2025 *Activity Report*

RESEARCH CENTRE: **Inria Centre at the University of Lille**

IN PARTNERSHIP WITH: **Université de Lille, IMT Nord Europe**

Project-Team

POPOPOP

Point Processes from Probability and Physics

In collaboration with Laboratoire Paul Painlevé (LPP)

Project-Team POPOPOP

Creation of the Project-Team: 2025 November 01

Each year, Inria research teams publish an Activity Report presenting their work and results over the reporting period. These reports follow a common structure, with some optional sections depending on the specific team. They typically begin by outlining the overall objectives and research programme, including the main research themes, goals, and methodological approaches. They also describe the application domains targeted by the team, highlighting the scientific or societal contexts in which their work is situated. The reports then present the highlights of the year, covering major scientific achievements, software developments, or teaching contributions. When relevant, they include sections on software, platforms, and open data, detailing the tools developed and how they are shared. A substantial part is dedicated to new results, where scientific contributions are described in detail, often with subsections specifying participants and associated keywords. Finally, the Activity Report addresses funding, contracts, partnerships, and collaborations at various levels, from industrial agreements to international cooperations. It also covers dissemination and teaching activities, such as participation in scientific events, outreach, and supervision. The document concludes with a presentation of scientific production, including major publications and those produced during the year.

Keywords

Computer sciences and digital sciences

A6.1.2. – Stochastic Modeling

A6.1.3. – Discrete Modeling (multi-agent, people centered)

A6.1.4. – Multiscale modeling

A6.5. – Mathematical modeling for physical sciences

A8.1. – Discrete mathematics, combinatorics

A8.7. – Graph theory

Other research topics and application domains

B1.1.6. – Evolutionnary biology

B1.1.8. – Mathematical biology

B2.3. – Epidemiology

B3.6. – Ecology

B3.6.1. – Biodiversity

B6.3. – Network functions

B6.5. – Information systems

Contents

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2 Overall objectives

In random models, dependence may be at least as desirable as independence. This will be our paradigm in the PoPoPoP research project. We aim at making clear that strong correlations and long-range dependence among points or other geometric structures can help us tackling challenges such as conceiving metamaterials with specific macroscopic properties through optimal placement of resonators, analyzing the dynamics of social networks through their geometry, or designing more effective stochastic algorithms through strongly correlated inputs. In probability theory or mathematical statistics, most models are built on independence or its avatars (Markovianity, mixing, short-range correlations and so forth) and are well understood. On the contrary, the study of random models including strong dependence is much more lacunary. For such structures, we need new and innovative mathematical descriptions through cutting-edge mathematical tools, from probability theory but also functional analysis, (Euclidean and hyperbolic) geometry or linear algebra. In the PoPoPoP research project, we will study the relevant random point configurations, usually called point processes, and the associated random geometric structures.

3 Research program

3.1 Repulsiveness and rigidity

Our challenges mostly involve point processes that are in an intermediate state between fully random, as Poisson point processes, and quasideterministic such as perturbed lattices. Intuitively, they exhibit some rigidity properties that we need to define and explore mathematically. A fruitful concept in this context is hyperuniformity. Hyperuniformity, viewed as a rigidity property of random configurations, manifests when the fluctuations of particles are orders of magnitude smaller than those expected in a pure random system of points. More formally, it means that the variance of the number of points within a domain Δ becomes negligible relative to the volume of Δ as Δ becomes large. Historically, hyperuniformity has been explored in the context of studying the compressibility of matter [40, 46]. Nevertheless, understanding the order of the variance of the number of points holds interdisciplinary significance. In material science, hyperuniformity plays a crucial role in characterizing naturally organized structures, such as crystals and quasicrystals [47]; in statistical physics, it appears necessary to achieve equilibrium in charged fluids due to the strong repulsion of the Coulomb interaction [43, 45]; in data science, low fluctuations of random inputs is known to improve the convergence rate of Monte Carlo algorithms [28]. In all these settings, it seems that hyperuniformity emerges primarily when the random points of the configuration strongly repel each other, while being compelled to remain close to each other by confining forces. To thoroughly understand the relationship between repulsiveness and rigidity is our primary objective in this axis.

Another key concept in this context is number-rigidity [38, 39]: a point process is said to be number-rigid if, for any bounded set Δ, the number of points within Δ is almost surely determined by the knowledge of the point configuration outside Δ. At first glance, it may seem that only the perturbed lattices, resembling a crystal-like structure, may inherit this property. However, this is not the case, and a large class of interacting points is known to satisfy this property, particularly point processes with a strong repulsiveness property as in the hyperuniform case. Understanding this number rigidity property and its links with hyperuniformity is also an important goal in this axis.

3.2 Percolation and infinite directed geodesics

One may also consider random graphs or other random geometric structures built (through various geometrical rules) on the point processes under consideration. Henceforth, a very natural question is percolation, that is the emergence of an unbounded cluster or the occurrence of infinite directed geodesics. Percolation constitutes a vast and dynamic field of research, centered around the exploration of connectivity properties within random graphs. Its significance spans across diverse disciplines including physics, biology, and computer science. In physics, it serves as a fundamental tool for characterizing phenomena such as fluid flow through porous materials or the behavior of disordered systems. In computer science, it encompasses the resilience of communication networks and the dissemination of information within social networks. In mathematics, the study of discrete, lattice-based models has been extremely successful and recognized by major international prizes. Continuum percolation which gathers models based on point processes in a continuous setting, has been much less explored. Most of the known results hold for an underlying Poisson point process. Leveraging our expertise in point processes, we aim to investigate more realistic models, with interactions. Recently, such dependent continuum percolation models have appeared as natural paradigms for Device-to-Device (D2D) networks; see [42] and the recent work of our team [3]. A very exciting perspective in this field consists in introducing dynamics in the random network with moving users. This aspect is currently underrepresented in the existing literature and its investigation will necessarily require new concepts since classical tools in percolation theory are purely static. With a similar flavour, we are also interested in Boolean percolation with an underlying repulsive point process such as the Ginibre ensemble.

