

2025 Activity Report

RESEARCH CENTRE: Inria Centre at the University of Lille
IN PARTNERSHIP WITH: Université de Lille

Project-Team

RAPSODI

Reliable numerical approximations of dissipative systems

In collaboration with Laboratoire Paul Painlevé (LPP)



Project-Team RAPSODI

Creation of the Project-Team: 2017 November 01

Each year, Inria research teams publish an Activity Report presenting their work and results over the reporting period. These reports follow a common structure, with some optional sections depending on the specific team. They typically begin by outlining the overall objectives and research programme, including the main research themes, goals, and methodological approaches. They also describe the application domains targeted by the team, highlighting the scientific or societal contexts in which their work is situated. The reports then present the highlights of the year, covering major scientific achievements, software developments, or teaching contributions. When relevant, they include sections on software, platforms, and open data, detailing the tools developed and how they are shared. A substantial part is dedicated to new results, where scientific contributions are described in detail, often with subsections specifying participants and associated keywords. Finally, the Activity Report addresses funding, contracts, partnerships, and collaborations at various levels, from industrial agreements to international cooperations. It also covers dissemination and teaching activities, such as participation in scientific events, outreach, and supervision. The document concludes with a presentation of scientific production, including major publications and those produced during the year.

Keywords

Computer sciences and digital sciences

- A6. – Modeling, simulation and control
 - A6.1. – Methods in mathematical modeling
 - A6.1.1. – Continuous Modeling (PDE, ODE)
 - A6.1.4. – Multiscale modeling
 - A6.1.5. – Multiphysics modeling
 - A6.2. – Scientific computing, Numerical Analysis & Optimization
 - A6.2.1. – Numerical analysis of PDE and ODE
 - A6.2.6. – Optimization
 - A6.5.2. – Fluid mechanics
 - A6.5.3. – Transport
 - A6.5.4. – Waves
 - A6.5.5. – Chemistry

Other research topics and application domains

- B1.1.8. – Mathematical biology
- B3. – Environment and planet
 - B3.3. – Geosciences
 - B3.3.1. – Earth and subsoil
 - B3.4. – Risks
 - B3.4.2. – Industrial risks and waste
- B4. – Energy
 - B4.2. – Nuclear Energy Production
 - B4.2.1. – Fission
- B9.5.2. – Mathematics
- B9.5.3. – Physics
- B9.5.4. – Chemistry

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2 Overall objectives

Together with the diffusion of scientific computing, there has been a recent and impressive increase of the demand for numerical methods. The problems to be addressed are everyday more complex and require specific numerical algorithms. The quality of the results has to be accurately assessed, so that in-silico experiments results can be trusted. Nowadays, producing such reliable numerical results goes way beyond the abilities of isolated researchers, and must be carried out by structured teams.

The topics addressed by the RAPSODI project-team belong to the broad theme of numerical methods for the approximation of the solutions to systems of partial differential equations (PDEs). Besides standard convergence properties, a good numerical method for approximating a physical problem has to satisfy at least the following three criteria:

- (a) preservation at the discrete level of some crucial features of the solution, such as positivity of solutions, conservation of prescribed quantities (e.g., mass), decay of physically motivated entropies, free energies, etc;
- (b) provide accurate numerical approximations at a reasonable computational cost (and ultimately maximize the accuracy at a fixed computational effort);
- (c) robustness with respect to physical conditions: the computational cost for a given accuracy should be essentially insensitive to a change in physical parameters.

We contribute to the development of methods fulfilling the above quality criteria for physical models which display, in their vast majority, a dissipative behavior, and that are motivated by industrial collaborations or multidisciplinary projects.

Ideally, we should allow ourselves to design entirely new numerical methods. For some applications however (often in the context of industrial collaborations), the members of the team have to work with existing codes. The numerical algorithms have thus to be optimized under this constraint.

Some technological bottlenecks related to points (a)–(c) mentioned above are well identified. In particular, it appears that a good numerical method should handle general meshes, so that dynamic mesh adaptation strategies can be used in order to achieve (b). But it should also be of the highest possible order while remaining stable in the sense of (a), and robust in the sense of (c). There have been numerous research contributions on each point of (a)–(c) in the last decades, in particular for solving each difficulty separately, but combining them still leads to unsolved problems of crucial interest. Yet, before addressing the above points (a)–(c) concerning the design of nice numerical methods, one has to identify the structural properties of (and possibly modify) the continuous model to be discretized.

In a nutshell, our goal is to take advantage of and extend the most recent breakthroughs of the mathematical community to tackle in an efficient way some application-guided problems coming either from academics or from industrial partners. To this end, we focus on the following objectives, which are necessary for the applications we work on and which define the three research axes of our project:

1. Study of the structural properties of continuous PDE models.
2. Design and numerical analysis of structure-preserving numerical methods.
3. Computational optimization of the numerical methods.

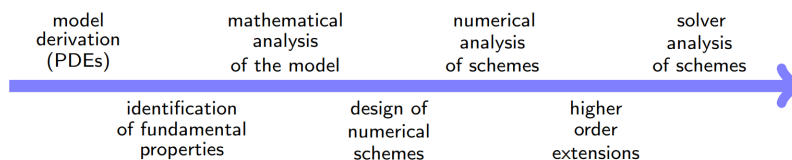


Figure 1: We consider the problem resolution in a global manner, starting from the model derivation (for instance based on physical principles) to the practical development of efficient numerical methods.

The originality of the RAPSODI research team is its holistic approach as illustrated on Figure 1.

The study of the structural properties of the models under consideration helps in designing robust and accurate numerical schemes. It also helps in the design of efficient solvers. Our research program in these three axes, which constitute the core of the RAPSODI research project, is now going to be further detailed.

3 Research program

3.1 Axis 1: Study of the structural properties of continuous models

A well-behaved continuous (PDE based) model is a requisite to a reliable numerical approximation. The fine understanding of a continuous model is indeed a prior to the design of a numerical scheme in order to identify the structural properties to be preserved at the discrete level. Further, the mathematical analysis carried out on the continuous model often paves the way for the theoretical foundations of the numerical methods, hence our strong interest in this first research axis. Analyzing time-dependent PDE systems relying on their mathematical (Hamiltonian, gradient flow, . . .) structure is very natural and unsurprisingly not new. However, the interest in this domain strongly increased in the last two decades. The focus of the RAPSODI project-team encompasses kinetic models, fluid mechanics, electromagnetism, multicomponent systems and applied calculus of variations. While carrying out theoretical analyses, we bear in mind that our results should be transposed to the discrete setting.

3.1.1 Thermodynamically consistent models for multi-component systems

Mathematical tools for the derivation and the mathematical analysis of models which are compatible with the second principle of thermodynamics and more specifically with Onsager’s reciprocal principle [146] widely developed in the last decade. The general setting proposed in [141, 147] offers a powerful and robust framework for obtaining such models in configurations where inertia can be neglected.

This situation is typical of porous media flows in the Darcy regime. When rich physics are considered, like in presence of multiple phases and components within deformable and fractured matrices, the derivation of such thermodynamically consistent models is a timely problem [153, 154, 156]. One of the goals of the RAPSODI project-team is to contribute to the mathematical understanding of such models [5].

Inertia is also commonly neglected in models for semi-conductors of van Roosbroeck type, as for instance in models for the corrosion of iron [87]. In the latter framework, the geometry of the domain varies along time and is one of the unknowns of the problem. The reference model proposed in [87] is not built from thermodynamical considerations. The design, the simulation and the calibration of a mathematical model which is consistent with the second principle is an ongoing task addressed by our team (see [104] for preliminary results).

3.1.2 Variational models and optimal transport

Optimal transport has a long history. In Monge’s seminal work “Théorie des déblais et des remblais” dating back to 1781 [155], the problem was roughly stated the following way: how should one proceed to move piles of sand from one point to another in order to minimize the workload? This problem is now often referred to as “Monge problem”. It has stayed with no solution in the general case until the 1940s, when Kantorovich inserted it into a proper framework [135], which allowed eventually to tackle it and provide solutions. A

major breakthrough was obtained by Brenier [96], who showed that under very broad assumptions, the general solution to the optimal transport problem had a very specific form and could be linked with the famous Monge–Ampère equation.

In recent years, optimal transport has become a very active field of research, due to the discovery of new and efficient algorithms [122] and to its wide range of applications, to economy, image processing, analysis of partial differential equations or data sciences (see e.g. references in [149]).

One of the focus of our team is the use of optimal transport for understanding the formation of bi-layer cellular membranes thanks to a model introduced in [148]. In this model, the global shape of micelles or liposomes should be the result of competing forces: a short-range attractive force given by the area of the boundary, and a non-local repulsive force of Wasserstein-type (that is, an optimal transport cost). Our aim is to explore numerically this problem and obtain theoretically the shape of the solutions in some asymptotic regimes.

3.1.3 Kinetic models of large particle systems and their asymptotic regimes

Nearly 150 years after Boltzmann paved the way to the kinetic representation of complex phenomena [95], this approach for modeling is more than ever active. This increasing interest is partially due to the development of computational facilities that make the numerical approximation of such models possible, but also to the great flexibility of the approach which can be applied in biology [152], economics [124] or social sciences [97], way beyond its original scope in physics of gases.

One focus of our team is the asymptotic analysis of these equations. The large-time limit of kinetic equations has been dealt with thanks to hypocoercivity techniques in [83] and new hydrodynamic limits have been derived from kinetic models for gases composed of macroscopic particles interacting via energy dissipative collisions [133, 112] and plasmas [132]. A future concern will be the study of the long time dynamics of a kinetic model for relativistic electron bunches in storage rings [150] (see Section 4.6).

3.2 Axis 2: Design and analysis of structure-preserving numerical methods

The more complex is the PDE system to be discretized, the smaller is the chance that a naive discretization preserves at the discrete level its mathematical structure. Since this structure was the cornerstone of the mathematical analysis (typically by providing the well-posedness and the stability of the continuous system under consideration), its preservation at the discrete level will be key to get well-behaved and theoretically certified numerical methods. Our team concentrates a large part of its research effort on the design of provably convergent numerical methods, either based on the popular two-point flux approximation finite volume method, or on more flexible yet less natural structure preserving methods. We also pay attention to the fact that our schemes are robust with respect to the parameters and in the long time limit. The numerical approximation of complex (inhomogeneous or with low compressibility) flows is one of the topics we address.

3.2.1 Structure-preserving methods for systems of dissipative PDEs and variational models

A very important part of the activity of our team concerns the design of structure-preserving numerical methods for dissipative PDEs.

In this regard, two-point flux approximation (TPFA) finite volume schemes are very popular in industrial codes since they enjoy remarkable stability properties at a moderate computational price. Our team has a strong experience in the design of provably convergent entropy stable TPFA finite volume schemes for complex problems [76, 101, 107, 130, 110, 4, 100, 116, 105, 108, 110]. Ongoing [106, 144] and future research directions are based on the discretization of some action functionals and their optimization to obtain so-called variational schemes based on TPFA finite volumes. We also mention the Lagrangian approach [128], for which the computation of some optimal tessellation based on semi-discrete optimal transportation shows subtle but deep similarities with Eulerian TPFA based methods.

The main drawback of TPFA finite volume schemes is that they (essentially) restrict to isotropic diffusion problems and to regular grids [127, 129]. The strong constraints on the meshes and on the isotropic nature of the problems under consideration for TPFA schemes motivated in the last decades the development of more robust finite volume schemes, see for instance [125] for a review on this topic. These developments were particularly motivated by applications in geosciences, where anisotropic porous media are common

and where (possibly poorly regular) meshes are often prescribed by data. Tuning such advanced methods, which do not enjoy monotonicity properties in general, to make them thermodynamically consistent is a key challenge for our team. An active research topic of our team is to propose new structure-preserving numerical schemes beyond TPFA. Our team has made important preliminary contributions on this topic through the design of structure-preserving control volume finite elements schemes [77, 78], vertex centered finite volume schemes [6], discrete duality finite volume schemes [103], finite element schemes [109] and hybrid finite volume schemes [115]. The latter numerical method also constitutes a first step towards high-order methods (see Section 3.2.2).

We intend to pursue our contributions in the development of structure-preserving numerical methods, both based on simple TPFA finite volumes and on more advanced robust methods. We will keep working on always more complex porous media flows, more advanced semi-conductor devices, or on non-standard flow models.

3.2.2 High-order structure-preserving numerical methods

Taking advantage of our experience in the design of low-order structure-preserving numerical schemes, we intend to improve the accuracy of our numerical methods by going to high-order discretizations in the space and time variables.

There are important contributors in the community in these topics. For instance, the development of high-order in space methods allowing for general grids for diffusion problems was pushed forward in the last decade by several groups, for instance around Lourenço Beirão da Veiga and Franco Brezzi [90] for the Virtual Element Methods (VEM), around Daniele A. Di Pietro and Alexandre Ern [10, 123] for the Hybrid High-Order (HHO) methods, or around Bernardo Cockburn [120] for the Hybridizable Discontinuous Galerkin (HDG) methods.

Our team aims at designing high-order in space and entropy stable schemes based on the HHO technology. We also aim at developing high-order methods on general meshes for electromagnetism in the context of a collaboration with EDF (French electricity supply company), with application to the non-invasive control of nuclear plants (see Section 4.4).

Concerning the time discretization strategies, we take inspiration in the works by Giovanni Samaey [136] for projective integration schemes, and in those by Christophe Besse [92] and Jie Shen [98] for relaxation methods. It appears that (explicit) Strong Stability Preserving high-order time discretizations [131], which are very popular in the community working on hyperbolic systems, are not suitable for solving dissipation driven (and typically parabolic) systems for which implicit methods are commonly used.

