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2025 *Activity Report*

RESEARCH CENTRE: **Inria Centre at the University of Lille**

IN PARTNERSHIP WITH: **Université de Lille, CNRS**

Project-Team

PARADYSE

PARticles And DYnamical SystEms

In collaboration with Laboratoire Paul Painlevé (LPP)

Project-Team PARADYSE

Creation of the Project-Team: 2020 March 01

Each year, Inria research teams publish an Activity Report presenting their work and results over the reporting period. These reports follow a common structure, with some optional sections depending on the specific team. They typically begin by outlining the overall objectives and research programme, including the main research themes, goals, and methodological approaches. They also describe the application domains targeted by the team, highlighting the scientific or societal contexts in which their work is situated. The reports then present the highlights of the year, covering major scientific achievements, software developments, or teaching contributions. When relevant, they include sections on software, platforms, and open data, detailing the tools developed and how they are shared. A substantial part is dedicated to new results, where scientific contributions are described in detail, often with subsections specifying participants and associated keywords. Finally, the Activity Report addresses funding, contracts, partnerships, and collaborations at various levels, from industrial agreements to international cooperations. It also covers dissemination and teaching activities, such as participation in scientific events, outreach, and supervision. The document concludes with a presentation of scientific production, including major publications and those produced during the year.

Keywords

Computer sciences and digital sciences

A6.1.1. – Continuous Modeling (PDE, ODE)

A6.1.2. – Stochastic Modeling

A6.1.4. – Multiscale modeling

A6.2.1. – Numerical analysis of PDE and ODE

A6.2.3. – Probabilistic methods

A6.5. – Mathematical modeling for physical sciences

A8.6. – Information theory

A8.6.1. – quantum information theory

A8.13.5. – Photonic quantum computing

Other research topics and application domains

B3.6. – Ecology

B3.6.1. – Biodiversity

B5.3. – Nanotechnology

B5.5. – Materials

B5.11. – Quantum systems

B6.2.4. – Optical networks

Contents

1 Team members, visitors, external collaborators

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- Guillaume Ferriere [INRIA, Researcher]

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- Vianney Combet [UNIV LILLE, Associate Professor Delegation, until Aug 2025]
- Stephan De Bièvre [UNIV LILLE, Professor, HDR]
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2 Overall objectives

The PARADYSE team gathers mathematicians from different communities with the same motivation: to provide a better understanding of dynamical phenomena involving particles. These phenomena are described by fundamental models arising from several fields of physics. We shall focus on model derivation, study of stationary states and asymptotic behaviors, as well as links between different levels of description (from microscopic to macroscopic) and numerical methods to simulate such models. Applications include non-linear optics, thermodynamics and ferromagnetism. Research in this direction has a long history, that we shall only partially describe in the sequel. We are confident that the fact that we come from different mathematical communities (PDE theory, mathematical physics, probability theory and numerical analysis), as well as the fact that we have strong and effective collaborations with physicists will bring new and efficient scientific approaches to the problems we plan to tackle and will make our team strong and unique in the scientific landscape. Our goal is to obtain original and important results on a restricted yet ambitious set of problems that we develop in this document.

3 Research program

3.1 Time asymptotics: Stationary states, solitons, and stability issues

The team investigates the existence of *solitons* and their link with the global dynamical behavior for non-local problems such as the Gross–Pitaevskii (GP) equation which arises in models of dipolar gases. These models, in general, also introduce non-zero boundary conditions which constitute an additional theoretical and numerical challenge. Numerous results are proved for local problems, and numerical simulations allow to verify and illustrate them, as well as making a link with physics. However, most fundamental questions are still open at the moment for non-local problems.

The non-linear Schrödinger (NLS) equation finds applications in numerous fields of physics. We concentrate, in a continued collaboration with our colleagues from the physics department (PhLAM) at Université de Lille (U-Lille) in the framework of the Laboratoire d'Excellence CEMPI, on its applications in non-linear optics and cold atom physics. Issues of orbital stability and modulational instability are central here (see Section 4.1 below).

Another typical example of problem that the team wishes to address concerns the Landau–Lifshitz (LL) equation, which describes the dynamics of the spin in ferromagnetic materials. This equation is a fundamental model in the magnetic recording industry [46] and solitons in magnetic media are of particular interest as a mechanism for data storage or information transfer [47]. It is a quasilinear PDE involving a function that takes values on the unit sphere $\mathbb{S}^2$ of $\mathbb{R}^3$. Using the stereographic projection, it can be seen as a quasilinear Schrödinger equation and the questions about the solitons, their dynamics and potential blow-up of solutions evoked above are also relevant in this context. This equation is less understood than the NLS equation: even the Cauchy theory is not completely understood [38, 45]. In particular, the geometry of the target sphere imposes that the solution has a norm equal to one everywhere, so in particular the boundary conditions cannot be zero, and, even in dimension one, there are kink-type solitons having different limits at $\pm\infty$.

3.2 Derivation of macroscopic laws from microscopic dynamics

The team investigates, from a microscopic viewpoint, the dynamical mechanism at play in the phenomenon of relaxation towards thermal equilibrium for large systems of interacting particles. For instance, a first step consists in giving a rigorous proof of the fact that a particle repeatedly scattered by random obstacles through a Hamiltonian scattering process will eventually reach thermal equilibrium, thereby completing previous works in this direction by the team. As a second step, models similar to the ones considered classically will be defined and analyzed in the quantum mechanical setting, and more particularly in the setting of quantum optics.

Another challenging problem is to understand the interaction of large systems with the boundary, which is responsible for most energy exchanges (forcing and dissipation), even though it is concentrated in very thin layers. The presence of boundary conditions to evolution equations sometimes lacks understanding from a physical and mathematical point of view. In order to legitimate the choice done at the macroscopic level of the mathematical definition of the boundary conditions, we investigate systems of particles with different local interactions. We apply various techniques to understand how diffusive and driven systems interact with the boundaries.

Finally, we aim at obtaining results on the macroscopic behavior of large scale interacting particle systems subject to kinetic constraints. In particular, we study the behavior in one and two dimensions of the Facilitated Exclusion Process (FEP), on which several results have already been obtained. The latter is a very interesting prototype for kinetically constrained models because of its unique mathematical features (explicit stationary states and absence of mobile cluster to locally shuffle the configuration). There are very few mathematical results on the FEP, which was put forward by the physics community as a toy model for phase separation.

Our goal is to develop collaboration at the interface between probability and PDE theory, and use the rich PDE background of the team to provide tools to be used on statistical physics problems put forward by the probability side of the team.

3.3 Numerical methods: analysis and simulations

The team addresses both questions of precision and numerical cost of discrete schemes for the numerical integration of non-linear evolution PDEs, such as the NLS equation. In particular, we aim at developing, studying and implementing numerical schemes with high-order convergence rates that are more efficient for these problems. We also want to contribute to the design and analysis of schemes with appropriate qualitative properties. These properties may as well be “asymptotic-preserving” properties, energy-preserving properties, or convergence to an equilibrium. Other numerical goals of the team include the numerical simulation of standing waves of non-linear non-local GP equations. We also keep on developing numerical methods to efficiently simulate and illustrate theoretical results on instability, in particular in the context of the modulational instability in optical fibers, where we study the influence of randomness in the physical parameters of the fibers.

