorn, Gale–Shapley, Bertsekas' auction) and this new perspective has the potential to unlock new convergence results, rates and accelerated methods.

Non Convex inverse problems The PhD [109] provided a first exploration of Unbalanced Sinkhorn Divergence in this context. Given enough resources, a branch of PySit, a public domain software to test misfit functions in the context of Seismic imaging will be created and will allow to test other signal processing strategies in Full Waveform Inversion. Likewise the numerical method tested for 1D reflectors in [50] coule be devellોped further (in particular in 2D).

Equilibrium and transport Equilibrium in labor markets can often be expressed in terms of the Kantorovich duality. In the context of urban modelling or spatial pricing, this observation can be fruitfully used to compute equilibrium prices or densities as fixed points of operators involving OT, this was used in [47] and [46]. Quentin Petit, Guillaume Carlier and Yves Achdou are currently developing a (non-variational) new semi-discrete model for the structure of cities with applications to tele-working.

Non-convex Principal-Agent problems Guillaume Carlier, Xavier Dupuis, Jean-Charles Rochet and John Thanassoulis are developing a new saddle-point approach to non-convex multidimensional screening problems arising in regulation (Barron-Myerson) and taxation (Mirrlees).

3.3 Inverse problems with structured priors

Participants: Irène Waldspurger , Antonin Chambolle , Vincent Duval , Faniriana Rakoto Endor , Annette Dumas.

Off-the-grid reconstruction of complex objects Whereas, very recently, some methods were proposed for the reconstruction of curves and piecewise constant images on a continuous domain ([58] and [67]), those are mostly proofs of concept, and there is still some work to make them competitive in real applications. As they are much more complex than point source reconstruction methods, there is room for improvements (parametrization, introduction of several atoms. . .). In particular, we are currently working on an improvement of the algorithm [58] for inverse problems in imaging which involve Optimal Transport as a regularizer (see [82] for preliminary results). Moreover, we need to better understand their convergence and the robustness of such methods, using sensitivity analysis.

Correctness guarantees for Burer-Monteiro methods The Burer-Monteiro strategy solves low-rank optimization problems by writing the low-rank matrix to recover as a product of two thinner matrices. It has an integer parameter, namely the number of columns of the product factors. It is important for numerical efficiency that the parameter is chosen as small as possible. Unfortunately, general correctness guarantees for the method are only available for values of the parameter above a certain threshold, much higher than the values typically used by practitioners [54], and we know that no such general guarantee can hold below the threshold [107]. Faniriana Rakoto Endor and Irène Waldspurger have shown in [102] that, below the threshold, it is still possible to prove that the the Burer-Monteiro strategy is correct (i.e. it does not have spurious critical points which could trap the numerical solver) for a large class of semidefinite problems. It explains the favorable behavior of the Burer-Monteiro strategy observed in a number of applications. Generalizations of this result are currently under investigation.

3.4 Geometric variational problems, and their interactions with transport

Participants: Vincent Duval , Paul Pegon , Antonin Chambolle , Joao-Miguel Machado .

Approximation of measures with geometric constraints Optimal Transport is a powerful tool to compare and approximate densities, but its interaction with geometric constraints is still not well understood. In applications such as optimal design of structures, one aims at approximating an optimal pattern while taking into account fabrication constraints [52]. In Magnetic Resonance Imaging (MRI), one tries to sample the Fourier transform of the unknown image according to an optimal density but the acquisition device can only proceed along curves with bounded speed and bounded curvature [95]. Our goal is to understand how OT interacts with energy terms which involve, e.g. the length, the perimeter or the curvature of the support... We want to understand the regularity of the solutions and to quantify the approximation error. Moreover, we want to design numerical methods for the resolution of such problems, with guaranteed performance.

Discretization of singular measures Beyond the (B)Lasso and the total variation (possibly off-the-grid), numerically solving branched transportation problems requires the ability to faithfully discretize and represent 1-dimensional structures in the space. The research program of A. Chambolle consists in part in developing the numerical analysis of variational problems involving singular measures, such as lower-dimensional currents or free surfaces. We will explore both phase-field methods (with P. Pegon, V. Duval) [70, 99] which easily represent non-convex problems, but lack precision, and (with V. Duval) precise discretizations of convex problems, based either on finite elements (and relying to the FEM discrete exterior calculus [44], cf [71] for the case of the total variation), or on finite differences and possibly a clever design of dual constraints as studied in [75, 72] again for the total variation.

Transport problems with metric optimization In urban planning models, one looks at building a network (of roads, metro or train lines, etc.) so as to minimize a transport cost between two distributions, penalized by the cost for building the network, usually its length. A typical transport cost is Monge cost MK_ω with a metric $\omega = \omega_\Sigma$ which is modified as a fraction of the euclidean metric on the network

Σ. We would like to consider general problems involving a construction cost to generate a conductance field σ (having in mind 1-dimensional integral of some function of σ), and a transport cost depending on this conductance field. The afore-mentioned case studied in [62] falls into this category, as well as classical branched transport. The biologically-inspired network evolution model of [88] seems to provide such an energy in the vanishing diffusivity limit, with a cost for building a 1-dimensional permeability tensor and an L^2 congested transport cost with associated resistivity metric ; such a cost seems particularly relevant to model urban planning. Finally, we would like to design numerical methods to solve such problems, taking advantage of the separable structure of the whole cost.

4 Application domains

4.1 Natural Sciences

FreeForm Optics, Fluid Mechanics (Incompressible Euler, Semi-Geostrophic equations), Quantum Chemistry (Density Functional Theory), Statistical Physics (Schroedinger problem), Porous Media.

4.2 Signal Processing and inverse problems

Full Waveform Inversion (Geophysics), Super-resolution microscopy (Biology), Satellite imaging (Meteorology)

4.3 Social Sciences

Mean-field games, spatial economics, principal-agent models, taxation, nonlinear pricing.

5 New results

5.1 Convergence Rates of the Regularized Optimal Transport : Disentangling Suboptimality and Entropy

Participants: Hugo Malamut, Maxime Sylvestre.