At the moment, most of our energy stable methods rely on merely first-order time discretizations, typically Backward Euler or minimizing movement schemes [134]. The extension to higher order time discretizations is already the purpose of ongoing studies in our team. Two tracks have been identified so far and are already studied: (i) the variational BDF2 scheme [140] which generalizes to second order in time the minimizing movement scheme, and (ii) Runge-Kutta type schemes obtained after having changed of main variables. Both approaches have their pros and their cons: (i) is difficult to implement, whereas (ii) might lead to non-conservative methods. Our goal for the next years on this topic is to be able to overpass the difficulties of (at least one of) these methods and to get efficient implementation strategies as well as theoretical foundations.

3.2.3 Asymptotic-preserving and multiscale numerical methods

It has been understood around year 2000 that the fine quantification of the entropy dissipation allows one to deduce fine properties on the asymptotic behavior of the solutions to dissipative PDEs [81, 111]. The extension to the discrete setting of this type of tools is more recent, see for instance [113, 93, 114]. Our team is involved in the study of the long-time behavior of numerical schemes for drift-diffusion models [94, 102, 7, 115] and kinetic equations [2, 84]. A future challenge is the study of the long time behavior properties for numerical schemes which can handle general meshes and high order accuracy (see Section 3.2.1 and Section 3.2.2).

We also contribute actively to the design and analysis of multiscale methods for highly heterogeneous (or highly oscillatory) diffusion problems. We have extended in [119] within the HHO framework the nonconforming multiscale method of [137] to arbitrary approximation orders (and its analysis also to the

case of general meshes). We have also established in [117] an equivalence result between our multiscale HHO method and the MHM method of [80], improving in passing on both methods.

In the future we will continue our effort in the design of numerical schemes which enjoy accurate asymptotic properties in large time, extreme regimes of parameters and multiscale settings. This effort, which aims at increasing the robustness of our numerical methods, is strongly correlated with the preservation of energy or entropy dissipation structures at the discrete level (cf. Section 3.2.1), as explained above. A particular effort has begun in the investigation of asymptotic preserving schemes to pass from kinetic models to van Roosbroeck type models for semiconductor devices.

3.2.4 Numerical methods for optimal transport

In the last years, optimal transport and its generalisations have proven to be a powerful tool to analyse a large class of models, exposing useful mathematical structures which are responsible of their properties [79], and guiding the conception of novel numerical schemes. Due to its geometrical and physical meanings, optimal transport has also emerged as a useful modelling tool in different contexts including economics, biology or social sciences [151].

Transportation problems can often be cast as global space-time optimisation problems such as in variational mean field games or optimal planning problems [91]. Part of the activity of our team in this area is concerned with the analysis of such models in the discrete setting, which poses significant difficulties both in terms of the conception of schemes (which often passes through the definition of discrete variational problems) [144][143], and of the devising of efficient optimization algorithms to compute their solutions [12].

As currently many of these problems are still intractable at the numerical level (due to their high dimensionality and lack of smoothness), our aim is to further improve the current approaches, for example by using unstructured meshes in space-time, which would pave the way for the development of adaptive refinement techniques. Importantly, such new schemes could be used as a building block to construct new discretisations for PDEs with a variational structure related to optimal transport (such as Wasserstein gradient flows [145][13]). In particular, this could represent an additional research path to reach our objectives mentioned in Section 3.2.1.

3.3 Axis 3: Computational optimization of the numerical methods

Good numerical methods shall enjoy strong theoretical foundations allowing to guarantee their behavior in very general situations. They shall also be highly efficient from a computational point of view, so that they can be used in practice for solving real-world problems.

To increase the efficiency and applicability of our methods, we work on three tracks. First, we aim at building a unified and optimized software platform to implement, test and diffuse our numerical methods. Second, since our methods often yield nonlinear systems, the improvement of the nonlinear solvers is also key to increase the efficiency of our methods. Finally, for high-dimensional models such as kinetic and particle systems, the curse of dimensionality makes numerical computations realistically feasible only if specific computationally efficient numerical strategies are deployed.

3.3.1 Development of the platform ParaSkel++

ParaSkel++ [89] is a C++ platform, conceived by Simon Lemaire and mainly developed since December 2022 by T. Zoto (succeeding L. Beaudé), which is freely distributed under LGPL v3.0. The ParaSkel++ platform aims at the high-performance, arbitrary-order, 2/3D numerical approximation of PDEs on general polytopal meshes using skeletal Galerkin methods (see [139]).

A first version (v1, August 2021) of the platform is operational, featuring a sequential implementation of all the main skeletal methods (Lagrange FE, VEM and HHO). The next, already ongoing, crucial development steps are the parallelization on shared and distributed memory, and the implementation of efficient quadrature formulas on polytopal cells. Eventually, the ParaSkel++ platform is expected to possess five main assets with respect to other codes of the same nature from the community: (i) a unified 2/3D implementation, (ii) the native support of any type of DOF (vertex-, edge-, face-, and cell-based), (iii) an ultra-factorized architecture (with common-to-all-methods local elimination and global assembly steps), (iv) the use of

efficient quadrature formulas on general polytopal cells (without the need for subtessellation), and (v) the embedding of parallel computation capabilities.

3.3.2 Design of robust nonlinear solvers

The entropy stable methods we build often yield nonlinear systems to be solved at each time step. It is therefore of paramount importance to have fast and robust nonlinear solvers at hand to address them, hence the increasing interest for so-called nonlinear preconditioning techniques. In [3], we proposed a strategy which consists in expressing the problem in terms of a new primary variable which, if well chosen, allows to remove the degeneracies of the system under consideration. This strategy has then been extended to the case of more complex systems arising in the context of porous media flows [86, 85]. In the framework of a collaboration with IFPEN, we develop new solvers for the computation of chemical equilibria (see [42]), with application to reactive transport. The methodology being developed in this framework will then be transposed to other contexts in an as generic as possible way.

4 Application domains

4.1 Subsurface CO₂ storage

The capture and storage in the subsurface of carbon dioxide is commonly acknowledged as a promising solution to mitigate the emission of greenhouse gas from localized production sites, as for instance cement plants. The safety assessment of the subsurface sequestration sites requires advanced numerical tools building on theoretically certified numerical models and algorithms which remain valid in the long time limit. Among the numerous difficulties encountered in such a setting, let us mention the high level of coupling between the mechanics of the (solid) porous matrix [121], the multiphase and multicomponent character of the fluid which flows therein [88], and chemical reactions with a wide range of characteristic times [138]. Despite important differences (at the level of chemistry especially), similar problems occur in the emerging topic of dihydrogen subsurface storage, or native dihydrogen (generated by the corrosion of iron in an aqueous environment) migration in the context of nuclear waste repository management.

Together with colleagues from applied research institutes and academics, the RAPSODI project-team contributes to the derivation of so-called *thermodynamically consistent models*, the stability of which in the long-time limit being guaranteed by the second principle. The team also contributes to the design and the analysis of numerical schemes for multiphase and multicomponent flows in complex geometries (possibly allowing for general meshes), and to the design of fast and robust solvers for chemical equilibria.

4.2 Material sciences

The team is interested in the theoretical and numerical analysis of mathematical models describing the degradation of materials, as concrete carbonation and corrosion. The study of such models is an important environmental and industrial issue. Atmospheric carbonation degrades reinforced concretes and limits the lifetime of civil engineering structures. Corrosion phenomena issues occur for instance in the reliability of nuclear power plants and the nuclear waste repository. The study of the long time evolution of these phenomena is of course fundamental in order to predict the lifetime of the structures.

4.3 Complex fluid flows

The team is interested in numerical methods for the simulation of systems of PDEs describing complex flows, like for instance mixture flows, granular gases, rarefied gases, or quantum fluids.

Variable-density, low-Mach flows have been widely studied in the recent literature because of their applicability in various phenomena such as flows in high-temperature gas reactors, meteorological flows, flows with convective and/or conductive heat transfer or combustion processes. In such cases, the resolution of the full compressible Navier–Stokes system is not adapted, because of the sound waves' speed. The Boussinesq incompressible model is not a better alternative for such low-speed phenomena, because the compressibility effects cannot be totally cancelled due to large variations of temperature and density. Consequently, some models have been formally derived, leading to the filtering of the acoustic waves by the use of some formal

asymptotic expansions and two families of methods have been developed in the literature in order to compute these flows. We are interested in particular in the so-called pressure-based methods, which are more robust than density-based solvers, although their range of validity is in general more limited.

Quantum models can be used to describe superfluids, quantum semiconductors, weakly interacting Bose gases, or quantum trajectories of Bohmian mechanics. They have attracted considerable attention in the last decades, due in particular to the development of nanotechnology applications. To describe quantum phenomena, there exists a large variety of models. In particular, there exist three different levels of description: microscopic, mesoscopic, and macroscopic. The quantum Navier–Stokes equations deal with a macroscopic description in which the quantum effects are taken into account through a third-order term called the quantum Bohm potential. This Bohm potential arises from the fluid dynamical formulation of the single-state Schrödinger equation. The nonlocality of quantum mechanics is approximated by the fact that the equations of state do not only depend on the particle density but also on its gradient. These equations were employed to model field emissions from metals and steady-state tunneling in metal-insulator-metal structures, and to simulate ultra-small semiconductor devices.

4.4 Electromagnetism for non-invasive control

The RAPSODI project-team works on the development of high-order polyhedral methods for electromagnetism. A well-known specificity in electromagnetism is that topology plays a crucial role in the well-posedness of the models. Dedicated analysis tools must then be deployed for their study [82]. We are interested in the devising of HHO methods in the curl/curl setting [118]. The mathematical analysis of HHO methods in this setting is particularly involved, as most of the needed discrete functional analysis tools are currently lacking. The design of (efficient and robust) multilevel linear solvers for statically condensed HHO approximations of electromagnetic models, as well as of computable (reliable and locally efficient) a posteriori error estimators on polyhedral cells are two other, still largely unexplored, aspects we are interested in. Our main target application, in the framework of a recently initiated collaboration with EDF (see Section 8.2), is the simulation of eddy current testing (ECT). ECT is used by EDF as a non-invasive control technique to assess the integrity of heat exchanger tubes in nuclear plants. The forward problem consists in solving the time-harmonic 3D Maxwell's equations in domains featuring more or less complex flaws. The use of high-order polyhedral methods is expected to yield improvements on two aspects. First, the support of polyhedral cells is expected to ease the full meshing process, and in particular enable to account for defects with complex geometries/topologies. Second, the increase in the approximation order is expected to yield a reduction of the noise on the computed control signal based on which the presence of a defect is inferred.

4.5 Large population models in epidemiology

Developing mathematical models to describe how infectious pathogens spread in animal populations is an essential step to identify the main biological mechanisms or environmental factors which contribute to the emergence of epidemics. When describing a pathogen spread at large scales, it is often relevant to model the distributions of hosts (which may represent individuals, animals or herds, for example) as spatially varying densities, and to model the pathogen dynamics via PDEs which describe the combined effect of neighbourhood interactions, large scale population dynamics, and environmental factors.

The team initiated a collaboration with INRAE (National Research Institute on Agriculture and Environment) on the modeling of different types of pathogen spread mechanisms in such continuous models. A first focus of this collaboration is on neighbourhood interactions, which are a dominant factor in the infection dynamics of many pathogens (a specific example is the Bovine Viral Diarrhea Virus, an endemic disease present worldwide among industrial cattle herds, and causing annually large economic losses), and which at large scales may be described via non-local (convolution) terms. The analysis of the resulting model requires the development of dedicated mathematical tools, and will lead to a better understanding of the influence of neighborhood interactions on the spatial features of the epidemic dynamics. A second focus is on arboviruses, which are pathogens transmitted to mammals by the bite of arthropod vectors, mainly mosquitoes, and causing diseases such as the Zika virus, the Rift Valley fever and the West Nile virus. In this case, the aim is to produce a comprehensive description of the viral dynamics both in the vectors and in the hosts as well as to model their interaction in space (via diffusion processes and nonlocal interactions), which will then be used to identify the main mechanisms driving the virus spread.

4.6 Particle accelerators

Relativistic electron bunches are used in storage rings to produce intense radiation in various ranges of frequencies. The dynamics of these bunches is nonlinear because of interactions between electrons in the bunch. Moreover, these interactions occur in an asymmetric fashion because of the relativistic nature of the dynamics. The stability properties of the bunch (and thus the long-time properties of the dynamics) have been shown, both theoretically and experimentally, to be crucial in the understanding of the intensity of the emitted radiation [99, 126].

From a mathematical point of view, the dynamics of the electron density in the phase space can be described by a Vlasov–Fokker–Planck type equation [99] with a well suited mean-field self-interaction term [142]. The mathematical understanding of the long time behavior of solutions to this equation and the design of adapted numerical schemes constitutes a challenging and physically important problem.

5 Social and environmental responsibility

An important part of the scientific activity of the RAPSODI research group relates to energy transition. For instance, our research on next generation semi-conductor devices aims at better understanding the behavior of perovskite solar cells. The research on complex porous media flows are motivated by CO₂ subsurface sequestration. The corrosion model we develop should be used by ANDRA for high fidelity simulation of the **CIGEO nuclear waste repository** (Bure, France).

The RAPSODI team members, and more particularly Claire Chainais-Hillairet and Simon Lemaire, are strongly involved in the promotion of science for young (high-school or bachelor) students. Claire Chainais-Hillairet co-organized the one-week (female only) internship *Les Fourmis {éclairées}*, which was held at Université de Lille in April 2024 and April 2025 and that will hold in April 2026. S. Lemaire was also involved in the 2025 edition. Simon Lemaire co-organized in October 2024 and October 2025 the *Rendez-vous des Jeunes Mathématiciennes et Informatiennes* (RJMI). This two-day event is specifically geared towards female high school students, and aims at promoting scientific careers amongst them. Simon Lemaire is also in charge of the local implementation of the program *1 scientifique, 1 classe / Chiche !*, for which he has realized a number of interventions in high school classrooms of the Hauts-de-France region.