In some cases, as the transmission of an information towards a central node or the spreading of a rumor in a social network, it is relevant to work with directed graphs: each edge $\{x, y\}$ is now considered with a direction, say from x to y, becoming (x, y) and meaning for instance that a tweet of x has been retweeted by y. The Directed Spanning Forest (DSF) [27] represents a complex system with strong geometric dependencies in which information travels towards a targeted node. In this context, any statistics about infinite directed geodesics (as their number, asymptotic directions, density or possible scaling limit) starting at a given node are very meaningful. Several results of this kind have been already obtained by members of the team for the DSF [6, 33] and its hyperbolic version [4, 14]. Since there is no reason for a rumor to spread according to a specific direction, we also investigate isotropic exploration of node sets. A way to do it is to connect each node x to its $\ell(x)$-th closest node (according to random labels $\ell(\cdot)$). Percolation properties of the resulting graph have been investigated by members of the team in an Euclidean setting. To mimick the structure of real-world social networks, a promising perspective would be to study an hyperbolic version of this model.

3.3 Concentration and sampling

In many areas of applied science and engineering, for example in Bayesian statistics, one is led to compute integrals of the form $\int f\, d\mu$ of a given function f with respect to a specified measure μ. It is of particular interest in high dimensions, and difficult even for simple measures μ such as uniform measures on a convex body. Monte-Carlo algorithms for integration aims at giving numerical approximations of the integral $\int f\, d\mu$ by sums of the form $\sum_{i=1}^{n} f(x_i)$ (or weighted versions of the latter), where $(x_1, \ldots, x_n)$ is a well-chosen (random) sample of points. In standard methods, the points are usually chosen to be independent and identically distributed (i.i.d.) random variables with distribution μ. In this case, the behavior of the empirical measure distribution of the x_i's, its fluctuations and concentration properties are well understood through the classical law of large numbers, central limit theorem and the machinery of concentration inequalities.

It is a very natural idea to replace the i.i.d. sample $(x_1, \ldots, x_n)$ by other point processes: introducing repulsiveness between points should improve the exploration of space and lead to open the path to better concentration properties towards μ. As a consequence, this should reduce the sample size required to achieve a desired precision in the approximation of the integral. However, this efficiency gain may come at the cost of increased sampling complexity, as point processes with interactions pose greater challenges in sampling compared to i.i.d. samples. It raises questions for which very few answers are known. For a given measure μ, how to generically design point processes for which $\sum_{i=1}^{n} f(x_i)$ converges at high speed towards the integral ? What is the link between the law of the point processes and the concentration properties of the empirical measure around the target measure μ ? How difficult is it to efficiently sample such point processes ?

A few results in this direction are already available in the literature. Our team has been involved in showing concentration inequalities for Coulomb gases with confining potential [2]. However, the links between the choice of the potential and the limiting target measure is not fully understood yet. Other natural candidates would be log-gases, determinantal processes, Ries gases, and there are a lot of mathematical questions to explore around the concentration properties of those. Moreover, even if we can design point processes well-adapted to a given integration task, it can be challenging to sample them efficiently. Numerical studies for Coulomb and log-gases, as well as propositions of new simulation algorithms based on Langevin dynamics, appear in [30]. The quality of Langevin-based simulation methods were investigated in [29, 36] but under hypotheses that are generally not satisfied by Coulomb or log-gases. Our team is involved in exploring such questions in collaboration with Rémi Bardenet (CNRS, CRIStAL, Lille University), in particular through a co-advised PhD thesis funded by Rémi Bardenet's ERC starting grant project Blackjack.

4 Application domains

4.1 Metamaterials

Participants: David Dereudre.

In the realm of materials engineering, a significant challenge lies in crafting materials with the capability to effectively absorb electromagnetic, acoustic, or optical waves across a wide, predefined bandwidth. The success in achieving these macroscopic properties is intricately tied to the microscopic arrangements of resonators embedded within the material [35]. This pursuit not only holds fundamental importance but also underscores its potential applications in diverse fields, such as telecommunications, sensing technologies, and energy harvesting. Our contribution in this field is in collaboration with teams at IEMN (Institute of Electronics, Microelectronics and Nanotechnology, Lille University).

4.2 D2D networks

Participants: David Coupier, Benoît Henry.

In the future, networks and their infrastructures must address numerous constraints arising from the demand for low-latency networks to support emerging applications, cope with the escalating volume of data, all the while minimizing the environmental impact of these networks. In this context, it is likely that the modeling of Device-to-Device (D2D) networks, including mobile agents, and the analysis of such models will prove valuable in the efficient design and operation of networks. This work has already started in the context of the Beyond 5G project in which IMT is involved: see [3]. It will benefit from the IMT environment with many experts covering all non-mathematical aspects of telecommunication networks and historic collaborations with the French telecommunications companies. Understanding how long-range connections in a D2D network evolve when users are now allowed to move in the urban media is a very natural, interesting and challenging question. Starting from the static picture of a percolating network at time $t = 0$ and letting users move when time goes, one may wonder how does the moving infinite clusters of users behave? Or, focusing on a mobile and targeted user, is it often connected to the moving infinite cluster (which ensures a long-range connection)? Let us finally point out that the possibility of long-range connection via D2D networks is of capital importance for these companies. Indeed, such a network relying on the users themselves, would only require (for the company) a small investment in the network infrastructure. This possibility of having a functional network without prior investment constitutes an opportunity for new companies or operators to enter the market and therefore a real economic threat for historic companies.

4.3 Public health and Sociology

Participants: Viet Chi Tran.