In [19], we study the convergence of the transport plans γ_ε towards γ_0 as well as the cost of the entropy-regularized optimal transport (c, γ_ε) towards (c, γ_0) as the regularization parameter ε vanishes in the setting of finite entropy marginals. We show that under the assumption of infinitesimally twisted cost and compactly supported marginals the distance $W^2(\gamma_\varepsilon, \gamma_0)$ is asymptotically greater than $C\sqrt{\varepsilon}$ and the suboptimality $(c, \gamma_\varepsilon) - (c, \gamma_0)$ is of order ε. In the quadratic cost case the compactness assumption is relaxed into a moment of order $2 + \delta$ assumption. Moreover, in the case of a Lipschitz transport map for the non-regularized problem, the distance $W^2(\gamma_\varepsilon, \gamma_0)$ converges to 0 at rate $\sqrt{\varepsilon}$. Finally, if in addition the marginals have finite Fisher information, we prove $(c, \gamma_\varepsilon) - (c, \gamma_0) \sim \varepsilon/2$ and we provide a companion expansion of $H(\gamma_\varepsilon)$. These results are achieved by disentangling the role of the cost and the entropy in the regularized problem.

5.2 Entropic approximations of the semigeostrophic shallow water equations

Participants: Jean-David Benamou, Hugo Malamut.

In [31], we develop a discretisation of the semigeostrophic rotating shallow water equations, based upon their optimal transport formulation, which takes the form of a Moreau-Yoshida regularisation via the Wasserstein metric, whose solution provides the shallow water layer depth represented as a measure, which is the push forward of an evolving measure under the semigeostrophic coordinate transformation. First, we propose and study an entropic regularisation of the rotating shallow water equations. Second, we discretise the regularised problem by replacing both measures with weighted sums of Dirac measures, and approximating the (squared) L 2 norm of the layer depth appearing as the potential energy. We propose an iterative method to solve the discrete optimisation problem relating the two measures, and analyse its convergence. The iterative method is demonstrated numerically and applied to the solution of the time-dependent shallow water problem in numerical examples.

5.3 Convergence rates for regularized unbalanced optimal transport

Participants: Luca Nenna, Paul Pegon, Louis Tocquec.

Following our study of convergence rates of entropy-regularized OT costs [65, 98], we have are moving on to the case of unbalanced OT. With Luca Nennna and our PhD student Louis Tocquec we have treated the case of unbalanced OT in the discrete case, where the input measures are finite sums of Dirac masses, and with a general class of entropy functions [39]. We analyze the asymptotic behavior of both primal transport plans and dual variables as the regularization parameter tends to infinity. Under general convexity and regularity assumptions on the marginal penalization, we establish explicit convergence rates: at least $O(\varepsilon)$ for the dual variables and $O(\sqrt{\varepsilon})$ for the primal solutions. These theoretical results are illustrated and validated through numerical experiments for standard divergences, including the Kullback-Leibler and quadratic marginal penalties.

5.4 Optimal quantization via branched optimal transport distance

Participants: Paul Pegon, Mircea Petrache.

In [20] we study optimal quantization with respect to branched optimal transport distances, addressing the problem of approximating a target measure by an atomic measure with NN atoms. Unlike the classical Wasserstein setting, the associated optimal partitions lack an explicit Voronoï structure and are expected to have fractal interfaces. We analyze the asymptotic behavior of optimal quantizers for absolutely continuous measures as $N \to +\infty$, establishing a branched transport analogue of Zador's theorem and identifying the limiting distribution of point clouds. Our approach relies on a Γ-convergence framework and on the uniform Hölder regularity of the landscape function, a branched counterpart of Kantorovich potentials. As an application, we derive uniform separation and covering estimates for optimal quantizers under Ahlfors regularity assumptions.

5.5 A Rockafellar Theorem in the quasi-convex setting

Participants: Paul Pegon, Luigi De Pascale.

In [40] we investigate a quasi-convex analogue of Rockafellar's theorem, addressing the problem of integrating cyclically quasi-monotone maps through quasi-convex potentials. We establish such an integration result for cyclically quasi-monotone maps that are C^1 and non-vanishing, in arbitrary dimension, as well as for general multi-valued maps in dimension one. The proof relies on the construction of suitable preference relations and totally ordered families of convex sets, leading to lower semicontinuous quasi-convex potentials whose normal cone operators contain the given maps. We also discuss connections with revealed preference theory in economics and with L^∞ optimal transport, and provide examples illustrating the remaining challenges in the general case.

5.6 Weak optimal transport with moment constraints: constraint qualification, dual attainment and entropic regularization

Participants: Guillaume Carlier , Hugo Malamut, Maxime Sylvestre.

In [33], we consider weak optimal problems (possibly entropically penalized) incorporating both soft and hard (including the case of the martingale condition) moment constraints. Even in the special case of the martingale optimal transport problem, existence of Lagrange multipliers corresponding to the martingale constraint is notoriously hard (and may fail unless some specific additional assumptions are made). We identify a condition of qualification of the hard moment constraints (which in the martingale case is implied by well-known conditions in the literature) under which general dual attainment results are established. We also analyze the convergence of entropically regularized schemes combined with penalization of the moment constraint and illustrate our theoretical findings by numerically solving in dimension one, the Brenier-Strassen problem of Gozlan and Juillet and a family of problems which interpolates between monotone transport and left-curtain martingale coupling of Beiglböck and Juillet.

5.7 Graph Alignment via Birkhoff Relaxation

Participants: Sushil Varma, Irène Waldspurger, Laurent Massoulié.