6 Latest software developments, platforms, open data

Participants: Simon Lemaire, Marien-Lorenzo Hanot.

6.1 Latest software developments

6.1.1 ParaSkel++

Keywords: PDEs, Polytopal meshes, High-order methods, HPC, C++

Functional Description: ParaSkel++ is a C++ toolbox for the arbitrary-order, 2D/3D polytopal approximation of PDEs using skeletal Galerkin methods. Skeletal methods form a vast family of numerical approaches for the approximation of PDE-based models, which satisfies the two following building principles:

- (1) the degrees of freedom (DoFs) of the method split into (i) skeletal DoFs, attached to the geometric entities (vertices, edges, faces) composing the mesh skeleton and common to all cells sharing the geometric entity in question, which prescribe the conformity properties of the underlying discrete functional space, and (ii) bulk DoFs (potentially), attached to the interior of the cells, which play no role in the prescription of the conformity properties of the underlying discrete space,
- (2) the global discrete bilinear form of the problem (possibly after linearization, if the problem is nonlinear) writes as a sum over the mesh cells of local (cell-wise) bilinear contributions.

The very structure underpinning skeletal methods grants them the property of being amenable to static condensation. Locally to each mesh cell, the bulk DoFs can indeed be eliminated in terms of the local skeletal DoFs. As a consequence, the final global system to be solved only writes in terms of the skeletal DoFs of the method. Examples of skeletal methods include standard FE methods, as well as more recent polytopal methods (VE, HDG, HHO...). Remark that (plain-vanilla) DG methods do not enter the skeletal framework. ParaSkel++ offers a high-performance, factorized C++ architecture for the implementation of arbitrary-order skeletal methods on general 2D/3D polytopal partitions.

A first version (v1) of the platform has been released in August 2021, which features a sequential implementation of all the main skeletal methods for toy PDE problems. Ongoing developments concern, apart from the implementation of skeletal schemes for more advanced PDE models (e.g., in electromagnetism), the parallelization of the code on shared and distributed memories, as well as the implementation of efficient quadrature formulas on polytopal cells.

Eventually, the ParaSkel++ platform is expected to possess five main assets: (i) a unified 2D/3D implementation, (ii) the native support of any type of DoFs (vertex-, edge-, face-, and cell-based), (iii) a factorized architecture (with common-to-all-methods local elimination and global assembly steps), (iv) the use of efficient quadrature formulas on general polytopal cells (without the need for subtessellation), and (v) the embedding of parallel computation capabilities.

The ParaSkel++ toolbox has been conceived and is maintained by Simon Lemaire. The main other core developers have been Laurence Beauce (from 02/2020 to 08/2021) and Thoma Zoto (from 12/2022 to 06/2024). The toolbox is freely distributed under GNU LGPL v3.0.

URL: <https://gitlab.inria.fr/simlemai/paraskel>

Publication: hal-03517921

Contact: Simon Lemaire

Participants: Emil Hössjer, Laurence Beauce, Thoma Zoto, Silvano Pitassi, Jeremy Dalphin

6.1.2 DivDivSpline

Keywords: Numerical analysis, Partial differential equation

Functional Description: Library implementing finite element spaces with matrix values constructed using spline tensor products. The spaces considered form a particular differential complex, called the DivDiv complex. This construction is applied to the numerical approximation of a linearization of Einstein's field equations (or more specifically, York's reformulation of the ADM equations).

Contact: Marien-Lorenzo Hanot

7 New results

7.1 Modeling, analysis and numerical simulation of multi-component systems

Participants: Alain Blaustein, Clément Cancès, Claire Chainais-Hillairet, Amélie Dupouy, Maxime Herda, Juliette Venel.

In [16], Maxime Herda, Claire Chainais-Hillairet *et al.* present the numerical analysis and simulations of a multi-dimensional memristive device model. Memristive devices and memtransistors based on two-dimensional (2D) materials have demonstrated promising potential for neuromorphic computing and next-generation memory technologies. Their charge transport model describes the drift-diffusion of electrons, holes, and ionic defects self-consistently in an electric field. They incorporate two types of boundary models: ohmic and Schottky contacts. The coupled drift-diffusion partial differential equations are discretized using a physics-preserving Voronoi finite volume method. It relies on an implicit time-stepping scheme

and the excess chemical potential flux approximation. They demonstrate that the fully discrete nonlinear scheme is unconditionally stable, preserving the free-energy structure of the continuous system and ensuring the nonnegativity of carrier densities. Novel discrete entropy-dissipation inequalities for both boundary condition types in multiple dimensions allow us to prove the existence of discrete solutions. They perform multi-dimensional simulations to understand the impact of electrode configurations and device geometries, focusing on the hysteresis behavior in lateral 2D memristive devices. Three electrode configurations – side, top, and mixed contacts – are compared numerically for different geometries and boundary conditions. These simulations reveal the conditions under which a simplified one-dimensional electrode geometry can well represent the three electrode configurations. This work lays the foundations for developing accurate, efficient simulation tools for 2D memristive devices and memtransistors, offering tools and guidelines for their design and optimization in future applications.

In [41], Maxime Herda *et al.* design, analyze and simulate a finite volume scheme for a cross-diffusion system which models chemotaxis with local sensing. This system has the same Lyapunov function (or entropy) as the celebrated minimal Keller-Segel system, but unlike the latter, its solutions are known to exist globally in 2D. The long-time behavior of solutions is only partially understood which motivates numerical exploration with a reliable numerical method. They propose a linearly implicit, two-point flux finite volume approximation of the system. They show that the scheme preserves, at the discrete level, the main features of the continuous system, namely mass conservation, non-negativity of solution, entropy dissipation, and duality estimates. These properties allow them to prove the wellposedness, unconditional stability and convergence of the scheme. They also show rigorously that the scheme possesses an asymptotic preserving (AP) property in the quasi-stationary limit. They complement their analysis with thorough numerical experiments investigating convergence and AP properties of the scheme as well as its reliability with respect to stability properties of steady solutions.

In [48] Alain Blaustein, Claire Chainais-Hillairet, Maxime Herda *et al.* consider a stationary drift-diffusion system with ionic charge carriers and external generation of electron and hole charge carriers. This system arises, among other applications, in the context of semiconductor modeling for perovskite solar cells. Thanks to truncation techniques and iterative energy estimates, they show the existence and uniform upper and lower bounds on the solutions. The dependency of the bounds on the various parameters of the model is investigated numerically on physically relevant test cases.

In [64], Juliette Venel *et al.* initiate a mathematical investigation of a PDE model for the transport of high voltage direct current via overhead lines. They prove the existence of infinitely many solutions, give necessary conditions for existence, explicitly compute the continuum of all radial solutions, and develop a new numerical algorithm for this problem.

In [53], Clément Cancès, Claire Chainais-Hillairet, and Amélie Dupouy study a toy model for the evolution of the oxygen concentration in an oxide layer. It consists in a transient convection diffusion equation in a one-dimensional domain of variable width. The motions of the boundaries are governed by the traces of the concentration. The authors exhibit a necessary and sufficient condition on the parameters involved in the model for the existence of a unique traveling-wave solution. Moreover, they show that the model admits some universal entropy structure, in the sense that any convex function of the concentration yields a dissipated free energy (up to exchanges with the outer environment at the boundaries). Then the authors propose an implicit in time arbitrary Lagrangian-Eulerian finite volume scheme based on Scharfetter-Gummel fluxes. It is shown to be unconditionally convergent, to preserve exactly the travelling wave, and to dissipate all the aforementioned free energies. Numerical experiments show that the scheme is first order accurate in time and second order in space, and that the transient solution converges in the long-time limit towards the traveling-wave solution.

The physical processes of heat conduction, liquid water percolation, and phase changes govern the transfer of mass and energy in snow. They are therefore at the heart of any physics-based snowpack model. In the last decade, the use of Richards' equation has been proposed to better represent liquid water percolation in snow. While this approach allows the explicit representation of capillary effects, it can also be problematic as it usually presents a large increase in numerical complexity and cost. This notably arises from the problem of applying a water retention curve in a fully-dry medium such as snow, leading to a divergence and degeneracy in Richards' equation. Moreover, the difficulty of representing both dry and wet snow in a single framework hinders the concomitant solving of heat conduction, phase changes, and liquid percolation. Rather, current models employ a sequential approach, which can be subject to non-physical overshoots. To treat these problems, Clément Cancès *et al.* propose in [66] the use of a regularized water retention curve (WRC),

that can be applied to dry snow. Combined with a variable switch technique, this opens the possibility of solving the energy and mass budgets in a fully consistent and tightly coupled manner, whether the snowpack contains dry and/or wet regions. To assess the behavior of the proposed scheme, the authors compare it to other implementations based on loose-coupling between processes and on the state-of-the-art strategies in snowpack models. Results show that the use of a regularized WRC with a variable switch greatly improve the robustness of the numerical implementation, consistently allowing for large timesteps, which results in faster and cheaper simulations.

Entropy and the second principle of thermodynamics are regularly used as an analysis tool in applied mathematics for physics-based numerical models. In essence, this approach states that the second principle (i.e. the non-destruction of entropy) is closely related to stability. Consequently, numerical models complying with the second principle are expected to be more robust than models that do not. A notable advantage of this method is its straight-forward generalization to nonlinear physics and to systems of coupled equations. The goal of the work [67] by Clément Cancès *et al.* is to thus investigate the added-value of such an entropybased analysis to the case of snowpack modelling. For that, the authors study the conditions under which the physics describing snowpacks respects the second principle and the numerical schemes that preserve this compliance after temporal and spatial discretization. Specifically, we consider three cases of increasing complexity: (i) a dry snowpack governed by heat conduction only (meant to be an example of the method for unfamiliar readers), (ii) a system composed of a canopy and a snowpack exchanging heat, and (iii) a dry snowpack with heat conduction, vapor diffusion, and ice-vapor phase changes.

Multiphase chemical equilibrium problems lead to nonlinear systems with complementarity constraints, which become particularly challenging when phases may vanish. In [72], Clément Cancès *et al.* introduce a new algebraic formulation of the equilibrium problem based on extended mole fractions, derived from the subdifferential of the Gibbs free energy, and establish its equivalence with the classical minimization problem. Their analysis provides new conditions ensuring the uniqueness of solutions, even when some phases disappear. Building on this formulation, the authors propose two parametrized Newton-based strategies: one reformulates the relation between species quantities and chemical potentials, while the other parametrizes the complementarity conditions directly. Numerical experiments on a system with 72 species and 22 phases confirm the robustness and efficiency of the proposed methods. In tests with randomized inputs, both strategies achieve success rates above 90% with moderate iteration counts, outperforming established approaches such as the Newton-min and Fischer-Burmeister complementarity functions, and interior-point methods.

7.2 Analysis and numerical simulation in electromagnetism and related fields

Participants: Théophile Chaumont-Frelet, Marien-Lorenzo Hanot, Simon Lemaire.

In [25] Théophile Chaumont-Frelet proposes a novel a posteriori error estimator for the Nédélec finite element discretization of time-harmonic Maxwell's equations. After the approximation of the electric field is computed, he proposes a fully localized algorithm to reconstruct approximations to the electric displacement and the magnetic field, with such approximations respectively fulfilling suitable divergence and curl constraints. These reconstructed fields are in turn used to construct an a posteriori error estimator which is shown to be reliable and efficient. Specifically, the estimator controls the error from above up to a constant that tends to one as the mesh is refined and/or the polynomial degree is increased, and from below up to constant independent of p . Both bounds are also fully-robust in the low-frequency regime. The properties of the proposed estimator are illustrated on a set of numerical examples.

In [57], Théophile Chaumont-Frelet, Jérôme Droniou and Simon Lemaire establish Maxwell compactness results for the Discrete De Rham (DDR) polytopal complex: sequences in this polytopal complex with bounded discrete $H(\text{curl})$ (resp. discrete $H(\text{curl})$) norm and orthogonal to discrete gradients (resp. discrete curls) have L^2 -relatively compact potential reconstructions. The proof of these results hinges on the design of novel quasi-interpolators, that map the minimal-regularity de Rham spaces onto the discrete DDR spaces and form a commuting diagram. A full set of (primal and adjoint) consistency properties is established for these quasi-interpolators, which paves the way to convergence proofs, under minimal-regularity assumptions, of DDR schemes for partial differential equations based on the de Rham complex. The analysis is performed with

generic mixed boundary conditions, also covering the cases of no boundary conditions or fully homogeneous boundary conditions, and leverages recently introduced liftings from the DDR complex to the continuous de Rham complex.

The CHDG method is a hybridizable discontinuous Galerkin (HDG) finite element method suitable for the iterative solution of time-harmonic wave propagation problems. Hybrid unknowns corresponding to transmission variables are introduced at the element interfaces and the physical unknowns inside the elements are eliminated, resulting in a hybridized system with favorable properties for fast iterative solution. In [75], Théophile Chaumont-Frelet *et al.* extend the CHDG method, initially studied for the Helmholtz equation, to the time-harmonic Maxwell equations. They prove that the local problems stemming from hybridization are well-posed and that the fixed-point iteration naturally associated to the hybridized system is contractive. They propose a 3D implementation with a discrete scheme based on nodal basis functions. The resulting solver and different iterative strategies are studied with several numerical examples using a high-performance parallel C++ code.