We have ongoing projects with sociologists and doctors (especially from Inserm and Lille CHU) on the role of social networks in the propagation of epidemics, with a particular interest in networks of people who inject drugs (PWID), opioids users and sexual networks. Addiction, overdoses and infectious diseases that play a major role in the mortality and morbidity burden in France and abroad are major societal topics. Nevertheless, the methodologies developed can be generalized to other epidemics or applications. Social networks contribute to the dynamics of epidemics in different ways, either because the disease itself spreads across the population by means of contacts between individuals (e.g. [31, 7]) or because the information shared along social networks can modify the population behaviours: we consider to include rumors and fake news on the one hand; prevention and information about at-risk practices and contexts on the other hand. Using mathematical models, our interest is to understand how the geometry, the topology of the graph and the interactions between individuals shape the dynamic of the subgraph of people contaminated or reached by the information or the disease, and how the propagation of the latter process can in turn modify the structure of the underlying network. Our past collaborations already lead to the launch of a 'respondent sampling' survey in 2018, a technique first used in France to our knowledge, with the purpose of inferring the social network of people who inject drugs. Two PhD theses and several papers followed from this collaboration (e.g. [34, 32]). With the Covid 19 crisis, several scientific groups, in France or abroad, have worked on modelling the spread of diseases on networks. Most of the points of view developed are on (stochastic) dynamical systems (R_0 considerations), statistical analysis or numerical simulations. The subjects of PoPoPoP bring new and interesting ways to handle these questions, and we benefit from the strong interactions with sociologists.

A workgroup on the interface between mathematics and biology and medicine has been created in September by C. Tran with Olivier Bou-Aziz and François Bachoc (University of Lille).

4.4 Ecology and evolution

Participants: Benoît Henry, Mylène Maïda, Viet Chi Tran.

We are also involved with biologists (mainly of institutions participating to the Chaire MMB, including members of the EEP laboratory at Université of Lille, and of the IEES lab. at Paris Sorbonne Université) on modelling the evolutions of populations structured by a trait that affects their reproductive and survival abilities. We have two main applications in view: 1) the description of pathogens and viruses evolution with in particular the development of antibiotic resistances, 2) the conservation of biodiversity. In both cases, our understanding can be improved by the observation and study of the past history and phylogenies (see e.g. [1, 9]). An important ingredient to take into account are the interactions (selection, mutualism, predation etc.) between the individuals. The latter are shaped by the traits in the population, that can vary from an individual to the other, and influence in return the traits evolution. From individual-centered random model where the dynamics is described at the level of individuals (birth, death, horizontal transfer, mutation) by mean of stochastic differential equations driven by point processes, we can describe the evolution and the extinction or conservation of the species in large ecological network at the macroscopic population level (see the review in [26]). In the framework of the Chaire MMB, we are often lead to discuss with biologists and companies interested in these questions (Veolia, EDF and others).

4.5 Monte-Carlo integration and sampling

Participants: Mylène Maïda, Martin Rouault.

Putting everything together in the Concentration and sampling axis of the research program would lead long term to be able, for a large number of numerical integration tasks, to design repulsive processes that are well adapted to the task and that we know how to sample efficiently. In addition, we aim at providing a library implementing these procedures, in open access and intended to mathematicians and physicists, as well as research engineers.

4.6 Denoising of signals

Participants: Raphaël Butez.

Denoising techniques based on the zeros of the spectrogram of the signal were introduced by Flandrin [37] and have been studied over the past 10 years with great success. These new techniques are mostly empirical and no theoretical studies guarantee they exist. As the spectrogram of the white noise is essentially the flat Gaussian Analytic Function, a good understanding of its zeros will allow us to refine the existing techniques and to provide criteria for a good reconstruction of the signal. More precisely, we want to obtain bounds on the signal-to-noise ratio which ensure a good reconstruction with controlled probability of failure.

5 New results

5.1 IDLA in $\mathbb{Z}^d$ with infinitely many sources

Participant: David Coupier.

According to an IDLA (Internal Diffusion Limited Aggregation) protocol, when a source launches a particle, this one performs a simple random walk on $\mathbb{Z}^d$ until exiting the current aggregate A through a site x which is added to the aggregate: A is updated to $A \cup \{x\}$. In [12], we consider a random growth model based on the IDLA protocol with infinitely many sources from a hyperplane of $\mathbb{Z}^d$, say $\mathcal{H}$. Precisely, n particles are launched from each source of $\mathcal{H}$ according to a prescribed order. This process provides an infinite aggregate $A_n[\infty]$ for which we prove a shape theorem as $n \to \infty$, in any dimension $d \geq 2$, with sublogarithmic fluctuations.

In [20], we use i.i.d. exponential clocks to launch the particles during a time interval $[0, n]$, so that the next source emitting a particle is choosen *uniformly* within the (infinite) set $\mathcal{H}$. Also, we not only retain the site x by which a given particle exits the current aggregate but also the edge used by the particle to reach x. This construction leads to a random forest, denoted by $\mathcal{F}_n[\infty]$ and called the IDLA forest. With respect to the aggregate $A_n[\infty]$, the construction of the IDLA forest $\mathcal{F}_n[\infty]$ is much more intricated. Hence, we state a stabilization result for $\mathcal{F}_n[\infty]$, based on tools coming from the percolation theory, which allows us to define properly the IDLA forest as an ergodic random graph.

5.2 Percolation of the two-neighbor graph on the planar lattice

Participants: David Coupier, Benoît Henry.

One of the aims of the PoPoPoP team is to investigate how dependencies can affect large scale connectivity in random graph. A first fundamental question is whether the imposition of rigid local constraints facilitates or hinders percolation compared to standard models. A model introduced recently concerns graphs with fixed out-degrees, known as k-neighbor graphs, where each vertex connects to exactly k nearest neighbors chosen uniformly at random. This creates a degenerate random environment with specific local dependencies. In a collaboration with Benedikt Jahnel and Jonas Köppl (WIAS), we investigated the behavior of this model in the planar case. In the preprint [5], we settle a conjecture regarding the case $k = 2$ in dimension $d = 2$. We prove that the planar 2-neighbor graph percolates, meaning the origin is connected to infinity with positive probability. Our proof relies on a specific exploration algorithm and a comparison to i.i.d. bond percolation, demonstrating mathematically that the rigid constraint of having exactly 2 neighbors reduces the randomness in a way that is strictly beneficial for percolation compared to average-degree equivalents.

5.3 Line-of-sight percolation on Poisson-Delaunay triangulations

Participants: David Coupier, Benoît Henry.