In [25], we consider the graph alignment problem, wherein the objective is to find a vertex correspondence between two graphs that maximizes the edge overlap. The graph alignment problem is an instance of the quadratic assignment problem (QAP), known to be NP-hard in the worst case even to approximately solve. In this paper, we analyze Birkhoff relaxation, a tight convex relaxation of QAP, and present theoretical guarantees on its performance when the inputs follow the Gaussian Wigner Model. More specifically, the weighted adjacency matrices are correlated Gaussian Orthogonal Ensemble with correlation $1/\sqrt{1+\sigma^2}$. Denote the optimal solutions of the QAP and Birkhoff relaxation by Π^* and X^* respectively. We show that $||X^* - \Pi^*||_F^2 = o(n)$ when $\sigma = o(n^{-1.25})$ and $||X^* - \Pi^*||_F^2 = \Omega(n)$ when $\sigma = \Omega(n^{-0.5})$. Thus, the optimal solution X^* transitions from a small perturbation of Π^* for small σ to being well separated from Π^* as σ becomes larger than $n^{-0.5}$. This result allows us to guarantee that simple rounding procedures on X^* align $1 - o(1)$ fraction of vertices correctly whenever $\sigma = o(n^{-1.25})$. This condition on σ to ensure the success of the Birkhoff relaxation is state-of-the-art.

5.8 Benign landscape for Burer-Monteiro factorizations of MaxCut-type semidefinite programs

Participants: Faniriana Rakoto Endor, Irène Waldspurger.

We study the Burer-Monteiro factorization, which is a well-known heuristic to reduce the computational cost of solving semidefinite programs (SDP) in the case where the solution is a priori known to be low rank. This factorization reduces the dimension of the SDP at the cost of its convexity, therefore possibly introducing spurious second-order critical points which could trap the optimization algorithm and prevent it from finding the desired minimizer.

In [21], for MaxCut-type SDP, we give a sharp condition on the conditioning of the associated Laplacian matrix under which any second-order critical point of the non-convex problem is a global minimizer. By applying our theorem, we improve on recent results about the correctness of the Burer-Monteiro factorization on $\mathbb{Z}^2$ -synchronization problems.

5.9 Nonnegative cross-curvature in infinite dimensions: synthetic definition and spaces of measures

Participants: Flavien Léger , Gabriele Todeschi, François-Xavier Vialard.

In [17] we develop a synthetic notion of nonnegative cross-curvature. Nonnegative cross-curvature (NNCC) is a geometric property of a cost function defined on a product space that originates in optimal transportation and the Ma-Trudinger-Wang theory. Motivated by applications in optimization, gradient flows and mechanism design, we propose a variational formulation of nonnegative cross-curvature on c-convex domains applicable to infinite dimensions and nonsmooth settings. The resulting class of NNCC spaces is closed under Gromov-Hausdorff convergence and for this class, we extend many properties of classical nonnegative cross-curvature: stability under generalized Riemannian submersions, characterization in terms of the convexity of certain sets of c-concave functions, and in the metric case, it is a subclass of positively curved spaces in the sense of Alexandrov. One of our main results is that Wasserstein spaces of probability measures inherit the NNCC property from their base space. Additional examples of NNCC costs include the Bures-Wasserstein and Fisher-Rao squared distances, the Hellinger-Kantorovich squared distance (in some cases), the relative entropy on probability measures, and the 2-Gromov-Wasserstein squared distance on metric measure spaces.

5.10 A Cahn–Hilliard–Willmore phase field model for non-oriented interfaces

Participants: Antonin Chambolle, Elie Bretin, Simon Masnou.

In [13], we investigate a new phase field model for representing non-oriented interfaces, approximating their area and simulating their area-minimizing flow. Our contribution is related to the approach proposed in arXiv:2105.09627 that involves ad hoc neural networks. We show here that, instead of neural networks, similar results can be obtained using a more standard variational approach that combines a Cahn-Hilliard-type functional involving an appropriate non-smooth potential and a Willmore-type stabilization energy. We show some properties of this phase field model in dimension 1 and, for radially symmetric functions, in arbitrary dimension. We propose a simple numerical scheme to approximate its L2-gradient flow. We illustrate numerically that the new flow approximates fairly well the mean curvature flow of codimension 1 or 2 interfaces in dimensions 2 and 3.

5.11 Convergence of discrete approximations of the mean curvature flow

Participants: Antonin Chambolle, , Daniele De Gennaro, , Massimiliano Morini.

In [35], we have pushed further an idea developed last year for designing convergent time and space-discrete approximations of the crystalline mean curvature flow, in any dimension. In the new paper, we define convolution-redistancing schemes *in the fully discrete setting* which we show to converge to the curvature flow. A non-linear generalization allows to explain why some machine-learning based approximations of the flow, developed by Bretin and collaborators in Lyon, actually approximate such flows, since we show that any sufficiently symmetric discrete convolution kernel can be used to approximate the curvature flow, provided the time and space discretization parameters are appropriately tuned.

5.12 Variational problems in linearized elasticity with cracks

Participants: Antonin Chambolle, Vito Crismale.

Our paper on a very general and complete compactness result for “$G(S)BD$” functions (which arise in variational models for fracture growth) was finally published this year [14]. Then, this year in [34], we gave a new characterization of these functions, which is much simpler than the original definition of Dal Maso (2011). We also review the theoretical results shown during the past 10 years on the existence and regularity for the Griffith problem, a free discontinuity problem arising in this theory, and describe some proofs with a very simple presentations, in the proceedings of the ICIAM 2023 conference [23].

5.13 Analysis of Flow Matching problems and their relationship to optimal transport

Participants: Antonin Chambolle, , Johannes Hertrich, , Julie Delon.

In [24] we investigate the connections between rectified flows, flow matching, and optimal transport. Flow matching is a recent approach to learning generative models by estimating velocity fields that guide transformations from a source to a target distribution. Rectified flow matching aims to straighten the learned transport paths, yielding more direct flows between distributions. Our first contribution is a set of invariance properties of rectified flows and explicit velocity fields. In addition, we also provide explicit constructions and analysis in the Gaussian (not necessarily independent) and Gaussian mixture settings and study the relation to optimal transport. Our second contribution addresses recent claims suggesting that rectified flows, when constrained such that the learned velocity field is a gradient, can yield (asymptotically) solutions to optimal transport problems. We study the existence of solutions for this problem and demonstrate that they only relate to optimal transport under assumptions that are significantly stronger than those previously acknowledged. In particular, we present several counter-examples that invalidate earlier equivalence results in the literature, and we argue that enforcing a gradient constraint on rectified flows is, in general, not a reliable method for computing optimal transport maps.