In [44], Simon Lemaire and Silvano Pitassi prove discrete versions of the first and second Weber inequalities on $H(\text{curl}) \cap H(\text{div})$ -like hybrid spaces spanned by polynomials attached to the faces and to the cells of a polyhedral mesh. The proven hybrid Weber inequalities are optimal in the sense that (i) they are formulated in terms of $H(\text{curl})$ - and $H(\text{div})$ -like hybrid semi-norms designed so as to embed optimally (polynomially) consistent face penalty terms, and (ii) they are valid for face polynomials in the smallest possible stability-compatible spaces. The results are valid on domains with general, possibly non-trivial topology. In a second part, related discrete Maxwell compactness results are also proved, still within a general topological setting.

In [35], Simon Lemaire, Silvano Pitassi *et al.* devise and analyze hybrid polyhedral methods of arbitrary order for the approximation of div-curl systems on three-dimensional domains with non-trivial topology. The div-curl systems under consideration stem from magnetostatics, and can either be first-order (for field formulations) or second-order (for vector potential formulations). The mathematical analysis crucially leverages the discrete Weber inequalities established in [44]. An in-depth numerical assessment of the approach is also performed, covering, in particular, the tricky case of non-simply-connected domains.

In [65], Marien-Lorenzo Hanot *et al.* prove discrete Poincaré inequalities that are uniform in mesh size for the discrete de Rham complex of differential forms, extending known inequalities for gradient, curl, and divergence operators to polytopal domains of arbitrary dimension and topology. A crucial part of the proof involves deriving Poincaré inequalities for the cochain complex supported on the polytopal mesh. These inequalities have independent significance, aiding in the existence and stability analysis of solutions to numerical schemes such as Mimetic Finite Differences, Compatible Discrete Operators, and Discrete Geometric Approaches.

In [70], Marien-Lorenzo Hanot *et al.* provide a reformulation of the linearized Arnowitt–Deser–Misner equations as a Hodge–Dirac wave system built upon the divdiv complex, a structure that naturally handles issues of gauge fixing, constraint propagation, and tensor symmetries in numerical relativity. The well-posedness of the resulting system is established, and a discretization strategy with error estimates under minimal assumptions is proposed.

7.3 Analysis of dissipative models and their discretization

Participants: Matthieu Alfaro, Alain Blaustein, Maxime Herda, Andrea Natale, Marc Pegon, Juliette Venel.

In [71], Maxime Herda, Marc Pegon *et al.* provide a result of exponential stability for several dissipative linear kinetic equations with heavy-tailed equilibria. The approach, inspired by the so-called L^2 -hypocoercivity method, is robust enough to provide estimates that are uniform in the anomalous diffusion limit. Moreover, it is able to deal with bounded domains with periodic boundary condition or general Maxwell boundary condition (from the pure specular to the pure diffusive case). In addition, this framework accommodates linear collisional operators that act simultaneously on the velocity and spatial variables.

In [18], Matthieu Alfaro, Maxime Herda, and Andrea Natale consider an epidemic model with distributed-contacts. When the contact kernel concentrates, one formally reaches a very degenerate Fisher-KPP equation with a diffusion term that is not in divergence form. They make an exhaustive study of its travelling waves.

For every admissible speed, there exist not only a unique non-saturated (smooth) wave but also infinitely many saturated (sharp) ones. Furthermore their tails may differ from what is usually expected. These results are thus in sharp contrast with their counterparts on related models.

In [50], Alain Blaustein *et al.* design an asymptotic preserving scheme for the Vlasov-Poisson system in the quasineutral regime. They first establish a convergence result for the continuous solution with optimal error estimates. Following this path, they propose a fully discrete numerical method and rigorously prove that it is uniformly consistent. Finally, they perform several numerical simulations to illustrate the behavior of the proposed scheme which confirm the theoretical findings: stability and asymptotic preservation.

In [74], Juliette Venel *et al.* study the the long-term safety of the geological repository of nuclear wastes. A diffusion equation with a moving free boundary in one dimension is introduced and studied. The model describes some mechanisms involved in corrosion processes at the surface of carbon steel canisters in contact with a claystone formation. The main objective of the paper is to prove the existence of weak solutions to the problem which are maximal in time. For this, a time semidiscrete minimizing movements scheme based on a Wasserstein-like distance is introduced. The existence of solutions to the scheme is proved. Then, using a priori estimates, it is shown that as the time step goes to zero these solutions converge up to extraction towards a maximal weak solution to the free boundary model.

7.4 A posteriori error analysis

Participants: Théophile Chaumont-Frelet.

In [24] Théophile Chaumont-frelet proposes new a posteriori error estimators for non-conforming finite element discretizations of second-order elliptic PDE problems. These estimators are based on novel reformulations of the standard Prager-Synge identity, and enable to prove efficiency estimates without extra stabilization terms in the error measure for a large class of discretization schemes. He proposes a residual-based estimator for which the efficiency constant scales optimally in polynomial degree, as well as two equilibrated estimators that are polynomial-degree-robust. One of the two estimators further leads to guaranteed error bounds.

In [55], Théophile Chaumont-Frelet considers second-order PDE problems set in unbounded domains and discretized by Lagrange finite elements on a finite mesh, thus introducing an artificial boundary in the discretization. Specifically, he considers the reaction diffusion equation as well as Helmholtz problems in waveguides with perfectly matched layers. The usual procedure to deal with such problems is to first consider a modeling error due to the introduction of the artificial boundary, and estimate the remaining discretization error with a standard a posteriori technique. A shortcoming of this method, however, is that it is typically hard to obtain sharp bounds on the modeling error. In this work, he proposes a new technique that allows to control the whole error by an a posteriori error estimator. Specifically, he proposes a flux-equilibrated estimator that is slightly modified to handle the truncation boundary. For the reaction diffusion equation, we obtain fully-computable guaranteed error bounds, and the estimator is locally efficient and polynomial-degree-robust provided that the elements touching the truncation boundary are not too refined. This last condition may be seen as an extension of the notion of shape-regularity of the mesh, and does not prevent the design of efficient adaptive algorithms. For the Helmholtz problem, as usual, these statements remain valid if the mesh is sufficiently refined. The theoretical findings are completed with numerical examples which indicate that the estimator is suited to drive optimal adaptive mesh refinements.

In [58], Théophile Chaumont-Frelet *et al.* analyse adaptive combined field integral equations for Helmholtz problems. While the exterior Helmholtz problem with Dirichlet boundary conditions is always well-posed, the associated standard boundary integral equations are not if the squared wavenumber agrees with an eigenvalue of the interior Dirichlet problem. Combined field integral equations are not affected by this spurious resonances but are essentially restricted to sufficiently smooth boundaries. For general Lipschitz domains, the latter integral equations are applicable through suitable regularization. Under fairly general assumptions on the regularizing operator, they propose a posteriori computable error estimators for corresponding Galerkin boundary element methods of arbitrary polynomial degree. We show that adaptive mesh-refining algorithms steered by these local estimators converge at optimal algebraic rate with respect to the number of underlying boundary mesh elements. In particular, they consider mixed formulations involving

the inverse Laplace-Beltrami as regularizing operator. Numerical examples highlight that in the vicinity of spurious resonances the proposed adaptive algorithm is significantly more performant when applied to the regularized combined field equation rather than the standard one.

In [59], Théophile Chaumont-Frelet *et al.* consider linear reaction-diffusion equations posed on unbounded domains, and discretized by adaptive Lagrange finite elements. To obtain finite-dimensional spaces, it is necessary to introduce a truncation boundary, whereby only a bounded computational subdomain is meshed, leading to an approximation of the solution by zero in the remainder of the domain. They propose a residual-based error estimator that accounts for both the standard discretization error as well as the effect of the truncation boundary. This estimator is shown to be reliable and efficient under appropriate assumptions on the triangulation. Based on this estimator, they devise an adaptive algorithm that automatically refines the mesh and pushes the truncation boundary towards infinity. They prove that this algorithm converges and even achieves optimal rates in terms of the number of degrees of freedom. They finally provide numerical examples illustrating the key theoretical findings.

In [60], Théophile Chaumont-Frelet *et al.* consider a two dimensional biharmonic problem and its discretization by means of a symmetric interior penalty discontinuous Galerkin method. Based on the "div-div" complex, a novel split of an error measure based on a generalized Hessian into two terms measuring the conformity and nonconformity of the scheme is proven. This splitting is the departing point for the design of a new reliable and efficient error estimator, which does not involve any stabilization. Such an error estimator can be bounded from above by the standard residual error estimator. Numerical results assess the theoretical predictions, including the efficiency of the proposed estimator.

In [61], Théophile Chaumont-Frelet *et al.* study a posteriori error estimates and their use for time-dependent acoustic scattering problems, formulated as a time-dependent boundary integral equation based on a single-layer ansatz. The integral equation is discretized by the convolution quadrature method in time and by boundary elements in space. They prove the reliability of an error estimator of residual type and study the resulting space-adaptive mesh refinements. Moreover, they present a simple modification of the convolution quadrature method based on temporal shifts, which recovers, for the boundary densities, the full classical temporal convergence order $2m - 1$ of the temporal convolution quadrature method based on the m -stage convolution quadrature semi-discretization. They numerically observe that the adaptive scheme yields asymptotically optimal meshes for an acoustic scattering problem in two dimensions.

7.5 Analysis and numerical simulation of variational models

Participants: Marc Pegon.

In [52], Marc Pegon *et al.* investigate generalized liquid drop models with screened Riesz-type interactions, focusing in particular on truncated Coulomb and Yukawa potentials in three dimensions. They establish the existence of non spherical minimizers for some values of the screening parameter. This gives the first evidence of such minimizers in the class of repulsive, radial, and radially nonincreasing kernels in three dimensions. They further show that in the classical Riesz case, the widely-believed conjecture that minimizers are either balls or do not exist remains consistent with their results, but only just. Indeed they observe that the energy-per-mass ratios of the best balls and of the best cylinders are surprisingly close.

7.6 Complementary topics in numerical and PDE analysis

Participants: Matthieu Alfaro, Claire Chainais-Hillairet, Cindy Guichard, Théophile Chaumont-Frelet, Emmanuel Creusé, Andrea Natale, Juliette Venel.

In [49], Matthieu Alfaro, Claire Chainais-Hillairet *et. al* consider the so-called field-road diffusion model in a bounded domain, consisting of two parabolic PDEs posed on sets of different dimensions and coupled through (symmetric) nonlinear exchange terms. They propose a new and rather direct functional inequalities approach to prove the exponential decay of a relative entropy, and thus the convergence of the solution towards the stationary state selected by the total mass of the initial datum.

In [30], Théophile Chaumont-Frelet *et al.* investigate the approximation properties of solutions to the Ginzburg-Landau equation (GLE) in finite element spaces. Special attention is given to how the errors are influenced by coupling the mesh size h and the polynomial degree p of the finite element space to the size of the so-called Ginzburg-Landau material parameter κ . As observed in previous works, the finite element approximations to the GLE are suffering from a numerical pollution effect, that is, the best-approximation error in the finite element space converges under mild coupling conditions between h and κ , whereas the actual finite element solutions possess poor accuracy in a large pre-asymptotic regime which depends on κ . In this paper, they provide a new error analysis that allows them to quantify the preasymptotic regime and the corresponding pollution effect in terms of explicit resolution conditions. In particular, they are able to prove that higher polynomial degrees reduce the pollution effect, i.e., the accuracy of the best-approximation is achieved under relaxed conditions for the mesh size. They provide both error estimates in the H^1 - and the L^2 -norm and illustrate their findings with numerical examples.

In [32], Théophile Chaumont-Frelet *et al.* design a quasi-interpolation operator from the Sobolev space H_0^1 to its finite-dimensional finite element subspace formed by piecewise polynomials on a simplicial mesh with a computable approximation constant. The operator 1) is defined on the entire H_0^1 , no additional regularity is needed; 2) allows for an arbitrary polynomial degree; 3) works in any space dimension; 4) is defined locally, in vertex patches of mesh elements; 5) yields optimal estimates for both the H^1 seminorm and the L^2 norm error; 6) gives a computable constant for both the H^1 seminorm and the L^2 norm error; 7) leads to the equivalence of global-best and local-best errors; 8) possesses the projection property. Its construction follows the so-called potential reconstruction from a posteriori error analysis. Numerical experiments illustrate that our quasi-interpolation operator systematically gives the correct convergence rates in both the H^1 seminorm and the L^2 norm and its certified overestimation factor is rather sharp and stable in all tested situations.

In [54], Théophile Chaumont-Frelet analysis the well-posedness (or lack thereof) of three-dimensional time-harmonic wave propagation problems modeled by the Helmholtz equation. It is well-known that if the problem is set in bounded domain with Dirichlet boundary conditions, then the Helmholtz problem is well-posed for all (real-valued) frequencies except for a sequence of countably many resonant frequencies that accumulate at infinity. In fact, if the domain is sufficiently smooth, this can be quantified further and Weyl's law states that the number of resonant frequencies less than a given $k > 0$ scales as k^3 . On the other hand, scattering problems set in \mathbb{R}^3 with a radiation condition at infinity and a bounded obstacle modeled by variations in the PDE coefficients are well-posed for all frequencies under mild regularity assumption on such coefficients. In 2001, Filinov provided a counter example of a rough coefficient such that the scattering problem is not well-posed for (at least) a single frequency k . In this contribution, he uses this result to show that for all $\varepsilon > 0$ one can design a rough coefficient corresponding to a compactly supported obstacle such that the scattering problem is ill-posed for a countable sequence of frequencies accumulating at infinity, and such that the number of such frequencies less than any given $k > 0$ scales as $k^{3-\varepsilon}$.