Stochastic geometry is a key tool for analyzing the macroscopic properties of telecommunication networks. In D2D networks, signal propagation is heavily constrained by the street layout, which dictates line-of-sight availability. In the work [3], in collaboration with D. Corlin Marchand (former Postdoc at IMT Nord Europe), we proposed a model where the street system is represented by a Poisson-Delaunay triangulation. Users are distributed according to a Cox process supported on the edges (streets) and vertices (crossroads) of this triangulation. We study the associated connectivity graph, where links are formed based on distance and alignment along the streets. We establish a complete phase diagram of the model which improves the results existing on similar existing models, eliminating the unknown intermediate regions often found in previous works.

5.4 Measure estimation on a manifold explored by a diffusion process

Participant: Viet Chi Tran.

In the article [16], we consider the problem of estimating the stationary measure μ of a diffusion process $(X_t)_{t\in[0,T]}$ on a compact connected d-dimensional manifold $\mathcal{M}$ without boundary, from the observation of one of its path. We base our estimation on the occupation measure μ_T of this process defined for all bounded measurable test function f by

$$\int_{\mathcal{M}} f(x)\mu_T(dx) = \frac{1}{T}\int_0^T f(X_s)\, ds.$$

Wang and Zhu [48] showed that for the Wasserstein metric W_2 and for $d \geq 5$, the convergence rate of $T^{-1/(d-2)}$ is attained when $(X_t)_{t\in[0,T]}$ is a Langevin diffusion. We extend their result in several directions. First, we show that the rate of convergence holds for a large class of diffusion paths, whose generators are uniformly elliptic. Second, the regularity of the density p of the stationary measure μ with respect to the volume measure of $\mathcal{M}$ can be leveraged to obtain faster estimators: when p belongs to a Sobolev space of order $\ell \geq 2$, smoothing the occupation measure by convolution with a kernel yields an estimator whose rate of convergence is of order $T^{-(\ell+1)/(2\ell+d-2)}$. We further show that this rate is the minimax rate of estimation for this problem.

5.5 Euclidean Directed Spanning Forest and Radial Spanning Tree

Participant: Tom Garcia-Sanchez.

The Euclidean Directed Spanning Forest (DSF) is a geometric random graph whose vertex set is given by a homogeneous Poisson Point Process $\mathcal{N}$ on $\mathbb{R}^d$, $d \geq 2$, and whose (directed) edges consist of all pairs $(x, y) \in \mathcal{N}^2$ such that y is the closest point to x in $\mathcal{N}$ for the ℓ^p distance, $p \in [1, \infty]$, among points with a strictly larger e_d coordinate (the d-th vector of the canonical basis). This geometric random graph introduced by Baccelli and Bordenave in [27] in the case $p = d = 2$ admits a natural forest structure: it is a collection of unrooted directed trees. In [22], we prove that for $p \in \{1, 2, \infty\}$, the graph is almost surely a tree when $d = 3$, and consists of infinitely many disjoint trees when $d \geq 4$. Additionally, we show that for all $p \in [1, \infty]$, the DSF in dimension 2 is almost surely a tree and, under appropriate diffusive scaling, converges weakly to the Brownian web, generalizing the result of [6] for $p = 2$.

The (Euclidean) Radial Spanning Tree (RST) is a radial version of the DSF in which each vertex $x \in \mathcal{N}$ has a unique outgoing edge pointing to the nearest point in $\mathcal{N} \cup \{0\}$ that lies closer to the origin (w.r.t. the Euclidean distance ℓ^2). By construction, it forms almost surely a tree rooted at 0. In [23], we prove that the RST is almost surely *straight* in any dimension $d \geq 2$, meaning roughly that its subtrees become thinner and thinner as their roots move away from the origin. As a by-product, using the strategy developed by Howard and Newman [41], we obtain that with probability 1 each infinite branch of the RST admits an asymptotic direction and each asymptotic direction is targeted by (at least) one infinite branch.

5.6 Thick trace at infinity for the Hyperbolic Radial Spanning Tree

Participants: David Coupier, Viet Chi Tran.

In [14], we study the hyperbolic Radial Spanning Tree (RST) in any dimension $d \geq 2$. The hyperbolic RST had already been introduced and studied by the same authors in [4]. In this new paper, we are interested in its fine topological properties, and in particular, its exceptional directions which are defined as the directions $\xi \in \mathbb{S}^{d-1}$ targeted by at least two infnite branches of the RST. Exploiting the features of the hyperbolic geometry, we state that any infinite subtree T of the RST almost surely admits a thick trace at infinity, i.e. the set of directions $\xi \in \mathbb{S}^{d-1}$ targeted by an infinite branch of T has a positive measure. In the case $d = 2$, this solves the N3G problem (for *No 3 Geodesics*) for the hyperbolic RST: the hyperbolic RST in dimension 2 does not contain 3 infinite branches with the same (random) asymptotic direction with probability one.

5.7 Gibbs point processes for numerical integration

Participants: Mylène Maïda, Martin Rouault.

Markov chain Monte Carlo algorithms (MCMC) are numerical integration algorithms that are ubiquitous in high-dimensional statistical inference and Bayesian machine learning. The crux is to sample a carefully-chosen Markov chain in the domain of integration, and average the evaluations of the integrand along that chain. However, estimators resulting from MCMC algorithms have a mean squared error that decreases as $1/n$, where n is the number of time steps in the Markov chain sample. When the integrand function is very expensive to evaluate, it is relevant to design an estimator with better guarantees than MCMC for the same number of evaluations of the function (giving the same guarantees with less points), even if it requires a higher computational budget to sample.