5.14 One-dimensional approximation of measures in Wasserstein distance

Participants: Antonin Chambolle,, Joao-Miguel Machado,, Vincent Duval.

In [15], we have proposed a variational approach to approximate measures with measures uniformly distributed over a 1-dimensional set. The problem consists in minimizing a Wasserstein distance as a data term with a regularization given by the length of the support. As it is challenging to prove existence of solutions to this problem, we propose a relaxed formulation, which always admits a solution. In the sequel we show that, under some assumption on the original measure, any solution to the relaxed problem is solution to the original one. Finally we prove that, whenever the original measure has a density in $L^{d/(d-1)}(\mathbb{R}^d)$, any optimal solution is supported by an Ahlfors regular set.

5.15 Phase-field approximation for 1-dimensional shape optimization problems

Participants: Joao-Miguel Machado.

In [18], we propose an unified framework for the phase field approximation of 1-dimensional shape optimization problems with connectedness constraints in any dimension. In particular, we focus on the average distance minimizers problem and the Wasserstein-H^1 problem recently introduced by Duval, Chambolle and Machado. The scheme relies on the p-Ambrosio-Tortorelli energy and the diffuse connectedness functional proposed by

Dondl et al. that penalizes how disconnected the level sets of phase fields are. We argue that choosing p>d, not only the optimal profiles coming from the Ambrosio Tortorelli term present sharper transitions, but it also allows us to control the level sets of phase fields, enabling the analysis of the connectedness functional. This leads to general Γ-liminf and limsup inequalities that are easily adaptable to prove Γ-convergence results for the average distance and Wasserstein-H^1 problems.

5.16 Memorization vs. Generalization in diffusion models with the U-Net architecture

Participants: Sylvain Topeza,, Antonin Chambolle,, Vincent Duval.

During the internship of Sylvain Topeza [41], we have reproduced and extended the experiments of Kadkhodaie, Guth, Simoncelli and Mallat [92] on generalization in diffusion models. The original study reported that two U-Net denoisers trained independently on non-overlapping subsets of CelebA produce nearly identical samples when seeded with the same noise, suggesting a form of deterministic convergence. We confirm this qualitative behavior under constrained settings (40×40 resolution) and develop a fully reproducible, memory-efficient preprocessing pipeline. Beyond replication, we explore its stability with respect to dataset correlation by introducing attribute-controlled (eyeglasses/no-eyeglasses, male/female) and identity-disjoint splits, and by assessing sample originality with a perceptual metric (LPIPS) instead of pixel correlation. Our findings indicate that convergence between independently trained denoisers remains strong when training subsets are correlated, but weakens as semantic heterogeneity increases. These observations suggest that both architectural inductive biases and dataset redundancy contribute to the stability of the phenomenon originally reported by Mallat and collaborators.

5.17 A variational method for curve extraction

Participants: Majid Arthaud,, Antonin Chambolle,, Vincent Duval.

In [22], we have addressed the problem of extracting one-dimensional objects (curves) from images, which is of interest, e.g. when segmenting the blood vessels in retina images. The originality of our approach is that it is variational and basd on the discretization of an energy and Smirnov's decomposition theorem for vector fields. It is used to design a bi-level minimization approach to automatically extract curves and 1D structures from an image, which is mostly unsupervised.

In [30], we have extended then the method to curvature-dependent energies, using a now classical lifting of the curves in the space of positions and orientations equipped with an appropriate sub-Riemanian or Finslerian metric.

5.18 A synthetic approach to comparison principles for variational problems, with applications to optimal transport

Participants: Flavien Léger, , Maxime Sylvestre.

In [9], we develop a synthetic, variational framework for deriving comparison principles in infinite-dimensional Banach spaces. Unlike traditional approaches that rely on the regularity of minimizers and Euler–Lagrange equations, our method exploits the order-theoretic structure of the energy. Central to our analysis is the notion of submodularity and its convex dual, substitutability, which we extend here to the infinite-dimensional setting. We prove a duality theorem establishing that a convex functional is submodular if and only if its conjugate is substitutable. We apply these results to problems in optimal transport, and derive comparison principles for Kantorovich potentials in standard, entropic, and unbalanced settings without requiring regularity assumptions on the cost or domain. Finally, we prove that general transport costs are substitutable, yielding comparison principles for JKO schemes driven by internal energies.

6 Partnerships and cooperations

6.1 International initiatives

6.1.1 Inria associate team not involved in an IIL or an international program

KarMA

Participants: Paul Pegon, Guillaume Carlier, Louis Tocquec.

Title: KarMa

Partner Institution(s):
- Inria Saclay, ParMA team
- University of Alberta, Kantorovich Initiative

Date/Duration: 2023-2025

Additionnal info/keywords: The KarMA Associate Team (2023–2025) is jointly led by Inria Saclay's ParMA team and the Kantorovich Initiative at University of Alberta. Although the project is not a Mokaplan initiative, three Mokaplan members –Paul Pegon, Guillaume Carlier and Louis Tocquec– are involved to contribute to the axes on entropic, multi-marginal and weak OT. A first visit by L. Nenna, Paul Pegon, and their PhD student L. Tocquec took place in June 2025, during which they began developing a selection principle for entropic OT based on a next-order Γ-convergence approach.

6.1.2 Participation in other International Programs

OPTIMA (ECOS-SUD)

Participants: Paul Pegon, Louis Tocquec.

Title: OPtimal Transport Innovations for Multidisciplinary Applications

Partner Institution(s):
- Université Paris-Dauphine, Paris (France)
- Pontificia Universidad Católica de Chile, Santiago (Chile)

Date/Duration: 2026-2028

Additionnal info/keywords: This ECOS-SUD project, led by Paul Pegon and Mircea Petrache and submitted in 2025, was recently accepted (January 2026). The project involves seven senior researchers and two young researchers, based in Santiago (Chile), Concepción (Chile), Paris (France), and Toulon (France). It focuses on branched optimal transport, urban planning, multi-marginal optimal transport, measure quantization, and optimal transport methods for diffusion models in deep learning.