The team also designs simulation methods to estimate the accuracy of the physical description via microscopic systems, by computing precisely the rate of convergence as the system size goes to infinity. One method under investigation is related to cloning algorithms, which were introduced very recently and turn out to be essential in molecular simulation.

4 Application domains

4.1 Optical fibers

Participants: Stephan De Bièvre, Guillaume Dujardin.

In the propagation of light in optical fibers, the combined effect of non-linearity and group velocity dispersion (GVD) may lead to the destabilization of the stationary states (plane or continuous waves). This phenomenon, known under the name of modulational instability (MI), consists in the exponential growth of small harmonic perturbations of a continuous wave. The study of MI has been pioneered in the 60s in the context of fluid mechanics, electromagnetic waves as well as in plasmas, and it has been observed in non-linear fiber optics in the 80s. In uniform fibers, MI arises for anomalous (negative) GVD, but it may also appear for normal GVD if polarization, higher order modes or higher order dispersion are considered. A different kind of MI related to a parametric resonance mechanism emerges when the dispersion or the non-linearity of the fiber are periodically modulated.

As a follow-up of our work on MI in periodically modulated optical fibers, we investigate the effect of random modulations in the diameter of the fiber on its dynamics. It is expected on theoretical grounds that such random fluctuations can lead to MI and this has already been illustrated for some models of randomness. We investigate precisely the conditions under which this phenomenon can be strong enough to be experimentally verified. For this purpose, we investigate different kinds of random processes describing the modulations, taking into account the manner in which such modulations can be created experimentally by our partners of the fiber facility of the PhLAM. This necessitates a careful modeling of the fiber and a precise numerical simulation of its behavior as well as a theoretical analysis of the statistics of the fiber dynamics.

4.2 Ferromagnetism

Participants: André De Laire Peirano, Guillaume Dujardin, Guillaume Ferrière.

The Landau–Lifshitz (LL) equation describes the dynamics of the spin in ferromagnetic materials. Depending on the properties of the material, the LL equation can include a dissipation term (the so-called Gilbert damping) and different types of anisotropic terms. The LL equation belongs to a larger class of non-linear PDEs which are often referred to as geometric PDEs, and some related models are the Schrödinger map equation and the harmonic heat flow. We focus on the following aspects of the LL equation.

Solitons In the absence of Gilbert damping, the LL equation is Hamiltonian. Moreover, it is integrable in the one-dimensional case and explicit formulas for solitons can be given. In the easy-plane case, the orbital and asymptotic stability of these solitons have been established. However, the stability in other cases, such as in biaxial ferromagnets, remains an open problem. In higher dimensional cases, the existence of solitons is more involved. In a previous work, a branch of semitopological solitons with different speeds has been obtained numerically in planar ferromagnets. A rigorous proof of the existence of such solitons is established using perturbation arguments, provided that the speed is small enough. However, the proof does not give information about their stability. We would like to propose a variational approach to study the existence of this branch of solitons, that would lead to the existence and stability of the whole branch of ground-state solitons as predicted. We also investigate numerically the existence of other types of localized solutions for the LL equation, such as excited states or vortices in rotation.

On the other hand, with the inclusion of the Gilbert damping, the Landau-Lifshitz-Gilbert (LLG) equation becomes (partially) dissipative. Interestingly, in the one-dimensional case, the same solitons, referred to as *domain walls*, emerge as significant structures. Not only do they demonstrate asymptotic stability, even in the presence of a small magnetic field ([42]), but they also serve as crucial building blocks for various stable configurations, such as 2-domain wall structures ([41]). Numerical simulations further suggest that any general solution should decompose over time into a superposition of domain walls, though this still presents an open problem at the theoretical level. Exploring the scenario of a notched nanowire ([40]) reveals yet another context where generalized domain walls manifest. They exhibit an even better asymptotic stability compared to their non-notched counterparts, which may lead to applications in information storage.

Approximate models An important physical conjecture is that the LL model is to a certain extent universal, so that the non-linear Schrödinger and Sine-Gordon equations can be obtained as its various limit cases. In a previous work, A. de Laire has proved a result in this direction and established an error estimate in Sobolev norms, in any dimension. A next step is to produce numerical simulations that will enlighten the situation and drive further developments in this direction.

Self-similar behavior Self-similar solutions have attracted a lot of attention in the study of non-linear PDEs because they can provide some important information about the dynamics of the equation. While self-similar expanders are related to non-uniqueness and long time description of solutions, self-similar shrinkers are related to a possible singularity formation. However, there is not much known about the self-similar solutions for the LL equation. A. de Laire and S. Gutierrez (University of Birmingham) have studied expander solutions and proved their existence and stability in the presence of Gilbert damping. We will investigate further results about these solutions, as well as the existence and properties of self-similar shrinkers.

4.3 Bose-Einstein condensates and nonlinear optics

Participants: Quentin Chauleur, André De Laire Peirano, Guillaume Dujardin, Guillaume Ferrière.

In quantum physics and nonlinear optics, the Gross-Pitaevskii equation with non-zero boundary conditions is employed to describe the behavior of quantum fluids and Bose-Einstein condensates. The primary challenges are to comprehend new realistic physical effects, such as nonlocal interactions, quasilinear effects and variations in the width of the domain.

In order to establish a rigorous understanding of the dynamics of these models, the study of particular solutions such as dark solitons, which play a key role in the large-time behavior, is a crucial first step. For instance, proving the stability of dark solitons, based on various physical considerations, implies that these structures are good candidates to be controlled experimentally and to be considered in new applications.

Although the properties of dark solitons are well-known in classical models described by the Gross-Pitaevskii equation, the situation becomes more intricate when adding terms to model new realistic physical effects. Each characteristic introduces a range of new theoretical and numerical difficulties. This complexity emphasizes the need for a careful and detailed examination to enhance our understanding of these intricate systems.

4.4 Cold atoms

Participants: Quentin Chauleur, Stephan De Bièvre, Guillaume Dujardin.

The cold atoms team of the PhLAM Laboratory is reputed for having realized experimentally the so-called Quantum Kicked Rotor, which provides a model for the phenomenon of Anderson localization. The latter was predicted by Anderson in 1958, who received in 1977 a Nobel Prize for this work. Anderson localization is the absence of diffusion of quantum mechanical wave functions (and of waves in general) due to the presence of randomness in the medium in which they propagate. Its transposition to the Quantum Kicked Rotor goes as follows: a freely moving quantum particle periodically subjected to a "kick" will see its energy saturate at long times. In this sense, it "localizes" in momentum space since its momenta do not grow indefinitely, as one would expect on classical grounds. In its original form, Anderson localization applies to non-interacting quantum particles and the same is true for the saturation effect observed in the Quantum Kicked Rotor.

The challenge is now to understand the effects of interactions between the atoms on the localization phenomenon. Transposing this problem to the Quantum Kicked Rotor, this means describing the interactions between the particles with a Gross–Pitaevskii equation, which is a NLS equation with a local (typically cubic) non-linearity. So the particle's wave function evolves between kicks following the Gross–Pitaevskii equation and not the linear Schrödinger equation, as is the case in the Quantum Kicked Rotor. Preliminary studies for the Anderson model have concluded that in that case the localization phenomenon gives way to a slow subdiffusive growth of the particle's kinetic energy. A similar phenomenon is expected in the non-linear Quantum Kicked Rotor, but a precise understanding of the dynamical mechanisms at work, of the time scale at which the subdiffusive growth will occur and of the subdiffusive growth exponent is lacking. It is crucial to design and calibrate the experimental setup intended to observe the phenomenon. The analysis of these questions poses considerable theoretical and numerical challenges due to the difficulties involved in understanding and simulating the long term dynamics of the non-linear system. A collaboration of the team members with the PhLAM cold atoms group is currently under way.