In the framework of the PhD thesis of Martin Rouault, co-supervised by R. Bardenet and Mylène Maïda, we exploited the intuition that dependence, or repulsiveness, between points brings qualitative variance reduction, an idea which is a key point of the third axis of PoPoPoP scientific project. In particular, we propose to draw n points according to a Gibbs distribution and study the convergence properties of the associated estimator for numerical integration. In a first work in 2024, we prove a fast concentration inequality for such Gibbs distributions with general bounded pairwise interaction. Given a fixed precision level on the integration error, this means that a smaller number of points n is required to get the same confidence regions as MCMC. In the communication [18], we illustrate how this strategy can be used in the context of Bayesian inference. Approximately sampling points from this Gibbs distribution however requires computing the so-called interaction potential of the target measure. In the preprint [19], we develop a two-step-procedure taking into account an empirical approximation of the potential. The model is mathematically more involved but through a quenched large deviation principle, we can show that the guarantees previously obtained still hold, at least asymptotically, including this approximation.

5.8 Hyperuniform versus Poisson Distributions in Random Metasurfaces at Infrared Wavelengths

Participant: David Dereudre.

Metamaterials are artificial materials made of very small structures that can control how light or other electromagnetic waves behave. Unlike traditional optical materials, their properties come from the shape and arrangement of these tiny elements, called meta-atoms, rather than from their chemical composition. Most early metamaterials were arranged in very regular, periodic patterns. However, researchers have recently become interested in disordered arrangements, where meta-atoms are placed in a more random way. This approach offers more flexibility in design and can simplify fabrication, especially for large-area optical devices. One basic type of random arrangement follows a Poisson distribution, similar to how raindrops fall on the ground. In this case, the meta-atoms can overlap, which may strongly affect their optical response because each meta-atom acts like a small resonator whose behavior depends on its size and shape.

Another type of disordered structure, called hyperuniform, lies between perfect order and complete randomness. These structures are found in nature and have attracted attention in many areas of wave physics, from acoustics to optics. Hyperuniform designs can prevent overlap between meta-atoms while still being disordered, and they have already led to improved performance in several optical and electromagnetic devices.

In [44], in collaboration with colleagues at IEMN, we compare metasurfaces based on Poisson and hyperuniform distributions. They study how these two types of disorder influence light extinction and absorption in the infrared range. By analyzing how individual meta-atoms absorb light and how they interact with each other, the study shows that both disordered designs can achieve strong and broadband optical performance for realistic material densities. In [11], we analyze the extinction and absorption properties of randomly distributed Metal-Insulator-Metal metasurfaces operating at an infrared wavelength. Our attention has been focused on the comparison between correlated disordered, namely, hyperuniform, and Poisson Point Process distributed structures. To this aim, we define absorption and extinction terms by considering Floquet modes within a super-cell simulated by a finite element approach. Then, the role of particle intercoupling in high-density random metasurfaces has been pointed out and analyzed as a function of linear polarization of the impinging wave. Finally, hyperuniform and Poisson Point Process distribution have been compared for metasurface filling fraction ranging between 5% and 50% and relative standard deviations up to 1:0, taking the square periodic array as a reference.

5.9 (Non)-hyperuniformity of perturbed lattices

Participant: David Dereudre.

In [8], we study hyperuniformity properties of random point processes obtained by perturbing regular lattices in Euclidean space. Hyperuniformity is defined through the asymptotic behavior of the number variance in large observation windows: a point process is said to be hyperuniform when density fluctuations grow sublinearly with the volume. We start from the fact that stationary lattices are extremal examples of hyperuniform point configurations, exhibiting the slowest possible growth of the number variance. We then investigate whether this property is preserved when each lattice point is randomly displaced, leading to a class of perturbed lattices. The perturbations are assumed to be identically distributed and stationary, while allowing for arbitrary dependence. Our main results show that the stability of hyperuniformity depends crucially on the dimension and on the moment assumptions on the perturbations: in dimensions one and two, if the perturbations have a finite moment of order equal to the dimension, the perturbed lattice remains hyperuniform. This condition is essentially sharp, since weaker assumptions may produce point processes that are not hyperuniform and may even exhibit infinite number variance in bounded regions. In dimensions three and higher, hyperuniformity is much more fragile: we show that arbitrarily small perturbations can already destroy hyperuniformity, leading to macroscopic density fluctuations.

Concerning the stronger notion of class-I hyperuniformity, we establish a sharp sufficient condition in dimension one based solely on the size of the perturbations, while we show that in dimension two such conditions are no longer sufficient. Finally, we prove that hyperuniformity does not impose a universal decay rate for the rescaled number variance: even within the class of hyperuniform perturbed lattices, this decay can be arbitrarily slow. Overall, our results provide a precise characterization of how random perturbations affect large-scale regularity in point configurations, and they highlight the delicate balance between disorder, dimension, and correlation structure in hyperuniform systems.

5.10 Rigidity of one-dimensional point processes via optimal transport

Participants: Rafaël Digneaux, David Dereudre.

In [21], we investigate rigidity phenomena in one-dimensional point processes. We show that the existence of an L^1 transport map from a stationary lattice or the Lebesgue measure to a point process is sufficient to guarantee the properties of Number-Rigidity and Cyclic-Factor. We then apply this result to non-singular Riesz gases with parameter $s \in (-2, -1]$, defined in infinite volume as accumulation points of stationarized finite-volume Riesz gases. This includes, for $s = -1$, the well-known one-dimensional Coulomb gas (also called Jellium plasma, or the one-component 1D plasma).

5.11 Liquid-gas phase transition for Gibbs point process with Quermass interaction

Participant: David Dereudre.

In [15], we prove the existence of a liquid-gas phase transition for continuous Gibbs point process in $\mathbb{R}^d$ with Quermass interaction. The Hamiltonian we consider is a linear combination of the volume $\mathcal{V}$, the surface measure $\mathcal{S}$ and the Euler-Poincaré characteristic χ of a halo of particles (i.e. an union of balls centred at the positions of particles). We show the non-uniqueness of infinite volume Gibbs measures for special values of activity and temperature, provided that the temperature is low enough. Moreover we show the non-differentiability of the pressure at these critical points. Our main tool is an adaptation of the Pirogov-Sinaï-Zahradnik theory for continuous systems with interaction exhibiting a saturation property.

5.12 Absence of percolation for infinite Poissonian systems of stopped paths

Participants: David Coupier, David Dereudre.