6.2 International research visitors

6.2.1 Visits of international scientists

Other international visits to the team

Luigi De Pascale

Status Researcher

Institution of origin: Università di Firenze

Country: Italy

Dates: 2 weeks in May and 2 weeks in September

Context of the visit: Collaboration with Paul Pegon and Guillaume Carlier

Mobility program/type of mobility: "Invited professor" program of CEREMADE (Dauphine)

6.2.2 Visits to international teams

Research stays abroad

Jean-David Benamou

Visited institution: Imperial College London.

Country: UK

Dates: February - March - April.

Context of the visit: ICL-CRNS fellowship

Mobility program/type of mobility: A series of lectures on Dynamic Optimal Transport was given. A research programm was develloped and finalized on the numerical resolution of shallow water demi-geostrophic models. This was a collaboration with Prof. Colin Cotter, Jacob Francis (PhD) and Hugo Malamut (PhD).

Flavien Léger

Visited institution: New York University Shanghai.

Country: China

Dates: April – September.

Mobility program/type of mobility: research stay.

Additional informations: The fields of optimal transport and mean field games have similar mathematical formulations, however their respective communities have limited interaction. The goal of this collaboration is to adapt the so-called forward-backward methods in optimal transport to mean field games. These methods make it possible to prove the existence of solutions and also provide numerical resolution methods. In particular, these numerical methods are efficient because they are time-dynamic, unlike the methods classically used for mean field games which rely on both space and time domains.

Paul Pegon

Visited institution: University of Alberta

Country: Canada

Dates: June (2 weeks)

Mobility program/type of mobility: research stay.

Visited institution: Università di Firenze

Country: Italy

Dates: September (3 weeks)

Mobility program/type of mobility: research stay.

6.3 National initiatives

PR[AI]RIE-PSAI (ANR-23-IACL-0008) (2024-2029) France 2030 funded project (ANR-23-IACL-0008), chair of Antonin Chambolle (2 PhD and 2 post-docs during the 5 years, managed by the CNRS at CEREMADE/Paris-Dauphine)

PDE AI (2023-2027) Antonin Chambolle is the main coordinator of the PDE-AI project, funded by the PEPR IA (France 2030, ANR) and gathering 10 groups throughout France working on PDEs and nonlinear analysis for artificial intelligence.

ANR GOTA (2023-2027) is a JCJC grant (253k€) carried by Luca Nenna (PI), Paul Pegon and Maxime Laborde, dealing with some generalizations and applications of Optimal Transport theory with a particular focus on three main topics: multi-marginal optimal transport, urban planning and multi-population models, and multi-marginal entropic optimal transport.

ANR ESSTOS (2025-2029) is a JCJC grant (215k€) carried by A . Monteil, managed by Université Paris Est-Créteil, on the relation between elliptic systems and geometric objects such as minimal surfaces or minimal graphs. Paul Pegon is one of the five members, involved in one of the two axes, which focuses on blending branched OT with the different formulations of classical OT and congested OT.

7 Dissemination

7.1 Promoting scientific activities

7.1.1 Scientific events: organisation

Member of the organizing committees

- Flavien Léger co-organized with Mathieu Laurière (NYU Shanghai) the "Workshop on Mean Field Games, Optimal Transport and Machine Learning" at NYU Paris, June 23-26 2025.
- Antonin Chambolle has co-organised (since 2017) the "PGMO days" (Optimization conference in EDF Saclay, 2 days each year in November.)
- Antonin Chambolle co-organised the workshop MFO#2506 "Mathematical Imaging and Surface Processing" at Oberwolfach, 2-7 Feb., 2025 with about 40 participants.
- Paul Pegon has co-organized mini-symposia at PGMO Days every year since 2023.
- Vincent Duval co-organized the JuliaCon Local Paris 2025 conference (170 participants)
- Vincent Duval co-organizes the Imaging in Paris monthly seminar at IHP.
- Guillaume Carlier and Antonin Chambolle are co-organizers of the (monthly) *Séminaire Parisien d'Optimisation*, (SPO).
- Guillaume Carlier is a member of the scientific commitee of the PGMO program.

7.1.2 Scientific events: selection

Reviewer Antonin Chambolle and Vincent Duval have reviewed contributions to the SSVM 2025 conference. Vincent Duval has reviewed contributions to the GRETSI 2025 conference.

7.1.3 Journal

Member of the editorial boards

- Antonin Chambolle is a co-editor of “Interfaces and Free Boundaries” (EMS Press). He is also a member of the editorial board of the Journal of the European Math. Society (JEMS, EMS Press), Mathematical Modeling and Numerical Analysis (ESAIM-M2AN), Control, Optimization and Calculus of Variations (ESAIM-COCV), IMA Journal of Numerical Analysis (IMAJNA), Applied Math. Optim (AMO), Inverse problems and imaging (IPI), Journal of Math. Imaging and Vision (JMIV), and a special issue of JOTA (Journal of Optimization Theory and Applications, Springer, special issue on optimization for energy).
- Vincent Duval is an associate editor of Journal of Mathematical Imaging and Vision (JMIV).
- Guillaume Carlier is an associate editor of Applied Mathematics and Optimization, Journal of Mathematical Analysis and Applications, Journal of Dynamics and Games and was associate editor of the Journal de l’Ecole Polytechnique until 2023. Since January 2026, Guillaume Carlier is in the editorial board of SIAM journal on Optimization.

Reviewer - reviewing activities

- Antonin Chambolle has reviewed papers for Archive for Rational Mechanics and Analysis (ARMA), Proc. AMS, Journal of Scientific Computing Journal of Mathematical Imaging and Vision (JMIV), Applied. Math. Optim, SIAM J. Imaging Science, Journal of Optimization Theory and Applications (JOTA), Numerical Algorithms, IMA Journal of Numerical Analysis, IEEE Transactions on Image Processing, IEEE Signal Processing Letters, Acta Applicandae Mathematicae.
- Flavien Léger has reviewed articles for *Annals of Probability*.
- Irène Waldspurger has reviewed articles for *Information and inference, a journal of the IMA*, *Advances in mathematics* and *Optimization letters*.
- Vincent Duval has reviewed articles for the Journal of *Foundations of Computational Mathematics (FoCM)*, *Information and inference, a journal of the IMA*, *SIAM Journal on Mathematics of Data Science (SIMODS)*, *SIAM Journal on Imaging Sciences (SIIMS)*, *Computational Optimization and Applications (COAP)*. He has also written one review for *Mathematical Reviews*.