In [62], Théophile Chaumont-Frelet *et al.* construct potentials for the exterior derivative, in particular, for the gradient, the curl, and the divergence operators, over domains with shellable triangulations. Notably, the class of shellable triangulations includes local patches (stars) in two or three dimensions. The operator norms of the potentials satisfy explicitly computable bounds that depend only on the geometry. They thus compute upper bounds for constants in Poincaré-Friedrichs inequalities and lower bounds for the eigenvalues of vector Laplacians. As an additional result with independent standing, they establish Poincaré-Friedrichs inequalities with computable constants for the L^p de Rham complex over bounded convex domains, derived as explicit operator norms of regularized Poincaré and Bogovskii potential operators. They express all our main results in the calculus of differential forms and treat the gradient, curl, and divergence operators as instances of the exterior derivative. Computational examples illustrate the theoretical findings.

In [63], Théophile Chaumont-Frelet *et al.* introduce a quasi-interpolation operator that maps the infinite-dimensional Sobolev space H^1 into its finite-dimensional Lagrange finite element subspace formed by piecewise polynomials on a tetrahedral mesh. This operator enjoys the following key properties: 1) it is defined over the entire H^1 and includes essentially boundary conditions imposed on a part of the boundary; 2) it is defined locally in a neighborhood of each tetrahedron of the mesh; 3) it is based on simple piecewise polynomial projections; 4) it is stable in the H^1 -seminorm; 5) it has optimal (local-best) approximation properties; 6) it commutes with its sibling operator on $H(\text{curl})$; 7) it is a projector.

In [56] Théophile Chaumont-Frelet propose an algorithm to numerically determined whether a second-order linear PDE problem satisfying a Gårding inequality is well-posed. This algorithm further provides

a lower bound to the inf-sup constant of the weak formulation, which may in turn be used for a posteriori error estimation purposes. The numerical lower bound is based on two discrete singular value problems involving a Lagrange finite element discretization coupled with an a posteriori error estimator based on flux reconstruction techniques. He shows that if the finite element discretization is sufficiently rich, the lower bound underestimates the optimal constant only by a factor roughly equal to two.

In [33], Emmanuel Creusé, Juliette Venel *et al.* consider the propagation of shock waves on nonuniform grids. This phenomenon appears, for example, in some impact problems of the fast-dynamics type, where a finer mesh can typically be employed in regions of interest, leading to mesh ratios that can range from 10 to 20 or even more. Unfortunately, unwanted spurious reflections occur for this type of problem using (standard) explicit finite element software. An optimized explicit Runge-Kutta-Nyström time-integration scheme is therefore built to minimize spurious wave reflections for shock wave propagation on these nonuniform grids.

In [51] Andrea Natale *et al.* study the problem of reconstructing a Laguerre tessellation from the volumes and the barycenters of its cells. They show that any vector of barycenters arising from a Laguerre tessellation with prescribed cell volumes can be understood as an exposed point of a convex set defined by convex order relations. Leveraging this characterization, they formulate a projection problem that can be solved efficiently and yields an approximation of the desired reconstruction. Importantly, the same method can also construct a Laguerre tessellation that fits prescribed volumes and barycenters in cases where these are not, a priori, related to any Laguerre tessellation. This strategy may find application in the analysis of experimental data from polycrystalline samples.

In [73] Andrea Natale *et al.* provide a precise characterization of the link between backward/forward Wasserstein projections in convex order and the recently introduced metric extrapolation problem. They use such a link to derive new quantitative stability estimates for both problems.

In [69] Cindy Guichard provides a general framework for second-order elliptic problems, which includes a variety of boundary conditions. She shows how one can apply mass-lumped mixed finite element to this problem, and she provides sufficient conditions for the convergence of such a method. In particular, she exhibits convergence results assuming two different type of assumptions: on one hand, she shows convergence properties following the standard analysis of mixed finite elements. On the other hand, she provides conditions on one of the approximation space, which also lead to some convergence properties. She then formulates Abstract Gradient Discretization method (AGDM) based on these mass-lumped mixed finite elements, enabling her to apply this type of discretization to a variety of nonlinear problems. Finally, she illustrates her results by two examples. The first one is a second-order elliptic problem with homogeneous Neumann boundary conditions, discretized by Raviart-Thomas finite elements. She shows on this example that mass-lumping leads to classical finite volume schemes. The second one is inspired by the elliptic part of a model of shallow water flows with dispersive terms. She applies on this example generalized operators, and she proves the convergence of the method used in the literature.

7.7 Habilitation (HDR) theses

Participants: Juliette Venel.

On December 16, Juliette Venel defended her Habilitation à Diriger des Recherches (HDR), entitled "Theoretical and Numerical Contributions to the Study of Differential Inclusions and Partial Differential Equations" at the Polytechnic University of Hauts-de-France.

8 Bilateral contracts and grants with industry

8.1 Bilateral contracts with industry

Participant: Clément Cances.

Clément Cancès heads the **MATHSOUT** project (1M euros, 2024–2029) together with I. Faille (IFPEN). This project involves academic partners (Inria, UniCA, CNRS) as well as **IFPEN** and the French Geological survey (BRGM). Ngoc Do Quyen Dang started her PhD at IFPEN in November 2024 in the framework of the **MATHSOUT** project. The contract follows the lines of the bilateral agreement between Inria and IFPEN.

8.2 Bilateral grants with industry

Participant: Simon Lemaire.

In 2023, the PRCE project **HIPOTHEC** (HIgh-order POLYhedral meTHods for Eddy Current testing simulations) has been funded in the generic ANR call. This 6-year project, which started in January 2024, is coordinated by Simon Lemaire and is a collaboration between Inria, EDF, and 3 additional academic partners. The aim of the project is to pursue, within an enlarged academic consortium, the research efforts initiated in the framework of the France Relance collaboration between the team and EDF R&D. More details about the **HIPOTHEC** project can be found in Section 9.3.2.

9 Partnerships and cooperations

9.1 International initiatives

9.1.1 Associate Teams

Participants: Alain Blaustein, Clément Cancès, Claire Chainais-Hillairet, Théophile Chaumont-Frelet, Maxime Herda, Andrea Natale.

Since 2025, Maxime Herda is the principal investigator of Inria **ARISE** associate research team. Alain Blaustein, Clément Cancès, Claire Chainais, Thomas Crozon and Amelie Dupouy are also members of the associated team. The **ARISE** project (Analysis of Robust Numerical Solvers for Innovative Semiconductors in View of Energy Transition) focuses on the development of advanced mathematical and numerical models for innovative semiconductor devices, such as perovskite solar cells and memristors, to support the global energy transition. This collaboration between the **RAPSODI** team at the Inria Centre at the University of Lille and the **NUMSEMIC team at WIAS Berlin** builds upon several years of joint research on drift-diffusion systems for charge transport. The project aims to address key challenges in modeling these devices, particularly involving nonlinear diffusion, boundary conditions, and multi-scale phenomena. By combining rigorous mathematical analysis, robust numerical methods, and practical applications, **ARISE** aims at enhancing the understanding and simulation of cutting-edge semiconductor technologies. Budget: 8k€/year for research visits.

9.2 International research visitors

9.2.1 Visits of international scientists

Participants: Alain Blaustein, Clément Cancès, Claire Chainais-Hillairet, Théophile Chaumont-Frelet, Marien-Lorenzo Hanot, Maxime Herda, Simon Lemaire, Andrea Natale.

Other international visits to the team

M. Ingremeau

Status Full prof.

Institution of origin: Institut Fourier

Country: France

Dates: 29-31 Feb.

Context of the visit: Work with Théophile Chaumont-Frelet in the context of the AEx POPEG.

Mobility program/type of mobility: research stay

A. Bespalov

Status Associate prof.

Institution of origin: Univ. Birmingham

Country: United-Kingdom

Dates: 10-13 March

Context of the visit: Work with Théophile Chaumont-Frelet in the context of the ANR APOWA.

Mobility program/type of mobility: research stay

G. Gantner

Status Associate prof.

Institution of origin: Univ. Bonn

Country: Germany

Dates: 10-13 March

Context of the visit: Work with Théophile Chaumont-Frelet in the context of the ANR APOWA.

Mobility program/type of mobility: research stay

Z. Dong

Status Junior researcher

Institution of origin: Inria Paris

Country: France

Dates: 3-4 July

Context of the visit: Work with Théophile Chaumont-Frelet on quad-curl equations.

Mobility program/type of mobility: research stay funded by Inria Paris

M. Melenk

Status Full prof.

Institution of origin: TU Wein

Country: Austria

Dates: 8 Sept. - 3 Oct.

Context of the visit: Work with Théophile Chaumont-Frelet on time-harmonic elastodynamic equations.

Mobility program/type of mobility: research stay as “Professeur invité”

J. Dörner

Status PhD student

Institution of origin: KIT

Country: Germany

Dates: 1 Sept. 2025 - 28. Feb 2026

Context of the visit: Work with Théophile Chaumont-Frelet on interface conditions for Maxwell equations.

Mobility program/type of mobility: 6 month research stay during PhD thesis

G. Gantner

Status Assistant prof.

Institution of origin: Univ. Bonn

Country: Germany

Dates: 10-14 Nov.

Context of the visit: Work with Théophile Chaumont-Frelet on a posteriori estimation in unbounded domains.

Mobility program/type of mobility: research stay funded by Univ. Bonn

Z. Dong

Status Junior researcher

Institution of origin: Inria Paris

Country: France

Dates: 24-27 Nov.

Context of the visit: Work with Théophile Chaumont-Frelet on quad-curl equations.

Mobility program/type of mobility: research stay funded by Inria Paris

L. Mascotto

Status Assistant Prof.

Institution of origin: Univ. Milano Bicocca

Country: Italy

Dates: 1-3 Dec.

Context of the visit: Work with Théophile Chaumont-Frelet on biharmonic equations

Mobility program/type of mobility: research stay funded by Univ. Milano Bicocca

L. Bronsard

Status Full Prof.

Institution of origin: McMaster University

Country: Canada

Dates: 12 May - 9 June

Context of the visit: Work with Marc Pegon on Ginzburg–Landau type models with heavily penalized divergence.

Mobility program/type of mobility: research stay as an invited professor funded by the CDP C2EMPI.

A. Zurek

Status Assistant Prof.

Institution of origin: UTC

Country: France

Dates: 19-20 May

Context of the visit: Work with Maxime Herda on structure-preserving scheme for local sensing chemotaxis equations.

Mobility program/type of mobility: research stay funded by PEPS JCJC 2025 (FR2037FMHF).

D. Abdel

Status Postdoc

Institution of origin: WIAS Berlin

Country: Germany

Dates: 19-20 May

Context of the visit: Work with Maxime Herda Analysis of perovskite semiconductor models.

Mobility program/type of mobility: research stay funded by Inria ARISE and LabEx CEMPI (ANR-11-LABX-0007).

H. Hivert

Status Junior researcher

Institution of origin: Inria Rennes

Country: France

Dates: 23-36 June

Context of the visit: Work with Maxime Herda Asymptotic-preserving schemes in the anomalous diffusion limit.

Mobility program/type of mobility: research stay funded by Univ. Rennes.

A. Zurek**Status** Assistant Prof.**Institution of origin:** UTC**Country:** France**Dates:** 1-5 Sept.**Context of the visit:** Work with Maxime Herda Discrete functional inequalities.**Mobility program/type of mobility:** research stay funded by PEPS JCJC 2025 (FR2037FMHF).**A. Trescases****Status** CNRS researcher**Institution of origin:** Univ. Toulouse**Country:** France**Dates:** 1-5 Sept.**Context of the visit:** Work with Maxime Herda Discrete functional inequalities.**Mobility program/type of mobility:** research stay funded by PEPS JCJC 2025 (FR2037FMHF).**K. Fourteau****Status** Postdoc**Institution of origin:** Météo-France & CNRS**Country:** France**Dates:** 22-26 Sept.**Context of the visit:** Work with Clément Cancès on the mathematical modeling of snowpacks thanks to thermodynamics.**Mobility program/type of mobility:** research stay funded by the S-NOW project supported by the Mathematical Institute for Planet Earth (IMPT).**S. Cardenas****Status** PhD student**Institution of origin:** Univ. Montpellier**Country:** France**Dates:** 8-10 Dec.**Context of the visit:** Work with Marien-Lorenzo Hanot on high-order finite volumes on manifolds.**Mobility program/type of mobility:** research stay funded by Rapsodi.**9.2.2 Visits to international teams****Research stays abroad**

Matthieu Alfaro

Visited institution: Univ. Bologna

Country: Italy

Dates: 20-30 June

Mobility program/type of mobility: research stay.

Alain Blaustein

Visited institution: WIAS Berlin

Country: Germany

Dates: 3-7 Nov.

Context of the visit: Annual team meeting of the Inria ARISE associate team.

Mobility program/type of mobility: research stay funded by Inria ARISE associate team.

Clément Cancès

Visited institution: Météo-France & CNRS, Grenoble

Country: France

Dates: 20-22 Jan.

Context of the visit: work with K. Fourteau on snow models.

Mobility program/type of mobility: research stay.

Clément Cancès

Visited institution: WIAS Berlin

Country: Germany

Dates: 3-7 Nov.

Context of the visit: Annual team meeting of the Inria ARISE associate team.

Mobility program/type of mobility: research stay funded by Inria ARISE associate team.

Clément Cancès

Visited institution: TU Munich

Country: Germany

Dates: 8-12 Dec.

Context of the visit: work with D. Matthes on dissipation driven free boundary problems.

Mobility program/type of mobility: research stay.

Claire Chainais-Hillairet

Visited institution: WIAS Berlin

Country: Germany

Dates: 3-7 Nov.

Context of the visit: Annual team meeting of the Inria ARISE associate team.

Mobility program/type of mobility: research stay funded by Inria ARISE associate team.

Théophile Chaumont-Frelet

Visited institution: Univ. Bonn

Country: Germany

Dates: 3-5 June

Context of the visit: work with G. Gantner on a posteriori error estimation in unbounded domains.

Mobility program/type of mobility: research stay funded by Univ. Bonn.