In [13], we study a stopped germ–grain model in the plane based on a homogeneous Poisson point process, where each point is marked with an independent continuous path. From each point, a grain grows along its path and stops at its first intersection with another grain. Under a mild moment condition on the paths, this interacting growth process is well defined.

Our main question concerns percolation of the random union of stopped grains. We introduce a geometric loop condition, requiring that loops can form with positive probability at arbitrarily small space and time scales, and we investigate whether this condition prevents percolation.

We prove that, under the loop condition and a finite second-moment assumption on the paths, the model does not percolate almost surely. Our result substantially generalizes previous non-percolation results for thin-grain germ–grain models by allowing general continuous paths and by weakening the integrability assumptions on the growth dynamics.

5.13 Probabilistic methods for studying the partition function of the two-dimensional Yang-Mills theory

Participant: Mylène Maïda.

Getting asymptotic expansions for matrix integrals is an active topic within random matrix theory. When the coefficients of the expansions are related to geometrical or topological invariants, these expansions are called topological expansions. It is in general a hard mathematical task to show that topological expansions are not only formal power series but that they are properly convergent. In an ongoing collaboration with Thibaut Lemoine (Université de Strasbourg), we are interested in a specific matrix integral, which happens to be the partition function of a model in quantum field theory called the two-dimensional Yang-Mills theory. In [17], using probabilistic arguments on random partitions, we get a full description of the topological expansion, when the underlying manifold is the torus and the gauge group is the unitary group. In the recent preprint [25], we generalize these results to other compact Lie groups and show that the corresponding asymptotic expansions are related to the enumeration of ramified coverings of the torus. This leads to establishing rigorously a string/gauge duality result predicted by physicists Gross and Taylor in the nineties.

Although this line of research is not directly related to the main lines of research of PoPoPoP, we want to emphasize that the matrix integral giving the partition function of the studied model is closely related to a diffusion over the groupe of unitary matrices, called the unitary Brownian motion, which exhibits very interesting dependence and repulsiveness and has Gibbs measure of the type described in the previous subsection as invariant measures.

5.14 Continuous limits of large plant-pollinator random networks and some applications

Participant: Viet Chi Tran.

In [10], we study a stochastic individual-based model of interacting plant and pollinator species through a bipartite graph: each species is a node of the graph, an edge representing interactions between a pair of species. The dynamics of the system depends on the between- and within-species interactions: pollination by insects increases plant reproduction rate but has a cost which can increase plant death rate, depending on the densities of pollinators. Pollinators reproduction is increased by the resources harvested on plants. Each species is characterized by a trait corresponding to its degree of generalism. This trait determines the structure of the interaction graph and the quantities of resources exchanged between species. Our model includes in particular nested or modular networks. Deterministic approximations of the stochastic measure-valued process by systems of ordinary differential equations or integro-differential equations are established and studied, when the population is large or when the graph is dense and can be replaced with a graphon. The long-time behaviors of these limits are studied and central limit theorems are established to quantify the difference between the discrete stochastic individual-based model and the deterministic approximations. Finally, studying the continuous limits of the interaction network and the resulting PDEs, we show that nested plant-pollinator communities are expected to collapse towards a coexistence between a single pair of species of plants and pollinators.

5.15 Goodness-of-fit testing for the stationary density of a size-structured PDE

Participant: Viet Chi Tran.

In [24], we consider two division models for structured cell populations, where cells can grow, age and divide. These models have been introduced in the literature under the denomination of 'mitosis' and 'adder' models. In the recent years, there has been an increasing interest in biology to understand whether the cells divide equally or not, as this can be related to important mechanisms in cellular aging or recovery. We are therefore interested in testing the null hypothesis H_0 where the division of a mother cell results into two daughters of equal size, against the alternative hypothesis H_1 where the division is asymmetric and ruled by a kernel that is absolutely continuous with respect to the Lebesgue measure. The sample consists of i.i.d. observations of cell sizes and ages drawn from the population, and the division is not directly observed. The hypotheses of the test are reformulated as hypotheses on the stationary size and age distributions of the models, which we assume are also the distributions of the observations. We propose a goodness-of-fit test that we study numerically on simulated data before applying it on real data.

6 Bilateral contracts and grants with industry

6.1 Bilateral contracts with industry

Participants: Viet Chi Tran.

- Viet Chi Tran works with the enterprise Meteors (now part of Fourseeds) on the marketing impact on purchases of the irritation of customers when confronted to too massive mailings.

  Using a stochastic individual-based model, we analyze the influence of marketing emails and SMS strategies on customer purchasing behavior and decisions to unsubscribe. When used effectively, email and SMS campaigns can spark interest in new products and encourage purchases. However, bombarding customers with excessive emails or messages can lead to negative outcomes such as reduced brand engagement and environmental consequences. Using duration models and point processes, we account for customers exposures to the brand which can differ from an individual to the other and evolve in time. This exposure corresponds to an ad-stock point of view and is influenced by the communication received, purchase activities, and decisions to unsubscribe. A customer's exposure level to the brand affects their rate of purchases and unsubscription decisions. A working paper in collaboration with the enterprise and Annabel Salerno (IAE Lille) is in progress. After discussing the model and its statistical inference, we perform, as a case study, numerical simulations to explore the possible outcomes of advertisement policies under various scenarios, including potential saturation or customer impatience and starting from actual data from a leather goods company. In the past, a contract had been signed with Lille University. A follow up with Inria is planned.

- In the context of the possible extension of the Chaire MMB, in the steering committee of which Viet Chi Tran belongs, a workshop Modélisation mathématique et biodiversité : enjeux et outils was organized with the purpose of reuniting colleagues from the academy and enterprises. The talks alternated between presentation of private companies on their questions regarding biodiverity and the associated modelling and of academics presenting possible solutions. Note that several Inria teams where represented additionnally to PoPoPoP: Merge and Simba. The possible collaborations are still under discussions.

7 Partnerships and cooperations

7.1 International initiatives

7.1.1 Participation in other International Programs

CEFIPRA project No.6901-1

Participants: David Coupier, Tom Garcia-Sanchez, Viet Chi Tran.