4.5 Modelling shallow water dynamics

Participants: André De Laire Peirano, Olivier Goubet.

The understanding of the propagation of waves in shallow water is of importance for the modelling of tsunamis and other rogue waves. This requires a better understanding of dispersive shallow water systems as ABCD systems, that are related to the classical Boussines systems, and classifying particular travelling waves solutions for these systems. To deal with systems is at forefront of research. Analogous questions for single equations as KdV equations are well-documented.

This project includes collaboration with researchers in Chile : C. Muñoz (Universidad de Chile), M. E. Martinez (University of Chile) and F. Poblete (Austral University of Chile). The applications for tsunamis is of interest for people in Chile.

4.6 Qualitative and quantitative properties of numerical methods

Participants: Quentin Chauleur, Guillaume Dujardin.

Numerical simulation of multimode fibers The use of multimode fibers is a possible way to overcome the bandwidth crisis to come in our worldwide communication network consisting in singlemode fibers. Moreover, multimode fibers have applications in several other domains, such as high power fiber lasers and femtosecond-pulse fiber lasers which are useful for clinical applications of non-linear optical microscopy and precision materials processing. From the modeling point of view, the envelope equations are a system of non-linear non-local coupled Schrödinger equations. For a better understanding of several physical phenomena in multimode fibers (e.g. continuum generation, condensation) as well as for the design of physical experiments, numerical simulations are a suitable tool. However, the huge number of equations, the coupled non-linearities and the non-local effects are very difficult to handle numerically. Some attempts have been made to develop and provide efficient numerical codes for such simulations. However, there is room for improvement: one may want to go beyond MATLAB prototypes, and to develop an alternative parallelization to the existing ones, which could use the linearly implicit methods that we plan to develop and analyze. In link with the application domain 4.1, we develop in particular a code for the numerical simulation of the propagation of light in multimode fibers, using high-order efficient methods. This code is to be used by the physics community.

Qualitative and long-time behavior of numerical methods We contribute to the design and analysis of schemes with good qualitative properties. These properties may as well be "asymptotic-preserving" properties, energy-preserving properties, decay properties, or convergence to an equilibrium. In particular, we contribute to the design and analysis of numerically hypocoercive methods for Fokker–Planck equations [44], as well as energy-preserving methods for Hamiltonian problems [39].

High-order methods We contribute to the design of efficient numerical methods for the simulation of non-linear evolution problems. In particular, we focus on a class of linearly implicit high-order methods, that have been introduced for ODEs and generalized to PDEs. We wish to extend their analysis in the context of PDEs, and analyze their qualitative properties in this case.

4.7 Mathematical modeling for ecology

Participant: Olivier Goubet.

This application domain is at the interface of mathematical modeling and numerics. Its object of study is a set of concrete problems in ecology. The landscape of the south of the Hauts-de-France region is made of agricultural land, encompassing forest patches and ecological corridors such as hedges. The issues are:

- the study of the invasive dynamics and the control of a population of beetles which damages the oaks and beeches of our forests;
- the study of native protected species (the purple wireworm and the pike-plum) which find refuge in certain forest species.

Running numerics on models co-constructed with ecologists is also at the heart of the project. In our model, the timescales of animals and plants compare: the life cycle of a tree is around one year, while (for animals) we consider mainly insects whose life cycle is also of one year, even for the propagation of insects. For instance, beetle larvae spend a few years in the earth before moving. As a by-product, the mathematical model may tackle other major issues such as the interplay between heterogeneity, diversity and invasibility.

The models use Markov chains at a mesoscopic scale and evolution advection-diffusion equations at a macroscopic scale.

5 Social and environmental responsibility

5.1 Footprint of research activities

The team is committed to addressing environmental challenges and try to minimize its carbon footprint. Whenever possible, train travel is prioritized for conferences and research visits. Several team members have adopted a no-flight policy for ecological reasons.

5.2 Impact of research results

Our work as applied mathematicians is often interdisciplinary. We work with other scientists on cold atoms problems, quantum information theory, forests modelling, etc. Our societal relevance mostly comes from our inputs in advances in these applied research directions.

6 Latest software developments, platforms, open data

6.1 Latest software developments

6.1.1 MM_Propagation

Name: MultiMode Propagation

Keywords: Optics, Numerical simulations, Computational electromagnetics

Functional Description: This C++ software, which is interfaced with MatLab, simulates the propagation of light in multimode optical fibers. It takes into account several physical effects such as dispersion, Kerr effect, Raman effect, coupling between the modes. It uses high order numerical methods that allow for precision at reasonable computational cost.

URL: `https://github.com/alexandreroget/MM_Propagation`

Contact: Alexandre Roget

7 New results

Some of the results presented below overlap several of the main research themes presented in Section 3. However, results presented in paragraphs 7.1-7.13 are mainly concerned with research axis 3.1. Paragraphs 7.14-7.14.6 are related to quantum information and computing, and Paragraphs 7.15-7.17 concern numerics-oriented results, so that they are all encompassed in axis 3.3.

7.1 On the stationary solution of the Landau-Lifshitz-Gilbert equation on a nanowire with constant external magnetic field

Participant: Guillaume Ferrière.

In the preprint [21], Guillaume Ferriere examines the Landau-Lifshitz-Gilbert (LLG) equation governing the magnetization dynamics in an infinite ferromagnetic nanowire with easy-axis anisotropy along the e_1 direction and subjected to a constant external magnetic field $h_0 e_1$. Under specific conditions on h_0, the study establishes the existence of stationary solutions with identical asymptotic behavior at infinity, their uniqueness up to the symmetries of the LLG equation, and the instability of their orbits under the LLG flow. These findings provide new insights into the behavior of solutions to the LLG equation, complemented by numerical simulations that explore the stability of 2-domain wall structures and the interactions between domain walls.

7.2 Existence and Uniqueness of Domain Walls for Notched Ferromagnetic Nanowires

Participant: Guillaume Ferrière.

In the paper [17], in collaboration with R. Côte (University of Strasbourg), C. Courtès (University of Strasbourg), Guillaume Ferriere, L. Godard-Cadillac (University of Bordeaux), and Y. Privat (University of Lorraine, Nancy) explore the existence and properties of domain walls in a model of notched ferromagnetic nanowires. They employ variational methods and critical point theory to investigate the energy functional describing the system.

The authors first establish the equivalence of the critical points of this functional and the critical points of another, more suitable functional through lifting. The existence of a minium is then achieved under the assumption that the residual cross-section area function s is strictly below 1 in a bounded interval and is equal to 1 outside this interval. They then demonstrate the uniqueness of the critical point under the proper constraints on the limits at $\pm\infty$ by leveraging a Mountain-Pass argument. The uniqueness requires stronger monotonicity assumptions, mainly that s is unimodal, all the more as it is expected that non-uniqueness should hold in the case of many notches.