Thomas Crozon

Visited institution: WIAS Berlin

Country: Germany

Dates: 3-7 Nov.

Context of the visit: Annual team meeting of the Inria ARISE associate team.

Mobility program/type of mobility: research stay funded by Inria ARISE associate team.

Andrea Natale

Visited institution: Univ. Edimbourg

Country: United-Kingdom

Dates: 1-15 Aug.

Context of the visit: work with K. Hu on finite element methods for numerical relativity.

Mobility program/type of mobility: Research stay funded by ERC GeoFEM.

Maxime Herda

Visited institution: International research lab CRM-CNRS, Montréal

Country: Canada

Dates: 10-28 Jan.

Context of the visit: work with A. Zurek on structure-preserving scheme for McKean-Vlasov equations.

Mobility program/type of mobility: research stay funded by ANR CAPPs (ANR-23-CE40-0004) and IRL CRM-CNRS.

Maxime Herda

Visited institution: Laboratoire Jean Dieudonné (Univ. Côte d'Azur)

Country: France

Dates: 30 June - 4 July

Context of the visit: work with I. Tristani on fractional hypocoercivity and anomalous diffusion limit.

Mobility program/type of mobility: research stay funded by CNRS.

Maxime Herda

Visited institution: IMT (Univ. Toulouse)

Country: France

Dates: 13-17 Oct.

Context of the visit: work with A. Trescases on discrete functional inequalities.

Mobility program/type of mobility: research stay funded by PEPS JCJC 2025 (FR2037FMHF).

Maxime Herda

Visited institution: WIAS Berlin

Country: Germany

Dates: 3-7 Nov.

Context of the visit: Annual team meeting of the Inria ARISE associate team.

Mobility program/type of mobility: research stay funded by Inria ARISE associate team.

Maxime Herda

Visited institution: IMT (Univ. Toulouse)

Country: France

Dates: 24-28 Nov.

Context of the visit: work with T. Crin-Barat on hypocoercivity for hyperbolic and kinetic equations.

Mobility program/type of mobility: research stay funded by ANR HEAD (ANR-24-CE40-3260).

Andrea Natale

Visited institution: University of British Columbia

Country: Canada

Dates: 4-23 April

Context of the visit: work with Young-Heon Kim to work on Wasserstein projections in convex order.

Mobility program/type of mobility: Research stay funded by the joint Inria team KarMA (Kantorovich Initiative and ParMA).

Andrea Natale**Visited institution:** Herriot-Watt University**Country:** United-Kingdom**Dates:** 6-12 July**Context of the visit:** work with David Bourne to work on Laguerre tessellation fitting problems.**Mobility program/type of mobility:** Research stay funded by the Herriot-Watt University.**Marc Pegon****Visited institution:** McMaster University**Country:** Canada**Dates:** 5 Feb. - 11 March**Context of the visit:** work with L. Bronsard on Ginzburg–Landau type models with heavily penalized divergence.**Mobility program/type of mobility:** research stay.**Marc Pegon****Visited institution:** Laboratoire Jean Dieudonné (Univ. Côte d’Azur)**Country:** France**Dates:** 30 June - 4 July**Context of the visit:** work with I. Tristani on fractional hypocoercivity and anomalous diffusion limit.**Mobility program/type of mobility:** research stay funded by CNRS.**9.3 National initiatives**

Participants: Alain Blaustein, Clément Cancès, Théophile Chaumont-Frelet, Cindy Guichard, Simon Lemaire, Andrea Natale, Marc Pegon.

9.3.1 France 2030 program

Clément Cancès is the co-PI, together with I. Faille (IFPEN), of the **MATHSOUT** project of the PEPR Mathematics in Interaction (Maths-VivES) funded by the ANR in the framework of the France 2030 program. The project essentially aims at lifting, thanks to mathematical studies, scientific locks identified by practitioners in Numerical Geosciences. The project involves 4 institutional partners (INRIA Univ. Lille, IFPEN, UniCA, and BRGM) as well as colleagues from CNRS—UGA and from Sorbonne Université.

- Title: MATHématiques SOUTerraines
- Type: PEPR
- ANR reference: **ANR-23-EXMA-0010**
- Duration: 2024–2029 (5 years)
- Budget: ~1 000 000 euros
- Coordinators: Clément Cancès and I. Faille (IFPEN)

9.3.2 ANR projects

ANR COOKIE. Alain Blaustein is a member of the [ANR project Cookie](#). The project runs from 2025 to 2030 and it has been granted 435 717 euros. The project focuses on the numerical simulation and analysis of kinetic equations and related models. It addresses both Lagrangian and Eulerian numerical methods and aims to establish new connections between the analysis of partial differential equations, numerical analysis, and computational aspects in kinetic theory and related models.

- Title: Calcul & Approximation d'équations cinétiques
- Type: PRC
- ANR committee: Mathématiques (CE40) - 2025
- Coordinator: F. Filbet (IMT, Univ. Toulouse)

ANR APOWA. Theophile Chaumont-Frelet is the PI of the ANR JCJC project [APOWA](#), funded in the 2023 generic call. The APOWA project aims at the development, analysis and implementation of a posteriori error estimators and adaptive finite element schemes for time-dependent wave propagation problems.

- Title: A Posteriori error estimates for Wave equations
- Type: JCJC
- ANR committee: Mathématiques (CE40) - 2023
- ANR reference: [ANR-23-CE40-0019](#)
- Duration: 2024–2027 (48 months)
- Budget: 266 725 euros
- Coordinator: Theophile Chaumont-Frelet

ANR HIPOTHEC. Simon Lemaire is the PI of the ANR PRCE project [HIPOTHEC](#), funded in the 2023 generic call. This project aims at developing accurate and flexible numerical solvers for eddy current testing simulations, based on high-order polyhedral methods. The target application, of industrial interest, is the non-invasive detection of flaws within steam generators in nuclear plants.

- Title: High-order Polyhedral methods for Eddy Current testing simulations
- Type: PRCE, in partnership with EDF
- ANR committee: Modèles numériques, simulation, applications (CE46) - 2023
- ANR reference: [ANR-23-CE46-0013](#)
- Duration: 2024–2028 (60 months)
- Budget: 604 193 euros (in total), with 143 464 euros for INRIA Lille
- Coordinator: Simon Lemaire
- Consortium: INRIA (Lille), UPHF (Valenciennes) - EDF (Palaiseau) - Université de Montpellier, Université Côte d'Azur (Nice)

In the framework of the HIPOTHEC project, it has been agreed that all the software developments will be implemented within the prototyping platform ParaSkel++ developed by RAPSODI.

ANR NUHEMIBA. Theophile Chaumont-Frelet is a member of the ANR JCJC project [NuHeMiBa](#), whose goal is to develop discretization methods for high-frequency wave propagation problems using semiclassical wavelets. This project is a follow up on the AEx POPEG previously granted to Théophile Chaumont-Frelet.

- Title: Numerics for Helmholtz equation with Microlocal Base
- Type: JCJC
- ANR committee: Mathématiques (CE40) - 2024
- ANR reference: [ANR-24-CE40-3760](#)
- Duration: 2024–2028 (48 months)
- Budget: 266 725 euros
- Coordinator: M. Ingremeau

ANR BARYFLOW. Andrea Natale is the PI of the ANR JCJC project BARYFLOW, funded in the 2025 generic call. This project aims at developing a new framework to compute interpolation and extrapolation of measures using Optimal Transport. We will use this to tackle data analysis applications involving measure-valued data (e.g., statistical regression), but also to develop accurate particle schemes to simulate gradient flow systems, focusing in particular on the development of fast methods for variational sampling.

- Title: Generalizing Wasserstein BARYcenters to discretize gradient FLOWS
- Type: JCJC
- ANR committee: Mathématiques (CE40) - 2025
- ANR reference: ANR-25-CE40-3242-01
- Budget: 237 215 euros
- Coordinator: Andrea Natale

ANR STOIQUES. Marc Pegon is a member of the ANR project STOIQUES ([ANR-24-CE40-2216](#)). The STOIQUES project led by Y. Privat is dedicated to the investigation of modern problems in shape optimizations. It addresses nonlinear PDE models with topological constraints, quantitative geometric inequalities, uncertainty in shape inverse problems, and numerical approaches that avoid the use of costly meshes.

- Title: Shape and Topology Optimization: Impactful Questions and Emerging Subjects
- Type: PRC
- ANR committee: Mathématiques (CE40) - 2024
- ANR reference: ANR-24-CE40-2216
- Budget: 463 080 euros
- Coordinator: Y. Privat (Univ. Lorraine)

ANR ESSTOS. Marc Pegon is a member of the ANR project ESSTOS. The ESSTOS project led by A. Monteil studies the links between nonlinear elliptic PDEs and geometric objects arising from energy concentration phenomena. At the interface of analysis, geometry, with applications to micromagnetics, liquid crystals, and urban planning, it focuses on characterizing singular sets and optimal profiles near singularities, with particular emphasis on transport networks and symmetry-driven structures.

- Title: Systèmes elliptiques à transitions abruptes sur des ensembles optimaux
- Type: JCJC
- ANR committee: Mathématiques (CE40) - 2025
- Coordinator: A. Monteil (Univ. Paris-Est)

9.3.3 LabEx CEMPI

Through their affiliation to the Laboratoire Paul Painlevé of University of Lille, RAPSODI team members benefit from the support of the LabEx **CEMPI**.

- Title: Centre Européen pour les Mathématiques, la Physique et leurs Interactions
- Partners: Laboratoire Paul Painlevé (LPP) and Laser Physics department (PhLAM), Université de Lille
- ANR reference: **11-LABX-0007**
- Duration: February 2012 - December 2024 (the project has been renewed in 2019)
- Budget: 6 960 395 euros
- Coordinator: E. Fricain (LPP, Université de Lille)

The "Laboratoire d'Excellence" CEMPI (Centre Européen pour les Mathématiques, la Physique et leurs Interactions), a project of the Laboratoire de mathématiques Paul Painlevé (LPP) and the laboratoire de Physique des Lasers, Atomes et Molécules (PhLAM), was created in the context of the "Programme d'Investissements d'Avenir" in February 2012. The association Painlevé-PhLAM creates in Lille a research unit for fundamental and applied research and for training and technological development that covers a wide spectrum of knowledge stretching from pure and applied mathematics to experimental and applied physics. The CEMPI research is at the interface between mathematics and physics. It is concerned with key problems coming from the study of complex behaviors in cold atoms physics and nonlinear optics, in particular fiber optics. It deals with fields of mathematics such as algebraic geometry, modular forms, operator algebras, harmonic analysis, and quantum groups, that have promising interactions with several branches of theoretical physics.

The PhD thesis of A.-A. Diallo, which started in November 2024 in collaboration with the PhLAM laboratory, and which is co-supervised by M. Herda, is funded by the LabEx CEMPI.

9.3.4 HQI: France Hybrid HPC Quantum Initiative

Cindy Guichard is a member of the WP4 of the **HQI initiative**. The HQI initiative aims at serving academic and industrial research scientists, who want to evaluate the potential of quantum and/or hybrid computing for their applications.

- Title: HQI: France Hybrid HPC Quantum Initiative
- Type: Programme d'Investissements d'Avenir (PIA4)
- Duration: January 2022 - December 2026
- Budget: 72 300 000 euros
- Coordinator of WP4: P. Tremblin (Maison de la Simulation & CEA)

9.3.5 Mathematical Institute for Planet Earth (IMPT)

Clément Cancès is the co-PI, together with K. Fourteau (Météo France Grenoble), of the S-NOW project (2023-2024) funded (15k€) by the Mathematical Institute for Planet Earth (IMPT). The goal of the S-NOW project is to assess the compatibility of the models used to describe snow with thermodynamics, and introduce as minor as possible corrections if needed.

Andrea Natale is leading with G. Beaunée (INRAE) the project “Calibration of epidemic models on graphs with Optimal Transport and derivative-free optimization” (2023-2024) financed (55k€) by the Mathematical Institute for Planet Earth (IMPT). The project deals with the design of inference methods for epidemiological models on graphs. State-of-the-art inference methods to determine unknown parameters in this context often rely on extensive manipulations of the model outputs to match simulated results with measurements, which limits their robustness and reliability. This project aims at developing optimal transport metrics to compare epidemic scenarios, and use these to construct more robust calibration techniques via minimum discrepancy estimators. The post-doc position of C. Sarrazin was funded in the framework of this project.

9.3.6 Exploratory Actions (AEx)

Theophile Chaumont-Frelet is the PI of the "Action exploratoire" **AEx POPEG** funded by INRIA. The aim of the POPEG project is to develop innovative Galerkin methods for high-frequency wave propagation problems. Specifically, the objective is to use Gaussian coherent state basis functions in order to drastically reduce the number of degrees of freedom compared to finite element basis functions for the same accuracy when the frequency is high. The PhD position of F. Proust (UniCA) is supported by this project.

9.3.7 Action de développement technologique (ADT)

Simon Lemaire is the instigator and coordinator of the ADT project ParaSkel++, funded by Inria and the Hauts-de-France region, which started in 2020. The aim of the project is to develop an optimized parallel C++ platform for the arbitrary-order numerical approximation of PDEs on general polytopal meshes. More details can be found in Section 6. All software developments realized in the framework of the HIPOTHEC project (see Section 9.3.2) are implemented within the ParaSkel++ platform.

9.3.8 PEPS-JCJC

Marc Pegon is the recipient of a PEPS-JCJC grant (~ 3.5k€) of the INSMI section of the CNRS. C. Sarrazin is also a member of this project. It is dedicated to the numerical resolution of isoperimetric problems involving non-local energies. The non-local energies considered can be either of Gamow-type (such as Riesz potentials or non-local perimeters) or related to optimal transport.