7.1.4 Invited talks

- Jean-David Benamou gave a talk at Séminaire ÉDP, Modélisation et Calcul Scientifique de Lyon-Saint Etienne (jannuary), three lectures at Imperial College London (ferbruary and march) as part of its ICL-CNRS Fellowships, a talk at “Inference on the equilibrium flow problem" (IASC Cargese,avril) , a talk at “Mean Field Games, Optimal Transport, and Machine Learning," (june, NYU Paris), a talk at the conference for the 60th birthday of Bruno Depres (june, Paris).
- Flavien Léger was an invited speaker at the Physics of AI algorithms workshop in Les Houches (2025) Wasserstein Gradient Flows in Math and Machine Learning workshop at BIRS (Banff, Canada, 2025).
- Irène Waldspurger gave seminar talks at laboratoire Paul Painlevé (Université de Lille), Université Catholique de Louvain, laboratoire Jacques-Louis Lions (Sorbonne Université) and ETH Zurich. She gave a talk at the SMF congress in Dijon, and a colloquium at CMAP (école Polytechnique).
- Paul Pegon gave siminar talks at University of Alberta (Edmonton, Canada), Università di Firenze and Université de Brest.
- Vincent Duval gave an invited plenary talk at the 4th Alps Adriatic Inverse Problems Workshop (AAIP) 2025. He also gave a talk at the Journée MoKarma (Paris).
- Guillaume Carlier gave talks at Séminaire d’Analyse Fonctionnelle de Jussieu, Séminaire au Moroccan Center for Game Theory (Rabat, Maroc), EYAWKADANAJKOS workshop in Lyon, colloquium du laboratoire Dieudonné, Nice, workshop Inference on the equilibrium flow problem, Cargese, Corse, 3rd Vienna Workshop on Computational Optimization, Workshop on Mean Field Games, Optimal Transport, and Machine Learning, NYU Paris, Mathphys analysis seminar, ISTA, Vienne, workshop en

l'honneur de Christian Léonard, Paris, 100 ème du séminaire SPOT (Toulouse), workshop *Geometry, duality and convexity in new OT problems* (Orsay), séminaire de l'EPFL.

7.1.5 Scientific expertise

Vincent Duval served as an expert for the National Science Center Poland for a grant proposal. He also reviewed a Programme Inria Quadrant (PIQ) proposal.

7.1.6 Research administration

- Antonin Chambolle is a member of the scientific council of Université Paris Dauphine-PSL. He is also a member of the scientfic council and of the board of the PGMO *Programme Gaspard Monge pour l'Optimisation et la Recherche Opérationnelle*.
- Irène Waldspurger was a member of the selection committee for a maîtresse de conférences position in Dijon.
- Paul Pegon is an elected member of the Board of Directors of the Université Paris Dauphine-PSL.
- Paul Pegon was a member of the selection committee for a MCF position in Paris 1.
- Vincent Duval is *délégué scientifique adjoint (DSA) for the Inria Paris Center.*
- Vincent Duval was a member of the Inria CRCN/ISFP selection committee for the Université Côte d'Azur (UCA) center at Sophia-Antipolis.

7.2 Teaching - Supervision - Juries - Educational and pedagogical outreach

7.2.1 Teaching

- Master : Vincent Duval, Optimisation, 20 h équivalent TD, M1 level, Université Paris Dauphine-PSL, FR
- Master : Irène Waldspurger, Non-convex inverse problems, 27 h équivalent TD, M2 MASH and M2 IASD, PSL.
- Licence : Irène Waldspurger, Analyse de données, 13 h équivalent TD, L3 IM2D, Université Paris Dauphine-PSL
- Master : Irène Waldspurger, Modèles linéaires, 24 h équivalent TD, M1, Université Paris Dauphine-PSL, FR
- Master : Paul Pegon, Advanced Convex Analysis, 56h équivalent TD, M1 level (lectures and exercise sessions), Université Paris Dauphine-PSL.
- Master : Guillaume Carlier, Variational problems and optimal transport, M2 Math and Masef, Université Paris Dauphine-PSL.
- L1 : Guillaume Carlier, raisonnement and analysis 1, Université Paris Dauphine-PSL.

7.2.2 Supervision

- Jean-David Benamou supervised the PhD of Hugo Malamut (Dauphine PSL U.).
- Antonin Chambolle and Irène Waldspurger supervise the PhD of Faniriana Rakoto Endor (Dauphine PSL U.).
- Antonin Chambolle supervises the PhD of Gaetano Agazzotti (Dauphine PSL), together with Clément Royer (Lamsade)

- Antonin Chambolle supervises the PhD of Hugo Koubbi (Dauphine PSL) together with Borjan Gehskovski (INRIA Paris)
- Antonin Chambolle and Vincent Duval supervise the PhD of Saja Salama (Dauphine PSL U.).
- Antonin Chambolle and Vincent Duval have supervised the L3 internship of Sylvain Topeza (2 months).
- Paul Pegon is co-encadrant of Louis Tocquec's PhD, and co-supervises Alessandro Cosenza's Postdoc since December.
- Guillaume Carlier supervised the PhD of Maxime Sylvestre, co-supervised the PhD of Hugo Malamut and is co-supervising with Q. Mérigot the PhD of Julien Guérin.