The identified critical point corresponds to a domain wall structure, *i.e.* a transition from $-e_1$ to e_1. The authors also prove that the transition is mainly performed inside the notch. Furthermore, the study analyzes the asymptotic behavior of the solution, showing that the magnetization decays to a uniform state at infinity. In the special case of a symmetric notch, additional insights are obtained using rearrangement techniques.

7.3 Minimal time of magnetization switching in small ferromagnetic ellipsoidal samples.

Participant: Guillaume Ferrière.

In the article [18], R. Côte (Université de Strasbourg), C. Courtès (Université de Strasbourg), Guillaume Ferriere and Y. Privat (Université de Lorraine & Institut Universitaire de France) studied the process of the switching of the magnetization of a small ferromagnetic ellipsoidal sample, assumed to be constant, by an external magnetic field. They first proved that the latter must be strong enough in order to be able to switch the magnetization. When this condition is fullfiled, they found an explicit expression for the minimal time of switching in cases where the sample satisfies some rotational symmetry property. Some numerical simulations complete the study.

7.4 The Spear and the Ring: Emergent Structures in Magnetic Colloidal Suspensions

Participant: Guillaume Ferrière.

In the preprint [32], R. Côte (Université de Strasbourg), C. Courtès (Université de Strasbourg), Guillaume Ferriere, Ludovic Godard-Cadillac (Université de Bordeaux) and Y. Privat (Université de Lorraine

& Institut Universitaire de France) studied from a mathematical point of view the nanoparticle model of a magnetic colloid presented by G. Klughertz, in which small ferromagnetic particles immerged in a fluid interact with each other through standard dipole-dipole magnetic interaction and soft sphere model. More specifically, they analyzed two specific structures: the spear (chain of aligned particles) and the ring (particles placed on a circle with N-fold rotationnal symmetry). Under some technical assumptions on the parameters of the interactions, they proved the existence and the uniqueness of these structures, along with bounds and sharp asymptotics (as the number of particles tends to infinity) of the distance between neighboring particles.

7.5 On the propagation of high regularity for the logarithmic Schrödinger equation

Participants: Quentin Chauleur, Guillaume Ferrière.

In the prepint [30], Quentin Chauleur and Guillaume Ferriere managed to understand the effect of cancellation points in the instantaneous loss of regularity of the logarithmic nonlinear Schrödinger equation, which was numerically observed and had been an open question for many years on the theoretical level.

7.6 On the dependence of the nonlinear Schrodinger flow upon the power of the nonlinearity

Participants: Quentin Chauleur, Guillaume Ferrière.

In collaboration with R. Carles (Université de Rennes), Quentin Chauleur and Guillaume Ferriere proved in [28] some precise convergence rates for the limit of classical nonlinear Schrödinger models with power nonlinearities to the logarithmic nonlinear Schrödinger equation equation in the subcritical regime and in large time.

7.7 On the ground state of the nonlinear Schrödinger equation: asymptotic behavior at the endpoint powers

Participants: Quentin Chauleur, Guillaume Ferrière.

In collaboration with R. Carles (Université de Rennes) and D. Pelinovsky (McMaster University), Quentin Chauleur and Guillaume Ferriere studied in [29] both limits of the stationary nonlinear Schrödinger equation when the power of the nonlinearity tends to zero or to the energy-critical exponent, up to some rescaling to infer non-trivial limits. They proved strong convergence results for both cases, and provide detailed numerical insights.

7.8 Gray and black solitons of nonlocal Gross-Pitaevskii equations

Participant: André De Laire Peirano.

In the paper [24], Andre De Laire Peirano and S. López-Martínez (Autonomous University of Madrid) continue the investigation started in previous works concerning the qualitative aspects of dark solitons of one-dimensional Gross-Pitaevskii equations with general nonlocal interactions. Under general conditions on the potential interaction term, they provide uniform bounds, demonstrate the existence of symmetric solitons, and identify conditions under which monotonicity is lost. Additionally, they present new properties of black solitons. Moreover, they establish the nonlocal-to-local convergence, i.e. the convergence of the soliton of the nonlocal model toward the explicit dark solitons of the local Gross-Pitaevskii equation.

7.9 Exotic traveling waves for a quasilinear Schrödinger equation with nonzero background

Participants: André De Laire Peirano, Erwan Le Quiniou.

Andre De Laire Peirano and Erwan Le Quiniou have studied a quasilinear Schrödinger equation with nonzero conditions at infinity in dimension one. This quasilinear model corresponds to a weakly nonlocal approximation of the nonlocal Gross–Pitaevskii equation, and can also be derived by considering the effects of surface tension in superfluids. When the quasilinear term is neglected, the resulting equation is the classical Gross–Pitaevskii equation, which possesses a well-known stable branch of subsonic traveling waves solution, given by dark solitons.

In the paper [23], they investigate how the quasilinear term affects the traveling-waves solutions. They provide a complete classification of finite energy traveling waves of the equation, in terms of the two parameters: the speed and the strength of the quasilinear term. This classification leads to the existence of dark and antidark solitons, as well as more exotic localized solutions like dark cuspons, compactons, and composite waves, even for supersonic speeds. Depending on the parameters, these types of solutions can coexist, showing that finite energy solutions are not unique. Furthermore, they prove that some of these dark solitons can be obtained as minimizers of the energy, at fixed momentum, and that they are orbitally stable.

7.10 Traveling waves for a quasilinear Schrödinger equation

Participant: Erwan Le Quiniou.

In the paper [26], Erwan Le Quiniou studies a quasilinear Schrödinger equation with nonzero conditions at infinity. In the previous work [23] with Andre De Laire Peirano, he obtained a continuous branch of traveling waves, given by dark solitons indexed by their speed. Neglecting the quasilinear term, one recovers the Gross–Pitaevskii equation, for which the branch of dark solitons is stable. It is known that the Vakhitov–Kolokolov (VK) stability criterion or momentum of stability criterion holds for general semilinear equations with nonvanishing conditions at infinity. In the quasilinear case, E. Le Quiniou proves that the VK stability criterion still applies and he deduces that the branch of dark solitons is stable for weak quasilinear interactions. For stronger quasilinear interactions, a cusp appears in the energy-momentum diagram, implying the stability of fast waves and the instability of slow waves.

7.11 Travelling waves for the Gross–Pitaevskii equation on the strip

Participant: André De Laire Peirano.

In one space dimension, the Gross-Pitaevskii equation possesses a family of finite energy travelling waves, called dark solitons. These solitons extend trivially to the strip given by the product space $\mathbb{R} \times \mathbb{T}_L$, where $L > 0$ and $\mathbb{T}_L$ is the torus $\mathbb{T}_L = \mathbb{R}/L\mathbb{Z}$. In this two-dimensional context, the dark solitons are called planar (or line) dark solitons. However, it is well-known in the physics literature that these planar solitons can be unstable due to the tendency to develop distortions in their transverse profile. In addition, experimental observations have shown that the dynamics of planar dark solitons are stable when they are sufficiently confined in the transverse direction L, but unstable otherwise. In the latter case, the creation of vortices can occur.