Maxime Herda is a member of PEPS JCJC 2025 (FR2037FMHF), PI: Antoine Zurek (UTC). The goal is to design structure-preserving scheme for local sensing chemotaxis equations. Budget 4k€ for research visits.

10 Dissemination

10.1 Promoting scientific activities

10.1.1 Organization of scientific events

Matthieu Alfaro is one of the organizers of the recurrent **Groupe de Travail BioMaths Normand**.

Caterina Calgaro and Maxime Herda co-organized the regional one-day conference “**Journée Analyse Appliquée des Haut-de-France**” which gathered around 40 participants and was held on Oct. 9.

Clément Cancès was the main organizer of the 2025 edition of the annual meeting of the MathSout project held at in Paris on Nov. 12. Seven scientific talks were given there, as a basis for informal scientific discussions between the members of the MathSout project.

Theophile Chaumont-Frelet co-organized the **Frontiers in Numerical Methods for nonlinear PDEs** workshop held in Lausanne the Oct. 8-10. He also co-organized the minisymposium “Numerical methods for complex wave propagation problems” at the **ISOCAHOM2025** conference.

Theophile Chaumont-Frelet and Marc Pegon are co-organizers of the ANEDP seminar held weekly at the Laboratoire Paul Painlevé.

Maxime Herda organized the ARISE team meeting workshop, WIAS Berlin in Nov 4-5. He is also the co-organiser of a one week **bilateral French-German team meeting** with two-day workshop.

Andrea Natale co-organized the Workshop “**Geometry, duality and convexity in new OT problems**” held on November 19-21 at the Institut de Mathématique d’Orsay.

Marc Pegon co-organized two mini-symposia on Calculus of Variations in the framework of the **SMAI biennial conference** in June 2025.

10.1.2 Journal

Claire Chainais-Hillairet is editor-in-chief of the book series **Mathématiques et applications** and member of the editorial boards of **ESAIM: Mathematical Modelling and Numerical Analysis**, **IMA Journal of Numerical Analysis** and **North-Western European Journal of Mathematics**.

10.1.3 Invited talks

Matthieu Alfaro gave a talk in **Recent Advances in Mathematical Modelling for Medicine and Biology** in Rouen in January. He also gave a talk in the Journée de l’équipe PARADYSE, INRIA Lille in April, in the **Bruno Pini Mathematical Analysis Seminar**. He gave a talk at Univ. Bologna in October. He also gave a talk in **A ReaDiNet workshop on deterministic and stochastic PDEs**, in Obernai in November.

Alain Blaustein gave a talk at the **Workshop of the ARISE associate team** that was held at the WIAS institute in Berlin on November 4-5. He also gave a talk at the **Mathematics seminar** of the LMAC in Compiègne on November 18. He also gave a talk at the **Journée du laboratoire Paul Painlevé**, in Lille on October 16. Finally, he gave a **seminar** at the Laboratoire de Mathématiques Jean Leray in Nantes on March 11.

Caterina Calgaro gave a talk at the workshop **AMRENA 2025** in May.

Clément Cancès gave seminars in the mathematics departments of the Université Littoral Côte d’Opale (Calais), the University of Rennes, the University of Oxford, and the University of Lille. He further gave talks in the following workshops: **Mixtures: Modeling, Analysis and Computing** (Prague), **EYAWKADANAJKOS workshop** (Lyon), **Gradient Flows Face-to-Face 5** (Granada), **ARISE workshop** (Berlin). Clément Cancès also gave scientific talks during the MATHSout and RAPSODI annual meeting (Nov. 14 and Nov. 25). Besides, he presented the MathSout project during the **kick-off meeting of the PEPR Maths-VivES program**.

Claire Chainais-Hillairet was invited to give an **Anneliese Niethammer lecture** in Stuttgart in February 2025. She also gave a talk during the workshops ARISE that were held in **Lille, June 17** and **Berlin, November 4-6**.

Theophile Chaumont-Frelet gave a talk at the **Conference on Mathematics of Wave Phenomena 2025** in Karlsruhe in February. He was invited online at the weekly numerical analysis seminar of the University of Hong Kong on Feb. 19. He was also invited to present his work at the **Numerical analysis seminar** at Univ. Basel in April. He was invited in June to give a talk in the **ICAN2025** conference in Hong-Kong. He participated to the CERAMATH weekly seminar at Univ. Valenciennes on June 26. He gave two talks at the **ICOSAHOM2025** conference, one talk at the **ENUMATH2025** conference, and one talk at the **ACOMEN2025** conference.

Pierre Gervais gave a talk at the **Journées jeunes EDPistes de France** on January 8th.

Maxime Herda gave invited talks at **The International Conference on emerging aspects of kinetic theory, nonlocal equations, and related applications** (Wuhan University, China), the **Workshop on Mathematical Models for Quantum and Semiclassical Dynamics** (Università di Firenze, Italy) and **Gradient flows face-to-face 5th edition** (Universidad de Granada, Spain). He also gave seminar talk at the MAC seminar (Univ. Toulouse) and the Applied Mathematics Seminar at CRM (McGill University and Univ. Montréal, Canada).

Marién-Lorenzo Hanot gave a talk at the SIAM Conference on Computational Science and Engineering **mini-symposium "DECFEEC: Discrete and Finite Element Exterior Calculus"** that was held in Forth Worth on March 3-7.

Simon Lemaire gave two 90-minute lectures during the **Indo-French (CEFIPRA) Workshop** on "Innovative Numerical Methods for Modern Engineering Problems" that was held at IIT Roorkee (India) in January 2025. He also gave invited talks during the **30th UK Biennial Numerical Analysis Conference** held in Glasgow

in June, in the framework of the **ICOSAHOM** and **ENUMATH** conferences respectively held in Montréal (Canada) and Heidelberg (Germany) in July and September, and during the **HyBOX Workshop** held at ENSTA (Palaiseau) in December 2025. In September 2025, he also gave an online talk in the **ABB Corporate Research** (Baden-Dättwil, Switzerland) internal seminar. Finally, he gave talks in the Inria SERENA team (Paris) internal seminar in March, for the **Kick-off Meeting** of the Cross-Disciplinary Project C2EMPI at Université de Lille also in March, and for the annual meeting of the RAPSODI team in November 2025.

Andrea Natale gave a talk at the University of British Columbia (Canada) and online as part of the **Kantorovich Initiative Seminar** series. He also gave a talk at the **Séminaire Parisien d'Optimisation** which takes place at the Institut Henri Poincaré in Paris.

Marc Pegon gave a talk at the **PDE and Analysis seminar** of McMaster University (Canada) in February 2025.

Juliette Venel gave a talk at the **37th seminar on Computational Fluid Dynamics** organized jointly by the CEA and the SMAI/GAMNI Thematic Group on January 27–28 at the Institut Poincaré in Paris.

10.1.4 Research administration

Matthieu Alfaro is a deputy director of the Doctoral School "Mathématiques, Information, Ingénierie des systèmes" (MIIS), handling about 200 students in Normandy.

Alain Blaustein participated to the "groupe de travail ATER" at the laboratoire Paul Painlevé at the University of Lille whose task was to examine the application files of candidates to the position opening and to submit recommendations to the "commission mixte restreinte".

Caterina Calgaro is a member of the management team of the Faculté des Sciences et Technologies at Université de Lille. She is also a member of the Conseil de Département de Mathématiques at the Faculté des Sciences et Technologies at Université de Lille.

Clément Cancès was a member until Nov. 2025 of the Bureau du Comité des Équipes-Projets (BCEP) of the INRIA Lille research center. He also heads the Research Jobs Committee (Commission des Emplois de Recherche, CER for short), and represents INRIA in the council of the MADIS graduate school. He also acts as a member of the Mathematical Domain Board (Bureau de Domaine) of the MADIS graduate school. Besides, he was a member of the Administration Council of the French Society for Applied and Industrial Mathematics (SMAI) until June 2025.

Claire Chainais-Hillairet is vice-director of the Laboratoire Paul Painlevé, in charge of human resources for researchers, professors and assistant professors since 2020. She was a member of the scientific committee of the **Institut Henri Poincaré** up to end of 2025.

Emmanuel Creusé has been Director of the Mathematics Department at CERAMATHS since 2022. He is also President of the jury for the Bachelor's degree in Mathematics and the CaSSAD Master's degree at UPHF and INSA Hauts-de-France.

Marien-Lorenzo Hanot is a member of "conseils du département", and of the "CPVA" pour the "master mention math et applications".

Maxime Herda is an elected member of the Conseil de Laboratoire and of the Commission Mixte related to the math laboratory and the math department, from 2020 to 2025. Maxime Herda is also substitute member of the INRIA Lille Center Committee since 2020, a nominated member of the Commission des Utilisateurs des Moyens Informatiques (CUMI) since 2021 and of the Commission sur les formations since 2025 at INRIA Lille.

Until 2025, Simon Lemaire was responsible, for the Inria Lille research center (15 research teams), of the coordination of the yearly campaign of activity reports. He is currently a substitute member of the Inria Lille Center Committee, as well as a member of the Catering Commission (in charge of the attribution of the catering market within the center).

Since 2023, Marc Pegon is responsible for the selection of students for a training program at the engineering school "Polytech Lille".

Juliette Venel is a member of the council of the Mathematical Federation of Hauts-de-France. She is the Head of the Mathematics Resource Center at INSA Hauts-de-France since September 2025.

10.1.5 Hiring committees

Matthieu Alfaro was a member of two hiring committees for associate professor (MCF) positions: one in Univ. Sorbonne Paris-Nord and one in Univ. Bretagne Occidentale.

Clément Cancès was an external member of the hiring committee for a full professor position in applied mathematics (PR26) at the Université de Pau et de Pays de l'Adour (UPPA). He was also chairing the hiring committee for a full professor in statistics (PR26) at the University of Lille.

Claire Chainais-Hillairet took part in two hiring committees for full professor positions: one in Lille (she was the head of the committee) and one in Nancy.

Claire Chainais-Hillairet, Theophile Chaumont-Frelet and Marc Pegon were members of the hiring committee for an associate professor position (MCF 26) within the AN-EDP team of the Laboratoire Paul Painlevé (Université de Lille).

Maxime Herda was a member of the hiring committee for an associate professor position (MCF 26) at Centrale Nantes.

Juliette Venel was a member of two recruitment committees for associate professor positions (MCF 26) at INSA Rouen and at INSA Hauts-de-France.

10.2 Teaching - Supervision - Juries - Educational and pedagogical outreach

10.2.1 Teaching

Alain Blaustein taught the course Refresher in Mathematics to the second year students of the Master data science of Centrale Lille and Université de Lille. The course lasted 24 hours (12 hours of lectures ; 12 hours of exercise session).

Clément Cancès was in charge of the course "Mathematical Tools for Simulation - Hyperbolic PDEs" (27h) of the M2 Scientific Computing at Université de Lille. He was also in charge of the course "Méthodes variationnelles et équations aux dérivées partielles" (24h) in the Master of Mathematics of the Université Libre de Bruxelles (Belgique).

Pierre Gervais taught a TD on "équations différentielles ordinaires" in L3 (36h).

Maxime Herda was in charge of the course "Scientific computing" (28h) for 1st year (L3) students in Centrale Lille.

In 2025, Simon Lemaire taught the course "Mathematical Tools for Simulation - Elliptic & Parabolic PDEs" (44h) in the framework of the Master 2 "Scientific Computing" at Université de Lille.

10.2.2 Supervision

Interns:

- The "contrat relais-thèse" of Mohammed Chaibi (Inria) was supervised by Théophile Chaumont-Frelet until October 2025. It concerned the *a posteriori error analysis of finite element discretizations to the time-dependent wave equation*. It was funded by the ANR project APOWA.
- The M2 internships of Lisa Grandin, Cyprien Magnier and Raphaël Soupart on *Study of the impact of thermal insulation on the internal temperature of a tiny house* were supervised by Emmanuel Creusé and Juliette Venel.
- The M1 internships of Kodjo Adjoguidji and Koffi Kpelly on *Numerical simulation of the driven cavity by the DF-MAC method* were supervised by Emmanuel Creusé.
- The M2 internship of Cyrian Marczewski (Univ. Lille) on *A posteriori error analysis for hybrid high-order methods*, was supervised by Théophile Chaumont-Frelet and Simon Lemaire, from January to June 2025. It was funded by an ENS Paris-Saclay fellowship.
- The L3 internship of Astel Molendi-Coste on the *Modelling and the effective computation of thermodynamical equilibria in wet snow* was supervised by Clément Cancès and Andrea Natale from May to June.

- The M1 internship of Brondol Nguetsa on *Solving an inverse problem for characterizing structures for ultrasound* was co-supervised by Emmanuel Creusé and Mamadou N'Diaye.
- The M1 internship of A.S.B. Sonko (Univ. Lille) on *Finite volume approach to conservation equations with application to biofilm dynamics* was supervised by Caterina Calgaro from June to July.
- The L3 internship M. Zellat (Univ. Lille) on *Theoretical study of some classical numerical methods for ODEs* was supervised by Caterina Calgaro in July.