7.2.3 Juries

- Jean-David Benamou was a member of the PhD jury of Siwan Boufadene (Gustave Eiffel U.) and Erwan Stampli (U. Paris Sud Orsay).
- Irène Waldspurger was a member of the PhD jury of Iskander Legharaba (Dauphine PSL U.) and Quentin Rebjock (EPFL, Lausanne).
- Paul Pegon was a jury member of the PhD of Alessandro Cosenza (Université Paris-Cité).
- Antonin Chambolle was a member of the jury of Maxime Sylvestre (Mokaplan), Benjamin Dubois-Taine (A. D'Aspremont), and reported for the thesis of Iyad Walwil (Telecom Paris, O. Fercoq).
- Vincent Duval reported for the PhD theses of Nathanaël Munier (Université de Toulouse) and Phuoc Truong Huynh (Alpen-Adria-Universität Klagenfurt, Autriche).
- Guillaume Carlier was a member of the PhD jury of Stefan Schrott (Vienne), Jianyu Ma, Thibault Moquet and coordinated the Habilitation thesis of Idriss Mazari.

7.2.4 Educational and pedagogical outreach

Irène Waldspurger, Antonin Chambolle, Christian Léonard and Vincent Duval have been involved in welcoming high school pupils ("classe de seconde") for their mandatory internships. They have proposed introductions to several topics of the team, general scientific topics (from image denoising to generative modeling, AI, scientific culture, climate change, disinformation and merchants of doubts...), and a few mathematical games which were highly appreciated.

7.3 Popularization

7.3.1 Participation in Live events

- Irène Waldspurger participated in the "après-midi des lycéennes", in Dauphine (an event aiming at encouraging female high school students to consider scientific studies) and presented scientific activities to 3ème interns in Dauphine, 2nde interns at Inria and middle school students from the Rallye mathématique des collèges de Bourgogne.

8 Scientific production

8.1 Major publications

[1] P.-C. Aubin-Frankowski, A. Korba and F. Léger. 'Mirror Descent with Relative Smoothness in Measure Spaces, with application to Sinkhorn and EM'. In: NeurIPS 2022 - Thirty-sixth Conference on Neural Information Processing Systems. New Orleans, United States, 2022. URL: https://hal.science/hal-03811583.

[2] J.-D. Benamou, G. Carlier, M. Cuturi, L. Nenna and G. Peyré. 'Iterative Bregman Projections for Regularized Transportation Problems'. In: *SIAM Journal on Scientific Computing* 2.37 (2015), A1111–A1138. DOI: 10.1137/141000439. URL: https://hal.science/hal-01096124.

[3] J.-D. Benamou, T. Gallouët and F.-X. Vialard. 'Second order models for optimal transport and cubic splines on the Wasserstein space'. In: *Foundations of Computational Mathematics* (Oct. 2019). DOI: 10.1007/s10208-019-09425-z. URL: https://hal.science/hal-01682107.

[4] C. Boyer, A. Chambolle, Y. de Castro, V. Duval, F. de Gournay and P. Weiss. 'On Representer Theorems and Convex Regularization'. In: *SIAM Journal on Optimization* 29.2 (9th May 2019), pp. 1260–1281. DOI: 10.1137/18M1200750. URL: https://hal.archives-ouvertes.fr/hal-01823135.

[5] G. Carlier, V. Duval, G. Peyré and B. Schmitzer. 'Convergence of Entropic Schemes for Optimal Transport and Gradient Flows'. In: *SIAM Journal on Mathematical Analysis* 49.2 (18th Apr. 2017). DOI: 10.1137/15M1050264. URL: https://hal.science/hal-01246086.

[6] G. Carlier, P. Pegon and L. Tamanini. *Convergence rate of general entropic optimal transport costs*. 7th June 2022. URL: https://hal.archives-ouvertes.fr/hal-03689945.

[7] A. Chambolle, V. Duval and J. M. Machado. 'One-dimensional approximation of measures in Wasserstein distance'. In: *Journal de l'École polytechnique — Mathématiques* 12 (1st Jan. 2025), pp. 101–145. DOI: 10.5802/jep.286. URL: https://hal.science/hal-04082932.

[8] F. Léger and P.-C. Aubin-Frankowski. *Gradient descent with a general cost*. 14th Dec. 2023. URL: https://hal.science/hal-04344054.

[9] F. Léger and M. Sylvestre. *A synthetic approach to comparison principles for variational problems, with applications to optimal transport*. 6th Feb. 2026. URL: https://hal.science/hal-05497486 (cit. on p. 15).

[10] F. Léger and F.-X. Vialard. *A geometric Laplace method*. 22nd Dec. 2022. URL: https://hal.science/hal-03911149.

[11] I. Waldspurger. 'Phase retrieval with random Gaussian sensing vectors by alternating projections'. In: *IEEE Transactions on Information Theory* 64.5 (2018), pp. 3301–3312. URL: https://hal.science/hal-01645081.

[12] I. Waldspurger and A. Waters. 'Rank optimality for the Burer-Monteiro factorization'. In: *SIAM Journal on Optimization* 30.3 (2020), pp. 2577–2602. DOI: 10.1137/19M1255318. URL: https://hal.science/hal-01958814.

8.2 Publications of the year

International journals

[13] É. Bretin, A. Chambolle and S. Masnou. 'A Cahn–Hilliard–Willmore phase field model for non-oriented interfaces'. In: *Interfaces and Free Boundaries : Mathematical Analysis, Computation and Applications* (2026). URL: https://hal.science/hal-04884780. In press (cit. on p. 13).

[14] A. Chambolle and V. Crismale. 'A general compactness theorem in <i>G(S)BD</i>'. In: *Indiana University Mathematics Journal* 74.1 (2025), pp. 233–249. DOI: 10.48550/arXiv.2210.04355. URL: https://hal.science/hal-03807668 (cit. on p. 14).

[15] A. Chambolle, V. Duval and J. M. Machado. 'One-dimensional approximation of measures in Wasserstein distance'. In: *Journal de l'École polytechnique — Mathématiques* 12 (1st Jan. 2025), pp. 101–145. DOI: 10.5802/jep.286. URL: https://hal.science/hal-04082932 (cit. on p. 14).

[16] A. Chambolle, I. Mazari-Fouquer and Y. Privat. 'Stability of optimal shapes and convergence of thresholding algorithms in linear and spectral optimal control problems'. In: *Mathematische Annalen* 392.3 (2025), pp. 4181–4219. DOI: 10.1007/s00208-025-03182-x. URL: https://hal.science/hal-04140177.