Title: Directed random networks and their scaling limits

Partner Institution(s):
- Institut Mines Télécom Nord Europe, France (with french PI D. Coupier).
- Ashoka University, India (with indian PI K. Saha).

Date/Duration: from 12.09.2023 to 11.09.2026, extended to 11.09.2027.

Additionnal info/keywords: The focus is to study the structure and scaling limited related questions for networks with complex dependencies and for networks undergoing certain types of modifications (branching, dynamic evolutions or annihilation) and to develop new convergence conditions if required and apply them to different network models. Our project is based on the funding work of some of the members [6].

7.2 National initiatives

- Mylène Maïda is a member of the ANR project *Large Objects Under Combinatorial Constraints and Outside Uniform Models* (LOUCCOUM) , funded by the ANR (French National Research Agency)
- Viet Chi Tran is in the steering committee of the Chaire MMB (Chaire Mathématique et Modélisation de la Biodiversité, of Veolia-Ecole Polytechnique-Museum National d'Histoire Naturelle-Fondation X).
- Mylène Maïda is the head of the axe MEGA of the Réseau Thématique Mathématiques et Physique (RT2173) funded by CNRS Mathématiques (primary) and CNRS Physics (secondary).
- Viet Chi Tran is in the steering committee of the Réseau Thématique Matrisk (RT2167) funded by CNRS Mathématiques.

8 Dissemination

8.1 Promoting scientific activities

8.1.1 Scientific events: organisation

Member of the organizing committees

- David Dereudre was the main organizer of the kick-off meeting of the CDP C2EMPI (March 2025).

Animation of seminars and workgroups

- Raphaël Butez was co-organizer of the probability and statistics weekly seminar at the LPP until August 2025. V.C.T. is organizer since September 2025.
- Benoît Henry and David Coupier are organizers of the weekly working group *Point Processes and related fields* at the LPP.
- Viet Chi Tran is organizer of the monthly working group on statistics and biology at the LPP.
- Raphaël Butez is the organizer of the MEGA seminar, affiliated to the axis MEGA (Matrices Et Graphes Aléatoires) of the RT Mathematics and Physics.

8.1.2 Journal

Member of the editorial boards

- Mylène Maïda is an associate editor of Electronic Journal of Probability. and Electronic Communications in Probability
- Mylène Maïda is a member of the editorial board of la Gazette de la Société mathématique de France.
- David Coupier is an associate editor of the Indian Journal of Statistics Sankhya.

Reviewer - reviewing activities All permanent members of the PoPoPoP team work as referees for many of the main scientific publications in probability and statistics, depending on their respective fields of expertise.

David Coupier is a reviewer for the popularization articles written by the recipients of the Neveu Prizes and published in the french journal Matapli.

8.1.3 Invited talks

- Viet Chi Tran was invited to give a talk at the Conference Dynamics of interacting populations and beyond, in Travemünde, Germany. September 2025.
- David Dereudre has been invited to the conference The 23rd Symposium Stochastic Analysis on Large Scale Interacting Systems, Tokyo. October 2025.
- David Dereudre has been invited to the conference Quantum Many-body Systems and Bose-Einstein Condensation: A Mathematical Physics Perspective, Vienne. November 2025.
- Viet Chi Tran was invited at the Rhein-Main Kolloquium Stochastik. December 2025.
- David Coupier was invited at the colloquium of the institute of mathematics of Osnabrück university. May 2025.
- David Coupier was invited at the Mathematics Colloquium of Ashoka university, India. April 2025.

8.1.4 Leadership within the scientific community

- David Dereudre is the principal investigator of the CDP C2EMPI (Cross Disciplinary Project, Univ. Lille Program).

8.1.5 Scientific expertise

- Viet Chi Tran is an expert of the International Expertise Unit of the European and International Affairs Delegation of the MESR responsible for evaluating projects applying for various funding programmes.

8.1.6 Research administration

- David Dereudre is a member of the executive commity of the CPER WaveTech.
- David Dereudre is a member of the commity of the HUB 3 "Monde numérique au service de l'humain".
- David Dereudre is a member of the "Conseil de laboratoire" for the Laboratoire Paul Painlevé.
- David Coupier is member of the Research Council of Institut Mines Télécom Nord Europe.
- M. Maïda is a member of the council of the Graduate School Madis and member of the *jury de domaine* for mathematics.
- Mylène Maïda is member of the executive board of the CDP C2EMPI in charge of the learning program.
- Mylène Maïda is a member of the "Conseil de département de mathématiques" and "Commission de concertation disciplainire en mathématiques", Université de Lille.
- David Dereudre is a member of the "Commission mixte" for the department of Math/FST.
- Mylène Maïda is a member of the council of the Graduate School Madis and member of the *jury de domaine* for mathematics.
- David Coupier is a member of the executive board of the "Groupe MAS".
- David Coupier has been a member of 1 *comité de sélection* (Univ. Sorbonne Paris Nord for the recruitement of a Maître de Conférences). Mylène Maïda has been a member of 3 *comités de sélection* (Lille as a president of the jury, Paris-Cité and Université Clermont-Auvergne). Viet Chi Tran has been a member of 1 *comité de sélection* (Lille for the recruitment of a Professor).