In the article [22], Andre De Laire Peirano, P. Gravejat (CY Cergy Paris University) and D. Smets (Sorbonne University) provide a rigorous framework for studying this kind of phenomenon. Precisely, they prove the existence of nonconstant finite energy travelling wave solutions to the Gross-Pitaevskii equation on the strip $\mathbb{R} \times \mathbb{T}_L$, obtained as minimizers of the energy at fixed momentum. Moreover, by studying the associated variational problem, they deduce that these minimizers are exactly the planar dark solitons when L is less than a critical value, and that they are genuinely two-dimensional solutions otherwise. In particular, planar solitons do not minimize the energy in the presence of a large transverse direction. The proof of the existence of minimizers is based on the compactness of minimizing sequences, relying on a new symmetrization argument that is well-suited to the periodic setting.

7.12 Standing wave for two-dimensional Schrödinger equations with discontinuous dispersion

Participants: Olivier Goubet.

In collaboration with B. Alouini (University of Monastir) and I. Manoubi (Université of Gabès) [15], Olivier Goubet has studied the existence and stability of standing wave for an evolution nonlinear Schrödinger equation with discontinuous dispersion, the discontinuity being supported by a straight line. Both pure power nonlinearities and logarithmic nonlinearities are considered. The discontinuity destroys the invariance by space translation for the equation. The main result is that when restricted to a suitable subspace that contains the standing waves, these waves are orbitally stable in the H^1 subcritical regime in the pure power case or in the logarithmic case, and strongly unstable in the critical or supercritical case.

7.13 Dynamics of generalized abcd Boussinesq solitary waves under a slowly variable bottom

Participants: André De Laire Peirano, Olivier Goubet.

In collaboration with the associated team PANDA, the team investigated the existence of generalized solitary waves and the corresponding collision problem in the physically relevant variable bottom regime, for the $abcd$-Boussinesq system. In [33], the authors provide a detailed description of weak long-range interactions and the evolution of the traveling wave without its destruction. They establish this result by constructing a new approximate solution that captures the interaction between the solitary wave and a slowly varying bottom.

7.14 Kirkwood-Dirac distributions

Participants: Stephan De Bièvre, Christopher Langrenez, Mateo Spriet.

The Kirkwood-Dirac (KD) quasiprobability distribution can describe any quantum state with respect to the eigenbases of two observables A and B. KD distributions behave similarly to classical joint probability distributions but can assume negative and nonreal values. In [43], Stephan De Bièvre provided an in-depth study of the notion of completely incompatible observables that he recently introduced and of its links to the support uncertainty and to the Kirkwood-Dirac nonpositivity of pure quantum states. The latter notion has recently been proven central to a number of issues in quantum information theory and quantum metrology. In this last context, it was shown that a quantum advantage requires the use of Kirkwood-Dirac nonclassical states.

Several papers have been published by members of PARADYSE on this subject in the last year. They are mentioned in the following sections.

7.14.1 Convex roofs witnessing Kirkwood-Dirac nonpositivity

In [25], Stephan De Bièvre, Christopher Langrenez and D. R. M. Arvidsson-Shukur (Hitachi Cambridge Laboratory) introduce and study two witnesses for KD nonpositivity, through a convex roof construction and the notion of *support uncertainty*.

7.14.2 Kirkwood-Dirac distribution as a tool in quantum information

The geometry of the set of Kirkwood-Dirac positive states is studied in [27]. Conditions are given guaranteeing that it equals the convex hull of the Kirkwood-Dirac pure positive states. This property is shown to hold with probability one. Examples are given where this is not the case.

7.14.3 The Kirkwood-Dirac Representation Associated to the Fourier Transform for Finite Abelian Groups: Positivity.

The Kirkwood-Dirac (KD) representations naturally associated to the Fourier transform of finite abelian groups G is constructed and the set of KD-positive states is studied. It is proven in [19] that, for $G = \mathbb{Z}_d$, with d a prime power, the convex set of KD-positive states contains states equals the convex hull of the pure KD-positive states.

7.14.4 Characterizing the Kirkwood-Dirac positivity on second countable LCA groups

We define in [35] the Kirkwood-Dirac quasiprobability representation of quantum mechanics associated with the Fourier transform over second countable locally compact abelian groups. We then show that the classical fragment of quantum mechanics associated with the Kirkwood-Dirac distribution is non-trivial if and only if the group has a compact connected component and we provide for connected compact abelian groups a complete geometric description of this classical fragment.

7.14.5 What is special about the Kirkwood-Dirac distributions?

It is shown in [34] that the Kirkwood-Dirac quasiprobability representation is the unique Born-rule compatible quasiprobability representation for which the associated conditional expectation of any observable coincides with its best predictor, as it does classically.

7.14.6 Nonpositivity is a Necessary Resource for Quantum Computing

Classical computers can simulate models of quantum computation with restricted input states. We cast in [36] a real-quantum-bit model of computation in terms of a Kirkwood-Dirac (KD) quasiprobability distribution. Algorithms, throughout which this distribution is a proper (positive) probability distribution can be simulated efficiently on a classical computer. We leverage recent results on the geometry of the set of KD-positive states to construct previously unknown classically-simulable (bound) states.

7.15 Strong convergence for the discrete nonlinear Klein-Gordon equation

Participant: Quentin Chauleur.

In [16], Quentin Chauleur extends the analysis of nonlinear dispersive equations, such as the nonlinear Klein-Gordon equation, on an infinite lattice $h\mathbb{Z}^d$ as the lattice spacing $h \to 0$ approaches the continuum limit. This work builds upon the framework established in previous works, employing bilinear estimates of the Shannon interpolation combined with controls on the growth of discrete Sobolev norms of the solutions. Additionally, Quentin Chauleur also provides some perspectives on uniform dispersive estimates for nonlinear waves on lattices.

7.16 Vortex nucleation in 2D rotating Bose–Einstein condensates

Participants: Quentin Chauleur, Guillaume Dujardin.

In [20], Guillaume Dujardin, I. Lacroix-Violet (University of Lorraine, Nancy) and A. Nahas (University of Lille) introduce a new numerical method for the minimization under constraints of a discrete energy modeling multicomponents rotating Bose-Einstein condensates in the regime of strong confinement and with rotation. Moreover, they consider both segregation and coexistence regimes between the components. It is well known that, depending on the regime, the minimizers may display different structures, sometimes with vorticity (from singly quantized vortices, to vortex sheets and giant holes). In order to study numerically the structures of the minimizers, the authors of [20] introduce a numerical algorithm for the computation of the indices of the vortices, as well as an algorithm for the computation of the indices of vortex sheets. Several computations are carried out, to illustrate the efficiency of the method, to cover different physical cases, to validate recent theoretical results as well as to support conjectures. Moreover, the new methods is compared with an alternative method from the literature. This work was part of A. Nahas' PhD thesis, co-advised by I. Lacroix-Violet (University of Lorraine, Nancy) and Guillaume Dujardin.

7.17 High order uniform in time schemes for weakly nonlinear Schrödinger equation and wave turbulence

Participant: Quentin Chauleur.

In the preprint [31], Quentin Chauleur and A. Mouzard (Université de Nanterre) has developed some uniform in time high order schemes for the discretization of NLS equations with a small (weak) nonlinearity. The motivation of such work lies in the theory of wave turbulence, which describes the nonlinear interactions of waves outside thermal equilibrium by a statistical approach, in analogy with Boltzmann kinetic theory of gases. This analysis aims for instance at understanding the behavior of waves propagating at the surface of the ocean, with the coexistence of waves of various wavelengths propagating in many directions. The scheme developed was further used in order to study such physical behaviors.