[17] F. Léger, G. Todeschi and F.-X. Vialard. 'Nonnegative Cross-Curvature in Infinite Dimensions: Synthetic Definition and Spaces of Measures'. In: *Geometric And Functional Analysis* 35.6 (11th Nov. 2025), pp. 1638–1711. DOI: 10.1007/s00039-025-00725-x. URL: https://hal.science/hal-05446552 (cit. on p. 13).

[18] J. M. Machado. 'Phase-field approximation for 1-dimensional shape optimization problems'. In: *SIAM Journal on Mathematical Analysis* 57.4 (1st July 2025), pp. 3488–3523. DOI: 10.1137/24M1672845. URL: https://hal.science/hal-04620380 (cit. on p. 14).

[19] H. Malamut and M. Sylvestre. 'Convergence Rates of the Regularized Optimal Transport : Disentangling Suboptimality and Entropy'. In: *SIAM Journal on Mathematical Analysis* 57.3 (2025), pp. 2533–2558. DOI: 10.1137/23M1591554. URL: https://hal.science/hal-04114127 (cit. on p. 10).

[20] P. Pegon and M. Petrache. 'Optimal quantization with branched optimal transport distances'. In: *SIAM Journal on Mathematical Analysis* (2025). URL: https://hal.science/hal-04208973. In press (cit. on p. 11).

[21] F. Rakoto Endor and I. Waldspurger. 'Benign landscape for Burer-Monteiro factorizations of MaxCut-type semidefinite programs'. In: *SIAM Journal on Optimization* (2026). URL: https://hal.science/hal-04797879. In press (cit. on p. 12).

International peer-reviewed conferences

[22] M. Arthaud, A. Chambolle and V. Duval. 'A variational method for curve extraction'. In: *SSVM 2025 (à paraitre)*. SSVM 2025 - 10th Scale-Space and Variational Methods in Computer Vision. Totnes, Devon, United Kingdom, 18th May 2025. URL: https://hal.science/hal-04980911 (cit. on p. 15).

[23] A. Chambolle. 'Existence of minimizers in the variational approach to brittle fracture'. In: *Recent Developments in Industrial and Applied Mathematics - ICIAM 2023 Springer Series*. ICIAM 2023 (International Conference on Industrial and Applied Mathematics). Vol. 1. ICIAM2023 Springer Series. Tokyo, Japan: Springer Nature Singapore, 12th Jan. 2026, pp. 69–91. DOI: 10.1007/978-981-95-1446-5_4. URL: https://hal.science/hal-04279405 (cit. on p. 14).

[24] J. Hertrich, A. Chambolle and J. Delon. 'On the Relation between Rectified Flows and Optimal Transport'. In: *Advances in Neural Information Processing Systems*. NeurIPS 2025 - 39th Annual Conference on Neural Information Processing Systems. San Diego (CA), United States, 2025. URL: https://hal.science/hal-05088241 (cit. on p. 14).

[25] S. M. Varma, I. Waldspurger and L. Massoulié. 'Graph Alignment via Birkhoff Relaxation'. In: *NeurIPS 2025*. NeurIPS 2025 - Thirty-Ninth Annual Conference on Neural Information Processing Systems. San Diego (CA), United States, 2nd Dec. 2025. URL: https://inria.hal.science/hal-05387695 (cit. on p. 12).

Conferences without proceedings

[26] D. Ratsimandresy, R. Tovey, V. Duval, M. Carioni, L. Muresan, N. Barry and J. Boulanger. 'New Approach to Particle Tracking as a Dynamic Inverse Problem'. In: Microscience Microscopy Congress 2025. Manchester (UK), United Kingdom, 1st July 2025. URL: https://hal.science/hal-05387253.

Doctoral dissertations and habilitation theses

[27] H. Malamut. 'Entropic optimal transport and semigeostrophic equations'. Université Paris sciences et lettres, 21st Nov. 2025. URL: https://theses.hal.science/tel-05406717.

[28] M. Sylvestre. 'Entropic regularization in optimal transport and beyond'. Université Paris sciences et lettres, 12th Dec. 2025. URL: https://theses.hal.science/tel-05474054.

Reports & preprints

[29] G. S. Alberti, R. Petit, C. Poon and I. Waldspurger. *On the non-convexity issue in the radial Calderón problem*. 4th July 2025. URL: https://hal.science/hal-05144000.

[30] M. Arthaud, A. Chambolle and V. Duval. *A variational method for curve extraction with curvature-dependent energies*. 27th Oct. 2025. URL: https://hal.science/hal-05386995 (cit. on p. 15).

[31] J.-D. Benamou, C. J. Cotter, J. J. M. Francis and H. Malamut. *Entropic approximations of the semigeostrophic shallow water equations*. 15th July 2025. URL: https://inria.hal.science/hal-05161741 (cit. on p. 10).

[32] G. Carlier, A. Figalli, Q. Mérigot and Y. Wang. *Sharp comparisons between sliced and standard 1-Wasserstein distances*. 18th Oct. 2025. URL: https://hal.science/hal-05501134.

[33] G. Carlier, H. Malamut and M. Sylvestre. *Weak optimal transport with moment constraints: constraint qualification, dual attainment and entropic regularization*. 3rd Dec. 2025. URL: https://hal.science/hal-05395845 (cit. on p. 12).

[34] A. Chambolle and V. Crismale. *A characterization of Generalized functions of Bounded Deformation*. 15th Feb. 2025. URL: https://hal.science/hal-04946846 (cit. on p. 14).

[35] A. Chambolle, D. de Gennaro and M. Morini. *Elementary discrete diffusion/redistancing schemes for the mean curvature flow*. 5th Dec. 2025. URL: https://hal.science/hal-05400630 (cit. on p. 13).

[36] N. Gozlan and M. Sylvestre. *Global Regularity Estimates for Optimal Transport via Entropic Regularisation*. 17th July 2025. URL: https://hal.science/hal-04894959.

[37] F. Léger and M. Sylvestre. *A synthetic approach to comparison principles for variational problems, with applications to optimal transport*. 6th Feb. 2026. URL: https://hal.science/hal-05497486.

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