PhD students:

- The PhD thesis of Mohammed Chaibi (Inria) is supervised by Théophile Chaumont-Frelet since November 2025. The topic concerns the *a posteriori error analysis of finite element discretizations to the time-dependent wave equation*. It is funded by the ANR project APOWA.
- The PhD thesis of Robin Colombier (UPHF, Valenciennes) is co-supervised by Emmanuel Creusé and Caterina Calgaro, since November 2022. The topic concerns *numerical schemes for solving the macroscopic quantum hydrodynamics equations*, and it is co-funded by UPHF and the Hauts-de-France region.
- The PhD thesis of Abdoul Aziz Diallo (Université de Lille) is co-supervised by Maxime Herda, S. Bielawski (PhLAM, Univ. Lille), and C. Évain (PhLAM, Univ. Lille), since November 2024. The topic concerns the *numerical simulation of relativistic beams of charged particles in particle accelerators*, and it is funded by the LabEx CEMPI.
- The PhD thesis of Amelie Dupouy (Inria) is supervised by Clément Cancès and Claire Chainais-Hillairet since October 2023. She works on the *mathematical and numerical analysis of dissipative problems with free boundaries*. This PhD is co-funded by INRIA and the Hauts-de-France region.
- The PhD thesis of Cyrian Marczewski (UPHF/Inria) is co-supervised by Théophile Chaumont-Frelet, Simon Lemaire and Serge Nicaise (UPHF), since September 2025. The topic concerns *a posteriori error analysis for hybrid high-order methods with application to the harmonic Maxwell equations*, and it is funded by the ANR project HIPOTHEC.

Postdoctoral fellows:

- The postdoctoral fellowship of Thomas Crozon is mentored by Clément Cancès and Claire Chainais-Hillairet since March 2025. He works on the numerical approximation of a thermodynamically consistent model for iron corrosion. He particularly focuses on the structure preserving character of the numerical method he developed. This post-doc is funded by the CPER WaveTech@HdF.
- The postdoctoral fellowship of Pierre Gervais (CNRS) was mentored by Maxime Herda from September 2023 to August 2025. It was concerned with *self-consistent Vlasov–Fokker–Planck equations, and their application to the modeling of electron beams*. The postdoc was funded by the CPER WaveTech. P. Gervais is now MCF in Univ. Toulouse.
- The postdoctoral fellowship of Sumit Mahajan is mentored by Théophile Chaumont-Frelet is mentored by Théophile Chaumont-Frelet and M. Vohralík since April 2025. It is concerned with the *a posteriori error estimation for the time-dependent wave equation on dynamic meshes*. The postdoc is funded by the ANR project APOWA.

10.2.3 Juries

HDR juries:

- Matthieu Alfaro was a referee of the HdR of Jimmy Garnier (CNRS, Univ. Savoie Mont-Blanc). The defense took place on June 4th.
- Clément Cancès took part to the Habilitation defense committee of Maxime Breden (École Polytechnique, June 30).

PhD juries:

- Matthieu Alfaro was a jury member of the PhD of Nathanaël Boutillon (Univ. Aix-Marseille and INRAE Avignon). The defense took place on June 11th. He was also a referee of the PhD of Sheila Permanes (Univ. Picardie). The defense took place on December 9th.
- Clément Cancès served as a reviewer and took part to the defense committees for the PhD theses of Thibault Caillet (Université Lyon 1, July 3) and Mohammed Laaziri (Université Côte d'Azur, Nov. 17).
- Claire Chainais-Hillairet was president of the PhD juries of des juries of Z. Wang (ENPC, 23/9) and N. Bourdineaud (Bordeaux, 17/12). She also was a member of the PhD jury of H. Malamut (PSL, Paris Dauphine, 21/11).
- Théophile Chaumont-Frelet was a jury member of the PhD of Simone Pescuma. The defense took place on November 20th.
- Emmanuel Creusé was a referee of the thesis manuscript and a member of the PhD defense committee of Sarah Serhal (Ecole centrale de Nantes and Université Saint Joseph de Beyrouth) on october 2025.
- Simon Lemaire served as Referee for the evaluation of the PhD thesis of Jia Jia Qian (Monash University, Australia) in July-August 2025. He also served as Examiner in the PhD defense committee of Farah Chaaban (ENSTA Paris) in December 2025.

10.3 Popularization

Alain Blaustein hosted a high-school student at the Laboratoire Paul Painlevé for a half day during his internship in October.

Claire Chainais-Hillairet is one of the organizers of the one-week internship "Les Fourmis {éclairées}", which was held at Université de Lille in April 2025, details are available [here](#).

Emmanuel Creusé co-organized the Sixth Valenciennes Mathematics Meetings. This event, bringing together mathematics students from the UPHF and preparatory class students from the Wallon (Valenciennes) and Kastler (Denain) high schools, allows participants to discover various aspects of mathematics through a series of scientific presentations given by teacher-researchers. The program for the 2025 edition is available [here](#).

Maxime Herda wrote popularization article on the [Inria ARISE research team](#).

Simon Lemaire is currently (co-)Scientific Officer in charge of Mediation for the Inria Lille research center. He is responsible for both internal (organization of the monthly scientific seminar "30 MIN. de sciences") and external (through different events, intended in particular for high school students) scientific outreach actions. He co-organized in October 2025 the "Rendez-vous des Jeunes Mathématiciennes et Informatiennes" (RJMI). This two-day event is specifically geared towards female high school students, and aims at promoting scientific careers amongst them. He also participated in the organization of the Maths and Computer Science one-week internship "Les Fourmis {éclairées}", which was held at Université de Lille in April 2025. He is finally invested in the national scientific outreach programme "1 scientifique, 1 classe / Chiche !", for which he realized an intervention in 2025.

In 2025, Marc Pegon co-animated two introductory research activities for high school students on the Kakeya needle problem.

On May 20, Juliette Venel presented the various responsibilities of an associate professor during the "Maths C for L" week, held at the University of Artois in Lens.

11 Scientific production

11.1 Major publications

- [1] M. Bessemoulin-Chatard and C. Chainais-Hillairet. 'Exponential decay of a finite volume scheme to the thermal equilibrium for drift-diffusion systems'. In: *Journal of Numerical Mathematics* 25.3 (2017), pp. 147–168. DOI: [10.1515/jnma-2016-0007](https://doi.org/10.1515/jnma-2016-0007). URL: <https://hal.archives-ouvertes.fr/hal-01250709>.

- [2] M. Bessemoulin-Chatard, M. Herda and T. Rey. ‘Hypocoercivity and diffusion limit of a finite volume scheme for linear kinetic equations’. In: *Mathematics of Computation* 89 (Jan. 2020), pp. 1093–1133. DOI: [10.1090/mcom/3490](https://doi.org/10.1090/mcom/3490). URL: <https://hal.archives-ouvertes.fr/hal-01957832> (cit. on p. 9).
- [3] K. Brenner and C. Cancès. ‘Improving Newton’s method performance by parametrization: the case of Richards equation’. In: *SIAM Journal on Numerical Analysis* 55.4 (2017), pp. 1760–1785. URL: <https://hal.archives-ouvertes.fr/hal-01342386> (cit. on p. 11).
- [4] C. Cancès, V. Ehrlacher and L. Monasse. ‘Finite Volumes for the Stefan-Maxwell Cross-Diffusion System’. In: *IMA Journal of Numerical Analysis* 44.2 (2024), pp. 1029–1060. DOI: [10.1093/imanum/drad032](https://doi.org/10.1093/imanum/drad032). URL: <https://hal.science/hal-02902672> (cit. on p. 8).
- [5] C. Cancès, T. Gallouët and L. Monsaingeon. ‘Incompressible immiscible multiphase flows in porous media: a variational approach’. In: *Analysis & PDE* 10.8 (2017), pp. 1845–1876. DOI: [10.2140/apde.2017.10.1845](https://doi.org/10.2140/apde.2017.10.1845). URL: <https://hal.archives-ouvertes.fr/hal-01345438> (cit. on p. 7).
- [6] C. Cancès and C. Guichard. ‘Numerical analysis of a robust free energy diminishing Finite Volume scheme for parabolic equations with gradient structure’. In: *Foundations of Computational Mathematics* 17.6 (2017), pp. 1525–1584. URL: <https://hal.archives-ouvertes.fr/hal-01119735> (cit. on p. 9).
- [7] C. Chainais-Hillairet and M. Herda. ‘Large-time behaviour of a family of finite volume schemes for boundary-driven convection-diffusion equations’. In: *IMA Journal of Numerical Analysis* 40.4 (1st Oct. 2020), pp. 2473–2505. DOI: [10.1093/imanum/drz037](https://doi.org/10.1093/imanum/drz037). URL: <https://hal.archives-ouvertes.fr/hal-01885015> (cit. on p. 9).
- [8] T. Chaumont-Frelet and M. Vohralík. ‘p-robust equilibrated flux reconstruction in $H(\text{curl})$ based on local minimizations. Application to a posteriori analysis of the curl-curl problem’. In: *SIAM Journal on Numerical Analysis* 61.4 (2023), pp. 1783–1818. DOI: [10.1137/21M141909X](https://doi.org/10.1137/21M141909X). URL: <https://inria.hal.science/hal-03227570>.
- [9] F. Chave, D. A. Di Pietro and S. Lemaire. ‘A discrete Weber inequality on three-dimensional hybrid spaces with application to the HHO approximation of magnetostatics’. In: *Mathematical Models and Methods in Applied Sciences* 32.1 (2022), pp. 175–207. DOI: [10.1142/S0218202522500051](https://doi.org/10.1142/S0218202522500051). URL: <https://hal.archives-ouvertes.fr/hal-02892526>.
- [10] D. A. Di Pietro, A. Ern and S. Lemaire. ‘An arbitrary-order and compact-stencil discretization of diffusion on general meshes based on local reconstruction operators’. In: *Computational Methods in Applied Mathematics* 14.4 (June 2014), pp. 461–472. DOI: [10.1515/cmam-2014-0018](https://doi.org/10.1515/cmam-2014-0018). URL: <https://hal.archives-ouvertes.fr/hal-00978198> (cit. on p. 9).
- [11] G. Dimarco, R. Loubère, J. Narski and T. Rey. ‘An efficient numerical method for solving the Boltzmann equation in multidimensions’. In: *Journal of Computational Physics* 353 (2018), pp. 46–81. DOI: [10.1016/j.jcp.2017.10.010](https://doi.org/10.1016/j.jcp.2017.10.010). URL: <https://hal.archives-ouvertes.fr/hal-01357112>.
- [12] E. Facca, G. Todeschi, A. Natale and M. Benzi. ‘Efficient preconditioners for solving dynamical optimal transport via interior point methods’. In: *SIAM Journal on Scientific Computing* 46.3 (2024). DOI: [10.1137/23M1570430](https://doi.org/10.1137/23M1570430). URL: <https://inria.hal.science/hal-03766668> (cit. on p. 10).
- [13] T. Gallouët, A. Natale and G. Todeschi. ‘From geodesic extrapolation to a variational BDF2 scheme for Wasserstein gradient flows’. In: *Mathematics of Computation* 93 (2024), pp. 2769–2810. DOI: [10.1090/mcom/3951](https://doi.org/10.1090/mcom/3951). URL: <https://hal.science/hal-03790981> (cit. on p. 10).
- [14] T. Gallouët, A. Natale and F.-X. Vialard. ‘Generalized compressible flows and solutions of the $H(\text{div})$ geodesic problem’. In: *Archive for Rational Mechanics and Analysis* (2020). DOI: [10.1007/s00205-019-01453-x](https://doi.org/10.1007/s00205-019-01453-x). URL: <https://hal.archives-ouvertes.fr/hal-01815531>.
- [15] B. Merlet. ‘A highly anisotropic nonlinear elasticity model for vesicles I. Eulerian formulation, rigidity estimates and vanishing energy limit’. In: *Arch. Ration. Mech. Anal.* 217.2 (2015), pp. 651–680. DOI: [10.1007/s00205-014-0839-5](https://doi.org/10.1007/s00205-014-0839-5). URL: <https://hal.archives-ouvertes.fr/hal-00848547>.

11.2 Publications of the year

International journals

- [16] D. Abdel, M. Herda, M. Ziegler, C. Chainais-Hillairet, B. Spetzler and P. Farrell. ‘Numerical analysis and simulation of lateral memristive devices: Schottky, ohmic, and multi-dimensional electrode models’. In: *Computers & Mathematics with Applications* 199 (1st Dec. 2025), pp. 286–308. doi: [10.1016/j.camwa.2025.09.034](https://doi.org/10.1016/j.camwa.2025.09.034). URL: <https://hal.science/hal-04850468> (cit. on p. 14).
- [17] M. Alfaro and C. Chainais-Hillairet. ‘Long time behavior of the field-road diffusion model: an entropy method and a finite volume scheme’. In: *Journal of Numerical Mathematics* 33.3 (2025), pp. 241–265. URL: <https://hal.science/hal-04217897>.
- [18] M. Alfaro, M. Herda and A. Natale. ‘Infinitely many saturated travelling waves for a degenerate Fisher-KPP equation not in divergence form’. In: *Journal of Differential Equations* 453.4 (5th Feb. 2026). doi: [10.1016/j.jde.2025.113890](https://doi.org/10.1016/j.jde.2025.113890). URL: <https://hal.science/hal-04951055> (cit. on p. 17).
- [19] M. Bessemoulin-Chatard, T. Laidin and T. Rey. ‘Discrete hypocoercivity for a nonlinear kinetic reaction model’. In: *IMA Journal of Numerical Analysis* 45.4 (2025), pp. 2204–2247. doi: [10.1093/imanum/drae058](https://doi.org/10.1093/imanum/drae058). URL: <https://hal.science/hal-04494454>. In press.
- [20] J. W. Both and C. Cancès. ‘A global existence result for weakly coupled two-phase poromechanics’. In: *SIAM Journal on Mathematical Analysis* 57.5 (1st Sept. 2025), pp. 4995–5038. doi: [10.1137/24M1688783](https://doi.org/10.1137/24M1688783). URL: <https://hal.science/hal-04316962>.
- [21] C. Cancès, J. Cauvin-Vila, C. Chainais-Hillairet and V. Ehrlacher. ‘Cross-diffusion systems coupled via a moving interface’. In: *Interfaces and Free Boundaries : Mathematical Analysis, Computation and Applications* (2025). doi: [10.4171/IFB/536](https://doi.org/10.4171/IFB/536). URL: <https://hal.science/hal-04654232>. In press.
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