8 Partnerships and cooperations

8.1 International initiatives

8.1.1 Associate Teams in the framework of an Inria International Lab or in the framework of an Inria International Program

Participants: André De Laire Peirano, Guillaume Dujardin, Olivier Goubet, Guillaume Ferriere, Erwan Le Quiniou , Sebastian Tapia Mandiola.

- PANDA
  - Title: Partial Differential Equations, Dispersive Models and Nonlinear Analysis
  - Duration: 3 years (2024 -> 2027)
  - Coordinator: Claudio Muñoz (cmunoz@dim.uchile.cl)
  - Partners: Universidad de Chile (Chili)
  - Inria contact: André de Laire

- Summary: PANDA is a collaborative project between Chilean and French teams, in the field of applied mathematics. The main subject is the study of systems of dispersive partial differential equations, based on nonlinear analysis and numerical simulation techniques. One of the main applications of this project concerns the modeling of the propagation of waves on the ocean surface. Webpage.

8.2 National initiatives

8.2.1 LabEx CEMPI

Through their affiliation to the Laboratoire Paul Painlevé of Université de Lille, PARADYSE team members benefit from the support of the LabEx CEMPI.

Title: Centre Européen pour les Mathématiques, la Physique et leurs Interactions

Partners: Laboratoire Paul Painlevé (LPP) and Laser Physics department (PhLAM), Université de Lille

ANR reference: 11-LABX-0007

Duration: February 2012 - December 2024 (the project has been renewed in 2019)

Budget: 6 960 395 euros

Coordinator: Emmanuel Fricain (LPP, Université de Lille)

The "Laboratoire d'Excellence" CEMPI (Centre Européen pour les Mathématiques, la Physique et leurs Interactions), a project of the Laboratoire de mathématiques Paul Painlevé (LPP) and the laboratoire de Physique des Lasers, Atomes et Molécules (PhLAM), was created in the context of the "Programme d'Investissements d'Avenir" in February 2012. The association Painlevé-PhLAM creates in Lille a research unit for fundamental and applied research and for training and technological development that covers a wide spectrum of knowledge stretching from pure and applied mathematics to experimental and applied physics. The CEMPI research is at the interface between mathematics and physics. It is concerned with key problems coming from the study of complex behaviors in cold atoms physics and nonlinear optics, in particular fiber optics. It deals with fields of mathematics such as algebraic geometry, modular forms, operator algebras, harmonic analysis, and quantum groups, that have promising interactions with several branches of theoretical physics.

8.2.2 Cross Disciplinary Project C2EMPI

The team is part of the "Cross Disciplinary Project" C2EMPI. The aim of the C2EMPI project is to adopt a resolutely interdisciplinary approach to develop mathematical, computational, theoretical, and experimental physics tools to address complexity in the modeling and design of new quantum, optical, and metamaterial devices, as well as in the ultrafast and large-scale processing of information.

8.2.3 PEPS JCJC

Participants: Quentin Chauleur, Guillaume Ferriere.

Quentin Chauleur is the Principal Investigator of a PEPS JCJC project from the CNRS (2500€), to support research on the analysis of numerical schemes for the Gross-Pitaevskii equation on general meshes. Other members of the project are Guillaume Ferriere and Julien Moatti (Associate Professor at Bordeaux INP).

8.2.4 MITI

Participant: Stephan De Bièvre.

Stephan De Bièvre is the Principal Investigator of the KIDIWI project, which received grants (40k€) from the MITI (Mission pour les Initiatives Transverses et Interdisciplinaires – CNRS) for its work on quasiprobability distributions in quantum mechanics, including funding for the QuiDiQua workshop in Lille in 2023, and for computing equipment. The funding includes in addition one doctoral student contract (2024-2027) by MITI.

8.2.5 ANR project DYNACQUS

Participant: Stephan De Bièvre.

Stephan De Bièvre is a member of the Cergy pole of the ANR project DYNACQUS ANR-24-CE40-5714. The long-time asymptotics of thermodynamically large systems is an important problem in non-equilibrium classical and quantum statistical mechanics. While the thermal equilibrium states are well understood. non-equilibrium steady states are not. The main goal of this project is to investigate the mechanisms of relaxation to these states in the framework of various physically relevant models.

8.2.6 ANR project SOS 2ID

Participant: Guillaume Dujardin.

Guillaume Dujardin is a member of the ANR project SOS 2ID (2025-2028) ANR-24-CE40-3786, between Pau (LMAP and UPPA) and Toulouse (IMT and INSA), which focuses on stochastic optimization schemes in finite and infinite dimensional settings. There are two main research directions in this project : analyze finite and infinite dimensional stochastic inertial optimization methods, and design and study new stochastic inertial optimization methods.

8.2.7 ANR project MOSICOF

Participant: Guillaume Ferriere.

Guillaume Ferriere is a member of the ANR project MOSICOF (2021-2025) ANR-21-CE40-0004. The aim of the project is to improve the modelling and simulation of ferromagnetic devices, taking into account their complex geometries (e.g. nanowire arrays, curved nanowires) and the multiphysical nature of the phenomena involved: electromagnetic, mechanical (magnetostriction) and thermal effects. The project brings together researchers from several French institutions.

8.2.8 PEPR FORESTT

Participant: Olivier Goubet.

Olivier Goubet is a member of the Made in France project of the PEPR FORESTT. The goal of this project is the understanding of the evolution of forest ecosystems in complex and changing environnement. O. Goubet is in charge of co-organizing WP 3 of this project, which deals with the developpement of a mechanistic model for meta-community dynamics in forest ecosystems.

8.3 Regional initiatives

8.3.1 CPER WaveTech

The team is part of the CPER WaveTech (2021-2027), hosted by the Physics Lab PhLAM in collaboration with the Maths Lab LPP of the Université de Lille.

9 Dissemination

9.1 Promoting scientific activities

9.1.1 Scientific events: organisation

Participants: Quentin Chauleur, Stephan De Bièvre, André De Laire Peirano, Guillaume Ferrière.

General chair, scientific chair

Participants: Stephan De Bièvre, Olivier Goubet.

- Stephan De Bièvre was the chair of the Scientific Committee of the QuidiQua conference held in Paris Nov. 2025.

Member of the organizing committees

- Andre De Laire Peirano was one of the organizers of the 2nd PANDA Workshop in Lille, from June 11 to June 13, 2025.
- Andre De Laire Peirano was one of the organizers of the 2nd PANDA Workshop in Santiago, Chile, Dec. 9, 2025.
- Quentin Chauleur co-organized the Journée du Laboratoire Paul Painlevé 2025 in October 2025.
- Guillaume Ferriere is a co-organizer of the conference New Trends in the Mathematical and Physical Aspects of Magnetism held in Strasbourg in June 2025.

9.1.2 Journal

Member of the editorial boards

- Stephan De Bièvre is associate editor of the Journal of Mathematical Physics (since January 2019);
- Olivier Goubet is the Editor in Chief of North-Western Europan Journal of Math., and member of the editorial board of Journal of Math. Studies, and of Boundary Value Problems. He just recently quit the editorial board of Advances in Nonlinear Analysis.