8.2 Teaching - Supervision - Juries - Educational and pedagogical outreach

8.2.1 Supervision

- David Coupier and Viet Chi Tran co-supervise the PhD thesis of Tom Garcia-Sanchez who works on the Directed Spanning Forest and the Radial Spanning Tree in arbitrary dimensions and with various L^p norms. He is interested in the number of topological ends in these models.
- David Coupier has co-supervised, with N. Chenavier (ULCO) and A. Rousselle (Université de Bourgogne) the PhD thesis of K. Penner, defended in July 2025, working on a random growth model with infinitely many sources.
- David Dereudre is supervising the PhD thesis of R. Digneaux, working on rigidity for Riesz gases.
- David Dereudre and M. Maïda are co-supervising the PhD thesis of G. Bellot, working on a model of spatial permutations for bosonic interactions.
- R. Bardenet (CRIStAL, Université de Lille) and Mylène Maïda are co-supervising the PhD thesis of Martin Rouault, working on the use of Gibbs point processes for numerical integration.
- Raphaël Butez and Mylène Maïda are co-supervising the PhD thesis of Adèle Deberge, working on diffusive methods for random matrix theory.
- S. Méléard (Ec. Polytechnique) and Viet Chi Tran co-supervise the PhD thesis of M. DeAngeli Bravo on the study of phylogenies in population dynamics with horizontal transfers.
- S. Robin (Paris Sorbonne Université) and Viet Chi Tran co-supervise the PhD thesis of B. Liu on the indentifiability and statistical inference for VAR models with applications to large Lotka-Volterra dynamics in Ecology.
- Benoît Henry is co-supervising with R. Azaïs, the PhD thesis of H. Péchoux, at Inria research team Mosaïc (Lyon), working on simulation algorithms in statistical physics.
- Raphaël Butez and R. Bardenet (CRIStAL, Université de Lille) are co-supervising the postdoc of Q. François, working on denoising on signals with zeros of Gaussian analytic functions.

8.2.2 Juries

- David Dereudre has been a member in 2025 of the PhD defense committee of C. Langrenez (Université de Lille).
- Mylène Maïda has been a member of the PhD defense committee of H. Lebeau (Université Grenoble Alpes), J. Zurcher (Université de Lille), M. Rousselot (Université de Poitiers), Q. François (PSL, referee), J. Giral (PSL).
- Viet Chi Tran has been a member of the PhD defense committee of A. Barnier (Inrae, referee), N. Dinh-Toàn (Université Gustave Eiffel, advisor). He was also a member of the HDR defense committee of C. Fritsch (Université de Lorraine, referee), S.V. Bitseki Penda (Université de Bourgogne, referee) and H. Mohamed (Université Paris Nanterre).

8.2.3 Educational and pedagogical outreach

- Mylène Maïda is the head of the research track of the Master 2 in Mathematics, Université de Lille.
- Mylène Maïda has taught the Course Advanced probability in the research track of the Master 2 in Mathematics, Université de Lille.
- Raphaël Butez and M. Maïda have taught a specialized course entitled *Determinantal Point Processes* in the research track of the Master 2 in Mathematics, Université de Lille.
- Raphaël Butez has taught the course Theory of Statistical learning of the Master 2 in Applied Mathematics, Université de Lille.

8.3 Popularization

8.3.1 Specific official responsibilities in science outreach structures

- Viet Chi Tran is Secretary of the CFEM (Commission Française pour l'Enseignement des Mathématiques). He host with L. Broze (Université de Lille) a working group on AI and Education.

8.3.2 Productions (articles, videos, podcasts, serious games, ...)

- Viet Chi Tran is the founder (with C. Baey) and host of the YouTube channel Briques2math. Briques2math tells maths degree students (but not only them!) about careers in maths and what maths is like after graduation! This channel may also be of interest to secondary school teachers and businesses...
- Raphaël Butez participated in events for high school students (introduction to research for high school students, presentation of mathematical careers in high schools).

9 Scientific production

9.1 Major publications

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[3] D. Corlin Marchand, D. Coupier and B. Henry. 'Line-of-sight Cox percolation on Poisson-Delaunay triangulation'. In: *Stochastic Processes and their Applications* (2024). URL: https://hal.science/hal-04452230 (cit. on pp. 6, 8, 10).

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9.2 Publications of the year

International journals

[10] S. Billiard, H. Leman, T. Rey and V.-C. Tran. 'Continuous limits of large plant-pollinator random networks and some applications'. In: *MathematicS In Action* (2025). URL: https://cnrs.hal.science/hal-03525607 (cit. on p. 14).

[11] R. Buisine, O. Vanbesien, D. Dereudre, L. Burgnies and E. Lheurette. 'Absorption properties of hyperuniform and Poisson Point Process distributed metasurfaces at infrared wavelengths'. In: *Journal of Applied Physics* 138.6 (14th Aug. 2025), p. 063101. DOI: 10.1063/5.0272612. URL: https://hal.science/hal-05374899 (cit. on p. 12).

[12] N. Chenavier, D. Coupier, K. Penner and A. Rousselle. 'IDLA with sources in a hyperplane of $\mathbb{Z}^d$'. In: *Electronic Journal of Probability* 30.136 (21st July 2025). DOI: 10.1214/25-EJP1389. URL: https://hal.science/hal-04506736 (cit. on p. 9).

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Conferences without proceedings

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Reports & preprints

[19] R. Bardenet, M. Maïda and M. Rouault. *Quenched large deviations for Monte Carlo integration with Coulomb gases*. 5th Aug. 2025. DOI: 10.48550/arXiv.2508.01392. URL: https://hal.science/hal-05213356 (cit. on p. 12).

[20] N. Chenavier, D. Coupier, K. Penner and A. Rousselle. *Construction of ergodic IDLA forests in $\mathbb{Z}^d$*. 11th June 2025. URL: https://hal.science/hal-05108139 (cit. on p. 10).

[21] D. Dereudre and R. Digneaux. *Rigidity of one-dimensional point processes via optimal transport*. 17th Oct. 2025. URL: https://hal.science/hal-05318149 (cit. on p. 13).

[22] T. Garcia-Sanchez. *The Directed Spanning Forest: coalescence versus dimension*. 17th July 2025. URL: https://hal.science/hal-05168383 (cit. on p. 11).

[23] T. Garcia-Sanchez. *The Radial Spanning Tree is straight in all dimensions*. 13th Jan. 2026. URL: https://hal.science/hal-05456932 (cit. on p. 11).

[24] v. H. Hoang, P. T. Nguyen, T. M. Pham Ngoc, V. Rivoirard and V. Chi Tran. *Goodness-of-fit testing for the stationary density of a size-structured PDE*. 10th June 2025. URL: https://hal.science/hal-05105010 (cit. on p. 15).

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9.3 Cited publications

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