Reviewer - reviewing activities All permanent members of the PARADYSE team work as referees for many of the main scientific publications in analysis, partial differential equations and statistical physics, depending on their respective fields of expertise.

9.1.3 Invited talks

All PARADYSE team members take active part in numerous scientific conferences, workshops and seminars, and in particular give frequent talks both in France and abroad.

9.1.4 Research administration

Participants: Stephan De Bièvre, André De Laire Peirano, Guillaume Dujardin, Olivier Goubet.

- Stephan De Bièvre presides the Scientific Committee of the conference series QuiDiQua.
- Andre De Laire Peirano is member of Comité national de la recherche scientifique, in Section 01 *Mathématiques et interactions des mathématiques*.
- Guillaume Dujardin is a member of the Comité Exécutif of the CPER Wavetech and a member of the Comité Exécutif of the CDP C2EMPI.
- Guillaume Dujardin is the vice-head of science for the Centre Inria de l'Université de Lille since January 2024, and he serves as a member of the Commission d'Évaluation of Inria, since September 2024.
- Olivier Goubet is member of the CA and president of the Scientific Comittee of SMAI.

9.2 Teaching - Supervision - Juries - Educational and pedagogical outreach

Participants: Quentin Chauleur, Stephan De Bièvre, André De Laire Peirano, Guillaume Dujardin, Olivier Goubet, Guillaume Ferrière.

9.2.1 Teaching

The PARADYSE team teaches various undergraduate level courses in several partner universities. We only make explicit mention here of the Master courses (level M1-M2) and the doctoral courses.

- Stephan De Bièvre taught
  - Univ. Lille. 2nd year of Master Applied Math. and Scientific Computing, 40h, *Modeling*;
  - Univ. Lille. Faculty of Sciences. PhD program. *Quantum information theory*, 24h.
- Andre De Laire Peirano taught
  - Univ. Lille, 2nd year of a Master's degree, 33h, *Nonlinear PDEs*.
  - Univ. Lille, 1st year of a Master's degree, 66h, *Étude de problèmes elliptiques et paraboliques*.
- Guillaume Dujardin taught
  - ~64h/year as "professeur chargé de cours" at École Polytechnique in 3rd year (corresponding to an M2-level) in the engineering cycle, and in the Bachelor (corresponding to an L1-level) of the École.
- Guillaume Ferriere taught
  - Université de Lille, 2nd year of a Master's degree, 33h, *Nonlinear PDEs*.
- Olivier Goubet taught
  - A specialized course in Master 2 on the mathematics for waterwaves 40h.

9.2.2 Supervision

- Stephan De Bièvre supervised the PhD thesis of Christopher Langrenez on "Kirkwood-Dirac nonclassicality" [27], during 2022-2025.
- Stephan De Bièvre is supervising the PhD thesis of Matéo Spriet "Mesures opérationnels de nonclassicalité" during 2024-2027.
- Andre De Laire Peirano and Olivier Goubet are supervising the PhD thesis of Erwan Le Quiniou on the "Study of a quasilinear Gross-Pitaevskii equation", during 2022-2025.
- Andre De Laire Peirano and Guillaume Dujardin are supervising the PhD thesis of Sebastian Tapia Mandiola on the "Theorical and numerical study of dark solitons for nonlinear Schrödinger equations" during 2024-2027.
- Guillaume Dujardin is supervising the PhD thesis of Abbas El Hajj during 2024-2027.
- Olivier Goubet is supervising the PhD thesis of Céline Wang during 2023-2026.
- Guillaume Ferriere has supervised the M2 internship of Mohamed Bensaid (Apr.-Sept. 2025) and is supervising his PhD thesis "Solitons, multi-solitons and long-time behavior for nonlinear Schrödinger equations with singular nonlinearities" during 2025-2028.
- Quentin Chauleur is supervising the internship (6 months) of Maxime Haberthur from September 2025.

9.2.3 Juries

- Guillaume Dujardin is a member of the jury of the agrégation externe de mathématiques, in charge with Frédérique Charles of the "Scientific Computing" option.

10 Scientific production

10.1 Major publications

[1] R. Ahmed, C. Bernardin, P. Gonçalves and M. Simon. 'A Microscopic Derivation of Coupled SPDE's with a KPZ Flavor'. In: *Annales de l'Institut Henri Poincaré* 58.2 (2022). DOI: 10.1214/21-AIHP1196. URL: https://hal.archives-ouvertes.fr/hal-02307963.

[2] C. Bernardin, P. Gonçalves, M. Jara and M. Simon. 'Interpolation process between standard diffusion and fractional diffusion'. In: *Annales de l'Institut Henri Poincaré (B) Probabilités et Statistiques* 54.3 (2018), pp. 1731–1757. DOI: 10.1214/17-AIHP853. URL: https://hal.archives-ouvertes.fr/hal-01348503.

[3] C. Besse, S. Descombes, G. Dujardin and I. Lacroix-Violet. 'Energy preserving methods for nonlinear Schrödinger equations'. In: *IMA Journal of Numerical Analysis* 41.1 (Jan. 2021), pp. 618–653. DOI: 10.1093/imanum/drz067. URL: https://hal.archives-ouvertes.fr/hal-01951527.

[4] O. Blondel, C. Erignoux and M. Simon. 'Stefan problem for a non-ergodic facilitated exclusion process'. In: *Probability and Mathematical Physics* 2.1 (2021). DOI: 10.2140/pmp.2021.2.127. URL: https://hal.inria.fr/hal-02482922.

[5] Q. Chauleur. 'Growth of Sobolev norms and strong convergence for the discrete nonlinear Schrödinger equation'. In: *Nonlinear Analysis: Theory, Methods and Applications* 242 (May 2024), p. 113517. DOI: 10.1016/j.na.2024.113517. URL: https://hal.science/hal-04142120.

[6] R. Côte and G. Ferriere. 'Asymptotic stability of 2-domain walls for the Landau-Lifshitz-Gilbert equation in a nanowire with Dzyaloshinskii-Moriya interaction'. In: *International Mathematics Research Notices* 2024.4 (21st Feb. 2024), pp. 3551–3600. DOI: 10.1093/imrn/rnad249. URL: https://hal.science/hal-03913853.

[7] S. De Bièvre. 'Complete Incompatibility, Support Uncertainty, and Kirkwood-Dirac Nonclassicality'. In: *Physical Review Letters* (2021). DOI: 10.1103/PhysRevLett.127.190404. URL: https://hal.archives-ouvertes.fr/hal-03464950.

[8] S. Dumont, O. Goubet and Y. Mammeri. 'Decay of solutions to one dimensional nonlinear Schrödinger equations with white noise dispersion'. In: *Discrete and Continuous Dynamical Systems - Series S* 14.8 (2021), pp. 2877–2891. DOI: 10.3934/dcdss.2020456. URL: https://hal.archives-ouvertes.fr/hal-02944262.

[9] C. Erignoux. 'Hydrodynamic limit for an active exclusion process'. In: *Mémoires de la Société Mathématique de France* 169 (May 2021). DOI: 10.24033/msmf.477. URL: https://hal.science/hal-01350532.

[10] C. Griffet, M. Arnhem, S. De Bièvre and N. Cerf. 'Interferometric measurement of the quadrature coherence scale using two replicas of a quantum optical state'. In: *Physical Review A* 108.2 (31st Aug. 2023), p. 023730. DOI: 10.1103/PhysRevA.108.023730. URL: https://hal.science/hal-03911306.

[11] A. Hertz and S. de Bièvre. 'Quadrature coherence scale driven fast decoherence of bosonic quantum field states'. In: *Physical Review Letters* (5th Mar. 2020). DOI: 10.1103/PhysRevLett.124.090402. URL: https://hal.science/hal-02394344.

[12] A. de Laire and P. Gravejat. 'The cubic Schrödinger regime of the Landau-Lifshitz equation with a strong easy-axis anisotropy'. In: *Revista Matemática Iberoamericana* 37.1 (2021), pp. 95–128. DOI: 10.4171/rmi/1202. URL: https://hal.archives-ouvertes.fr/hal-01954762.

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[14] A. de Laire and S. López-Martínez. 'Existence and decay of traveling waves for the nonlocal Gross-Pitaevskii equation'. In: *Communications in Partial Differential Equations* 47.9 (2022), pp. 1732–1794. DOI: 10.1080/03605302.2022.2070853. URL: https://hal.archives-ouvertes.fr/hal-03422447.

10.2 Publications of the year

International journals

[15] B. Alouini, O. Goubet and I. Manoubi. 'Standing wave for two-dimensional schrödinger equations with discontinuous dispersion'. In: *Applicable Analysis* 104.13 (25th Feb. 2025), pp. 2414–2435. DOI: 10.1080/00036811.2025.2468506. URL: https://hal.science/hal-04854005 (cit. on p. 15).

[16] Q. Chauleur. 'Continuum limit of the discrete nonlinear Klein-Gordon equation'. In: *ESAIM: Proceedings and Surveys* (2025). URL: https://hal.science/hal-04470275. In press (cit. on p. 16).

[17] R. Côte, C. Courtès, G. Ferriere, L. Godard-Cadillac and Y. Privat. 'Existence and uniqueness of domain walls for notched ferromagnetic nanowires'. In: *ESAIM: Control, Optimisation and Calculus of Variations* 31 (2025), p. 60. DOI: 10.1051/cocv/2025051. URL: https://hal.science/hal-05170804 (cit. on p. 12).

[18] R. Côte, C. Courtès, G. Ferriere and Y. Privat. 'Minimal time of magnetization switching in small ferromagnetic ellipsoidal samples'. In: *Journal de l'École polytechnique — Mathématiques* Tome 12 (17th Jan. 2025), pp. 147–184. DOI: 10.5802/jep.287. URL: https://hal.science/hal-03931579 (cit. on p. 12).

[19] S. De Bièvre, C. Langrenez and D. Radchenko. 'The Kirkwood-Dirac Representation Associated to the Fourier Transform for Finite Abelian Groups: Positivity'. In: *Annales Henri Poincaré* (2nd Sept. 2025). DOI: 10.1007/s00023-025-01614-7. URL: https://hal.science/hal-05250921 (cit. on p. 16).

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[22] A. de Laire, P. Gravejat and D. Smets. ‘Minimizing travelling waves for the Gross-Pitaevskii equation on $\mathbb{R} \times \mathbb{T}$’. In: *Annales de la Faculté des Sciences de Toulouse. Mathématiques.* 34.1 (23rd June 2025), pp. 135–192. DOI: 10.5802/afst.1808. URL: https://hal.science/hal-03586112 (cit. on p. 15).

[23] A. de Laire and E. Le Quiniou. ‘Exotic traveling waves for a quasilinear Schrödinger equation with nonzero background’. In: *Nonlinear Analysis: Theory, Methods and Applications* (2025). DOI: 10.48550/arXiv.2311.08918. URL: https://hal.science/hal-04287188. In press (cit. on p. 14).

[24] A. de Laire and S. López-Martínez. ‘Gray and black solitons of nonlocal Gross-Pitaevskii equations: existence, monotonicity and nonlocal-to-local limit’. In: *Calculus of Variations and Partial Differential Equations* 64.4 (18th Apr. 2025), p. 138. DOI: 10.1007/s00526-025-03000-x. URL: https://hal.science/hal-04669342 (cit. on p. 13).

[25] C. Langrenez, D. Arvidsson-Shukur and S. De Bièvre. ‘Convex roofs witnessing Kirkwood-Dirac nonpositivity’. In: *Journal of Physics A: Mathematical and Theoretical* 58.21 (21st May 2025), p. 215301. DOI: 10.1088/1751-8121/add58f. URL: https://hal.science/hal-04649474 (cit. on p. 16).

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Doctoral dissertations and habilitation theses

[27] C. Langrenez. ‘Kirkwood-Dirac distribution as a tool in quantum information’. Université de Lille, 30th June 2025. URL: https://hal.science/tel-05395404 (cit. on pp. 16, 22).

Reports & preprints

[28] R. Carles, Q. Chauleur and G. Ferriere. *On the dependence of the nonlinear Schrodinger flow upon the power of the nonlinearity*. 29th Sept. 2025. URL: https://hal.science/hal-05288185 (cit. on p. 13).

[29] R. Carles, Q. Chauleur, G. Ferriere and D. Pelinovsky. *On the ground state of the nonlinear Schrödinger equation: asymptotic behavior at the endpoint powers*. 10th Dec. 2025. URL: https://hal.science/hal-05409500 (cit. on p. 13).

[30] Q. Chauleur and G. Ferriere. *On the propagation of high regularity for the logarithmic Schrödinger equation*. 15th May 2025. URL: https://hal.science/hal-05068643 (cit. on p. 13).

[31] Q. Chauleur and A. Mouzard. *High order uniform in time schemes for weakly nonlinear Schrödinger equation and wave turbulence*. 2025. DOI: 10.48550/arXiv.2507.02662. URL: https://hal.science/hal-05164591 (cit. on p. 17).

[32] R. Côte, C. Courtès, G. Ferriere, L. Godard-Cadillac and Y. Privat. *The Spear and the Ring: Emergent Structures in Magnetic Colloidal Suspensions*. 2nd Apr. 2025. URL: https://hal.science/hal-05016866 (cit. on p. 12).

[33] A. de Laire, O. Goubet, M. E. Martínez, C. Muñoz and F. Poblete. *Dynamics of generalized abcd Boussinesq solitary waves under a slowly variable bottom*. 24th Nov. 2025. URL: https://hal.science/hal-05379838 (cit. on p. 15).

[34] M. Spriet, C. Langrenez, R. Brummelhuis and S. de Bièvre. *What is special about the Kirkwood-Dirac distributions?* 5th Nov. 2025. URL: https://hal.science/hal-05362489 (cit. on p. 16).

[35] M. Spriet. *Characterizing the Kirkwood-Dirac positivity on second countable LCA groups*. 8th Oct. 2025. URL: https://hal.science/hal-05303406 (cit. on p. 16).

[36] J. Thio, S. Yang, S. De Bièvre, C. Barnes and D. Arvidsson-Shukur. *Kirkwood-Dirac Nonpositivity is a Necessary Resource for Quantum Computing*. 2025. DOI: 10.48550/arXiv.2506.08092. URL: https://hal.science/hal-05362444 (cit. on p. 16).

Other scientific publications

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10.3 Cited publications

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